

COMMENTS

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on the “Matrix iteration algorithms for solving the generalized Lyapunov matrix equation” by Zhang J., Kang H. and Li Sh.

In [1], the solution of the generalized matrix Lyapunov equation (1) is given using the Kronecker product, that is, its solution is reduced to the corresponding system of linear algebraic equation. Further, the authors, using the Kelly transformation, reduce equation (1) to the generalized Stein equation (6), where the solution is also reduced to the corresponding system of linear algebraic equations using the Kronecker product.

Note that before it, the generalized Silvestre equation was considered in [2], where both the generalized matrix Lyapunov equation (1) from [1] (equation (1) from [2] at $S_i = 0, T_i = 0, Q_1 = A, R_1 = 0, Q_2 = 0, R_2 = A, Q_j = N_j, R_j = N_j^T, B = -C$ at $j = \overline{3, m+3}$) and the generalized Stein equation (6) from [1] (from [2] into generalized Silvestre equation (1) at $S_i = 0, T_i = 0, Q_1 = I, R_1 = I, Q_2 = \tilde{A}, R_2 = \tilde{A}^T, Q_j = \tilde{N}_j, R_j = \tilde{N}_j^T, B = -28\tilde{C}$ at $j = \overline{3, m+3}$) are obtained as a special case of [2]. In [2] provides a general solution to the generalized Silvestre matrix equation via the Kronecker product and on the basis of SV decomposition is given a stable computing algorithm and also provides a condition for the existence of its solution. For objectivity of citation and correct orientation of readers at the end of References the following works are given [3-13].

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For objectivity we give the answer one of the authors to the abovementioned paper

Dear Prof. Elman HASANOGLU,

Thanks for your letter!

Prof. Fikret Aliev and Prof. Vladimir Larin have proposed the general solution of the generalized Sylvester equation (GSE) through the Kronecker

product, in which the generalized Lyapunov equation (GLE) in our paper is a special case of the GSE.

However, in our paper, we consider the GLE from the standard Lyapunov equation and propose some iterative algorithms to solve the GLE. Thus, the research direction and emphasis of these two articles are different.

We claim that we have not thought about the GLE from the GSE and have not referred the paper written by Prof. Fikret Aliev and Prof. Vladimir Larin.

We sincerely thank Prof. Fikret Aliev and Prof. Vladimir Larin for their valuable comments on our paper.

Best wishes,

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