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TOTAL COLORINGS OF CORE-SATELLITE, COCKTAIL PARTY AND MODULAR PRODUCT GRAPHS

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Abstract. A total coloring of a graph G is a combination of vertex and edge colorings of G. In other words, is an assignment of colors to the elements of the graph G such that no two adjacent elements (vertices and edges) receive a same color. The *total chromatic number* of a graph G, denoted by $\chi''(G)$, is the minimum number of colors that suffice in a total coloring. Total coloring conjecture (TCC) was proposed independently by Behzad and Vizing that for any graph G, $\Delta(G) + 1 \leq \chi''(G) \leq \Delta(G) + 2$, where $\Delta(G)$ is the maximum degree of G. In this paper, we prove TCC for Core Satellite graph, Cocktail Party graph, Modular product of paths and Shrikhande graph.

Keywords: Total coloring, Modular product graph, Core Satellite graph, Cocktail Party graph, Shrikhande graph.

AMS Subject Classification: 05C15

1. Introduction

Let $G = (V(G), E(G))$ be a graph with vertex set $V(G)$ and edge set $E(G)$. In this paper, graph is simple, that is, it has no multiple edges or loops. A *total coloring* of G is a mapping $f : V(G) \cup E(G) \to C$, where C is a set of colors, satisfying the following three conditions (a)-(c):

(a) $f(u) \neq f(v)$ for any two adjacent vertices $u, v \in V(G)$,
(b) $f(e) \neq f(e')$ for any two adjacent edges $e, e' \in E(G)$ and
(c) $f(v) \neq f(e)$ for any vertex $v \in V(G)$ and any edge $e \in E(G)$ incident with v.

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The *total chromatic number* of a graph G, denoted by $\chi''(G)$, is the minimum number of colors that suffice in a total coloring. It is clear that $\chi''(G) \geq \Delta(G)+1$, where $\Delta(G)$ is the maximum degree of G. Behzad [6] and Vizing [19] indepndently conjectured (Total Coloring Conjecture (TCC)) that for every graph G, $\Delta(G)+1 \leq \chi''(G) \leq \Delta(G)+2$. If $\chi'(G) = \Delta(G)$ then G is called class-I graph and if $\chi'(G) = \Delta(G)+1$ then G is called class-II graph, where $\chi'(G)$ is the edge chromatic number of G. For example, K_{2n} is class-I where as K_{2n+1} is class-II. Also, any bipartite graph is class-I. We call a graph G is total colorable if it has a total coloring with $\Delta(G)+2$ colors. Analogously, if a graph G is total colorable with $\Delta(G)+1$ colors then the graph is called type-I, and if it is not total colorable with $\Delta(G)+1$ colors but $\Delta(G)+2$ colors, then it is type - II. The following theorems are due to Yap [21]. Arindam Dey et al. studied the concept of vertex and edge coloring on simple vague graphs [1]. Also, they studied the colorings of fuzzy graph in [2][3][4][5].

Theorem 1.1. *For any complete graph K_n,* $\chi''(K_n) = \begin{cases} n, & \text{if } n \text{ is odd} \\ n+1, & \text{if } n \text{ is even.} \end{cases}$

Theorem 1.2. *For any cycle C_n,* $\chi''(C_n) = \begin{cases} 3, & \text{if } n \text{ is multiple of } 3 \\ 4, & \text{otherwise.} \end{cases}$

In [14], Molly and Reed gave a probabilistic approach to prove that for sufficiently large $\Delta(G)$, the total chromatic number is at most $\Delta(G) + 1026$. It is known [13] that the problem of finding a minimal total coloring of a graph is in general case NP-complete. The problem remains NP-complete even for cubic bipartite graphs. For general classes of graphs, the total colouring would be harder than edge colouring. A lot of work was done on various topics related to graph products, but on the other hand there are still many questions open. Geetha and Somasundaram [10] verified TCC for certain classes of direct product, strong product and lexicographic product graphs. In [9] Geetha and Somasundaram studied the total coloring conjecture for certain classes of Generalized Sierpiński graphs. An exhaustive survey on total colorings is given [11]. This survey includes all the recent results and open problems in total colorings.
Recently, Vignesh et al. verified TCC for certain classes of deleted lexicographic product graphs [16] and corona product graphs [17].

If we color a graph G with $\Delta(G)+2$ colors then at each vertex $v \in V(G)$, at least one color is not used among $\Delta(G)+2$ colors. These colors are called missing colors of v. In particular, if n is even and we color the graph K_n with $n+1$ colors (Theorem 1.1), then at each vertex $v \in V(K_n)$ there will be exactly one color missing and the missing colors at the vertices are distinct. In this paper, let $c(v_i)$ denote the color of the vertex v_i and $c(v_iv_j)$ denote the color of the edge v_iv_j.

2. Core - Satellite Graph

Let c, s and η be positive integers, $c > s$ and $\eta \geq 2$. The core-satellite graph (introduced by Estrada and Benzi [8]) is $\Theta(c, s, \eta) \cong K_c \vee (\eta K_s)$. That is, they are the graphs consisting of η copies of K_s (the satellite cliques) meeting in a common clique K_c (the core clique). This can be generalized in the following way: There are k satellite cliques $K_{s_1}, K_{s_2}, ..., K_{s_k}$, each satellite clique K_{s_i} is join with the core clique K_c. It is also denoted as $\Theta(c, 1_{s_1}, 1_{s_2}, ..., 1_{s_k})$.
For example, $\Theta(6, 1_5, 1_4, 1_3)$ consists of three satellite cliques K_5, K_4, K_3 and a core clique

K_6, which is shown in Fig. 1. In this section, we prove that the core-satellite graph is total colorable.

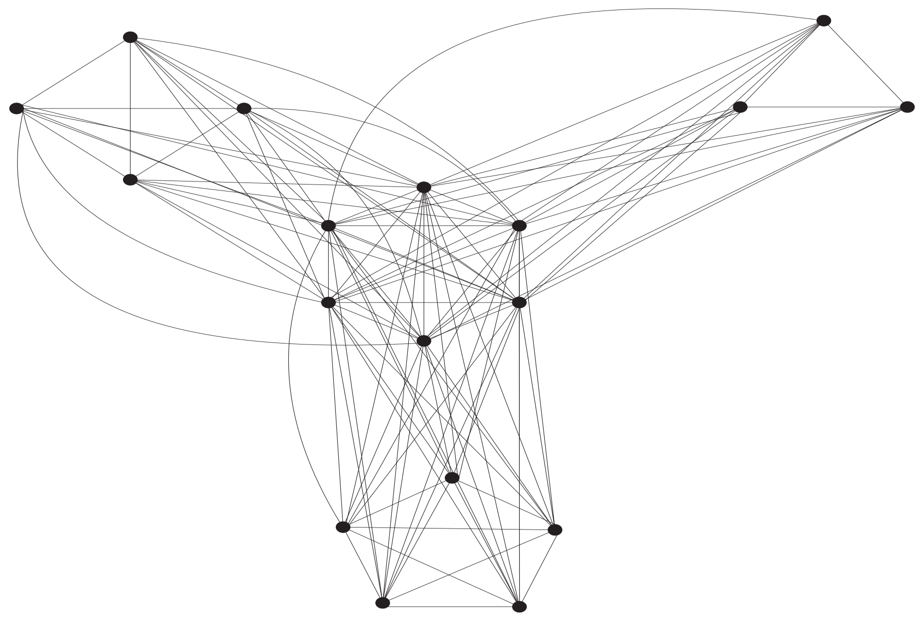

FIG. 1. $\Theta(6, 1_5, 1_4, 1_3)$

Theorem 2.1. *The core - satellite graph is total colorable.*

Proof. Consider the core-satellite graph $\Theta(c, 1_{s_1}, 1_{s_2}, ..., 1_{s_k}), c > s_i \geq 2, i = 1, 2, ..., k$.
Here, the maximum degree is $\Delta(\Theta(c, 1_{s_1}, 1_{s_2}, ..., 1_{s_k})) = (c-1) + s_1 + s_2 + ... + s_k$.
Case (a). c is even.
Subcase (i). Suppose all the satellite cliques are same size k.
From the Theorem (1.1), we know that the core clique K_c requires $c+1$ colors to color its elements. In this coloring assignment, one color is not assigned to any of the vertices of K_c, let it be the color a. Assign s_i colors to the edges between the core clique K_c and the satellite cliques K_{s_i}, $i = 1, 2, ..., k$. Since the satellites cliques are disjoint to each other, color the satellite clique K_{s_i} with s_k colors and the color a, $1 \leq i \leq k-1$. Since $c > s_k$, we assign the edge coloring of K_{s_k} from the edge coloring of the core K_c. There will be exactly one missing color at each vertices in K_c and these missing colors are distinct. Now, take a set of matching edges (s_k edges) between K_c and K_{s_k} and remove the colors of the matching edges and assign the removed colors to the corresponding vertices in K_{s_k}. Color the matching edges with the missing colors at the corresponding vertices in K_c. Here, we used $(c+1) + s_1 + s_2 + ... + s_k$=$\Delta(\Theta(c, 1_{s_1}, 1_{s_2}, ..., 1_{s_k})) + 2$ colors.
Subcase (ii). Suppose not all s_i are same size.
Let $s_p = \max\{s_i | i = 1, 2, ..., k\}$.
We assign $c+1$ colors to the element of the core clique K_c (Theorem (1.1)). Assign s_i colors to the edges between the core clique K_c and K_{s_i}, $i = 1, 2, ..., k$. Since the satellites cliques are disjoint to each other, color the satellite clique K_{s_i} with s_p colors, $1 \leq i \leq k, i \neq p$. Since $c > s_p$, we assign the edge coloring of K_{s_p} from the edge coloring of the core K_c. There will be exactly one missing color at each vertices in K_c and these missing colors are distinct. Now, take a set of matching edges (s_p edges) between K_c and K_{s_p} and remove the colors of the matching edges and assign the removed colors to the corresponding vertices in

K_{s_p}. Color the matching edges with the missing colors at the corresponding vertices in K_c.

Case (b). c is odd.
From the Theorem (1.1), we know that for odd values of n, K_n requires only n colors for its total colorings. Here, we give $n+2$ colors to the elements of K_n, n is odd, in the following way:
We embed K_n such that its vertices $v_1, v_2, ..., v_n$ are situated equidistantly on a circle. In the first step, we color all edges incident with v_1 such that the color of $c(v_1v_i) = c_i$, for $i = 2, 3, ..., n$ and the vertex v_1, $c(v_1) = c_1$. Next we consider the vertex v_2: one edge is already colored with c_2, so we put $c(v_2) = c_3$ and $c(v_2v_i) = c_{i+1}$, for $i = 3, 4, ..., n$. In general $c(v_j) = c_{2j-1 \mod (n+2)}$ and $c(v_jv_i) = c_{(j+i-1) \mod (n+2)}$, for $i = j+1, ..., n$ (here c_0 is the color $n+2$). This gives a proper total coloring of K_n with $n+2$ colors. Now at each vertex in K_n, we have exactly two distinct missing colors, they are $(c_{n+1}, c_{n+2}), (c_{n+2}, c_1), (c_1, c_2), ..., (c_{n-2}, c_{n-1})$ respectively at $v_1, v_2, ..., v_n$. In general, the two distinct missing colors at vertex v_i are $c_{(n+i) \mod (n+2)}$ and $c_{(n+i+1) \mod (n+2)}$, $1 \leq i \leq n$.

Now, we have $s_1 + s_2 + ... + s_k - 1$ remaining colors. We take $s_1 - 1$ colors and a missing color at the vertices in K_c to color the edges between K_c and K_{s_1}. We use the new s_i colors to color the edges between K_c and $K_{s_i}, i = 2, 3, ..., k$. Similar to the previous case, we color all the satellite cliques with s_p colors.
Therefore the core-satellite graph is total colorable.

□

Corollary 2.1. *If the core and all the satellite cliques are type -I, then the core-satellite graph is also type-I.*

3. Cocktail Party Graph

Cocktail party graph is a graph consisting of two rows of paired vertices in which all vertices except the paired ones are connected with an edge and is denoted by T_v. The cocktail party graph is also called the hyperoctahedral graph [7] or Roberts graph. It is complement of the ladder rung graph L'_n and the dual graph of the hypercube graph Q_n. It is the skeleton of the n-cross polytope. The cocktail party graph of order n is isomorphic to complement of C_{2n}. It is belonging to the class of claw-free perfect graphs. The n-cocktail party graph is also the $(2n, n)$-Turán graph. Vijayalakshmi [18] found the number of multiplicity of triangles in cocktail party graphs. In [12] Gregory et al. obtained the clique partitions of the cocktail party. Spectral characterization of generalized cocktail-party graphs is given by Wang and Huang [20]. The Cocktail party graph T_5 is shown in Fig.2. $T_2 \cong C_4$ is type-II (Theorem 1.2). In the next theorem, we prove that cocktail party graph with n vertices, $n \geq 3$, is type-I.

Theorem 3.1. *If G is a cocktail party graph with n vertices, $n \geq 3$, then G is type-I.*

Proof. Let G be a cocktail party graph with n vertices, $n \geq 3$. We know that $G \cong \overline{C_{2n}}$. The maximum degree of G is $\Delta(G) = 2(n-1)$. We decompose the G into two complete graphs with n vertices, K_n^1 and K_n^2, and a bipartite graph $H_{n,n}$ (decomposition of T_5 is shown in Fig.3).
Suppose n is odd then we color the elements of K_n^1 and K_n^2 with n colors (by Theorem 1.1). The maximum degree of the bipartite graph $H_{n,n}$ is $n-1$ and we know that $n-1$ colors are sufficient for edge colorings of the edges of $H_{n,n}$. Therefore, we need only $2n-1$

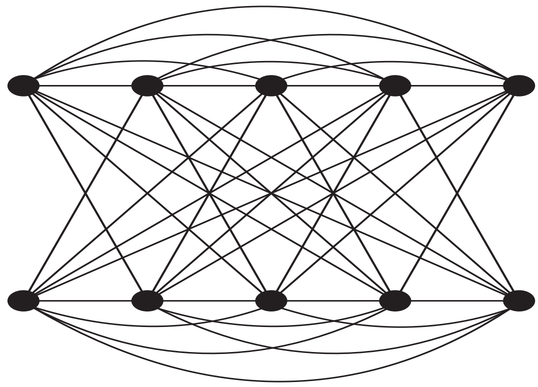

FIG. 2. Cocktail party graph T_5.

colors for the total coloring of G.
Suppose n is even then we color the element of K_n^1 and K_n^2 with $n+1$ colors (by Theorem 1.1). When we color the graph K_n with $n+1$ colors, at each vertex $v \in V(K_n)$, exactly one color is missing and these missing colors are the vertices are distinct. Now, permute the colors of K_n^2 such that i^{th} vertex in K_n^1 and $(i+1)^{th}$ vertex in K_n^2 are having same missing color, $1 \leq i \leq n$. Assign the missing color to the edge between i^{th} vertex in K_n^1 and $(i+1)^{th}$ vertex in K_n^2, $1 \leq i \leq n$. We have remaining $n-2$ colors and using these $n-2$ colors we color the uncolored edges of $H_{n,n}$. □

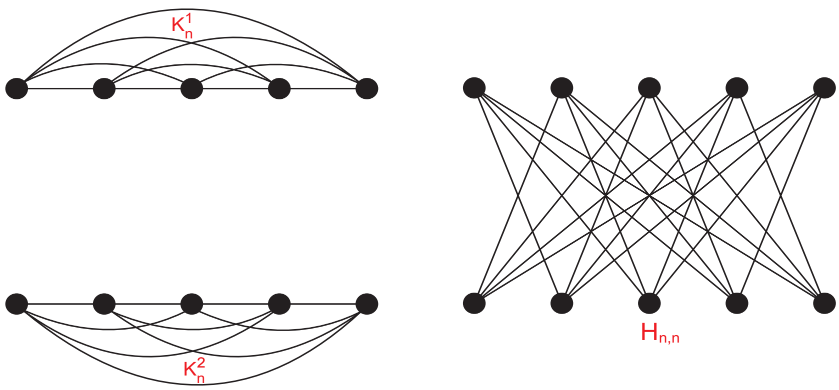

FIG. 3. Decomposition of cocktail party graph .

4. Modular Product Graph

There are four standard product of graphs namely, cartesian, direct, strong and lexicographic product. Strong product is the union of cartesian and direct products. In this section, we considered the modular product. Modular product is the combination of strong product edges and edges corresponding to the non-adjacent vertices. The formal definition as follows:
The modular product [22] $G \Diamond H$ of two graphs G and H is the graph with vertex set $V(G) \times V(H)$, in which a vertex (v, w) is adjacent to a vertex (v', w') if and only if either (i) $v = v'$ and w is adjacent to w', or (ii) $w = w'$ and v is adjacent to v', or (iii) v is adjacent to v' and w is adjacent to w', or (iv) v is not adjacent to v' and w is not adjacent to w'.

It is interesting to see that, if either G or H is a complete graph, then $G\Diamond H \simeq G \boxtimes H$. Fig.4 shows the graph $C_4 \Diamond C_4$.

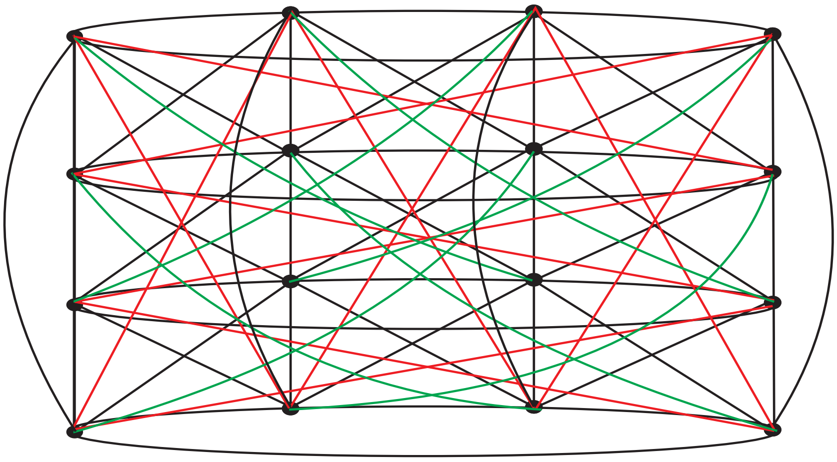

FIG. 4. $C_4 \Diamond C_4$

Let G and H be two connected graphs with m and n vertices, respectively. Let $x = (u, v)$ be a vertex in $G\Diamond H$. Let $d_u = deg_G(u)$ and $d_v = deg_H(v)$. It is easy to see that the maximum degree is $\Delta(G\Diamond H) = max_{u\in G, v\in H} \{d_u + d_v + d_u d_v + (m - d_u - 1)(n - d_v - 1)\}$.

Theorem 4.1. *The graphs $P_3 \Diamond P_n$ and $P_3 \Diamond C_n$ are total colorable graph.*

Proof. We know that $P_2 \Diamond P_2 \cong C_4$ and C_4 is type-II graph (Theorem 1.2). For $n \geq 3$, Geetha and Somasundaram [10] proved that $P_2 \Diamond P_n \cong P_2 \boxtimes P_n$ and $P_2 \Diamond C_2 \cong P_3 \boxtimes C_n$ are type-I graphs.

Let us consider the graph $P_3 \Diamond P_n$, $n \geq 3$. The graph $P_3 \Diamond P_n$ is taken as three layers of P_n. Here, the maximum degree

$$\Delta(P_3 \Diamond P_n) = \max_{u\in P_3, v\in P_n} \{d_u + d_v + d_u d_v + (3 - d_u - 1)(n - d_v - 1)\}.$$

$$= \begin{cases} 8, & n = 3, ..., 6, \\ n + 2, & n \geq 7. \end{cases}$$

Let u_{ij} be the j^{th} vertex in i^{th} layer, $i = 1, 2, 3$ and $j = 1, 2, ..., n$. For $n = 3, ..., 6$, the maximum degree attains at the vertices like u_{22}. It is easy to prove the theorem in a direct way for $n = 3, .., 6$.

For $n \geq 7$, the maximum degree attains at u_{1j} and u_{3j}, where $j = 2, 3, ..., n - 1$. Let $C = \{a, b, c, d, x, y, z, c_1, c_2, ..., c_{n-3}\}$ be a set of colors with $n + 4$ colors.
Color all the edges of the path in the first layer with the colors x and y and color all the edges in third layer with the colors y and z. At each vertex in the first layer, the color z is missing and at each vertex in the third layer, the color x is missing. Color the edges between the corresponding vertices of the first and second layers with z and between the second and third layers with x. Color the direct product edges between the first and second layers with colors a, b and second and third layers with c, d. Now the vertices in the first layer are colored with c, d and the vertices in the third layer are colored with a, b. The edge between the vertices u_{11} and u_{3n} is colored with the color a and the edge between

the vertices u_{1n} and u_{31} is colored with the color d. The remaining edges between the first and third layers (the edges correspond nonadjacent vertices) are colored with the colors $c_1, c_2, ..., c_{n-3}$. The edges and vertices of the path in the second layer is colored with the colors c_1, c_2, c_3, c_4. Therefore, $\Delta(P_3 \Diamond P_n) \leq n + 4$.

Similar way, we can prove $P_3 \Diamond C_n$ is total colorable graph if n is even. If n is odd, color all the vertices and edges of the first layer with the colors c, d, x, y and color all the vertices and edges of the third layer with the colors a, b, y, z. At each vertex in the first layer, the color z is missing and at each vertex in the third layer, the color x is missing. Color the edges between the corresponding vertices of the first and second layers with z and between the second and third layers with x. Color the direct product edges between the first and second layers with colors a, b and second and third layers with c, d. The edges corresponding to strong product are colored with the colors a, b, c, d, x, y, z and the edges corresponding to non-adjacency are colored with the colors $c_1, c_2, ..., c_{n-3}$. The edges and vertices of the path in the second layer are colored with the colors c_1, c_2, c_3 and c_4. In this case also, we have used $n+4$ colors to color the elements of $P_3 \Diamond C_n$. Hence $P_3 \Diamond C_n$ is total colorable.

□

5. **Shrikhande Graph**

In the mathematical field of graph theory, the Shrikhande graph is a named graph discovered by S. S. Shrikhande in 1959 [15]. It is a strongly regular graph with 16 vertices and 48 edges, with each vertex having degree 6. Every pair of nodes has exactly two other neighbors in common. The chromatic number of Shrikhande graph is 4 and it is Hamiltonian. The symmetric structure of Shrikhande graph is shown in Fig.5.

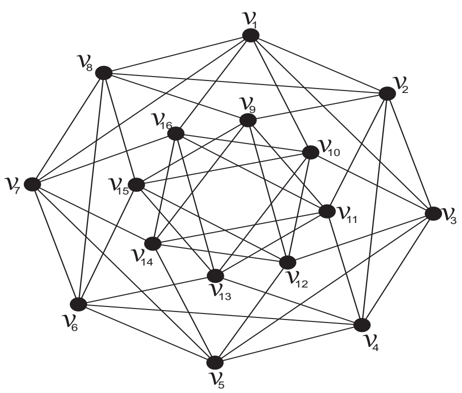

FIG. 5. Shrikhande graph G.

Theorem 5.1. *Shrikhande graph G is type-I.*

Proof. Let G be the Shrikhande graph and $V(G) = \{v_1, v_2, v_3, ..., v_{16}\}$ be the set of vertices. Let $C = \{1, 2, 3, 4, 5, 6, 7\}$ be a set of colors. We decompose the graph G in to three cycles C_1 ($v_9, v_{11}, v_{13}, v_{15}$), C_2 ($v_{10}, v_{12}, v_{14}, v_{16}$) and C_3 (v_1 to v_8). Based on the decomposition, we color the elements of cycles with 7 colors. Table 1 shows a total coloring of the Shrikhande graph.

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}	v_{11}	v_{12}	v_{13}	v_{14}	v_{15}	v_{16}
v_1	1	4	5	-	-	-	6	3	-	7	-	-	-	-	-	2
v_2	4	2	1	5	-	-	-	6	3	-	7	-	-	-	-	-
v_3	5	1	3	2	6	-	-	-	-	4	-	7	-	-	-	-
v_4	-	6	2	4	3	5	-	-	-	-	1	-	7	-	-	-
v_5	-	-	6	3	1	4	5	-	-	-	-	2	-	7	-	-
v_6	-	-	-	5	4	2	1	6	-	-	-	-	3	-	7	-
v_7	6	-	-	-	5	1	3	2	-	-	-	-	-	4	-	7
v_8	3	5	-	-	-	6	2	4	7	-	-	-	-	-	1	-
v_9	-	3	-	-	-	-	-	7	1	-	2	6	-	5	4	-
v_{10}	7	-	4	-	-	-	-	-	-	2	-	3	5	-	6	1
v_{11}	-	7	-	1	-	-	-	-	2	-	3	-	4	6	-	5
v_{12}	-	-	7	-	2	-	-	-	6	3	-	4	-	1	5	-
v_{13}	-	-	-	7	-	3	-	-	-	5	4	-	1	-	2	6
v_{14}	-	-	-	-	7	-	4	-	5	-	6	1	-	2	-	3
v_{15}	-	-	-	-	-	7	-	1	4	6	-	5	2	-	3	-
v_{16}	2	-	-	-	-	-	7	-	-	1	5	-	6	3	-	4

TABLE 1. Total coloring of Shrikhande graph G

□

6. CONCLUSIONS

A total coloring of a graph G is an assignment of colors to the elements of the graph G such that no two adjacent elements (vertices and edges) receive a same color. Total coloring problem belongs to NP-Hard classes. The total coloring conjecture is well known conjecture, which states that for any graph G, $\Delta(G) + 1 \leq \chi''(G) \leq \Delta(G) + 2$, where $\Delta(G)$ is the maximum degree of G. In this paper, we proved the conjecture for Core Satellite graph, Cocktail Party graph, Modular product of paths and Shrikhande graph. Total colorings of modular product of other classes of graphs are open.

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