DESIGN AND OPTIMIZATION OF MULTIUSER COOPERATIVE PROTOCOLS FOR WIRELESS NETWORKS

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Abstract

Recently, the main trend in wireless communications has shifted from voice transmission to data communication. Naturally, this shift caused an increase in the data rate and bandwidth requirements. Being a limited and thus expensive resource, wireless spectrum needs to be used efficiently. For higher spectral and data rate efficiency, new spectrum-allocation policies as well as new transmission techniques are needed. Cooperative and cognitive radios are promising emerging technologies, which can enable efficient spectrum resource utilization as well as high data rate transmission in the next generation wireless networks.

This thesis addresses wireless relay networks consisting of multiple cooperative and/or cognitive nodes. The main contribution of this thesis is the extension of cooperative strategies from two users to three user settings, focusing on mutual cooperation. In the first part of this dissertation, we concentrate on a superposition block Markov encoding based three user cooperative communication scheme for a fading Gaussian multiple access channel. We consider all possible channel conditions between users and propose a channel adaptive block Markov encoding strategy. In the second part of this thesis, for the same three user MAC model we discuss a new channel non-adaptive superposition block Markov encoding structure which enables all three users to cooperate collectively as well as in pairs. The proposed cooperation models not only provide increased diversity to all participating users, but also contain as special cases the multiple relay channel and the multiple access relay channel. In the third part, we focus on the joint use of cognition and cooperation. In particular, we consider overlay and underlay cooperative-cognitive radio models with one primary and two secondary users. We extend the cooperative encoding models developed for the three user MAC to the cognitive set-up. In all three problems studied in this thesis, we obtain new and improved achievable rate regions, by cognition and/or cooperation.

KABLOSUZ AĞLAR İÇİN ÇOK KULLANICILI İŞBİRLİKLİ PROTOKOL TASARIMI VE ENİYİLENMESİ

Özet

Son zamanlarda, kablosuz haberleşme alanındaki başlıca trend, ses iletişimden veri iletişimine doğru kaymıştır. Doğal olarak bu kayma veri hızı ve band genişliğin de artan bir ihtiyaca neden olmuştur. Mevcut kaynakların limitli ve pahalı olması sebebiyle de, kablosuz spektrumun efektif bir şekilde kullanılması gerekmektedir. Daha yüksek spektrum ve veri hızı verimliliği için, yeni iletişim tekniklerine ihtiyaç olduğu gibi yeni spektrum paylaşım politikalarına da ihtiyaç bulunmaktadır.

Bu tez, çoklu işbirlikli ve/veya bilişsel birimleri kapsayacak şekilde kablosuz röle ağlara yönelik olarak hazırlanmıştır. Bu tezin en önemli katkısı, işbirlikli stratejilerin iki kullanıcıdan üç kullanıcının ortak işbirliğine dayalı olarak geliştirilmesidir. Bu tezde ilk kısımda, birçok yeni işbirlikli Gauss çoklu erişim kanalı iki kullanıcılı üste bindirmeli blok Markov kodlamanın önemli bir şekilde geliştirilmesi ile üç kullanıcı için işbirliği stratejileri önerilmiştir. Bu tezin ilk kısmında, sönümlenmeli Gauss çoklu erişim kanalı için üç kullancılı işbirlikli haberleşme modeline bağlı olarak üste bindirmeli blok Markov kodlama teknikleri geliştirilmiştir. Kullanıcılar arası tüm kanal durumlarını göz önünde bulundurulmuş ve kanal adaptif bir blok Markov kodlama stratejisi önerilmiştir. Tezin ikinci kısmında ise, aynı üç kullanıcılı ÇEK modeli için, tüm kullanıcıların eşzamanlı olarak ikili ve üçlü işbirliğine adanmış farklı mesajlar üzerinden işbirliği yapabildikleri yeni kanal adaptif olmayan bir blok Markov kodlama yapısı önerilmiştir. Önerilen işbirlikli modeller sadece eş zamanlı çok kullanıcılı işbirliğine imkan tanımakla kalmayıp, aynı zamanda çoklu röle ve çoklu erişim röle kanalını da özel birer durum olarak içermektedir. Üçüncü kısımda bilişsellik ve işbirliğinin ortak kullanımına odaklanılmıştır. Özellikle, alta ve üste serim işbirlikli-bilişsel radyo modelleri, bir birincil kullanıcı ve iki ikincil kullanıcı ile ele alınmıştır. Üç kullanıcılı ÇEK için geliştirilen işbirlikli kodlama modelleri bilişsel kuruluma doğru genişletilmiştir. Bu tezde çalışılmış olan her bir problemde, bilişsel ve/veya işbirlikli yapının kullanılmasıyla yeni ve geliştirilmiş ulaşılabilir veri alanları elde edilmiştir.

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To my family...

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List of Abbreviations

AF	**A**mplify and **F**orward
BC	**B**roadcast **C**hannel
BME	**B**lock **M**arkov **E**ncoding
BD	**B**ackward **D**ecoding
CCR	**C**ausal **C**ognitive **R**adio
CF	**C**ompress and **F**orward
CR	**C**ognitive **R**adio
DF	**D**ecode and **F**orward
DMC	**D**iscrete **M**emoryless **C**hannel
DM-MAC	**D**iscrete **M**emoryless **M**ultiple **A**ccess **C**hannel
EF	**E**stimate **F**orward
i.i.d.	**I**ndependent and **I**dentically **D**istributed
MAC	**M**ultiple **A**ccess **C**hannel
MAC-GF	**M**ulti **A**ccess **C**hannel with **G**eneralized **F**eedback
MARC	**M**ultiple **A**ccess **R**elay **C**hannel
MIMO	**M**ultiple **I**nput **M**ultiple **O**utput
PDF	**P**robability **D**sensity **F**unction
PU	**P**rimary **U**ser
SNR	**S**ignal to **N**oise **R**atio
SU	**S**econdary **U**ser

Chapter 1

Introduction and Structure of the Thesis

1.1 Introduction

With the increasing user demand, next generation wireless networks aim to achieve higher data rates and more reliable communication. However, due to multipath fading, severe shadowing and pathloss, wireless communication systems face some fundamental capacity limits. In order to push these limits further, several diversity techniques, such as frequency diversity, time diversity, spatial diversity and recently cooperation diversity have been proposed.

Frequency diversity technique can be used to combat frequency selective fading effects. Meanwhile, time diversity can be an effective way to combat time selective fading if error control and interleaving methods are employed together. The spatial diversity technique can combat both frequency selective fading and time selective fading via multiple antennas. In other words, the main benefit of spatial diversity is that it provides a way to prevent effects of multi path fading. Although spatial diversity is clearly advantageous on a cellular base station, it may not be practical for other scenarios (e.g. mobile phones). Specifically, due to size, cost, or hardware limitations, a wireless device may not be able to support multiple transmit antennas. Therefore, alternative means of achieving spatial diversity are needed. One such alternative is to employ user cooperation to develop diversity gain in the absence of more than one antenna at the same mobile device. User cooperation makes efficient use of the available resources, in that it exploits what comes for free in wireless networks, i.e, side information, to create additional diversity in transmissions, by forming a virtual antenna array for transmissions. This thesis invariably focuses on this type of diversity, i.e., user cooperation diversity.

Roots of user cooperation date back to the introduction of the relay channel. The communication from a single source to a single destination with the help of any other communicating terminal is called relay communication. Basic information theoretic treatment of the relay channel dates back to 1970s and has been carried out in many ways by various researchers. The three-terminal relay channel (set-up in Figure 1.1) which is basis of cooperative communication was introduced by van der Meulen [1], [2]. For the general single relay channel, several capacity upper and lower bounds were obtained in [3]. In the case of the degraded relay channel in which the communication channel between the source and the relay is physically better than the source-destination link, the capacity can be obtained [3] but the capacity of the setup in Figure 1.1, where the destination is able to hear both source and relay remains unsolved in general case. In terms of relaying strategy, the relay may simply forward the signal received from the source terminal (amplify-and-forward (AF)) or retransmit the estimates of the received symbols obtained by detection (decode-and-forward (DF)) or quantize the signal received from the source (compress-and-forward (CF)). Usually, in cooperative communication systems, two basic relaying modes are used: AF and DF. These two transmission schemes were discussed by Laneman and Wornell in [4]. In AF scheme, the relays amplify the received signal subject to a power constraint and retransmit it to the destination. In DF scheme, the relay performs hard decisions before retransmission. It decodes the received source message, re-encodes it, and forwards the resulting signal to the destination. Note that, since the relay must perfectly decode the source message, the achievable rates are accordingly bounded by the capacity of the channel between the source and relay. In [5] different types of AF relay settings are studied and general expressions for the aggregate SNR at the destination are derived for a varying number of relaying nodes. The approach in [5] is motivated by the previous observations that AF relays can sometimes

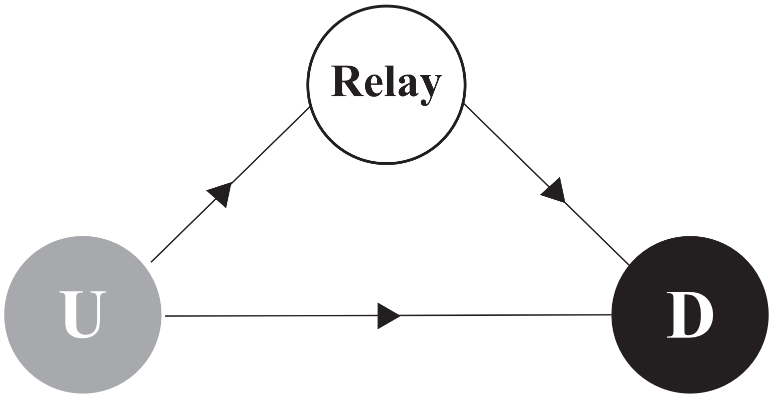


Figure 1.1: Three-terminal relay channel

approach or exceed the performance of their DF counter parts [6]. QF (quantize-and-forward (QF)) relay implementation is considered by Katz and Shamai in [7] and shown to be superior, in terms of average throughput, with respect to the DF and AF relays, in the presence of a direct link, and a fixed known channel gain on the relay destination which models a two co-located user cooperation.

The efforts in obtaining achievable rates for multi-terminal cooperative communication are certainly not limited to two transmitter scenarios. The three-terminal relay channel can be extended to models consisting of two or more transmitters that intend to transmit independent messages to a decoder in the presence of one relay, namely multiple access relay channel (MARC). The MARC was introduced by Kramer et al. in [8] and has been extensively studied from a channel coding perspective. Several encoding schemes with achievable rate regions have been presented for the MARC in references [9], [10] and [11]. In [9], Kramer et al. derived an achievable rate region for the MARC with independent messages. The coding scheme employed in [9] is based on DF relaying, and uses regular encoding, successive decoding at the relay, and backward decoding at the destination. In [11], it was shown that, in contrast to the three-terminal relay channel, in a MARC, different DF schemes yield different rate regions. In particular, backward decoding can support a larger rate region than sliding window decoding. Outer bounds on the capacity region of MARCs were obtained in [10]. More recently, capacity regions for two classes of MARCs were characterized in [12].

In contrast to MARC, in multiple relay communication (MRC), a group of relays help the communication between one source and one destination. The results of Cover and El-Gamal [3] were generalized to networks with multiple relays by El-Gamal a few years later in [13]. In subsequent studies, the deterministic relay networks with no interference, and deterministic broadcast relay networks and their rate region were obtained by Tse [14], Cover [15] and Lifang [16], respectively. Further studies in recent works on the capacity of multiple relay communication appeared in [6], [10], [17], [18], [19], [20], [21], [22], [23], [24]. As discussed above, extension of cooperative strategies from two to multiple users in MARC and MRC have been widely investigated, but the main focus has been on strategies relying on dedicated relaying rather than mutual cooperation perhaps due to the difficulty of generalizing the encoding strategies, and more importantly, of characterizing the seemingly much more complicated achievable rate regions.

The model where two transmitters transmit to a common destination and these transmitters also receive a common feedback from the destination was introduced by King in [25] . In [26], Carleial generalized this model to include different feedback to the two transmitters. It is easy to see that the relay channel is a special case of Carleial's model in [26]. Remarkably, as discussed by Kramer et al. in [9], Carleial introduced a coding scheme that is different from, and in some respects preferable to the superposition block-Markov encoding introduced by Cover and El-Gamal, in [3].

Recently, a new class of relaying methods named, multi-user cooperative communication, have been proposed. These methods enable single antenna mobiles in a multi-user environment to share their antennas and generate a virtual multiple-antenna transmitter that allows them to achieve transmitting diversity without bandwidth expansion. Multi-user cooperative communication is also one of the fastest growing areas of research, and it is enabling for efficient spectrum use in the future. Multi-user cooperation is possible whenever there is at least one additional node willing to aid in communication as illustrated in Figure 1.2. The

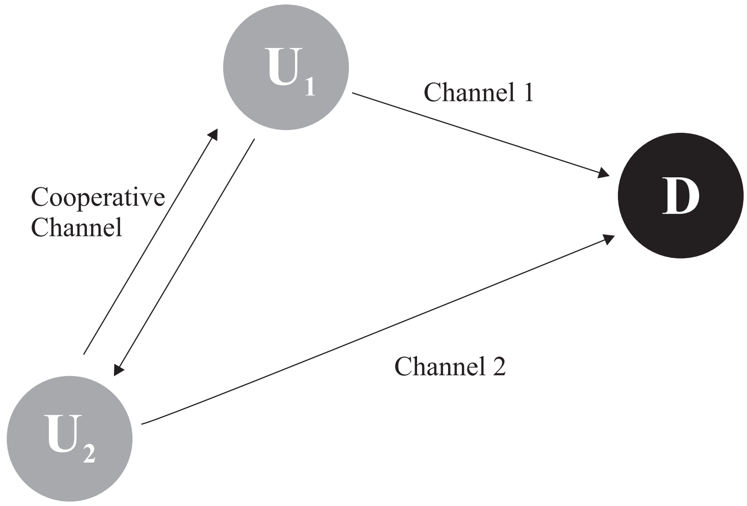


Figure 1.2: Basic scheme for two user's cooperative communication with each user node as both a user and a relay.

relays (users) have their own information to send, so all terminals help one another to communicate by acting as relays for each other. Multi user cooperation roots from toy information theoretic model, called multiple access channel with generalized feedback (MAC-GF). A seminal work on user-cooperation which appeared in the recent years is Sendoranis, et al. [27], [28]. In this work, the authors apply the MAC-GF model to a wireless setup, and propose user cooperation as a form of diversity in a mobile uplink scenario, and show its benefits using various metrics. In this proposed scenario, the partners can overhear each other's transmissions though the wireless medium, process this information and re-transmit to collaborate. This provides extra observations of the source signals at the destinations, the observations which are dispersed in space and usually discarded by current implementations of cellular, wireless LAN or ad-hoc systems. Beside this, there exists several remarkable contributions by Laneman et al. and Janani et al. in [29] [30], which study the performance of important relaying protocols in fading environments.

User cooperation can push the achievable data rates close to their fundamental limits within the allocated spectrum. However, the spectrum allocation itself is still very rigid. One possible way of utilizing the available spectrum more efficiently is to allow secondary user to access the already used spectrum, by sharing the resources of primary users that are already in the network. This idea is called as cognitive radio, and it is based on the principle of providing service to secondary users without adversely affecting the primary user communication quality. The term cognitive radio was first suggested by [31] where cognitive radio is presented as an extension of software defined radio enhancing flexibility of personal wireless services. Haykin [32] and Le et al. [33] provide a good overview of cognitive radio research and the semantics of cognitive radio. Information theoretic approaches are applied in [34], [35] [36] [37] where three approaches to cognitive radio emerge: underlay, overlay and interweave. Generally, in each cognitive radio approaching network, two forms of users exist, primary users and secondary users. Primary users have higher priority than the secondary users in the utilization of the spectrum. Both underlay and overlay approaches allow concurrent primary and secondary user transmissions. Each of them propose a different level of cognition, leading to different challenges in wireless medium [37]. Underlay or interference control model allows concurrent transmission of primary and secondary users in ultra wideband (UWB) fashion where the primary users are protected by enforcing spectral masks on the secondary signals so that the generated interference is below the noise floor for the primary user. However, underlay allows only short-range communication due to the power constraints. Overlay or known interference model also allows concurrent transmission of primary and secondary users. The secondary users use part of their transmission power for relaying the data of primary users and part of the power for their own secondary transmission.

The objective in theoretic approaches is to characterize the achievable rates in a cognitive radio network under various assumptions on how the secondary users interfere with the primary users. The article [37] provides an overview of the different approaches for the primary/secondary data transmission and their impact on the achievable performance. The mathematical models of the overlay approaches can be found in [34] and [36]. Development of cognitive models and resource allocation under these models, remain as open problems and are addressed by this thesis

Motivated by above, in this thesis, we propose several three user cooperation strategies, based on non-trivial extensions of two user block Markov superposition encoding for a cooperative Gaussian multiple access channel (MAC) and also focus cooperative-cognitive radio in underlay/overlay set-ups. We obtain the expressions for the resulting achievable rate regions and maximize these regions as a function of user transmit powers. We demonstrate through simulations that the participation of an extra user in cooperation provides significant rate improvements.

1.2 Structure of the Thesis

This thesis is organized as follows. In Chapter 2, we introduce the related background materials about wireless communication technology and give a brief overview of the main information theoretic results found in literature on the relay channel with and without cooperation. Further, we discuss some capacity bounds that have been derived for two user cooperative channel.

In Chapter 3, we introduce our three user cooperative MAC model, and propose encoding and decoding policies that rely on a non-trivial extension of the well known block Markov superposition coding. We characterize, and evaluate the rate region achievable by our proposed encoding-decoding techniques. We demonstrate that the added diversity due to the presence of an additional user may translate into significant rate gains, especially near the sum rate point. It has to be noted that our propositions and derivations here are only preliminary results on a wide open and relatively untouched problem, and many variations to the encoding policy can be developed.

In Chapter 4, in contrast to proposed system model in Chapter 3, we introduce a new three user block Markov encoding strategy, in which pairwise and collective cooperation is performed based on dedicated sub-messages, and obtain the resulting achievable rate regions. We demonstrate that these regions are larger than achievable rate regions for two user cooperative MAC, two user one relay MARC, and may even be larger than those for known adaptive three user encoding/decoding policies.

In Chapter 5, based on our fundamental studies in Chapter 3 and 4, we propose a cooperative-cognitive system where underlay and overlay cognitive approaches are considered together with one PU and two SUs. In our proposed system models, switching from an overlay mode to an underlay mode enable to guarantee the rate of primary user $R_P \geq B^\star$ and obtain the maximum sum rate. The proposed system model is normally working in overlay mode and thus the maximum sum rate can be obtained while holding the rate of primary user $R_P \geq B^\star$. However, since the rate of primary user does not hold $R_P \geq B^\star$, the proposed system model operates in an underlay mode and secondary users are cooperatively allowed to send their packets to a destination even though the primary user is directly transmitting. In such a case, switching to an underlay mode is beneficial to primary user to guarantee the rate constraint $R_P \geq B^\star$. We obtain and optimized the SU achievable rate regions, while keeping the PU rate at its single user optimum. The overlay protocol, which requires only a slight modification of PU transmissions in the form of power control, provides significant rate improvements, especially when the PU is far from the destination, compared to the SUs. Through mathematical and numerical analysis, we compare the performance of our proposed overlay system to that of proposed underlay scheme.

Most of the works explained in these chapters can be found in reviewed research papers. Chapter 3 appeared in [38],[39], Chapter 4 in [40] and Chapter 5 in [41].

Chapter 2

Background

In this chapter we give a brief overview of information theoretic aspects of single / multi user channel(s). Throughout this section, we state the basic system models for both discrete memoryless and Gaussian multiple-access channels, along with known achievable rate results in different communication and encoding/decoding strategies.

2.1 Fading

The communication channel provides the connection between the transmitter and the receiver. The physical connection can be established in different wayS such as a pair of wires that carry the electrical signal, or an optical fiber that carries the information on a modulated light beam, or an underwater ocean channel in which the information is transmitted acoustically, or free space over which the information-bearing signal is radiated by use of an antenna. The common problem in propagating a signal through any channel is additive noise. In general, additive noise is generated internally by components such as a resistor or solid-state devices. This type of noise is sometimes called the thermal noise. Other types of noise and interference may cause externally, such as interference from other users of channel. Besides noise and interference, other types of the signal degradation are signal attenuation, amplitude and phase distortion, and multipath distortion [42]. These effects are illustrated in Figure 2.1. These are of several basic types

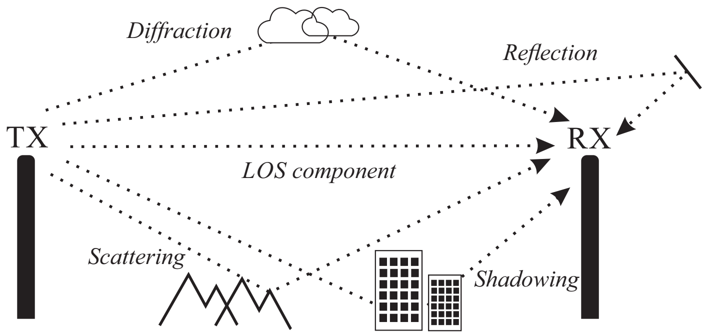


Figure 2.1: Illustration of multipath effects in wireless communication

for propagation loses in wireless channels: one of them is shadow fading. Shadow fading results from the presence of objects (buildings, walls, etc.) between transmitter and receiver. Shadow fading is typically modeled by attenuation in signal amplitude that follows a log-normal distribution. The variation in this fading is specified by the standard deviation of the logarithm of this attenuation. Multipath refers to multiple copies of a transmitted signal are received at the receiver due to the presence of multiple radio paths between the transmitter and receiver. These multiple paths arise due to reflections from objects in the wireless channel. Multi-path from scatterers that are spaced very close together will cause a random change in the amplitude of the received signal. Due to central-limit type effects, the resulting received amplitude is often modeled as being a complex Gaussian random variable. This results in random amplitude whose envelope has a Rayleigh distribution, and termed as Rayleigh fading.

When the scatterers are spaced so that the differences in their corresponding path lengths are significant relative to a wavelength of the carrier, then the signals arriving at the receiver along different paths can add constructively or destructively. This gives rise to fading that depends on the wavelength or equivalently the frequency of radiation, which is thus called frequency-selective fading. When there is relative motion between the transmitter and receiver, this type of fading also depends on time, since the path length is a function of the radio geometry. This results in time-selective fading. (Such motion also causes signal distortion due to Doppler effects.) The time delay of arrival along different paths is significant relative to a symbol interval. This results in dispersion of the transmitted signal, and causes inter-symbol interference (ISI); contributions from multiple symbols arrive at the receiver at the same time.

The statistical characterization of multi-path communication channel should be developed to determine effects of propagation. The starting point of the development of the characteristics, is determining a channel impulse response $h(t;\tau)$, which is characterized as a complex-valued random process in the t variable. If $h(t;\tau)$ is assumed to be wide-sense- stationary, then the autocorrelation function of $h(t;\tau)$ can be defined as

$$R_c(\tau_1, \tau_2; \Delta t) = \frac{1}{2} E[h^*(\tau_1; t) h(\tau_2; t + \Delta t)] \tag{2.1}$$

Generally, the attenuation and phase shift of the channel associated with path delay τ_1 is uncorrelated with the attenuation and phase shift associated with path delay τ_2 in the communication. This case is called as uncorrelated-scattering. From this assumption which are scattering at two different delay is uncorrelated and incorporate it into (2.1) the following equation can be obtained.

$$\frac{1}{2} E[h^*(\tau_1; t) h(\tau_2; t + \Delta t)] = R_c(\tau_1; \Delta t)\delta(\tau_1 - \tau_2)$$

If $\Delta t = 0$, the autocorrelation can be defined as $R_c(\tau; 0) \equiv R_c(\tau)$ and a function of time delay τ. Beside this, $R_c(\tau; \Delta t)$ is called as the multi-path intensity profile or the delay power spectrum of the channel. To characterize the channel, there is need to pass to the frequency domain by taking Fourier transform of $R_c(\tau; t)$. In case of taking Fourier transform of $R_c(\tau; t)$ the time variant transfer function $H(f, t)$ is obtained by using following equation.

$$H(f; t) = \int_{-\infty}^{\infty} h(t; \tau) e^{-j2\pi f \tau} d\tau \tag{2.2}$$

Since $h(t;\tau)$ is complex-valued zero-mean Gaussian random variable, the transfer function $H(f;t)$ has the same statistical property. Under this consideration, the autocorrelation function can be expressed as

$$R_c(f_1, f_2; \Delta t) = \frac{1}{2}E[H^*(f_1; t)H(f_2; t + \Delta t)]$$

Since $H(f; t)$ is Fourier transform of $h(\tau; t)$, it is easy to establish a relationship between $R_c(f_1, f_2; \Delta t)$ and $R_c(\tau_1, \tau_2; \Delta t)$ by using (2.2). Thus,

$$\begin{aligned}
R_c(f_1, f_2; \Delta t) &= \frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[h^*(t; \tau)h(\tau_2; t + \Delta t)]e^{-j2\pi(f_1\tau_1 - f_2\tau_2)}d\tau_1 d\tau_2 \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_c(\tau_1; \Delta t)\delta(\tau_1 - \tau_2)e^{-j2\pi(f_1\tau_1 - f_2\tau_2)}d\tau_1 d\tau_2 \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_c(\tau_1; \Delta t)e^{-j2\pi(f_1\tau_1 - f_2\tau_2)}d\tau_1 d\tau_2 \\
&= \int_{-\infty}^{\infty} R_c(\tau_1; \Delta t)e^{-j2\pi(f_1 - f_2)\tau_1}d\tau_1 \\
&= \int_{-\infty}^{\infty} R_c(\tau_1; \Delta t)e^{-j2\pi(\Delta f)\tau_1}d\tau_1 \\
&\equiv R_c(\Delta f; \Delta t) \qquad (2.3)
\end{aligned}$$

where $\Delta f = f_1 - f_2$. The function $R_c(\Delta f; \Delta t)$ is closely related to frequency difference Δf. For this reason, it is called as the spread-frequency, space time correlation function of the channel. In case of choosing $\Delta t = 0$ in (2.3) $R_c(\Delta f; \Delta t)$ and $R_c(\tau; \Delta t)$ can be determined as $R_c(\Delta f; 0) \equiv R_c(\Delta f)$ and $R_c(\tau; 0) \equiv R_c(\tau)$ with the following relationship.

$$R_c(\Delta f) = \int_{-\infty}^{\infty} R_c(\tau)e^{-j2\pi\Delta f\tau}d\tau \qquad (2.4)$$

where $R_c(\Delta f)$ is a metric of frequency coherence of the channel. From (2.4), the relationship between Δf and τ can be approximately denoted as

$$\Delta f_{coherence} \approx \frac{1}{T_m}$$

where $\Delta f_{coherence}$ is the coherence bandwidth. Depending on the value of $\Delta f_{coherence}$, the channel can be divided into two categories. When $\Delta f_{coherence} < bandwidth$, the channel is called as frequency-selective. In this case, the signal is distorted

	$\Delta f_{coherence} < bandwidth$	$\Delta f_{coherence} > bandwidth$
$\Delta t_{coherence} < symbol\ duration$	Frequency selective fast fading	Frequency non-selective fast fading
$\Delta t_{coherence} > symbol\ duration$	Frequency selective slow fading	Frequency non-selective slow fading

Table 2.1: Classifications of fading channel.

by the channel. In the second case when $\Delta f_{coherence} > bandwidth$, the channel is denoted as frequency-nonselective. In addition to Δf in $R_c(\Delta f; \Delta t)$, Δt is the time variations of the channel which is related to Doppler effect. The value range of Doppler frequency is called as Doppler spread B_d of the channel. And the relationship between B_d and τ can be approximately denoted as

$$\Delta t_{coherence} \approx \frac{1}{B_d}$$

where $\Delta t_{coherence}$ is the coherence time. If the symbol duration is smaller than $\Delta t_{coherence}$, then the channel is classified as slow fading. Slow fading channels are very often modeled as time-invariant channels over a number of symbol intervals. On the other hand, if it is smaller than the symbol duration, the channel is considered to be fast fading. In general, it is difficult to estimate the channel parameters in a fast fading channel. Due to explanations in above, a fading channel can be classified into four different types which is listed in Table 2.1.

2.1.1 Ergodicity

A process is ergodic if the time averages may be used to replace ensemble averages. In practical term, this means that the channel varies sufficiently rapidly over the duration of the transmission. Ergodicity allows one to apply the concept of averages since the channel's average mutual information over all (infinitely long codewords) is the same. In this thesis, the proposed three user cooperative channel is considered under Gaussian fading and can be described by the ergodic capacity.

As detailed in Section 2.1, the channel can be categorized as slow or fast fading channel. The ergodic capacity can be used if the coherence time is much shorter than the code length and the codeword experiences all the fading states (refers to fast fading channel). This type of channel capacity is also defined by Shannon capacity. There are two different consideration for channel gain availability, only available at the decoder and available at both the encoder and decoder. In our proposed studies, we consider coding under channel gain availability at the encoder and decoder.

2.2 Capacity in single user and single destination

One important way of characterizing of a proposed system model is that finding its performance indicators such as mutual information and the characterization of performance limits through system capacity. Due to it, in this section, we represent the channel capacity in a ergodic channel. Considering the information provided by the outcome x of a discrete random variable X is defined as

$$I_x(x) = \log \frac{1}{P[X = x]} = -\log P[X = x] \tag{2.5}$$

where $P[X = x]$ is the probability of the outcome $X = x$. i. The communication process is inherently a process relating more than one random variable (the input and the output of a proposed system). The mutual information in communication system, which for two discrete random variables X and Y is defined as

$$I(X;Y) = \sum_{x\in X}\sum_{y\in Y} P[X = x, Y = y] \log \frac{P[X = x, Y = y]}{P[X = x]P[Y = y]}$$

where $P[X = x, Y = y]$ is the joint probability mass function and $P[X = x]$ and $P[Y = y]$ are marginal probability mass functions. Applying Bayes theorem ($P[X = x, Y = y] = P[X = x|Y = y]P[Y = y]$, with $P[X = x|Y = y]$ being the conditional probability mass function of X given that $Y = y$), the mutual information can also be written as

$$I(X;Y) = \sum_{x \in X} \sum_{y \in Y} P[X = x, Y = y] \log \frac{P[X = x|Y = y]}{P[X = x]}$$

Then, we can write

$$\begin{aligned} I(X;Y) = & -\sum_{x \in X} \log P[X = x] \sum_{y \in Y} P[X = x, Y = y] \\ & + \sum_{x \in X} \sum_{y \in Y} P[X = x, Y = y] \log P[X = x|Y = y] \\ = & -\sum_{x \in X} \log P[X = x] \\ & + \sum_{x \in X} \sum_{y \in Y} P[X = x, Y = y] \log P[X = x|Y = y] \end{aligned} \tag{2.6}$$

The first term in this result is called the entropy of the random variable X

$$H(X) = -\sum_{x \in X} P[X = x] \log P[X = x]$$

and the second term can be written in terms of the conditional entropy of X.

$$H(X|Y) = -\sum_{x \in X} P[X = x, Y = y] \log P[X = x|Y = y]$$

Considering (2.5), we can say that the entropy of the random variable can be determined as the mean value of the information provided by all its outcomes. Likewise, the conditional entropy can be regarded as the mean value of the information provided by all the outcomes of a random variable (X) given than the outcome of a second random variable (Y) is known, or how much uncertainty about a random variable (X) remains after knowing the outcome of a second random variable (Y). Therefore, the mutual information as in (2.6) can now be rewritten as

$$I(X;Y) = H(X) - H(X|Y)$$

The value of $I(X;Y)$ is maximized over the set of probability of input alphabet $P[X = x]$ and is a quantity that depends on the characteristics of the DMC through the conditional probabilities $P[X = x|Y = y]$. This quantity is called the capacity of the channel and denoted by C. So the capacity of the DMC is defined as below.

$$
\begin{aligned}
C = &\max_{P[X=x]} I(X;Y) \\
&\max_{P[X=x]} \sum_{x \in X} \sum_{y \in Y} P[X = x, Y = y] \log \frac{P[X = x|Y = y]}{P[X = x]}
\end{aligned}
$$

under constraints $P[X = x] \geq 0$ and $\sum_{x \in X} P[X = x] = 1$.

The same concepts can be applied to continuous random variables with the only differences that the sums are replaced by integrals and the probability mass functions by probability density functions. Let X be a continuous random variable with probability density function (pdf) $p(x)$. The differential entropy $h(X)$ is a concave function of $p(x)$ and is defined as

$$
\begin{aligned}
h(x) &= -\int P(x) \log P(x) dx \\
&= -E(\log P(X))
\end{aligned}
$$

The capacity of the channel per unit time has been defined as

$$
C = \max_{P(x)} \iint P(x) P(y|x) \log \frac{P(x|y) P(x)}{P(y) P(x)} \tag{2.7}
$$

Consider the point-to-point discrete-time additive white Gaussian noise channel model. The channel output corresponding to the input X is $Y = \alpha X + N$ where α is the channel gain, or path loss, and N is the Gaussian noise with zero mean $N_0/2$ variance. Also assume that an average transmission power constraint is $\sum x^2 \leq \bar{P}$. The capacity of the Gaussian channel is a simple function of the received SNR.

$$C = \frac{1}{2}\log\left(1 + \frac{|\alpha|^2 P}{N_0}\right) \tag{2.8}$$

where $E[*]$ is the expectation operator (operating on the random channel attenuation) and $|\alpha|^2$ is the envelope of the channel attenuation. This capacity formula for the discrete Gaussian channel under average power constraint investigated by Shannon (1948). The discrete-time Gaussian channel is the model for continuous-time band-limited Gaussian channel with bandwidth. Consider a band-limited waveform channel with additive white Gaussian noise. The capacity has is defined by

$$C = \lim_{T\to\infty} \max_{P(x)} \frac{1}{T} I(X;Y)$$

where the average mutual information given in (2.7). In (2.7) X and Y are random variables with joint PDF $p(x,y)$ and marginal PDFs $p(x)$ and $p(y)$. The maximum value of $I(X;Y)$ over marginal PDFs $p(x)$ of the input x is characterized by zero-mean Gaussian random variable as

$$p(x) = \frac{1}{\sqrt{2\pi\sigma_x}} e^{-(y_i - x_i)/2\sigma_x^2}$$

Then, by using (2.7) the mutual information can be expressed as

$$\max_{P(x)} I(X;Y) = WT \log\left(1 + \frac{2\sigma_x^2}{N_0}\right)$$

where channel noise variance is $N_0/2$. In case of putting power constraint on the average transmission power of x

$$\begin{aligned} \bar{P} &= \frac{1}{T}\int_0^T E[x^2(t)]dt \\ &= \frac{1}{T} E[x^2(t)] \\ &= \frac{\sigma_x^2}{T} \end{aligned}$$

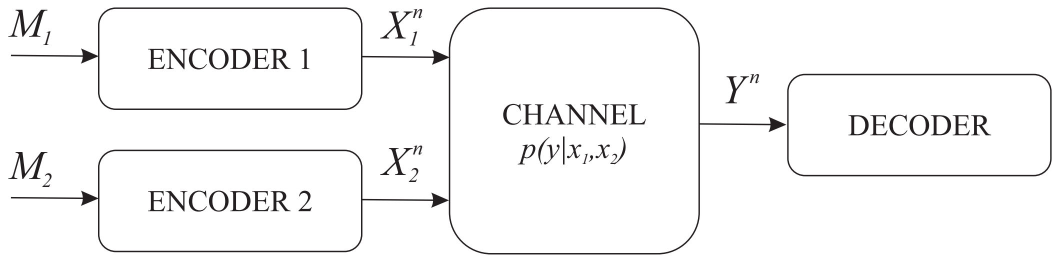


Figure 2.2: Multiple access channel with two user - one destination

After some mathematical simplification, the variance of x can be rewritten $\sigma_x^2 = \bar{P}/2W$. By substitution of variance of x

$$\max_{P(x)} I(X;Y) = WT \log\left(1 + \frac{\bar{P}}{WN_0}\right)$$

the channel capacity of per unit can be obtained by dividing T.

$$C = W \log\left(1 + \frac{\bar{P}}{WN_0}\right)$$

2.3 Capacity in multi-user communication

Beside single user channel capacity explained in Section 2.2, the multi user channel achievable rate region is related to multiple user techniques such as multiple access channel, broadcast channel, relay channel, multiple access relay channel and multi user cooperative channel which is used to share information in multiple channel.

2.3.1 Achievable rate region of two user DM-MAC

First, consider the discrete memoryless multiple access channel (DM-MAC) system model $\mathcal{X}_1 \times \mathcal{X}_2, p(y|x_1x_2)$ depicted in Figure 2.2. Each sender wishes to communicate an independent message $\mathcal{X}_1$ and $\mathcal{X}_2$ reliably to a common destination. Theorem 2.1 gives us an achievable rate region for two user DM-MAC.

Theorem 2.1. The achievable rate region of the two user DM-MAC, [43]

An achievable rate region for the system given in Section 2.3.1 is the closure of the convex hull of all rate pairs (R_1, R_2)

$$R_1 \leq I(X_1; Y|X_2)$$
$$R_2 \leq I(X_2; Y|X_1)$$
$$R_2 \leq I(X_1, X_2; Y|X_1)$$

Before giving the proof of Theorem 2.1, it is needed to introduce some definitions.

Definition 2.2. A sequence of channels $\{\mathcal{W}_n : \mathcal{X}^n \rightarrow \mathcal{Y}^n\}_{n=1}^{\infty}$ is called a discrete memory less channel (DMC) with transition probability matrix P_w if $P_{w_n}(y|x) = \prod_{i=1}^{n} P_w(y_i|x_i)$.

Definition 2.3. An $(n; M)$ code for a DMC $\{\mathcal{W} : \mathcal{X} \rightarrow \mathcal{Y}\}$ consists of an encoding function $f : \{1, 2, ..., M\} \rightarrow \mathcal{X}^n$ and a decoding function $\gamma : \mathcal{Y}^n \rightarrow \{1, 2, ..., M\}$. The sequence $f(i) \in \mathcal{X}^n$ is called a codeword and the set $\{f(i) : i = 1, ..., M\}$ is called the codebook.

Definition 2.4. A rate $R < C = \max_{p(x)} I(X, Y)$ is said to be achievable if there exits a sequence of 2^{nR} codes with $P_e^{(n)} \rightarrow 0$ as $n \rightarrow \infty$.

Proof. As a proof of two user DM-MAC in Theorem 2.1, the following steps determine random codebook generation.
-Fix a distribution $p(x_1)p(x_2)$.
-Randomly and independently generate $[2^{n(R_1}]$ length codewords from distribution $p(w_1^n)$ intended for User 1. Assign each codeword to a distinct message $\{x_1^n(w)$ and call as X_1.
-Randomly and independently generate $[2^{n(R_2}]$ length codewords from distribution $p(w_2^n)$ intended for User 2. Assign each codeword to a distinct message $\{x_2^n(w)$ and call as X_2.

These generated sequences constitute the random codebook C. The decoder uses joint typicality decoding to find an estimate for each message separately. □

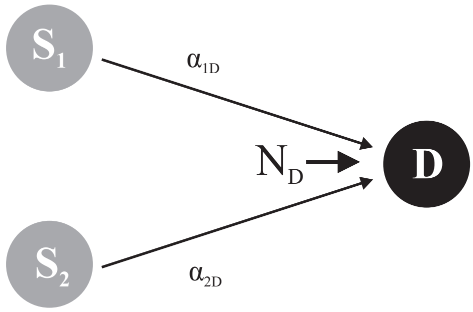


Figure 2.3: Gaussian multiple access channel with two user - one destination

2.3.2 Achievable rate region of two user Gaussian MAC

Now, consider the two user discrete-time additive white Gaussian noise multiple access channel illustrated in Figure 2.3 The channel output Y corresponding to the inputs X_1 and X_2 is

$$Y = \alpha_{1D}X_1 + \alpha_{2D}X_2 + N_D$$

where N_D is the Gaussian noise $N_D \sim \mathcal{N}(0, N_0/2)$, α_{1D} and α_{2D} are channel gains.

Theorem 2.5. The achievable rate region of the two user Gaussian MAC, [44]

An achievable rate region for the system given in Section 2.3 is the closure of the convex hull of all rate pairs (R_1, R_2)

$$R_1 \leq \frac{1}{2}\log\left(1 + \frac{\alpha_{1D}^2 X_1}{N_D}\right)$$
$$R_2 \leq \frac{1}{2}\log\left(1 + \frac{\alpha_{2D}^2 X_2}{N_D}\right)$$
$$R_1 + R_2 \leq \frac{1}{2}\log\left(1 + \frac{\alpha_{1D}^2 X_1 + \alpha_{2D}^2 X_2}{N_D}\right)$$

Proof. As a proof of two user Gaussian MAC in Theorem 2.5, the mutual information for User 1 and User 2 is

$$
\begin{aligned}
I(X_1;Y|X_2) &= h(Y|X_2) - h(Y|X_1,X_2) \\
&= h(\alpha_{1D}^2 X_1 + \alpha_{2D}^2 X_2 + N_D|X_2) - h(\alpha_{1D}^2 X_1 + \alpha_{2D}^2 X_2 + N_D|X_1,X_2) \\
&= h(\alpha_{1D}^2 X_1 + N_D) - h(N_D) \\
&= \frac{1}{2}\log\left(1+\alpha_{1D}^2 X_1\right) - \frac{1}{2}\log\left(N_D\right) \\
&= \frac{1}{2}\log\left(1+\frac{\alpha_{1D}^2 X_1}{N_D}\right)
\end{aligned}
$$

$$
\begin{aligned}
I(X_2;Y|X_1) &= h(Y|X_1) - h(Y|X_1,X_2) \\
&= h(\alpha_{1D}^2 X_1 + \alpha_{2D}^2 X_2 + N_D|X_1) - h(\alpha_{1D}^2 X_1 + \alpha_{2D}^2 X_2 + N_D|X_1,X_2) \\
&= h(\alpha_{2D}^2 X_2 + N_D) - h(N_D) \\
&= \frac{1}{2}\log\left(1+\alpha_{2D}^2 X_2\right) - \frac{1}{2}\log\left(N_D\right) \\
&= \frac{1}{2}\log\left(1+\frac{\alpha_{2D}^2 X_2}{N_D}\right)
\end{aligned}
$$

$$
\begin{aligned}
I(X_1,X_2;Y) &= h(Y) - h(Y|X_1,X_2) \\
&= h(\alpha_{1D}^2 X_1 + \alpha_{2D}^2 X_2 + N_D) - h(\alpha_{1D}^2 X_1 + \alpha_{2D}^2 X_2 + N_D|X_1,X_2) \\
&= h(\alpha_{1D}^2 X_1 + \alpha_{2D}^2 X_2 + N_D) - h(N_D) \\
&= \frac{1}{2}\log\left(1+\alpha_{1D}^2 X_1 + \alpha_{2D}^2 X_2\right) - \frac{1}{2}\log\left(N_D\right) \\
&= \frac{1}{2}\log\left(1+\frac{\alpha_{1D}^2 X_1 + \alpha_{2D}^2 X_2}{N_D}\right)
\end{aligned}
$$

The rest of achievability can be proved by following Theorem 2.1. □

2.3.3 Achievable rate region of two user Gaussian BC

Consider the one user and two destinations discrete-time additive white Gaussian noise broadcast channel (Gaussian BC) illustrated in Figure 2.4 The channel

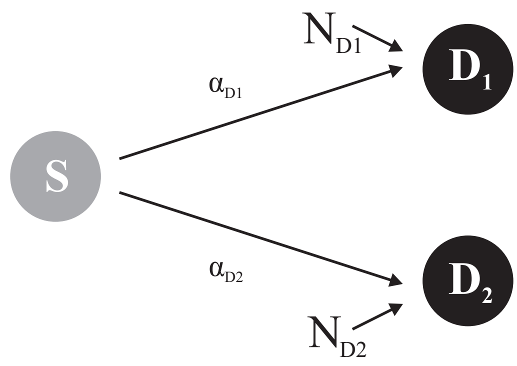


Figure 2.4: Gaussian broadcast channel with one user two destinations

output Y_1 and Y_2 corresponding to the inputs X is

$$Y_1 = \alpha_{D_1} X + N_{D_1}$$
$$Y_2 = \alpha_{D_2} X + N_{D_2}$$

where N_{D_1} and N_{D_2} is the Gaussian noises $N_{D_1} \sim \mathcal{N}(0, N_0/2)$, $N_{D_2} \sim \mathcal{N}(0, N_0/2)$. α_{D_1} and α_{D_2} are channel gains.

Theorem 2.6. The achievable rate region of the two user Gaussian BC [45]

An achievable rate region for the system given in Section 2.3.3 is the closure of the convex hull of all rate pairs (R_1, R_2)

$$R_1 \leq \frac{1}{2}\log\left(1 + \beta S_1\right)$$
$$R_2 \leq \frac{1}{2}\log\left(1 + \frac{(1-\beta)S_1}{\beta\ S_2 + 1}\right)$$

where $\beta \in [0, 1]$.

Proof. As a proof of two user Gaussian BC in Theorem 2.6, the broadcast channel in Figure 2.4 can be determined as a stochastically degraded channel and its achievable rate region can be similarly proved as that of the physically degraded Gaussian BC.

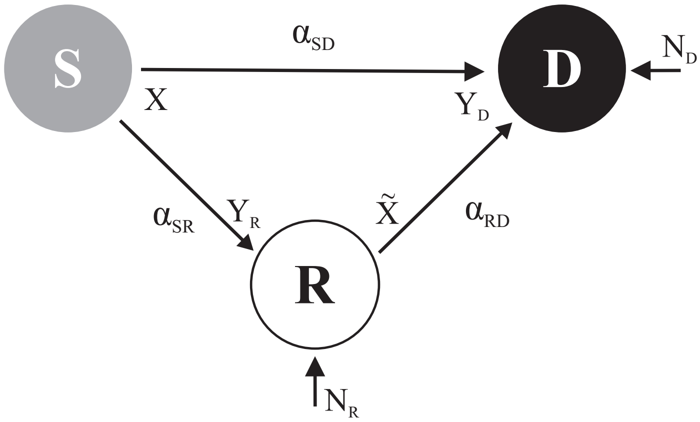


Figure 2.5: Detailed scheme for one user one relay communication

The channel output Y_1 and Y_2 can be rewritten as

$$Y_1 = X + N_{D_1}$$
$$Y_2 = Y_1 + \tilde{N}_{D_2}$$

where $\tilde{N}_{D_2}$ independent Gaussian random variable $\tilde{N}_{D_2} \sim \mathcal{N}(0, N_{D_2} - N_{D_1})$. Let define the received SNR for each destination as $S_1 = P/N_1$ and $S_2 = P/N_2$. Assuming the source has two independent codewords U and V for $D1$ and $D2$, respectively, the mutual information at the $D1$

$$I(X; Y_1|U) = \frac{1}{2}\log\left(1 + \frac{\beta P}{N_1}\right)$$
$$I(U; Y_2) = \frac{1}{2}\log\left(1 + \frac{(1-\beta)P}{\beta P + N_2}\right)$$

□

2.4 Relaying Strategies

The relaying methods can roughly be classified into two groups, these are transparent and regenerative relaying protocols. Using the transparent relaying, the relay does not modify the information represented by a chosen waveform. Very simple operations are usually performed, such as simple amplification, phase rotation, etc. Since no digital operations are performed on the signal, the analog signal is received in one frequency band, amplified and retransmitted on another frequency band. The most prominent example belonging to the transparent relaying family is:

Amplify and Forward (AF): Constituting one of the simplest and most popular relaying methods, the signal received by the relay is amplified, frequency translated and retransmitted. One important property of amplify and forward relays is the amplification of the receiver noise at the relay . In a fixed AF relaying protocol, which is often simply called an AF protocol, the relay scales the received version and transmits an amplified version of it to the destination.

Considering regenerative relaying protocols, information (bits) or waveform (samples) is modified. This requires digital baseband operations and thus more powerful hardware. Hence, regenerative relays usually outperform than transparent ones. The most prominent examples of regenerative relaying are:

Decode and Forward (DF): Being the prominent counter protocol to the transparent AF protocol, DF detects the received signal, re-encode it, and then retransmit it to the receiver. This kind of relaying is termed as a fixed decode-and-forward (DF) scheme, which is often simply called a DF scheme without the confusion from the selective DF relaying scheme. If the decoded signal at the relay is denoted by X, the transmitted signal from the relay can be denoted by $\sqrt{P}\tilde{X}$ given that X has unit variance. Note that the decoded signal at the relay may be incorrect. If an incorrect signal is forwarded to the destination, the decoding at the destination is meaningless. It is clear that for such a scheme the diversity achieved is only one, because the performance of the system is limited by the worst link from the source-relay and source-destination. Although fixed DF relaying has the advantage over AF relaying in reducing the effects of additive noise at the relay, it entails the possibility of forwarding erroneously detected signals to the destination, causing error propagation that can diminish the performance of the system. The mutual information between the source and the destination is limited by the mutual information of the weakest link between the source-relay and the combined channel from the source-destination and relay-destination.

Compress and Forward (CF): Besides the two most common techniques for fixed relaying, there are other techniques without requirement of decoding at the relay such as compress cooperation (CF) and coded cooperation. The main difference between CF and DF/AF is that while in the later the relay transmits a copy of the received message, in compress and forward the relay transmits a quantized and compressed version of the received message. Therefore, the destination node will perform the reception functions by combining the received message from the source node and its quantized and compressed version from the relay node. This protocol is similar to the estimate-and-forward (EF) protocol in that it relays a compressed version of the detected information stream to the destination. This involves some form of source coding on the sampled signal samples and was shown to be capacity/performance optimum for the compressing node being close to the destination.

The most common techniques are the fixed AF relaying protocol and the fixed relaying DF protocol. Throughout this section, we only give the background information for two common relaying strategies namely AF and DF. The relaying channel model for both protocols and the achievable rate region for DF can be found in Section 2.4.1 and Section 2.4.2, respectively.

2.4.1 System Model for AF and DF

The system model is illustrated in Figure 2.5 as the simplest relay network. It consists of 3 nodes, the source S, the relay R and the destination D. The source transmits the signal X and the received signal at the relay Y_R can be found as

$$Y_R = \alpha_{SR}\sqrt{P_{SR}}X + N_R$$

where P_{SR} denotes the signal power transmitted by the source and α_{SR} the channel coefficient between source and relay. N_R is the additive white Gaussian noise at the relay with zero mean and variance $\sigma^2_{N_R}$ As explained above, depending on the signal processing performed by the relay the different relay operation modes can be distinguished. In amplify and forward mode the relay amplifies the received signal and forwards it to the destination. The received signal at the destination can be found as

$$Y_D = \alpha_{SR}\alpha_{RD}\sqrt{\frac{P_{SR}P_{SR}}{P_{SR}+\sigma^2_{N_R}}}X + \tilde{N}_D$$

where P_{RD} denotes the signal power transmitted by the relay and $\sqrt{\frac{1}{P_{SR}+\sigma^2_{N_R}}}$ is a power normalization term used at the relay. α_{RD} denotes the channel coefficient between the relay and the destination. $\tilde{N}_D$ denotes the effective noise contained in the received signal, which consists of the noise at the relay, amplified by $\sqrt{\frac{P_{RD}}{P_{SR}+\sigma^2_{N_R}}}$ and filtered by α_{RD} as well as the additive white Gaussian noise N_D with zero mean and variance $\sigma^2_{N_D}$

$$\tilde{N}_D = \alpha_{RD}N_R\sqrt{\frac{P_{RD}}{P_{SR}+N_0}} + N_D$$

In decode and forward mode the relay decodes the signal received from the source and forwards it to the destination. The received signal at the destination can be found as

$$Y_D = \alpha_{RD}\sqrt{P_{RD}}\tilde{X} + N_D$$

As noticed that the relay does not forward the same signal X to the destination. The relay may change the modulation and coding scheme when forwarding the signal, sometimes referred to as decode and re-encode or it might not be able to decode the signal correctly. As explained above there are two important properties of decode and forward relaying. The flexibility to change the modulation and coding scheme allows the relay network to adapt to different channel qualities on the links to the source and to the destination. Secondly, decoding errors at the relay will also propagate to the destination.

In general the destination may be able to receive both the signal from the source and from the relay. Assuming that the source does not transmit the new information while the relay transmits the received signal at the destination can be found as

$$Y_D = \alpha_{SD}\sqrt{P_{SD}}X + \alpha_{RD}\sqrt{P_{RD}}\tilde{X} + N_D$$

where P_{SD} denotes the average power to transmit the information from the source to destination and α_{SD} denotes the channel coefficient between the source and the destination.

2.4.2 Achievable rate in Decode & Forward

In general the capacity of the relay channel is not known, except for the degraded relay channel, where the noise at the relay is also added to the signal transmitted by the source to the destination. The following upper bounds and achievable rates for fixed channel gains have been derived in [46]. For all the equations, we assume an independent additive white Gaussian noise with unit variance at both the relay and the destination, i.e. $\sigma_r = \sigma_d = 1$.

The upper bound for the full-duplex relay channel can be derived by applying the maximum flow minimum cut theorem to relay networks as suggested by [3]. The relay network in Figure 2.5 has two possible cuts, around the source (broadcast) $(S), (R, D)$ and around the destination (multiple access) $(S, R)(D)$. Clearly the capacity cannot be greater than the maximum flow through either of these cuts. The upper bound for the full-duplex relay channel can be found as [3]

$$C^+ = \max_{0<\beta<1} \min \Big\{ 1/2 \log(1 + (1-\beta)\alpha_{sr}^2 P_{sr} + \alpha_{sd}^2 P_{sd})) \;, \\ 1/2 \log(1 + \alpha_{sd}^2 P_{sd} + \alpha_{rd}^2 P_{rd} + 2\sqrt{\beta \alpha_{sd}^2 \alpha_{rd}^2 P_{sd} P_{rd}}) \Big\}$$

where the first term limits the maximum information transfer from the source to the relay and the destination. The second term limits the maximum information transfer from the source and relay to the destination. The codebook for each block used by the source and the relay is assumed to be Gaussian transmitted with powers P_S and P_R and $E[X_1, X_2] = \sqrt{\beta P_1 P_2}$. Note that for the first block transmitted by the source, no cooperation between source and relay is possible but by assuming that $n \to \infty$ blocks are transmitted the resulting loss can be neglected.

An achievable rate for decode and forward relaying can be found as in [3]

$$
\begin{aligned}
R =& \max(R_1, R_2) \\
R_1 =& \max_{0<\beta<1} \min \left\{ 1/2 \log(1 + (1-\beta)\alpha_{sr}^2 P_{sr} + \alpha_{sd}^2 P_{sd})) \right., \\
& \left. 1/2 \log(1 + \alpha_{sd}^2 P_{sd} + \alpha_{rd}^2 P_{rd} + 2\sqrt{\beta \alpha_{sd}^2 \alpha_{rd}^2 P_{sd} P_{rd}}) \right\} \\
R_2 =& 1/2 \log \left(1 + \alpha_{sd}^2 P_{sd} + \frac{\alpha_{sd}^2 P_{sd}}{1 + (1 + \alpha_{sd}^2 P_{sd} + \alpha_{sr}^2 P_{sr} / \alpha_{rd}^2 P_{rd})} \right)
\end{aligned}
$$

where Block Markov encoding is assumed, i.e. the destination is only able to decode the signal after receiving all the encoded blocks. For a better signal quality of the source signal at the relay compared to the destination the relay will cooperate with the source, i.e. β will be close to one. For a worse quality of the source signal received at the relay than at the destination the relay will only have a limited contribution to the signal transmitted by the source and β will be close to zero.

2.5 Capacity in Multi-user Cooperative Communication

In this section, we discuss the system models and achievable rates of multi-terminal cooperative communication models for MARC and two-user cooperation which are basis of our proposed studies on multi user cooperative protocols.

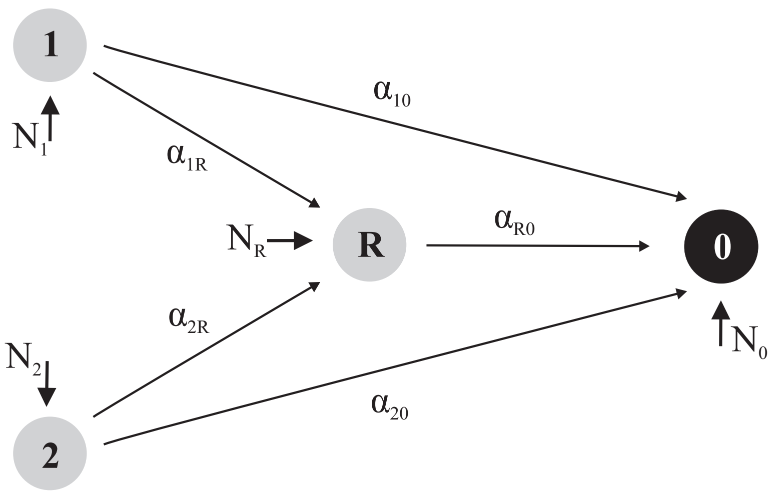


Figure 2.6: Gaussian MARC with two user, one relay and one destination

2.5.1 MARC

We now introduce a multiple access relay channel with two user and one relay as a sub-model of our proposed strategy in Section 4.1. There are two users, a relay and a single destination, denoted 1, 2, R and 0 respectively. Based on MARC scheme illustrated in Figure 2.6, the system is modeled by

$$Y_0 = \sqrt{h_{10}}X_1 + \sqrt{h_{20}}X_2 + \sqrt{h_{r0}}X_R + N_0 \tag{2.9}$$

$$Y_R = \sqrt{h_{1r}}X_1 + \sqrt{h_{2r}}X_2 + N_R \tag{2.10}$$

with $N_0 \sim \mathcal{N}(0, N_0/2)$, $N_R \sim \mathcal{N}(0, N_0/2)$. User 1 and 2 divide their information W_1, W_2 into two part: W_{10}, W_{20} which are sent directly to the destination, and W_{1R}, W_{2R} which are sent to relay. The signals transmitted by users and relay can be generated as follows, satisfying the average power constraints $E[X_1^2] \leq P_1$, $E[X_2^2] \leq P_2$ and $E[X_R^2] \leq P_R$.

$$X_1 = \sqrt{P_{10}}X_{10} + \sqrt{P_{1R}}X_{1R} \tag{2.11}$$

$$X_2 = \sqrt{P_{20}}X_{20} + \sqrt{P_{2R}}X_{2R} \tag{2.12}$$

$$X_R = \sqrt{P_{R_1}}X_{R_1} + \sqrt{P_{R_2}}X_{R_2} \tag{2.13}$$

In this MARC model, only relay and destination have decoding capabilities and should satisfy the traditional MAC constraints to decode all received messages as given Theorem 2.7.

Theorem 2.7. MARC (Two user and one relay) [10, Theorem 1]

An achievable rate region for the system given in Section 2.5.1 is the closure of the convex hull of all rate pairs (R_1, R_2, R_R) *such that* $R_1 = R_{10}+R_{1R}$, $R_2 = R_{20}+R_{2R}$ *and* $R_R = R_{1R} + R_{2R}$ *where* $\{R_{1R}, R_{20}, R_{2R}, R_{1R}, R_{2R}\}$ *satisfy the constraints*

$$R_{1R} \leq E\left[\log\left(1+s_{1r}P_{1R}\right)\right] \tag{2.14}$$

$$R_{2R} \leq E\left[\log\left(1+s_{2r}P_{2R}\right)\right] \tag{2.15}$$

$$R_{1R} + R_{2R} \leq E\left[\log\left(1+s_{1r}P_{1R}+s_{2r}P_{2R}\right)\right] \tag{2.16}$$

$$R_{10} + R_{R_1} \leq E\left[\log\left(1+s_{10}P_1+s_{r0}P_{R_1}+2\sqrt{s_{10}s_{r0}P_{10}P_{R_1}}\right)\right] \tag{2.17}$$

$$R_{20} + R_{R_2} \leq E\left[\log\left(1+s_{20}P_1+s_{r0}P_{R_2}+2\sqrt{s_{20}s_{r0}P_{20}P_{R_2}}\right)\right] \tag{2.18}$$

$$\begin{aligned} R_1 + R_2 + R_R \leq E\Big[\log\Big(&1+s_{10}P_1+s_{20}P_{20}+s_{r0}P_R \\ &+2\sqrt{s_{10}s_{r0}P_{10}P_{R_1}}+2\sqrt{s_{20}s_{r0}P_{20}P_{R_2}}\Big)\Big] \end{aligned} \tag{2.19}$$

2.5.2 Block-Markov Encoder

In this section, we give a brief introduction of the block-Markov encoding (BME), which is used to prove the achievability theorem given in the Section 2.5.3. Since the relay/user's codeword is statistically depend on the message transmitted in the previously block, this coding scheme is called as block Markov coding. As illustrated in Figure 2.7, transmission using block-Markov encoding operates over a number of B blocks. In each block, with the exception of the first or the last block, a new message is sent. The sub-signals in (2.26)-(2.27) are expressed as a function of some not only the current messages $(X_{10}, X_{12}, X_{20}, X_{21})$, but also the messages from the previous block (U_1, U_2). The sub-signals intended for user 1, X_{10}, X_{12} and U_1 are detailed in the following equations, and summarized in

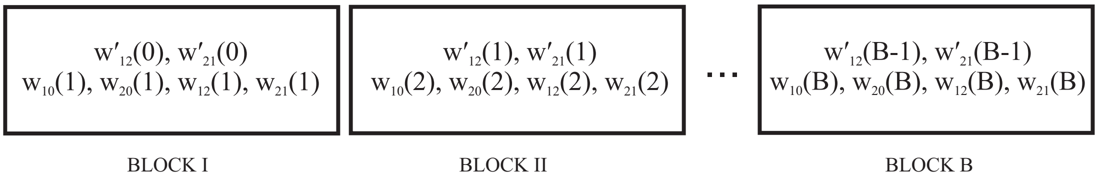


Figure 2.7: Block-Markov encoder for two user cooperative communication

	Block I	Block II
User I	$X_{10}(w_{10}(1), U_1(0))$	$X_{10}(w_{10}(2), U_1(1))$
	$X_{12}(w_{12}(1), U_1(0))$	$X_{12}(w_{12}(2), U_1(1))$
	$U_1(w_{12}(0), w_{21}(0))$	$U_1(w_{12}(1), w_{21}(1))$
User II	$X_{20}(w_{20}(1), U_1(0))$	$X_{20}(w_{20}(2), U_1(1))$
	$X_{21}(w_{21}(1), U_1(0))$	$X_{21}(w_{21}(2)U_1(1))$
	$U_2(w_{12}(0), w_{21}(0))$	$U_1(w_{12}(1), w_{21}(1))$

Table 2.2: Block-Markov encoder for two user cooperative channel [27]

Table 2.2.

$$X_{10} = \sqrt{P_{10}} X_{10}(W_{10}(i), W_{12}^{'}(i-1), W_{21}^{'}(i-1)) \tag{2.20}$$

$$X_{12} = \sqrt{P_{12}} X_{12}(W_{12}(i), W_{12}^{'}(i-1), W_{21}^{'}(i-1)) \tag{2.21}$$

$$U_1 = \sqrt{P_{U1}}(W_{12}^{'}(i-1), W_{21}^{'}(i-1)) \tag{2.22}$$

where i and $i-1$ indicate the current block and previous block, respectively, $i \in \{1, 2, ...B\}$. The message of each user is divided into two independent messages, W_{10}, W_{20} are the parts of sending information intended towards destination and W_{12}, W_{21} are the parts of information intended both towards its partner and destination, taking values from index $W_{10} \triangleq \{1, .., 2^{nR_{10}}\}$, $W_{12} \triangleq \{1, .., 2^{nR_{12}}\}$, $W_{20} \triangleq \{1, .., 2^{nR_{20}}\}$, $W_{21} \triangleq \{1, .., 2^{nR_{21}}\}$.

2.5.3 Multi-user Cooperative Communication (Two user cooperation case)

Here, we introduce two user cooperative channel as a reference model and it is well examined in [27]. Let's start considering the channel model of Figure 2.8. There are two users and a single destination, numbered 1, 2, and 0 respectively. Both

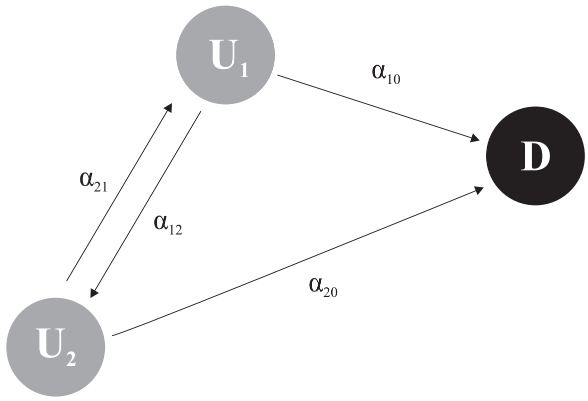


Figure 2.8: Detailed scheme for two user cooperative communication

transmitters can overhear each other, and are willing to cooperate by forwarding information from the other. Transmitters are capable of full-duplex communication. The system model for this channel is given by the following equations.

$$Y_0 = \sqrt{\alpha_{10}}X_1 + \sqrt{\alpha_{20}}X_2 + N_0 \tag{2.23}$$

$$Y_1 = \sqrt{\alpha_{S_{21}}}X_2 + N_1 \tag{2.24}$$

$$Y_2 = \sqrt{\alpha_{S_{12}}}X_1 + N_2 \tag{2.25}$$

with $N_0 \sim \mathcal{N}(0, \sigma_0^2)$, $N_1 \sim \mathcal{N}(0, \sigma_1^2)$ and $N_2 \sim \mathcal{N}(0, \sigma_2^2)$. User 1 generates its information W_1 as a function of W_{10}, which is sent directly to the destination, and W_{12}, which is sent to user 2 and then forwarded by user 2 to the destination. The following signals transmitted by user 1 can be generated by block-Markov encoding structure detailed in Section 2.5.2.

$$X_1 = \sqrt{P_{10}}X_{10} + \sqrt{P_{12}}X_{12} + \sqrt{P_{U1}}U_1 \tag{2.26}$$

$$X_2 = \sqrt{P_{20}}X_{20} + \sqrt{P_{21}}X_{21} + \sqrt{P_{U2}}U_2 \tag{2.27}$$

Here, although not detailed as in Section 2.5.2, we briefly mention block-Markov encoding structure. In equation (2.26)-(2.27), X_{10} uses power P_{10} to send W_{10} at rate R_{10} directly to the destination, X_{12} uses power P_{12} to send W_{12} to User 2 at rate R_{12}. U_1 refers to the part of the signal that carries cooperative information and uses power P_{U1} to send cooperative information to the destination. User 2 similarly structures its transmit signal X_2 and divides its total power P_2. Theorem 2.8 gives us an achievable rate region.

Theorem 2.8. Two user cooperative channel [27, Theorem 1]

An achievable rate region for the system given in Section 2.5.3 is the closure of the convex hull of all rate pairs (R_1, R_2) *such that* $R_1 = R_{10} + R_{12}$ *and* $R_2 = R_{20} + R_{21}$ *where* $\{R_{10}, R_{12}, R_{20}, R_{21}\}$ *satisfy the constraints*

$$R_{12} \leq E\left[\log\left(1+\frac{\alpha_{12}^2 P_{12}}{\alpha_{12}^2 P_{10}+N_1}\right)\right] \tag{2.28}$$

$$R_{21} \leq E\left[\log\left(1+\frac{\alpha_{21}^2 P_{21}}{\alpha_{21}^2 P_{20}+N_2}\right)\right] \tag{2.29}$$

$$R_{10} \leq E\left[\log\left(1+\frac{\alpha_{10}^2 P_{10}}{N_0}\right)\right] \tag{2.30}$$

$$R_{20} \leq E\left[\log\left(1+\frac{\alpha_{20}^2 P_{20}}{N_0}\right)\right] \tag{2.31}$$

$$R_{10}+R_{20} \leq E\left[\log\left(1+\frac{\alpha_{10}^2 P_{10}+\alpha_{20}^2 P_{20}}{N_0}\right)\right] \tag{2.32}$$

$$R_1+R_2 \leq E\left[\log\left(1+\frac{\alpha_{10}^2 P_{10}+\alpha_{20}^2 P_{20}+2\alpha_{10}\alpha_{20}\sqrt{P_{U1}P_{U2}}}{N_0}\right)\right] \tag{2.33}$$

Proof. The proof of achievability follows by examining random codebook generation, jointly typicality and backward decoding (BD) at destination side.

Codebook Generation:

As a proof of two user cooperative channel in Theorem 2.8, the following steps determine random codebook generation.

- Fix a distribution $p(x_1)p(x_2)p(x_1|x_2)p(x_2|x_1)$.
- Generate $[2^{n(R_{12}+R_{21})}]$ length codewords from distribution $p(u_1^n)$ intended for User 1. Assign each codeword to a distinct message $\{w_{12}^{'}, w_{21}^{'}\} \in W_{U1} \times W_{U2}$ and call them as U_1.
- Generate $[2^{n(R_{12}+R_{21})}]$ length codewords from distribution $p(u_2^n)$ intended for User 2. Assign each codeword to a distinct message $\{w_{12}^{'}, w_{21}^{'}\} \in W_{U1} \times W_{U2}$ and call them as U_2.

- Generate $[2^{nR_{10}}]$ length codewords for every U_1 from distribution $p(x_{10}|u_1^n)$ and $[2^{nR_{12}}]$ length codewords for every U_1 from distribution $p(x_{12}|u_1^n)$ for User 1.

- Generate $[2^{nR_{20}}]$ length codewords for every U_2 from distribution $p(x_{20}|u_2^n)$ and $[2^{nR_{21}}]$ length codewords for every U_2 from distribution $p(x_{21}|u_2^n)$ for User 2.

Decoding:

The decoding operation will perform at both the user and destination side.

- User side: Assuming that transmission is completed over B blocks and i defines the current block $i \in \{1, 2, ...B\}$. After User 1 and 2 estimate $w_{21}^{'}$, $w_{12}^{'}$ from the previous block $i-1$ with error free, the codewords U_1, U_2 are already known. Using these codewords and jointly typicality, each user decodes w_{21}, w_{12}. Since only decoding w_{21}, w_{12}, U_1, U_2, the codewords w_{20} and w_{10} treat as a noise component at User 1 and 2, respectively.

- Destination side: The destination must wait until all blocks are received and starts decoding from the last block B, proceeding backwards. In the last block of code sequence w_{10}, w_{20}, w_{12}, w_{21} contain no new information and they can be set $w_{10}(B)$, $w_{20}(B)$, $w_{12}(B)$, $w_{21}(B) = (0, 0, 0, 0)$. Then the decoding algorithm starts decoding from the B'th block to first block. Since transmitting no new information at the last block, the total information rate reduces by a coefficient of $(B-1)/B$. However, the reduction of information ratio can be undervalued at large B . Contrary to the block-Markov encoding scheme, in the backward decoding the destination wants to decode message by help of block $(i+1)$.

□

Chapter 3

Channel Adaptive Encoding and Decoding Strategies and Rate Regions for the Three User Cooperative Multiple Access Channel

The growing demand for high data rate mobile applications challenges researchers to develop wireless communication systems which are able to accommodate a higher number of concurrent users, communicating reliably at improved rates. Although an increase in the number of users in a system seems to cause more interference and hence worse performance, this interference may actually be viewed as free side information, which is distributed to all communicating parties thanks to the propagative nature of the wireless communication channel. Therefore, if the users are allowed to make use of the free side information and cooperate in sending each other's messages, the diversity provided to the participating users will increase with increasing number of users, potentially leading to higher rates.

The multi-user cooperative communication can push the rate regions close to higher data rate demand. The most interesting area in this communication scheme is extending the 2 user MAC-GF to three user. Beside this, in our proposed models, we assume that both the user and destination know the channel states. Taking advantage of knowledge of the channel states between the users, we can propose an encoding structure based on channel quality. The encoding structure for multi-user cooperative communication based on Shannon capacity defines the achievable rate region that can be transmitted over a wireless channel with small error probability since the codeword length is sufficient long. However, in any non-adaptive encoding structure, the rate for each user is constant since due to a lack of transmitting strategy relevant to channel gains. Thus poor channel states between the user reduce the user capacity because the encoding strategy can not incorporate the effect of these poor gain. Motivated above, to overcome the poor gain effect and meet higher data rate requirements, we propose an channel adaptive block Markov encoding structure for three user that adaptively transmit codeword and decide on which coding scheme should be used to improve the achievable rate without increasing the interference significantly.

The rest of chapter is dedicated to introducing channel adaptive encoding policies and decoding strategies and rate regions for the three user cooperative multiple access channel. We first establish the adaptive encoding structures focusing channel state information between three users. Next, we turn our attention to the achievable rate constraints for each proposed encoding structure. We then show that usefulness of the proposed three user cooperation strategies under several fading scenarios, and compare it to the corresponding two user cooperative system.

3.1 Adaptive encoding policies for three user cooperative multiple access channel

A short description on the proposed policies and encoding structure, moving from the two user MAC studied in [27] to our three user MAC, the block Markov encoding structure is non-trivial generalized. Furthermore, considering the possible channel conditions between users, the block Markov encoding takes a symmetric/asymmetric structure, and is more sophisticated than the two user scheme. We therefore define two policies where two different encoding structures are employed to cover all possible conditions. The possible channel conditions are explained in Table 3.1. s_{ij} is normalized fading coefficients between the user i and j. To proceed further, the message generation, generalized for our proposed policies is detailed in Table 3.3 and 3.6.

		Possible Channel Dist.			Strongest	Normal	Weakest
Policy I	State I	$s_{12} > s_{13}$	$s_{21} > s_{23}$	$s_{31} > s_{32}$	User 1	User 2	User 3
	State II	$s_{12} < s_{13}$	$s_{21} > s_{23}$	$s_{31} > s_{32}$	User 1	User 3	User 2
	State III	$s_{12} > s_{13}$	$s_{21} > s_{23}$	$s_{31} < s_{32}$	User 2	User 1	User 3
	State IV	$s_{12} > s_{13}$	$s_{21} < s_{23}$	$s_{31} < s_{32}$	User 2	User 3	User 1
	State V	$s_{12} < s_{13}$	$s_{21} < s_{23}$	$s_{31} > s_{32}$	User 3	User 1	User 2
	State VI	$s_{12} < s_{13}$	$s_{21} < s_{23}$	$s_{31} < s_{32}$	User 3	User 2	User 1
Policy II	State VII	$s_{12} > s_{13}$	$s_{21} < s_{23}$	$s_{31} > s_{32}$	equal decoding capability		
	State VIII	$s_{12} < s_{13}$	$s_{21} > s_{23}$	$s_{31} < s_{32}$			

Table 3.1: Summary of Decoding strategy at the transmitters, Policy I-II

As well defined in Table 3.1, considering better inter-user link quality than direct link, we propose that in each of states I-VI, only one user decodes all messages transmitted by others, and other users attempt to decode only part of the transmitted messages. Based on common information established in previous block, the block Markov encoder generates one common message U and pairwise cooperative messages $U_i, i \in 1, 2, 3$ to obtain coherent combining at destination side. We call this encoding structure as Policy I (Section 3.2). For channel states, which do not obey I-VI, one can recognize that only one inter-user link is better for each user. This means that, all users have symmetric message decoding capabilities, this encoding structure namely Policy II (Section 3.3) therefore generates more pairwise cooperative messages than Policy I as well as generating of common message U. Accordingly, it is aimed to increase the coherent combining terms at the destination side. The policies and encoding structure will be well defined in the next sections.

3.2 Policy I - channel adaptive encoding structure for three user cooperative MAC

Throughout this section, we propose two new superposition block Markov encoding based cooperation scheme for a three user Gaussian multiple access channel

(MAC). Our scheme allows the three users to simultaneously cooperate both in pairs, and collectively, by dividing the transmitted messages into sub-messages intended for each cooperating partner. The proposed encoding and decoding at the transmitters take into account the relative qualities of the cooperation links between the transmitters. We obtain and evaluate the achievable rate region based on our encoding strategies, and compare them with the achievable rates for the two user cooperative MAC. We demonstrate that the added diversity by the presence of the third user improves the region of achievable rates, and this improvement is especially significant as far as the sum rate of the system is concerned.

3.2.1 System model

We consider a three user fading Gaussian MAC, where both the receiver and the transmitters receive noisy versions of the transmitted messages, as illustrated in Figure 3.1. The transmitters are assumed to be operating in the full duplex mode. The system is modeled by

$$Y_0 = \sqrt{h_{10}}X_1 + \sqrt{h_{20}}X_2 + \sqrt{h_{30}}X_3 + N_0 \tag{3.1}$$

$$Y_1 = \sqrt{h_{21}}X_2 + \sqrt{h_{31}}X_3 + N_1 \tag{3.2}$$

$$Y_2 = \sqrt{h_{12}}X_1 + \sqrt{h_{32}}X_3 + N_2 \tag{3.3}$$

$$Y_3 = \sqrt{h_{13}}X_1 + \sqrt{h_{23}}X_2 + N_3 \tag{3.4}$$

where X_i is the symbol transmitted by node i, Y_i is the symbol received at node i, and the receiver is denoted by $i = 0$; N_i is the zero-mean additive white Gaussian noise at node i, having variance σ_i^2, and h_{ij} are the random fading coefficients, the instantaneous realizations of which are assumed to be known by both the transmitters and the receiver. We further define the normalized fading coefficients $s_{ij} = \frac{h_{ij}}{\sigma_{ij}^2}$, for the simplicity of our discussions.

Throughout this section, we assume that the normalized channel gains satisfy $s_{ij} > s_{i0}$, $\forall\ i, j \in \{1, 2, 3\}$, $i \neq j$; that is, the inter-user cooperation links are

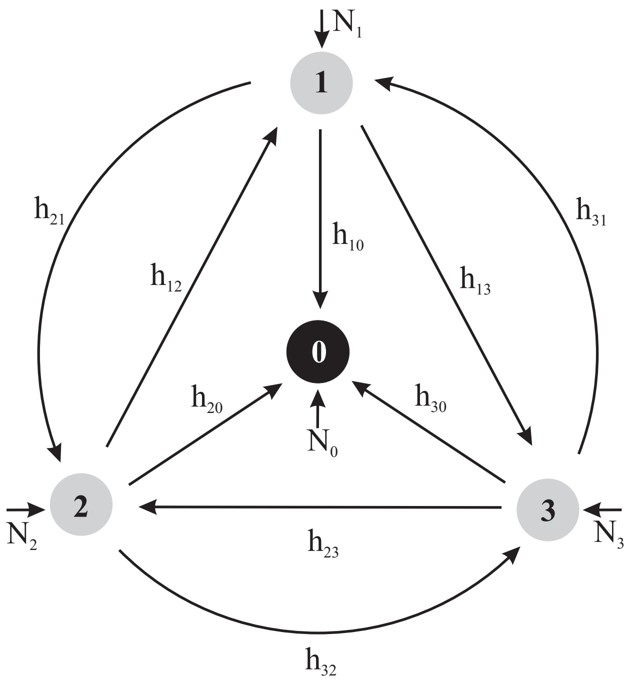


Figure 3.1: Three user cooperative channel model

uniformly stronger than the direct links. This particular case is of practical interest since the cooperating transmitters are likely to be closely located with less number of scatterers and obstructions on the paths connecting them, when compared to their paths to the receiver, and thus have better channel conditions among each other.

3.2.2 Encoding strategy

Moving from the two user MAC with generalized feedback to its three user counterpart, the block Markov encoding strategy is not trivially generalized, as the presence of the extra user presents a choice among a multitude of new and more complicated cooperation strategies. In this case, the following questions need to be answered: how should the users form their codewords, so as to allow for cooperation among each other? Will the users cooperate in pairs, all together, or using a mixture of both? What kind of cooperation signals will be used, and which signal is to be decoded by which terminal? There are many answers to each of these questions, and in this subsection we only focus on one seemingly logical approach in which we design the encoding and decoding strategies based on the knowledge of channel states among the users. Following the development in the case of two user MACGF, we divide the messages of the users into sub-messages intended for each receiver, i.e., $w_1 = \{w_{10}, w_{12}, w_{13}\}$, $w_2 = \{w_{20}, w_{21}, w_{23}\}$, $w_3 = \{w_{30}, w_{31}, w_{32}\}$, where w_{ij} denotes the message of User i intended for User j in each block of transmission. These sub-messages will then be used in the next block to create common cooperation signals, which will be sent to the ultimate receiver (0). It becomes immediately obvious upon crowding the list of these sub-messages that, except for the ultimate receiver, each receiver has two sub-messages intended for it, and six sub-messages which will cause interference to it, should the code-words that will be used to transmit these sub-messages be treated as noise, as it is done in [27]. Moreover, as will become clearer from our oncoming exposition, the pairwise cooperation signals, that shall be generated after the messages are decoded at their intended receivers, will cause additional interference in the upcoming block. This is because of the fact that not all sub-messages will be known to all users, unlike the two user MACGF, where both cooperating nodes get to know each other's cooperation signals after each block. In order to avoid the added interference at the transmitters, we instead propose a modified block Markov encoding strategy, in which the users try to decode as many messages as they can, before forming their cooperation signals. To this end, we first start by assuming without loss of generality that the normalized inter-user link gains s_{ij} are distributed so as to satisfy.

$$s_{12} > s_{13}, \quad s_{21} > s_{23}, \quad s_{32} > s_{31} \tag{3.5}$$

We will make use of this particular ordering in deciding which sub-messages are to be decoded by which users. Although our encoding strategy does depend on this ordering, it can be easily modified to at any other ordering of the channel gains. Our proposed encoding and decoding strategy is inspired by the capacity achieving encoding/decoding for Gaussian broadcast channels, where the stronger receiver decodes not only its own message, but also the weaker users' messages.

It is evident from (3.5) that User 2 is the stronger receiver for transmissions from both User 1 and User 3. For one moment let us assume that User 1 was broadcasting alone: User 2 would be able to decode correctly not only its intended message w_{12}, but also the message w_{13} intended for User 3, provided the message w_{13} were sent at a rate that is supported by User 3. A similar argument would apply to the message transmitted by User 3, as User 2 would be able to decode w_{31}, in addition to its intended message w_{32}. Likewise, since User 1 is in a stronger position than User 3 when User 2 is transmitting alone, it would be able to resolve the message w_{23}, as long as User 3 itself can resolve this same message.

Motivated by the above argument, we propose a decoding strategy for the transmitters, which is summarized in Table 3.2. This table provides a list of messages known to each user after common information is established in each block. The rate requirements for reliable decoding of each message at the corresponding receiver will be given in the next section. Looking at Table 3.2, it is easy to observe that the messages w_{13}, w_{23} and w_{31} are known to all transmitters, the messages w_{12}, w_{21} are only known to the transmitters 1 and 2, and the message w_{32} is only known to the transmitters 2 and 3. This grouping of common information immediately suggests a way to form the cooperation signals: we shall use one cooperation signal common to all users, and two other cooperation signals common to pairs $\{1,2\}$ and $\{2,3\}$ respectively. Following the notation in [27], [47] and suitably extending the codebook generation process described therein, the signals transmitted by each user can be generated by block Markov superposition encoding as follows:

$$X_1 = \sqrt{P_{10}}X_{10} + \sqrt{P_{12}}X_{12} + \sqrt{P_{13}}X_{13} + \sqrt{P_{1U_1}}U_1 + \sqrt{P_{1U}}U \tag{3.6}$$

$$X_2 = \sqrt{P_{20}}X_{20} + \sqrt{P_{21}}X_{21} + \sqrt{P_{23}}X_{23} + \sqrt{P_{2U_1}}U_1 + \sqrt{P_{2U_3}}U_3 + \sqrt{P_{1U}}U \tag{3.7}$$

$$X_3 = \sqrt{P_{30}}X_{30} + \sqrt{P_{31}}X_{31} + \sqrt{P_{32}}X_{32} + \sqrt{P_{3U_3}}U_3 + \sqrt{P_{3U}}U \tag{3.8}$$

Here, the signals X_{i0} carry the fresh information intended for the receiver, X_{ij} carry the information intended for transmitter j for cooperation in the next block,

	Decoded Messages	Own Messages
User I	w_{21}, w_{31}, w_{23}	w_{12}, w_{13}
User II	w_{12}, w_{32}, w_{13}, w_{31}	w_{21}, w_{23}
User III	w_{13}, w_{23}	w_{31}, w_{32}

Table 3.2: Decoding strategy a the transmitter, before forming the common cooperation signals

and U, U_1, U_3 are the common information sent by groups of three, two and two transmitters respectively for the resolution of the remaining uncertainty from the previous block, all chosen from unit power Gaussian distributions. The transmit power is thus captured by the powers associated with each component, which are required to satisfy the average power constraints.

$$P_{10} + P_{12} + P_{13} + P_{1U_1} + P_{1U} \leq P_1 \tag{3.9}$$

$$P_{20} + P_{21} + P_{23} + P_{2U_1} + P_{2U_3} + P_{2U} \leq P_2 \tag{3.10}$$

$$P_{30} + P_{31} + P_{32} + P_{3U_3} + P_{3U} \leq P_3 \tag{3.11}$$

The encoding strategy, and the dependency of the transmitted codewords on the messages are depicted in more detail in Table 3.3. In Table 3.3, the sub-messages w_{ij} stand for the messages received in the previous block: the cooperation signals depend on the messages received in previous block, and new information is also encoded into codewords X_{ij}, taking into account the messages received in the previous block. Once all information blocks are transmitted using the modified block Markov superposition encoding, the receiver decodes the messages of all users starting from the cooperation signals in the last block, using backwards decoding, as in [27], [47]. The conditions on the rates of each sub-message for reliable decoding both at the transmitters and at the receiver is obtained in the next section.

	Block I	Block II
User I	$X_{10}(w_{10}(1), U_1(0), U(0))$	$X_{10}(w_{10}(2), U_1(1), U(1))$
	$X_{12}(w_{12}(1), U_1(0), U(0))$	$X_{12}(w_{12}(2), U_1(1), U(1))$
	$X_{13}(w_{13}(1), U(0))$	$X_{13}(w_{13}(2), U(1))$
	$U(w_{13}(0), w_{31}(0), w_{23}(0))$	$U(w_{13}(1), w_{31}(1), w_{23}(1))$
	$U_1(w_{12}(0), w_{21}(0))$	$U_1(w_{12}(1), w_{21}(1))$
User II	$X_{20}(w_{20}(1), U_1(0), U_3(0), U(0))$	$X_{20}(w_{20}(2), U_1(1), U_3(0), U(1))$
	$X_{21}(w_{21}(1), U_1(0), U(0))$	$X_{21}(w_{21}(2), U_1(1), U(1))$
	$X_{23}(w_{23}(1), U(0))$	$X_{23}(w_{23}(2), U(1))$
	$U(w_{13}(0), w_{31}(0), w_{23}(0))$	$U(w_{13}(1), w_{31}(1), w_{23}(1))$
	$U_1(w_{12}(0), w_{21}(0))$	$U_1(w_{12}(1), w_{21}(1))$
	$U_3(w_{32}(0))$	$U_3(w_{32}(2))$
User III	$X_{30}(w_{30}(1), U_3(0), U(0))$	$X_{30}(w_{30}(2), U_3(1), U(1))$
	$X_{31}(w_{31}(1), U_3(0), U(0))$	$X_{31}(w_{31}(2), U_3(1), U(1))$
	$X_{32}(w_{32}(1), U(0))$	$X_{32}(w_{32}(2), U(1))$
	$U(w_{13}(0), w_{31}(0), w_{23}(0))$	$U(w_{13}(1), w_{31}(1), w_{23}(1))$
	$U_3(w_{32}(0))$	$U_3(w_{32}(1))$

Table 3.3: Block Markov encoding for policy I: mapping of codewords to messages

3.2.3 Achievable rates

Before proceeding to characterize the achievable rate region, we would like to make a final simplification in our encoding scheme. For a two user cooperative MAC where channel state information is available to the transmitters, it has recently been shown in [48] that, when the inter-user cooperation links are uniformly stronger than the direct links to the receiver, to maximize the achievable rates the signals X_{i0} should never be transmitted and all the available power should be allocated to cooperative signals. In order to prove a similar statement for the three user MAC in question here, the achievable rate region first needs to be characterized, and then optimized over the transmit powers. However, in view of our assumption about the strength of inter-user links when compared to user-destination links, and the results in the two user case [48] we simply choose to drop the signals X_{i0} from our encoding rule, so that the achievable rate region expressions are more tractable, and more easily evaluated using simulations.

The cooperative communication proceeds reliably if the rates at which we transmit each of the sub-messages are supported both on the inter-user links while building up common information, and on the user-to-ultimate-receiver links, where the users's messages are decoded with the help of cooperation signals. The rate at which a message w_{ij} is transmitted is denoted by R_{ij}. For notational convenience, we first define the following variables which will be used to simplify the rate expressions before giving Theorem 3.1.

$$\begin{aligned}
A =& s_{21}P_{2U_3} + s_{31}(P_{32} + P_{3U_3}) + 2\sqrt{s_{21}s_{31}P_{2U_3}P_{3U_3}} + 1 \\
B =& s_{13}(P_{12} + P_{1U_1}) + s_{23}(P_{21} + P_{2U_1}) + 2\sqrt{s_{13}s_{23}P_{1U_1}P_{2U_1}} + 1 \\
C =& 2\sqrt{s_{10}s_{20}P_{1U_1}P_{2U_1}} \\
D =& 2\sqrt{s_{20}s_{30}P_{2U_3}P_{3U_3}} \\
E =& 2(\sqrt{s_{10}s_{20}P_{1U}P_{2U}} + \sqrt{s_{10}s_{30}P_{1U}P_{3U}} + \sqrt{s_{20}s_{30}P_{2U}P_{3U}})
\end{aligned}$$

Theorem 3.1. Adaptive Encoding For Three User Cooperative Multiple Access Channel-Policy I:

A rate region for the system given in Section 3.2 is the closure of the convex hull of all rate pairs (R_1, R_2, R_3) such that $R_1 = R_{12} + R_{13}$, $R_2 = R_{21} + R_{23}$, and $R_3 = R_{31} + R_{32}$ where $\{R_{12}, R_{13}, R_{21}, R_{23}, R_{31}, R_{32}\}$ satisfy the constraints

$$R_{12} \leq E\left[\log\left(1+s_{12}P_{12}\right)\right] \tag{3.12}$$

$$R_{13} \leq E\left[\log\left(1+s_{12}P_{13}\right)\right] \tag{3.13}$$

$$R_{31} \leq E\left[\log\left(1+s_{32}P_{31}\right)\right] \tag{3.14}$$

$$R_{32} \leq E\left[\log\left(1+s_{32}P_{32}\right)\right] \tag{3.15}$$

$$R_{1} \leq E\left[\log\left(1+s_{12}(P_{12}+P_{13})\right)\right] \tag{3.16}$$

$$R_{12}+R_{31} \leq E\left[\log\left(1+s_{12}P_{12}+s_{32}P_{31}\right)\right] \tag{3.17}$$

$$R_{12}+R_{32} \leq E\left[\log\left(1+s_{12}P_{12}+s_{32}P_{32}\right)\right] \tag{3.18}$$

$$R_{13}+R_{31} \leq E\left[\log\left(1+s_{12}P_{13}+s_{32}P_{31}\right)\right] \tag{3.19}$$

$$R_{13}+R_{32} \leq E\left[\log\left(1+s_{12}P_{13}+s_{32}P_{32}\right)\right] \tag{3.20}$$

$$R_{3} \leq E\left[\log\left(1+s_{32}(P_{31}+P_{32})\right)\right] \tag{3.21}$$

$$R_{1}+R_{31} \leq E\left[\log\left(1+s_{12}(P_{12}+P_{13})+s_{32}P_{31}\right)\right] \tag{3.22}$$

$$R_1+R_{32} \leq E\left[\log\left(1+s_{12}(P_{12}+P_{13})+s_{32}P_{32}\right)\right] \tag{3.23}$$

$$R_{12}+R_3 \leq E\left[\log\left(1+s_{12}P_{12}+s_{32}(P_{31}+P_{32})\right)\right] \tag{3.24}$$

$$R_{13}+R_3 \leq E\left[\log\left(1+s_{12}P_{13}+s_{32}(P_{31}+P_{32})\right)\right] \tag{3.25}$$

$$R_1+R_3 \leq E\left[\log\left(1+s_{12}(P_{12}+P_{13})+s_{32}(P_{31}+P_{32})\right)\right] \tag{3.26}$$

$$R_{21} \leq E\left[\log\left(1+\frac{s_{21}P_{21}}{A}\right)\right] \tag{3.27}$$

$$R_{23} \leq E\left[\log\left(1+\frac{s_{21}P_{23}}{A}\right)\right] \tag{3.28}$$

$$R_{31} \leq E\left[\log\left(1+\frac{s_{31}P_{31}}{A}\right)\right] \tag{3.29}$$

$$R_2 \leq E\left[\log\left(1+\frac{s_{21}(P_{21}+P_{23})}{A}\right)\right] \tag{3.30}$$

$$R_{21}+R_{31} \leq E\left[\log\left(1+\frac{s_{21}P_{21}+s_{31}P_{31}}{A}\right)\right] \tag{3.31}$$

$$R_{23}+R_{31} \leq E\left[\log\left(1+\frac{s_{21}P_{23}+s_{31}P_{31}}{A}\right)\right] \tag{3.32}$$

$$R_2+R_{31} \leq E\left[\log\left(1+\frac{s_{21}(P_{21}+P_{23})+s_{31}P_{31}}{A}\right)\right] \tag{3.33}$$

$$R_{13} \leq E\left[\log\left(1+\frac{s_{13}P_{13}}{B}\right)\right] \tag{3.34}$$

$$R_{23} \leq E\left[\log\left(1+\frac{s_{23}P_{23}}{B}\right)\right] \tag{3.35}$$

$$R_{13}+R_{23} \leq E\left[\log\left(1+\frac{s_{13}P_{13}+s_{23}P_{23}}{B}\right)\right] \tag{3.36}$$

$$R_{32} \leq E\left[\log\left(1+s_{20}P_{2U_3}+s_{30}(P_{32}+P_{3U_3})+D\right)\right] \tag{3.37}$$

$$R_{12}+R_{21} \leq E\left[\log\left(1+s_{10}(P_{12}+P_{1U_1})+s_{20}(P_{21}+P_{2U_1})+C\right)\right] \tag{3.38}$$

$$\begin{aligned} R_{13}+R_{23}+R_{31} \leq\, & E\left[\log\left(1+s_{10}(P_{13}+P_{1U})+s_{20}(P_{23}+P_{2U})\right.\right. \\ & \left.\left.+s_{30}(P_{31}+P_{3U})+E\right)\right] \end{aligned} \tag{3.39}$$

$$\begin{aligned} R_{12}+R_{21}+R_{32} \leq\, & E\left[\log\left(1+s_{10}(P_{12}+P_{1U_1})+s_{20}(P_{21}+P_{2U_1}+P_{2U_3})\right.\right. \\ & \left.\left.+s_{30}(P_{32}+P_{3U_3})+C+D\right)\right] \end{aligned} \tag{3.40}$$

$$\begin{aligned} R_{13}+R_{23}+R_3 \leq\, & E\left[\log\left(1+s_{10}(P_{13}+P_{1U})+s_{20}(P_{23}+P_{2U}+P_{2U_3})\right.\right. \\ & \left.\left.+s_{30}P_3+D+E\right)\right] \end{aligned} \tag{3.41}$$

$$\begin{aligned} R_1+R_2+R_{31} \leq\, & E\left[\log\left(1+ s_{10}P_1+s_{20}(P_{21}+P_{23}+P_{2U}+P_{2U_1})\right.\right. \\ & \left.\left.+s_{30}(P_{31}+P_{3U})+C+E\right)\right] \end{aligned} \tag{3.42}$$

$$R_1+R_2+R_3 \leq E\left[\log\left(1+s_{10}P_1+s_{20}P_2+s_{30}P_3+C+D+E\right)\right] \qquad (3.43)$$

Proof. We first start focusing on the decoding of the messages at the transmitters. From Table 3.2, it is easy to see that User 2 simultaneously decodes all messages (those remaining after dropping direct messages X_{i0}) in the system. Therefore, the rates of the messages w_{12}, w_{13}, w_{31}, w_{32} should satisfy the traditional MAC constraints in (3.12)-(3.26). User 1 does not intend to decode the message w_{32}, and therefore treats it as noise. Moreover, unlike the two user cooperative MAC, the cooperation signal U_3 is also unknown to the User 1, as it involves the message w_{32} not affiliated with this user. Therefore, the coherently combined version of the cooperation signals U_3 from users two and three should also be treated as noise at User 1. Then, the reliable decoding of all other messages is possible if (3.27)-(3.33) are satisfied. Lastly, since User 3 is only interested in decoding the two messages directly intended for itself, we only require the rates of these messages to satisfy the two user MAC capacity bound, where all other signals, including the cooperation signal U_1, are treated as noise. All MAC constraints for User 3 are given in (3.34)-(3.36).

Once the common information is reliably established at the transmitters, it remains to make sure that the transmitted messages are also reliably decoded at the ultimate receiver. Note that, as we backwards decode the cooperative signals using joint typicality decoding at the receiver, the message groups $\{w_{12}, w_{21}\}$, w_{32} and $\{w_{13}, w_{23}, w_{31}\}$ appear jointly in cooperative codewords, and they will be decoded jointly as if each group is a single message. Having this in mind, traditional arguments on MAC capacity can be used to obtain the set of constraints on the rates that should be satisfied for achievability at the ultimate receiver. These constraints are given in equations (3.37)-(3.43). Note that, the constraints (3.39), (3.41), (3.42) are dominated by the tighter constraint (3.43), which has the same right hand side but bounds more rate components. Therefore, the inequalities (3.39), (3.41), (3.42) can be omitted from the final solution. □

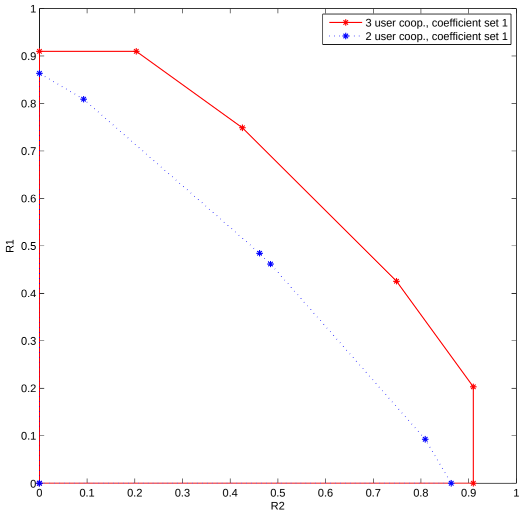
Fading Distribution		
Link Gains	Coefficient Set I	Coefficient Set II
$s_{10},\ s_{20},\ s_{30}$	$\{0.1 : 0.2 : 0.9\}$	$\{0.5 : 0.05 : 0.7\}$
$s_{13},\ s_{23},\ s_{31}$	$\{1.1 : 0.2 : 1.9\}$	$\{0.8 : 0.05 : 1.0\}$
$s_{12},\ s_{21},\ s_{32}$	$\{2.1 : 0.2 : 2.9\}$	$\{1.1 : 0.05 : 1.3\}$

Table 3.4: Fading Distribution

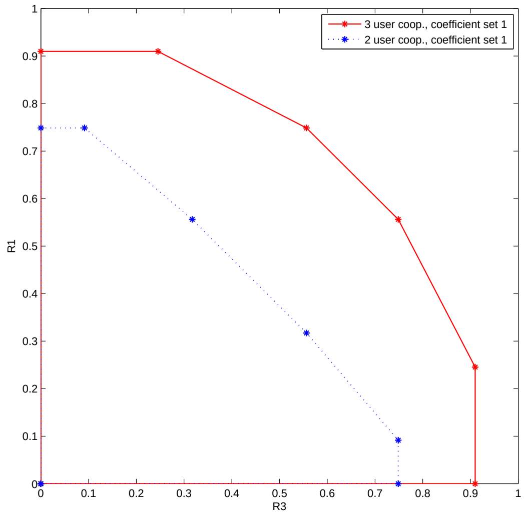
3.2.4 Simulation results

In this section we numerically evaluate the achievable rate region described by the equations (3.12)-(3.42) for the three user cooperative MAC. Since the achievable rate region obtained as a result of the three dimensional convex hull operation turns out to be hard to visualize, we simply plot its cross-sections on $R_1 - R_2$, $R_1 - R_3$ and $R_2 - R_3$ planes. This enables us to compare the three user achievable rate region with the corresponding two user cooperation strategies, obtained by the encoding/decoding structure in [27].

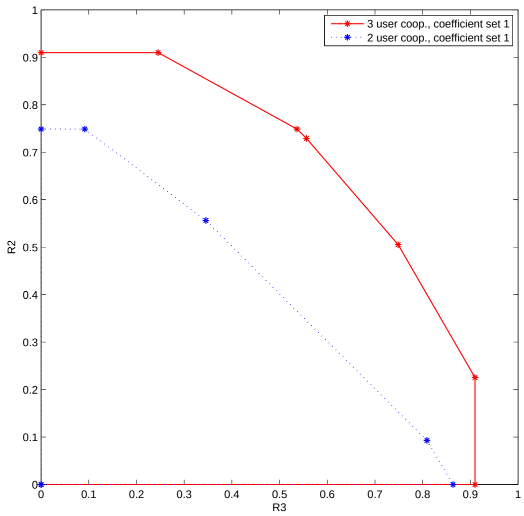
The achievable rate region is generated for two sets of channel state distributions, each of which is chosen uniformly to satisfy the assumption in (3.5), as well as the assumption about the cooperative links being stronger than the direct links: $s_{10},\ s_{20},\ s_{30}$ are i.i.d uniform random variables taking the values from the set $\{0.1 : 0.2 : 0.9\}$, $s_{13},\ s_{23},\ s_{31}$ are i.i.d taking values from $\{1.1 : 0.2 : 1.9\}$ and s_{12}, s_{21}, s_{32} are also i.i.d with values $\{2.1 : 0.2 : 2.9\}$. The average transmit power for each user is chosen to be 1. The resulting sets of achievable rate pairs are plotted in Figure 3.2, along with the two user cooperation strategy of Sendonaris et al. in [27]. We see in all three figures that the existence of a third user improves the set of achievable rates significantly, especially for rate tuples near the sum rate. When we search for the active constraints for the points on the axes in all figures, we see that each single user rate is bounded by the rate constraint coming from the inter-user links, rather than the direct links for this selection of fading coefficients.



(a) Rate region for Users 1 and 2, with User 3 acting as a relay



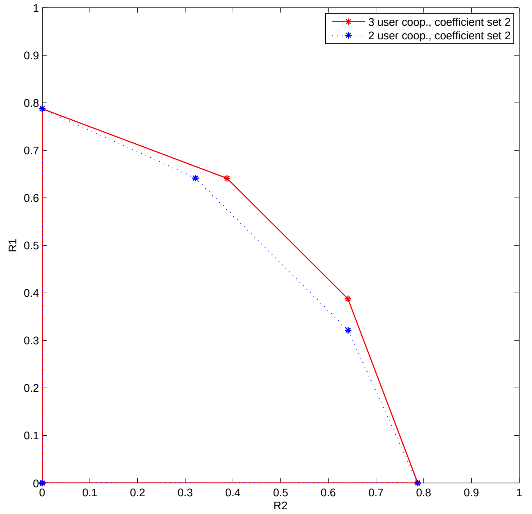
(b) Rate region for Users 1 and 3, with User 2 acting as a relay



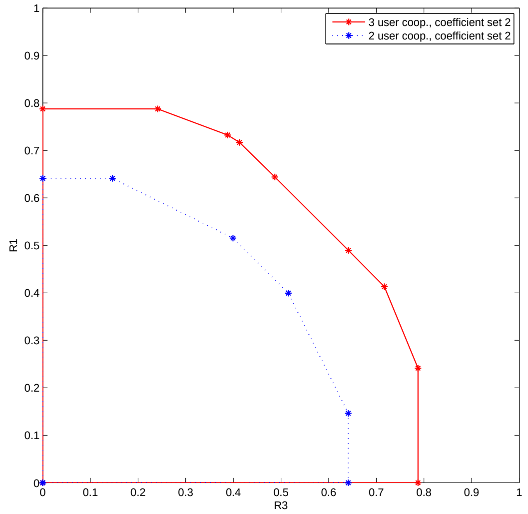
(c) Rate region for Users 2 and 3, with User 1 acting as a relay

Figure 3.2: Achievable rate regions for the three user cooperative MAC, coefficients obeying set I

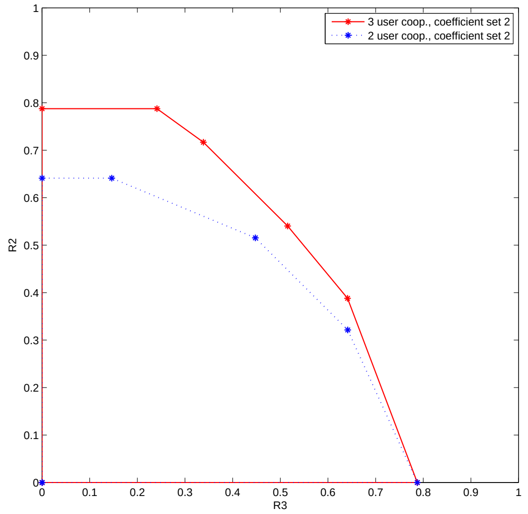
As an example, let us consider Figure 3.2(a). The main advantage of having a third user in the system, as far as the maximum achievable rate R_1 is concerned, is that User 1 does not have to allocate any part of its power to a cooperation signal; it is able to use its power solely to establish common information, while Users 2 and 3 send only cooperation signals to establish a coherent combining gain. This way, the rate constraint on the direct link becomes loose, and the rate R_1 can be pushed all the way to the rate on the inter-user link. Another interesting



(a) Rate region for Users 1 and 2, with User 3 acting as a relay



(b) Rate region for Users 1 and 3, with User 2 acting as a relay



(c) Rate region for Users 2 and 3, with User 1 acting as a relay

Figure 3.3: Achievable rate regions for the three user cooperative MAC, coefficients obeying set II

observation is from Figure 3.2(c): although the rate region is asymmetric for the two user cooperative MAC, it is symmetric for the three user MAC, with the same maximum achievable rate for all users. In this case, the channel coefficients s_{32} are better than the coefficients s_{23}, therefore it is expected that the two user cooperation will yield higher rates for User 3, which has a better outgoing link. However, the presence of a third user creates additional diversity by making a better channel condition, namely s_{21}, available to User 2, which is no longer constrained to cooperate solely with User 3, and the resulting achievable rates are increased. Note that, fixing one of the rate components in our region is simply equivalent to considering a multiple access relay channel. Meanwhile, the points where our rate regions intersect the axes correspond to the case of two parallel relays.

The second set of uniform fading distributions; s_{10}, s_{20}, s_{30} i.i.d from $\{0.5 : 0.05 : 0.7\}$, s_{13}, s_{23}, s_{31} i.i.d from $\{0.8 : 0.05 : 1\}$ and s_{12}, s_{21}, s_{32} i.i.d from $\{1.1 : 0.05 : 1.3\}$, yield a more interesting set of achievable rate regions, depicted in Figure 3.3. Namely, it can be seen in Figure 3.3(a) that the two and three user cooperation strategies give the same maximum individual rates. This is because of the fact that it is no longer profitable to use the cooperation links as far as the individual rates are concerned. However as we get closer to the sum rate point, the common cooperation signal becomes useful, as it is a function of many sub-messages coming from all users. At the points where the two and three user cooperative rate regions coincide, the employed power distribution for both strategies are the same, and the extra user is treated as if it is not present in the system. Lastly, since the two user cooperation strategy is simply a subset of its three user counterpart, we always expect to have the achievable rate region of the former to be also a subset of the latter.

3.3 Policy II - channel adaptive encoding structure for three user cooperative MAC

In this section, we have propose a new block Markov type encoding strategy for the three user multiple access channel. The encoding and decoding policies is developed by making use of a specific ordering of the channel states, complementing our existing results in Section 3.2, thereby yielding a complete channel adaptive encoding policy. We obtain the rate constraints for reliable decoding of messages for the three user multiple access channel under our proposed encoding and decoding strategies, and evaluate them to obtain three user achievable rate regions.

We demonstrate through simulations that, going from the two user cooperative multiple access channel to its three user counterpart, the achievable rates increase significantly due to the additional diversity provided by the existence of an extra user.

3.3.1 System model

We consider a three user fading Gaussian MAC, where both the receiver and the transmitters receive noisy versions of the transmitted messages, as illustrated in Figure 3.1. The transmitters are assumed to be operating in the full duplex mode. The system is modeled by

$$Y_0 = \sqrt{h_{10}}X_1 + \sqrt{h_{20}}X_2 + \sqrt{h_{30}}X_3 + N_0 \tag{3.44}$$

$$Y_1 = \sqrt{h_{21}}X_2 + \sqrt{h_{31}}X_3 + N_1 \tag{3.45}$$

$$Y_2 = \sqrt{h_{12}}X_1 + \sqrt{h_{32}}X_3 + N_2 \tag{3.46}$$

$$Y_3 = \sqrt{h_{13}}X_1 + \sqrt{h_{23}}X_2 + N_3 \tag{3.47}$$

where X_i is the symbol transmitted by node i, Y_i is the symbol received at node i, and the receiver is denoted by $i = 0$; N_i is the zero-mean additive white Gaussian noise at node i, having variance σ_i^2, and h_{ij} are the random fading coefficients, the instantaneous realizations of which are assumed to be known by both the transmitters and the receiver. We further define the normalized fading coefficients $s_{ij} = \frac{h_{ij}}{\sigma_{ij}^2}$, for the simplicity of our discussions.

Throughout this section, we assume that the normalized channel gains satisfy $s_{ij} > s_{i0}$, $\forall\ i, j \in \{1, 2, 3\}$, $i \neq j$; that is, the inter-user cooperation links are uniformly stronger than the direct links. This particular case is of practical interest since the cooperating transmitters are likely to be closely located with less number of scatterers and obstructions on the paths connecting them, when compared to their paths to the receiver, and thus have better channel conditions among each other.

3.3.2 Encoding strategy

For a multiple access channel with generalized feedback, the extension of the block Markov encoding policy from the two user channel to three users is non-trivial, since the cooperation options become more diverse, as the number of cooperating users increases. For the two user MAC-GF, the encoding is performed by dividing each user's message w_i into two sub-messages, one used solely to introduce fresh information intended for the receiver, and the other used for simultaneously transmitting cooperative information to both the cooperating partner and the receiver [47], [27]. For a three user MAC-GF, it is natural to extend this strategy by simply including additional cooperative sub-messages intended for each user from each transmitter, i.e.

$$w_i = (w_{i0}, w_{ij}, w_{ik}), \quad i \neq j \neq k, \quad \forall\, i, j \in \{1, 2, 3\} \tag{3.48}$$

However, with the introduction of new sub-messages, one has to decide on how to encode these messages into cooperative codewords, i.e., which messages should be used by which users for cooperation. This also requires a decision about which cooperative messages should be decoded by which receivers. If the User i were to decode only the messages w_{ji} solely intended for itself, treating all other signals as noise as it is done in the two user MAC-GF in [27], it would face a significant amount of interference in reception over the inter-user links. This would eventually degrade the quality of the inter-user links and possibly reduce the rate advantage due to the cooperation among the users. In order to control the interference over the inter user links, we have proposed in Section 3.3 an extension of the block Markov coding strategy, based on relative receive-link qualities for the users. The encoding and decoding strategies in Section 3.3 were limited in the sense that they were designed only for a specific ordering of the inter-user links,

$$s_{ij} > s_{ik}, \quad s_{ji} > s_{jk}, \quad s_{kj} > s_{ki}, \quad i \neq j \neq k \tag{3.49}$$

	Decoded Messages	Own Messages
User I	w_{21}, w_{31}, w_{32}	w_{12}, w_{13}
User II	w_{12}, w_{32}, w_{13}	w_{21}, w_{23}
User III	w_{13}, w_{23}, w_{21}	w_{31}, w_{32}

Table 3.5: Decoding strategy a the transmitter, before forming the common cooperation signals

that lead to User j being the strongest, and decoding all the information sent over the channel, and User k being the weakest, and decoding only the information intended for itself. The resulting decoding strategies for each participating transmitter which we called Policy I, are summarized in Table 3.3.

Now, we introduce a new encoding/decoding policy, which is designed for the remaining possible orderings of the instantaneous channel states, i.e

$$s_{ij} > s_{ik}, \quad s_{jk} > s_{ji}, \quad s_{ki} > s_{kj}, \quad i \neq j \neq k \tag{3.50}$$

As in Section 3.2, our proposed encoding and decoding strategy is inspired by the capacity achieving encoding/decoding for Gaussian broadcast channels, where the stronger receiver decodes not only its own message, but also the weaker users' messages. It is easy to check that, there are a total of eight possible orderings for the receive-link qualities at the users, six of which obey (3.49), and two of which obey (3.50). Unlike the asymmetric situation caused by ordering (3.49), when the channel qualities satisfy (3.50), each user has better reception quality on one of the underlying broadcast channels, and worse on the other. For the simplicity of the exposition, we will assume from now on without loss of generality that

$$s_{12} > s_{13}, \quad s_{23} > s_{21}, \quad s_{31} > s_{32} \tag{3.51}$$

Based on this assumption, User 2 has the stronger receive link for the transmission of User 1. If User 1 were broadcasting alone, User 2 would be able to correctly decode not only its own intended message w_{12} but also the message w_{13} intended for User 3, provided message w_{13} was being transmitted at a rate that is supported at User 3. The same argument holds for all other broadcast scenarios, in each of which only a distinct user is the stronger one. Motivated by these observations, we propose a variation of the decoding policies for the transmitters suited for the ordering in (3.50). This new decoding policy, which we will call Policy II, is summarized in Table 3.5. The decoding policy for the specific ordering in(3.51) is then simply obtained by substituting $i = 1$, $j = 2$, $k = 3$ in Table 3.5.

Although the derivation of decoding strategies closely followed the ideas in Section 3.2, the structure of the resulting cooperation signals are significantly different. From Table 3.5, one can observe that the messages w_{13}, w_{21} and w_{32} are known to all transmitters, but there are no pairs of messages known to more than one transmitter. The message w_{12} is only known to the transmitters 1 and 2; w_{23} only to 2 and 3, and w_{31} only to 1 and 3. This grouping of common information calls for the following new way to form the cooperation signals. We use one cooperation signal common to all users, which is a function of three sub-messages, and three other cooperation signals common to each pair of users, which are functions of just one sub-message each, a little reminiscent of the coding for the relay channel. By a suitable extension of the codebook generation process described in [27], [47], we perform the codebook generation and encoding as summarized in Table 3.6. In Table 3.6, the sub messages w_{ij} denote the messages received in the previous block: the cooperation signals depend on the messages received in previous block, and new information is also encoded into codewords X_{ij}, taking into account the messages received in the previous block. The order in the codebook generation is also observed in Table III: the collective cooperation signals U are generated first, then the pairwise cooperation signals, and so on.

Then, the signals transmitted by each user can be generated by block Markov superposition encoding as follows:

$$X_1 = \sqrt{P_{10}}X_{10} + \sqrt{P_{12}}X_{12} + \sqrt{P_{13}}X_{13} + \sqrt{P_{1U_1}}U_1 + \sqrt{P_{1U_3}}U_3 + \sqrt{P_{1U}}U \quad (3.52)$$
$$X_2 = \sqrt{P_{20}}X_{20} + \sqrt{P_{21}}X_{21} + \sqrt{P_{23}}X_{23} + \sqrt{P_{2U_1}}U_1 + \sqrt{P_{2U_2}}U_2 + \sqrt{P_{1U}}U \quad (3.53)$$
$$X_3 = \sqrt{P_{30}}X_{30} + \sqrt{P_{31}}X_{31} + \sqrt{P_{32}}X_{32} + \sqrt{P_{3U_2}}U_2 + \sqrt{P_{3U_3}}U_3 + \sqrt{P_{3U}}U \quad (3.54)$$

	Block I	Block II
User I	$X_{10}(w_{10}(1), X_{12}(1), X_{13}(1))$	$X_{10}(w_{10}(2), X_{12}(2), X_{13}(2))$
	$X_{12}(w_{12}(1), U_1(0), U(0))$	$X_{12}(w_{12}(2), U_1(1), U(1))$
	$X_{13}(w_{13}(1), U_3(0), U(0))$	$X_{12}(w_{13}(2), U_3(1), U(1))$
	$U(w_{13}(0), w_{21}(0), w_{32}(0))$	$U(w_{13}(1), w_{21}(1), w_{32}(1))$
	$U_1(w_{12}(0), U(0))$	$U_1(w_{12}(1), U(1))$
	$U_3(w_{31}(0), U(0))$	$U_3(w_{31}(1), U(1))$
User II	$X_{20}(w_{20}(1), X_{21}(1), X_{23}(1))$	$X_{20}(w_{20}(2), X_{21}(2), X_{23}(2))$
	$X_{21}(w_{21}(1), U_1(0), U(0))$	$X_{21}(w_{21}(2), U_1(1), U(1))$
	$X_{23}(w_{23}(1), U_2(0), U(0))$	$X_{23}(w_{23}(2), U_3(1), U(1))$
	$U(w_{13}(0), w_{21}(0), w_{32}(0))$	$U(w_{13}(1), w_{21}(1), w_{32}(1))$
	$U_1(w_{12}(0), U(0))$	$U_1(w_{12}(1), U(1))$
	$U_2(w_{23}(0), U(0))$	$U_2(w_{23}(1), U(1))$
User III	$X_{30}(w_{30}(1), X_{31}(1), X_{32}(1))$	$X_{30}(w_{30}(2), X_{31}(2), X_{32}(2))$
	$X_{31}(w_{31}(1), U_3(0), U(0))$	$X_{31}(w_{31}(2), U_3(1), U(1))$
	$X_{32}(w_{32}(1), U_2(0), U(0))$	$X_{32}(w_{32}(2), U_2(1), U(1))$
	$U(w_{13}(0), w_{21}(0), w_{32}(0))$	$U_1(w_{13}(1), w_{21}(1), w_{32}(1))$
	$U_2(w_{23}(0), U(0))$	$U_2(w_{23}(1), U(1))$
	$U_3(w_{31}(0), U(0))$	$U_3(w_{31}(1), U(1))$

Table 3.6: Block Markov encoding for policy II: mapping of codewords to messages

Here, the codewords X_{i0} carry the fresh information intended for the receiver, X_{ij} carry the information intended for transmitter j for cooperation in the next block. The cooperation codeword U carries the common information sent by all three users; and the cooperation codewords U_1, U_2 and $U3$ relay the sub-messages common to each pair of users, for the resolution of the remaining uncertainty from the previous block. All codewords are chosen from unit-power Gaussian distributions. The transmit powers are then captured by the powers associated with each component, which are required to satisfy the average power constraints.

$$P_{10}+P_{12}+P_{13}+P_{1U_1}+P_{1U} \leq P_1 \tag{3.55}$$

$$P_{20}+P_{21}+P_{23}+P_{2U_1}+P_{2U_3}+P_{2U} \leq P_2 \tag{3.56}$$

$$P_{30}+P_{31}+P_{32}+P_{3U_3}+P_{3U} \leq P_3 \tag{3.57}$$

Note that, encoding and decoding Policies I and II described in Table 3.2 and 3.5 respectively are sufficient to cover all possible channel state orderings, and can be used adaptively based on the channel state information, to maximize the rates.

Therefore, the proposed policy in this section complements the policy of Section 3.2, thereby yielding a channel adaptive three user block Markov encoding policy.

3.3.3 Achievable rates

Before proceeding to characterize the rate region, we make one further simplification to the encoding policy. In [48], it has been shown that when the cooperative links are stronger than the direct links, the optimum strategy of the users is to send only cooperative information, and discard X_{i0}. Although in order to prove a similar result for the three user cooperative MAC, the rate regions need to be established, and then power optimized for the general case; we simply choose to assume that the cooperative links are uniformly stronger than the direct links of the users, and drop the codewords X_{i0} from our encoding policy, so that the rate regions are easier to obtain and simulate.

The rate constraints bounding the achievable rate region are easiest viewed in two groups: those necessary for reliable decoding at the transmitters, and those necessary for reliable decoding at the ultimate receiver.

Theorem 3.2. Adaptive Encoding For Three User Cooperative Multiple Access Channel-Policy II:

An achievable rate region for the system given in Section 3.3 is the closure of the convex hull of all rate pairs (R_1, R_2, R_3) *such that* $R_1 = R_{12}+R_{13}$, $R_2 = R_{21}+R_{23}$, *and* $R_3 = R_{31} + R_{32}$ *where* $\{R_{12}, R_{13}, R_{21}, R_{23}, R_{31}, R_{32}\}$ *satisfy the constraints*

$$R_{21} \leq E\left[\log\left(1+\frac{s_{21}P_{21}}{A}\right)\right] \tag{3.58}$$

$$R_{31} \leq E\left[\log\left(1+\frac{s_{31}P_{31}}{A}\right)\right] \tag{3.59}$$

$$R_{32} \leq E\left[\log\left(1+\frac{s_{31}P_{32}}{A}\right)\right] \tag{3.60}$$

$$R_{12} \leq E\left[\log\left(1+\frac{s_{12}P_{12}}{B}\right)\right] \tag{3.61}$$

$$R_{13} \leq E\left[\log\left(1+\frac{s_{12}P_{13}}{B}\right)\right] \tag{3.62}$$

$$
\begin{aligned}
R_{32} &\leq E\left[\log\left(1+\frac{s_{32}P_{32}}{B}\right)\right] && (3.63)\\
R_{13} &\leq E\left[\log\left(1+\frac{s_{13}P_{13}}{C}\right)\right] && (3.64)\\
R_{23} &\leq E\left[\log\left(1+\frac{s_{23}P_{21}}{C}\right)\right] && (3.65)\\
R_{23} &\leq E\left[\log\left(1+\frac{s_{23}P_{23}}{C}\right)\right] && (3.66)\\
R_{21}+R_{31} &\leq E\left[\log\left(1+\frac{s_{21}P_{21}+s_{31}P_{31}}{A}\right)\right] && (3.67)\\
R_{21}+R_{32} &\leq E\left[\log\left(1+\frac{s_{21}P_{21}+s_{31}P_{32}}{A}\right)\right] && (3.68)\\
R_{3} &\leq E\left[\log\left(1+\frac{s_{31}(P_{31}+P_{32})}{A}\right)\right] && (3.69)\\
R_{21}+R_{3} &\leq E\left[\log\left(1+\frac{s_{21}P_{21}+s_{31}(P_{31}+P_{32})}{A}\right)\right] && (3.70)\\
R_{1} &\leq E\left[\log\left(1+\frac{s_{12}(P_{12}+P_{13})}{B}\right)\right] && (3.71)\\
R_{12}+R_{32} &\leq E\left[\log\left(1+\frac{s_{12}P_{12}+s_{32}P_{32}}{B}\right)\right] && (3.72)\\
R_{13}+R_{32} &\leq E\left[\log\left(1+\frac{s_{12}P_{13}+s_{32}P_{32}}{B}\right)\right] && (3.73)\\
R_{1}+R_{32} &\leq E\left[\log\left(1+\frac{s_{12}(P_{12}+P_{13})+s_{32}P_{32}}{B}\right)\right] && (3.74)\\
R_{13}+R_{21} &\leq E\left[\log\left(1+\frac{s_{13}P_{13}+s_{23}P_{21}}{C}\right)\right] && (3.75)\\
R_{13}+R_{23} &\leq E\left[\log\left(1+\frac{s_{13}P_{13}+s_{23}P_{23}}{C}\right)\right] && (3.76)\\
R_{2} &\leq E\left[\log\left(1+\frac{s_{23}(P_{21}+P_{23})}{C}\right)\right] && (3.77)\\
R_{13}+R_{2} &\leq E\left[\log\left(1+\frac{s_{13}P_{13}+s_{23}(P_{21}+P_{23})}{C}\right)\right] && (3.78)\\
R_{12} &\leq E\left[\log\left(1+s_{10}(P_{12}+P_{1U1})+s_{20}(P_{21}+P_{2U1})+D\right)\right] && (3.79)\\
R_{31} &\leq E\left[\log\left(1+s_{10}(P_{13}+P_{1U3})+s_{30}(P_{31}+P_{3U3})+E\right)\right] && (3.80)\\
R_{23} &\leq E\left[\log\left(1+s_{20}(P_{23}+P_{2U2})+s_{30}(P_{32}+P_{3U2})+F\right)\right] && (3.81)\\
R_{12}+R_{31} &\leq E\left[\log\left(1+s_{10}(P_{12}+P_{13}+P_{1U1}+P_{1U3})\right.\right.\\
&\qquad \left.\left.+s_{20}(P_{21}+P_{2U1})+s_{30}(P_{31}+P_{3U3})+D+E\right)\right] && (3.82)\\
R_{23}+R_{31} &\leq E\left[\log\left(1+s_{10}(P_{13}+P_{1U3})+s_{20}(P_{23}+P_{2U2})\right.\right.
\end{aligned}
$$

$$+s_{30}(P_{31}+P_{32}+P_{3U2}+P_{3U3})+E+F)] \quad (3.83)$$

$$R_{12}+R_{23} \leq E\left[\log\left(1+s_{10}(P_{12}+P_{1U1})\right.\right.$$
$$+s_{20}(P_{21}+P_{23}+P_{2U1}+P_{2U2})$$
$$+s_{30}(P_{32}+P_{3U2})+D+F)] \quad (3.84)$$

$$R_{12}+R_{23}+R_{31} \leq E\left[\log\left(1+s_{10}(P_{12}+P_{13}+P_{1U1}+P_{1U3})\right.\right.$$
$$+s_{20}(P_{21}+P_{23}+P_{2U1}+P_{2U2})$$
$$+s_{30}(P_{31}+P_{32}+P_{3U2}+P_{3U3})$$
$$+D+E+F)] \quad (3.85)$$

$$R_1+R_2+R_3 \leq E\left[\log\left(1+s_{10}P_1+s_{20}P_2+s_{30}P_3+D+E+F+G\right)\right] \quad (3.86)$$

Here, the interference plus noise terms A, B, C and coherent combining gains D, E, F, G are defined as,

$$A = s_{21}P_{2U2}+s_{31}P_{3U2}+2\sqrt{s_{21}s_{31}P_{2U2}P_{3U2}}+1 \quad (3.87)$$

$$B = s_{12}P_{1U3}+s_{32}P_{3U3}+2\sqrt{s_{12}s_{32}P_{1U3}P_{3U3}}+1 \quad (3.88)$$

$$C = s_{13}P_{1U1}+s_{23}P_{2U1}+2\sqrt{s_{13}s_{23}P_{1U1}P_{2U1}}+1 \quad (3.89)$$

$$D = 2\sqrt{s_{10}s_{20}P_{1U1}P_{2U1}} \quad (3.90)$$

$$E = 2\sqrt{s_{10}s_{30}P_{1U3}P_{3U3}} \quad (3.91)$$

$$F = 2\sqrt{s_{20}s_{30}P_{2U2}P_{3U2}} \quad (3.92)$$

$$G = 2(\sqrt{s_{10}s_{20}P_{1U}P_{2U}}+\sqrt{s_{10}s_{30}P_{1U}P_{3U}}+\sqrt{s_{20}s_{30}P_{2U}P_{3U}}) \quad (3.93)$$

Proof. In the proof of Theorem 3.1, it is easy to see that for each transmitter we have a multiple access channel with a group of independent messages that need to be decoded, and an extra message which will be treated as noise. Classical arguments on achievable regions for multiple access channels [15] can be used to obtain the rate constraints in Theorem 3.2, corresponding to the decodings at Users 1, 2 and 3 respectively.

It is interesting to note that the users suffer from the effect of coherent combining in the interference terms. The major difference from the rate region in Section

Fading Distribution, obeying (3.51)		
Link Gains	Coefficient Set I	Coefficient Set II
$s_{10},\ s_{20},\ s_{30}$	$\{0.1, 0.3, 0.5, 0.7, 0.9\}$	$\{0.5, 0.55, 0.6, 0.65, 0.7\}$
$s_{13},\ s_{21},\ s_{32}$	$\{1.1, 1.3, 1.5, 1.7, 1.9\}$	$\{0.8, 0.85, 0.9, 0.95, 1.0\}$
$s_{12},\ s_{23},\ s_{31}$	$\{2.1, 2.3, 2.5, 2.7, 2.9\}$	$\{1.1, 1.15, 1.2, 1.25, 1.3\}$
Fading Distribution, obeying (3.49)		
Link Gains	Coefficient Set III	Coefficient Set VI
$s_{10},\ s_{20},\ s_{30}$	$\{0.1, 0.3, 0.5, 0.7, 0.9\}$	$\{0.5, 0.55, 0.6, 0.65, 0.7\}$
$s_{13},\ s_{23},\ s_{31}$	$\{1.1, 1.3, 1.5, 1.7, 1.9\}$	$\{0.8, 0.85, 0.9, 0.95, 1.0\}$
$s_{12},\ s_{21},\ s_{32}$	$\{2.1, 2.3, 2.5, 2.7, 2.9\}$	$\{1.1, 1.15, 1.2, 1.25, 1.3\}$

Table 3.7: Coefficient Distribution, obeying (3.49) and (3.51)

3.2 is that, the rate constraints are now symmetric, and the rates of User 3 are now less prone to interference, whereas there is some added interference at User 2, due to the decoding assumption. In the simulation results section, we will demonstrate that sometimes the more symmetric Policy II may in fact produce better achievable rates, even for channel states it is not designed for, i.e., those satisfying (3.49), which justifies the novelty and usefulness of Policy II proposed in this section.

The rate constraints for error free decoding at the receiver are also obtained by using capacity results for the traditional MAC. However, one has to take into account the effect of backwards decoding: in a given block, the receiver first decodes the cooperative information, which consists of sub-messages encoded in groups into codewords U, U_1, U_2 and U_3. Therefore, the sub-messages w_{13}, w_{21} and w_{32}, should be treated as one single message and should be jointly decoded. Keeping this in mind, the rate constraints that need to be satisfied at the receiver are obtained as given in equations (3.79)-(3.86). □

3.3.4 Simulation results

In this section, we demonstrate the usefulness of the proposed three user cooperation strategies by evaluating the achievable rate region for several fading scenarios, and comparing it to the corresponding two user cooperative system, as

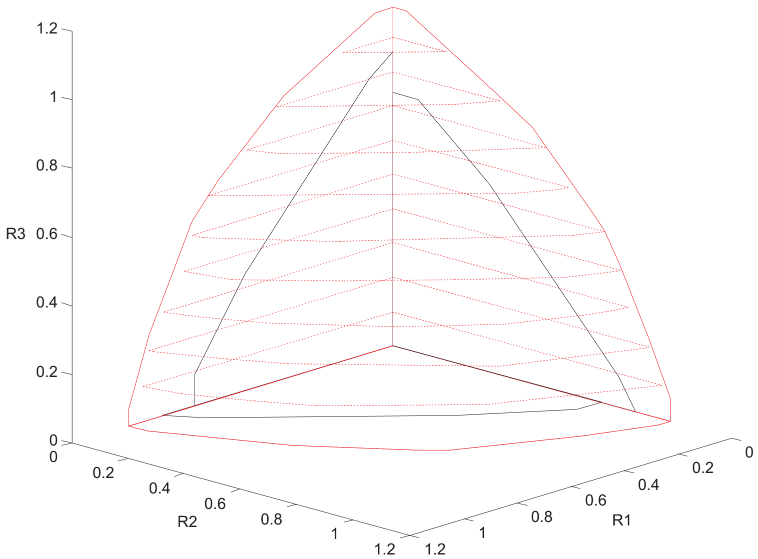
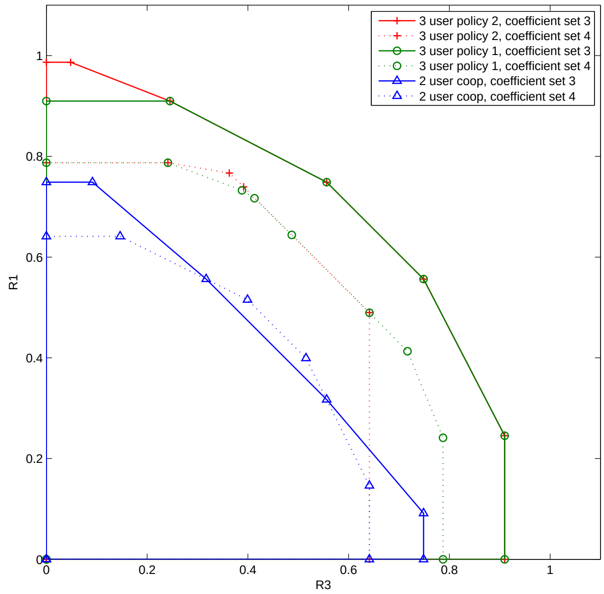
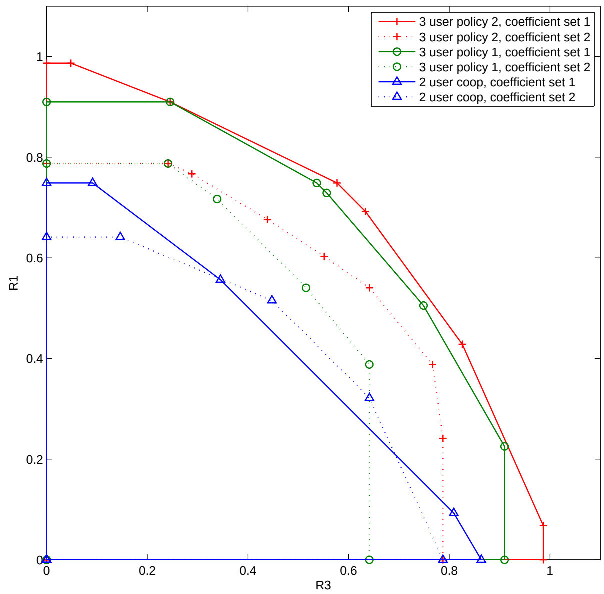


Figure 3.4: The 3-D achievable rate region for the three user cooperative MAC, compared with two user cooperative rate regions

well as the encoding/decoding policy proposed in section 3.2. We first evaluate the rate region achievable by Policy II, for a fading distribution which satisfies the assumption in (3.51), that is, s_{10}, s_{20}, s_{30} are i.i.d uniform random variables taking the values from the set $\{0.1, 0.3, 0.5, 0.7, 0.9\}$, s_{13}, s_{21}, s_{32} are i.i.d taking values from $\{1.1, 1.3, 1.5, 1.7, 1.9\}$ and s_{12}, s_{23}, s_{31} are also i.i.d with values $\{2.1, 2.3, 2.5, 2.7, 2.9\}$. The average transmit power for each user is chosen to be 1 in all simulations in this section. The 3-D achievable rate region is plotted in Figure 3.4 (outer region), along with the 2-D two user cooperative MAC achievable rate regions [27] (inner regions only on $R_i - R_j$ planes), evaluated for the same fading distributions. There are two important observations: firstly, the presence of the third user improves the achievable rates significantly, when compared to the two user strategy: simply compare the two strategies on the planes corresponding to $R_i = 0$. Secondly, the maximum values for individual rates are asymmetric for the two user cooperation case, due to the asymmetry in the inter user links. Therefore, the separately obtained 2-D achievable rate regions on each plane has different intersections with the corresponding axes. However, in three user cooperation, the presence of the third user helps the user with the worse cooperative link by presenting another option to relay its information, thereby symmetrizing the achievable rate region, and providing a fairer rate distribution.



(a) Rate region for Users 1 and 3, with User 2 acting as a relay, coefficients obeying (3.49)



(b) Rate region for Users 1 and 3, with User 2 acting as a relay, coefficients obeying (3.51)

Figure 3.5: Achievable rate regions for the three user cooperative MAC

We next compare the rate regions achievable by policies I and II, under four different fading distributions; two of which obey (3.49), and the remaining two of which obey (3.51). The fading distributions are again chosen as independent uniform random variables, as summarized in Tables 3.7.

Figure 3.5(a) illustrates the rate regions achievable by Policy I, Policy II, and the two user cooperative MAC, under the assumption that the fading distributions obey (3.51). This is the ordering for which the three user cooperation Policy II in this section is designed. Therefore, it is expected that the proposed policy gives significantly larger achievable rates than the other policies, for the same channel set. It is worth mentioning that, the Policy I of section 3.2 performed surprisingly poorly when compared to the 2 user cooperation strategy, which is simply a special case of Policy I encoding-wise. The reason for this phenomenon is that, the decoding rules are strictly dictated in Policy I, and we force User 2, which has a relatively poor incoming link from User 3, to decode all sub-messages. Therefore, even if it will not participate in the transmission, User 2 creates a bottleneck for the rate of User 3. One last remark: the rate plane $R_1 - R_3$ was chosen arbitrarily for the comparison, all other rate regions also look similar.

Figure 3.5(b), two alternative sets of fading distributions, each of which obey (3.49) are considered. Surprisingly, especially for coefficient set III, our proposed Policy II performs better than Policy I, although it was not designed for the assumed ordering of the fading values. This shows that, enforcing User 2 to decode all messages, while treating most messages as noise at User 3 may be more limiting than letting each user decode an equal number of sub-messages, under certain situations. When the potential channel states get closer to each other, as in coefficient set 4, Policy II partly outperforms Policy I. Also, Policy II outperforms the two user cooperative strategy under this ordering, as expected (decoding at User 2 is no longer a bottleneck).

Now, we evaluate the achievable rate region in a Rayleigh fading channel that changes over time. At the beginning of this section, we compared our proposed channel adaptive encoding structure to the corresponding two user cooperative

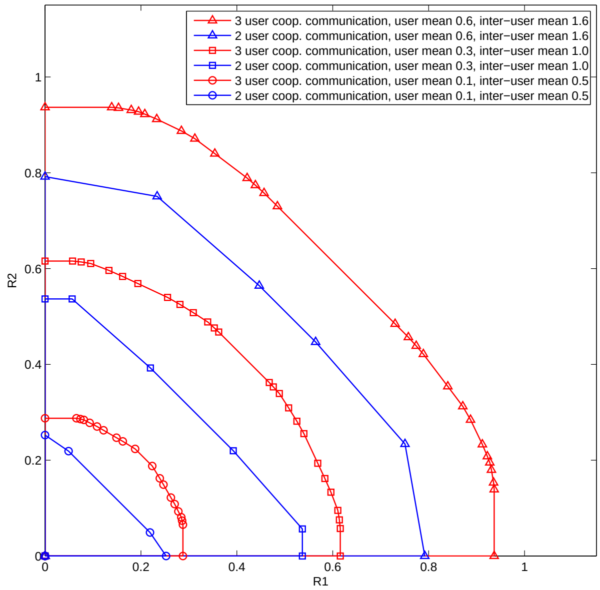


Figure 3.6: The 3-D achievable rate region for the three user cooperative MAC in a Rayleigh fading channel, compared with two user cooperative rate regions

Rayleigh Fading Parameters			
LINK GAINS	PARAMETER SET I	PARAMETER SET II	PARAMETER SET III
s_{10}, s_{20}, s_{30}	$\{0.1\}$	$\{0.3\}$	$\{0.6\}$
s_{12}, s_{13}, s_{21}	$\{0.5\}$	$\{1\}$	$\{1.6\}$
s_{23}, s_{31}, s_{32}	$\{0.5\}$	$\{1\}$	$\{1.6\}$

Table 3.8: Coefficient Distribution for Rayleigh Fading

system under uniformly distributed fading scenarios. The main reason of performing under uniform distribution was that our proposed encoding structure must have been performed in Policy I and II for certain to evaluate and create comparable simulation results.

We assume the same encoding structures as proposed in Section 3.2 and 3.3. The Rayleigh parameters for direct links s_{10}, s_{20}, s_{30} are i.i.d random variables taking the values from the parameter set $\{0.1, 0.3, 0.6\}$, and for inter-user links s_{12}, s_{13}, s_{21}, s_{23}, s_{31}, s_{32} are also i.i.d taking values from the parameter $\{0.5, 1, 1.6\}$.

In Figure 3.6, we see that the achievable rate region under the Rayleigh channel outperforms better than two user cooperative system for each Rayleigh fading parameter set that is because the proposed adaptive encoding structure adapts the current inter-user fading state assigned to different policy in Table 3.1.

Chapter 4

Pairwise and Collective Encoding and Decoding Strategies and Rate Regions for the Three User Cooperative Multiple Access Channel

In this chapter, we propose a new block Markov superposition encoding strategy for a three user cooperative Gaussian multiple access channel (MAC), which enables all three users to cooperate collectively as well as in pairs. We obtain the resulting achievable rate expressions and compare them with existing two and three user cooperative strategies. We demonstrate that significant rate gains may be possible, without resorting to adaptive encoding/decoding techniques. We investigate the contributions from pairwise and collective cooperation signals while achieving tuples on the rate region boundary, and compare by simulations the sum rates achievable by two user versus three user grouping in cooperative MACs with fixed total resources.

An attempt to generalize the results of [27], [47] to more than two users was made in Section 3.2 and 3.2. There, we have introduced the three user MAC with generalized feedback model which contains multiple access relay channel (MARC), multiple relay network (MRN) and parallel relay network (PRN) with three users and one receiver as its special cases, and obtained achievable rate regions based on three user channel adaptive block Markov encoding (BME) and backwards decoding. The proposed BME strategy assigned varying roles to users in cooperation: users with stronger receive links decoded more cooperative signals, and therefore participated in cooperative transmissions more actively. Although this non-trivial extension of BME was shown to produce considerable rate gains, it has the drawback that it requires instantaneous adaptation of encoding/decoding policies, which increases system complexity. A MIMO extension of the three user cooperative MAC was considered more recently in [49], but there either conferencing encoders were assumed, or the common messages among the users were assumed to be already established, thereby not requiring any resources, and not causing any constraints for inter-user transmissions.

4.1 System Model

We consider a fading three user cooperative Gaussian MAC with full duplex operation. The system is modeled by

$$Y_0 = \sqrt{h_{10}}X_1 + \sqrt{h_{20}}X_2 + \sqrt{h_{30}}X_3 + N_0 \tag{4.1}$$

$$Y_1 = \sqrt{h_{21}}X_2 + \sqrt{h_{31}}X_3 + N_1 \tag{4.2}$$

$$Y_2 = \sqrt{h_{12}}X_1 + \sqrt{h_{32}}X_3 + N_2 \tag{4.3}$$

$$Y_3 = \sqrt{h_{13}}X_1 + \sqrt{h_{23}}X_2 + N_3 \tag{4.4}$$

where X_i is the symbol transmitted by node i, satisfying $E[X_i^2] \leq P_i$, Y_j is the effective received symbol at node j after subtraction of any self interference, and the receiver is denoted by $j = 0$; $N_j \sim N(0, 1)$ is the white Gaussian noise at node j, and $\sqrt{s_{ij}}$ are the normalized (for unit noise power) Rayleigh distributed fading coefficients with parameters γ_{ij}, the realizations of which are assumed to be known by both the transmitters and the receiver. A few words on the notation used throughout this section: from now on, we use the user indices $i \neq j \neq k$ to denote distinct elements of the set $S = \{1, 2, 3\}$.

4.1.1 Encoding Strategy

The decoded and own messages for each user are summarized in Table 4.1. Each user selects the codewords corresponding to these messages, from the generated

	Decoded Messages	Own Messages
User I	w_{21}, w_{31}, w_{2U},w_{3U}	w_{12}, w_{13}, w_{1U}
User II	w_{12}, w_{32} , w_{1U},w_{3U}	w_{21}, w_{23}, w_{2U}
User III	w_{13}, w_{23}, w_{1U},w_{2U}	w_{31}, w_{32}, w_{3U}

Table 4.1: Decoding strategy at the transmitter, before forming the common cooperation signals

codebooks, as shown in Table 4.2. Then, the overall transmitted codeword of each user is obtained by scaling the codewords listed in Table 4.2 to have the desired power levels, and superposing them, i.e., yielding our proposed three user BME strategy. Note that, the BME strategy employs two main types of codewords: X_{12}, X_{13}, X_{1U}, X_{21}, X_{23}, X_{2U}, X_{31}, X_{32}, X_{3U}, which are used to transmit fresh information at a higher data rate than normally decodable by the receiver, and U_{12}, U_{13}, U_{21}, U_{23}, U_{31}, U_{32} and U, which are cooperatively used to resolve the receiver's remaining uncertainty from previous transmissions. In Table 4.2, the specific tasks of these codewords are also explained.

	Block I	Block II
User I	$X_{10}(w_{10}(1), U_{12}(1), U_{13}(1), X_{1U}(1))$	$X_{10}(w_{10}(2), U_{12}(2), U_{13}(2), X_{1U}(2))$
	$X_{12}(w_{12}(1), U_{12}(1), U(1))$	$X_{12}(w_{12}(2), U_{12}(2), U(2))$
	$X_{13}(w_{13}(1), U_{13}(1), U(1))$	$X_{13}(w_{13}(2), U_{13}(2), U(2))$
	$X_{1U}(w_{1U}(1), U(1))$	$X_{1U}(w_{1U}(2), U(2))$
	$U_{12}(w_{12}(0), w_{21}(0), U(1))$	$U_{12}(w_{12}(1), w_{21}(1), U(2))$
	$U_{13}(w_{13}(0), w_{31}(0), U(1))$	$U_{13}(w_{13}(1), w_{31}(1), U(2))$
	$U(w_{1U}(0), w_{2U}(0), w_{3U}(0))$	$U(w_{1U}(1), w_{2U}(1), w_{3U}(1))$
User II	$X_{20}(w_{20}(1), U_{12}(1), U_{23}(1), X_{2U}(1))$	$X_{20}(w_{20}(2), X_{12}(2), X_{23}(2), X_{2U}(2))$
	$X_{21}(w_{21}(1), U_{12}(1), U(1))$	$X_{21}(w_{21}(2), U_{12}(2), U(2))$
	$X_{23}(w_{23}(1), U_{23}(1), U(1))$	$X_{23}(w_{23}(2), U_{23}(2), U(2))$
	$X_{2U}(w_{2U}(1), U(1))$	$X_{2U}(w_{2U}(2), U(2))$
	$U_{12}(w_{12}(0), w_{21}(0), U(1))$	$U_{12}(w_{12}(1), w_{21}(1), U(2))$
	$U_{23}(w_{23}(0), w_{32}(0), U(1))$	$U_{23}(w_{23}(1), w_{32}(1), U(2))$
	$U(w_{1U}(0), w_{2U}(0), w_{3U}(0))$	$U(w_{1U}(1), w_{2U}(1), w_{3U}(1))$
User III	$X_{30}(w_{30}(1), U_{13}(1), U_{23}(1), X_{3U}(1))$	$X_{30}(w_{30}(2), X_{13}(2), X_{23}(2), X_{3U}(2))$
	$X_{31}(w_{31}(1), U_{13}(1), U(1))$	$X_{31}(w_{31}(2), U_{13}(2), U(2))$
	$X_{32}(w_{32}(1), U_{23}(1), U(1))$	$X_{32}(w_{32}(2), U_{23}(2), U(2))$
	$X_{3U}(w_{3U}(1), U(1))$	$X_{3U}(w_{3U}(2), U(2))$
	$U_{13}(w_{13}(0), w_{31}(0), U(1))$	$U_{13}(w_{13}(1), w_{31}(1), U(2))$
	$U_{23}(w_{23}(0), w_{32}(0), U(1))$	$U_{23}(w_{23}(1), w_{32}(1), U(2))$
	$U(w_{1U}(0), w_{2U}(0), w_{3U}(0))$	$U(w_{1U}(1), w_{2U}(1), w_{3U}(1))$

Table 4.2: Block Markov encoding: mapping of codewords to messages

$$X_1 = \sqrt{P_{10}} X_{10} + \sqrt{P_{12}} X_{12} + \sqrt{P_{13}} X_{13} + \sqrt{P_{1XU}} X_{1U} + \sqrt{P_{U_{12}}} U_{12} \tag{4.5}$$
$$+ \sqrt{P_{U_{13}}} U_{13} + \sqrt{P_{1U}} U$$
$$X_2 = \sqrt{P_{20}} X_{20} + \sqrt{P_{21}} X_{21} + \sqrt{P_{23}} X_{23} + \sqrt{P_{2XU}} X_{2U} + \sqrt{P_{U_{21}}} U_{21} \tag{4.6}$$
$$+ \sqrt{P_{U_{23}}} U_{23} + \sqrt{P_{2U}} U$$
$$X_3 = \sqrt{P_{30}} X_{30} + \sqrt{P_{31}} X_{31} + \sqrt{P_{32}} X_{32} + \sqrt{P_{3XU}} X_{3U} + \sqrt{P_{U_{31}}} U_{31} \tag{4.7}$$
$$+ \sqrt{P_{U_{32}}} U_{32} + \sqrt{P_{3U}} U$$

Although there are many similarities to the previously proposed policies in Section 3.2 and 3.2, the main difference of our new proposed block Markov encoder is that the sub-messages are decoded only at their intended receivers. As detailed in Table 4.2, the signals are categorized into several parts, namely X_{i0}, X_{iU} and X_{ij}. X_{i0} is a function of $w_{i0}, U_{ij}, U_{ik}, U$ and carries the fresh information intended for the receiver captured by the powers associated with each component as follow.

$$P_{10} + P_{12} + P_{13} + P_{1XU} + P_{1U_{12}} + P_{1U_{13}} + P_{1U} \leq P_1 \tag{4.8}$$
$$P_{20} + P_{21} + P_{23} + P_{2XU} + P_{2U_{21}} + P_{2U_{23}} + P_{2U} \leq P_2 \tag{4.9}$$
$$P_{30} + P_{31} + P_{32} + P_{3XU} + P_{3U_{31}} + P_{3U_{32}} + P_{1U} \leq P_3 \tag{4.10}$$

Besides, X_{ij} is a function of fresh information w_{ij} intended for transmitter j, pairwise cooperative information U_{ij} and the common cooperative information U. These sub-messages will then be used in the next block to create common cooperation signals, which will be sent to the receiver. The signals U, U_{ij}, U_{ik}, U_{jk} are the cooperative codewords mapped to the common information from the previous block $(b-1)$ and are sent by groups of three, two, two and two transmitters respectively for the resolution of the remaining uncertainty from the previous block. As a main difference from BME in section(3.2) and (3.3), X_{iU} is also added to block Markov encoder and will be decoded by all users. Incooperating the codewords X_{iU} in the transmitted signal, we aim to create a common information at each user, unlimiting the rates of messages w_{12}, w_{13},

w_{21}, w_{23}, w_{31}, and w_{32}, but also these increase the number of messages that will potentially cause additional interference at the user side.

4.1.2 Achievable Rates

Theorem 4.1. Pairwise and Collective Encoding For Three User Cooperative Multiple Access Channel:

An achievable rate region for the system given in Section 4.1 is the closure of the convex hull of all rate pairs (R_1, R_2, R_3) *such that* $R_1 = R_{12} + R_{13} + R_{1U}$, $R_2 = R_{21} + R_{23} + R_{2U}$, *and* $R_3 = R_{31} + R_{32} + R_{3U}$ *where* $\{R_{12}, R_{13}, R_{1U}, R_{21}, R_{23}, R_{2U}, R_{31}, R_{32}, R_{3U}\}$ *satisfy the constraints.*

$$R_{21} \leq E\left[\log\left(1+\frac{s_{21}P_{21}}{A}\right)\right] \tag{4.11}$$

$$R_{2U} \leq E\left[\log\left(1+\frac{s_{21}P_{2XU}}{A}\right)\right] \tag{4.12}$$

$$R_{31} \leq E\left[\log\left(1+\frac{s_{31}P_{31}}{A}\right)\right] \tag{4.13}$$

$$R_{3U} \leq E\left[\log\left(1+\frac{s_{31}P_{3XU}}{A}\right)\right] \tag{4.14}$$

$$R_{21}+R_{2U} \leq E\left[\log\left(1+\frac{s_{21}(P_{21}+P_{2XU})}{A}\right)\right] \tag{4.15}$$

$$R_{21}+R_{31} \leq E\left[\log\left(1+\frac{s_{21}P_{21}+s_{31}P_{31}}{A}\right)\right] \tag{4.16}$$

$$R_{21}+R_{3U} \leq E\left[\log\left(1+\frac{s_{21}P_{21}+s_{31}P_{3XU}}{A}\right)\right] \tag{4.17}$$

$$R_{2U}+R_{31} \leq E\left[\log\left(1+\frac{s_{21}P_{2XU}+s_{31}P_{31}}{A}\right)\right] \tag{4.18}$$

$$R_{2U}+R_{3U} \leq E\left[\log\left(1+\frac{s_{21}P_{2XU}+s_{31}P_{3XU}}{A}\right)\right] \tag{4.19}$$

$$R_{31}+R_{3U} \leq E\left[\log\left(1+\frac{s_{31}(P_{31}+P_{3XU})}{A}\right)\right] \tag{4.20}$$

$$R_{21}+R_{2U}+R_{31} \leq E\left[\log\left(1+\frac{s_{21}(P_{21}+P_{2XU})+s_{31}P_{31}}{A}\right)\right] \tag{4.21}$$

$$R_{21}+R_{2U}+R_{3U} \leq E\left[\log\left(1+\frac{s_{21}(P_{21}+P_{2XU})+s_{31}P_{3XU}}{A}\right)\right] \tag{4.22}$$

$$R_{21}+R_{31}+R_{3U} \leq E\left[\log\left(1+\frac{s_{21}P_{21}+s_{31}(P_{31}+P_{3XU})}{A}\right)\right] \tag{4.23}$$

$$R_{2U}+R_{31}+R_{3U} \leq E\left[\log\left(1+\frac{s_{21}P_{2XU}+s_{31}(P_{31}+P_{3XU})}{A}\right)\right] \tag{4.24}$$

$$R_{21}+R_{2U}+R_{31}+R_{3U} \leq E\left[\log\left(1+\frac{s_{21}(P_{21}+P_{2XU})+s_{31}(P_{31}+P_{2XU})}{A}\right)\right] \tag{4.25}$$

$$R_{12} \leq E\left[\log\left(1+\frac{s_{12}P_{12}}{B}\right)\right] \tag{4.26}$$

$$R_{1U} \leq E\left[\log\left(1+\frac{s_{12}P_{1XU}}{B}\right)\right] \tag{4.27}$$

$$R_{32} \leq E\left[\log\left(1+\frac{s_{32}P_{32}}{B}\right)\right] \tag{4.28}$$

$$R_{3U} \leq E\left[\log\left(1+\frac{s_{32}P_{3XU}}{B}\right)\right] \tag{4.29}$$

$$R_{12}+R_{1U} \leq E\left[\log\left(1+\frac{s_{12}(P_{12}+P_{1XU})}{B}\right)\right] \tag{4.30}$$

$$R_{12}+R_{32} \leq E\left[\log\left(1+\frac{s_{12}P_{12}+s_{32}P_{32}}{B}\right)\right] \tag{4.31}$$

$$R_{12}+R_{3U} \leq E\left[\log\left(1+\frac{s_{12}P_{12}+s_{32}P_{3XU}}{B}\right)\right] \tag{4.32}$$

$$R_{1U}+R_{32} \leq E\left[\log\left(1+\frac{s_{12}P_{1XU}+s_{32}P_{32}}{B}\right)\right] \tag{4.33}$$

$$R_{1U}+R_{3U} \leq E\left[\log\left(1+\frac{s_{12}P_{1XU}+s_{32}P_{3XU}}{B}\right)\right] \tag{4.34}$$

$$R_{32}+R_{3U} \leq E\left[\log\left(1+\frac{s_{32}(P_{32}+P_{3XU})}{B}\right)\right] \tag{4.35}$$

$$R_{12}+R_{1U}+R_{32} \leq E\left[\log\left(1+\frac{s_{12}(P_{12}+P_{1XU})+s_{32}P_{32}}{B}\right)\right] \tag{4.36}$$

$$R_{12}+R_{1U}+R_{3U} \leq E\left[\log\left(1+\frac{s_{12}(P_{12}+P_{1XU})+s_{32}P_{3XU}}{B}\right)\right] \tag{4.37}$$

$$R_{12}+R_{32}+R_{3U} \leq E\left[\log\left(1+\frac{s_{12}P_{12}+s_{32}(P_{32}+P_{3XU})}{B}\right)\right] \tag{4.38}$$

$$R_{1U}+R_{32}+R_{3U} \leq E\left[\log\left(1+\frac{s_{12}P_{1XU}+s_{32}(P_{32}+P_{3XU})}{B}\right)\right] \tag{4.39}$$

$$R_{12}+R_{1U}+R_{32}+R_{3U} \leq E\left[\log\left(1+\frac{s_{12}(P_{12}+P_{1XU})+s_{32}(P_{32}+P_{3XU})}{B}\right)\right] \tag{4.40}$$

$$R_{13} \leq E\left[\log\left(1+\frac{s_{13}P_{13}}{C}\right)\right] \tag{4.41}$$

$$R_{1U} \leq E\left[\log\left(1+\frac{s_{13}P_{1XU}}{C}\right)\right] \tag{4.42}$$

$$R_{23} \leq E\left[\log\left(1+\frac{s_{23}P_{23}}{C}\right)\right] \tag{4.43}$$

$$R_{2U} \leq E\left[\log\left(1+\frac{s_{23}P_{2XU}}{C}\right)\right] \tag{4.44}$$

$$R_{13}+R_{1U} \leq E\left[\log\left(1+\frac{s_{13}(P_{13}+P_{1XU})}{C}\right)\right] \tag{4.45}$$

$$R_{13}+R_{23} \leq E\left[\log\left(1+\frac{s_{13}P_{13}+s_{23}P_{23}}{C}\right)\right] \tag{4.46}$$

$$R_{13}+R_{2U} \leq E\left[\log\left(1+\frac{s_{13}P_{13}+s_{23}P_{2XU}}{C}\right)\right] \tag{4.47}$$

$$R_{1U}+R_{23} \leq E\left[\log\left(1+\frac{s_{13}P_{1XU}+s_{23}P_{23}}{C}\right)\right] \tag{4.48}$$

$$R_{1U}+R_{2U} \leq E\left[\log\left(1+\frac{s_{13}P_{1XU}+s_{23}P_{2XU}}{C}\right)\right] \tag{4.49}$$

$$R_{23}+R_{2U} \leq E\left[\log\left(1+\frac{s_{23}(P_{23}+P_{2XU})}{C}\right)\right] \tag{4.50}$$

$$R_{13}+R_{1U}+R_{23} \leq E\left[\log\left(1+\frac{s_{13}(P_{13}+P_{1XU})+s_{23}P_{23}}{C}\right)\right] \tag{4.51}$$

$$R_{13}+R_{1U}+R_{2U} \leq E\left[\log\left(1+\frac{s_{13}(P_{13}+P_{1XU})+s_{23}P_{2XU}}{C}\right)\right] \tag{4.52}$$

$$R_{13}+R_{23}+R_{2U} \leq E\left[\log\left(1+\frac{s_{13}P_{13}+s_{23}(P_{23}+P_{2XU})}{C}\right)\right] \tag{4.53}$$

$$R_{1U}+R_{23}+R_{2U} \leq E\left[\log\left(1+\frac{s_{13}P_{1XU}+s_{23}(P_{23}+P_{2XU})}{C}\right)\right] \tag{4.54}$$

$$R_{13}+R_{1U}+R_{23}+R_{2U} \leq E\left[\log\left(1+\frac{s_{13}(P_{13}+P_{1XU})+s_{23}(P_{23}+P_{2XU})}{C}\right)\right] \tag{4.55}$$

$$\begin{aligned} R_{12}+R_{21} \leq E\,[\log\,(1+s_{10}(P_{12}+P_{1U1}) \\ +s_{20}(P_{21}+P_{2U1})+D)] \end{aligned} \tag{4.56}$$

$$\begin{aligned} R_{13}+R_{31} \leq E\,[\log\,(1+s_{10}(P_{13}+P_{1U3}) \\ +s_{30}(P_{31}+P_{3U3})+E)] \end{aligned} \tag{4.57}$$

$$\begin{aligned} R_{23}+R_{32} \leq E\,[\log\,(1+s_{20}(P_{23}+P_{2U2}) \\ +s_{30}(P_{32}+P_{3U2})+F)] \end{aligned} \tag{4.58}$$

$$\begin{aligned} R_{12}+R_{21}+R_{13}+R_{31} \leq E\,[\log\,(1+s_{10}(P_{12}+P_{13}+P_{1U1}+P_{1U3}) \\ +s_{20}(P_{21}+P_{2U1}) \\ +s_{30}(P_{31}+P_{3U3})+D+E)] \end{aligned} \tag{4.59}$$

$$R_{12}+R_{21}+R_{23}+R_{32} \leq E\,[\log\,(1+s_{10}(P_{12}+P_{1U1})$$

$$
\begin{aligned}
&+s_{20}(P_{21}+P_{23}+P_{2U1}+P_{2U2}) \\
&+s_{30}(P_{32}+P_{3U2})+D+F)]
\end{aligned}
\tag{4.60}
$$

$$
\begin{aligned}
R_{23}+R_{32}+R_{13}+R_{31} \leq E\,[\log(1+s_{10}(P_{13}+P_{1U3}) \\
+s_{20}(P_{23}+P_{2U2}) \\
+s_{30}(P_{31}+P_{32}+P_{3U2}+P_{3U3})+E+F)]
\end{aligned}
\tag{4.61}
$$

$$
\begin{aligned}
R_{12}+R_{21}+R_{23}+R_{32}+R_{13}+R_{31} \leq E\,[\log(1+s_{10}(P_{12}+P_{13}+P_{1U1}+P_{1U3}) \\
+s_{20}(P_{21}+P_{23}+P_{2U1}+P_{2U2}) \\
+s_{30}(P_{31}+P_{32}+P_{3U2}+P_{3U3}) \\
+D+E+F)]
\end{aligned}
\tag{4.62}
$$

$$
\begin{aligned}
R_1+R_2+R_3 \leq E\,[\log(1+s_{10}P_1+s_{20}P_2+s_{30}P_3 \\
+D+E+F+G)]
\end{aligned}
\tag{4.63}
$$

Proof. The decoding at the users is executed at the end of each block, based on joint typicality check. For example, the channel towards User 1 may be viewed as a two user MAC, where users 2 and 3 transmit several independent messages, but User 1 only decodes the messages w_{21}, w_{31}, w_{2U} and w_{3U}, while treating codewords devoted to other messages, namely w_{23}, w_{32} and $U_{23} = U_{32}$ as noise. Note that, assuming the previous decoding stages were error free, User 1 already knows the codewords $U_{12} = U_{21}$ and $U_{13} = U_{31}$, U and X_1 . Therefore, User 1 searches for w_{21}, w_{31}, w_{2U} and w_{3U}, that make $\{X_{21}, X_{31}, X_{2U}, X_{3U}, Y_1\}$ jointly typical, given $U_{12} = U_{21}$, $U_{13} = U_{31}$, U and X_1 . Then, using traditional results on the capacity of a MAC [15], it is straightforward to show that the probability of User 1 decoding the messages listed above incorrectly goes to zero, if the rates of these messages satisfy for each user as in Theorem 4.1.

The decoding at the receiver is performed after all B blocks of information are received, using backwards decoding. As commonly done in BME, no fresh information is transmitted in block B, hence the codewords X_{12}, X_{21}, X_{13}, X_{31}, X_{23}, X_{32}, U_{12}, U_{21}, U_{13}, U_{31}, U_{23} and U_{32} are all used to decode the pair $\{w_{12}(B-1), w_{21}(B-1)\}$, $\{w_{13}(B-1), w_{31}(B-1)\}$ and $\{w_{23}(B-1), w_{32}(B-1)\}$.

Similarly, $w_{1U}(B-1)$, $w_{2U}(B-1)$ and $w_{3U}(B-1)$ are also decoded in the last block, by using all of the received codewords, as each codeword is also a function of U. Since the messages are jointly decoded, asymptotically error free decoding is possible. □

Here, the interference plus noise terms A, B, C and coherent combining gains D, E, F, G are defined as

$$A = 1+s_{21}(P_{23}+P_{2U_{23}})+s_{31}(P_{32}+P_{3U_{23}})+2\sqrt{s_{21}s_{31}P_{2U_{23}}P_{3U_{23}}} \quad (4.64)$$

$$B = 1+s_{12}(P_{13}+P_{1U_{13}})+s_{32}(P_{31}+P_{3U_{13}})+2\sqrt{s_{12}s_{32}P_{1U_{13}}P_{3U_{13}}} \quad (4.65)$$

$$C = 1+s_{13}(P_{12}+P_{1U_{12}})+s_{23}(P_{21}+P_{2U_{12}})+2\sqrt{s_{13}s_{23}P_{1U_{12}}P_{2U_{12}}} \quad (4.66)$$

$$D = 2\sqrt{s_{10}s_{20}P_{1U_{12}}P_{2U_{12}}} \quad (4.67)$$

$$E = 2\sqrt{s_{10}s_{30}P_{1U_{13}}P_{3U_{13}}} \quad (4.68)$$

$$F = 2\sqrt{s_{20}s_{30}P_{2U_{23}}P_{3U_{23}}} \quad (4.69)$$

$$G = 2(\sqrt{s_{10}s_{20}P_{1U}P_{2U}} + \sqrt{s_{10}s_{30}P_{1U}P_{3U}} + \sqrt{s_{20}s_{30}P_{2U}P_{3U}}) \quad (4.70)$$

4.2 Simulation Results

The rate region compactly characterized in (4.11)-(4.63) is in fact governed by a total of 53 simultaneous constraints on the rates, and its simulation is a challenge in its own right. In this section, we evaluate the rate constrains under several fading scenarios, to compare our rate region to some known results, and to further investigate the usefulness of each cooperative codeword component in achieving rate tuples on the rate region boundary. In Figure 4.1, we compare our proposed strategy to adaptive BME in Section 3.3 , 2-user cooperation in [27], and an outer bound, which assumes co-located transmitters. The achievable rate regions are obtained under Rayleigh fading, with varying direct link and inter-user link average SNRs (0.5 vs 1, 0.5 vs 2 and 0.5 vs 5, respectively). For the ease of demonstration, only a slice of each of the 3-D three user rate regions, with $R_3 = 0$, (the case when User 3 acts like a relay for the other two users) are shown.

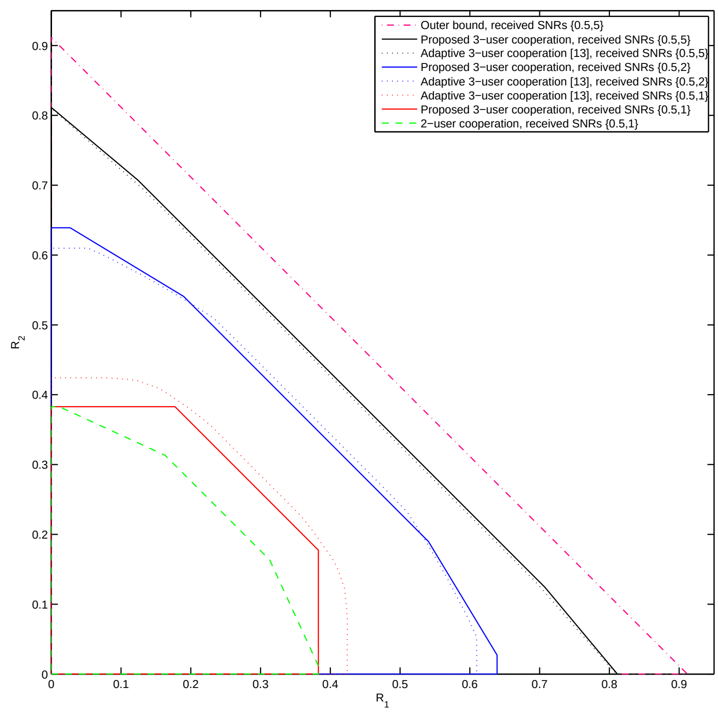


Figure 4.1: Comparison of achievable rates for two user cooperation, three user cooperation with channel adaptive BME in Section 3.3, and dedicated three user cooperation

First observation, based on the innermost three regions (SNRs 0.5 vs 1) is that, both three user cooperative strategies expectedly surpass two user cooperation in terms of achievable rates. At these SNRs, when the direct links are moderately stronger than cooperative links, the adaptive strategy of Section 3.3 gives the largest rate region. For fading set 2 (SNRs 0.5 vs 2), the proposed strategy, although non-adaptive, performs nearly as well as the adaptive strategy near the sum rate point, and even better near the single user rates. For set 3, where the cooperative links are much stronger than the direct links (SNRs 0.5 vs 5), our proposed dedicated cooperation strategy outperforms the adaptive strategy of Section 3.3 for all rate pairs. This can be explained by a closer look at the structure of the encoding policy: the collective cooperation in our BME strategy is established through dedicated messages, w_{iU}, which are then mapped to the codeword U; whereas in Section 3.2 and 3.3, the users decode as many of the pairwise cooperative messages as possible, even those not intended for themselves, to form the collective cooperative codewords. When the channel gains among the

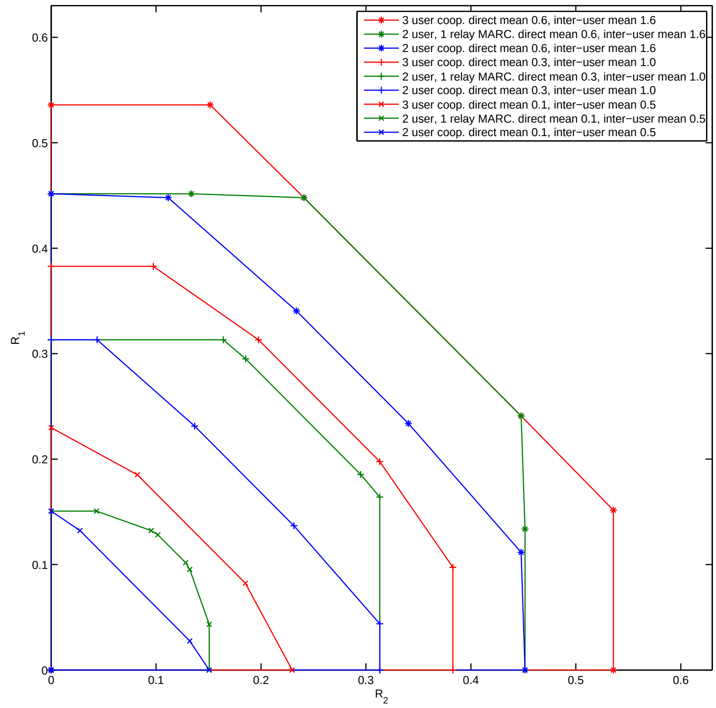


Figure 4.2: Comparison of achievable rates for two user cooperation, two-usernone-relay MARC, and dedicated three user cooperation

users are equally very strong, the latter approach puts additional unnecessary constraints on the rates at the transmitters, while for our proposed non-adaptive approach, the power distribution achieving the points on the capacity region dictates that the users should not use the pairwise cooperation signals (X_{ij}), and instead they should only cooperate collectively via X_{iU} and U. Such encoding results in looser rate constraints at the transmitters (no noise terms due to unintended messages), and overall a better rate region. Finally, we observe that the achievable rate region is not very far from the outer bound, which is obtained under the unfair assumption that all transmitters are co-located, with common information.

In Figure 4.2, we compare the rate regions achieved by our proposed three user cooperation strategy, with those for a two user cooperative MAC, and a two user one relay MARC [11] under several Rayleigh fading scenarios with means indicated in the Figure, and equal user powers. Note that the MARC is the closest model to three user cooperative MAC: if one of the users rate is set to 0 in our achievable rate region, so as to force dedicated relaying like in the MARC,

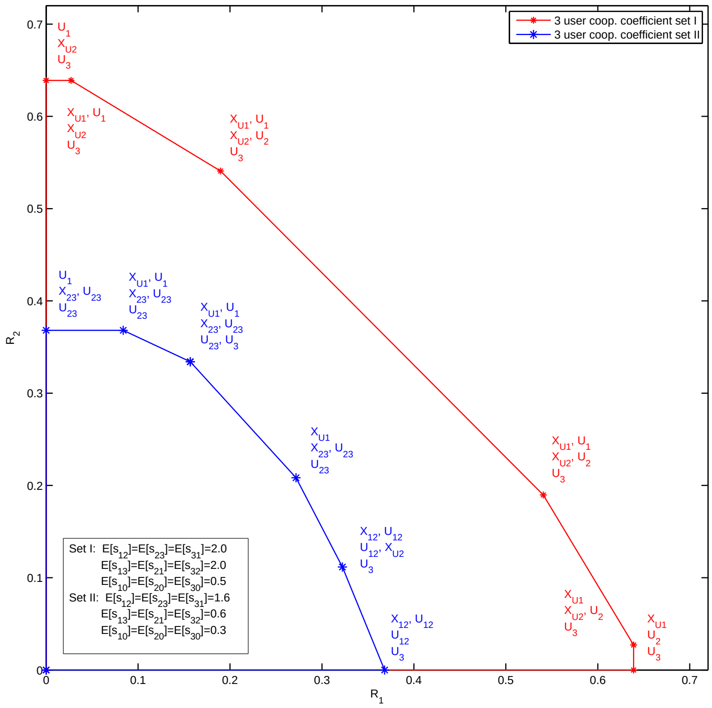


Figure 4.3: Demonstration of active codewords in three user cooperation under symmetric vs asymmetric fading.

we obtain a MARC, but with cooperating encoders. The results in Figure 2 are particularly interesting, because the comparison of our rate region with that of the MARC demonstrates the additional gain due to cooperation among the two transmitters, while being helped by a relay; whereas the comparison of our rate region with that of two user cooperation demonstrates e gain obtained by further assigning a dedicated relay to a already cooperating pair of users. We see that, the maximum individual rates (R_i intercepts in the Figure) of the users are not improved by MARC versus two-user cooperation, as in each setup there is one dedicated relay. In contrast, three user cooperation provides an additional relay per user, hence the single user rates improve, especially when the direct link gains are relatively low, which makes cooperation more valuable. MARC gives improvement versus two user cooperation near the sum rate point, because additional relay's power can be used to relay the users messages, while in two user cooperation, the users have to allocate some of their own power for cooperation. However, especially when the direct links are weak, and the inter-user links are

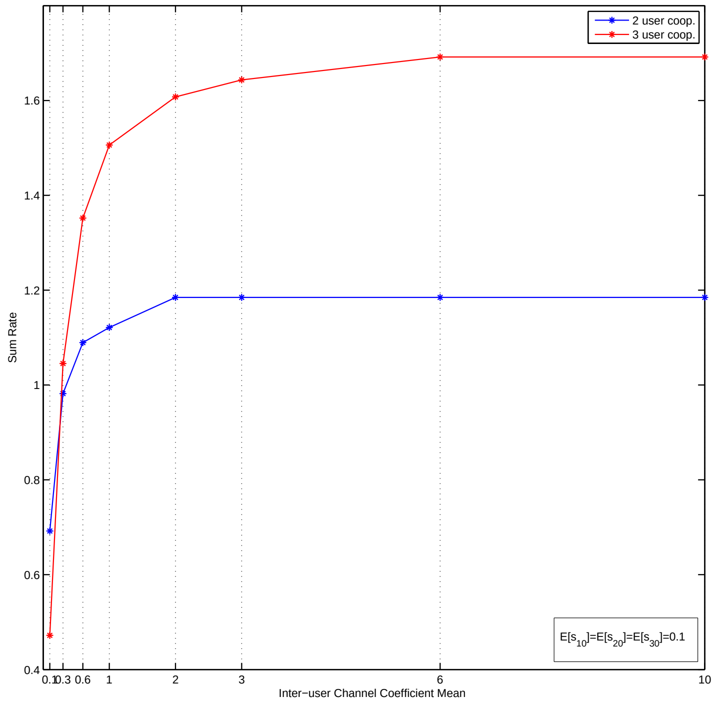


Figure 4.4: Comparison of sum rates achievable by cooperation within two user and three user partitions of a multiuser system, with fixed total resources (power and bandwidth).

strong, the cost of establishing common information is low, and additional three-way coherent combining gain of our proposed strategy improves the entire rate region, compared to MARC with the same resources. Note that, in three user cooperation, it is also possible to select a non-zero rate for the third user, in expense of some of the rate gains in R_1 and R_2, which is fairer than the MARC.

In Figure 4.3, we investigate which codewords are active while achieving several points on the rate region boundary, under symmetric and asymmetric fading. The codewords that are assigned non-zero powers are listed next to the rate tuples. In the symmetric setup (set I), collective cooperation is used throughout, and pairwise cooperation signal powers are set to zero. In the asymmetric setup (set II), the fading coefficients favor User 3 relaying messages of User 2, while User 2 relays messages of User 1, especially near R_2 and R_1 axes respectively. Near the sum rate point, we see pairwise and collective cooperation signals are both being used; interestingly User 2 decodes message of User 1, but does not relay it, and

User 3 uses the common cooperation signal $U_3 = U_1 = U$ to help User 1 while it uses pairwise cooperation signal $U_{32} = U_{23}$ to help User 2

In Figure 4.4, we investigate the gain from three user cooperation versus two user cooperation on a fairer ground: we consider a six user setup with fixed total bandwidth and identical user powers, and consider grouping the users into two triplets versus three pairs. The channels are assumed to be symmetric, therefore it is immaterial which users go into which group. The sum rate of the system is plotted against the common mean inter-user link gain, while keeping the mean direct link gain constant. We see that except very low quality inter-user links, when the extra cooperating user causes additional interference during user-side decoding, three user cooperation with the same resources is always more beneficial

Chapter 5

Cognitive Cooperative MAC with One Primary and Two Secondary Users: Achievable Rates and Optimal Power Control

In this section we consider a three user cognitive cooperative multiple access channel (MAC) with one primary, two secondary transmitters. We propose two encoding/decoding strategies with varying levels of cooperation, based on block Markov superposition encoding and backwards decoding. The first is an overlay model, where the secondary users (SUs) aid the transmission of the primary user (PU) by causally decoding part of the PU message, and forwarding it, while also cooperating among each other. The second is an underlay model, where the SUs cooperate by decoding and forwarding each others' messages, while treating the signal received from the PU as noise. In either case, the PU is guaranteed to operate at its maximum achievable single user rate. We characterize the achievable SU rate region in a fading scenario for both models, and then maximize this region as a function of transmit powers. The simulation results indicate that, the SU rate region can be significantly enlarged, especially using the overlay model.

Cognitive radio (CR) and user cooperation are both advanced techniques that rely on the propagative nature of the wireless channel, and presence of sophisticated nodes which are aware of their surroundings. Therefore, it is quite natural to design wireless protocols which are based on their joint use. While the original premise of CR, introduced in [31], was to use the unoccupied licensed spectrum (later termed interweave CR), more aggressive approaches such as underlay and overlay CR which lead to spectrum sharing were quickly developed. An excellent survey of information theoretical approaches to CR can be found in [50].

Recently, there has been significant work on overlay cognitive radio, where the system is information theoretically modeled as an interference channel with co-operating transmitters [34], [36], [51],[52], [53], [54]. These works mainly focus on cognitive networks with separate primary and secondary receivers, with different assumptions on message sharing (causal vs non-causal), and often use sophisticated encoding techniques at the transmitters to cooperatively cancel interference at the receiver. However, in cognitive setups it is often desired not to tamper with the PU transmissions, and such sophisticated techniques for interference channels are not practical for CR. There is also a line of work which focuses on resource allocation for cognitive relaying, where the goal is to buy transmission time/rights for SUs, by first speeding up PU transmissions by relaying their messages, and then using the created temporal gaps for SU message transmission (see [55], and references therein). Yet, the focus is again on interference channels, and moreover simultaneous transmissions and cooperation among SUs are not considered.

In this chapter, we focus instead on a more practical model where the PU and the SUs belong to the same network, and communicate with the same receiver. This model alleviates the problem of sharing channel state information between the primary and secondary networks, as the common receiver can coordinate transmissions. The model we use is a cognitive cooperative MAC, with one primary, two secondary users. There has been some work on throughput maximization for cognitive MAC in the literature. In [56], optimal power control policies were obtained for a cognitive MAC, but without cooperative relaying of data. In [57], power allocation for a cognitive MAC with PU-SU cooperation was considered for a non-fading scenario, but the SUs were assumed to non-causaly know the PU message, and did not cooperate among each other. In [58], a cognitive cooperative MAC with only one SU was considered, and optimal power control to guarantee

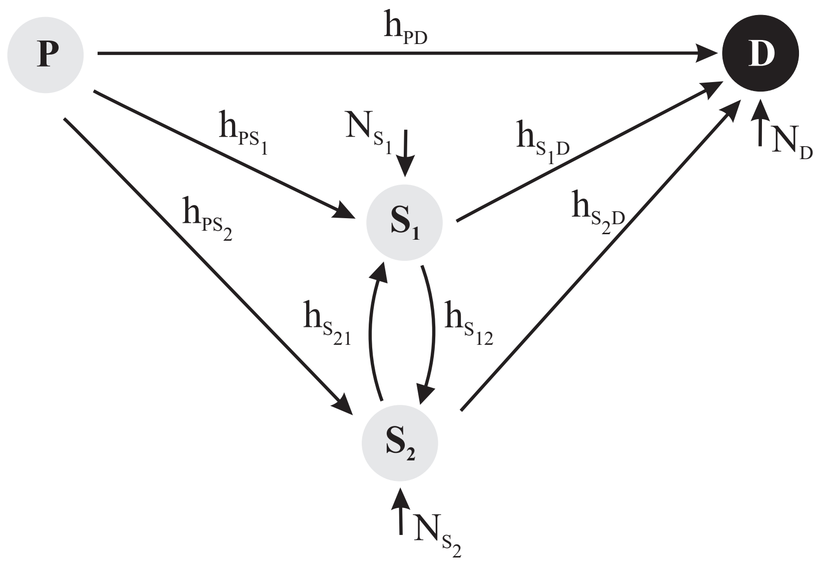


Figure 5.1: Overlay system model: cooperative behaviour towards primary user

no rate loss for the PU was derived; yet the SU rate was not significantly improved. In this letter, without assuming prior knowledge of the PU message, and without orthogonalizing user transmissions, we propose two cooperation models based on MACs with generalized feedback [47]: overlay and underlay cooperation. We allow two SUs to simultaneously cooperate among each other and with the PU, thereby increasing diversity. We characterize and optimize the corresponding SU long term achievable rate regions, as a function of the channel adaptive transmit powers, and compare them via simulations.

5.1 System Model

We consider a fading three user cognitive cooperative Gaussian MAC illustrated in Fig. 5.1. The general system model consists of four nodes: the primary user P, the secondary users S_1 and S_2 and the destination D. The received signals Y_D, Y_{S_1} and Y_{S_2} at the destination, S_1 and S_2 respectively are given by

$$Y_D = \sqrt{h_{PD}}X_P + \sqrt{h_{S_1D}}X_{S_1} + \sqrt{h_{S_2D}}X_{S_2} + N_D \tag{5.1}$$

$$Y_{S_1} = \sqrt{h_{PS_1}}X_P + \sqrt{h_{S_{21}}}X_{S_2} + N_{S_1} \tag{5.2}$$

$$Y_{S_2} = \sqrt{h_{PS_2}}X_P + \sqrt{h_{S_{12}}}X_{S_1} + N_{S_2} \tag{5.3}$$

where h_{PD}, h_{S_iD}, $h_{S_{ij}}$, h_{PS_i} denote the PU-destination, S_i-destination, S_i-S_j and PU-S_i channel coefficients respectively, where $i, j \in \{1, 2\}$ and $i \neq j$; N_D, N_{S_i} denote the zero mean additive white Gaussian noise components at the destination and SUs. The noise variances are normalized to 1 without loss of generality, by absorbing the noise power in channel coefficients. The variables X_P, X_{S_1} and X_{S_2} are the codewords transmitted by the PU, S_1 and S_2. The joint channel state $\mathbf{h}$ consisting of all instantaneous fading coefficients is assumed to be known at all nodes, as the channel states can be estimated by each receiving node, and then collected and re-distributed by the common receiver. Depending on how the overheard information is processed at the SUs, we propose two cognitive cooperative models: overlay and underlay cooperation, and characterize their corresponding achievable rates.

5.2 Overlay Cooperation Model

In the overlay system model, we employ a causal cognitive radio (CCR) approach, where the SUs first decode each other's and PU's messages from previous block; and then forward them in the following block. The CCR model can be viewed as a special case of three user cooperation [40], if the decoding function at one of the users is disabled (thereby making it the PU), and the cooperation signals are formed accordingly. We propose the following encoding/decoding policy at the users: the PU generates the codewords X_{P_U}, and uses them to convey its message w_P to the SUs. Both SUs decode this codeword, and form the common codeword U to further assist the receiver in decoding w_P. The SUs also cooperate with each other, by first establishing common information by exchanging their messages w_{S_1}, w_{S_2} using codewords $X_{S_{12}}$ and $X_{S_{21}}$ respectively, and then sending the pairwise cooperation signal U_S which is mapped to the messages w_{S_1}, w_{S_2} shared in the previous block. The block Markov encoding (BME) procedure, obtained by a suitable modification of the three user BME structure introduced in [40, Table 4.2] to fit the cognitive radio set-up, generates unit power Gaussian codewords and maps them to distinct messages from the current and previous block, as summarized in Table 5.1. The transmitted codewords $\{X_P, X_{S_1}, X_{S_2}\}$ are then obtained by superposing the scaled versions of the component codewords, i.e.,

$$\begin{aligned} X_P &= \sqrt{P_P(\mathbf{h})} X_{P_U} \\ X_{S_1} &= \sqrt{P_{S_{12}}(\mathbf{h})} X_{S_{12}} + \sqrt{P_{U_{S_1}}(\mathbf{h})} U_S + \sqrt{P_{U_1}(\mathbf{h})} U \\ X_{S_2} &= \sqrt{P_{S_{21}}(\mathbf{h})} X_{S_{21}} + \sqrt{P_{U_{S_2}}(\mathbf{h})} U_S + \sqrt{P_{U_2}(\mathbf{h})} U. \end{aligned} \tag{5.4}$$

Let $\mathbf{P}(\mathbf{h}) \triangleq [P_P(\mathbf{h}), P_{S_{12}}(\mathbf{h}), P_{U_{S_1}}(\mathbf{h}), P_{U_1}(\mathbf{h}), P_{S_{21}}(\mathbf{h}), P_{U_{S_2}}(\mathbf{h}), P_{U_2}(\mathbf{h})]$ denote the vector of power variables, and let $P_{S_{ij}}(\mathbf{h}) + P_{U_{S_i}}(\mathbf{h}) + P_{U_i}(\mathbf{h}) \triangleq P_{S_i}(\mathbf{h})$. Then, the powers used in (5.4) must take values from the following feasible set:

$$\begin{aligned} \mathcal{P}_{feasible} = \big\{\mathbf{P}(\mathbf{h}) : \quad & E[P_P(\mathbf{h})] \le P_P, E[P_{S_1}(\mathbf{h})] \le P_{S_1}, \\ & E[P_{S_2}(\mathbf{h})] \le P_{S_2}, \mathbf{P}(\mathbf{h}) \ge \mathbf{0}\big\} \end{aligned} \tag{5.5}$$

It is worth noting that the involvement of the PU in the cooperation process is minimal. It sends only a single codeword, as if it were transmitting alone, and scales it by a power level that is determined at the receiver and fed back. As a result, it even does not have to know the individual channel states of the SUs, thereby making our model suitable for cognitive transmissions.

The decoding at the SUs is carried out at the end of each block $b \in \{1, \cdots B-1\}$. In each block b, each SU S_i decodes $w_P(b)$ and $w_{S_j}(b)$ by joint typicality check. Since the codewords U and U_S that depend on previous block messages are already known at each SU S_i, error free decoding is possible if the rates satisfy

$$R_P \le I(X_P; Y_{S_i} | X_{S_j}, U, U_S, \mathbf{h}) \tag{5.6}$$

$$R_{S_j} \le I(X_{S_j}; Y_{S_i} | X_P, U, U_S, \mathbf{h}) \tag{5.7}$$

$$R_P + R_{S_j} \le I(X_P, X_{S_j}; Y_{S_i} | U, U_S, \mathbf{h}) \tag{5.8}$$

The receiver employs backwards decoding starting with the last block B, to jointly decode $w_P(B-1)$ and the pair $\{w_{S_1}(B-1), w_{S_2}(B-1)\}$. No fresh information

	Block I	Block II
Primary	$X_{P_U}(w_P(1), w_P(0))$	$X_{P_U}(w_P(2), w_P(1))$
Secondary I	$X_{S_{12}}(w_{S_1}(1), U_S(1), U(1))$ $U_S(w_{S_1}(0), w_{S_2}(0), U(1))$ $U(w_P(0))$	$X_{S_{12}}(w_{S_1}(2), U_S(2), U(2))$ $U_S(w_{S_1}(1), w_{S_2}(1), U(2))$ $U(w_P(1))$
Secondary II	$X_{S_{21}}(w_{S_2}(1), U_{S_{21}}(1), U(1))$ $U_S(w_{S_1}(0), w_{S_2}(0), U(1))$ $U(w_P(0))$	$X_{S_{21}}(w_{S_2}(2), U_S(1), U(2))$ $U_S(w_{S_1}(1), w_{S_2}(1), U(2))$ $U(w_P(1))$

Table 5.1: Block Markov Coding Structure for Overlay System Model

is transmitted in the last block, therefore error free decoding at the receiver is possible if

$$R_P \leq I(X_P, X_{S_1}, X_{S_2}; Y_D|\mathbf{h}) \tag{5.9}$$

$$R_{S_1} + R_{S_2} \leq I(X_{S_1}, X_{S_2}; Y_D|X_P, U, \mathbf{h}) \tag{5.10}$$

$$R_P + R_{S_1} + R_{S_2} \leq I(X_P, X_{S_1}, X_{S_2}; Y_D|\mathbf{h}). \tag{5.11}$$

In obtaining (5.9)-(5.11) we have used the fact that all codewords are functions of $w_P(B-1)$, and that the pair $\{w_{S_1}(B-1), w_{S_2}(B-1)\}$ is decoded as a single message. Note that, (5.11) dominates (5.9), hence (5.9) can be dropped. Once $\{w_{S_1}(B-1), w_{S_2}(B-1)\}$ are decoded, the messages in block $B-2$, $B-3$, etc. can be decoded provided the same conditions hold. Finally, evaluating (5.6)-(5.11) for Gaussian codewords (5.4), we obtain the following result in Section 5.2.1.

5.2.1 Achievable Rates

Adding the ability to decode the primary user's codewords, the following achievable rate expressions are obtained at the secondary users and the MAC constraints should be satisfied by using backward decoding structure at the destination side.

Theorem 5.1. *An achievable rate region for the overlay cooperation model given in* (5.4) *is the closure of the convex hull of all rate triplets* (R_P, R_{S_1}, R_{S_2}) *which satisfy the constraints.*

$$R_P \leq E[\log(1+h_{PS_1}P_P(\mathbf{h}))] \tag{5.12}$$

$$R_P \leq E[\log(1+h_{PS_2}P_P(\mathbf{h}))] \tag{5.13}$$

$$R_{S_1} \leq E[\log(1+h_{S_{12}}P_{S_{12}}(\mathbf{h}))] \tag{5.14}$$

$$R_{S_2} \leq E[\log(1+h_{S_{21}}P_{S_{21}}(\mathbf{h}))] \tag{5.15}$$

$$R_P+R_{S_1} \leq E[\log(1+h_{PS_2}P_P(\mathbf{h})+h_{S_{12}}P_{S_{12}}(\mathbf{h}))] \tag{5.16}$$

$$R_P+R_{S_2} \leq E[\log(1+h_{PS_1}P_P(\mathbf{h})+h_{S_{21}}P_{S_{21}}(\mathbf{h}))] \tag{5.17}$$

$$\begin{aligned} R_{S_1}+R_{S_2} \leq E\big[\log\big(1&+h_{S_1D}(P_{S_{12}}(\mathbf{h})+P_{U_{S_1}}(\mathbf{h})) \\ &+h_{S_2D}(P_{S_{21}}(\mathbf{h})+P_{U_{S_2}}(\mathbf{h}))+C_p\big)\big] \end{aligned} \tag{5.18}$$

$$R_P+R_{S_1}+R_{S_2} \leq E[\log(C)] \tag{5.19}$$

where $C = 1+h_{PD}P_P(\mathbf{h})+h_{S_1D}P_{S_1}(\mathbf{h})+h_{S_2D}P_{S_2}(\mathbf{h}) + C_p+C_t$,
$C_t = 2\sqrt{h_{S_1D}h_{S_2D}P_{U_1}(\mathbf{h})P_{U_2}(\mathbf{h})}$ *and* $C_p = 2\sqrt{h_{S_1D}h_{S_2D}P_{U_{S_1}}(\mathbf{h})P_{U_{S_2}}(\mathbf{h})}$.

Proof. In the proof of Theorem 5.1, we consider the decoding at the user sides is executed at the end of each block, based on joint typicality check. Since the PU is broadcasting, the codeword w_P can be decoded correctly by both SUs if (5.12)-(5.13) are satisfied for broadcast channel at each SU, respectively. The SUs also decode each other's codewords w_{S_1} and w_{S_2}, satisfying the rate constraints (5.14)-(5.15) for broadcast channel and (5.16)-(5.17) for the traditional MAC .

We can see the effect of pairwise and threefold coherent combining for the receiver side rate constraints in (5.18) - (5.19), considering all codewords are decoded at the end of transmission. The receiver first decodes the cooperative codewords U and U_S, then jointly w_{S_1}, w_{S_2} to obtained rate constraints (5.18) - (5.19) by using capacity results for the traditional MAC. ∎

Theorem 5.1 alone does not provide any rate guarantees for the PU. Had the PU been transmitting alone, it would achieve

$$R_P = E[\log(1 + h_{PD}P_P^*(h_{PD}))] \triangleq B^* \tag{5.20}$$

where $P_P^*(h_{PD})$ is the optimal single user power allocation, obtained by water-filling. Therefore, the overlay model proposed in this section is applicable only if the rate region given in Theorem 5.1, optimized as a function of transmit powers, satisfies (5.20). The rate optimization problem, including the cognitive rate constraint (5.20), is solved in the next section. In the event that (5.20) cannot be satisfied using overlay cooperation model, an underlay cooperation model, which involves cooperation of SUs only, can be employed as a back up alternative. Such an underlay cooperation model, and the resulting achievable rates will be presented in Section 5.3.

5.2.2 Achievable Rate Maximization for Overlay Cooperation

In this section, we propose a method to obtain the optimal power allocation policy that maximizes the SU achievable rate region. Setting $R_P = B^*$, the SU achievable rate region maximization problem can be stated as

$$\max_{\mathbf{P}(\mathbf{h})\in P_{feasible}} \quad \mu_1 R_{S_1} + \mu_2 R_{S_2} \tag{5.21}$$

$$\text{s.t.} \quad R_P = B^\star$$

$$R_{S_1} \leq E\left[\log\left(1 + h_{S_{12}}P_{S_{12}}(\mathbf{h})\right)\right] \tag{5.22}$$

$$R_{S_2} \leq E\left[\log\left(1 + h_{S_{21}}P_{S_{21}}(\mathbf{h})\right)\right] \tag{5.23}$$

$$R_{S_1} \leq E\left[\log\left(1 + h_{PS_2}P_P(\mathbf{h}) + h_{S_{12}}P_{S_{12}}(\mathbf{h})\right)\right] - B^* \tag{5.24}$$

$$R_{S_2} \leq E\left[\log\left(1 + h_{PS_1}P_P(\mathbf{h}) + h_{S_{21}}P_{S_{21}}(\mathbf{h})\right)\right] - B^* \tag{5.25}$$

$$R_{S_1} + R_{S_2} \leq E\Big[\log\Big(1 + h_{S_1D}(P_{S_{12}}(\mathbf{h}) + P_{U_{S_1}}(\mathbf{h})) \tag{5.26}$$

$$+ h_{S_2D}(P_{S_{21}}(\mathbf{h}) + P_{U_{S_2}}(\mathbf{h})) + C_P\Big)\Big]$$

$$R_{S_1} + R_{S_2} \leq E\left[\log\left(C\right)\right)] - B^* \tag{5.27}$$

$$E\left[P_P(\mathbf{h})\right] \leq \bar{P}_P,$$

$$E\left[P_{S_{12}}(\mathbf{h})+P_{U_{S_1}}(\mathbf{h})+P_{U_1}(\mathbf{h})\right]\leq\bar{P}_{S_1},$$
$$E\left[P_{S_{21}}(\mathbf{h})+P_{U_{S_2}}(\mathbf{h})+P_{U_2}(\mathbf{h})\right]\leq\bar{P}_{S_2}$$
$$P_P(\mathbf{h}),\ P_{S_{12}}(\mathbf{h}),\ P_{U_{S_1}}(\mathbf{h}),\ P_{U_1}(\mathbf{h}),\ P_{S_{21}}(\mathbf{h}),\ P_{U_{S_2}}(\mathbf{h}),\ P_{U_2}(\mathbf{h})\geq 0,\forall h$$

where, by varying μ_1 and μ_2, all points on the convex rate region can be obtained. Assuming $\mu_1 > \mu_2$ without loss of generality, the problem can be restated as

$$\begin{aligned}\max_{\mathbf{P}(\mathbf{h})\in P_{feasible}} \quad & (\mu_1-\mu_2)\min\{(5.22),(5.24),(5.26),(5.27)\}\\ & +\mu_2\min\{(5.22)+(5.23),(5.22)+(5.25),(5.23)+(5.24),\\ & (5.24)+(5.25),(5.26),(5.27)\}\end{aligned} \tag{5.28}$$

where each equation number in (5.28) denotes the right hand sides of the corresponding constraint. Let the cost function in (5.28) be represented by R_μ. Since R_μ is the minimum of monotone increasing concave functions of powers, it is concave in powers. The constraint set $P_{feasible}$ is convex. However, due to the minimum operation, R_μ is in general discontinuous, and the gradient of R_μ is not well defined. Instead, we use the method of projected subgradients, to iteratively maximize R_μ. A close examination of (5.28) reveals that R_μ takes one of 24 functional forms, depending on which constraint is active. The gradient of each functional form can be computed in each iteration n, and the one corresponding to the active constraint will form a subgradient, say G_n of the objective function. Then, the update in the $n+1^{st}$ iteration of the subgradient algorithm is given by

$$P(\mathbf{h})^{n+1} = \left[P(\mathbf{h})^n + \beta_n G_n / \left\|G_n\right\|^{\dagger}\right] \tag{5.29}$$

where β_n is the step size parameter $[.]^{\dagger}$ denotes projection onto the feasible set. This update is guaranteed to converge to the global optimum, for diminishing step size β_n, due to concavity. The rate regions obtained by subgradient algorithm will be provided in Section 5.4.

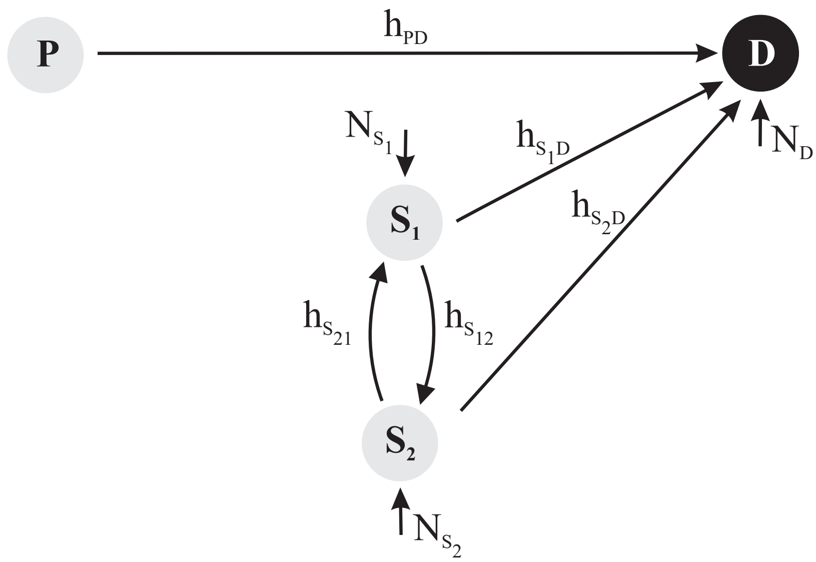


Figure 5.2: Underlay system model: egoistic behaviour towards primary user

5.3 Underlay Cooperation Model

Considering current studies, the most common information theoretical model for underlay cognitive cooperative MAC model is the interference channel. As commonly determined, in a classical underlay cognitive scenario, three users can communicate towards a destination where each of them independent transmitter-receiver pairs communicate interfering each other. However, in this proposed system model, we introduce an three user communication model where primary user directly transmits its information, while secondary users (S_1, S_2) are cooperatively communicated each other, hence they cause interference in primary user. In this section, we propose an underlay cooperation model, where the PU transmission, including power control, is not altered at all by the presence the SU's. This mode can be used if selecting $R_P = B^*$ violates (5.12) or (5.13) in the overlay scenario. The PU transmits its signal X_P to be decoded at only the destination, allocating its power using single user waterfilling. The SUs can overhear each other and they cooperate as in the overlay scenario, to convey each other's message to the destination. The resulting encoding policy is given by

$$X_P = \sqrt{P_P^*(h_{PD})} X_{PD} \tag{5.30}$$

$$X_{S_1} = \sqrt{P_{S_{12}}(\mathbf{h})} X_{S_{12}} + \sqrt{P_{U_{S_1}}(\mathbf{h})} U_S \tag{5.31}$$

$$X_{S_2} = \sqrt{P_{S_{21}}(\mathbf{h})} X_{S_{21}} + \sqrt{P_{U_{S_2}}(\mathbf{h})} U_S \tag{5.32}$$

where $E\left[P_P(\mathbf{h})\right] \leq \bar{P}_P,\ E\left[P_{S_{12}}(\mathbf{h}) + P_{U_{S_1}}(\mathbf{h})\right] \leq \bar{P}_{S_1}\ E\left[P_{S_{21}}(\mathbf{h}) + P_{U_{S_2}}(\mathbf{h})\right] \leq \bar{P}_{S_2}$, and the component codewords are mapped to the messages as given in Table 5.2. More specifically, the primary user only uses the codeword X_{PD} as function of w_P that carries fresh information to the destination side, referring to single user broadcasting. Giving closely followed the ideas in [27], we introduce two user cooperative channel for the secondary users. Both secondary users can overhear each other, and are willing to cooperate by forwarding information from the other. We start dividing each message $X_{S_{12}}$ and $X_{S_{21}}$ into two sub messages: $\{w_{S_1}, U_S\}$ and $\{w_{S_2}, U_S\}$. w_{S_1} and w_{S_2} are used to transmit directly to destination. U_S refers to the part of the message that carries cooperative information between secondary users. The transmitting signals $\{X_P, X_{S_1}, X_{S_2}\}$ to carry these messages.

By treating the PU signal as noise, SU Si decodes $w_{Sj}(b)$ in each block, and cooperatively sends this information in the next block. The receiver first decodes the SU messages by backwards decoding, treating the PU signal as noise, then cancels interference from SU's, and decodes the PU as if it is alone in the system. The SUs then form a well known two user cooperative MAC [27], and the rate region follows directly from [27], [48] by treating the interference as additional noise, as stated below.

5.3.1 Achievable Rates

In the proposed underlay system model, only secondary users and destination have decoding capabilities and should satisfy the traditional MAC constraints to decode all received messages as given in Theorem 5.2.

Theorem 5.2. *An achievable rate region for the SUs based on the encoding given in (5.30) is the closure of the convex hull of all rate pairs (R_{S_1}, R_{S_2}) that satisfy* *the constraints.*

$$R_{S_1} \leq E\left[\log\left(1+h_{S_{12}}P_{S_{12}}(\mathbf{h})/I_{S_2}\right)\right] \quad (5.33)$$

$$R_{S_2} \leq E\left[\log\left(1+h_{S_{21}}P_{S_{21}}(\mathbf{h})/I_{S_1}\right)\right] \quad (5.34)$$

$$R_{S_1}+R_{S_2} \leq E\left[\log\left(1+\frac{h_{S_1D}P_{S_1}(\mathbf{h})+h_{S_2D}P_{S_2}(\mathbf{h})+C_p}{I_D}\right)\right] \quad (5.35)$$

where the interference at node $\Gamma \in \{S_1, S_2, D\}$ *is given by* $I_\Gamma = 1 + h_{P\Gamma}P_P^*(h_{PD})$.

Proof. The proof follows the same idea in the Theorem 5.1. The proposed BME has a different coding structure than the previously proposed model in Section 5.2.1 and can improve secondary users sum rate.

Considering the single user / multiple access gaussian channel capacity theorems and using the proposed encoding structure in Table 5.2, we can proof this theorem as follow.

We know that the secondary users use the block Markov encoder and decoding operation will be proceed at the end of each received block. Since the codeword X_{PD} is not intended in the secondary users, the achievable rate constraint R_{S_1} and R_{S_2} at the secondary users (S_1, S_2) can be obtained with the interference term. As mentioned, the primary user directly transmits its information to the destination side in this proposed model. Thus, only single user gaussian channel theorem could be satisfactory for proofing the achievable rate region at the intended destination.

Taking account of the secondary users rate region, we should know that, the destination use backward decoding structure to resolve the received codewords and obtains achievable rate constraints by utilizing Gaussian MAC capacity theorem. □

Finally, since fixing the PU transmit policy reduces the model to a two user cooperative MAC, the SU rate region is maximized by suitably modifying the

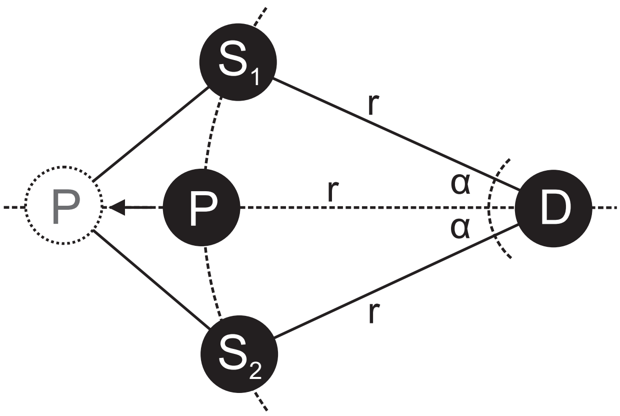


Figure 5.3: Simulation setup for Overlay and Underlay system model

	Block I	Block II
Primary	$X_{P_D}(w_P(1)$	$X_{P_D}(w_P(2))$
Secondary I	$X_{S_{12}}(w_{S_1}(1), U_S(1))$ $U_S(w_{S_1}(0), w_{S_2}(0))$	$X_{S_{12}}(w_{S_1}(2), U_S(2))$ $U_S(w_{S_1}(1), w_{S_2}(1))$
Secondary II	$X_{S_{21}}(w_{S_2}(1), U_{S_{21}}(1))$ $U_S(w_{S_1}(0), w_{S_2}(0))$	$X_{S_{21}}(w_{S_2}(2), U_{S_{21}}(1))$ $U_S(w_{S_1}(1), w_{S_2}(1))$

Table 5.2: Block Markov Coding Structure for Underlay System Model

subgradient technique proposed in [48], to include the interference caused by the PU. The resulting rate region will be evaluated in the next section.

5.4 Simulation Results

In this section, we evaluate the achievable rates obtained by power optimization for both overlay and underlay cooperation models. We focus on several user geometries, and investigate the effect of varying channel conditions on the achievable

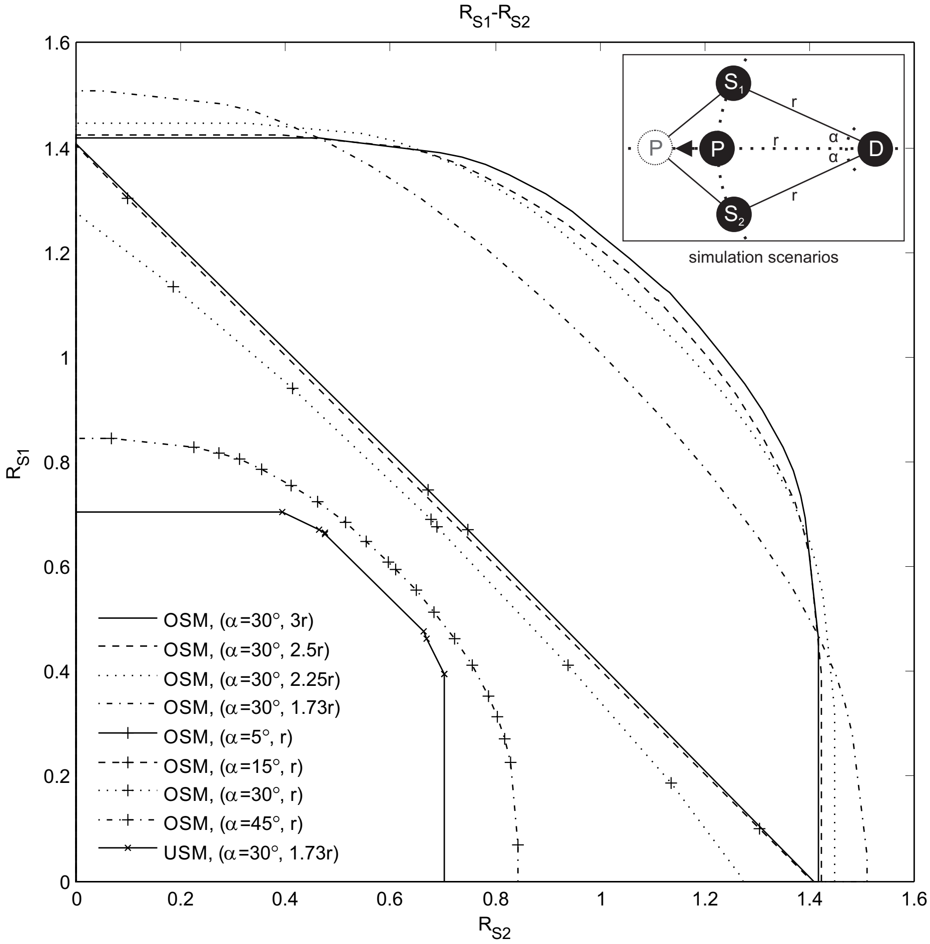


Figure 5.4: Comparison of achievable rates for Overlay System Model with BME in Section 5.2.1 and Underlay System Model with BME in Section 5.3 dedicated for three user cognitive cooperative system model

rate regions. We employ a simplified path loss model, with a path loss exponent of 3, to determine the mean value of Rayleigh fading coefficients. The transmit powers, the radius r and the noise variances are normalized to give an average received SNR of 4.8dB from each secondary user.

In the first scenario illustrated in Figure 5.3, we fix the SU positions on a circle of radius r, with $\alpha = 30°$ as illustrated in Figure 5.4, and vary the location of the PU to take the values r, $\sqrt{3}r$, $2.25r$, $2.5r$ and $3r$ respectively. The achievable rate regions for the overlay system model (OSM) are evaluated by optimal power allocation, and the results are as shown in Figure 5.4. We observe that as the PU moves away from the receiver, the SU sum rate, which is dictated by constraint (5.27), increases, as the interference at the receiver due to the PU, as well as the

PU rate requirement B^* decreases. On the other hand, the individual SU rates show an interesting behavior: among the simulated distance parameters, they are maximized when the PU is $\sqrt{3}r$ away from the receiver, and they decrease when the PU moves toward or away from the receiver. When the PU is close, the PU rate requirement is too high, making (5.27) active for single user rates. When the PU is far, the SUs should spend more power in relaying its message to compensate for the interference they cause, and they still face interference from the PU while decoding each other's message, making (5.24) and (5.25) active, thereby explaining the variable nature of the SU individual rate. Note that, when the PU is at $4r$ or more, the B^* constraint can no longer be satisfied using the OSM, and underlay system model (USM) should be used instead. A comparison of OSM and USM is given in Figure 5.4 for a symmetrical structure with PU located at $\sqrt{3}r$, and it is clear that the overlay cooperation provides a very large rate gain over underlay cooperation.

In the second scenario, the PU and SUs are placed on a circle. By varying the angle α as shown in Figure 5.4, we adjust the average qualities of inter-user link gains, and investigate the effect on the achievable rate region. For simplicity, we only demonstrate the overlay rate regions. As the SU's get closer to each other and the PU, the gain from cooperation increases, as expected. In the limit, the rate region approaches to that of a MISO upper bound, where the co-located SU's can have free access to each other's message without spending power to establish common information.

Conclusion

This thesis has investigated the capacity region of three user cooperative communication for Gaussian multiple access channel proposing different encoding structures and considering pairwise and collective cooperative messages.

The main contributions of this thesis is to:

- develop information theoretic encoding and decoding policies that allow multi user cooperation , in particular three user cooperation.
- obtain achievable rates for the three user cooperative MAC-GF.
- determine how much rate we gain by increasing number of cooperating users.
- establish cognitive-cooperative communication in a cognitive radio setting.

In Chapter 3, we introduced a three user cooperative MAC model, and proposed encoding and decoding policies that rely on a non-trivial extension of the well known block Markov superposition coding. In particular, we proposed the use of channel adaptive encoding which depends on the relative receive link qualities; among the cooperating users. We characterized and evaluated the rate region achievable by our proposed encoding-decoding techniques. We demonstrated through simulations that, going from the two user cooperative multiple access channel to its three user counterpart, the achievable rates increased significantly due to the additional diversity provided by the existence of an extra user.

In Chapter 4, we proposed a novel non-adaptive three user BME strategy to simultaneously establish and send common information. In our encoding model, each user divided its message into submessages, each of which is dedicated for either pairwise or collective (three user) cooperation, but not both. Using this strategy, we obtained a set of achievable rates for the three user cooperative MAC, and shown that this set not only improves upon two user cooperation or MARC significantly, but it could even outperform the adaptive BME strategy, when the cooperative links were much stronger than direct links to the receiver. The channel non-adaptive strategy is more complex in that it uses more cooperation signals, but it is less complex in that it does not require as much channel state information as the adaptive version, and in that it does not require frequent changes to the used codewords.

In Chapter 5, we focused instead on a more practical model where the PU and the SUs belong to the same network, and communicate with the same receiver. This model alleviated the problem of sharing channel state information between the primary and secondary networks, as the common receiver can coordinate transmissions. There, without assuming prior knowledge of the PU message, and without orthogonalizing user transmissions, we proposed two cooperation models based on MACs with generalized feedback [47]: overlay and underlay cooperation. We allowed two SUs to simultaneously cooperate among each other and with the PU, thereby increasing diversity. We characterized and optimized the corresponding SU long term achievable rate regions, as a function of the channel adaptive transmit powers, and compared them via simulations.

The overall conclusion of this thesis based on the observation in all three problems solved are:

- The capacity rate region is increased by adding the third user with reliable encoding and decoding strategies.
- Three user cooperation can be more beneficial method than to use two user cooperative scheme. In particular,

  - adaptive model can be used at both low and high inter user links to obtain better on individual rates and sum rates.

- non adaptive model can be used at only high inter user links, also gives lower complexity than adaptive model.

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