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Modulation of Electron-Acoustic Waves in a Plasma with Vortex Electron Distribution

Abstract: In the present work, employing a one-dimensional model of a plasma composed of a cold electron fluid, hot electrons obeying a trapped/vortex-like distribution and stationary ions, we study the amplitude modulation of electron-acoustic waves by use of the conventional reductive perturbation method. Employing the field equations with fractional power type of nonlinearity, we obtained the nonlinear Schrödinger equation as the evolution equation of the same order of nonlinearity. Seeking a harmonic wave solution with progressive wave amplitude to the evolution equation it is found that the NLS equation with fractional power assumes envelope type of solitary waves.

Keywords: nonlinear Schrödinger equation, electron-acoustic waves, solitary waves

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1 Introduction

The concept of electro-acoustic mode had been conceived by Fried and Gould [1] during the numerical solutions of the linear electrostatic dispersion equation in an unmagnetized, homogeneous plasma. Besides the well-known Langmuir and ion-acoustic waves, they noticed the existence of a heavily damped acoustic-like solution of dispersion equation. It was later shown that in the presence of two distinct groups (cold and hot) of electrons and immobile ions, one indeed obtains a weakly damped electron-acoustic mode (Watanabe and Taniuti [2]), the properties of which significantly differ from those of the Langmuir waves.

To study the properties of electron-acoustic solitary wave structure Dubouloz et al. [3] considered a one-dimensional, unmagnetized collisionless plasma consisting of cold electrons, Maxwellian hot electrons and stationary ions. However, in practice, the hot electrons may not follow a Maxwellian distribution, due to the formation of phase space holes caused by the trapping of hot electrons in a wave potential. Accordingly, in most space plasma the hot electrons follow the trapped/vortex-like distribution (Schamel [4, 5], Abbasi et al. [6]). Therefore, in the present work, we shall consider a plasma model consisting of a cold electron fluid, hot electrons obeying a non-isothermal (trapped/vortex-like) distribution, and stationary ions.

The propagation of small-but-finite amplitude waves in a one-dimensional ion-acoustic model had been studied by several researchers (see, for instance, Washimi and Taniuti [7]) and one-dimensional electron-acoustic model by Schamel [4, 5], Mamun and Shukla [8] by use of the classical reductive perturbation method (Taniuti [9]) and Demiray [10] by use of the modified PLK (Poincaré-Lighthill-Kuo) method, wherein the contribution of higher order terms is also investigated.

Due to its central importance to the theory of quantum mechanics, the nonlinear equations of Schrödinger type are of great interest. They arise in many nonlinear problems such as water waves [11–15], waves in plasmas [16–20], nonlinear waves in fluid-filled elastic or viscoelastic tubes [21–23] and other nonlinear waves of similar nature. In all these works the nonlinear equations with integer power had been taken into consideration. However, when the nonlinearity is of the type of fractional order of certain field quantities much more careful analysis of the problem has to be made.

In the present work, employing the one-dimensional model of a plasma composed of a cold electron fluid, hot electrons obeying a trapped/vortex-like distribution and stationary ions, we study the amplitude modulation of electron-acoustic waves through the use of the reductive perturbation method. Due to physics of this problem, the field equations involve nonlinearity of fractional power (3/2) of the electrostatic potential and the resulting evolution equation is found to be the nonlinear Schrödinger (NLS) equation of the same fractional order of nonlinearity. Seeking harmonic wave solution with progressing

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amplitude to the evolution equation it is found that the NLS equation with fractional power assumes envelope type of solitary waves.

2 Governing equations

We consider a one-dimensional, collisionless plasma consisting of a cold electron fluid, hot electrons obeying a trapped/vortex-like distribution and stationary ions. The dynamics of electron-acoustic waves is governed by the following equations:

$$\frac{\partial n_c}{\partial t}+\frac{\partial}{\partial x}(u_c n_c)=0, \tag{1}$$

$$\frac{\partial u_c}{\partial t}+u_c\frac{\partial u_c}{\partial x}-\alpha\frac{\partial \phi}{\partial x}=0, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}=\frac{1}{\alpha}n_c+n_h-(1+1\alpha), \tag{3}$$

where n_c is the normalized cold electron number density, n_h is the normalized hot electron number density, u_c is the cold electron fluid velocity, ϕ is the electrostatic potential and the coefficient α is defined by $\alpha=n_{h0}/n_{c0}$, where n_{c0} and n_{h0} are the equilibrium values of the cold and hot electron number densities, respectively. The hot electron number density n_h (for $\beta<0$) can be expressed by (Schamel [4])

$$n_h=I(\phi)+\frac{2}{\sqrt{-\pi\beta}}W_D(\sqrt{-\beta\phi}), \tag{4}$$

with the definitions

$$\begin{aligned} I(x)&=[1-erf(\sqrt{x})]\exp(x),\\ W_D(x)&=\exp(-x^2)\int_0^x \exp(y^2)dy, \end{aligned} \tag{5}$$

where $erf(x)$ is the error function. For $\phi\ll 1$, eq. (4) gives

$$\begin{aligned} n_h=1+\phi-\frac{4}{3\sqrt{\pi}}(1-\beta)\phi^{3/2}+\frac{\phi^2}{2}\\ -\frac{8}{15\sqrt{\pi}}(1-\beta^2)\phi^{5/2}+\frac{\phi^3}{6}+\cdots \end{aligned} \tag{6}$$

Here we note that n_h depends on the fractional power of the electrostatic potential. Denoting the fluctuation of the cold electron number density from its equilibrium value by n, i.e., $n_c=1+n$, eqs (1–3) can be written as

$$\frac{\partial n}{\partial t}+\frac{\partial u_c}{\partial x}+\frac{\partial}{\partial x}(u_c n)=0, \tag{7}$$

$$\frac{\partial u_c}{\partial t}+u_c\frac{\partial u_c}{\partial x}-\alpha\frac{\partial \phi}{\partial x}=0, \tag{8}$$

$$\begin{aligned} \frac{\partial^2 \phi}{\partial x^2}=\frac{1}{\alpha}n+\phi-\frac{4}{3\sqrt{\pi}}(1-\beta)\phi^{3/2}+\frac{\phi^2}{2}\\ -\frac{8}{15\sqrt{\pi}}(1-\beta^2)\phi^{5/2}+\frac{\phi^3}{6}+\cdots \end{aligned} \tag{9}$$

These equations will be used as we study the amplitude modulation of nonlinear waves propagating in such a plasma.

3 Modulation of nonlinear waves

In this section we shall study the amplitude modulation of nonlinear waves propagating in such a plasma medium. For this purpose we introduce the following slow variables:

$$\xi=\epsilon(x-\lambda t)\quad \tau=\epsilon^2 t, \tag{10}$$

where ϵ is a small parameter characterizing the bandwidth of superposed waves and λ is an unknown constant to be determined from the solution. The field variables are assumed to be functions of the fast variables (x,t) as well as the slow variables(ξ,τ). Then the following differential relations hold true:

$$\frac{\partial}{\partial t}\to\frac{\partial}{\partial t}-\epsilon\lambda\frac{\partial}{\partial \xi}+\epsilon^2\frac{\partial}{\partial \tau},\quad \frac{\partial}{\partial x}\to\frac{\partial}{\partial x}+\epsilon\frac{\partial}{\partial \xi}. \tag{11}$$

Introducing eq. (11) into the field equations (7–9) the following equations are obtained:

$$\begin{aligned} &\frac{\partial n}{\partial t}-\epsilon\lambda\frac{\partial n}{\partial \xi}+\epsilon^2\frac{\partial n}{\partial \tau}+\frac{\partial u_c}{\partial x}+\epsilon\frac{\partial u_c}{\partial \xi}\\ &+\frac{\partial}{\partial x}(nu_c)+\epsilon\frac{\partial}{\partial \xi}(nu_c)=0, \end{aligned}$$

$$\begin{aligned} &\frac{\partial u_c}{\partial t}-\epsilon\lambda\frac{\partial u_c}{\partial \xi}+\epsilon^2\frac{\partial u_c}{\partial \tau}-\alpha\frac{\partial \phi}{\partial x}-\epsilon\alpha\frac{\partial \phi}{\partial \xi}\\ &+u_c\frac{\partial u_c}{\partial x}+\epsilon u_c\frac{\partial u_c}{\partial \xi}=0, \end{aligned}$$

$$\frac{\partial^2 \phi}{\partial x^2}+2\epsilon\frac{\partial^2 \phi}{\partial \xi\partial x}+\epsilon^2\frac{\partial^2 \phi}{\partial \xi^2}=\frac{n}{\alpha}+\phi-\frac{4}{3\sqrt{\pi}}(\phi)^{3/2}+\cdots \tag{12}$$

For our future purposes it is convenient to assume that the field quantities can be expanded into a power series of ϵ in the following form:

$$n=\epsilon^4(n^{(1)}+\epsilon n^{(2)}+\epsilon^2 n^{(3)}+\cdots),$$

$$u_c = \epsilon^4 (u_c^{(1)} + \epsilon u_c^{(2)} + \epsilon^2 u_c^{(3)} + \cdots),$$

$$\phi = \epsilon^4 (\phi^{(1)} + \epsilon \phi^{(2)} + \epsilon^2 \phi^{(3)} + \cdots), \tag{13}$$

where $n^{(1)}, \ldots\ldots, \phi^{(3)}$ are functions of the fast (x, t) as well as the slow (ξ, τ) variables.

Introducing the expansion (13) into the field equations (12) and setting the coefficients of like powers of ϵ equal to zero, the following sets of differential equations are obtained:

O(ϵ^4) equations:

$$\frac{\partial n^{(1)}}{\partial t} + \frac{\partial u_c^{(1)}}{\partial x} = 0, \quad \frac{\partial u_c^{(1)}}{\partial t} - \alpha \frac{\partial \phi^{(1)}}{\partial x} = 0,$$
$$\frac{\partial^2 \phi^{(1)}}{\partial x^2} - \frac{n^{(1)}}{\alpha} - \phi^{(1)} = 0. \tag{14}$$

O(ϵ^5) equations:

$$\frac{\partial n^{(2)}}{\partial t} + \frac{\partial u_c^{(2)}}{\partial x} - \lambda \frac{\partial n^{(1)}}{\partial \xi} + \frac{\partial u_c^{(1)}}{\partial \xi} = 0,$$

$$\frac{\partial u_c^{(2)}}{\partial t} - \alpha \frac{\partial \phi^{(2)}}{\partial x} - \lambda \frac{\partial u_c^{(1)}}{\partial \xi} - \alpha \frac{\partial \phi^{(1)}}{\partial \xi} = 0,$$

$$\frac{\partial^2 \phi^{(2)}}{\partial x^2} + 2 \frac{\partial^2 \phi^{(1)}}{\partial \xi \partial x} - \frac{n^{(2)}}{\alpha} - \phi^{(2)} = 0. \tag{15}$$

O(ϵ^6) equations:

$$\frac{\partial n^{(3)}}{\partial t} + \frac{\partial u_c^{(3)}}{\partial x} - \lambda \frac{\partial n^{(2)}}{\partial \xi} + \frac{\partial u_c^{(2)}}{\partial \xi} + \frac{\partial n^{(1)}}{\partial \tau} = 0,$$

$$\frac{\partial u_c^{(3)}}{\partial t} - \alpha \frac{\partial \phi^{(3)}}{\partial x} - \lambda \frac{\partial u_c^{(2)}}{\partial \xi} - \alpha \frac{\partial \phi^{(2)}}{\partial \xi} + \frac{\partial u_c^{(1)}}{\partial \tau} = 0,$$

$$\frac{\partial^2 \phi^{(3)}}{\partial x^2} + 2 \frac{\partial^2 \phi^{(2)}}{\partial \xi \partial x} + \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} - \frac{n^{(3)}}{\alpha} - \phi^{(3)}$$
$$+ \frac{4}{3\sqrt{\pi}} (1 - \beta)(\phi^{(1)})^{3/2} = 0. \tag{16}$$

3.1 Solution of the field equations

For the solution of eq. (14), we shall seek a solution of the form

$$(n^{(1)}, u_c^{(1)}, \phi^{(1)}) = (N^{(1)}, U_c^{(1)}, \varphi_1) \exp(i\theta) + c.c.,$$
$$\theta = \omega t - kx, \tag{17}$$

where ω is the angular frequency, k is the wave number, $N^{(1)}$, $U_c^{(1)}, \varphi_1$ are some complex functions of the slow variables (ξ, τ) and c.c. stands for the complex conjugate of the corresponding quantities. Introducing eq. (17) into the field equations (14) we obtain

$$U_c^{(1)} = -\alpha \frac{k}{\omega} \varphi_1, \quad N^{(1)} = -\alpha \frac{k^2}{\omega^2} \varphi_1, \tag{18}$$

provided that the following dispersion relation is satisfied:

$$\omega^2 = \frac{k^2}{k^2 + 1}, \tag{19}$$

where $\varphi_1(\xi, \tau)$ is an unknown complex function whose governing equation will be obtained later.

Introducing eqs (17) and (18) into the field equations (15) we have

$$\frac{\partial n^{(2)}}{\partial t} + \frac{\partial u_c^{(2)}}{\partial x} + \alpha \frac{k}{\omega} \left(\lambda \frac{k}{\omega} - 1 \right) \frac{\partial \varphi_1}{\partial \xi} \exp(i\theta) + \text{c.c.} = 0,$$

$$\frac{\partial u_c^{(2)}}{\partial t} - \alpha \frac{\partial \phi^{(2)}}{\partial x} + \alpha \left(\lambda \frac{k}{\omega} - 1 \right) \frac{\partial \varphi_1}{\partial \xi} \exp(i\theta) + \text{c.c.} = 0,$$

$$\frac{\partial^2 \phi^{(2)}}{\partial x^2} - 2ik \frac{\partial \varphi_1}{\partial \xi} \exp(i\theta) - \frac{n^{(2)}}{\alpha} - \phi^{(2)} + \text{c.c.} = 0. \tag{20}$$

The form of eq. (20) suggests us to seek a solution to the field variables $n^{(2)}$, $u_c^{(2)}$, $\phi^{(2)}$ of the following form:

$$(n^{(2)}, u_c^{(2)}, \phi^{(2)}) = (N^{(2)}, U_c^{(2)}, \varphi_2) \exp(i\theta) + \text{c.c.}, \tag{21}$$

where $N^{(2)}, U_c^{(2)}, \varphi_2$ are complex functions of the slow variables (ξ, τ).

Introducing eq. (21) into the field equations (20) we obtain the following set of differential equations:

$$i(\omega N^{(2)} - k U_c^{(2)}) + \alpha \frac{k}{\omega} \left(\lambda \frac{k}{\omega} - 1 \right) \frac{\partial \varphi_2}{\partial \xi} = 0,$$

$$i(\omega U_c^{(2)} + \alpha k \varphi_2) + \alpha \left(\lambda \frac{k}{\omega} - 1 \right) \frac{\partial \varphi_2}{\partial \xi} = 0,$$

$$-(k^2 + 1)\varphi_2 - \frac{N^{(2)}}{\alpha} - 2ik \frac{\partial \varphi_1}{\partial \xi} = 0. \tag{22}$$

From the solution of the set (22) one obtains

$$U_c^{(2)} = -\alpha \frac{k}{\omega} \varphi_2 + i \frac{\alpha}{\omega} \left(\lambda \frac{k}{\omega} - 1 \right) \frac{\partial \varphi_1}{\partial \xi},$$

$$N^{(2)} = -\alpha \frac{k^2}{\omega^2} \varphi_2 + 2i\alpha \frac{k}{\omega^2} \left(\lambda \frac{k}{\omega} - 1 \right) \frac{\partial \varphi_1}{\partial \xi}, \tag{23}$$

$$-2ik \left(\lambda \frac{k}{\omega^3} - \frac{1}{k^2} \right) \frac{\partial \varphi_1}{\partial \xi} = 0.$$

Here, in obtaining the last equation in eq. (23) we have utilized the dispersion relation (19). In order to have a non-zero solution for φ_1 the coefficient of $\partial\varphi_1/\partial\xi$ must vanish, i.e.,

$$\lambda\frac{k}{\omega^2}-\frac{1}{k^2}=0,\quad \text{or}\quad \lambda=\frac{\omega^3}{k^3}. \tag{24}$$

Here λ is the group velocity.

To obtain the solution for $O(\epsilon^6)$ equations, we introduce the solutions (18) and (23) into eqs (16) we obtain

$$\begin{aligned}&\frac{\partial n^{(3)}}{\partial t}+\frac{\partial u_c^{(3)}}{\partial x}+\Big[\alpha\frac{k}{\omega}\Big(\lambda\frac{k}{\omega}-1\Big)\frac{\partial\varphi_2}{\partial\xi}\\&+i\frac{\alpha}{\omega}\Big(\lambda\frac{k}{\omega}-1\Big)\Big(1-2\lambda\frac{k}{\omega}\Big)\frac{\partial^2\varphi_1}{\partial\xi^2}-\alpha\frac{k^2}{\omega^2}\frac{\partial\varphi_1}{\partial\tau}\Big]\exp(i\theta)\\&+\text{c.c.}=0,\end{aligned} \tag{25}$$

$$\begin{aligned}&\frac{\partial u_c^{(3)}}{\partial t}-\alpha\frac{\partial\phi^{(3)}}{\partial x}+\Big[\alpha\Big(\lambda\frac{k}{\omega}-1\Big)\frac{\partial\varphi_2}{\partial\xi}-i\frac{\alpha}{\omega}\lambda\Big(\lambda\frac{k}{\omega}-1\Big)\frac{\partial^2\varphi_1}{\partial\xi^2}\\&-\alpha\frac{k}{\omega}\frac{\partial\varphi_1}{\partial\tau}\Big]\exp(i\theta)+\text{c.c.}=0,\end{aligned} \tag{26}$$

$$\begin{aligned}&\frac{\partial^2\phi^{(3)}}{\partial x^2}-\frac{n^{(3)}}{\alpha}-\phi^{(3)}+\frac{4}{3\sqrt{\pi}}(1-\beta)(\phi^{(1)})^{3/2}\\&+\Big[-2ik\frac{\partial\varphi_2}{\partial\xi}+\frac{\partial^2\varphi_1}{\partial\xi^2}\Big]\exp(i\theta)+\text{c.c.}=0.\end{aligned} \tag{27}$$

For our future purposes we need only the equations related to the coefficients of *exp*$(i\theta)$ terms. In order to obtain such an equation we have to examine the term $(\phi^{(1)})^{3/2}$, which can be written as

$$\begin{aligned}(\phi^{(1)})^{3/2}&=|\varphi_1|^{3/2}|e^{i(s+\theta)}+e^{-i(s+\theta)}|^{3/2}\\&=2\sqrt{2}|\varphi_1|^{3/2}|\cos^{3/2}(s+\theta)|,\end{aligned} \tag{28}$$

where $|\varphi_1|$ is the modulus and s is the argument of the complex variable φ_1.

In order to ensure that $(\phi^{(1)})^{3/2}$ remains real we must restrict the variation of $(s+\theta)$ as $|(s+\theta)|\le\pi/2$. Since $cos(s+\theta)$ is a periodic function in $(s+\theta)$ we can expand the expression of $(\phi^{(1)})^{3/2}$ into a Fourier cosine series of the following form:

$$2\sqrt{2}|\varphi_1|^{3/2}|\cos(s+\theta)|^{3/2}=\sum_{n=0}^{\infty}a_n\cos n(s+\theta), \tag{29}$$

where the coefficient a_n is defined by

$$a_n=\frac{4\sqrt{2}}{\pi}|\varphi_1|^{3/2}\int_0^{\pi/2}\cos^{3/2}(s+\theta)\cos n(s+\theta)d(s+\theta). \tag{30}$$

For our future purposes we need only the coefficient a_1, which can be expressed as

$$a_1=\frac{4\sqrt{2}}{\pi}|\varphi_1|^{3/2}\int_0^{\pi/2}\cos^{3/2}(s+\theta)\cos(s+\theta)d(s+\theta). \tag{31}$$

Noting the integral relation

$$\int_0^{\pi/2}(\cos x)^{5/2}dx=0.718884$$

the expression of a_1 becomes

$$a_1\approx 1.3|\varphi_1|^{3/2}. \tag{32}$$

If we write the Fourier cosine series in complex notation we have

$$\sum_{n=0}^{\infty}a_n\cos n(s+\theta)=\sum_{n=0}^{\infty}[c_ne^{i(s+\theta)}+c_ne^{-i(s+\theta)}]$$

we obtain $c_1=a_1/2=0.65|\varphi_1|^{3/2}$. Returning to eqs. (25–27), the equations related to the coefficients of $\exp(i\theta)$ terms we can write

$$\begin{aligned}&i\omega N^{(31)}-ikU_c^{(31)}+\alpha\frac{k}{\omega}\Big(\lambda\frac{k}{\omega}-1\Big)\frac{\partial\varphi_2}{\partial\xi}\\&+i\alpha\Big(-2\lambda^2\frac{k^2}{\omega^3}+3\lambda\frac{k}{\omega^2}-\frac{1}{\omega}\Big)\frac{\partial^2\varphi_1}{\partial\xi^2}-\alpha\frac{k^2}{\omega^2}\frac{\partial\varphi_1}{\partial\tau}=0,\end{aligned} \tag{33}$$

$$\begin{aligned}&i\omega U^{(31)}+i\alpha k\phi^{(31)}+\alpha\Big(\lambda\frac{k}{\omega}-1\Big)\frac{\partial\varphi_2}{\partial\xi}\\&+i\alpha\Big(-\lambda^2\frac{k}{\omega^2}+\frac{\lambda}{\omega}\Big)\frac{\partial^2\varphi_1}{\partial\xi^2}-\alpha\frac{k}{\omega}\frac{\partial\varphi_1}{\partial\tau}=0,\end{aligned} \tag{34}$$

$$\begin{aligned}&-(k^2+1)\phi^{(31)}-\frac{N^{(31)}}{\alpha}+0.65\frac{4}{3\sqrt{\pi}}(1-\beta)|\varphi_1|^{1/2}\varphi_1\\&-2ik\frac{\partial\varphi_2}{\partial\xi}+\frac{\partial^2\varphi_1}{\partial\xi^2}=0,\end{aligned} \tag{35}$$

Here we sought a solution for $N^{(3)}, U_c^{(3)}$ and $\phi^{(3)}$ of the following form:

$$(N^{(3)},U_c^{(3)},\phi^{(3)})=\sum_{n=0}^{\infty}(N^{(3n)},U_c^{(3n)},\phi^{(3n)}\exp(in\theta)+\text{c.c.}. \tag{36}$$

Eliminating $N^{(31)}, U^{(31)}, \phi^{(31)}$ and φ_2 between these equations through the use of dispersion relation and the definition of group velocity λ, we obtain the following NLS equation:

$$i\frac{\partial\varphi_1}{\partial\tau}+\mu_1\frac{\partial^2\varphi_1}{\partial\xi^2}+\mu_2|\varphi_1|^{1/2}\varphi_1=0, \tag{37}$$

where the coefficients μ_1 and μ_2 are defined by

$$\mu_1 = \frac{3\lambda k}{2(k^2+1)}, \quad \mu_2 = \frac{1.3\lambda k}{3\sqrt{\pi}}(1-\beta). \tag{38}$$

3.2 Progressive wave solution

In this sub-section we shall give a progressive wave solution to the evolution equation (37). As is well known, the form of the progressive wave solution of the NLS equation depends on the sign of the product of the coefficients μ_1 and μ_2. As is seen from eq. (38) this product is positive for all wave numbers. Thus, we shall seek a progressive wave solution to the evolution equation of the form

$$\varphi_1 = f(\zeta)\exp[i(\Omega\tau - K\xi)], \quad \zeta = c(\xi + 2\mu_1 K\tau), \tag{39}$$

where $f(\zeta)$ is a real function of its argument c, Ω and K are some constants. Introducing eq. (39) into eq. (37) one has

$$\mu_1 c^2 f'' - (\Omega + \mu_1 K^2)f + \mu_2 f^{3/2} = 0. \tag{40}$$

Here, a prime denotes the differentiation of the corresponding quantity with respect to ζ. Since the coefficients μ_1 and μ_2 satisfy the inequality $\mu_1\mu_2 > 0$, the solution for $f(\zeta)$ may be given by

$$f(\zeta) = a \ \mathrm{sech}^4\zeta, \tag{41}$$

where a is the constant amplitude of the solitary wave and other quantities are defined by

$$c = \left(\frac{\mu_2}{20\mu_1}\right)^{1/2} a^{1/4}, \quad \Omega = \frac{4\mu_2}{5}a^{1/2} - \mu_1 K^2. \tag{42}$$

This result shows that the NLS equation with (3/2)th order nonlinearity assumes the envelope solitary wave solution as given in eq. (41). One should also note that the frequency of the harmonic wave is proportional to the square root of the solitary wave amplitude.

4 Conclusions

In the present work, employing a one-dimensional model of a plasma composed of a cold electron fluid, hot electrons obeying a trapped/vortex-like distribution and stationary ions, we studied the amplitude modulation of electron-acoustic waves. Due to the nature of the physics of the problem the nonlinearity of the field equations is of fractional order (3/2), which causes serious difficulties in studying modulation problems. To surmount this difficulty, we expanded this nonlinear term of fractional order into Fourier cosine series of the phase function and obtained the NLS equation of the same fractional order of nonlinearity as the evolution equation. Seeking a harmonic wave with progressive amplitude to the evolution equation it is found that the NLS equation with fractional order of nonlinearity assumes an envelope type of solitary wave solution.

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