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Magnetoresistivity of a Weakly-Screened, Low-Density, Two-Dimensional Electron Liquid

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We probe the strength of electron–electron interactions at intermediate magnetic fields in the classical regime using magnetoconductivity measurements of two-dimensional, non-degenerate electrons on liquid helium. We span both the independent-electron regime, where the data are qualitatively described by the self-consistent Born approximation (SCBA), and the strongly-interacting electron (Drude) regime. We observe a crossover from SCBA to Drude theory at finite magnetic fields as a function of electron density. The SCBA magnetoresistance is found to be density dependent. Our data confirm the theory for magnetoresistivity in a weakly screened, two-dimensional electron gas, and demonstrate that electron–electron interactions are important to very low densities.

KEYWORDS: magnetoresistance, self-consistent Born approximation, electrons on helium

1. Introduction

Electrons supported by a liquid-helium surface form a low-density, non-degenerate, two-dimensional (2D) electron gas. Aside from the non-degeneracy, it differs from other 2D electron systems in the strength of the electron–electron interaction.[1–7] The Coulomb interaction is weakly screened by metallic plates that are separated from the electron layer by about 1 mm. It is an ideal system for testing the properties of strongly-interacting electrons.

The critical assumption of the many-electron theory is strong electron–electron correlations. By attaining very low densities, we are able to explore the independent-electron magnetoresistance described by the SCBA theory in the intermediate and low field regimes that could not be previously accessed. We observe the effect of electron–electron interactions on altering the orbital paths of electrons in small magnetic fields, which results in Drude behavior. This crossover from Drude to SCBA behavior has been confirmed by earlier experiments in the quantum limit,[2,4,5,7–11] $\hbar\omega_c > kT$, at high densities. Here $\omega_c = eB/m$. We probe the low-density electron liquid near $\Gamma = 1$, where $\Gamma = e^2(\pi n)^{1/2}/4\pi\epsilon_0 kT$ is the plasma parameter. The system is considered an electron gas for $\Gamma < 1$. We fit our results with the theory of Dykman and Lea.[2–7] Preliminary results without this analysis were presented elsewhere.[12]

2. Theory

In this system, electron-helium atom scattering dominates at temperatures above 0.8 K. This scattering is quasi-elastic, and helium atoms in the vapor act as short-range scatterers. The magnetoconductivity of electrons, $\sigma_{xx}(B)$, is given in the Drude model as

$$\sigma_{xx}(B) = \frac{\sigma_{xx}(0)}{[1 + (\mu_0 B)^2]}, \quad \sigma_{xx}(0) = ne\mu_0, \qquad (1)$$

where $\sigma_{xx}(0)$, $\mu_0 = e\tau_0/m$, and τ_0 are the zero field conductivity, mobility, and scattering time, respectively, and n is the electron density.

In a magnetic field transverse to the 2D electron array electron states are confined to the Landau levels (LL) with a width Δ. The contribution[13–15] to Δ from scattering is $\Delta_s = \hbar/\tau_B$. The enhancement of the density of states (DOS) on the LL increases the scattering rate, τ_B^{-1}. When considering only broadening by scattering and for the case $\mu_0 B \gg 1$, τ_B^{-1} is given by

$$\frac{1}{\tau_B} = \frac{1}{\tau_0}\kappa(\mu_0 B)^{1/2}, \qquad (2)$$

where κ is a number of order unity.

We interpret our data with the orbit diffusion approach to the SCBA theory by Dykman and Lea.[2–7] For intermediate fields, an electron undergoes repeated scattering from the same atom before its orbit is shifted by an electron wavelength. This "localization" destroys Drude behavior. However, there is an electric field, E_f, due to the electron assembly that fluctuates at the plasma frequency, $\omega_p = (n^{3/2}e^2/2m\bar{\varepsilon}\varepsilon_0)^{1/2}$. Here $\bar{\varepsilon} = 1.028$ is the effective dielectric constant for the layer. At smaller fields, below a characteristic field B_0, the fluctuating field of the electron assembly causes a shift in the electron orbit by an amount equal to the thermal wavelength[16] of the electron, $\lambda_T = h/(2mkT)^{1/2}$. This is sufficient to destroy repeated scattering from the same atom and restore Drude behavior.

The drift velocity of the electron is $\mathbf{v}_d = -\mathbf{E} \times \mathbf{B}/B^2$. The fluctuating field strength is given by[6]

$$\langle E_f^2 \rangle = \frac{F(\Gamma)kTn^{3/2}}{4\pi\bar{\varepsilon}\epsilon_0}, \qquad (3)$$

where $F(\Gamma)$ is obtained from Monte Carlo simulations[6,7] and for our densities has a numerical value of approximately ten. The parameter B_0 is defined by $v_d/\omega_c = \lambda_T$. It is proportional to $\langle E_f^2 \rangle^{1/4}$, and its numerical value is $B_0 = 3.87 \times 10^{-6} F^{1/4} T^{1/2} n^{3/8}$.

The typical shift of the orbit is the classical orbit radius, $R_c = (2mkT)^{1/2}/eB$. The diffusion coefficient is $D = R_c^2/4\tau_B$. With the use of the Einstein relation and the definitions of R_c, τ_B, and $\sigma_{xx}(0)$, we obtain,

$$\frac{\sigma_{xx}(0)}{\sigma_{xx}(B)} = \gamma(\mu_0 B)^{3/2}, \qquad (4)$$

where $\gamma \approx 1/\kappa$. The above derivation describes the SCBA theory in the classical limit.

The fluctuating field contributes an amount Δ_e to the broadening of the LL. An interpolation formula for the width, Δ, of the LL is

$$\Delta^2 = \Delta_s^2 + \Delta_e^2; \quad \Delta_e = eE_f\lambda_T. \tag{5}$$

For our electron densities the parameter $\Delta_e \ll \Delta_s$, and its contribution to the broadening of the LL is negligible. The role of electron–electron interactions at high electron densities is to broaden the LL through the parameter Δ_e, which is included in Eq. (2). For the low electron densities used in this experiment, the role of electron–electron interactions is to disrupt the circular orbit of a given electron in the applied magnetic field and cause a shift of repeated scattering from one scattering site (helium atom) to another scattering site.

More rigorous derivations of expressions for the SCBA magnetoresistance have been given for semi-elliptical[8] and Gaussian[10,17] DOS profiles. Their expressions for the classical limit are in agreement with Eq. (4) with numerical coefficients of $(\mu_0 B)^{3/2}$ that are approximately 1.5.

3. Experimental Results

We measured the inverse magnetoconductivity $1/\sigma_{xx}(B)$ of electrons as a function of a magnetic field perpendicular to the 2D electron layer in a Corbino geometry. Electrons were deposited over a ~1 mm thick helium film from a gaseous discharge. The density of electrons was controlled by carefully adjusting the dc holding voltage on the Corbino electrodes below the liquid helium surface. Most of our data were taken at $T = 1.22$ K and at probing frequencies 1–10 kHz. The mobility was $25\,\text{m}^2\,\text{V}^{-1}\,\text{s}^{-1}$. We work in the classical regime. The values of $\hbar\omega_c/kT$ for all data shown herein were less than 0.12. The quantum limit is reached at $B = 0.91$ T.

We do not observe a clean B^2 Drude region at low densities. Therefore we obtain the zero-field mobility and the density for the highest density curve from a fit to the Drude theory in small fields. Then, we calculate the electron densities for the lower density curves using the measured zero-field resistivity, $1/ne\mu_0$, and assuming the same zero-field mobility of $\mu_0 = 25\,\text{m}^2\,\text{V}^{-1}\,\text{s}^{-1}$ obtained for the highest density curve.

We show the normalized inverse magnetoconductivity $\sigma_{xx}(0)/\sigma_{xx}(B)$ as a function of $B^{3/2}$ for our lowest density, $n_{10} = 0.21$, corresponding to $\Gamma = 1.1$, in Fig. 1. We give our densities in units of $n_{10} = 10^{10}\,\text{m}^{-2}$. The approach to $B^{3/2}$ behavior from below is asymptotic, and a quantitative value of onset field for SCBA behavior cannot be defined. However, we note that the data deviate from a strictly $B^{3/2}$ variation below about 0.04 T. This is more than the value of $B_0 = 0.025$ T. The curve is expanded in the inset. We note that the SCBA magnetoresistivity does not extrapolate to the origin.

In Fig. 2 we present the normalized inverse magnetoconductivity $\sigma_{xx}(0)/\sigma_{xx}(B)$ as a function of $(\mu_0 B)^{3/2}$ for six electron densities. We infer from Fig. 2 that there is a density dependence in both the slope (i.e., γ) and the intercept, defined by extrapolating the $B^{3/2}$ region to zero field. The dashed line is the normalized theoretical Drude magnetoresistivity calculated using the experimental parameters of the highest density curve. As the density is increased, the fully developed SCBA behavior shifts to higher fields. This is consistent with the effect of the fluctuating field on the electron paths. For a fixed field the dependence shifts from $B^{3/2}$ to B^2 as the density is increased.

We also have limited data at higher temperatures. We find coincidentally at all temperatures an apparent small decrease in Drude mobilities with decreasing density at the densities where the $B^{3/2}$ regime occurs at low fields. At lower densities the exponent of $(\mu_0 B)$ in Eq. (1) is less than two. We conjecture that these features result from a distortion of the Drude region due to the crossover to SCBA. A smooth transition between the Drude and SCBA regions reduces the resistivity in the Drude region leading to an apparent lower mobility.

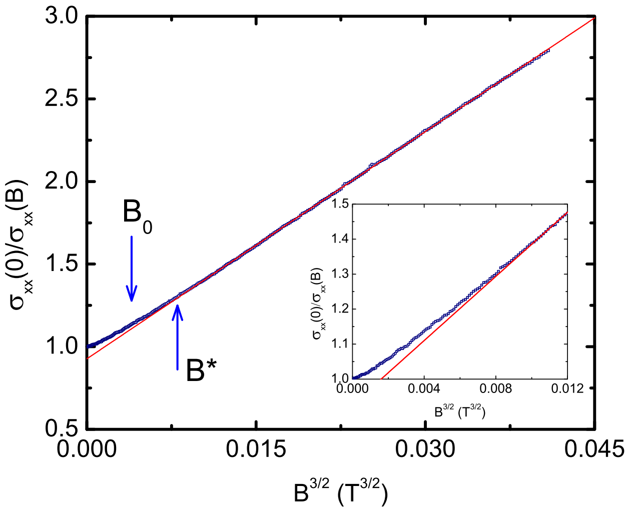

Fig. 1. (Color online) Normalized inverse magnetoconductivity vs $B^{3/2}$ for $n_{10} = 0.21$. The definition of B^*, denoted by an arrow, is defined below. The inset is a blow-up of the plot in the low field region. The solid line is a qualitative fit given by Eq. (4).

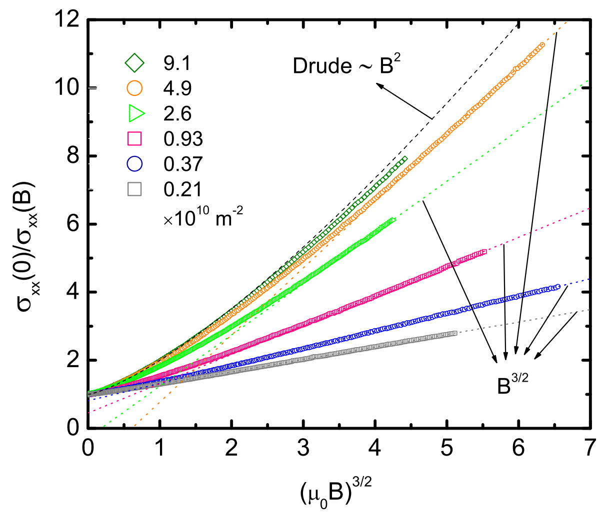

Fig. 2. (Color online) Normalized inverse-magnetoconductivity plotted as a function of $(\mu_0 B)^{3/2}$ at $T = 1.22$ K.

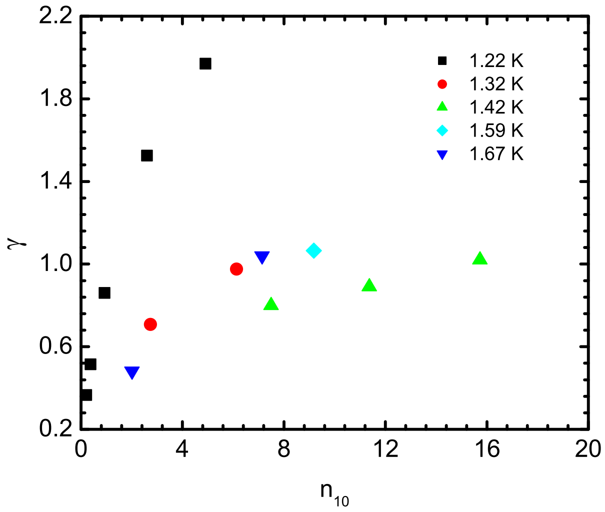

Fig. 3. (Color online) The parameter γ versus density at various temperatures.

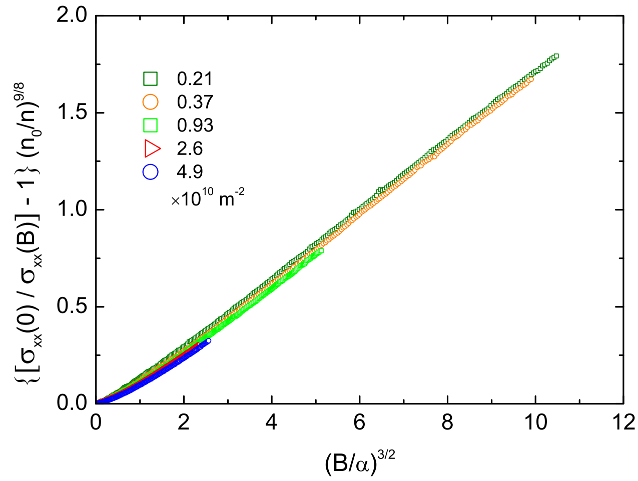

Fig. 4. (Color online) Scaled magnetoresistivity as a function of reduced field. Scaling is described in the text.

The SCBA theory has no density dependence. However, we have traces with both the Drude mobility and values of γ determined by using the Drude mobility in Eq. (4) at various temperatures. Our values, shown in Fig. 3, vary with density and temperature. The highest density data points at each temperature represent values from traces with a valid Drude mobility.

4. Discussion

Experimental data confirm that the value of γ is not constant in contradiction with theory. In addition, for higher densities, the Drude region extends to a higher field and to a higher value of resistivity. This requires a higher value of γ unless the zero-field intercept is more positive. At a given temperature γ increases with density. The derivative of gamma with respect to density decreases with increasing temperature except for an anomalous data point at 1.67 K. The variation of the high-field SCBA behavior with electron density has been observed for scattering from vapor atoms[4] and from ripplons.[11]

Figure 4 shows all of the data on a universal curve obtained by scaling the normalized resistivity by $(n_0/n)^{9/8}$, $n_0 = 0.21 \times 10^{10}\,\mathrm{m}^{-2}$, and scaling the field by a best fit parameter α^{-1}. The scaling of the ordinate is set so that the slopes in Fig. 4 are density independent. We included the term (-1) in the quantity plotted on the ordinate to shift the origin of all curves to the origin of the graph. The inclusion of (-1) does not change the slopes of the curves. The value of α is within experimental error proportional to B_0. Its value is arbitrarily set to 0.025 at $n = n_0$. The parameter α is plotted versus B_0 in Fig. 5. The solid line is a linear fit with a slope of unity. This scaling of the data clearly demonstrates the predicted influence of the fluctuating field.

Our values of γ deduced from the experimental curves at $T = 1.22$ K vary as $0.07 + 0.8n^{9/16}$. This is consistent with the normalization used in Fig. 4. The parameter $F(\Gamma)$ that enters the expression for E_f is weakly density dependent, $F(\Gamma) \approx n^{-0.05}$, in the low density regime.[7] Then the density dependence of $\alpha \propto B_0 \propto F(\Gamma)^{1/4} n^{3/8}$ is slightly less than $n^{3/8}$. Thus scaling of the abscissa by $n^{-9/16}$ and the ordinate

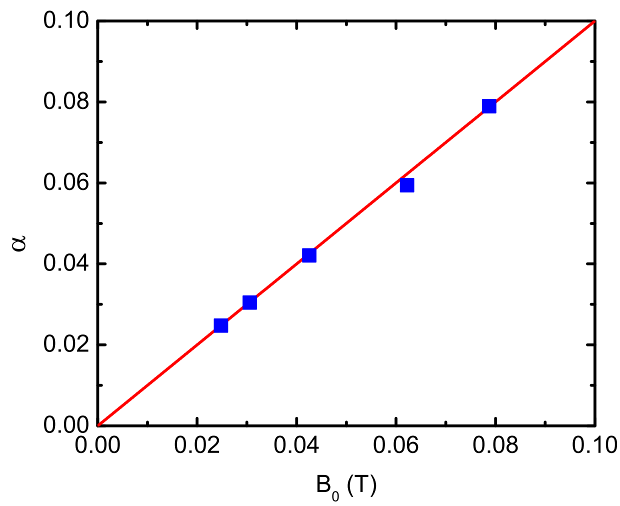

Fig. 5. (Color online) Scaling parameter α vs B_0.

by $n^{-9/8}$ leads to a normalized slope that is density independent.

The curves approach the $B^{3/2}$ region asymptotically as the field is increased. Lacking a theoretical dependence of the magnetoresistivity through the crossover from Drude to SCBA, we find it convenient to define a value of B^*, shown in Fig. 1, above which the $B^{3/2}$ dependence is fully developed. We define B^* as the point where the deviation from a linear dependence on $B^{3/2}$ is one-half of the height of the data symbols of the same size for all plots. The parameter B^* is a relative measure for comparing curves. The value of B^* should be greater than the value of B_0, since there is a range of fields around B_0 over which repeated scattering is disrupted. The ratio B^*/B_0 for temperatures of 1.22 K is 1.6 ± 0.5. The constancy of the ratio B^*/B_0 is consistent with the scaling of data for different densities. Note that for a phenomenological theory the value of B_0 is not given precisely.

We find the ratios $B^*/B_0 \sim 0.5$ at $T = 1.32$ and 1.42 K. They differ from the values at 1.22 K and are less than unity. For all data at $T > 1.22$ K, the value of $(\mu B^*) = 0.55 \pm 0.1$.

This value appears to be inconsistent with the limitations on self-consistency given by Eq. (2) for a value of κ of order unity. We are unable to resolve these discrepancies.

Scattering from helium atoms dominates the linewidth of the Landau levels at 1.22 K and therefore the value of τ_B. The contribution of Δ_e to the LL line width is small for our parameters. We find no deviations from the $B^{3/2}$ dependence of the resistivity in the SCBA region to indicate a contribution of Δ_e to the linewidth.

5. Conclusions

Our very low density values allow us to study the effect of electron–electron interactions on the magnetoresistance in the classical regime and in intermediate and low fields. Our data demonstrate the fully developed SCBA $B^{3/2}$ dependence in a weakly screened classical two-dimensional electron gas. The data are in excellent qualitative agreement with the theory of Dykman and Lea. The scaling of the curves with parameter B_0 supports the correctness of the expression for the fluctuating field. We observe that electron–electron interactions are important to a very low density, $\sim 10^9$ m^{-2}.

We find the coefficient of $(\mu_0 B)^{3/2}$ to be density dependent. We have no explanation for this dependence. A density-dependent magnetoresistivity is compatible with the fact that the Drude resistivity extends to higher values for higher densities.

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