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OPTIMAL PROJECT DURATION FOR RESOURCE LEVELING

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ELİF EREN
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ELİF EREN

APPROVED BY:

Assist. Prof. Dr. S. Tankut Atan (Thesis Supervisor)	Işık University	
Assist. Prof. Dr. Demet Ö. Ünlüakın	Işık University	
Assist. Prof. Dr. Burak Çavdaroğlu	Kadir Has University	

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Abstract

Resource leveling is important in project management as resource fluctuations are costly and undesired. Typically, schedules with better resource profiles are obtained using the schedule found by Critical Path Method and then shifting the activities within their float times. However, if the project duration can be extended, it is plausible to find a schedule with enhanced resource leveling since a longer duration allows for more float time for all activities. In this thesis, we investigate what the duration for the best leveled schedule should be. We provide mixed-integer linear models for several objectives including the Release and Rehire metric. We show that not all metrics used for leveling under fixed durations may be appropriate when the project duration becomes a decision variable. Optimal solutions from smaller problems are used to find the magnitude of the extension needed and benefits obtained thereby. Since the problem is a NP-hard problem for which exact solutions cannot be obtained for big networks, we modify Burgess-Killebrew heuristic to solve larger problems.

Computational experiments with benchmark problems from the literature indicate that the more the number of resource types is increased, the less leveling benefits are gained from extending the project. The optimal project durations can also be significantly different for different metrics.

Keywords: Resource leveling, project scheduling, mixed-integer linear programming, Burgess-Killebrew, Release and Rehire metric

KAYNAK DENGELEME İÇİN EN İYİ PROJE SÜRESİ

Özet

Kaynak dengeleme, kaynak çizelgelerindeki maliyetli ve istenmeyen dalgalanmalardan dolayı proje yönetiminde önemlidir. Genellikle Kritik Yol Metodu (KYM) kullanılarak bulunan çizelgede aktiviteler bolluk süreleri içinde kaydırılarak daha iyi kaynak profilli çizelgeler elde edilir. Ancak, eğer proje süresi uzatılırsa, uzatılan süre tüm aktiviteler için daha fazla bolluk süresi sağlayacağı için daha iyi kaynak dengelemesi yapılmış bir çizelge bulunması olasıdır. Bu çalışmada en iyi dengelenmiş çizelgenin süresinin ne olması gerektiği incelenmektedir. Küçük boyutlu problemlerin en iyi çözümleri gerekli uzatmanın büyüklüğünü ve uzatmadan elde edilen faydaları bulmak için kullanılmıştır. Büyük boyutlu problemleri çözmek için Burgess Killebrew sezgisel yaklaşımı uygulanmıştır.

Sayısal deneyler, kaynak sayısı arttıkça proje uzatılmasından elde edilen dengelemenin faydasının azalacağını, en iyi proje süresinin ve proje süresinin uzatılmasından elde edilen dengeleme faydalarının farklı ölçütler için farklı olduğunu göstermiştir.

Anahtar kelimeler: Kaynak dengeleme, proje çizelgeleme, karışık tamsayılı doğrusal programlama, Burgess-Killebrew, Release and Rehire

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To my family...

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List of Abbreviations

BK	**B**urgess **K**illebrew
CPM	**C**ritical **P**ath **M**ethod
EST	**E**arliest **S**tart **T**ime
GA	**G**enetic **A**lgorithm
KYM	**K**ritik **Y**ol **M**etodu
LST	**L**atest **S**tart **T**ime
PSPLIB	**P**roject **S**cheduling **LIB**rary
RCPSP	**R**esource **C**onstrained **P**roject **S**cheduling **P**roblem
RLP	**R**esource **L**eveling **P**roblem
ROP	**R**esource **O**verload **P**roblem
RRH	**R**elease **R**e **H**ire

Chapter 1

Introduction

Resource Leveling Problem (RLP) is the problem of adjusting activity start times of a project such that the variation of resource utilization over time is minimized while satisfying precedence constraints among activities. RLP is important since resulting daily resource usages from the timing of the activities have an impact on the cost of a project because of factors such as idle resource times, and release and rehire of temporary workforce. Thus, better leveled projects help reducing such costs. In RLP, resources are commonly assumed to be unconstrained.

Normally, resource leveling literature concentrates on finding the schedule of project activities with minimum fluctuations in resource usage or utilization for a fixed project duration. This fixed project duration is obtained by multiplying the duration found through the Critical Path Method (CPM) with a factor, usually between 1.0 and 2.0. Some activities may have predecessors, and they can start only after all of their preceding activities finish. Such precedence relationships among tasks force activities to start only within a time range between their Earliest Start Time (EST) and Latest Start Time (LST). Some activities are critical, i.e. their EST and LST are the same; they do not have any float time. But EST and LST of noncritical activities differ. Thus, in RLP, noncritical activity start times are shifted within their float times so that the resource profiles are leveled best. Different objectives can be used to measure how well a project is leveled. Depending on the measure chosen, different schedules may be obtained. Some examples of objectives are minimizing sum of squares of daily usages, minimizing sum of absolute (or just positive) deviations from daily target values, minimizing maximum daily usage, minimizing the total number of released and rehired workers.

It is possible that extending a project's duration will give a better leveled project schedule since this additional time generates additional float time for all activities. While time extension may be desirable from a resource leveling perspective, a longer duration may not meet the project deadline, and can also have additional cost implications. However, when the number of resources cannot be easily changed over time and resource fluctuations have significant cost impacts, allowing a project extension with better leveling can be beneficial especially when the project deadline can still be met. In this work, we are interested in finding the minimal extended project duration that best levels the resource profiles. To that end, we allow the project duration to be variable rather than assuming that it is fixed as in the RLP literature so far. We answer several questions that arise from making the project duration variable:

- What metrics are proper to be used with a variable project duration when comparing leveling in resource profiles of different schedules?
- How can we find the duration for an optimally leveled schedule?
- How much improvement in resource profiles can be obtained by a project duration extension?
- How far away is the smallest optimal duration from the duration found by CPM?
- Does the choice of fitness metric have a significant impact on the optimal duration?
- How does the number of resources affect the duration extension?

Answers to these questions can be useful to project managers in deciding a project schedule with a variable duration when resource leveling is important.

This thesis is organized as follows. After reviewing relevant literature in Chapter 2, we show that not all metrics used with fixed project durations are appropriate when the project duration is a decision variable in Section 3.1. Section 3.2 details a mixed-integer linear model for solving RLP with a fixed duration which forms a basis for new models in later sections. Section 3.2 also includes a new model for the Release and Rehire metric under fixed duration. Section 3.3 gives new models for finding the optimal extended project duration. In the computational experiments, these models are used for finding exact solutions for smaller problems. Since large problems cannot be solved exactly in reasonable time, we give a modified Burgess-Killebrew (BK) heuristic [1] in Section 3.4 for solving large problems. Results of numerical experiments are reported in Section 4 followed by the Conclusion.

Chapter 2

Literature Review

Rieck and Zimmermann [2] review and discuss the exact methods for RLP, and also provide an extensive computational study for several mixed-integer linear models with different objectives. The studies in Rieck and Zimmermann [2] complement the work in Rieck et al. [3] where some of the models in the later study have also been computationally tested. Rieck and Zimmermann [2] also investigate the Resource Overload Problem (ROP) where the concern is leveling the resource profile so that only positive rather than all deviations from the targeted number of resources are penalized. Due to difficulties in finding an exact solution for bigger networks, many researchers developed heuristic algorithms for RLP. A review of these methods can be found in Christodoulou et al. [4]. The problem of minimizing maximum daily usage, a common metric used in RLP studies, is known as Resource Availability Cost Problem and Resource Investment Problem in project scheduling. Exact methods for this problem are discussed in Rodrigues and Yamashita [5] whereas Petegham and Vanhoucke [6] deal with heuristic methods for the same problem.

Few researchers dealt with extending the project duration beyond the CPM duration and looked on the effect of such extensions on resource profiles. However, these studies only assume a predetermined extended duration and are not focused on finding the minimal duration that gives the best leveling as in our study. Kim et al. [7] apply the Minimum Moment Heuristic for resource leveling (Harris [8]) to extended project durations and compare the resulting profiles to the resource profile obtained when the project duration is kept fixed at the CPM value. On a sample network with 12 activities, they obtain a better resource profile with a one day extension. Liao et al. [9] propose extension of project deadline with penalty as a possible topic for future study for RLP. Ponz-Tienda et al. [10] actually implement this idea in their adaptive genetic algorithm for RLP. Their algorithm also uses the Weibull distribution to guess the value of the optimal solution which is used as a termination criterion. Rieck et al. [3] and Rieck and Zimmermann [2] report computational results for their problem instances where the project deadline has been increased from the CPM duration by a fixed factor. Their concern is how computation times are affected by these increases. Recently, Rahman and Elazouni [11] demonstrated on two sample networks, one with 30 and another with 120 activities, that enhanced resource leveling schedules are indeed possible by extending the project duration. They also provide a genetic algorithm in which the CPM schedule is extended by a fixed amount and the activities are scheduled within the new duration. The schedules obtained with extended duration were better leveled than the schedules obtained via the genetic algorithm using the CPM duration for the sample networks. All of their reported results were approximate solutions generated with the genetic algorithm. In this work, we also provide the optimal solutions of the sample networks from Rahman and Elazouni [11] for certain extended durations.

Resource-Constrained Project Scheduling Problem (RCPSP) is a related problem where the minimum project duration that observes activity interdependencies and resource capacities is sought after. RCPSP has an extensive literature which is not reviewed here since it is a different problem. In RLP, as opposed to RCPSP, resources are assumed to be unconstrained. Furthermore, RCPSP schedules are not necessarily well leveled. In fact, Roca et al. [12] provide an evolutionary algorithm to solve a multi-objective problem where resources are leveled and makespan is minimized under scarce resources.

Relevant literature is summarized in Table 2.1.

Table 2.1: Review of the literature on RLP

Publication Year	Author	Problem Type	Method Type	Remarks
2000	Neumann and Zimmermann	RLP and Net Present Value Problems	Exact and Heuristic Procedures	There are three test sets with 1, 3, and 5 different resources. The first set and second set involve 270 problem instances. Problem instances for the first set include 10,15, and 20 activities while the second set contains problem instances with 100, 200, and 500 activities. The third set involves 7200 problem instances with 10, 20, 30, 50, and 100 activities. The activity durations are considered uniformly selected (from 1 to 10).
2012	Ponz-Tienda et. al.	RLP	GA	GA coded in VBA for Excel 2010. Three problem sets were used. The total number of jobs for the poblem sets are equal to 32, 62 and 122 respectively. Each set involves 480 problem instances.
2012	Rieck et. al.	RLP	Mixed-integer linear programming	Two test sets were tested. First set contains 810 problem instances while the second set contains 600 instances. First set involves 270 problem instances with 10,20 and 30 activities and one resource is required for each activity. The second set involves 120 problem instances with 10, 15, 20, 30, 50 activities and more than one resource might be needed for each activity.
2013	Ranjbar	RLP	Metaheuristic	A test set which contains 600 problem instances with 100, 200, 300, 400, and 500 activities was used. For each instance, four project deadline values were considered. First project deadline value is equal to the earliest start time of dummy end activity. The other values are 1.1, 1.25, and 1.5 times of the earliest start time of dummy end activity, respectively.
2015	Li et. al.	Robust RLP	GA	GA coded in Visual C++ 2012. 810 randomly generated problem instances with 30, 60 and 90 activities were tested. The number of resource types used for each instances is 4.
2015	Rahman et. al.	RLP	GA	GA model coded in MATLAB. Two randomly generated networks of 30 and 120 activities with 10 and 20 day extension were solved.
2015	Coughlan et. al.	Multi-mode RLP	Branch-Price-and-Cut Algorithm	Instances with 50 multi-mode jobs were solved optimally.

Chapter 3

Mathematical Models

3.1 Leveling Metrics with Variable Project Duration

There are several resource leveling metrics that can be used to measure how well resources are leveled in a project. Some commonly used examples are minimizing sum of squares of daily resource usages (classical resource leveling problem), minimizing sum of absolute deviations of daily resource usages from daily target values, and minimizing maximum daily usage among others. However, not all metrics normally used under fixed duration scenarios are suitable for finding the optimal duration when one is also concerned about not extending the project duration too much. As shown next, minimizing sum of squares and minimizing maximum daily usage will favour longer durations to achieve smaller daily usage amounts. It is assumed that the daily resource usage of an activity is the same for all days.

Proposition 1. Under minimization of sum of squares of daily resource usages metric, an optimal project duration is obtained by scheduling the activities back-to-back with no overlaps.

Proof. Say all activities are scheduled serially with no parallel processing. Take any two jobs with resource usages and durations of r_1, r_2, d_1, and d_2 respectively. Let them overlap for t time periods. With no overlap the objective function penalty due to these two jobs is equal to $d_1 r_1^2 + d_2 r_2^2$. When the activities overlap by t time periods, the penalties become $(d_1 - t)r_1^2 + (d_2 - t)r_2^2 + t(r_1 + r_2)^2$. After expanding, it can be seen that the new penalties have an additional amount of $2tr_1r_2$. Thus, an optimal duration is obtained with no activity overlaps by avoiding these additional penalties. Serial schedules with idle times between the activities will provide alternative optimal solutions but of longer durations. □

Proposition 2. Under minimization of maximum daily resource usage metric, an optimal project duration is obtained by scheduling the activities back-to-back with no overlaps.

Proof. Trivial since the smallest maximum daily resource usage amount that can be achieved when scheduling activities is equal to the maximum daily usage of individual activities. Any activity overlap may increase this minimum. However, there can be alternative solutions with shorter durations as long as overlapping activities do not consume more resources than the activity with the maximum resource usage. Hence, the activity with maximum resource usage cannot overlap with another activity in these alternative solutions. □

Model 1 in Section 3.2 uses the sum of absolute deviations from a given target daily resource usage as RLP metric. This target value can be determined based on business conditions, or based on total resource usage of all activities and the project duration by finding the average resource utilization. Moreover, the target value can be kept equal to the target value found when scheduling the activities according to the CPM duration, or can be lowered as the project duration is extended. Model 2 and Model 3 of Section 3.3 deal with changing target values.

In this work, we also investigate the Release and Rehire (RRH) metric. This metric has been introduced by El-Rayes and Jun [13] and it measures the total amount of resources that need to be temporarily released during low demand periods and rehired at a later stage during high demand periods. Rahman and Elazouni [11] also used this metric in their resource leveling study of extended projects. With Model RRH, we provide a mixed-integer linear model for this new metric in Section 3.2. Kreter et al. [14] model and investigate the Total Adjustment Cost problem where the cost function is different than RRH metric and only penalizes weighted positive adjustments in resource levels.

3.2 Integer Linear Models with Fixed Project Duration

Let's assume that a project with N (real) activities and precedence relationships among the activities has been scheduled using CPM method. Thus, earliest and latest start times that adhere to the precedence relationships have been obtained, and the minimum number of days to finish the project, D, has also been determined. The project activities require R different resource types. For modeling purposes, two artificial activities with zero duration and zero resource usage have been added to the project network; one dummy activity precedes all real activities whereas another dummy activity follows all other activities. These artificial activities have been numbered as 0 and $N + 1$.

One commonly used objective for finding the best leveling is to minimize the weighted sum of absolute differences of daily resource usages from target values. These target values could be chosen equal to the (rounded) average resource utilizations. The following discrete-time based integer linear model (Model 1) uses this measure. Discrete-time based models in RLP were inspired by Pritsker et al. [15]. For ease of understanding the model, we prefer expressing the model by using extra auxiliary variables although some of them may be omitted from the formulation as in Rieck et al. [3] and Rieck and Zimmermann [2] for more efficiency. The computational studies in Rieck et al. [3] and Rieck and Zimmermann [2] indicate that this formulation type performs well.

Model 1 forms the basis of the new models provided in this work, and is also used in computational studies within an iterative solution approach. Note that regardless of the model formulation, an exact approach will be limited to solving problems only up to a certain size. However, optimal solutions obtained by exact methods for small problems give valuable insight to the problem at hand. Furthermore, they also help in understanding the quality of solutions generated by approximate algorithms. Thus, we will also attempt to solve benchmark problems exactly using formulations in our study. Next, the notation is followed by the model with a fixed project duration.

Sets

I = Project activities, $i = 0, ..., N + 1$.

K = Resources needed by activities, $k = 1, ..., R$.

T = Days in the project, $t = 0, ..., D$.

Parameters

EST_i = Earliest start time (day) of activity i. These values are found via CPM.

LST_i = Latest start time (day) of activity i. These values are found via CPM.

d_i = Duration of activity i in days.

$r_{i,k}$ = Amount of resource k used daily by activity i.

a_k = Targeted daily usage for resource k. While it is assumed that these targets are the same for each day, the formulation can easily be changed to where target values differ from one day to another.

w_k = Weight of resource k.

D = Project duration. The duration is found via CPM.

$p_{i,j}$ = 1 if activity j precedes activity i; 0 otherwise.

Decision Variables

z = The weighted sum of absolute deviations of total daily resource usages from targeted daily resource usages.

f_i = The starting day of activity i.

$u_{t,k}$ = Amount of resource k used on day t.

$x_{t,k}$ = The excess amount of resource k on day t when compared to the targeted amount (a_k).

$y_{t,k}$ = The shortage amount of resource k on day t when compared to the targeted amount (a_k).

$\phi_{t,i}$ = 1 if activity i is active on day t; 0 otherwise.

$\sigma_{t,i}$ = 1 if activity i starts on day t; 0 otherwise.

3.2.1 Model 1

$$min \quad z = \sum_{t<D} \sum_{k} w_k(x_{t,k} + y_{t,k}) \tag{3.1}$$

subject to:

$$u_{t,k} - a_k = x_{t,k} - y_{t,k} \quad \forall t \in T, \forall k \in K \tag{3.2}$$

$$\sum_{i} r_{i,k}\phi_{t,i} = u_{t,k} \quad \forall t \in T, \forall k \in K \tag{3.3}$$

$$p_{i,j}f_i \geq p_{i,j}(f_j + d_j) \quad \forall i, j \in I, i \neq j \tag{3.4}$$

$$\sum_{EST_i \leq t \leq LST_i} \sigma_{t,i} = 1 \quad \forall i \in I \tag{3.5}$$

$$\phi_{t,i} = \sum_{t_1=max(EST_i,t-d_i+1)}^{min(LST_i,t)} \sigma_{t_1,i} \quad \forall i \in I, EST_i \leq t \leq LST_i + d_i - 1 \tag{3.6}$$

$$\phi_{t,i} = 0 \quad \forall i \in I, t < EST_i \tag{3.7}$$

$$\phi_{t,i} = 0 \quad \forall i \in I, t > LST_i + d_i - 1 \tag{3.8}$$

$$\sum_{EST_i \leq t \leq LST_i} t\sigma_{t,i} = f_i \quad \forall i \in I \tag{3.9}$$

$$f_i \geq EST_i \quad \forall i \in I \tag{3.10}$$

$$f_i \leq LST_i \quad \forall i \in I \tag{3.11}$$

$$f_0 = 0 \tag{3.12}$$

$$\sigma_{0,0} = 1 \tag{3.13}$$

$$f_{N+1} \leq D \tag{3.14}$$

$$u_{t,k}, x_{t,k}, y_{t,k} \in \mathbb{Z}_0^+ \quad \forall t \in T, \forall k \in K \tag{3.15}$$

$$f_i \in \mathbb{Z}_0^+ \quad \forall i \in I \tag{3.16}$$

$$\phi_{t,i} \in \{0, 1\} \quad \forall t \in T, \forall i \in I \tag{3.17}$$

$$\sigma_{t,i} \in \{0, 1\} \quad \forall t \in T, \forall i \in I \tag{3.18}$$

The objective function aims to minimize the total absolute deviation from targeted daily resource usage amounts. The absolute value function has been linearized via Constraint (3.2) and $u_{t,k} - a_k$ terms (deviations from target values) have been expressed as differences of two sets of nonnegative variables. The formulation can easily be adapted to only penalizing overloads, i.e. exceeding the target values. The first and last activity are dummy activities with zero duration and resource usage, and they mark the beginning and the end of the project. Constraint (3.3) finds the daily resource usages. Activities can only use a resource when they are active. Constraint (3.4) enforces the predecessor activities to finish before their successors. Constraint (3.5) states that an activity can only start between its earliest and latest start times. Constraint (3.6) finds the days activities are active, and enforces those days to be consecutive. Constraint (3.7) and Constraint (3.8) make sure that activities are not active outside of their possible time windows. Constraint (3.9) finds on which day an activity starts. Constraint (3.10) states that activities cannot start before their earliest start times whereas Constraint (3.11) prevents activities to start after their latest start times. Constraint (3.12) is used for starting the first (dummy) activity at the beginning of the project. Constraint (3.13) says that activity 0 is active at time $t = 0$. With Constraint (3.14) the last (dummy) activity is forced to start (and finish) before the project's finish time. $u_{t,k}, x_{t,k}, y_{t,k}, f_i$ are nonnegative integer variables whereas $\phi_{t,i}$, and $\sigma_{t,i}$ are binary variables.

3.2.2 Model 1 (Modified)

While auxiliary variables used in Model 1 increase the readability of the model, they can be left out from the formulation to yield a model with less variables. Rieck et al. [3] and Rieck and Zimmermann [2] provided an efficient such model for ROP given below. Note that the modified model doest not include $y_{t,k}$ variables in the objective function as both objective functions yield the same optimal solutions (Rieck et al. [3]).

$$min \quad z = \sum_{t<D} \sum_{k} w_k x_{t,k} \tag{3.19}$$

subject to:

$$\sum_{i} r_{i,k} \sum_{t_1=max(EST_i, t-d_i+1)}^{min(LST_i,t)} \sigma_{t_1,i} - a_k \leq x_{t,k} \quad EST_i \leq t \leq LST_i + d_i - 1, \forall k \in K \tag{3.20}$$

$$p_{i,j} \sum_{EST_i \leq t \leq LST_i} t\sigma_{t,i} \geq p_{i,j} \left(\sum_{EST_j \leq t \leq LST_j} t\sigma_{t,j} + d_j \right) \quad \forall i, j \in I, i \neq j \tag{3.21}$$

$$\sum_{EST_i \leq t \leq LST_i} \sigma_{t,i} = 1 \quad \forall i \in I \tag{3.22}$$

$$\sigma_{0,0} = 1 \tag{3.23}$$

$$x_{t,k} \geq 0 \quad \forall t \in T, \forall k \in K \tag{3.24}$$

$$\sigma_{t,i} \in \{0, 1\} \quad \forall t \in T, \forall i \in I \tag{3.25}$$

The objective function aims to minimize total resource usage above the targeted daily resource usage amounts. Activities can only use a resource when they are active. The first part of Constraint (3.20) finds the daily resource usages by summing over activities on days that they are active. When the resource usage on a given day is less than the target value, the LHS of Constraint (3.20) becomes negative and the respective $x_{t,k}$ will be set to zero. Otherwise, $x_{t,k}$ will need to become positive. Constraint (3.21) enforces the predecessor activities to finish before their successors start by making sure that the start time of an activity is greater than or equal to the finish times of its predecessors. Constraint (3.22) states that an activity can only start between its earliest and latest start times. Constraint (3.23) says that activity 0 is active at time $t = 0$. $x_{t,k}$ are nonnegative continuous variables whereas $\sigma_{t,i}$ are binary variables.

3.2.3 Integer Linear Model for RRH Metric

As mentioned before, RRH is another metric we solve for as a RLP measure. RRH is given for a single resource in El-Rayes and Jun [13]. To deal with multiple resource types with weights, we adjust the metric as follows. Here, $u_{t,k}$ represents the amount of resource k used on day t.

$$RRH = \sum_{k} w_k \left(0.5(u_{0,k} + \sum_{t=0}^{D-2} |u_{t,k} - u_{t+1,k}| + u_{D-1,k}) - \max_{t<D} u_{t,k} \right) \tag{3.26}$$

As the name suggests, RRH calculates the number of resources (workers) that are dismissed (released) before the project ends but then reused (rehired) during the project. Consider only one resource type and let the maximum resource usage during the project be equal to $umax$. Let I and J denote the resource level increases and decreases along the envelope of the resource profile. From Figure 3.1, we see that the the sum of increases and decreases in resource levels is equal to $(I_1+I_2+I_3)+(J_1+J_2+J_3)+2(R_1+R_2)$. But this is equal to $2(umax+R_1+R_2)$. Thus, the total number of released and rehired resources, $R_1 + R_2$ in Figure 3.1, is found by subtracting $umax$ from half of the sum of total resource adjustments. Intuitively, thinking only of one resource, the first part of Equation 3.26 gives the total number of increases by multiplying the total resource adjustments by 0.5. But each resource (worker) utilized (hired) in the project has to be first added to the project even when it is not released until the end of the project. The number of these initial hires equals to the maximum resource usage. The rest of the increases are due to the rehires. Thus, one reaches at the number of resources that are dismissed but then rehired by subtracting the maximum resource usage from the sum of all increases. A similar analysis applies to all resource types.

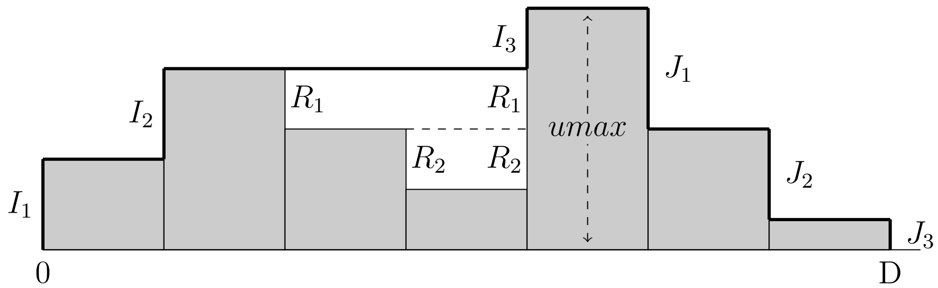

Figure 3.1: RRH metric

The RRH function is nonlinear. Firstly, there are absolute terms in the objective function. Moreover, the maximum daily resource usage is subtracted from the absolute sum of increases and decreases. The sum of absolute differences from a target level can be linearized with the help of two sets of nonnegative, auxiliary variables x and y by setting the difference between the daily resource usage and the target level equal to $x - y$. The first part of the RRH objective function can be handled this way. Normally the maximum of a discrete set of values can be linearized by defining a single new variable to replace the maximum and specifying in the constraints that this new variable should be greater than each u (daily resource usage value) variable. However, due to the negative sign, the objective function becomes unbounded with this linearization method since the bigger the auxiliary maximum value gets the smaller the objective function becomes. Thus, additional (binary) variables and constraints are needed to prevent the objective function from becoming unbounded. Next, we only show additional parts/changes that are needed in Model 1 to accommodate RRH metric.

New Parameters

M_k = A big number that daily usage for resource type k cannot exceed. It can be set to the sum of resource type k usage of all activities.

New Decision Variables

$q_{t,k}$ = 1 if daily resource usage for resource type k on day t is greater than usage on another day that is being compared to the usage on day t; 0 otherwise.

$umax_k$ = the maximum daily resource usage for resource type k.

3.2.3.1 Model RRH

$$min \quad z = \sum_k w_k \left(0.5(u_{0,k} + \sum_{t=1}^{D-2} (x_{t,k} + y_{t,k}) + u_{D-1,k}) - umax_k \right) \tag{3.27}$$

subject to:

$$u_{t,k} = \sum_i r_{i,k} \sum_{t_1 = max(EST_i, t-d_i+1)}^{min(LST_i,t)} \sigma_{t_1,i} \quad \forall t \in T, \forall k \in K \tag{3.28}$$

$$u_{t,k} - u_{t+1,k} = x_{t+1,k} - y_{t+1,k} \quad \forall t \in T, t < D-1, \forall k \in K \tag{3.29}$$

$$u_{t_1,k} - u_{t_2,k} \geq -M_k(1 - q_{t_1,k}) \quad \forall t_1, t_2 \in T, t_1 \neq t_2, \forall k \in K \tag{3.30}$$

$$\sum_t q_{t,k} = 1 \quad \forall k \in K \tag{3.31}$$

$$umax_k \geq u_{t,k} \quad \forall t \in T, \forall k \in K \tag{3.32}$$

$$umax_k \leq u_{t,k} + M_k(1 - q_{t,k}) \quad \forall t \in T, \forall k \in K \tag{3.33}$$

$$x_{t,k}, y_{t,k} \geq 0 \quad \forall t \in T, \forall k \in K \tag{3.34}$$

$$q_{t,k} \in \{0,1\} \quad \forall t \in T, \forall k \in K \tag{3.35}$$

$$u_{t,k} \in \mathbb{Z}_0^+ \quad \forall t \in T, \forall k \in K \tag{3.36}$$

$$umax_k \in \mathbb{Z}_0^+ \quad \forall k \in K \tag{3.37}$$

The first part of Equation (3.27) adds the increases and decreases in daily resource usage levels together. The first day's (increase) and last day's (decrease) usage levels are added directly whereas intermediate increases and decreases are added via auxiliary variables calculated by Constraint (3.29). This sum is then multiplied by 0.5 to account only for increases. Then, the maximum daily usage for resource type k is subtracted from the sum of increases. By using different weights for resource types one could distinguish between more and less important (expensive) resource types. Constraint (3.29) replaces Constraint (3.2). Constraint (3.30) through Constraint (3.33) are introduced to handle the subtraction of the daily maximum usage. In Constraint (3.30) setting a specific $q_{t_1,k}$ to 1 indicates that the daily usage on day t_1 is greater than or equal to the daily usage on t_2 for resource type k. However, when $q_{t_1,k}$ is set to 0, the equation becomes redundant. Due to Constraint (3.31) the model can set only one $q_{t,k}$ variable to 1. However, to guarantee the feasibility of the constraint set in Constraint (3.30) the chosen day has to be one of the days with maximum usage. Constraint (3.32) says that the maximum daily usage value for resource type k has to be greater than or equal to the daily usages on all other days. Constraint (3.33) makes sure that $umax_k$ is bounded by the actual maximum daily usage which results from scheduling the activities in the project. Since only the respective $q_{t,k}$ value that corresponds to when the maximum daily usage occurs will be set to 1, this constraint only becomes active for a day with maximum usage. For other days with equal or smaller usage values it is redundant.

3.3 Integer Models with Variable Project Duration

In this section, we allow the project duration to be variable. Our objective is to find the project duration that results in the best resource profile for the chosen metric. The idea is that a better leveled resource usage can be obtained by lengthening the project and shifting certain activities forward in time. In effect, longer duration increases the slack times for each activity thus allowing more scheduling options to achieve a better leveled project. First, we assume that the daily target resource levels do not depend on the project duration. For example, a manager may have already committed using a certain number of resources on which the project duration has no impact. Later, the model will be changed so that the targeted number of resources depends on the project duration and is equal to the rounded average resource utilizations. Before moving to the formulations, some results are provided regarding the optimal schedule.

Proposition 3. The variable target levels a_k are nonincreasing in D for all $k \in K$.

Proof. Trivial since $\sum_i r_{i,k} d_i$ are constant. Note that the target levels can remain the same if rounding is applied. The largest target level for resource k will be $\sum_i r_{i,k} d_i / D_{CPM}$. □

Proposition 4. For the optimal project duration, there exists a schedule with no idle times.

Proof. The existence of an optimal solution with no idle times is dependent on the leveling metric chosen. Let S be a schedule of duration D with no idle times. Adding idle time to S does not change the daily resource usages on active days. When the target levels are kept the same, additional penalties incurred for the idle days will make the objective function value actually worse. With RRH, inserting idle times also worsens the objective function value by penalizing the sum of the daily usage just before and just after the idle times rather than only their difference if the idle times did not exist. Hence, one can remove any idle time from a schedule to obtain at least as good of a schedule for both of these metrics. However, showing the existence of a schedule with no idle times is not as trivial with decreasing daily average resource utilizations as targets.

Let S_{idle} be a schedule of length $D+1$ obtained by inserting one time unit of idle time into S. Thus, the daily resource usages remain the same for both schedules on active days; however, they may have shifted by a day. Since it does not matter where this idle time has been inserted, we will assume that it occurs on the last day of S_{idle} for notational convenience. The new daily target value for resource k becomes $\frac{\sum_i r_{i,k} d_i}{(D+1)}$.The difference between the old and new averages equals $\frac{\sum_i r_{i,k} d_i}{D(D+1)}$. The total penalties for resource type k in schedule S are

$$\sum_{t=0}^{D-1} \left| u_{t,k} - \frac{\sum_i r_{i,k} d_i}{D} \right|$$

whereas for schedule S_{idle} they are

$$\sum_{t=0}^{D-1} \left| u_{t,k} - \frac{\sum_i r_{i,k} d_i}{D} + \frac{\sum_i r_{i,k} d_i}{D(D+1)} \right| + \frac{\sum_i r_{i,k} d_i}{(D+1)}.$$

Comparing the penalty functions for S and S_{idle}, it can be seen that the penalties can be decreased by at most $\frac{\sum_i r_{i,k} d_i}{D(D+1)}$ in schedule S_{idle} on days where the daily resource usage is below the average in schedule S. On days where the daily resource usage exceeds or is equal to the average in schedule S a penalty decrease is not possible. When one assumes that the maximum penalty decrease is realized on each day except the idle day, the upper bound on the achievable decrease in penalties is $D\frac{\sum_i r_{i,k} d_i}{D(D+1)}$. But this is equal to the additional penalty incurred on the idle day. Thus, the penalties for S_{idle} have to be at least as much as the penalties in S. In fact, they are only equal when all resource usages are equal to zero. The same analysis is true for all resource types.

However, when the target values change according to rounded average resource utilizations it is possible to have lesser penalties by inserting some idle time and increasing the project duration. A numerical example with two activities is shown in Figures 3.2, 3.3 and 3.4. In the example, the rounded average resource usage per day stays equal to 4 for up to 27 days, and then drops to 3. However, this mathematical possibility which occurs when rounding is applied blindly does not make sense to apply in practice. An idle time can always be pushed to the end of a project thus allowing finishing a project earlier and incurring even less penalties by using the same target value. In the example, the penalties become only 5 by pushing the idle time to the end and finishing the project at 25 with a target value of 3.

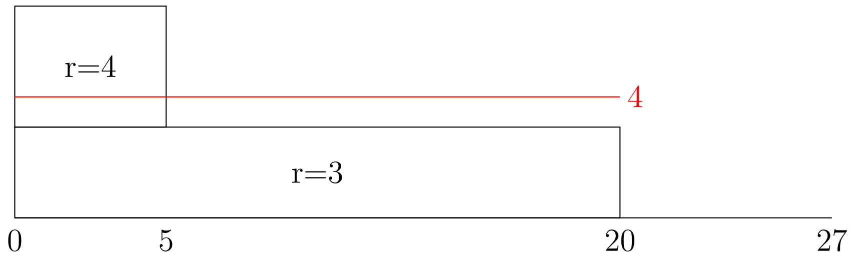

Figure 3.2: CPM solution - Penalties = 30

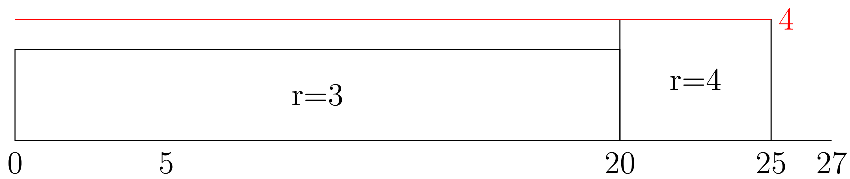

Figure 3.3: Extended solution with no idle time - Penalties = 20

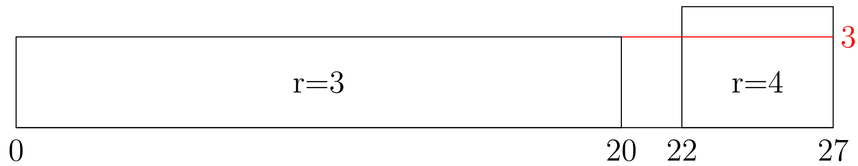

Figure 3.4: Optimal solution - Penalties = 11

Note that when only overloads are penalized the idle times can also be removed. Since the averages are decreasing (nonincreasing when rounded), the overload penalties cannot get better by adding idle times. On another note, when there are general temporal constraints which impose time lags among start times of activities (for example for allowing setups) then some idle time may exist. When the available number of resources is dictated by business conditions and changes with time, one also cannot assert the nonexistence of idle times.

□

Corollary 1. The sum of activity durations is an upper bound on the optimal project duration.

Proof. Since there is an optimal schedule with no idle times, the longest possible project duration in such a schedule occurs when all activities are done serially. □

In the following formulation incorporating a variable project duration, the differences from Model 1 with a fixed duration are shown in bold.

New Sets

T = Days in the project, $t = 0, ..., \mathbf{D_{max}}$.

New Parameters

D_{CPM} = Project's earliest possible finish time. This duration is found via CPM.

D_{max} = Project's latest possible finish time. This could be set to the sum of activity durations.

M = a big penalty coefficient.

New Decision Variables

z = The weighted sum of absolute deviations of total daily resource usages from targeted daily resource usages.

D = Project duration.

f_i = The starting day of activity i.

$\phi_{t,i}$ = 1 if activity i is active on day t; 0 otherwise.

b_t = 1 if time t is after project's finish time D; 0 otherwise.

3.3.1 Model 2

$$min \quad z = \sum_{\mathbf{t}} \sum_{k} w_k(x_{t,k} + y_{t,k}) - \sum_{t} \sum_{k} w_k a_k b_t \tag{3.38}$$

subject to:

$$u_{t,k} - a_k = x_{t,k} - y_{t,k} \quad \forall t \in T, \forall k \in K \tag{3.39}$$

$$u_{t,k} = \sum_i r_{i,k}\phi_{t,i} \quad \forall t \in T, \forall k \in K \tag{3.40}$$

$$p_{i,j}f_i \geq p_{i,j}(f_j + d_j) \quad \forall i, j \in I, i \neq j \tag{3.41}$$

$$\sum_{EST_i \leq t} \sigma_{t,i} = 1 \quad \forall i \in I \tag{3.42}$$

$$\phi_{t,i} = \sum_{t_1=max(EST_i, t-d_i+1)}^{\mathbf{t}} \sigma_{t_1,i} \quad \forall t \in T, \forall i \in I, EST_i \leq t \tag{3.43}$$

$$\phi_{t,i} = 0 \quad \forall t \in T, \forall i \in I, t < EST_i \tag{3.44}$$

$$f_i = \sum_{EST_i \leq t} t\sigma_{t,i} \quad \forall i \in I \tag{3.45}$$

$$f_i \geq EST_i \quad \forall i \in I \tag{3.46}$$

$$f_0 = 0 \tag{3.47}$$

$$\sigma_{0,0} = 1 \tag{3.48}$$

$$f_{N+1} \leq D \tag{3.49}$$

$$D \geq D_{CPM} \tag{3.50}$$

$$D \leq t + M(1 - b_t) \quad \forall t \in T \tag{3.51}$$

$$u_{t,k}, x_{t,k}, y_{t,k} \geq 0 \quad \forall t \in T, \forall k \in K \tag{3.52}$$

$$f_i \in \mathbb{Z}_0^+ \quad \forall i \in I \tag{3.53}$$

$$\phi_{t,i} \in \{0, 1\} \quad \forall t \in T, \forall i \in I \tag{3.54}$$

$$\sigma_{t,i} \in \{0, 1\} \quad \forall t \in T, \forall i \in I \tag{3.55}$$

$$b_t \in \{0, 1\} \quad \forall t \in T \tag{3.56}$$

$$D \in \mathbb{Z}_0^+ \tag{3.57}$$

In Model 2 references to LST_i have been removed. When the project duration is variable, it is not possible to determine the latest start times for activities a priori since they are dependent on the project's overall finish time. The first term in the objective function determines the sum of absolute deviations from the daily target resource usages over the project's maximum possible duration, D_{max}. Hence, the first term includes penalties for times beyond the project's actual finish time, D. The second term in the objective function removes those unnecessary penalties. Constraint (3.49) says that the last (dummy) activity should finish with the project's finish time. Constraint (3.50) states that project should not finish earlier than the duration determined by its critical path. Constraint (3.51) determines days which are beyond the project's finish time by setting the relevant b_t variables to 1. Since the objective function value is decreased with every b_t variable that is set to 1, the model will try to set D to a day as early as possible unless there is a tie in the objective function value of several durations. For days before the project's finish time D, related b_t variables cannot be set to 1 because the constraint will become infeasible.

3.3.2 Variable Target Resource Levels

When lengthening the project duration, it is also possible to allow the target resource levels to be in line with the project duration and be equal to the average daily resource needs. Thus, the target levels will decrease as the project duration is extended. Making the target resource levels a_k also a decision variable leads to a nonlinear formulation with quadratic constraints and objective function.

3.3.2.1 Model 3

When a_k are positive integer variables, Model 3 is obtained which differs from Model 2 by the following constraints. Together with these new constraints, the objective function is now also quadratic due to the terms involving a_k.

$$a_k \geq \sum_i r_{i,k} d_i / D \quad \forall k \in K \tag{3.58}$$

$$a_k \in Z^+ \quad \forall k \in K \tag{3.59}$$

Since a_k are integer, Constraint (3.58) enforces the average daily resource usage on the RHS to be rounded up and used as a target value. This rounding can cause certain solutions that are actually optimal to be missed. These are solutions where some idle time exists in the schedule as discussed in Proposition 4. Thus, one could actually remove Constraint (3.58) from the formulation and leave it to the model to decide appropriate values for a_k. Furthermore, the objective function which involves multiplication of integer valued variables with binary variables can easily be linearized.

3.3.2.2 Variable Project Duration and RRH Metric

When the best project duration is itself a decision variable, on top of the changes to the constraints regarding D given in Model 2, the objective function also needs to change as follows to accommodate the RRH metric.

$$min \quad z = \sum_k w_k \left(0.5(u_{0,k} + \sum_{\mathbf{t=1}}^{\mathbf{D_{max}-1}} (x_{t,k} + y_{t,k})) - umax_k \right) \tag{3.60}$$

The optimal project finish time with this formulation may not be set to the earliest possible time. To penalize setting the project's finish time to a later time than necessary, term D with a small coefficient should be added to the objective function. The small coefficient is needed for preventing the project finish time to be shortened at the expense of the RRH value. Project finish time D is bounded by D_{max}. Furthermore, Constraint (3.51) and variables b_t are not needed when the objective function involves the RRH metric.

3.4 Modified Burgess-Killebrew Heuristic

Burgess-Killebrew heuristic [1] (BK) is a well-known greedy heuristic for leveling projects with fixed durations. We adapt this simple heuristic for finding the optimal project duration with the best resource leveling. The pseudocode of our modified BK heuristic is given in Figure 3.5.

In the new heuristic, we increase the project duration iteratively starting from the duration found by CPM until the duration upper bound is reached. At each iteration, the LST of each activity is increased by one time unit and BK heuristic is applied. The duration which leads to the best solution from all iterations is then reported. In the algorithm, activities are tried according to an activity priority list. Rather than using a single activity priority list we also randomly generate several priority lists (number of lists was set to 100 in computational studies) and use the best solution among them. A single-pass BK heuristic with a given priority list and fixed duration works as follows: Starting with the last activity in the list, the best starting time for each activity within its slack time is found and fixed. Activity start times are fixed as late as possible within their float time to allow for more flexibility for earlier activities. When shifting an activity, the precedence (temporal) relationships are checked and only if no precedence (temporal) relationship is violated, a shift is allowed. Observe that when a project deadline is extended beyond the duration found by CPM all activities become noncritical as their LSTs increase with extended duration.

3.5 Iterative Approach

Rather than directly solving for the optimal duration, it is also possible to adapt an iterative approach where the deadline of a project is increased by one time unit until a duration upper bound is reached while solving a fixed duration model (Model 1) at each iteration. Furthermore, at each iteration the best objective

Via CPM calculate activities' EST_i and LST_i, and D_{CPM}
Set D_{max} to the sum of activity durations
Set $D_{best} = BIGNUMBER$; $z_{best} = BIGNUMBER$
for TFinish = D_{CPM} to D_{max}
    Generate $nLists$-many random activity priority lists
    Set $z_{TFinish} = BIGNUMBER$
    for list = 1 to $nLists$
        Apply BK heuristic using the current TFinish and list; find z_{list}
        if $z_{list} < z_{TFinish}$ then $z_{TFinish} = z_{list}$
    End
    if $z_{TFinish} < z_{best}$ then $z_{best} = z_{TFinish}$; D_{best} = TFinish
    Set $LST_i = LST_i + 1$ for all activities
End
Report D_{best} and z_{best}

Figure 3.5: Modified Burgess-Killebrew heuristic

function value up to that iteration can be used as an upper bound. When the duration upper bound is reached, the best objective function value from all iterations is reported together with the corresponding duration as the optimal solution.

Chapter 4

Computational Studies

4.1 Experiments

For our numerical experiments we use problem sets in the PSPLIB [16] library. The library contains randomly generated 40 problem instances with 10, 15, 20, 30 and 50 activities each. There are different sets with 1, 3 and 5 resource types. Since PSPLIB problems involve general temporal relationships rather than precedence relationships only, we adjusted our models to reflect those temporal relationships when solving the problems. Specifically, Constraint (3.4) and Constraint (3.41) no longer require exactly the total duration of preceding activities to elapse before the successors can begin. Differences in starting times were dictated by input files. We used the sum of activity times as D_{max} in the experiments although it may not be a valid upper bound since a project may have to take longer than the sum of the durations with temporal lags. When that occurred for a few instances, we simply set the upper bound by adding enough extra time to the EST of the last dummy activity and increased D_{max}. Gurobi 6.5 ([17]) was used for optimally solving the mixed-integer linear models. Gurobi was chosen as it provides a state-of-the-art mixed-integer linear and quadratic solver, and it is also free to use for academic purposes. All model interfaces to Gurobi and algorithms were coded in Java.

First we tried to solve benchmark problems exactly. While we could solve all problem instances for Model 2 for up to 30 activities, most instances with 50 activities could not be solved within 10 hours, a time limit we have set for all problems. However, since RLP is NP-hard it is hardly surprising that problems become intractable after a certain size. Solving Model 3 and Model RRH to optimality proved to be impractical as in many cases the time limits were exceeded. In case of Model RRH, problems even with 10 activities were difficult to solve. Thus, we tried to solve Model RRH only for particular problems to obtain as good of a solution as possible. These Model RRH results are helpful in measuring how well the modified BK heuristic is performing for RRH metric.

As expected, the average run times increased with the number of activities, the number of resource types and the complexity of the models. Average run times for Model 1, Model 2 and Model 3 with up to 20 activities are given in Table 4.1. The first column of Table 4.1 show the problem sets which were used for our numerical experiments. In the second row, rlp_10_1 represents a problem set which includes 40 problem instances with 10 activities and 1 resource.

Table 4.1: Average times (seconds)

	Model 1	Model 2	Model 3
rlp_10_1	0.05	1.17	25.63
rlp_10_3	0.06	3.29	71.21
rlp_10_5	0.10	6.54	138.14
rlp_15_1	0.20	17.94	851.39
rlp_15_3	0.22	55.21	1372.43
rlp_15_5	0.31	60.16	1487.74
rlp_20_1	0.36	170.87	4859.36
rlp_20_3	2.23	315.84	7844.35
rlp_20_5	1.54	651.21	13218.01

Moreover, from Table 4.2 one can see that average run times for Model 1 and modified Model 1 do not show very significant differences.

As can be seen from Table 4.3 and Table 4.4, the leveling benefits obtained from extending the duration decrease when the number of resource types increases.

Table 4.2: Average times (seconds) for Model 1 and Model 1 (modified)

	Model 1	Model 1 (modified)
rlp_10_1	0.05	0.02
rlp_10_3	0.06	0.05
rlp_10_5	0.10	0.10
rlp_15_1	0.20	0.19
rlp_15_3	0.22	0.24
rlp_15_5	0.31	0.23
rlp_20_1	0.36	0.32
rlp_20_3	2.23	1.94
rlp_20_5	1.54	2.94

Furthermore, the optimal durations get closer to the CPM duration when the number of resource types increases. Comparison of values in both tables shows that if daily target resource usages are updated according to the averages indicated by the project duration rather than keeping the target values the same regardless of the project duration, better leveling can be obtained at the expense of extending the project more. Moreover, while percent changes seem to become less with increasing number of activities they do not show very significant differences. Table 4.5 indicates that with RRH metric the duration extensions needed to obtain the best leveled profile may be much more compared to other metrics but more leveling benefits can be achieved. However, optimal solutions for bigger problems could not be obtained so it is not appropriate to make a generalization from this single result.

Table 4.3: Optimal average percent changes with same targets

	Average increase in duration	Average decrease in penalties
rlp_10_1	12.00	16.56
rlp_10_3	7.05	3.92
rlp_10_5	3.05	1.04
rlp_15_1	10.83	14.21
rlp_15_3	6.87	4.82
rlp_15_5	4.10	2.02
rlp_20_1	8.92	13.56
rlp_20_3	7.21	3.99
rlp_20_5	3.73	1.36

Table 4.4: Optimal average percent changes with different targets

	Average increase in duration	Average decrease in penalties
rlp_10_1	29.08	34.13
rlp_10_3	12.18	8.74
rlp_10_5	10.55	3.36
rlp_15_1	27.91	34.45
rlp_15_3	15.38	10.25
rlp_15_5	9.06	5.91
rlp_20_1	21.23	32.11
rlp_20_3	11.47	8.78
rlp_20_5	7.20	3.77

Table 4.5: Optimal average percent changes with RRH

	Average increase in duration	Average decrease in penalties
rlp_10_1	64.59	48.71

Percent changes with modified BK can be found in Table 4.6 and Table 4.7. The percent changes provided in these tables are changes from values found by the heuristic using the CPM duration. As expected, percent changes decrease with increasing number of resource types. In addition, percent changes decrease while number of activities increase excluding experiments with 20 activities. The heuristic provided better performance for finding optimal values, if daily target resource usages are keeping the same regardless of the project duration, better resource leveling and cost reductions were obtained if daily target resource usages are updated according to the averages indicated by the project duration. Optimal values could not be obtained solving the modified BK heuristic as BK heuristic is one of the first heuristic for resource leveling. However, we used BK heuristic because an advantage of the modified BK heuristic is that it can be applied for any chosen metric. Table 4.8 gives that modified BK results for RRH metric.

Table 4.6: Average percent changes with modified BK heuristic (same targets)

	Average increase in duration	Average decrease in penalties	Optimal found
rlp_10_1	4.28	5.55	10
rlp_10_3	1.23	0.63	7
rlp_10_5	0.73	0.13	10
rlp_15_1	2.86	3.54	0
rlp_15_3	1.25	0.91	2
rlp_15_5	0.80	0.22	6
rlp_20_1	6.90	3.74	2
rlp_20_3	3.05	0.24	0
rlp_20_5	5.15	0.25	1

Table 4.7: Average percent changes with modified BK heuristic (different targets)

	Average increase in duration	Average decrease in penalties	Optimal found
rlp_10_1	24.73	17.92	3
rlp_10_3	16.52	9.11	1
rlp_10_5	13.37	5.25	0
rlp_15_1	21.26	16.34	0
rlp_15_3	15.17	6.83	1
rlp_15_5	11.56	4.95	0
rlp_20_1	16.66	12.91	0
rlp_20_3	13.94	5.22	0
rlp_20_5	19.12	8.08	0

Table 4.8: Average percent changes with modified BK heuristic (RRH)

	Average increase in duration	Average decrease in penalties
rlp_10_1	8.74	21.26
rlp_10_3	19.88	16.28
rlp_10_5	26.33	17.27
rlp_15_1	10.89	20.58
rlp_15_3	12.73	10.32
rlp_15_5	15.63	10.19
rlp_20_1	12.05	19.04
rlp_20_3	8.83	7.23
rlp_20_5	18.29	5.70

4.2 Iterative Solutions

Rather than directly solving for the optimal duration, it is also possible to adapt an iterative approach where the deadline of a project is increased by one time unit until a duration upper bound is reached while solving a fixed duration model (Model 1) at each iteration. Furthermore, at each iteration the best objective function value up to that iteration can be used as an upper bound. When the duration upper bound is reached, the best objective function value from all iterations is reported together with the corresponding duration as the optimal solution.

All problem instances with 10, 15 and 20 activities were solved iteratively. Table 4.9 and Table 4.10 show that average run times for iterative and direct approaches. As can be seen from Table 4.9, solving Model 2 directly is slower than solving Model 1 iteratively. This means that, the iterative approach with Model 1 must be chosen instead of the direct approach with Model 2. When the cut off values were not provided, all iterations are completed and then solving Model 1 iteratively becomes slower than solving Model 2 directly. Average run times for iterative approach with Model 1 and direct approach for Model 3 are given in Table 4.10. The direct approach for Model 3 is slower than solving Model 1 iteratively. Model 3 includes quadratic terms and this terms slow the proceess.

Table 4.9: Average time comparison (seconds) between iterative and direct approaches (same targets)

	Direct	Iterative
rlp_10_1	1.17	2.35
rlp_10_3	3.29	4.37
rlp_10_5	6.54	6.92
rlp_15_1	17.94	12.41
rlp_15_3	55.21	19.89
rlp_15_5	60.16	26.93
rlp_20_1	170.87	159.63
rlp_20_3	315.84	132.55
rlp_20_5	651.21	184.79

Table 4.10: Average time comparison (seconds) between iterative and direct approaches (different targets)

	Direct	Iterative
rlp_10_1	25.63	7.09
rlp_10_3	71.21	9.04
rlp_10_5	138.14	15.59
rlp_15_1	851.39	188.58
rlp_15_3	1372.43	83.61
rlp_15_5	1487.74	55.43
rlp_20_1	4859.36	1268.37
rlp_20_3	7844.35	818.93
rlp_20_5	13218.01	625.16

Chapter 5

Conclusion

Under circumstances where resource level fluctuations are costly, cost reductions can be obtained by extending the project duration. As indicated by the experiments sometimes even a moderate duration increase can bring about a significant improvement in resource profiles. Thus, when circumstances allow such as when a deadline extension occurs, a project manager can actually use this to her/his advantage by rescheduling project activities and obtaining a project plan with a better resource profile instead of keeping to the original schedule. However, one has to be careful when choosing the measure for leveling. For example, the classical metric of summing the squares of daily resource usages will result in the longest possible project duration. Furthermore, the amount of benefits obtained for different measures can be significantly different.

In this study, mix-integer linear and quadratic models are presented to find optimal duration with best resource profiles. First, we tried to solve mix-integer linear models for our problem sets. We tested Model 1 on all problem sets. Many problem instances with 50 activities for Model 2 could not be solved due to the time limit that we have set for all problems. While the quadratic model (Model 3) was solved, run times for all problem sets increase due to quadratic terms. Many problem instances with 30 and 50 activities exceeded the time limits and could not be solved. Experiments show that the average run times increased while the number of activities, the number of resource types and the complexity of models increased. The optimal durations obtained from Model 2 and Model 3 get closer to the CPM duration and benefiting from better leveling becomes more difficult as the number of resource types increases. Moreover, better leveling benefits can be obtained if daily target resource usages are updated according to the averages indicated by the project duration. To solve larger problems where an exact approach fails, we offered a modified Burgess-Killebrew heuristic. BK heuristic was preferred as it can be applied for any chosen metric. We also adjusted an iterative approach using Model 1. The fixed duration value found by CPM in Model 1 was increased by one time unit and Model 1 was solved at each iteration. When the duration value reached the upper bound, the best objective function value and its duration value were reported. The average run times for iterative approach were compared with the average run times for direct approach. Our results indicate that the iterative approach provided better performance. Benefiting from better resource leveling by extending the project duration becomes more difficult aas the number of resources increase.

As future research, the BK heuristic can be replaced by a state-of-the-art heuristic such as Ballestin et al. [18] since BK heuristic's performance is mediocre. Several findings in this thesis only applies with precedence relationships but they are not valid with general temporal relationships. Theoretical work on the optimal project duration in the presence of general temporal relationships can also be looked at.

Chapter 6

Curriculum Vitae

Elif Eren was born on 13 April 1990, in Denizli. She graduated from Denizli Anatolian High School in 2008. She received her B.S. degree in Industrial Engineering from Işık University in 2014 as 3rd ranking student. She completed her M.S. degree in 2017 in Industrial Engineering and Operations Research from Işık University. From 2014 to present she has been working as a teaching assistant at Işık University Industrial Engineering department.

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Appendices

Appendix A

Computational Results for Same Targets

Table A.1: 10 activities with 1 resource, same targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_10_1_01	40	40	0.00	88	88	0.00	0.047	0.234
rlp_10_1_02	30	35	16.67	102	102	0.00	0.109	0.718
rlp_10_1_03	27	28	3.70	86	84	2.33	0.047	0.780
rlp_10_1_04	18	21	16.67	152	136	10.53	0.000	0.796
rlp_10_1_05	34	36	5.88	195	185	5.13	0.655	2.309
rlp_10_1_06	39	43	10.26	59	47	20.34	0.031	0.343
rlp_10_1_07	13	15	15.38	152	120	21.05	0.000	0.452
rlp_10_1_08	21	24	14.29	108	69	36.11	0.000	0.452
rlp_10_1_09	22	27	22.73	284	208	26.76	0.000	1.607
rlp_10_1_10	19	19	0.00	52	52	0.00	0.125	0.842
rlp_10_1_11	24	24	0.00	60	60	0.00	0.187	3.385
rlp_10_1_12	20	22	10.00	140	116	17.14	0.000	0.655
rlp_10_1_13	40	52	30.00	396	300	24.24	0.016	1.295
rlp_10_1_14	18	20	11.11	80	72	10.00	0.000	0.952
rlp_10_1_15	25	28	12.00	252	128	49.21	0.000	2.356
rlp_10_1_16	34	38	11.76	208	176	15.38	0.016	0.328
rlp_10_1_17	32	34	6.25	220	190	13.64	0.203	2.387
rlp_10_1_18	37	37	0.00	86	86	0.00	0.094	0.156
rlp_10_1_19	27	29	7.41	20	20	0.00	0.031	0.452
rlp_10_1_20	31	31	0.00	52	52	0.00	0.078	1.966
rlp_10_1_21	43	50	16.28	288	212	26.39	0.031	0.390
rlp_10_1_22	40	47	17.50	108	84	22.22	0.016	0.218
rlp_10_1_23	44	49	11.36	188	148	21.28	0.016	0.047
rlp_10_1_24	26	31	19.23	256	224	12.50	0.016	3.089
rlp_10_1_25	47	53	12.77	308	236	23.38	0.016	0.047
rlp_10_1_26	34	43	26.47	249	168	32.53	0.000	0.109
rlp_10_1_27	27	30	11.11	86	68	20.93	0.000	2.371
rlp_10_1_28	22	25	13.64	171	150	12.28	0.000	0.655
rlp_10_1_29	35	41	17.14	276	156	43.48	0.000	0.515
rlp_10_1_30	32	38	18.75	71	59	16.90	0.016	5.476
rlp_10_1_31	34	36	5.88	235	235	0.00	0.094	3.073
rlp_10_1_32	57	64	12.28	381	336	11.81	0.047	0.343
rlp_10_1_33	32	37	15.63	270	245	9.26	0.016	0.686
rlp_10_1_34	22	23	4.55	66	48	27.27	0.000	0.842
rlp_10_1_35	39	48	23.08	44	31	29.55	0.016	0.312
rlp_10_1_36	20	26	30.00	180	92	48.89	0.016	0.640
rlp_10_1_37	37	38	2.70	142	134	5.63	0.000	2.590
rlp_10_1_38	23	23	0.00	36	36	0.00	0.000	0.328
rlp_10_1_39	33	37	12.12	174	132	24.14	0.016	1.451
rlp_10_1_40	39	45	15.38	82	64	21.95	0.094	1.092

D_{CPM} : CPM duration, D_{opt} : optimal duration
Z_{CPM} : CPM cost, Z_{opt} : optimal cost
T_{CPM} : CPM run times (seconds), T_{opt} : optimal run times (seconds)

Table A.2: 10 activities with 3 resources, same targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_10_3_01	26	34	30.77	544	478	12.13	0.078	10.951
rlp_10_3_02	31	32	3.23	339	339	0.00	0.109	3.323
rlp_10_3_03	27	35	29.63	493	369	25.15	0.016	34.694
rlp_10_3_04	24	25	4.17	322	309	4.04	0.031	4.165
rlp_10_3_05	23	24	4.35	248	244	1.61	0.140	1.716
rlp_10_3_06	19	19	0.00	370	370	0.00	0.031	1.638
rlp_10_3_07	22	22	0.00	215	215	0.00	0.000	0.874
rlp_10_3_08	24	28	16.67	348	304	12.64	0.016	4.805
rlp_10_3_09	21	22	4.76	183	183	0.00	0.016	0.733
rlp_10_3_10	33	35	6.06	388	372	4.12	0.187	3.463
rlp_10_3_11	27	28	3.70	376	363	3.46	0.047	7.098
rlp_10_3_12	34	37	8.82	614	614	0.00	0.156	1.903
rlp_10_3_13	50	50	0.00	580	580	0.00	0.062	0.328
rlp_10_3_14	18	19	5.56	167	159	4.79	0.016	1.747
rlp_10_3_15	25	28	12.00	329	296	10.03	0.016	1.732
rlp_10_3_16	24	31	29.17	312	305	2.24	0.047	0.749
rlp_10_3_17	39	39	0.00	425	425	0.00	0.374	4.274
rlp_10_3_18	23	23	0.00	453	453	0.00	0.125	4.228
rlp_10_3_19	18	19	5.56	208	200	3.85	0.109	2.340
rlp_10_3_20	37	38	2.70	550	548	0.36	0.156	1.342
rlp_10_3_21	53	55	3.77	772	748	3.11	0.031	0.125
rlp_10_3_22	33	36	9.09	611	597	2.29	0.031	3.682
rlp_10_3_23	41	41	0.00	553	553	0.00	0.031	0.328
rlp_10_3_24	46	56	21.74	1294	1098	15.15	0.016	0.733
rlp_10_3_25	27	31	14.81	494	444	10.12	0.000	7.862
rlp_10_3_26	35	35	0.00	319	319	0.00	0.031	0.749
rlp_10_3_27	51	51	0.00	941	941	0.00	0.156	0.421
rlp_10_3_28	38	41	7.89	427	406	4.92	0.109	3.136
rlp_10_3_29	31	31	0.00	523	523	0.00	0.016	4.446
rlp_10_3_30	28	28	0.00	400	400	0.00	0.016	0.671
rlp_10_3_31	27	27	0.00	253	253	0.00	0.078	3.026
rlp_10_3_32	48	53	10.42	421	386	8.31	0.047	0.078
rlp_10_3_33	29	32	10.34	400	353	11.75	0.016	3.806
rlp_10_3_34	18	19	5.56	478	470	1.67	0.031	1.576
rlp_10_3_35	25	30	20.00	845	790	6.51	0.078	2.980
rlp_10_3_36	47	47	0.00	907	907	0.00	0.016	0.281
rlp_10_3_37	35	35	0.00	338	338	0.00	0.047	0.811
rlp_10_3_38	27	30	11.11	414	378	8.70	0.016	1.934
rlp_10_3_39	49	49	0.00	695	695	0.00	0.031	1.388
rlp_10_3_40	22	22	0.00	334	334	0.00	0.047	1.404

Table A.3: 10 activities with 5 resources, same targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_10_5_01	36	36	0.00	758	758	0.00	0.094	0.094
rlp_10_5_02	25	26	4.00	828	816	1.45	0.016	5.788
rlp_10_5_03	38	38	0.00	827	827	0.00	0.452	43.004
rlp_10_5_04	32	32	0.00	747	747	0.00	0.203	7.671
rlp_10_5_05	28	29	3.57	888	876	1.35	0.031	2.231
rlp_10_5_06	37	37	0.00	781	781	0.00	0.250	0.390
rlp_10_5_07	22	22	0.00	878	878	0.00	0.047	6.262
rlp_10_5_08	26	28	7.69	523	503	3.82	0.031	4.118
rlp_10_5_09	24	25	4.17	674	659	2.23	0.109	3.463
rlp_10_5_10	29	29	0.00	690	690	0.00	0.343	2.948
rlp_10_5_11	23	23	0.00	349	349	0.00	0.047	3.292
rlp_10_5_12	24	24	0.00	577	577	0.00	0.203	3.838
rlp_10_5_13	21	21	0.00	732	732	0.00	0.031	3.962
rlp_10_5_14	38	38	0.00	743	743	0.00	0.374	1.014
rlp_10_5_15	10	10	0.00	331	331	0.00	0.016	0.702
rlp_10_5_16	30	30	0.00	671	671	0.00	0.140	11.903
rlp_10_5_17	36	39	8.33	844	838	0.71	0.172	25.206
rlp_10_5_18	37	39	5.41	577	551	4.51	0.140	2.855
rlp_10_5_19	31	31	0.00	999	999	0.00	0.062	4.508
rlp_10_5_20	25	26	4.00	507	503	0.79	0.125	1.810
rlp_10_5_21	44	44	0.00	1356	1356	0.00	0.140	45.520
rlp_10_5_22	35	38	8.57	1084	1002	7.56	0.203	7.316
rlp_10_5_23	37	40	8.11	1243	1226	1.37	0.000	5.242
rlp_10_5_24	38	40	5.26	892	890	0.22	0.016	1.747
rlp_10_5_25	57	57	0.00	1084	1084	0.00	0.031	0.094
rlp_10_5_26	32	33	3.13	843	832	1.30	0.031	7.550
rlp_10_5_27	37	37	0.00	975	975	0.00	0.047	1.045
rlp_10_5_28	34	43	26.47	1412	1329	5.88	0.031	0.702
rlp_10_5_29	24	24	0.00	598	598	0.00	0.000	2.309
rlp_10_5_30	28	28	0.00	795	795	0.00	0.062	1.373
rlp_10_5_31	40	40	0.00	713	713	0.00	0.031	0.718
rlp_10_5_32	26	27	3.85	893	848	5.04	0.016	4.961
rlp_10_5_33	34	37	8.82	1216	1176	3.29	0.016	26.816
rlp_10_5_34	21	21	0.00	621	621	0.00	0.016	11.887
rlp_10_5_35	41	47	14.63	967	955	1.24	0.062	1.123
rlp_10_5_36	65	66	1.54	2131	2128	0.14	0.031	0.858
rlp_10_5_37	15	15	0.00	506	506	0.00	0.000	2.028
rlp_10_5_38	40	40	0.00	1396	1396	0.00	0.094	0.390
rlp_10_5_39	22	22	0.00	477	477	0.00	0.016	1.981
rlp_10_5_40	47	49	4.26	1095	1087	0.73	0.094	2.668

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_15_1_01	32	32	0.00	270	270	0.00	0.047	4.383
rlp_15_1_02	31	35	12.90	325	235	27.69	0.125	19.328
rlp_15_1_03	45	50	11.11	135	95	29.63	0.577	63.428
rlp_15_1_04	36	39	8.33	360	300	16.67	0.172	1.794
rlp_15_1_05	47	52	10.64	78	76	2.56	0.140	9.469
rlp_15_1_06	61	64	4.92	162	135	16.67	0.515	92.552
rlp_15_1_07	49	55	12.24	110	78	29.09	0.296	34.148
rlp_15_1_08	34	34	0.00	114	114	0.00	0.109	3.978
rlp_15_1_09	31	32	3.23	235	220	6.38	0.172	32.556
rlp_15_1_10	46	50	8.70	52	46	11.54	0.234	1.576
rlp_15_1_11	36	36	0.00	150	150	0.00	0.172	3.884
rlp_15_1_12	42	45	7.14	340	284	16.47	0.172	36.144
rlp_15_1_13	46	47	2.17	152	140	7.89	0.624	5.569
rlp_15_1_14	39	39	0.00	476	476	0.00	0.031	4.727
rlp_15_1_15	49	61	24.49	182	158	13.19	0.047	16.271
rlp_15_1_16	25	32	28.00	304	248	18.42	0.016	2.496
rlp_15_1_17	25	25	0.00	72	72	0.00	1.295	4.774
rlp_15_1_18	46	46	0.00	50	50	0.00	1.045	73.990
rlp_15_1_19	22	24	9.09	110	98	10.91	0.031	1.451
rlp_15_1_20	32	34	6.25	192	174	9.38	0.390	28.657
rlp_15_1_21	56	62	10.71	97	73	24.74	0.125	1.919
rlp_15_1_22	69	73	5.80	675	635	5.93	0.016	0.515
rlp_15_1_23	51	62	21.57	505	355	29.70	0.062	10.655
rlp_15_1_24	52	61	17.31	387	321	17.05	0.016	45.115
rlp_15_1_25	48	52	8.33	140	112	20.00	0.078	2.886
rlp_15_1_26	30	35	16.67	72	50	30.56	0.016	2.028
rlp_15_1_27	45	55	22.22	472	328	30.51	0.016	40.637
rlp_15_1_28	26	29	11.54	95	75	21.05	0.000	2.371
rlp_15_1_29	42	57	35.71	411	372	9.49	0.016	7.004
rlp_15_1_30	76	76	0.00	508	508	0.00	0.047	0.530
rlp_15_1_31	48	58	20.83	470	350	25.53	0.109	23.213
rlp_15_1_32	48	57	18.75	240	189	21.25	0.062	14.461
rlp_15_1_33	45	55	22.22	87	81	6.90	0.016	28.501
rlp_15_1_34	58	66	13.79	530	340	35.85	0.031	32.401
rlp_15_1_35	54	63	16.67	184	146	20.65	0.062	28.345
rlp_15_1_36	65	69	6.15	375	357	4.80	0.062	1.092
rlp_15_1_37	64	72	12.50	369	309	16.26	0.016	0.562
rlp_15_1_38	41	46	12.20	420	308	26.67	0.000	1.685
rlp_15_1_39	54	57	5.56	288	288	0.00	0.234	25.240
rlp_15_1_40	36	38	5.56	200	190	5.00	0.983	7.067

Table A.4: 15 activities with 1 resource, same targets

Table A.5: 15 activities with 3 resources, same targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_15_3_01	59	67	13.56	607	585	3.62	0.530	199.645
rlp_15_3_02	57	67	17.54	1022	988	3.33	0.265	30.170
rlp_15_3_03	32	34	6.25	527	491	6.83	0.218	21.107
rlp_15_3_04	32	37	15.63	791	580	26.68	0.031	224.590
rlp_15_3_05	41	43	4.88	409	399	2.44	0.905	50.927
rlp_15_3_06	53	54	1.89	773	769	0.52	1.092	291.295
rlp_15_3_07	30	31	3.33	516	516	0.00	0.062	7.129
rlp_15_3_08	47	49	4.26	579	553	4.49	0.359	221.958
rlp_15_3_09	28	29	3.57	345	343	0.58	0.031	3.198
rlp_15_3_10	54	60	11.11	590	482	18.31	0.187	47.252
rlp_15_3_11	44	48	9.09	783	701	10.47	0.796	6.068
rlp_15_3_12	27	28	3.70	617	607	1.62	0.031	16.973
rlp_15_3_13	41	44	7.32	343	334	2.62	0.343	58.921
rlp_15_3_14	36	38	5.56	649	641	1.23	0.359	170.165
rlp_15_3_15	40	41	2.50	552	543	1.63	0.390	30.592
rlp_15_3_16	41	48	17.07	1169	1093	6.50	0.016	53.227
rlp_15_3_17	38	41	7.89	512	476	7.03	0.468	73.117
rlp_15_3_18	54	58	7.41	661	645	2.42	0.421	21.887
rlp_15_3_19	30	31	3.33	459	447	2.61	0.062	75.723
rlp_15_3_20	30	30	0.00	319	319	0.00	0.172	10.904
rlp_15_3_21	48	49	2.08	998	995	0.30	0.062	2.699
rlp_15_3_22	50	61	22.00	792	685	13.51	0.016	14.680
rlp_15_3_23	120	120	0.00	1208	1208	0.00	0.234	2.933
rlp_15_3_24	66	72	9.09	1351	1237	8.44	0.031	0.452
rlp_15_3_25	42	44	4.76	562	550	2.14	0.094	68.422
rlp_15_3_26	40	43	7.50	514	496	3.50	0.016	26.504
rlp_15_3_27	46	46	0.00	1421	1421	0.00	0.016	6.552
rlp_15_3_28	46	47	2.17	432	430	0.46	0.234	21.481
rlp_15_3_29	50	52	4.00	934	928	0.64	0.031	81.541
rlp_15_3_30	60	60	0.00	778	778	0.00	0.094	35.318
rlp_15_3_31	43	46	6.98	670	622	7.16	0.187	47.440
rlp_15_3_32	61	73	19.67	791	635	19.72	0.016	5.554
rlp_15_3_33	50	53	6.00	1265	1235	2.37	0.125	90.277
rlp_15_3_34	34	42	23.53	880	664	24.55	0.062	13.447
rlp_15_3_35	46	48	4.35	719	707	1.67	0.031	8.549
rlp_15_3_36	41	45	9.76	479	467	2.51	0.016	26.972
rlp_15_3_37	60	60	0.00	655	655	0.00	0.047	0.437
rlp_15_3_38	43	44	2.33	807	796	1.36	0.016	7.036
rlp_15_3_39	75	75	0.00	1226	1226	0.00	0.125	2.387
rlp_15_3_40	43	45	4.65	507	499	1.58	0.421	130.900

Table A.6: 15 activities with 5 resources, same targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_15_5_01	51	52	1.96	1033	1028	0.48	0.240	34.180
rlp_15_5_02	57	57	0.00	1091	1091	0.00	0.236	22.682
rlp_15_5_03	25	28	12.00	1156	1062	8.13	0.049	14.430
rlp_15_5_04	39	39	0.00	1046	1046	0.00	0.521	41.012
rlp_15_5_05	27	31	14.81	621	563	9.34	0.331	38.875
rlp_15_5_06	48	51	6.25	1637	1586	3.12	0.403	155.485
rlp_15_5_07	24	26	8.33	561	543	3.21	0.092	10.577
rlp_15_5_08	34	37	8.82	1011	972	3.86	0.080	138.138
rlp_15_5_09	28	28	0.00	670	670	0.00	0.175	12.215
rlp_15_5_10	39	39	0.00	1105	1105	0.00	0.143	110.526
rlp_15_5_11	53	53	0.00	767	767	0.00	0.912	873.524
rlp_15_5_12	40	42	5.00	827	821	0.73	0.602	30.061
rlp_15_5_13	70	70	0.00	1883	1883	0.00	1.042	4.649
rlp_15_5_14	37	37	0.00	584	584	0.00	0.340	47.003
rlp_15_5_15	49	49	0.00	2158	2158	0.00	0.722	65.707
rlp_15_5_16	28	28	0.00	962	962	0.00	0.260	359.737
rlp_15_5_17	35	43	22.86	1181	971	17.78	0.030	11.513
rlp_15_5_18	27	30	11.11	771	732	5.06	0.090	24.274
rlp_15_5_19	56	56	0.00	1834	1834	0.00	3.469	120.073
rlp_15_5_20	35	36	2.86	1009	989	1.98	0.060	18.673
rlp_15_5_21	47	48	2.13	1209	1206	0.25	0.238	4.945
rlp_15_5_22	68	69	1.47	1908	1891	0.89	0.070	11.965
rlp_15_5_23	62	62	0.00	1088	1088	0.00	0.400	14.446
rlp_15_5_24	53	57	7.55	1672	1648	1.44	0.065	20.015
rlp_15_5_25	63	63	0.00	2114	2114	0.00	0.529	1.997
rlp_15_5_26	57	61	7.02	1419	1335	5.92	0.020	2.340
rlp_15_5_27	50	50	0.00	1398	1398	0.00	0.030	1.061
rlp_15_5_28	48	50	4.17	1843	1803	2.17	0.100	14.368
rlp_15_5_29	43	46	6.98	1045	1020	2.39	0.070	62.104
rlp_15_5_30	57	66	15.79	1815	1690	6.89	0.018	22.049
rlp_15_5_31	55	55	0.00	952	952	0.00	0.030	6.848
rlp_15_5_32	56	58	3.57	1597	1535	3.88	0.080	8.502
rlp_15_5_33	70	70	0.00	1636	1636	0.00	0.130	9.376
rlp_15_5_34	32	33	3.13	756	733	3.04	0.030	11.185
rlp_15_5_35	76	76	0.00	2049	2049	0.00	0.150	6.802
rlp_15_5_36	64	64	0.00	1063	1063	0.00	0.060	2.886
rlp_15_5_37	50	55	10.00	2537	2532	0.20	0.230	27.019
rlp_15_5_38	48	52	8.33	1499	1497	0.13	0.130	40.638
rlp_15_5_39	62	62	0.00	1638	1638	0.00	0.142	0.889
rlp_15_5_40	68	68	0.00	2080	2080	0.00	0.250	3.432

Table A.7: 20 activities with 1 resource, same targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_20_1_01	53	54	1.89	125	125	0.00	4.633	178.667
rlp_20_1_02	41	41	0.00	47	47	0.00	0.250	20.233
rlp_20_1_03	73	83	13.70	412	340	17.48	0.078	109.568
rlp_20_1_04	43	45	4.65	31	25	19.35	0.140	21.060
rlp_20_1_05	41	42	2.44	168	164	2.38	0.530	29.078
rlp_20_1_06	58	64	10.34	500	440	12.00	0.624	121.275
rlp_20_1_07	74	81	9.46	420	370	11.90	0.920	527.109
rlp_20_1_08	34	37	8.82	365	275	24.66	0.047	6.552
rlp_20_1_09	41	48	17.07	69	64	7.25	0.109	12.995
rlp_20_1_10	46	47	2.17	90	90	0.00	1.076	68.047
rlp_20_1_11	39	40	2.56	144	141	2.08	0.343	26.692
rlp_20_1_12	45	49	8.89	174	150	13.79	0.842	63.055
rlp_20_1_13	38	40	5.26	260	210	19.23	0.218	172.848
rlp_20_1_14	48	49	2.08	63	61	3.17	0.359	572.849
rlp_20_1_15	60	66	10.00	450	370	17.78	0.203	58.797
rlp_20_1_16	41	44	7.32	81	62	23.46	0.187	25.303
rlp_20_1_17	39	44	12.82	178	144	19.10	0.094	11.482
rlp_20_1_18	49	55	12.24	160	128	20.00	0.296	93.647
rlp_20_1_19	34	36	5.88	132	100	24.24	0.047	15.865
rlp_20_1_20	60	71	18.33	260	140	46.15	0.328	267.806
rlp_20_1_21	69	79	14.49	567	489	13.76	0.016	41.590
rlp_20_1_22	52	58	11.54	129	103	20.16	0.062	66.909
rlp_20_1_23	64	71	10.94	122	108	11.48	0.031	61.542
rlp_20_1_24	78	87	11.54	486	432	11.11	0.031	57.627
rlp_20_1_25	70	81	15.71	1255	1025	18.33	0.016	789.783
rlp_20_1_26	49	50	2.04	177	176	0.56	0.016	79.061
rlp_20_1_27	61	65	6.56	490	450	8.16	0.062	67.486
rlp_20_1_28	57	62	8.77	540	480	11.11	0.265	299.864
rlp_20_1_29	79	83	5.06	320	296	7.50	1.559	88.281
rlp_20_1_30	89	99	11.24	296	264	10.81	0.062	113.163
rlp_20_1_31	94	101	7.45	980	920	6.12	0.062	130.011
rlp_20_1_32	88	95	7.95	248	212	14.52	0.328	1937.461
rlp_20_1_33	86	93	8.14	605	495	18.18	0.125	17.566
rlp_20_1_34	70	81	15.71	220	178	19.09	0.109	55.786
rlp_20_1_35	36	37	2.78	67	66	1.49	0.016	17.956
rlp_20_1_36	76	92	21.05	620	460	25.81	0.156	361.063
rlp_20_1_37	63	73	15.87	652	540	17.18	0.031	177.544
rlp_20_1_38	47	53	12.77	492	404	17.89	0.031	43.914
rlp_20_1_39	48	49	2.08	39	38	2.56	0.109	4.727
rlp_20_1_40	75	82	9.33	372	288	22.58	0.047	20.530

Table A.8: 20 activities with 3 resources, same targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_20_3_01	66	67	1.52	629	621	1.27	33.090	1296.393
rlp_20_3_02	40	44	10.00	965	817	15.34	0.031	288.741
rlp_20_3_03	47	50	6.38	941	936	0.53	0.655	158.465
rlp_20_3_04	41	42	2.44	513	512	0.19	0.265	462.915
rlp_20_3_05	37	40	8.11	711	657	7.59	0.140	87.173
rlp_20_3_06	56	60	7.14	695	661	4.89	1.950	497.922
rlp_20_3_07	53	69	30.19	1006	860	14.51	0.733	577.310
rlp_20_3_08	45	48	6.67	465	432	7.10	0.172	197.621
rlp_20_3_09	33	34	3.03	658	642	2.43	0.078	69.467
rlp_20_3_10	84	84	0.00	936	936	0.00	15.758	211.209
rlp_20_3_11	55	56	1.82	527	526	0.19	0.640	402.949
rlp_20_3_12	43	44	2.33	725	711	1.93	0.109	563.239
rlp_20_3_13	44	46	4.55	281	265	5.69	3.760	202.676
rlp_20_3_14	63	64	1.59	761	759	0.26	12.509	1152.733
rlp_20_3_15	62	64	3.23	1578	1550	1.77	0.218	406.381
rlp_20_3_16	58	64	10.34	1397	1381	1.15	0.858	116.704
rlp_20_3_17	42	46	9.52	960	910	5.21	0.094	40.123
rlp_20_3_18	40	50	25.00	996	986	1.00	0.062	121.867
rlp_20_3_19	67	71	5.97	1100	1048	4.73	14.338	427.800
rlp_20_3_20	40	45	12.50	1208	1002	17.05	0.062	437.721
rlp_20_3_21	63	64	1.59	862	832	3.48	0.437	125.611
rlp_20_3_22	61	67	9.84	704	696	1.14	0.172	442.027
rlp_20_3_23	52	52	0.00	1084	1084	0.00	0.172	194.018
rlp_20_3_24	79	81	2.53	912	906	0.66	0.062	145.969
rlp_20_3_25	54	54	0.00	1015	1015	0.00	0.359	195.546
rlp_20_3_26	51	51	0.00	659	659	0.00	0.203	391.295
rlp_20_3_27	54	56	3.70	1234	1190	3.57	0.125	305.542
rlp_20_3_28	53	57	7.55	1225	1219	0.49	0.140	153.582
rlp_20_3_29	92	94	2.17	1186	1170	1.35	0.062	56.675
rlp_20_3_30	84	84	0.00	788	788	0.00	0.811	120.027
rlp_20_3_31	73	81	10.96	1302	1174	9.83	0.031	76.081
rlp_20_3_32	140	142	1.43	2630	2622	0.30	0.218	83.729
rlp_20_3_33	66	77	16.67	575	535	6.96	0.047	288.039
rlp_20_3_34	66	70	6.06	1149	1133	1.39	0.250	132.382
rlp_20_3_35	53	54	1.89	790	788	0.25	0.031	116.392
rlp_20_3_36	44	47	6.82	735	702	4.49	0.094	83.523
rlp_20_3_37	64	78	21.88	1393	1359	2.44	0.062	102.835
rlp_20_3_38	70	80	14.29	844	778	7.82	0.062	115.518
rlp_20_3_39	44	52	18.18	781	723	7.43	0.078	894.849
rlp_20_3_40	57	63	10.53	1260	1068	15.24	0.047	890.621

Table A.9: 20 activities with 5 resources, same targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_20_5_01	42	46	9.52	1091	1039	4.77	1.342	1417.871
rlp_20_5_02	43	43	0.00	1090	1090	0.00	0.905	158.356
rlp_20_5_03	53	54	1.89	1208	1188	1.66	1.030	1489.662
rlp_20_5_04	45	45	0.00	1300	1300	0.00	1.732	726.446
rlp_20_5_05	49	49	0.00	1315	1315	0.00	0.562	518.483
rlp_20_5_06	58	58	0.00	1135	1135	0.00	33.090	2824.307
rlp_20_5_07	42	44	4.76	1605	1593	0.75	0.421	1210.781
rlp_20_5_08	50	51	2.00	1288	1278	0.78	0.530	268.274
rlp_20_5_09	26	27	3.85	694	674	2.88	0.172	14.040
rlp_20_5_10	45	46	2.22	1288	1283	0.39	0.827	1215.367
rlp_20_5_11	36	39	8.33	574	523	8.89	0.343	48.500
rlp_20_5_12	58	58	0.00	927	927	0.00	4.243	729.988
rlp_20_5_13	38	39	2.63	1045	1008	3.54	1.108	419.922
rlp_20_5_14	62	62	0.00	1752	1752	0.00	0.577	810.406
rlp_20_5_15	35	35	0.00	602	602	0.00	0.952	214.656
rlp_20_5_16	38	41	7.89	1481	1458	1.55	0.172	3372.835
rlp_20_5_17	49	56	14.29	1518	1513	0.33	0.952	2856.537
rlp_20_5_18	57	57	0.00	1621	1621	0.00	0.749	183.581
rlp_20_5_19	56	56	0.00	977	977	0.00	1.310	1121.548
rlp_20_5_20	58	58	0.00	1203	1203	0.00	2.902	498.140
rlp_20_5_21	67	70	4.48	1377	1300	5.59	2.153	227.214
rlp_20_5_22	84	85	1.19	1301	1295	0.46	0.499	855.537
rlp_20_5_23	58	60	3.45	1528	1488	2.62	0.265	413.869
rlp_20_5_24	80	80	0.00	1551	1551	0.00	0.109	295.090
rlp_20_5_25	50	54	8.00	947	903	4.65	0.047	100.074
rlp_20_5_26	78	78	0.00	2261	2261	0.00	0.187	162.958
rlp_20_5_27	67	67	0.00	2777	2777	0.00	0.031	554.924
rlp_20_5_28	63	63	0.00	2763	2763	0.00	2.106	281.456
rlp_20_5_29	68	70	2.94	1670	1652	1.08	0.047	68.515
rlp_20_5_30	62	65	4.84	1650	1630	1.21	0.078	92.149
rlp_20_5_31	85	89	4.71	3630	3570	1.65	0.094	134.379
rlp_20_5_32	51	51	0.00	1667	1667	0.00	0.608	131.836
rlp_20_5_33	45	54	20.00	1290	1254	2.79	0.047	243.158
rlp_20_5_34	42	51	21.43	769	736	4.29	0.047	175.329
rlp_20_5_35	81	83	2.47	2613	2601	0.46	0.078	67.923
rlp_20_5_36	93	95	2.15	2849	2809	1.40	0.328	192.754
rlp_20_5_37	71	71	0.00	2489	2489	0.00	0.328	74.209
rlp_20_5_38	53	53	0.00	1911	1911	0.00	0.265	57.549
rlp_20_5_39	57	63	10.53	2109	2067	1.99	0.172	685.075
rlp_20_5_40	55	58	5.45	1822	1808	0.77	0.047	1.134.559

Appendix B

Computational Results for Different Targets

Table B.1: 10 activities with 1 resource, different targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_10_1_01	40	40	0.00	88	88	0.00	0.047	0.172
rlp_10_1_02	30	30	0.00	102	102	0.00	0.109	2.886
rlp_10_1_03	27	41	51.85	92	34	63.04	0.047	16.427
rlp_10_1_04	18	30	66.67	152	40	73.68	0.000	14.056
rlp_10_1_05	34	36	5.88	195	185	5.13	0.655	13.135
rlp_10_1_06	39	43	10.26	59	47	20.34	0.031	0.484
rlp_10_1_07	13	16	23.08	172	112	34.88	0.000	20.311
rlp_10_1_08	21	24	14.29	108	69	36.11	0.000	4.056
rlp_10_1_09	22	38	72.73	284	124	56.34	0.000	81.449
rlp_10_1_10	19	32	68.42	52	22	57.69	0.125	11.248
rlp_10_1_11	24	29	20.83	60	33	45.00	0.187	27.518
rlp_10_1_12	20	25	25.00	148	60	59.46	0.000	131.727
rlp_10_1_13	40	52	30.00	452	300	33.63	0.016	4.742
rlp_10_1_14	18	21	16.67	92	64	30.43	0.000	13.588
rlp_10_1_15	25	48	92.00	256	104	59.38	0.000	138.809
rlp_10_1_16	34	43	26.47	208	108	48.08	0.016	12.168
rlp_10_1_17	32	37	15.63	220	100	54.55	0.203	58.126
rlp_10_1_18	37	37	0.00	86	86	0.00	0.094	0.125
rlp_10_1_19	27	35	29.63	20	17	15.00	0.031	0.858
rlp_10_1_20	31	45	45.16	62	36	41.94	0.078	48.844
rlp_10_1_21	43	50	16.28	268	212	20.90	0.031	3.058
rlp_10_1_22	40	47	17.50	108	84	22.22	0.016	0.218
rlp_10_1_23	44	49	11.36	188	148	21.28	0.016	0.140
rlp_10_1_24	26	49	88.46	256	144	43.75	0.016	21.186
rlp_10_1_25	47	53	12.77	308	236	23.38	0.016	0.062
rlp_10_1_26	34	43	26.47	291	168	42.27	0.000	0.125
rlp_10_1_27	27	37	37.04	80	56	30.00	0.000	14.742
rlp_10_1_28	22	30	36.36	171	123	28.07	0.000	9.142
rlp_10_1_29	35	41	17.14	276	156	43.48	0.000	7.441
rlp_10_1_30	32	37	15.63	81	59	27.16	0.016	10.483
rlp_10_1_31	34	41	20.59	235	160	31.91	0.094	115.175
rlp_10_1_32	57	64	12.28	366	342	6.56	0.047	0.390
rlp_10_1_33	32	37	15.63	270	245	9.26	0.016	4.571
rlp_10_1_34	22	23	4.55	66	48	27.27	0.000	33.400
rlp_10_1_35	39	48	23.08	67	31	53.73	0.016	0.608
rlp_10_1_36	20	28	40.00	172	60	65.12	0.016	2.324
rlp_10_1_37	37	53	43.24	142	106	25.35	0.000	145.189
rlp_10_1_38	23	42	82.61	36	13	63.89	0.000	13.026
rlp_10_1_39	33	37	12.12	174	132	24.14	0.016	35.630
rlp_10_1_40	39	45	15.38	81	64	20.99	0.094	7.504

D_{CPM} : CPM duration, D_{opt} : optimal duration
Z_{CPM} : CPM cost, Z_{opt} : optimal cost
T_{CPM} : CPM run times (seconds), T_{opt} : optimal run times (seconds)

Table B.2: 10 activities with 3 resources, different targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_10_3_01	26	34	30.77	544	400	26.47	0.078	23.306
rlp_10_3_02	31	33	6.45	361	351	2.77	0.109	173.004
rlp_10_3_03	27	35	29.63	502	362	27.89	0.016	1122.640
rlp_10_3_04	24	25	4.17	332	309	6.93	0.031	112.991
rlp_10_3_05	23	25	8.70	250	248	0.80	0.140	125.877
rlp_10_3_06	19	25	31.58	380	340	10.53	0.031	13.463
rlp_10_3_07	22	22	0.00	199	199	0.00	0.000	1.997
rlp_10_3_08	24	28	16.67	366	272	25.68	0.016	40.170
rlp_10_3_09	21	21	0.00	183	183	0.00	0.016	5.756
rlp_10_3_10	33	35	6.06	410	372	9.27	0.187	88.686
rlp_10_3_11	27	32	18.52	390	303	22.31	0.047	167.950
rlp_10_3_12	34	34	0.00	614	614	0.00	0.156	46.816
rlp_10_3_13	50	50	0.00	578	578	0.00	0.062	2.839
rlp_10_3_14	18	20	11.11	181	161	11.05	0.016	30.576
rlp_10_3_15	25	31	24.00	357	248	30.53	0.016	139.963
rlp_10_3_16	24	31	29.17	312	307	1.60	0.047	7.644
rlp_10_3_17	39	40	2.56	465	436	6.24	0.374	51.324
rlp_10_3_18	23	24	4.35	465	450	3.23	0.125	62.104
rlp_10_3_19	18	19	5.56	208	200	3.85	0.109	58.344
rlp_10_3_20	37	38	2.70	562	560	0.36	0.156	21.637
rlp_10_3_21	53	55	3.77	816	800	1.96	0.031	0.328
rlp_10_3_22	33	36	9.09	629	597	5.09	0.031	23.275
rlp_10_3_23	41	43	4.88	580	569	1.90	0.031	5.086
rlp_10_3_24	46	53	15.22	1334	1134	14.99	0.016	1.388
rlp_10_3_25	27	31	14.81	547	444	18.83	0.000	50.840
rlp_10_3_26	35	38	8.57	412	330	19.90	0.031	14.056
rlp_10_3_27	51	51	0.00	946	946	0.00	0.156	36.176
rlp_10_3_28	38	42	10.53	455	443	2.64	0.109	25.771
rlp_10_3_29	31	40	29.03	556	510	8.27	0.016	18.720
rlp_10_3_30	28	28	0.00	412	412	0.00	0.016	14.134
rlp_10_3_31	27	41	51.85	253	221	12.65	0.078	184.361
rlp_10_3_32	48	53	10.42	457	437	4.38	0.047	0.281
rlp_10_3_33	29	32	10.34	391	335	14.32	0.016	56.254
rlp_10_3_34	18	22	22.22	484	448	7.44	0.031	30.342
rlp_10_3_35	25	30	20.00	775	750	3.23	0.078	22.948
rlp_10_3_36	47	47	0.00	943	943	0.00	0.016	0.983
rlp_10_3_37	35	35	0.00	331	331	0.00	0.047	22.168
rlp_10_3_38	27	33	22.22	441	324	26.53	0.016	27.986
rlp_10_3_39	49	60	22.45	855	701	18.01	0.031	7.332
rlp_10_3_40	22	22	0.00	318	318	0.00	0.047	8.939

Table B.3: 10 activities with 5 resources, different targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_10_5_01	36	36	0.00	858	858	0.00	0.094	0.920
rlp_10_5_02	25	33	32.00	843	817	3.08	0.016	232.394
rlp_10_5_03	38	44	15.79	827	783	5.32	0.452	534.707
rlp_10_5_04	32	32	0.00	759	759	0.00	0.203	682.688
rlp_10_5_05	28	30	7.14	904	878	2.88	0.031	73.008
rlp_10_5_06	37	37	0.00	870	870	0.00	0.250	14.414
rlp_10_5_07	22	30	36.36	928	894	3.66	0.047	263.937
rlp_10_5_08	26	27	3.85	589	505	14.26	0.031	216.450
rlp_10_5_09	24	26	8.33	660	636	3.64	0.109	151.211
rlp_10_5_10	29	29	0.00	681	681	0.00	0.343	538.357
rlp_10_5_11	23	25	8.70	362	358	1.10	0.047	91.728
rlp_10_5_12	24	25	4.17	659	653	0.91	0.203	132.444
rlp_10_5_13	21	22	4.76	834	794	4.80	0.031	216.076
rlp_10_5_14	38	38	0.00	721	721	0.00	0.374	29.531
rlp_10_5_15	10	10	0.00	327	327	0.00	0.016	110.869
rlp_10_5_16	30	34	13.33	717	673	6.14	0.140	128.310
rlp_10_5_17	36	36	0.00	840	840	0.00	0.172	235.295
rlp_10_5_18	37	40	8.11	726	578	20.39	0.140	70.107
rlp_10_5_19	31	35	12.90	1081	1077	0.37	0.062	208.510
rlp_10_5_20	25	26	4.00	507	503	0.79	0.125	40.966
rlp_10_5_21	44	45	2.27	1314	1313	0.08	0.140	99.918
rlp_10_5_22	35	39	11.43	1113	1022	8.18	0.203	142.257
rlp_10_5_23	37	40	8.11	1243	1226	1.37	0.000	31.216
rlp_10_5_24	38	38	0.00	912	912	0.00	0.016	22.136
rlp_10_5_25	57	57	0.00	1209	1209	0.00	0.031	0.718
rlp_10_5_26	32	34	6.25	853	849	0.47	0.031	127.608
rlp_10_5_27	37	37	0.00	1037	1037	0.00	0.047	54.491
rlp_10_5_28	34	43	26.47	1446	1329	8.09	0.031	42.448
rlp_10_5_29	24	30	25.00	622	546	12.22	0.000	117.328
rlp_10_5_30	28	28	0.00	791	791	0.00	0.062	18.049
rlp_10_5_31	40	45	12.50	761	752	1.18	0.031	44.288
rlp_10_5_32	26	36	38.46	899	767	14.68	0.016	132.975
rlp_10_5_33	34	37	8.82	1284	1176	8.41	0.016	57.283
rlp_10_5_34	21	26	23.81	611	595	2.62	0.016	59.280
rlp_10_5_35	41	47	14.63	1031	955	7.37	0.062	9.142
rlp_10_5_36	65	68	4.62	2237	2192	2.01	0.031	37.440
rlp_10_5_37	15	27	80.00	522	520	0.38	0.000	179.634
rlp_10_5_38	40	40	0.00	1378	1378	0.00	0.094	8.003
rlp_10_5_39	22	22	0.00	503	503	0.00	0.016	18.268
rlp_10_5_40	47	47	0.00	1154	1154	0.00	0.094	351.141

Table B.4: 15 activities with 1 resource, different targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_15_1_01	32	48	50.00	270	130	51.85	0.047	4580.792
rlp_15_1_02	31	42	35.48	360	140	61.11	0.125	8138.784
rlp_15_1_03	45	50	11.11	180	95	47.22	0.577	110.557
rlp_15_1_04	36	41	13.89	360	245	31.94	0.172	177.092
rlp_15_1_05	47	61	29.79	112	68	39.29	0.140	154.908
rlp_15_1_06	61	67	9.84	201	144	28.36	0.515	1680.622
rlp_15_1_07	49	55	12.24	140	78	44.29	0.296	1144.434
rlp_15_1_08	34	74	117.65	114	90	21.05	0.109	2292.627
rlp_15_1_09	31	46	48.39	270	110	59.26	0.172	289.568
rlp_15_1_10	46	50	8.70	52	46	11.54	0.234	246.480
rlp_15_1_11	36	50	38.89	150	110	26.67	0.172	191.319
rlp_15_1_12	42	73	73.81	348	128	63.22	0.172	456.145
rlp_15_1_13	46	47	2.17	152	140	7.89	0.624	130.900
rlp_15_1_14	39	43	10.26	476	440	7.56	0.031	634.516
rlp_15_1_15	49	67	36.73	192	124	35.42	0.047	125.970
rlp_15_1_16	25	52	108.00	324	96	70.37	0.016	378.847
rlp_15_1_17	25	30	20.00	72	52	27.78	1.295	1141.813
rlp_15_1_18	46	72	56.52	50	30	40.00	1.045	1250.717
rlp_15_1_19	22	28	27.27	110	22	80.00	0.031	839.812
rlp_15_1_20	32	40	25.00	192	126	34.38	0.390	928.997
rlp_15_1_21	56	62	10.71	115	73	36.52	0.125	13.556
rlp_15_1_22	69	71	2.90	730	680	6.85	0.016	1.638
rlp_15_1_23	51	66	29.41	530	245	53.77	0.062	169.323
rlp_15_1_24	52	59	13.46	429	321	25.17	0.016	53.102
rlp_15_1_25	48	56	16.67	140	100	28.57	0.078	46.972
rlp_15_1_26	30	36	20.00	72	40	44.44	0.016	72.977
rlp_15_1_27	45	53	17.78	468	328	29.91	0.016	55.817
rlp_15_1_28	26	38	46.15	95	33	65.26	0.000	629.820
rlp_15_1_29	42	58	38.10	453	291	35.76	0.016	3.385
rlp_15_1_30	76	76	0.00	508	508	0.00	0.047	1.232
rlp_15_1_31	48	66	37.50	480	210	56.25	0.109	309.629
rlp_15_1_32	48	57	18.75	264	189	28.41	0.062	301.564
rlp_15_1_33	45	58	28.89	87	74	14.94	0.016	139.371
rlp_15_1_34	58	66	13.79	530	340	35.85	0.031	6544.180
rlp_15_1_35	54	63	16.67	192	146	23.96	0.062	76.815
rlp_15_1_36	65	69	6.15	390	357	8.46	0.062	48.656
rlp_15_1_37	64	72	12.50	369	309	16.26	0.016	1.264
rlp_15_1_38	41	59	43.90	420	172	59.05	0.000	193.284
rlp_15_1_39	54	55	1.85	336	288	14.29	0.234	93.335
rlp_15_1_40	36	38	5.56	200	190	5.00	0.983	404.883

Table B.5: 15 activities with 3 resources, different targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_15_3_01	59	67	13.56	732	585	20.08	0.530	227.013
rlp_15_3_02	57	57	0.00	1041	1041	0.00	0.265	440.971
rlp_15_3_03	32	37	15.63	605	536	11.40	0.218	191.431
rlp_15_3_04	32	59	84.38	823	454	44.84	0.031	24615.802
rlp_15_3_05	41	43	4.88	441	428	2.95	0.905	1039.414
rlp_15_3_06	53	53	0.00	784	784	0.00	1.092	2976.594
rlp_15_3_07	30	34	13.33	522	498	4.60	0.062	875.224
rlp_15_3_08	47	53	12.77	586	473	19.28	0.359	2650.679
rlp_15_3_09	28	40	42.86	349	271	22.35	0.031	252.112
rlp_15_3_10	54	60	11.11	590	482	18.31	0.187	337.163
rlp_15_3_11	44	50	13.64	857	707	17.50	0.796	116.766
rlp_15_3_12	27	27	0.00	624	624	0.00	0.031	317.585
rlp_15_3_13	41	44	7.32	395	334	15.44	0.343	2168.841
rlp_15_3_14	36	38	5.56	667	641	3.90	0.359	944.722
rlp_15_3_15	40	42	5.00	608	564	7.24	0.390	477.688
rlp_15_3_16	41	52	26.83	1186	1049	11.55	0.016	355.509
rlp_15_3_17	38	41	7.89	504	476	5.56	0.468	2567.502
rlp_15_3_18	54	59	9.26	677	651	3.84	0.421	443.790
rlp_15_3_19	30	41	36.67	475	391	17.68	0.062	1419.571
rlp_15_3_20	30	31	3.33	367	327	10.90	0.172	276.651
rlp_15_3_21	48	50	4.17	1056	1012	4.17	0.062	40.685
rlp_15_3_22	50	61	22.00	792	685	13.51	0.016	67.111
rlp_15_3_23	120	120	0.00	1208	1208	0.00	0.234	2462.886
rlp_15_3_24	66	72	9.09	1351	1237	8.44	0.031	8.752
rlp_15_3_25	42	56	33.33	562	382	32.03	0.094	262.439
rlp_15_3_26	40	56	40.00	514	490	4.67	0.016	531.587
rlp_15_3_27	46	46	0.00	1421	1421	0.00	0.016	724.247
rlp_15_3_28	46	48	4.35	470	440	6.38	0.234	125.019
rlp_15_3_29	50	51	2.00	934	911	2.46	0.031	347.771
rlp_15_3_30	60	60	0.00	778	778	0.00	0.094	1581.156
rlp_15_3_31	43	46	6.98	738	622	15.72	0.187	454.663
rlp_15_3_32	61	75	22.95	826	630	23.73	0.016	23.197
rlp_15_3_33	50	53	6.00	1345	1310	2.60	0.125	962.693
rlp_15_3_34	34	51	50.00	876	544	37.90	0.062	1135.292
rlp_15_3_35	46	48	4.35	719	707	1.67	0.031	67.189
rlp_15_3_36	41	45	9.76	517	467	9.67	0.016	104.349
rlp_15_3_37	60	60	0.00	655	655	0.00	0.047	70.824
rlp_15_3_38	43	72	67.44	831	774	6.86	0.016	828.065
rlp_15_3_39	75	75	0.00	1277	1277	0.00	0.125	75.130
rlp_15_3_40	43	51	18.60	490	476	2.86	0.421	2329.068

Table B.6: 15 activities with 5 resources, different targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_15_5_01	51	53	3.92	1078	1046	2.97	0.240	116.064
rlp_15_5_02	57	58	1.75	1253	1139	9.10	0.236	245.856
rlp_15_5_03	25	28	12.00	1197	1002	16.29	0.049	3026.125
rlp_15_5_04	39	40	2.56	1126	1086	3.55	0.521	946.064
rlp_15_5_05	27	31	14.81	656	563	14.18	0.331	374.104
rlp_15_5_06	48	49	2.08	1589	1576	0.82	0.403	5333.712
rlp_15_5_07	24	35	45.83	557	388	30.34	0.092	3504.484
rlp_15_5_08	34	40	17.65	991	897	9.49	0.080	5458.356
rlp_15_5_09	28	28	0.00	670	670	0.00	0.175	414.337
rlp_15_5_10	39	39	0.00	1088	1088	0.00	0.143	430.264
rlp_15_5_11	53	53	0.00	759	759	0.00	0.912	1907.197
rlp_15_5_12	40	45	12.50	873	868	0.57	0.602	145.579
rlp_15_5_13	70	70	0.00	1883	1883	0.00	1.042	5052.459
rlp_15_5_14	37	37	0.00	591	591	0.00	0.340	1411.849
rlp_15_5_15	49	49	0.00	2139	2139	0.00	0.722	644.578
rlp_15_5_16	28	38	35.71	984	930	5.49	0.260	1354.457
rlp_15_5_17	35	47	34.29	1181	704	40.39	0.030	2489.593
rlp_15_5_18	27	30	11.11	826	744	9.93	0.090	232.674
rlp_15_5_19	56	56	0.00	1794	1794	0.00	3.469	1078.258
rlp_15_5_20	35	42	20.00	1074	983	8.47	0.060	16532.503
rlp_15_5_21	47	48	2.13	1197	1156	3.43	0.238	118.810
rlp_15_5_22	68	69	1.47	1970	1957	0.66	0.070	118.560
rlp_15_5_23	62	66	6.45	1242	1160	6.60	0.400	903.288
rlp_15_5_24	53	61	15.09	1866	1674	10.29	0.065	104.988
rlp_15_5_25	63	63	0.00	2114	2114	0.00	0.529	661.348
rlp_15_5_26	57	61	7.02	1516	1385	8.64	0.020	32.370
rlp_15_5_27	50	50	0.00	1498	1498	0.00	0.030	18.190
rlp_15_5_28	48	54	12.50	1979	1883	4.85	0.100	257.946
rlp_15_5_29	43	50	16.28	1141	1018	10.78	0.070	1466.917
rlp_15_5_30	57	66	15.79	1832	1690	7.75	0.018	76.300
rlp_15_5_31	55	55	0.00	958	958	0.00	0.030	55.099
rlp_15_5_32	56	60	7.14	1693	1597	5.67	0.080	203.690
rlp_15_5_33	70	72	2.86	1766	1656	6.23	0.130	148.029
rlp_15_5_34	32	44	37.50	750	704	6.13	0.030	1116.338
rlp_15_5_35	76	76	0.00	2133	2133	0.00	0.150	78.125
rlp_15_5_36	64	65	1.56	1209	1118	7.53	0.060	45.926
rlp_15_5_37	50	57	14.00	2591	2550	1.58	0.230	394.306
rlp_15_5_38	48	52	8.33	1545	1473	4.66	0.130	2319.256
rlp_15_5_39	62	62	0.00	1710	1710	0.00	0.142	6.193
rlp_15_5_40	68	68	0.00	2152	2152	0.00	0.250	685.559

Table B.7: 20 activities with 1 resource, different targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_20_1_01	53	63	18.87	125	70	44.00	4.633	1460.693
rlp_20_1_02	41	57	39.02	47	30	36.17	0.250	2163.911
rlp_20_1_03	73	83	13.70	424	340	19.81	0.078	558.809
rlp_20_1_04	43	46	6.98	42	26	38.10	0.140	233.142
rlp_20_1_05	41	45	9.76	168	76	54.76	0.530	36000.000
rlp_20_1_06	58	75	29.31	570	400	29.82	0.624	568.699
rlp_20_1_07	74	83	12.16	460	370	19.57	0.920	2779.800
rlp_20_1_08	34	53	55.88	365	150	58.90	0.047	1385.111
rlp_20_1_09	41	48	17.07	80	64	20.00	0.109	458.142
rlp_20_1_10	46	51	10.87	90	80	11.11	1.076	7996.605
rlp_20_1_11	39	56	43.59	201	99	50.75	0.343	678.679
rlp_20_1_12	45	60	33.33	219	81	63.01	0.842	21664.694
rlp_20_1_13	38	53	39.47	290	180	37.93	0.218	6387.681
rlp_20_1_14	48	61	27.08	73	62	15.07	0.359	36000.000
rlp_20_1_15	60	66	10.00	550	370	32.73	0.203	216.185
rlp_20_1_16	41	46	12.20	81	48	40.74	0.187	12999.206
rlp_20_1_17	39	69	76.92	178	54	69.66	0.094	4141.916
rlp_20_1_18	49	73	48.98	158	84	46.84	0.296	36000.000
rlp_20_1_19	34	36	5.88	156	100	35.90	0.047	3018.340
rlp_20_1_20	60	76	26.67	272	124	54.41	0.328	6632.679
rlp_20_1_21	69	79	14.49	567	489	13.76	0.016	222.706
rlp_20_1_22	52	66	26.92	129	67	48.06	0.062	793.667
rlp_20_1_23	64	74	15.63	122	70	42.62	0.031	481.542
rlp_20_1_24	78	87	11.54	486	432	11.11	0.031	187.388
rlp_20_1_25	70	81	15.71	1245	1025	17.67	0.016	778.207
rlp_20_1_26	49	52	6.12	186	180	3.23	0.016	312.765
rlp_20_1_27	61	73	19.67	490	375	23.47	0.062	414.399
rlp_20_1_28	57	71	24.56	540	295	45.37	0.265	3108.493
rlp_20_1_29	79	83	5.06	388	296	23.71	1.559	460.185
rlp_20_1_30	89	106	19.10	296	226	23.65	0.062	149.760
rlp_20_1_31	94	101	7.45	980	920	6.12	0.062	914.739
rlp_20_1_32	88	95	7.95	268	212	20.90	0.328	2527.048
rlp_20_1_33	86	93	8.14	605	495	18.18	0.125	469.951
rlp_20_1_34	70	81	15.71	268	178	33.58	0.109	814.524
rlp_20_1_35	36	48	33.33	67	39	41.79	0.016	139.636
rlp_20_1_36	76	87	14.47	730	460	36.99	0.156	301.455
rlp_20_1_37	63	75	19.05	652	528	19.02	0.031	256.121
rlp_20_1_38	47	66	40.43	492	356	27.64	0.031	322.624
rlp_20_1_39	48	57	18.75	39	29	25.64	0.109	281.097
rlp_20_1_40	75	82	9.33	372	288	22.58	0.047	93.850

Table B.8: 20 activities with 3 resources, different targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_20_3_01	66	67	1.52	613	601	1.96	33.090	15119.650
rlp_20_3_02	40	51	27.50	989	802	18.91	0.031	36000.000
rlp_20_3_03	47	50	6.38	996	936	6.02	0.655	1899.756
rlp_20_3_04	41	41	0.00	461	461	0.00	0.265	9088.467
rlp_20_3_05	37	51	37.84	702	545	22.36	0.140	7892.101
rlp_20_3_06	56	62	10.71	709	675	4.80	1.950	2085.895
rlp_20_3_07	53	69	30.19	1070	823	23.08	0.733	21500.524
rlp_20_3_08	45	48	6.67	527	432	18.03	0.172	5263.688
rlp_20_3_09	33	37	12.12	710	598	15.77	0.078	3809.432
rlp_20_3_10	84	84	0.00	950	950	0.00	15.758	1237.793
rlp_20_3_11	55	55	0.00	532	532	0.00	0.640	9988.385
rlp_20_3_12	43	46	6.98	833	727	12.73	0.109	36000.000
rlp_20_3_13	44	48	9.09	445	269	39.55	3.760	845.787
rlp_20_3_14	63	69	9.52	761	753	1.05	12.509	36000.000
rlp_20_3_15	62	64	3.23	1588	1586	0.13	0.218	1104.045
rlp_20_3_16	58	58	0.00	1293	1293	0.00	0.858	1854.734
rlp_20_3_17	42	48	14.29	1054	812	22.96	0.094	5528.993
rlp_20_3_18	40	51	27.50	1052	990	5.89	0.062	3573.061
rlp_20_3_19	67	71	5.97	1100	1048	4.73	14.338	36000.000
rlp_20_3_20	40	59	47.50	1216	878	27.80	0.062	17416.713
rlp_20_3_21	63	64	1.59	880	856	2.73	0.437	2161.852
rlp_20_3_22	61	61	0.00	688	688	0.00	0.172	8400.100
rlp_20_3_23	52	62	19.23	1090	1050	3.67	0.172	954.410
rlp_20_3_24	79	81	2.53	958	956	0.21	0.062	3684.071
rlp_20_3_25	54	54	0.00	1015	1015	0.00	0.359	1379.729
rlp_20_3_26	51	59	15.69	646	619	4.18	0.203	1726.455
rlp_20_3_27	54	57	5.56	1244	1201	3.46	0.125	1757.749
rlp_20_3_28	53	53	0.00	1192	1192	0.00	0.140	1534.013
rlp_20_3_29	92	95	3.26	1324	1176	11.18	0.062	429.063
rlp_20_3_30	84	84	0.00	726	726	0.00	0.811	1421.537
rlp_20_3_31	73	81	10.96	1389	1199	13.68	0.031	1185.540
rlp_20_3_32	140	140	0.00	2734	2734	0.00	0.218	3895.436
rlp_20_3_33	66	77	16.67	575	514	10.61	0.047	6730.008
rlp_20_3_34	66	70	6.06	1165	1133	2.75	0.250	4615.783
rlp_20_3_35	53	55	3.77	811	806	0.62	0.031	1506.120
rlp_20_3_36	44	56	27.27	729	657	9.88	0.094	1151.641
rlp_20_3_37	64	78	21.88	1443	1331	7.76	0.062	277.665
rlp_20_3_38	70	80	14.29	832	778	6.49	0.062	158.434
rlp_20_3_39	44	62	40.91	837	623	25.57	0.078	6216.920
rlp_20_3_40	57	64	12.28	1385	1071	22.67	0.047	12378.388

Table B.9: 20 activities with 5 resources, different targets

	D_{CPM}	D_{opt}	$Inc(\%)$	Z_{CPM}	Z_{opt}	$Dec(\%)$	T_{CPM}	T_{opt}
rlp_20_5_01	42	54	28.57	1119	1005	10.19	1.342	36000.000
rlp_20_5_02	43	44	2.33	1156	1110	3.98	0.905	5612.016
rlp_20_5_03	53	59	11.32	1269	1194	5.91	1.030	36000.000
rlp_20_5_04	45	47	4.44	1335	1326	0.67	1.732	30110.003
rlp_20_5_05	49	49	0.00	1315	1315	0.00	0.562	36000.000
rlp_20_5_06	58	58	0.00	1135	1135	0.00	33.090	36000.000
rlp_20_5_07	42	42	0.00	1629	1629	0.00	0.421	9737.584
rlp_20_5_08	50	50	0.00	1258	1258	0.00	0.530	1940.643
rlp_20_5_09	26	34	30.77	694	540	22.19	0.172	19557.380
rlp_20_5_10	45	52	15.56	1293	1285	0.62	0.827	15976.862
rlp_20_5_11	36	39	8.33	628	536	14.65	0.343	561.695
rlp_20_5_12	58	58	0.00	957	957	0.00	4.243	36000.000
rlp_20_5_13	38	39	2.63	1005	985	1.99	1.108	9115.252
rlp_20_5_14	62	62	0.00	1772	1772	0.00	0.577	36000.000
rlp_20_5_15	35	44	25.71	647	575	11.13	0.952	12755.586
rlp_20_5_16	38	46	21.05	1551	1371	11.61	0.172	4290.366
rlp_20_5_17	49	50	2.04	1538	1537	0.07	0.952	36000.000
rlp_20_5_18	57	57	0.00	1714	1714	0.00	0.749	1968.146
rlp_20_5_19	56	58	3.57	1009	989	1.98	1.310	17473.653
rlp_20_5_20	58	61	5.17	1299	1256	3.31	2.902	36000.000
rlp_20_5_21	67	71	5.97	1449	1301	10.21	2.153	6862.686
rlp_20_5_22	84	85	1.19	1333	1324	0.68	0.499	36000.000
rlp_20_5_23	58	59	1.72	1740	1652	5.06	0.265	1994.978
rlp_20_5_24	80	80	0.00	1557	1557	0.00	0.109	817.972
rlp_20_5_25	50	56	12.00	987	931	5.67	0.047	1306.611
rlp_20_5_26	78	85	8.97	2393	2293	4.18	0.187	3831.055
rlp_20_5_27	67	67	0.00	2940	2940	0.00	0.031	599.821
rlp_20_5_28	63	65	3.17	2882	2815	2.32	2.106	1404.127
rlp_20_5_29	68	70	2.94	1670	1652	1.08	0.047	818.643
rlp_20_5_30	62	64	3.23	1650	1630	1.21	0.078	1735.799
rlp_20_5_31	85	89	4.71	3778	3734	1.16	0.094	335.104
rlp_20_5_32	51	51	0.00	1655	1655	0.00	0.608	1600.968
rlp_20_5_33	45	55	22.22	1352	1206	10.80	0.047	3978.849
rlp_20_5_34	42	56	33.33	773	703	9.06	0.047	7126.186
rlp_20_5_35	81	81	0.00	2622	2622	0.00	0.078	1789.245
rlp_20_5_36	93	95	2.15	2876	2830	1.60	0.328	10027.588
rlp_20_5_37	71	71	0.00	2495	2495	0.00	0.328	4826.726
rlp_20_5_38	53	53	0.00	1956	1956	0.00	0.265	2955.472
rlp_20_5_39	57	68	19.30	2175	2046	5.93	0.172	19738.980
rlp_20_5_40	55	58	5.45	1874	1808	3.52	0.047	3870.476