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TRANSLATION AND SCALAR MULTIPLICATION ON PICTURE FUZZY IDEAL OF A PS ALGEBRA

S. DOGRA[1*], M. PAL[1], §

Abstract. In this paper, the notion of picture fuzzy ideal of PS algebra is introduced as a generalization of fuzzy ideal of PS algebra. The study of properties of picture fuzzy ideal of PS algebra is done in the environment of scalar multiplication with picture fuzzy set, translation of picture fuzzy set and extension of picture fuzzy set.

Keyword: Picture fuzzy ideal, scalar multiplication with picture fuzzy set, translation of picture fuzzy set, extension picture fuzzy ideal.

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1. Introduction

In the purpose of handling uncertainty in real life field, Zadeh [29] introduced fuzzy set. It is important to make reference to that fuzzy set includes only the grade of membership. For the generalization of fuzzy set, Atanassov [3] provided the notion of intuitionistic fuzzy set where intuitionistic fuzzy set includes the grade of membership and the grade of non-membership. Iseki and Tanaka [12] initiated the notion of BCK algebra. The concept of BCI algebra was provided by Iseki [13]. Combining the ideas of fuzzy set and BCK algebra, the study of BCK algebra based on fuzzy set was done by Xi [28]. The concept of fuzzy set associated with BCI algebra was given by Ahmad [1]. Neggers and Kim [20] provided the notion of d-algebra. With the headway of time, a lot of works on BCK/BCI/d-algebra and ideals associated with fuzzy set were done by several researchers [18, 2, 23, 26]. Intuitionistic fuzzy subalgebra and ideals in BCK algebra were studied by Jun and Kim [19] as an extension of fuzzy set concept associated with BCK algebra. As the time goes, BCK/BCI algebra and ideals were studied by Senapati et al. [25, 27] in context of intuitionistic fuzzy set in various directions. Jana et al. [15, 17] studied BCK/BCI under different set environment and Senapati [24] studied translation

[1] Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Midnapore, 721102, India.
e-mail: shovansd39@gmail.com; ORCID: https://orcid.org/0000-0002-0768-1764.
[*] Corresponding author.
e-mail: mmpalvu@gmail.com; ORCID: https://orcid.org/0000-0002-6709-836X.
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on B-algebra under intuitionistic fuzzy environment. A new kind of algebra named PS algebra was initiated by Priya and Ramachandran [21] as a generalization of BCI/BCK/d-algebra. They [22] also studied PS algebra applying the notion of fuzzy set in PS algebra. Including the grade of neutral membership and extending the notion of intuitionistic fuzzy set, the concept of picture fuzzy set was provided by Cuong [6]. Later on various types of research works under picture fuzzy circumstances were performed by several researchers [4, 5, 11, 14, 16, 7, 8, 9, 10]. In this article, we will provide the concept of picture fuzzy ideal and study its properties in context of translation of picture fuzzy set, multiplication of scalar with picture fuzzy set and extension picture fuzzy set.

2. Preliminaries

Here, we will call back again some basic concepts of BCK/BCI/d-algebra, PS algebra, ideal of PS algebra, fuzzy set (FS), fuzzy ideal (FI) of PS algebra, intuitionistic fuzzy set (IFS), picture fuzzy set (PFS) and some operations on picture fuzzy sets (PFSs).
Iseki [13] introduced BCI algebra.

Definition 2.1. [13] *An algebra $(A, \diamond, 0)$ is said to be BCI-algebra if for any $a, b, c \in A$, the below stated conditions are meet.*

(i) $[(a \diamond b) \diamond (a \diamond c)] \diamond (c \diamond b) = 0$
(ii) $[a \diamond (a \diamond b)] \diamond b = 0$
(iii) $a \diamond a = 0$
(iv) $a \diamond b = 0$ *and* $b \diamond a = 0 \Rightarrow a = b$.

A BCI algebra satisfying the condition $0 \diamond a = 0$ for all $a \in A$ is called BCK algebra. Neggers and Kim [20] introduced d-algebra as a related algebraic structure of BCK/BCI-algebra.

Definition 2.2. [20] *An algebra $(A, \diamond, 0)$ is said to be d-algebra if the below stated conditions are fulfilled.*

(i) $a \diamond a = 0$
(ii) $0 \diamond a = 0$
(iii) $a \diamond b = 0$ *and* $b \diamond a = 0 \Rightarrow a = b$ *for all* $a, b \in A$.

As the generalization of BCK/BCI/d-algebra, Priya and Ramachandran [21] defined ideal of PS algebra.

Definition 2.3. [21] *An algebra $(A, \diamond, 0)$ is said to be PS algebra if the below stated conditions are satisfied.*

(i) $a \diamond a = 0$
(ii) $a \diamond 0 = 0$
(iii) $a \diamond b = 0$ *and* $b \diamond a \Rightarrow a = b$ *for all* $a, b \in A$.

A binary relation '$\leqslant$' on A is defined as: $a \leqslant b$ holds iff $b \diamond a = 0$.

Definition 2.4. [21] *Let $(A, \diamond, 0)$ be a PS algebra and $C \subseteq A$ is non-empty. Then C is called ideal of A if the below stated conditions are meet.*

(i) $0 \in C$
(ii) $b \diamond a \in C$ *and* $b \in C \Rightarrow a \in C$

Definition 2.5. [29] *Let A be the set of universe. Then a FS P over A is of the form $P = \{(a, \mu_P(a)) : a \in A\}$, where $\mu_P(a) \in [0, 1]$ is the grade of membership of a in P.*

Realizing the absence of grade of non-membership in FS and generalizing FS, Atanassov [3] provided the definition IFS in the following way.

Definition 2.6. [3] *Let A be the set of universe. An IFS P over A is of the form $P = \{(a, \mu_P(a), v_P(a)) : a \in A\}$, where $\mu_P(a) \in [0,1]$ is the grade of membership of a in P and $v_P(a) \in [0,1]$ is the grade of non-membership of a in P such that the $0 \leqslant \mu_P(a)+v_P(a) \leqslant 1$ for all $a \in A$ is fulfilled.*

Applying the notion of fuzzy set (FS) to PS algebra, Priya and Ramachandran [22] provided the definition of fuzzy ideal (FI) in PS algebra.

Definition 2.7. [22] *Let $(A, \diamond, 0)$ be a PS algebra and P be a FS in A. Then P is called FI of A if the below stated conditions are satisfied.*
(i) $\mu_P(0) \geqslant \mu_P(a)$
(ii) $\mu_P(a) \geqslant \mu_P(b \diamond a) \wedge \mu_P(b)$ *for all* $a, b \in A$.

Including more possible types of uncertainty, Cuong [6] defined PFS generalizing the concepts of FS and IFS.

Definition 2.8. [6] *Let A be the set of universe. Then a PFS P over the universe A is defined as $P = \{(a, \mu_P(a), \eta_P(a), v_P(a)) : a \in A\}$, where $\mu_P(a) \in [0,1]$ is the grade of positive membership of a in P, $\eta_P(a) \in [0,1]$ is the grade of neutral membership of a in P and $v_P(a) \in [0,1]$ is the grade of negative membership of a in P such that $0 \leqslant \mu_P(a)+\eta_P(a)+v_P(a) \leqslant 1$ for all $a \in A$ is fulfilled. For all $a \in A$, $1-(\mu_P(a)+\eta_P(a)+v_P(a))$ is the grade of denial membership a in P.*

Definition 2.9. [6] *Let $P = \{(a, \mu_P(a), \eta_P(a), v_P(a)) : a \in A\}$ and $Q = \{(a, \mu_Q(a), \eta_Q(a), v_Q(a)) : a \in A\}$ be two PFSs over the universe A. Then intersection between P and Q is denoted by $P \cap Q$ and is defined in the following way.*
$P \cap Q = \{(a, \min(\mu_P(a), \mu_Q(a)), \min(\eta_P(a), \eta_Q(a)), \max(v_P(a), v_Q(a))) : a \in A\}$

Throughout the paper, we write PFS $P = \{(a, \mu_P(a), \eta_P(a), v_P(a)) : a \in A\}$ as $P = (\mu_P, \eta_P, v_P)$.

3. Scalar Multiplication on Picture Fuzzy Ideal

Scalar multiplication with PFS is an important operation. In the current section, scalar multiplication with PFS is defined and some important properties of PFI in the light of scalar multiplication with PFS has been highlighted here.
Now, it is the time to introduce PFI of PS algebra as an extension of FI of PS algebra.

Definition 3.1. *Let A be a PS algebra. A PFS $P = (\mu_P, \eta_P, v_P)$ is called PFI of A if the below stated conditions are meet.*
(i) $\mu_P(0) \geqslant \mu_P(a)$, $\eta_P(0) \geqslant \eta_P(a)$ *and* $v_P(0) \leqslant v_P(a)$
(ii) $\mu_P(a) \geqslant \mu_P(b \diamond a) \wedge \mu_P(b)$, $\eta_P(a) \geqslant \eta_P(b \diamond a) \wedge \eta_P(b)$ *and* $v_P(a) \leqslant v_P(b \diamond a) \vee v_P(b)$ *for all* $a, b \in A$.

Example 3.1. *Let us consider the PS algebra $(A, \diamond, 0)$ as follows.*

$\diamond$	0	a	b
0	0	a	b
a	0	0	b
b	0	a	0

Let us suppose a PFS $P = (\mu_P, \eta_P, v_P)$ in A such that

$$\mu_P(0) = 0.35, \eta_P(0) = 0.3 \text{ and } v_P(0) = 0.2$$
$$\mu_P(a) = 0.25, \eta_P(a) = 0.2 \text{ and } v_P(a) = 0.3$$
$$\mu_P(b) = 0.15, \eta_P(b) = 0.15 \text{ and } v_P(b) = 0.4$$

It can be clearly shown that P is a PFI of A.

Now, we are going to initiate the notion of scalar multiplication with PFS where the scalar lies in the interval $[0,1]$. This can be obtained by multiplying the scalar with three picture fuzzy components individually and it again produces a PFS.

Definition 3.2. *Let $P = (\mu_P, \eta_P, v_P)$ be a PFS over the set of universe A. Then for a scalar $k \in [0,1]$, scalar multiplication with PFS P is denoted by kP and is defined as $kP = (\mu_{kP}, \eta_{kP}, v_{kP})$, where $\mu_{kP}(a) = k\mu_P(a)$, $\eta_{kP}(a) = k\eta_P(a)$ and $v_{kP}(a) = kv_P(a)$ for all $a \in A$.*

Now, two important results on PFI on the basis of scalar multiplication with PFS are studied by following two Propositions. In this case, non-zero scalar has been taken into account during multiplication.

Proposition 3.1. *Let $(A, \diamond, 0)$ be a PS algebra and $P = (\mu_P, \eta_P, v_P)$ be a PFI of A. Then kP is a PFI of A for $k \in (0,1]$.*

Proof: Let $kP = Q = (\mu_Q, \eta_Q, v_Q)$ where $\mu_Q(a) = k\mu_P(a)$, $\eta_Q(a) = k\eta_P(a)$, $v_Q(a) = kv_P(a)$ for all $a \in A$ and for $k \in (0,1]$.

$$\begin{aligned}
\text{Now } \mu_Q(0) &= k\mu_P(0) \\
&\geqslant k\mu_P(a) \text{ [as } P \text{ is a PFI of } A] \\
&= \mu_Q(a), \\[1em]
\eta_Q(0) &= k\eta_P(0) \\
&\geqslant k\eta_P(a) \text{ [as } P \text{ is a PFI of } A] \\
&= \eta_Q(a) \\[1em]
\text{and } v_Q(0) &= kv_P(0) \\
&\leqslant kv_P(a) \text{ [as } P \text{ is a PFI of } A] \\
&= v_Q(a) \text{ for all } a \in A.
\end{aligned}$$

$$\begin{aligned}
\text{Also, } \mu_Q(a) &= k\mu_P(a) \\
&\geqslant k(\mu_P(b \diamond a) \wedge \mu_P(b)) \text{ [as } P \text{ is a PFI of } A] \\
&= (k\mu_P(b \diamond a)) \wedge (k\mu_P(b)) \\
&= \mu_Q(b \diamond a) \wedge \mu_Q(b), \\[1em]
\eta_Q(a) &= k\eta_P(a) \\
&\geqslant k(\eta_P(b \diamond a) \wedge \eta_P(b)) \text{ [as } P \text{ is a PFI of } A] \\
&= (k\eta_P(b \diamond a)) \wedge (k\eta_P(b)) \\
&= \eta_Q(b \diamond a) \wedge \eta_Q(b) \\[1em]
\text{and } v_Q(a) &= kv_P(a) \\
&\leqslant k(v_P(b \diamond a) \vee v_P(b)) \text{ [as } P \text{ is a PFI of } A] \\
&= (kv_P(b \diamond a)) \vee (kv_P(b)) \\
&= v_Q(b \diamond a) \wedge v_Q(b) \text{ for all } a \in A.
\end{aligned}$$

Thus, $Q = kP$ is a PFI of A.

Proposition 3.2. *Let $(A, \diamond, 0)$ be a PS algebra and $P = (\mu_P, \eta_P, v_P)$ be a PFS in A such that kP is a PFI of A for $k \in (0, 1]$. Then P is a PFI of A.*

Proof: Let $kP = Q = (\mu_Q, \eta_Q, v_Q)$ where $\mu_Q(a) = k\mu_P(a)$, $\eta_Q(a) = k\eta_P(a)$, $v_Q(a) = kv_P(a)$ for all $a \in A$ and for $k \in (0, 1]$.

$$\begin{aligned} &\text{Here, } \mu_Q(0) \geqslant \mu_Q(a) \text{ [as } Q \text{ is a PFI of } A]\\ &\text{i.e. } k\mu_P(0) \geqslant k\mu_P(a)\\ &\text{i.e. } \mu_P(0) \geqslant \mu_P(a) \text{ [as } k > 0], \end{aligned}$$

$$\begin{aligned} &\eta_Q(0) \geqslant \eta_Q(a) \text{ [as } Q \text{ is a PFI of } A]\\ &\text{i.e. } k\eta_P(0) \geqslant k\eta_P(a)\\ &\text{i.e. } \eta_P(0) \geqslant \eta_P(a) \text{ [as } k > 0] \end{aligned}$$

$$\begin{aligned} &\text{and } v_Q(0) \leqslant v_Q(a) \text{ [as } Q \text{ is a PFI of } A]\\ &\text{i.e. } kv_P(0) \leqslant kv_P(a)\\ &\text{i.e. } v_P(0) \leqslant v_P(a) \text{ [as } k > 0]. \end{aligned}$$

$$\begin{aligned} &\text{Also, } \mu_Q(a) \geqslant \mu_Q(b \diamond a) \wedge \mu_Q(b) \text{ [as } Q \text{ is a PFI]}\\ &\text{i.e. } k\mu_P(a) \geqslant (k\mu_P(b \diamond a)) \wedge (k\mu_P(b))\\ &\text{i.e. } k\mu_P(a) \geqslant k(\mu_P(b \diamond a) \wedge \mu_P(b))\\ &\text{i.e. } \mu_P(a) \geqslant (\mu_P(b \diamond a) \wedge \mu_P(a)) \text{ [as } k > 0], \end{aligned}$$

$$\begin{aligned} &\eta_Q(a) \geqslant \eta_Q(b \diamond a) \wedge \eta_Q(b) \text{ [as } Q \text{ is a PFI]}\\ &\text{i.e. } k\eta_P(a) \geqslant (k\eta_P(b \diamond a)) \wedge (k\eta_P(b))\\ &\text{i.e. } k\eta_P(a) \geqslant k(\eta_P(b \diamond a) \wedge \eta_P(b))\\ &\text{i.e. } \eta_P(a) \geqslant (\eta_P(b \diamond a) \wedge \eta_P(a)) \text{ [as } k > 0] \end{aligned}$$

$$\begin{aligned} &\text{and } v_Q(a) \leqslant v_Q(b \diamond a) \vee v_Q(b) \text{ [as } Q \text{ is a PFI]}\\ &\text{i.e. } kv_P(a) \leqslant (kv_P(b \diamond a)) \vee (kv_P(b))\\ &\text{i.e. } kv_P(a) \leqslant k(v_P(b \diamond a) \vee v_P(b))\\ &\text{i.e. } v_P(a) \leqslant (v_P(b \diamond a) \vee v_P(b)) \text{ [as } k > 0] \text{ for all } a, b \in A. \end{aligned}$$

Thus, P is a PFI of A whenever kP is a PFI of A for $k \in (0, 1]$.

4. Picture fuzzy ideal under translation of picture fuzzy set

In the current section, translation of PFS is defined as a generalization of translation of FS and some results on PFI under translation of PFS are studied.

Definition 4.1. *Let $(A, \diamond, 0)$ be the set of universe and $P = (\mu_P, \eta_P, v_P)$ be a PFS in A. Then (r, s, t)-translation of P is defined as the PFS $P^T_{r,s,t} = (\mu^T_P, \eta^T_P, v^T_P)$, where $\mu^T_P(a) = \mu_P(a) + r$, $\eta^T_P(a) = \eta_P(a) + s$, $v^T_P(a) = v_P(a) - t$ and $r, s, t \in [0, 1]$ with $t = inf\{v(a) : a \in A\}$ and $r + s = t$.*

Proposition 4.1. *Let $(A, \diamond, 0)$ be PS algebra and $P = (\mu_P, \eta_P, v_P)$ be a PFI of A. Then $P^T_{r,s,t}$ is a PFI of A.*

Proof: Let $P^T_{r,s,t} = (\mu^T_P, \eta^T_P, v^T_P)$.

$$\begin{aligned}\text{Now, } \mu^T_P(0) &= \mu_P(0) + r\\ &\geqslant \mu_P(a) + r \text{ [as } P \text{ is a PFI of } A]\\ &= \mu^T_P(a),\end{aligned}$$

$$\begin{aligned}\eta^T_P(0) &= \eta_P(0) + s\\ &\geqslant \eta_P(a) + s \text{ [as } P \text{ is a PFI of } A]\\ &= \eta^T_P(a)\end{aligned}$$

$$\begin{aligned}\text{and } v^T_P(0) &= v_P(0) - t\\ &\leqslant v_P(a) - t \text{ [as } P \text{ is a PFI of } A]\\ &= v^T_P(a) \text{ for all } a \in A.\end{aligned}$$

$$\begin{aligned}\text{Also, } \mu^T_P(a) &= \mu_P(a) + r\\ &\geqslant \mu_P(b \diamond a) \wedge \mu_P(b) + r \text{ [as } P \text{ is a PFI of } A]\\ &= (\mu_P(b \diamond a) + r) \wedge (\mu_P(b) + r)\\ &= \mu^T_P(b \diamond a) \wedge \mu^T(b),\end{aligned}$$

$$\begin{aligned}\eta^T_P(a) &= \eta_P(a) + s\\ &\geqslant \eta_P(b \diamond a) \wedge \eta_P(b) + s \text{ [as } P \text{ is a PFI of } A]\\ &= (\eta_P(b \diamond a) + s) \wedge (\eta_P(b) + s)\\ &= \eta^T_P(b \diamond a) \wedge \eta^T_P(b)\end{aligned}$$

$$\begin{aligned}\text{and } v^T_P(a) &= v_P(a) - t\\ &\leqslant v_P(b \diamond a) \vee v_P(b) - t \text{ [as } P \text{ is a PFI of } A]\\ &= (v_P(b \diamond a) - t) \vee (v_P(b) - t)\\ &= v^T_P(b \diamond a) \vee v^T_P(b) \text{ for all } a, b \in A.\end{aligned}$$

Consequently, $P^T_{r,s,t}$ is a PFI of A.

Proposition 4.2. *Let $(A, \diamond, 0)$ be a PS algebra and $P = (\mu_P, \eta_P, v_P)$ be a PFS in A. Then $P = (\mu_P, \eta_P, v_P)$ is a PFI of A if $P^T_{r,s,t}$ is a PFI of A.*

Proof: Let $P^T_{r,s,t} = Q = (\mu_Q, \eta_Q, v_Q)$.

$$\begin{aligned}\text{Now, } \mu_Q(0) &\geqslant \mu_Q(a) \text{ [as } Q \text{ is a PFI of } A]\\ \text{i.e. } \mu_P(0) + r &\geqslant \mu_P(a) + r\\ \text{i.e. } \mu_P(0) &\geqslant \mu_P(a),\end{aligned}$$

$$\begin{aligned}\eta_Q(0) &\geqslant \eta_Q(a) \text{ [as } Q \text{ is a PFI of } A]\\ \text{i.e. } \eta_P(0) + s &\geqslant \mu_P(a) + s\\ \text{i.e. } \eta_P(0) &\geqslant \eta_P(a)\end{aligned}$$

$$\begin{aligned}
\text{and } v_Q(0) &\leqslant v_Q(a) \text{ [as } Q \text{ is a PFI of } A]\\
\text{i.e. } v_P(0) - t &\leqslant v_P(a) - t\\
\text{i.e. } v_P(0) &\leqslant v_P(a).
\end{aligned}$$

$$\begin{aligned}
\text{Also, } \mu_Q(a) &\geqslant \mu_Q(b \diamond a) \wedge \mu_Q(b) \text{ [as } Q \text{ is a PFI of } A]\\
\text{i.e. } \mu_P(a) + r &\geqslant (\mu_P(b \diamond a) + r) \wedge (\mu_P(b) + r)\\
\text{i.e. } \mu_P(a) + r &\geqslant (\mu_P(b \diamond a) \wedge \mu_P(b)) + r\\
\text{i.e. } \mu_P(a) &\geqslant \mu_P(b \diamond a) \wedge \mu_P(a),
\end{aligned}$$

$$\begin{aligned}
\eta_Q(a) &\geqslant \eta_Q(b \diamond a) \wedge \eta_Q(b) \text{ [as } Q \text{ is a PFI of } A]\\
\text{i.e. } \eta_P(a) + s &\geqslant (\eta_P(b \diamond a) + s) \wedge (\eta_P(b) + s)\\
\text{i.e. } \eta_P(a) + s &\geqslant (\eta_P(b \diamond a) \wedge \eta_P(b)) + s\\
\text{i.e. } \eta_P(a) &\geqslant (\eta_P(b \diamond a) \wedge \eta_P(a))
\end{aligned}$$

$$\begin{aligned}
\text{and } v_Q(a) &\leqslant v_Q(b \diamond a) \vee v_Q(b) \text{ [as } Q \text{ is a PFI of } A]\\
\text{i.e. } v_P(a) - t &\leqslant (v_P(b \diamond a) - t) \vee (v_P(b) - t)\\
\text{i.e. } v_P(a) - t &\leqslant (v_P(b \diamond a) \vee v_P(b)) - t\\
\text{i.e. } v_P(a) &\leqslant (v_P(b \diamond a) \vee v_P(b)) \text{ for all } a, b \in A.
\end{aligned}$$

Consequently, P is a PFI of A

Example 4.1. *Let us consider the Example 3.1. Here, t=inf* $\{0.2, 0.3, 0.4\} = 0.2$. *Depending on t, choose* $r = 0.15$ *and* $s = 0.05$. *Then translation of PFI P is defined as the PFS* $P^T_{r,s,t} = (\mu^T_P, \eta^T_P, v^T_P)$ *where*

$$\begin{aligned}
\mu^T(0) &= \mu_P(0) + r = 0.35 + 0.15 = 0.5\\
\eta^T_P(0) &= \eta_P(0) + s = 0.3 + 0.05 = 0.35\\
\textit{and } v^T_P(0) &= v_P(0) - t = 0.2 - 0.2 = 0\\
\mu^T(a) &= \mu^T(a) + r = 0.25 + 0.15 = 0.4\\
\eta^T_P(a) &= \eta_P(a) + s = 0.2 + 0.05 = 0.25\\
\textit{and } v^T_P(a) &= v_P(a) - t = 0.3 - 0.2 = 0.1\\
\mu^T(b) &= \mu_P(b) + r = 0.15 + 0.15 = 0.3\\
\eta^T_P(b) &= \eta_P(b) + s = 0.15 + 0.05 = 0.2\\
\textit{and } v^T_P(b) &= v_P(b) - t = 0.4 - 0.2 = 0.2
\end{aligned}$$

5. Extension picture fuzzy ideal of PS algebra

In the current section, the notion of extension PFI is introduced generalizing the notion of extension of FI.

Definition 5.1. *Let A be the set of universe and* $P = (\mu_P, \eta_P, v_P)$ *be a PFI in A. Now, a PFS* $Q = (\mu_Q, \eta_Q, v_Q)$ *in A is said to be an extension PFI for P if*

(i) *Q is a PFI of A*

(ii) $\mu_Q(a) \geqslant \mu_P(a)$, $\eta_Q(a) \geqslant \eta_P(a)$ *and* $v_Q(a) \leqslant v_P(a)$ *for all* $a \in A$.

Proposition 5.1. *Let* $(A, \diamond, 0)$ *be a PS algebra and* $P = (\mu_P, \eta_P, v_P)$ *be a PFI. Then* $P^T_{r,s,t}$ *is an extension PFI for P.*

Proof: It is known from Proposition 4.1 that $P^T_{r,s,t}$ is a PFI of A.

$$\begin{aligned} \text{Now, } \mu^T_P(a) &= \mu_P(a) + r \geqslant \mu_P(a) \\ \eta^T_P(a) &= \mu_P(a) + s \geqslant \eta_P(a) \\ \text{and } v^T_P(a) &= v_P(a) - t \leqslant v_P(a) \text{ for all } a \in A. \end{aligned}$$

Consequently, $P^T_{r,s,t}$ is an extension PFI for P.

Proposition 5.2. *Let $(A, \diamond, 0)$ be a PS algebra and $P = (\mu_P, \eta_P, v_P)$ be a PFI of A. Also, let $Q = (\mu_P, \eta_P, v_P)$, $R = (\mu_R, \eta_R, v_R)$ be two extension PFIs for P. Then $Q \cap R$ is an extension PFI for P.*

Proof: Let $Q \cap R = S = (\mu_S, \eta_S, v_S)$, where $\mu_S(a) = \mu_Q(a) \wedge \mu_R(a)$, $\eta_S(a) = \eta_Q(a) \wedge \eta_R(a)$ and $v_S(a) = v_Q(a) \vee v_R(a)$ for all $a \in A$.

$$\begin{aligned} \text{Now, } \mu_S(0) &= \mu_Q(0) \wedge \mu_R(0) \\ &\geqslant \mu_Q(a) \wedge \mu_R(a) \text{ [as } Q, R \text{ are PFIs of } A] \\ &= \mu_S(a), \end{aligned}$$

$$\begin{aligned} \eta_S(0) &= \eta_Q(0) \wedge \eta_R(0) \\ &\geqslant \eta_Q(a) \wedge \eta_R(a) \text{ [as } Q, R \text{ are PFIs of } A] \\ &= \eta_S(a) \end{aligned}$$

$$\begin{aligned} \text{and } v_S(0) &= v_Q(0) \vee v_R(0) \\ &\leqslant v_Q(a) \vee v_R(a) \text{ [as } Q, R \text{ are PFIs of } A] \\ &= v_S(a) \text{for all } a, b \in A \end{aligned}$$

$$\begin{aligned} \text{Also, } \mu_S(a) &= \mu_Q(a) \wedge \mu_R(a) \\ &\geqslant (\mu_Q(b \diamond a) \wedge \mu_Q(b)) \wedge (\mu_R(b \diamond a) \wedge \mu_R(b)) \text{ [as } Q, R \text{ are PFIs of } A] \\ &= (\mu_Q(b \diamond a) \wedge \mu_R(b \diamond a)) \wedge (\mu_Q(b) \wedge \mu_R(b)) \\ &= \mu_S(b \diamond a) \wedge \mu_S(a), \end{aligned}$$

$$\begin{aligned} \eta_S(a) &= \eta_Q(a) \wedge \eta_R(a) \\ &\geqslant (\eta_Q(b \diamond a) \wedge \eta_Q(b)) \wedge (\eta_R(b \diamond a) \wedge \eta_R(b)) \text{ [as } Q, R \text{ are PFIs of } A] \\ &= (\eta_Q(b \diamond a) \wedge \eta_R(b \diamond a)) \wedge (\eta_Q(b) \wedge \eta_R(b)) \\ &= \eta_S(b \diamond a) \wedge \eta_S(b) \end{aligned}$$

$$\begin{aligned} \text{and } v_S(a) &= v_Q(a) \vee v_R(a) \\ &\leqslant (v_Q(b \diamond a) \vee v_Q(b)) \vee (v_R(b \diamond a) \vee v_R(b)) \text{ [as } Q, R \text{ are PFIs of } A] \\ &= (v_Q(b \diamond a) \vee v_R(b \diamond a)) \vee (v_Q(b) \vee v_R(b)) \\ &= v_S(b \diamond a) \vee v_S(b) \text{ for all } a, b \in A. \end{aligned}$$

Thus, it is obtained that $\mu_S(a) \geqslant \mu_S(b \diamond a) \wedge \mu_S(b)$, $\eta_S(a) \geqslant \eta_S(b \diamond a) \wedge \eta_S(b)$ and $v_S(a) \leqslant v_S(b \diamond a) \vee v_S(b)$ for all $a, b \in A$.

Consequently, $S = Q \cap R$ is a PFI of A.

$$\begin{aligned} \text{Now } \mu_S(a) &= \mu_Q(a) \wedge \mu_R(a) \\ &\geqslant \mu_P(a) \wedge \mu_P(a) \text{ [because } Q, R \text{ are extension PFIs for } P] \\ &= \mu_P(a), \end{aligned}$$

$$\begin{aligned} \eta_S(a) &= \eta_Q(a) \wedge \eta_R(a) \\ &\geqslant \eta_P(a) \wedge \eta_P(a) \text{ [because } Q, R \text{ are extension PFIs for } P] \\ &= \eta_P(a) \end{aligned}$$

$$\begin{aligned} \text{and } v_S(a) &= v_Q(a) \vee v_R(a) \\ &\leqslant v_P(a) \vee v_P(a) \text{ [because } Q, R \text{ are extension PFIs for } P] \\ &= v_P(a) \text{ for all } a \in A. \end{aligned}$$

Thus, $S = Q \cap R$ is an extension PFI for P.

6. Conclusion

In this article, the notions of PFI of PS algebra, scalar multiplication with PFS, translation of PFS and extension PFI of PS algebra have been introduced. Here, it has been observed that scalar multiplication of a PFI over a PS algebra is a PFI. Also, it has been shown that if scalar multiplication of a PFS over a PS algebra is a PFI then PFS will be a PFI. We have proved that the translation of a PFI over a PS algebra is a PFI. We have also shown that if the translation of a PFS over a PS algebra is a PFI then PFS will be a PFI. Here, the translation of a PFI has been presented here with a suitable example. We have proved that translation of a PFI is an extension PFI and intersection of two extension PFIs is an extension PFI.

References

[1] Ahmad, B., (1991), Fuzzy BCI-algebras, Journal of Fuzzy Mathematics, 1(2), pp. 445-452.

[2] Akram, M., (2005), On fuzzy d-algebras, Punjab University Journal of Mathematics, 37, pp. 61-76.

[3] Atanassov, K. T., (1965), Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, pp. 87-96.

[4] Cuong, B. C., (2014), Picture fuzzy sets, Journal of Computer Science and Cybernetics, 30, pp. 409-420.

[5] Cuong, B. C. and Hai, P. V. (2015), Some fuzzy logic operators for picture fuzzy sets, Seventh International Conference on Knowledge and Systems Engineering.

[6] Cuong, B. C. and Kreinovich, V., (2013), Picture fuzzy sets - a new concept for computational intelligence problems, Proceedings of the Third World Congress on Information and Communication Technologies WIICT.

[7] Dogra, S. and Pal, M., (2020), Picture fuzzy subring of a crisp ring, Proc. Natl. Acad. Sci., India, Sect. A Phys. Sci., https://doi.org/10.1007/s40010-020-00668-y.

[8] Dogra, S. and Pal, M., (2020), Picture fuzzy matrix and its application, Soft Computing, 24, pp. 9413-9428, https://doi.org/10.1007/s00500-020-05021-4.

[9] Dogra, S. and Pal, M., (2020), m-polar picture fuzzy ideal of a BCK algebra, International Journal of Computational Intelligence Systems, 13(1), pp. 409-420.

[10] Dogra, S. and Pal, M., (2023), Picture fuzzy subspace of a crisp vector space, Kragujevac Journal of Mathematics, 47(4), pp. 577-597.

[11] Dutta, P. and Ganju, S., (2020), Some aspects of picture fuzzy set, Transactions of A. Razmandze Mathematical Institute, 172, pp. 164-175.

[12] Iseki, K. and Tanaka, S., (1978), An introduction to the theory of BCK-algebras, Math Japonica, 23, pp. 1-26.

[13] Iseki, K., (1980), On BCI-algebras, Math. Seminar Notes, 8, pp. 125-130.

[14] Jana, C. and Pal, M., (2019), Assessment of enterprise performance based on picture fuzzy Hamacher aggregation operators, Symmetry, 11(75), pp. 1-15.
[15] Jana, C., Senapati, T., Pal, M. and Saeid, A. B., (2020), Different types of cubic ideals in BCI algebra based on fuzzy points, Afrika Matematika, 31, pp. 367-381.
[16] Jana, C., Senapati, T., Pal, M. and Yager, R. R., (2019), Picture fuzzy Dombi aggregation operators: Application to MADM process, Appl. Soft Comput., 74, pp. 99-109.
[17] Jana, C., Senapati, T., Shum , K.P. and Pal, M., (2019), Bipolar fuzzy soft subalgebras and ideals of BCK/BCI-algebra based on bipolar fuzzy points, Journal of Intelligent and Fuzzy Systems, 37(2), pp. 2785-2795.
[18] Jun, Y. B. and Roh E. H., (1994), Fuzzy commutative ideals of BCK algebra, Fuzzy Sets and Systems, 64, pp. 401-405.
[19] Jun, Y. B. and Kim, K. H., (2000), Intuitionistic fuzzy ideals of BCK-algebras, International Journal of Mathematics and Mathematical Science, 24, pp. 839-849.
[20] Neggers, J. and Kim, H. S., (1999), On d-algebras, Mathematica Slovaca, 49, pp. 19-26.
[21] Priya, T. and Ramachandran, T., (2014), Classification of PS algebras, International Journal of Innovative Science, Engineering and Technology, 1(7), pp. 193-199.
[22] Priya T. and Ramachandran, T., (2014), A note on fuzzy PS-ideals in PS-algebra and its level-subsets, International Journal of Advanced Mathematical Sciences, 2(2), pp. 101-106.
[23] Saeid, A. B., (2012), Fuzzy dot BCK/BCI-algebras, International Journal of Algebra, 4(7), pp. 341-352.
[24] Senapati, T., (2015), Translations of intuitionistic fuzzy B-algebras, Fuzzy Information and Engineering, 7(4), 389-404.
[25] Senapati, T., Bhowmik, M. and Pal, M., (2013), Atanassov's intuitionistic fuzzy translations of intuitionistic fuzzy H-ideals in BCK/BCI-algebras, Notes on Intuitionistic Fuzzy Sets, 19(1), pp. 32-47.
[26] Senapati, T., Bhowmik, M., Pal, M. and Davvaz, B., (2015), Fuzzy translations of fuzzy H-ideals in BCK/BCI-Algebras, Indonesian Mathematical Society, 21(1), pp. 45-58.
[27] Senapati, T., Bhowmik, M., Pal, M., Davvaz, B., (2015), Atanassov's intuitionistic fuzzy translations of intuitionistic fuzzy subalgebras and ideals in BCK/BCI-algebras, Eurasian Mathematical Journal, 6(1), pp. 96-114.
[28] Xi, O. G, (1991), Fuzzy BCK-algebras, Mathematica Japonica, 36, pp. 935-942.
[29] Zadeh, L.A., (1965), Fuzzy sets, Information and Control, 45, pp. 338-353.

Shovan Dogra received his BSc. in mathematics in 2013 from Kharagpur College and MSc. in 2015 from Vidyasagar University, West Bengal, India. Currently he is doing research at the same university. His research interest includes Advanced Fuzzy Algebraic Structures. Moreover, he is currently serving as a lecturer of Mathematics and HOD of Department of Science and Humanities at Medinipur Sadar Govt. Polytechnic.

Madhumangal Pal for the photography and short autobiography, see TWMS J. App. and Eng. Math. V.9, N.3.