TWMS J. App. and Eng. Math. V.12, N.2, 2022, pp. 588-599

SUM DIVISOR CORDIAL LABELING IN THE CONTEXT OF GRAPHS OPERATIONS ON BISTAR

DIVYA G. ADALJA[1], §

Abstract. A sum divisor cordial labeling of a graph G with vertex set $V(G)$ is a bijection $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ such that an edge $e = uv$ is assigned the label 1 if $2|[f(u)+f(v)]$ and 0 otherwise, then the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. If a graph admits a sum divisor cordial labeling, then it is called sum divisor cordial graph. In this paper we prove that bistar $B_{m,n}$, splitting graph of bistar $B_{m,n}$, degree splitting graph of bistar $B_{m,n}$, shadow graph of bistar $B_{m,n}$, restricted square graph of bistar $B_{m,n}$, barycentric subdivision of bistar $B_{m,n}$ and corona product of bistar $B_{m,n}$ with K_1 admit sum divisor cordial labeling.

Keywords: Sum divisor cordial labeling, Bistar.

AMS Subject Classification: 05C78.

1. Introduction

Throughout this work, by a graph we mean a simple, finite, undirected graph $G = (V, E)$ of order p and size q. For terms and notatisons related to graph theory which are not defined here, we refer to Gross and Yellen[7] and for standard terminology and notations related to number theory we refer to Burton[2]. This paper includes the results on sum divisor cordial labeling, which is a particular type of graph labeling. The concept of graph labeling was introduced by Rosa.

Definition 1.1 ([10]). *If the vertices or edges or both of the graph are assigned valued subject to certain conditions it is known as graph labeling.*

For a dynamic survey on various graph labeling problems along with an extensive bibliography we refer to Gallian[5].

Cordial labeling was introduced by Cahit as a weaker version of graceful and harmonious labeling of graphs. Combining the concept of divisibility from number theory and cordial labeling from graph labeling, Varatharajan et al.[12] introduced the concept of divisor cordial labeling of a graph.

[1] Department of Mathematics, Marwadi University, Rajkot, Gujarat, India.
e-mail: divya.adalja@marwadieducation.edu.in; ORCID: https://orcid.org/0000-0001-7052-5345.
§ Manuscript received: March 14, 2020; accepted: July 24, 2020.

Definition 1.2 ([12]). *A bijection $f : V(G) \to \{1, 2, \ldots, p\}$ is said to be divisor cordial labeling of a graph G if the induced function $f^* : E(G) \to \{0, 1\}$ defined by*

$$f^*(e = uv) = \begin{cases} 1; & \text{if } f(u) \mid f(v) \text{ or } f(v) \mid f(u) \\ 0; & \text{otherwise} \end{cases}$$

satisfies the condition $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ = number of edges labeled with 0 under f^* and $e_f(0)$ = number of edges labeled with 1 under f^*.
A graph with a divisor cordial labeling is called a divisor cordial graph.

Varatharajan et al.[12] proved that the graphs such as path, cycle, wheel, star, some complete bipartite graphs, some special classes of graphs such as full binary tree, dragon, corona, $G * K_{2,n}$ and $G * K_{3,n}$ are divisor cordial. Ghodasara and Adalja[6] derived divisor cordial labeling for ringsum of some standard graphs with star graph.

Definition 1.3 ([8]). *Let $f : V(G) \to \{1, 2, 3, \ldots, |V(G)|\}$ be a bijection and let the induced function $f^* : E(G) \to \{0, 1\}$ be defined as*

$$f^*(e = uv) = \begin{cases} 1; & \text{if } 2 \mid [f(u) + f(v)] \\ 0; & \text{otherwise.} \end{cases}$$

Then f is called sum divisor cordial labeling of graph G if $|e_f(0) - e_f(1)| \leq 1$.
A graph with a sum divisor cordial labeling is called sum divisor cordial graph.

Lourdusamy et al.[8] introduced the concept of sum divisor cordial labeling of graphs. In [9] the same authors have proved that shadow graph and splitting graph of $K_{1,n}$, shadow graph, subdivision graph, splitting graph and degree splitting graph of $B_{n,n}$, corona of ladder and triangular ladder with K_1, closed helm are sum divisor cordial graphs. In[1] Adalja and Ghodasara derived sum divisor cordial labeling of cycle, cycle with one chord, cycle with twin chords, cycle with triangle, wheel, helm, web, shell, flower and double fan.

Definition 1.4 ([5]). *Bistar $B_{m,n}$ is the graph obtained by joining the center(apex) vertices of $K_{1,m}$ and $K_{1,n}$ by an edge.*

Definition 1.5 ([5]). *Splitting graph $S'(G)$ of a graph G is the graph constructed by adding to each vertex v, a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G, i.e. $N(v) = N(v')$.*

Definition 1.6 (Selvaraju et al.[11]). *Let $G = (V, E)$ be a graph with $V(G) = S_1 \bigcup S_2 \bigcup S_3 \ldots S_t \bigcup T$, where each S_i is a set of vertices having at least two vertices of the same degree and $T = V \setminus \bigcup_{i=1}^{t} S_i$. The degree splitting graph of G denoted by $DS(G)$ is obtained from G by adding vertices $w_1, w_2, w_3 \ldots w_t$ and joining them to each vertex of S_i, for $1 \leq i \leq t$.*

Definition 1.7 (Gallian[5]). *Shadow graph $D_2(G)$ of a connected graph G is the graph constructed by taking two copies of G say G' and G'' and join each vertex u' in G' to the neighbors of the corresponding vertex v' in G''.*

Definition 1.8 (Clark and Holton[3]). *Square G^2 of a graph G is the graph with same vertex set as $V(G)$ and two vertices are adjacent in G^2 if and only if the distance between them is at most 2 in G.*

Definition 1.9 (Gallian[5]). *Restricted square of bistar $B_{n,n}$ is the graph G with vertex set $V(G) = V(B_{n,n})$ and edge set $E(G) = E(B_{n,n}) \bigcup \{uv_i, vu_i / 1 \leq i \leq n\}$.*

Definition 1.10 (Gross and Yellen[7]). *Let $G = (V, E)$ be a graph with p vertices and q edges. An edge $e = uv$ is said to be subdivided if a new vertex w is added such that w is adjacent to u and v. i.e. edge $e = uv$ is subdivided into two edges $e_1 = uw$ and $e_2 = wv$. The barycentric subdivision of given graph G is constructed by subdividing each edge of given graph G.In other words barycentric subdivision is the graph obtained by inserting a vertex of degree 2 into every edge of original graph. The barycentric subdivision of any graph G is denoted by $S(G)$.*

Definition 1.11 (Frucht and Harary[4]). *Corona $G \odot H$ of two graphs G and H is defined as the graph acquired by taking one copy of G (having p_1 vertices) and p_1 copies of H and joining one copy of H at every vertex of G by an edge.*

2. Main Results

Theorem 2.1. *The bistar $B_{m,n}$ is a sum divisor cordial graph.*

Proof. Let $G = B_{m,n}$ be the bistar with vertex set $V(G) = \{u_0, v_0, u_i, v_j \mid 1 \le i \le m, 1 \le i \le n\}$, where u_i and v_j are pendant vertices. We note that $|V(G)| = m + n + 2$ and $|E(G)| = m + n + 1$. Without loss of generality we can assume that $m \le n$ because $B_{m,n}$ and $B_{n,m}$ are isomorphic graphs.
We define vertex labeling $f : V(G) \to \{1, 2, \ldots, m + n + 2\}$ as follows.

$$\begin{aligned} f(u_0) &= 1; \\ f(v_0) &= 2; \\ f(u_i) &= 2 + i, \qquad 1 \le i \le m; \\ f(v_i) &= 2 + m + i, \quad 1 \le i \le n. \end{aligned}$$

The following table describes the results of edge labels obtained due to the above labeling pattern.

Cases of n	Edge results
m is odd and n is even	$e_f(0) = \frac{m+n+1}{2} = e_f(1)$
m is even and n is odd	$e_f(0) = \frac{m+n+1}{2} = e_f(1)$
m and n both are odd	$e_f(1) = \lceil \frac{m+n+1}{2} \rceil, e_f(0) = \lfloor \frac{m+n+1}{2} \rfloor$
m and n both are even	$e_f(0) = \lceil \frac{m+n+1}{2} \rceil, e_f(1) = \lfloor \frac{m+n+1}{2} \rfloor$

Thus $|e_f(0) - e_f(1)| \le 1$.
Hence, the bistar $B_{m,n}$ is a sum divisor cordial graph. □

Example 2.1. *Sum divisor cordial labeling of the graph $B_{4,8}$ is shown in the Figure 1.*

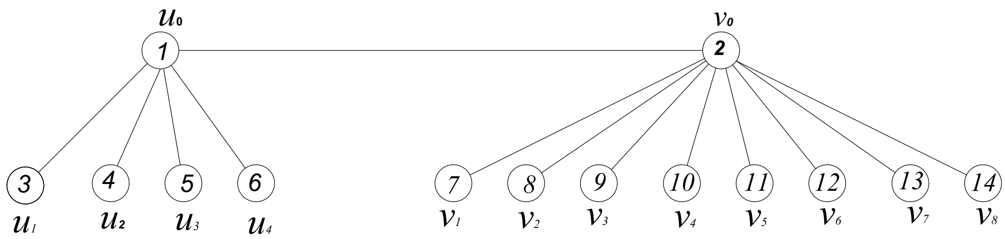

Figure 1. : Sum divisor cordial labeling of $B_{4,8}$

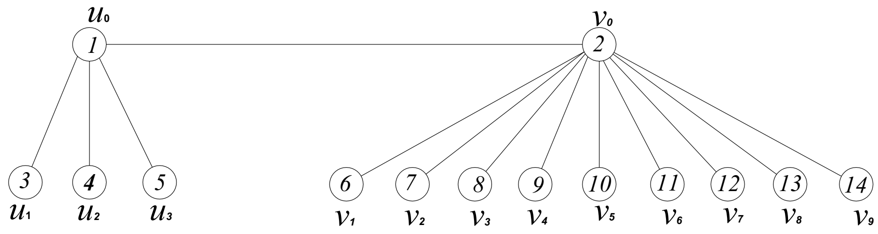

FIGURE 2. : Sum divisor cordial labeling of $B_{3,9}$

Example 2.2. *Sum divisor cordial labeling of the graph* $B_{3,9}$ *is shown in the Figure 2.*

Example 2.3. *Sum divisor cordial labeling of the graph* $B_{3,8}$ *is shown in the Figure 3.*

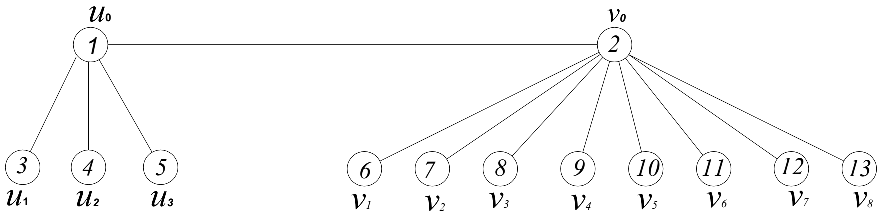

FIGURE 3. : Sum divisor cordial labeling of $B_{3,8}$

Example 2.4. *Sum divisor cordial labeling of the graph* $B_{5,5}$ *is shown in the Figure 4.*

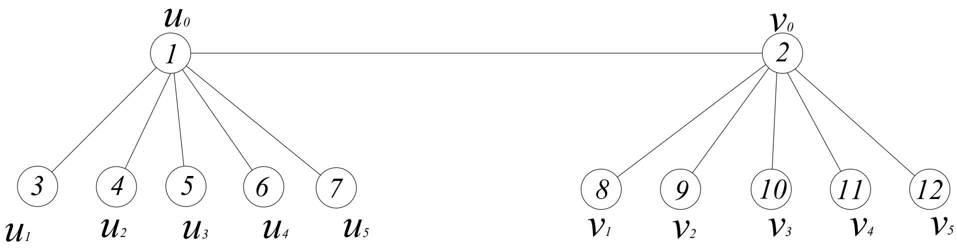

FIGURE 4. : Sum divisor cordial labeling of $B_{5,5}$

Theorem 2.2. $S'(B_{m,n})$ *is a sum divisor cordial graph.*

Proof. Let $B_{m,n}$ be the bistar with vertex set $V(B_{m,n}) = \{u_0, v_0, u_i, v_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$, where u_i and v_j are pendant vertices. Let u_0', v_0', u_i' and v_j' be the newly added vertices in order to obtain $G = S'(B_{m,n})$, where $1 \leq i \leq m$ and $1 \leq j \leq n$.
We note that $|V(G)| = 2m + 2n + 4$ and $|E(G)| = 3m + 3n + 3$. Without loss of generality we can assume that $m \leq n$ because $S'(B_{m,n})$ and $S'(B_{n,m})$ are isomorphic graphs.
We define vertex labeling $f : V(G) \to \{1, 2, \ldots, 2m + 2n + 4\}$ as follows.

If m, n are odd:

$$\begin{aligned} f(u_0) &= 1; \\ f(v_0) &= 3; \\ f(u'_0) &= 2; \\ f(v'_0) &= 4; \\ f(u_i) &= 4+i, & 1 \le i \le m; \\ f(v_i) &= 4+m+i, & 1 \le i \le n; \\ f(u'_i) &= 4+m+n+i, & 1 \le i \le m; \\ f(v'_i) &= 4+2m+n+i, & 1 \le i \le n. \end{aligned}$$

If m is odd and n is even:

$$\begin{aligned} f(u_0) &= 2; \\ f(v_0) &= 4; \\ f(u'_0) &= 1; \\ f(v'_0) &= 3, \\ f(u_i) &= 4+i, & 1 \le i \le m; \\ f(v_i) &= 4+m+i, & 1 \le i \le n; \\ f(u'_i) &= 4+m+n+i, & 1 \le i \le m; \\ f(v'_i) &= 4+2m+n+i, & 1 \le i \le n. \end{aligned}$$

The following table describes the results of edge labels obtained due to the above labeling pattern.

Cases of n	Edge results
m is odd and n is even	$e_f(0) = \frac{3m+3n+3}{2} = e_f(1)$
m and n both are odd	$e_f(0) = \lceil \frac{3m+3n+3}{2} \rceil, e_f(1) = \lfloor \frac{3m+3n+3}{2} \rfloor$

Thus $|e_f(0) - e_f(1)| \le 1$.
Hence, $S'(B_{m,n})$ is a sum divisor cordial graph. □

Example 2.5. *Sum divisor cordial labeling of the graph $S'(B_{3,7})$ is shown in the Figure 5.*

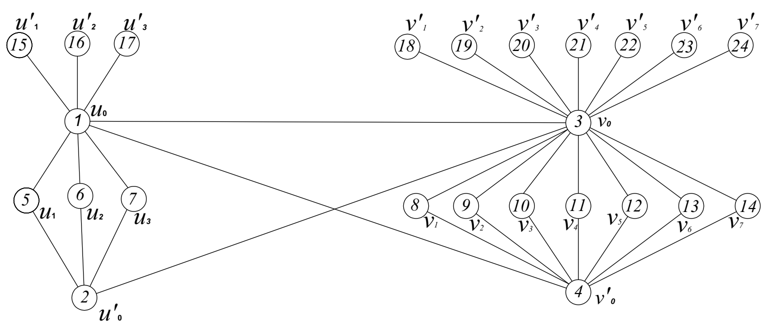

FIGURE 5. : Sum divisor cordial labeling of $S'(B_{3,7})$

Example 2.6. *Sum divisor cordial labeling of the graph $S'(B_{1,8})$ is shown in the Figure 6.*

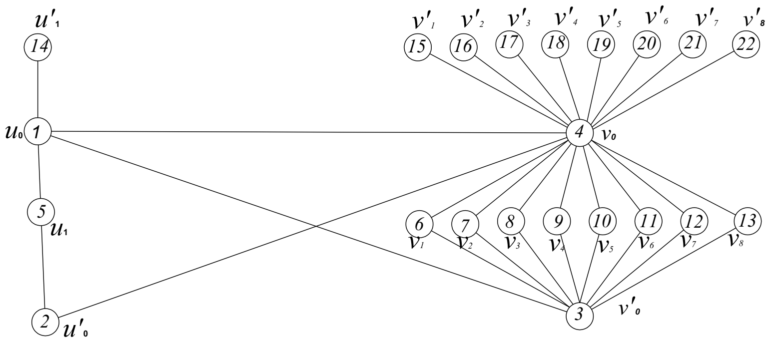

FIGURE 6. : Sum divisor cordial labeling of $S'(B_{1,8})$

Example 2.7. *Sum divisor cordial labeling of the graph $S'(B_{5,5})$ is shown in the Figure 7.*

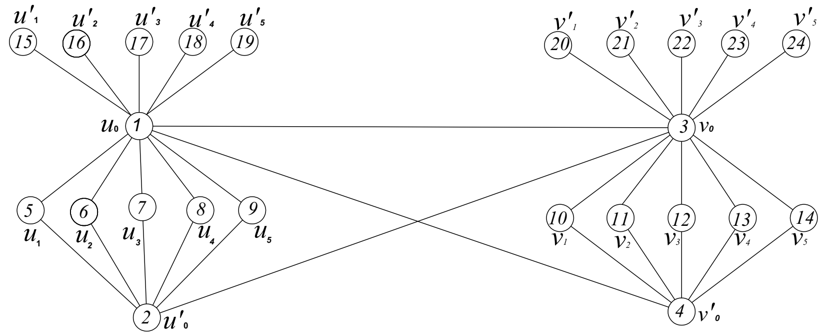

FIGURE 7. : Sum divisor cordial labeling of $S'(B_{5,5})$

Theorem 2.3. *The bistar $DS(B_{m,n})$ is a sum divisor cordial graph.*

Proof. Let $B_{m,n}$ be the bistar with vertex set $V(B_{m,n}) = \{u_0, v_0, u_i, v_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$, where u_i and v_j are pendant vertices. Here $V(B_{m,n}) = V_1 \bigcup V_2$, where $V_1 = \{u_i, v_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$ and $V_2 = \{u_0, v_0\}$.
Without loss of generality we can assume that $m \leq n$ because $DS(B_{m,n})$ and $DS(B_{n,m})$ are isomorphic graphs.
Now in order to obtain $G = DS(B_{m,n})$ from $B_{m,n}$ n, we consider following two cases.
Case 1: $m = n$.
We add w_1, w_2 corresponding to V_1, V_2. Then $|V(G)| = 2n+4$ and $E(G) = E(B_{m,n}) \bigcup \{u_0w_2, v_0w_2, u_iw_1, v_iw_1 \mid 1 \leq i \leq n\}$. So, $|E(G)| = 4n + 3$.

We define vertex labeling $f : V(G) \to \{1, 2, \ldots, 2n+4\}$ as follows.

$$\begin{aligned} f(u_0) &= 3; \\ f(v_0) &= 1; \\ f(w_1) &= 2; \\ f(w_2) &= 2n+4; \\ f(u_i) &= 2i+2, \quad 1 \le i \le m; \\ f(v_i) &= 2i+3, \quad 1 \le i \le n. \end{aligned}$$

The following table describes the results of edge labels obtained due to the above labeling pattern. Thus $|e_f(0) - e_f(1)| \le 1$.

Case 2: $m < n$.

We add w_1 corresponding to V_1. Then $|V(G)| = m+n+3$ and $E(G) = E(B_{m,n}) \bigcup \{u_i w_1, v_j w_1 \mid 1 \le i \le m, 1 \le i \le n\}$. So, $|E(G)| = 2m + 2n + 1$. We define vertex labeling $f : V(G) \to \{1, 2, \ldots, m+n+3\}$ as follows.

$$\begin{aligned} f(u_0) &= m+n+3; \\ f(v_0) &= 1; \\ f(w_1) &= 2; \\ f(w_2) &= m+n+2; \\ f(u_i) &= 2i+2, \qquad 1 \le i \le m; \\ f(v_i) &= 2i+1, \qquad 1 \le i \le m; \\ f(v_i) &= 2m+2+i, 1 \le i \le n. \end{aligned}$$

The following table describes the results of edge labels obtained due to the above labeling pattern. Thus $|e_f(0) - e_f(1)| \le 1$.

Hence, $DS(B_{m,n})$ is a sum divisor cordial graph. □

Example 2.8. *Sum divisor cordial labeling of the graph $DS(B_{4,4})$ is shown in the Figure 8.*

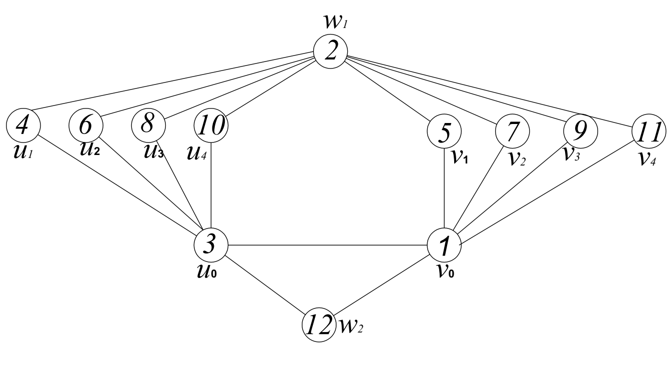

FIGURE 8. : Sum divisor cordial labeling of $DS(B_{4,4})$

Example 2.9. *Sum divisor cordial labeling of the graph $DS(B_{3,6})$ is shown in the Figure 9.*

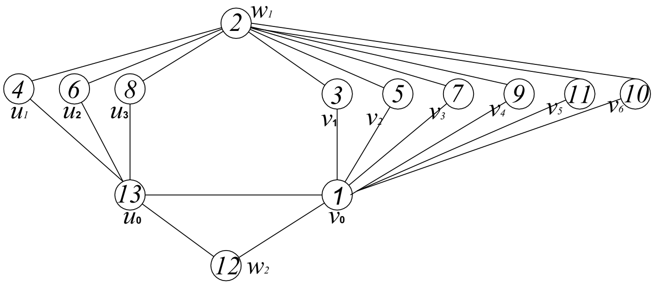

FIGURE 9. : Sum divisor cordial labeling of $DS(B_{3,6})$

Theorem 2.4. *The bistar $D_2(B_{m,n})$ is a sum divisor cordial graph.*

Proof. Let G' and G'' be two copies of bistar $B_{m,n}$ Let $V(G') = \{u'_0, v'_0, u'_i, v'_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$ and $V(G'') = \{u''_0, v''_0, u''_i, v''_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$.
Let $G = D_2(B_{m,n})$. We note that $|V(G)| = 2m + 2n + 4$ and $|E(G)| = 4m + 4n + 4$. Without loss of generality we can assume that $m \leq n$ because $D_2(B_{m,n})$ and $D_2(B_{n,m})$ are isomorphic graphs.
We define vertex labeling $f : V(G) \to \{1, 2, \ldots, 2m + 2n + 4\}$ as follows.

$$\begin{aligned}
f(u'_0) &= 1; \\
f(v'_0) &= 3; \\
f(u''_0) &= 2; \\
f(v''_0) &= 4; \\
f(u'_i) &= 4 + i, \qquad 1 \leq i \leq m; \\
f(v'_i) &= 4 + 2m + i, \quad 1 \leq i \leq n; \\
f(u''_i) &= 4 + m + i, \quad 1 \leq i \leq m; \\
f(v''_i) &= 4 + 2m + n + i, 1 \leq i \leq n.
\end{aligned}$$

The following table describes the results of edge labels obtained due to the above labeling pattern. Thus $|e_f(0) - e_f(1)| \leq 1$.
Hence, the bistar $D_2(B_{m,n})$ is a sum divisor cordial graph. □

Example 2.10. *Sum divisor cordial labeling of the graph $D_2(B_{4,8})$ is shown in the Figure 10.*

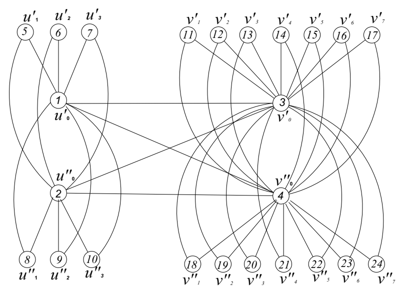

FIGURE 10. : Sum divisor cordial labeling of $D_2(B_{4,8})$

Theorem 2.5. *Restricted $B^2_{m,n}$ is a sum divisor cordial graph.*

Proof. Let $B_{m,n}$ be the bistar with vertex set $V(B_{m,n}) = \{u_0, v_0, u_i, v_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$, where u_i and v_j are pendant vertices. Let G be the resricted $B^2_{m,n}$ graph with $V(G) = V(B_{m,n})$ and $E(G) = E(B_{m,n}) \bigcup \{v_0u_i, u_0v_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$.
We note that $|V(G)| = m+n+2$ and $|E(G)| = 2m+2n+1$. Without loss of generality we can assume that $m \leq n$ because resricted $B^2_{m,n}$ and resricted $B^2_{n,m}$ are isomorphic graphs. We define vertex labeling $f : V(G) \to \{1, 2, \ldots, m+n+2\}$ as follows.

$$\begin{aligned} f(u_0) &= 2; \\ f(v_0) &= 1; \\ f(u_i) &= 2+i, \qquad 1 \leq i \leq m; \\ f(v_i) &= 2+m+i, \quad 1 \leq i \leq n. \end{aligned}$$

The following table describes the results of edge labels obtained due to the above labeling pattern. Thus $|e_f(0) - e_f(1)| \leq 1$.
Hence, the bistar $B^2_{m,n}$ is a sum divisor cordial graph. □

Example 2.11. *Sum divisor cordial labeling of the graph $B^2_{2,6}$ is shown in the Figure 11.*

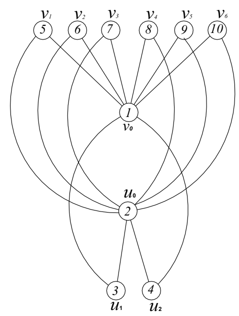

FIGURE 11. : Sum divisor cordial labeling of $B^2_{2,6}$

Example 2.12. *Sum divisor cordial labeling of the graph $B^2_{1,8}$ is shown in the Figure 12.*

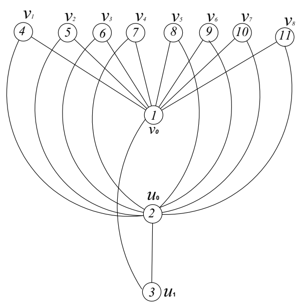

FIGURE 12. : Sum divisor cordial labeling of $B^2_{1,8}$

Theorem 2.6. *The barycentric subdivision $S(B_{m,n})$ of the bistar $B_{m,n}$ is a sum divisor cordial graph.*

Proof. Let $B_{m,n}$ be the bistar with vertex set $V(B_{m,n}) = \{u_0, v_0, u_i, v_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$, where u_i and v_j are pendant vertices and edge set $E(B_{m,n}) = \{u_0v_0, u_0u_i, v_0v_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$. Let $w_0, w_1, w_2, \ldots, w_m, w'_0, w'_1, w'_2, \ldots, w'_n$ be the newly added vertices to obtain $G = S(B_{m,n})$. where w_0 is added between u_0 and v_0, w_i is added between u_0 and u_i for $1 \leq i \leq m$ and w'_j is added between v_0 and v_j for $1 \leq j \leq n$.
We note that $|V(G)| = 2m + 2n + 3$ and $|E(G)| = 2m + 2n + 2$.
We define vertex labeling $f : V(G) \to \{1, 2, \ldots, 2m + 2n + 3\}$ as follows.

$$
\begin{aligned}
f(u_0) &= 2; \\
f(v_0) &= 1; \\
f(w_0) &= 3; \\
f(u_i) &= 2i + 2, && 1 \leq i \leq m; \\
f(w_i) &= 2i + 3, && 1 \leq i \leq m; \\
f(v_i) &= 2m + 2i + 2, && 1 \leq i \leq n; \\
f(w'_i) &= 2m + 3 + 2i, && 1 \leq i \leq n.
\end{aligned}
$$

The following table describes the results of edge labels obtained due to the above labeling pattern. Thus $|e_f(0) - e_f(1)| \leq 1$.
Hence, the bistar $S(B_{m,n})$ is a sum divisor cordial graph. □

Example 2.13. *Sum divisor cordial labeling of the graph $S(B_{3,7})$ is shown in the Figure 13.*

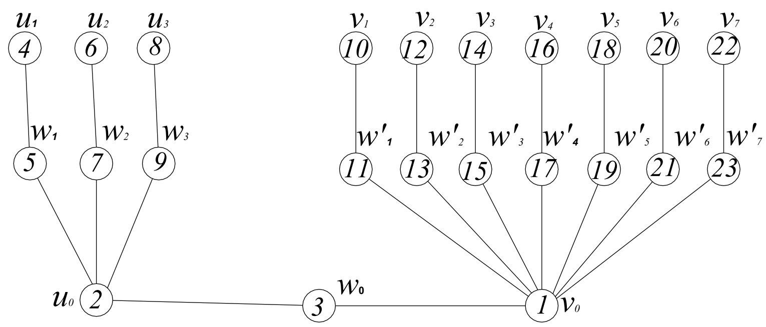

FIGURE 13. : Sum divisor cordial labeling of$S(B_{3,7})$

Theorem 2.7. *$B_{m,n} \odot K_1$ is a sum divisor cordial graph.*

Proof. Let $B_{m,n}$ be the bistar with vertex set $V(B_{m,n}) = \{u_0, v_0, u_i, v_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$, where u_i and v_j are pendant vertices. Let $u'_0, v'_0, u'_1, u'_2, \ldots, u'_m, v'_1, v'_2, \ldots, v'_n$ be the newly added vertices to obtain $G = B_{m,n} \odot K_1$.
We note that $V(G) = V(B_{m,n}) \bigcup \{u'_0, v'_0, u'_i, u'_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$ and $E(G) = E(B_{m,n}) \bigcup \{u_0u'_0, u_iu'_i, v_jv'_j \mid 1 \leq i \leq m, 1 \leq i \leq n\}$.
Hence $|V(G)| = 2m + 2n + 4$ and $|E(G)| = 2m + 2n + 3$.

We define vertex labeling $f : V(G) \to \{1, 2, \ldots, 2m + 2n + 4\}$ as follows.

$$\begin{aligned} f(u_0) &= 2; \\ f(v_0) &= 1; \\ f(u'_0) &= 2m + 2n + 4; \\ f(v'_0) &= 3; \\ f(u_i) &= 2i + 2, \qquad 1 \le i \le m; \\ f(v_i) &= 2m + 2i + 3, \quad 1 \le i \le n; \\ f(u'_i) &= 2i + 3, \qquad 1 \le i \le m; \\ f(v'_i) &= 2m + 2i + 2, \quad 1 \le i \le n. \end{aligned}$$

The following table describes the results of edge labels obtained due to the above labeling pattern. Thus $|e_f(0) - e_f(1)| \le 1$.
Hence, the bistar $B_{m,n} \odot K_1$ is a sum divisor cordial graph. □

Example 2.14. *Sum divisor cordial labeling of the graph $B_{5,7} \odot K_1$ is shown in the Figure 14.*

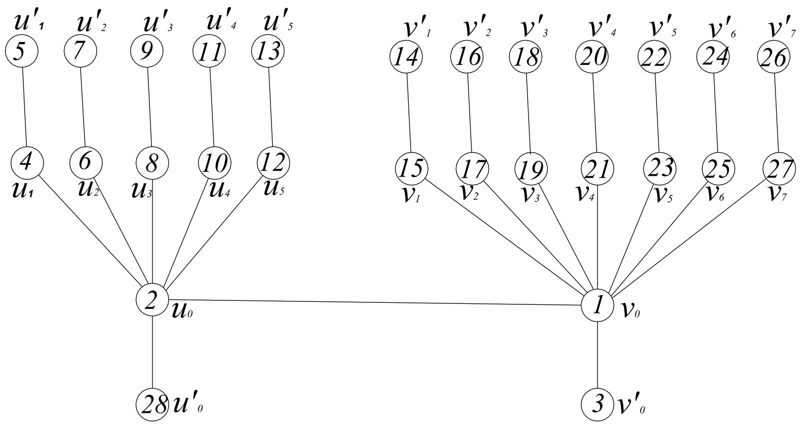

FIGURE 14. : Sum divisor cordial labeling of $B_{5,7} \odot K_1$

3. Concluding Remarks

Here, we have investigated some new results related to the graph operation duplication of graph for sum divisor cordial labeling technique. To explore some new sum divisor cordial graphs in the context of other graph operations is an open area of research.

References

[1] Adalja, D. G., and Ghodasara, G. V., (2018), Some New Sum Divisor Cordial Graphs, International Journal of Applied Graph Theory, 2, (1), pp. 19-33.
[2] Burton, D. M., (1990), Elementary Number Theory, Brown Publishers, Second Edition.
[3] Clark, J., Holton, D. A., (1991), A first look at graph theory, World Scientific Publishing Company.
[4] Frucht, R., Harary, F., 1970, On the corona of two graphs, Aequationes Mathematicae (Springer), Volume 4, (3), 322-325.
[5] Gallian, J. A., (2018), A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 21, # DS6.

[6] Ghodasara, G. V., Adalja, D. G., (2017), Divisor Cordial Labeling in Context of Ring Sum of Graphs, International Journal of Mathematics and Soft Computing, 7, (1), pp. 23-31

[7] Gross, J., Yellen, J., (2004), Graph Theory and Its Applications, CRC Press.

[8] Lourdusamy, A., Patrick, F., Shiama, J., (2016), Sum Divisor Cordial Graphs, Proyecciones Journal of Mathematics, 35, (1), pp 119-136.

[9] Lourdusamy, A., Patrick, F., (2016), Sum Divisor Cordial Labeling for Star and Ladder Related Graphs, Proyecciones Journal of Mathematics, 35, (4), pp. 437-455.

[10] Rosa, A., (1967), On Certain Valuations of The Vertices of a Graph, Theory of Graphs (Internat. Symposium, Rome, July (1966) Gordon and Breach, N. Y. and Dunod Paris, pp. 349-355.

[11] Selvaraju, P., Balaganesan, P., Renuka, J., Balaj, V., 2012, Degree splitting graph on graceful, felicitous and elegant labeling, The International Journal Mathematical Combinatorics, V. 2, pp. 96-102.

[12] Varatharajan, R., Navanaeethakrishnan, S., Nagarajan, K., (2011), Divisor Cordial Graphs, International J. Math. Combin., 4, pp. 15-25.

[13] Yegnanaryanan, V., Vaidhyanathan, P., (2012), Some Interesting Applications of Graph Labelings, J. Math. Comput. Sci., 2, (5), pp. 1522-1531.

Divya Ghanshyambhai Adalja has completed her B.Sc. and M.Sc. from Saurashtra University, Rajkot, Gujarat, India and Ph.D. from Gujarat Technological University, Ahmedabad, Gujarat, India. At present, she is working as an assistant professor in the Department of Mathematics at Marwadi Engineering College, Rajkot, Gujarat, India. Graph Labeling is her area of interest.