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M.S. Thesis

2018

MARKDOWN OPTIMIZATION IN APPAREL RETAIL SECTOR

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B.S., Industrial Engineering, IŞIK UNIVERSITY, 2015

Submitted to the Graduate School of Science and Engineering
in partial fulfillment of the requirements for the degree of
Master of Science
in
Industrial Engineering

IŞIK UNIVERSITY
2018

IŞIK UNIVERSITY
GRADUATE SCHOOL OF SCIENCE AND ENGINEERING

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APPROVAL DATE: 16/01/2018

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Abstract

Over last decades, rapidly growing textile and apparel industry has become an important sector. The branding and ever-changing fashion sense have been a trigger for a major competition environment. Nowadays, increasing competition has accompanied by rapidly changing demand. This changing demand leads to imbalances between aggregate demand and inventory when combined with long lead times. In a sector where lead time is longer than the season, such as fashion sector, the elapsed time between demand and lead time makes dynamic prices important. Dynamic pricing is to change the sales price of the product over time, taking into account the remaining inventory and taking into account the up-to-date customer demand. In this way, the inventory level of the products during the season can be controlled through demand and the cost of stock keeping and transportation can be reduced.

In this study, an empirical model is developed by using the empirical sales data which consisting of different product groups over multiple selling season. As distinct from the literature, weighted least square estimation is used as a regression method in order to generate empirical demand model. Developed empirical model is used in markdown optimization as dynamic demand for revenue maximization. Mathematical model determines the optimal discount level for each product and also, determines when markdown prices should be applied while maximizing company revenue and considering inventory goals. As a result, it is observed that when the markdowns were applied to the products, the sales increases and the inventory level of each product is used up until the end of the season.

Keywords: markdown optimization, demand forecasting, dynamic pricing

HAZIR GİYİM PERAKENDECİLİK SEKTÖRÜNDE FİYAT OPTİMİZASYONU

Özet

Hızla gelişen tekstil ve hazır giyim sektörü en önemli perakendecilik sektörlerinden biridir. Markalaşma ve sürekli değişen moda anlayışı büyük bir rekabet ortamının tetikleyicisi olmuştur. Günümüzde bu sektörde giderek artan rekabet koşulları, hızla değişen talepleri de beraberinde getirmiştir. Talepteki bu değişiklik yüksek tedarik süreleriyle birleştiği zaman toplam talep ile envanter arasında dengesizliklere sebep olmaktadır. Moda sektörü gibi tedarik süresinin sezona göre daha uzun olduğu sektörlerde talep ile tedarik süresi arasında geçen süre dinamik fiyatlandırmayı önemli hale getirmiştir. Dinamik fiyatlandırma, tüketicinin istek ve ihtiyaçlarını göz önünde bulundurup satıcının karını gözeterek, ürünün satış fiyatının zamana bağlı olarak değiştirilmesidir. Bu sayede, sezon içinde ürünlerin stok seviyesi talep vasıtasıyla kontrol edilerek stok tutma ve taşıma maliyetleri azaltılabilir.

Talep tahmini yapmak için işbirliği yapılan şirketten gelen farklı ürün gruplarına ait alınan verileri kullanarak ampirik bir model geliştirilmiştir. Literatürden farklı olarak, ampirik model ağırlıklı en küçük kareler yöntemi kullanılarak oluşturulmuştur. Geliştirilen ampirik model indirim optimizasyon modelinde gelir maksimizasyonu amacıyla dinamik bir talep olarak kullanılmıştır. Matematiksel model, envanter hedeflerini göz önünde bulundurarak, şirketin karını maksimize edecek şekilde her bir ürün için en uygun indirim seviyesini ve zamanını belirlemektedir. Sonuç olarak, ürünlere indirim uygulandığında, satışlarının arttığı ve envanter seviyelerinin sezon sonuna kadar tükendiği görülmüştür.

Anahtar kelimeler: indirim optimizasyonu, talep tahmini, dinamik fiyatlandırma

Acknowledgements

Foremost, I would first like to express my eternal gratitude to my supervisor Assist. Prof. Dr. Mustafa Hekimoğlu for the endless support of my master study and research. His wise guidance helped me to write this thesis.

Additionally, I would like to thank Işık University Industrial Engineering Department staff. I learned a lot of things from them during my education life.

Last but not the least, I would like to thank my parent Serdar Yıldız and Saadet Yıldız for trusting and supporting me throughout my life. Also, I would like to thank my lovely sister Ecem Yıldız who always understands and believes in me.

To my family...

Table of Contents

List of Tables

List of Figures

List of Abbreviations

MAPE **M**ean **A**bsolute **P**ercentage **E**rror
GAMS **G**eneral **A**lgebraic **M**odeling **S**system
NLP **N**on**L**inear **P**rogramming
VBA **V**isual **B**asic for **A**pplications

Chapter 1

Introduction

Dynamic pricing is a critical part of pricing strategy for retailers. With dynamic pricing strategy, retailers can change their prices within a fixed time periods. The definition of the dynamic pricing is constantly changing prices to meet individual customer demand by considering remaining inventory. Nowadays, in growing competition, retailers compete with competitors by using this strategy while at the same time they make profit. Airline companies where flight tickets are change constantly can be an example of the competition. Also, dynamic pricing strategy is widely used in fast-fashion sector. In this sector, demand is changed rapidly depending on ever-changing fashion sense. Therefore, products must be sold in a short time period. By way of managing the optimal prices of products for the short selling season, retailers can increase their revenue.

In the challenging competition environment, markdown prices is the good way for retailers to sell more products in the short selling season by considering inventory control in order to maximize their revenue in apparel sector. Unlike the dynamic pricing, markdown prices are permanent. It does not increase and decrease at the same time. In other words, when markdown is applied the products, it cannot be taken back. For example, a product price is reduced from 49.99 to 29.99. The price of the product cannot be 49.99 again in the same selling season. Therefore, well implemented markdown optimization plan provides retailers to increase the sales of products and to reduce inventory on hand.

In this study, we developed a markdown optimization tool using empirical sales data from a store of Turkish apparel retailer of fast-fashion products. The company provided us sales data of different products in five years time horizon. Our study has two main parts. First part of the study is estimating customer demand by conducting regression model. Second part of the study is conducting mathematical model for revenue maximization in order to make a markdown plan.

Forecasting customer demand is the first part of this study. An empirical model is developed by conducting proper regression analysis for demand estimation. As distinct from literature, in this study, weighted least square estimation method is used in regression analysis, because in the apparel sector, products usually do not sell much at the beginning of the season. The motivation behind considering weighted least squares is that price sensitivity of product demand changes over time in a selling season. Hence it might be beneficial to give more importance some parts of the selling season rather than treating all sales instances indifferently. The formula of weights is based on exponential smoothing formula from the forecasting literature [1]. To sum up, different regression models are developed by using both weighted least square method and least square estimation methods for linear and nonlinear models with different independent variables to find best fit value. Cross validation test is applied after parameter estimation of forecasting model. The model is selected to select best model. The results show that weighted nonlinear least square estimation method is outperformed among other regression method for demand estimation.

The second part of this study is to conduct a mathematical model for markdown optimization that maximizes the total revenue. The selected empirical model is used as price dependent demand in the mathematical model. Our model is constructed as a nonlinear problem. Nonlinearity is arisen from the objective function and a constraint which defines the dynamic customer demand. Therefore, we had to ensure that the objective function is concave on a convex set for the global optimum solution [2]. Hence, concavity analysis is made for the objective function. In our model analysis, we find that objective function is concave.

For the solution of the model, two approaches are considered to solve the optimization problem: static approach and dynamic approach. First approach is applied to the whole sample dataset for one selling season for different inventory level. Even we solved our model by using the static approach, it is a dynamic problem. Therefore, we developed approximate dynamic programming approach. In this approach, inventory level and demand are updated for every week in this approach. Hence, this approach is more suitable to solve markdown optimization problem. This method is applied to one product that selected from the sample dataset randomly for the selected inventory level.

The rest of the thesis is organized as follows. Chapter 2 presents literature review. Chapter 3 describes the dataset that we used in order to predict customer demand. Chapter 4 presents demand prediction models in detail. Chapter 5 describes the markdown optimization model. Conclusion and future research are presented Chapter 6.

Chapter 2

Literature Review

Over last decades, pricing management becomes a significant research area. In this area, there are many important researches, see Elmagrahraby and Keskinocak (2003) [3] and Talluri and van Ryzin (2005) [4] for general overview of this research area in detailed. Our research consists of two main parts: demand modeling and markdown optimization model. We will review the relevant literature for both parts.

The demand modeling and forecasting literature includes of many studies considering different aspects of customer demand. Among those studies, we reviewed the most relevant four for our research. Ferreira et al.(2015) [5] used machine learning method in order to estimate customer demand and historical lost sales. They developed a nonparametric approximation for demand forecasting by using different regression methods which are least squares, partial least squares, principle component, multiplicative and semilogarithmic regression and regression trees for multiple products. They performed fivefold cross validation test for comparison of regression models. After evaluation of all models, they chose regression tree. They created an algorithm to optimize pricing. Then, they developed a decision support tool for an online retailer's daily use by implementing the algorithm. As a result, they improved the test group revenue of the online retailer around about 9.7 percent. By the virtue of this article, we reviewed various regression models for our study in order to estimate customer demand. Also similarly, we used cross validation to compare different regression models. Bitran and Candentey (2003) [6] made in-depth overview about dynamic pricing and analyzed relationship between dynamic pricing and revenue management. This research involves revenue management that stochastic demand with price sensitivity is satisfied by perishable and nonrenewable products over a finite time horizon. They analyzed deterministic and stochastic demand model for both cases of single and multiproduct. That article provides us a significant overview of deterministic and stochastic demand processes for our study. By Smith, McIntyre and Achabal (1994) [1] , a two stage prediction process is built up. Their prediction process updates repeatedly sales and response estimation in order to promote marketing decision. The first stage of the study, seasonal profile and initial prediction of marketing reaction parameters are produced for each product group by conducting regression model on pooled sales data. Pooling generates a data set that is big enough to predict only the effects that are observed once for each product. In the second stage of the study, discounted least square by using smoothing is performed for the developed procedure that updates parameters in order to keep in step with the marketing conditions. In the updating procedure, they used smoothing method which is the widely use approach to update forecasts for changing marketing conditions. They measured the performance of their technique by applying the technique to big weekly sales data. As mentioned in Montgomery and Johnson (1976, pp. 77-78) [7], one dimensional exponential smoothing could reproduce the weights of the weighted least squares method. They expended this information to develop a multidimensional smoothing method to update parameters of market response. We applied exponential smoothing in weighted least square in our study.

The second part of the literature is markdown optimization. Caro and Gallien [8] developed and implemented a decision making procedure for price markdowns in collaboration with one of the fast-fashion retailer, Zara. Forecasting model is developed by using logarithmic regression in their study. After determining the customer demand model, the markdown optimization model which is fed is solved weekly basis. They implemented their markdown optimization model two stores of Zara in 2008. They increased Zara's revenue by approximately 6 percent. That paper is much related to our study with respect to both demand prediction model and markdown optimization model. They also used approximate dynamic programming to solve markdown optimization problem. Chen et al. (2015) [9] considered a markdown optimization problem for multiple store considering inventory allocation. The probability distribution of the demand is not known by retailer. Knowing all information about probability distribution of the demand improves over time. Even the deterministic demand model, the problem is still NP-hard at the beginning of each time period for a single store or a single time. Hence, they developed a heuristic approach in which demand is determined as stochastic by using discrete scenarios of demand depending on latest demand distribution information. They developed mixed integer programming model for the problem. To solve the problem they employed Lagrangian relaxation and compared that solution with several widely known benchmark approaches. Their solution approach gave better results in comparison with benchmark approaches. Their study is important for us, because it the part of the research stream that we would like to contribute for future works. Coşgun et al. (2012) [10] studied markdown optimization which is about the one of the main problem for retail chain in apparel sector. The problem is originated substitute products. Once the markdown policy is applied to one product, other products' sales are affected. In other words, demands of all products are affected when one product price is changed, i.e. strong cross price elasticity between products are applied markdown policy. They determined cross price elasticity of demand prediction by using multinomial logit model. Then, they developed a mathematical model by using approximate dynamic programming. The result of their study demonstrates that the high selling prices and lower discount are found when substitution effect increases.

To sum up briefly, our research area is consist of two parts; demand prediction model and deterministic markdown optimization that use the prediction model for revenue maximization of an apparel retail company. Articles are reviewed for both parts of the study and the most relative papers are explained in detailed as above. Within these articles, the most relevant study is Caro and Gallien (2012) [8] for our study.

Chapter 3

Data Analysis

We obtained a dataset from apparel retailer with six thousands stores all around the world. We worked with data set of one store in Turkey. The given dataset consists of date of sale, product identity number, brand identity number, selling years, selling seasons, group identity number, product number, collection identity number, collection name, gender information, product name, sales quantity, sales price and list price for every product sold between 2011 and 2015.

This dataset, there are three different products groups. In this study, we focus on only one product group. Dataset of one product group has 87619 rows belong to variables that mentioned as above. There are approximately three thousand products in the dataset. Every product has a unique product identity number which is obtained from combining variables of brand identity number, year, season, group identity number and product number. For example; in the product identity number of 1413089J3Z33, 1413 comes from products brand identity number, year which shows that how many years have passed since the product thrown into the market, season which shows that which season the product belongs and group identity number which shows that which product group it belongs respectively. The rest of the number, 089J3Z33 is the products number. There are four different brand identity numbers and four different seasons which are winter, spring, summer and fall. Each product belongs to a product group. Therefore each has a group identity number. The dataset also has 85 different collection identity numbers and collections name for all season which winter, spring, summer and fall. Basic denim, basic urban, summer evening, rock festival, office smart, smart black etc. can be shown as examples. The dataset consists of three kinds of gender information, male, female and unisex. Also, there is a column for product name for each product. Finally, the last three columns are very important for this study in order to estimate customer demands. These are daily sales quantity, sales price which is discounted prices for each product if there is any and list price which show that initial price for each products without any discounts, respectively.

Ten products are chosen randomly from the original data for the sample dataset. Chosen products are basic products, but each product has different aspect in order to represent the whole dataset such as belonging to different collections, different season, different gender etc. Selected products have various selling years among each other, i.e. product 1 is sold for 4 years, product 2 is sold for 2 years etc. In total, the sample dataset includes 856 rows and 6 variables.

Chapter 4

Demand Prediction Model

Within the sample dataset, it is known that all information about products such as selling prices, sales quantity, selling years, product identity number etc. except demands of products. By using those variables, customer demand is estimated.

In order to predict demands, we use regression analysis. Different types of regression models are developed and compared to find a proper demand model which gives the best fits for our study. Regression models are mainly divided two groups which are multiple linear least square regression and multiple nonlinear least square regression. However, in this study, we also considered weighted least squares in demand modeling with regression. For linear and nonlinear regression groups, several demand models are conducted by changing predictor variables and after that weighted least square estimation method is applied all conducted demand models for both, linear and nonlinear, regression groups. Eventually, regression models can be grouped under four titles which are multiple linear regression models, multiple nonlinear regression models, multiple linear regression models with weighted least square method and multiple nonlinear regression models with weighted least square method.

4.1 Regression Models Analysis

Several regression models are developed for different groups of regression models by using the sample. Results of regression analysis for each models compared with their R^2 values which is coefficient of multiple determination, and MAPE, mean absolute percentage error, which is calculated in cross validation test that we made for evaluate the effectiveness of demand estimation models.

Regression models are generated by using various predictor variables. In the regression equations, *weeks* presents the selling weeks of the products for one year. *Markdown* is calculated by using the sales prices and list prices from the data as *sales prices / list prices* . Inventory is assumed as total sales quantity minus current week sales price. *Year* shows that how many years have passed since the product thrown into the market which is cycle time. *ProductNo* is the numerical value of each product. *Previous* represents previous week sales. *Markdown*Year* is an interaction term of these two variables. *FirstWeekofaMonth*, *SecondWeekofaMonth*, *ThirdWeekofaMonth*, *FourthWeekofaMonth* are the categorical variables that emphasizes the first, second, third and fourth week of a month. TT is calculated as $(t-t^*)^2$ where t is the current week and t^* is the middle of the selling year. By using this variable, the middle of the selling year emphasized when the sales are increased. *Introduction, Maturity and Decline* are the categorical variables in which the beginning, the middle and the end of the selling year are emphasized respectively. Using these variables with various combinations and various demand prediction method, approximately sixty regression models are developed for this study. All regression models are generated in R programming.

In order to ensure the validation of each regression models, cross validation test is made for them. Within different types of cross validation method, k-fold cross validation test is more suitable for this study. Therefore, we used three-fold cross validation method to measure performance of the regression equations. Results are evaluated according to MAPE. Cross validation test is encoded in R programming.

All conducted regression equations are explained in detailed as following.

4.1.1 Multiple Linear Regression Models

Multiple linear least square regression supposes that there is a linear relationship between dependent variable Y, explanatory variable $x_i, i \in \{1, 2, ..., I\}$, and a error term ε. Equation of multiple linear least square regression is as follows.

$$Y = \beta_0 + \beta_1 * x_1 + \beta_2 * x_2 + ... + \beta_I * x_I + \varepsilon \tag{4.1}$$

where β_0 is the constant, $\beta_i, i \in \{1, 2, ..., I\}$ are the regression coefficients of explanatory variables.

4.1.1.1 Regression Model 1

First regression model is conducted as multiple linear regression to predict customer demand. In this model, there are seven independent variables. These are *SalesWeeks*, *Markdown*, *Invetory*, *ProductNumber*, *PreviousWeekSales*, *Year* and *Markdown*Year*. The regression equation of this model is as below.

$$Demand = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory +$$

$$\beta_4 * ProductNumber + \beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * Markdown * Year \tag{4.2}$$

Results of first demand prediction model can be shown as following tables.

Coefficients	Values
β_0	1.2027961
β_1	0.0735124
β_2	-1.3273061
β_3	0.0074011
β_4	0.2226894
β_5	0.7764790
β_6	-2.3841764
β_7	2.4121289

Table 4.1: Coefficients of constant term and independent variables of first regression model.

R-Squared	Adjusted R-Squared	P-Value
0.8345	0.8327	0.00

Table 4.2: Results of first regression model.

Adjusted R^2 is 0.8327 which is close to 1. Hence, there is a strong relationship between dependent variable, demand, and independent variables which is expressed in the model. In other respects, P-value is 0.00 for the first model. This demonstrates that model is significant according to p-value is smaller than 0.05.

4.1.1.2 Regression Model 2

Second regression model is conducted as multiple linear regression to predict customer demand. In this model, there are ten independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $Introduction$, $Maturity$, $Decline$ and $Markdown * Year$. The regression equation of this model is as below.

$$Demand = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$
$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * Introduction + \beta_8 * Maturity +$$
$$\beta_9 * Decline + \beta_1 0 * Markdown * Year \tag{4.3}$$

Results of second demand prediction equation can be shown as following tables.

Coefficients	**Values**
β_0	4.6517416
β_1	0.0764765
β_2	-7.8104773
β_3	0.0085686
β_4	0.1570488
β_5	0.7437158
β_6	-4.2212278
β_7	-2.8869354
β_8	4.9986489
β_9	NA
β_{10}	4.5074713

Table 4.3: Coefficients of constant term and independent variables of second regression model.

R-Squared	**Adjusted R-Squared**	**P-Value**
0.8398	0.8376	0.00

Table 4.4: Results of second regression model.

Adjusted R^2 is 0.8376 which is close to 1. Hence, there is a strong relationship between dependent variable, demand, and independent variables which is expressed in the model. In other respects, P-value is 0.00 for the second model.

This demonstrates that model is significant according to p-value is smaller than 0.05.

4.1.1.3 Regression Model 3

Third regression model is conducted as multiple linear regression to predict customer demand. In this model, there are eight independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, TT and $Markdown * Year$. The regression equation of this model is as below.

$$Demand = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$

$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * TT + \beta_8 * Markdown * Year \quad (4.4)$$

Results of third demand prediction equation can be shown as following tables.

Coefficients	**Values**
β_0	2.399718
β_1	0.193159
β_2	-2.111316
β_3	0.009362
β_4	0.091946
β_5	0.735966
β_6	-2.254132
β_7	-0.023244
β_8	2.588347

Table 4.5: Coefficients of constant term and independent variables of third regression model.

R-Squared	Adjusted R-Squared	P-Value
0.8389	0.8369	0.00

Table 4.6: Results of third regression model.

Adjusted R^2 is 0.8369 which is close to 1. Hence, there is a strong relationship between dependent variable, demand, and independent variables which is expressed in the model. In other respects, P-value is 0.00 for the third model. This demonstrates that model is significant according to p-value is smaller than 0.05.

4.1.1.4 Regression Model 4

Fourth regression model is conducted as multiple linear regression to predict customer demand. In this model, there are eleven independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $FirstWeekofaMonth$, $SecondWeekofaMonth$, $ThirdWeekofaMonth$, $FourthWeekofaMonth$ and $Markdown * Year$. The regression equation of this model is as below.

$$Demand = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$

$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * FirstWeekofaMonth + \beta_8 * SecondWeekofaMonth +$$

$$\beta_9 * ThirdWeekofaMonth + \beta_1 0 ForthWeekofaMonth + \beta_1 1 * Markdown * Year \tag{4.5}$$

Results of fourth demand prediction equation can be shown as following tables.

Coefficients	Values
β_0	-1.0503165
β_1	0.0800884
β_2	-1.4793839
β_3	0.0074186
β_4	0.2268850
β_5	0.7763590
β_6	-2.3736323
β_7	3.2602772
β_8	2.5238126
β_9	0.2958963
β_{10}	3.1450044
β_{11}	2.3760731

Table 4.7: Coefficients of constant term and independent variables of fourth regression model.

R-Squared	Adjusted R-Squared	P-Value
0.8354	0.8327	0.00

Table 4.8: Results of fourth regression model.

Adjusted R^2 is 0.8327 which is close to 1. Hence, there is a strong relationship between dependent variable, demand, and independent variables which is expressed in the model. In other respects, P-value is 0.00 for the fourth model. This demonstrates that model is significant according to p-value is smaller than 0.05.

4.1.1.5 Regression Model 5

Fifth regression model is conducted as multiple linear regression to predict customer demand. In this model, there are twelve independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $FirstWeekofaMonth$, $SecondWeekofaMonth$, $ThirdWeekofaMonth$, $FourthWeekofaMonth$, TT and $Markdown * Year$. The regression equation of this model is as below.

$$Demand = \beta_0+\beta_1*SalesWeek+\beta_2*Markdown+\beta_3*Inventory+\beta_4*ProductNumber+$$

$$\beta_5*PreviousWeekSales+\beta_6*Year+\beta_7*FirstWeekofaMonth+\beta_8*SecondWeekofaMonth+$$

$$\beta_9*ThirdWeekofaMonth+\beta_10 ForthWeekofaMonth+\beta_1 1*TT+\beta_1 2*Markdown*Year \tag{4.6}$$

Results of fifth demand prediction equation can be shown as following tables.

Coefficients	**Values**
β_0	4.2076025
β_1	0.1927357
β_2	-2.3848045
β_3	0.0093714
β_4	0.0841556
β_5	0.7368579
β_6	-2.2867821
β_7	-0.6270337
β_8	-1.3928452
β_9	-3.5694456
β_{10}	-0.7766533
β_{11}	-0.0234474
β_{12}	2.6406803

Table 4.9: Coefficients of constant term and independent variables of fifth regression model.

R-Squared	Adjusted R-Squared	P-Value
0.8398	0.8368	0.00

Table 4.10: Results of fifth regression model.

Adjusted R^2 is 0.8368 which is close to 1. Hence, there is a strong relationship between dependent variable, demand, and independent variables which is expressed in the model. In other respects, P-value is 0.00 for the fifth model. This demonstrates that model is significant according to p-value is smaller than 0.05.

4.1.2 Multiple Nonlinear Regression Models

After linear regression models are conducted, we tried nonlinear transformation by taking square root of dependent variable. Advantage of this transformation method is to reduced variance among sales quantity data that used to estimate dependent variable as customer demand. Because like fashion sector, it can be large difference among sales quantity from the beginning to end of the season. Customers generally are not willing to buy a product when it is first released to the market. Therefore, at the beginning of the season sales are very low but, at the end of the season sales are very high. This may cause the fluctuation of variance in sales quantity data. For this reason, this nonlinear transformation technique is more relevant than other transformation technique. Nonlinear regression supposes that there is a nonlinear relationship between dependent variable Y, independent variables $x_i, i \in \{1, 2, ..., I\}$, and a error term ε. Equation of multiple nonlinear regression is as follows.

$$\sqrt{Y} = \beta_0 + \beta_1 * x_1 + \beta_2 * x_2 + ... + \beta_I * x_I + \varepsilon \tag{4.7}$$

Where β_0 is the constant, $\beta_i, i \in \{1, 2, ..., I\}$ are the regression coefficients of explanatory variables.

4.1.2.1 Regression Model 6

Sixth regression model is conducted as multiple nonlinear regression to predict customer demand. In this model, there are seven independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$ and $Markdown * Year$. The regression equation of this model is as below.

$$\sqrt{Demand} = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory +$$

$$\beta_4 * ProductNumber + \beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * Markdown * Year \tag{4.8}$$

Results of sixth demand prediction model can be shown as following tables.

Coefficients	**Values**
β_0	3.1094531
β_1	-0.0012881
β_2	-1.1189505
β_3	0.0005971
β_4	-0.0070566
β_5	0.0586791
β_6	-0.6665660
β_7	0.9208713

Table 4.11: Coefficients of constant term and independent variables of sixth regression model.

R-Squared	**Adjusted R-Squared**	**P-Value**
0.7969	0.7948	0.00

Table 4.12: Results of sixth regression model.

Adjusted R^2 is 0.7948 which is close to 1. Hence, there is a strong relationship between dependent variable, demand, and independent variables which is expressed in the model. In other respects, P-value is 0.00 for the sixth model. This demonstrates that model is significant according to p-value is smaller than 0.05.

4.1.2.2 Regression Model 7

Seventh regression model is conducted as multiple linear regression to predict customer demand. In this model, there are ten independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $Introduction$, $Maturity$, $Decline$ and $Markdown * Year$. The regression equation of this model is as below.

$$\sqrt{Demand} = \beta_0+\beta_1*SalesWeek+\beta_2*Markdown+\beta_3*Inventory+\beta_4*ProductNumber+$$

$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * Introduction + \beta_8 * Maturity+$$

$$\beta_9 * Decline + \beta_1 0 * Markdown * Year \tag{4.9}$$

Results of seventh demand prediction equation can be shown as following tables.

Coefficients	Values
β_0	3.5710000
β_1	0.0010370
β_2	-2.1600000
β_3	0.0007843
β_4	-0.0156300
β_5	0.0532400
β_6	-0.9703000
β_7	-0.3689000
β_8	0.8774000
β_9	NA
β_{10}	1.2620000

Table 4.13: Coefficients of constant term and independent variables of seventh regression model.

R-Squared	Adjusted R-Squared	P-Value
0.8206	0.8181	0.00

Table 4.14: Results of seventh regression model.

Adjusted R^2 is 0.8181 which is close to 1. Hence, there is a strong relationship between dependent variable, demand, and independent variables which is expressed in the model. In other respects, P-value is 0.00 for the seventh model. This demonstrates that model is significant according to p-value is smaller than 0.05.

4.1.2.3 Regression Model 8

Eighth regression model is conducted as multiple linear regression to predict customer demand. In this model, there are eight independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$,

$Year$, TT and $Markdown * Year$. The regression equation of this model is as below.

$$Demand = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber + \beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * TT + \beta_8 * Markdown * Year \quad (4.10)$$

Results of eighth demand prediction equation can be shown as following tables.

Coefficients	**Values**
β_0	3.3020000
β_1	0.0179500
β_2	-1.2450000
β_3	0.0009125
β_4	-0.0280800
β_5	0.0521600
β_6	-0.6457000
β_7	-0.0037380
β_8	0.9492000

Table 4.15: Coefficients of constant term and independent variables of eighth regression model.

R-Squared	**Adjusted R-Squared**	**P-Value**
0.8157	0.8135	0.00

Table 4.16: Results of eighth regression model.

Adjusted R^2 is 0.8135 which is close to 1. Hence, there is a strong relationship between dependent variable, demand, and independent variables which is expressed in the model. In other respects, P-value is 0.00 for the eighth model.

This demonstrates that model is significant according to p-value is smaller than 0.05.

4.1.2.4 Regression Model 9

Ninth regression model is conducted as multiple linear regression to predict customer demand. In this model, there are eleven independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $FirstWeekofaMonth$, $SecondWeekofaMonth$, $ThirdWeekofaMonth$, $FourthWeekofaMonth$ and $Markdown * Year$. The regression equation of this model is as below.

$$\sqrt{Demand} = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$
$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * FirstWeekofaMonth + \beta_8 * SecondWeekofaMonth +$$
$$\beta_9 * ThirdWeekofaMonth + \beta_1 0 ForthWeekofaMonth + \beta_1 1 * Markdown * Year \tag{4.11}$$

Results of ninth demand prediction equation can be shown as following tables.

Coefficients	Values
β_0	2.3680000
β_1	0.0002999
β_2	-1.1100000
β_3	0.0006016
β_4	-0.0055750
β_5	0.0585100
β_6	-0.6576000
β_7	0.8222000
β_8	0.7076000
β_9	0.5561000
β_{10}	0.7995000
$\beta_1 1$	0.9041000

Table 4.17: Coefficients of constant term and independent variables of ninth regression model.

R-Squared	Adjusted R-Squared	P-Value
0.7988	0.7954	0.00

Table 4.18: Results of ninth regression model.

Adjusted R^2 is 0.7954 which is close to 1. Hence, there is a strong relationship between dependent variable, demand, and independent variables which is expressed in the model. In other respects, P-value is 0.00 for the nineth model. This demonstrates that model is significant according to p-value is smaller than 0.05.

4.1.2.5 Regression Model 10

Tenth regression model is conducted as multiple linear regression to predict customer demand. In this model, there are twelve independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $FirstWeekofaMonth$, $SecondWeekofaMonth$, $ThirdWeekofaMonth$, $FourthWeekofaMonth$, TT and $Markdown * Year$. The regression equation of this model is as below.

$$\sqrt{Demand} = \beta_0+\beta_1*SalesWeek+\beta_2*Markdown+\beta_3*Inventory+\beta_4*ProductNumber+$$

$$\beta_5*PreviousWeekSales+\beta_6*Year+\beta_7*FirstWeekofaMonth+\beta_8*SecondWeekofaMonth+$$

$$\beta_9*ThirdWeekofaMonth+\beta_10 ForthWeekofaMonth+\beta_11*TT+\beta_12*Markdown*Year \tag{4.12}$$

Results of tenth demand prediction equation can be shown as following tables.

Coefficients	**Values**
β_0	3.2010000
β_1	0.0181500
β_2	-1.2540000
β_3	0.0009111
β_4	-0.0281900
β_5	0.0522500
β_6	-0.6438000
β_7	0.2061000
β_8	0.0869000
β_9	-0.0564500
β_{10}	0.1780000
β_{11}	-0.0037160
β_{12}	0.9461000

Table 4.19: Coefficients of constant term and independent variables of tenth regression model.

R-Squared	Adjusted R-Squared	P-Value
0.8169	0.8135	0.00

Table 4.20: Results of tenth regression model.

Adjusted R^2 is 0.8135 which is close to 1. Hence, there is a strong relationship between dependent variable, demand, and independent variables which is expressed in the model. In other respects, P-value is 0.00 for the tenth model. This demonstrates that model is significant according to p-value is smaller than 0.05.

4.1.3 Multiple Linear Regression Models with Weighted Least Square Method

In contrast to the regression of linear and nonlinear least squares, the weighted least squares regression is not dependent on a particular function used to define the relationship among variables. Instead, the weighted least squares reflect the behavior of random errors in the model; and can be used for linear or nonlinear functions in parameters [11]. In this study, weighted least square estimation method is applied to both linear and nonlinear least square regression. The reason we choose this method is that lay emphasis on end of the season of the fashion sector. The increasing rank of weights is given to end of season rather than at the beginning of the season for markdown prices, because of the fact that markdowns are usually implemented at the end of the season. Weights are determined as a exponential smoothing formula from forecasting literature. There are five weights level calculated by changing α level. Formulation of weights is below.

$$w = ((1-\alpha)^{(t-t_c)})^2 \tag{4.13}$$

where t is the total sales weeks, t_c is the current sales week and α is chosen $\{0.1, 0.2, 0.5, 0.8\}$. Four weights are generated by using given α levels and last weight is generated by using α=0.1 and also we emphasis that years of product by multiplying with constant number. For example, a product spent four year in market. we multiplied its first, second, third and fourth year by 0.2, 0.4, 0.6 and 0.8 respectively. The graphical representation of the five weights belongs to product 1 is given as below as an example.

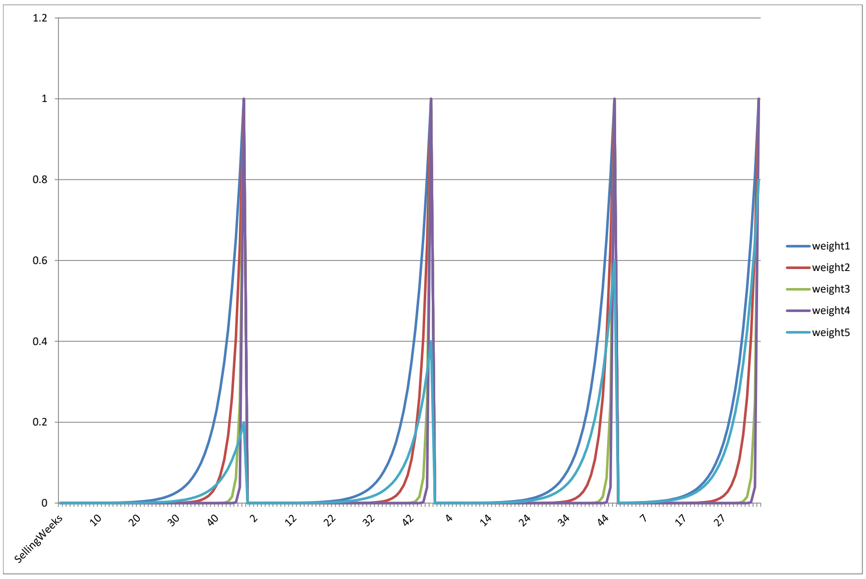

Figure 4.1: Graphical representation of weights of the product 1

Figure 4.1 represents all weights of the product 1 from the sample data for four selling years. As can be shown in the figure, at the end of the each selling years are emphasized by given higher value to each weight.

4.1.3.1 Regression Model 11

Eleventh regression model is conducted as multiple linear regression with weighted least square method by using five different weights as explained as above to predict customer demand. In this model, there are seven independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$ and $Markdown * Year$. The regression equation of this model is as below.

$$Demand = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory +$$
$$\beta_4 * ProductNumber + \beta_5 * PreviousWeekSales + \beta_6 * Year +$$
$$\beta_7 * Markdown * Year, Weight = w_i, i \in \{1, 2, ..., 5\} \tag{4.14}$$

Results of eleventh demand prediction model can be shown as following tables.

Coefficients	**w1**	**w2**	**w3**	**w4**	**w5**
β_0	10.19736	13.776569	23.454455	19.5800000	11.719727
β_1	0.01566	0.022274	-0.005798	0.0006091	0.002278
β_2	-10.52468	-12.877513	-18.005728	-12.7000000	-11.844906
β_3	0.02367	0.087076	0.147674	0.1361000	0.024753
β_4	0.05930	0.109425	0.103057	0.2248000	0.022262
β_5	0.70487	0.461522	0.134425	0.0618200	0.700356
β_6	-3.77115	-3.961740	-4.726916	-3.5960000	-4.493172
β_7	3.87744	3.616206	3.292605	1.3270000	4.844174

Table 4.21: Coefficients of constant term and independent variables of eleventh regression model.

	weight 1	**weight 2**	**weight 3**	**weight 4**	**weight 5**
R-Squared	0.8428	0.632	0.1816	0.2063	0.8337
Adjusted R-Squared	0.8412	0.6281	0.1729	0.1979	0.8319
P-Value	0.00	0.00	0.00	0.00	0.00

Table 4.22: Results of eleventh regression model.

4.1.3.2 Regression Model 12

Twelveth regression model is conducted as multiple linear regression with weighted least square method by using five different weights as explained as above to predict customer demand. In this model, there are ten independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $Introduction$, $Maturity$, $Decline$ and $Markdown * Year$. The regression equation of this model is as below.

$$Demand = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$

$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * Introduction + \beta_8 * Maturity +$$

$$\beta_9 * Decline + \beta_1 0 * Markdown * Year \tag{4.15}$$

Results of twelveth demand prediction equation can be shown as following tables.

Coefficients	**weight 1**	**weight 2**	**weight 3**	**weight 4**	**weight 5**
β_0	9.977358	13.888751	23.679067	19.6000000	11.701920
β_1	0.037092	0.016958	-0.007307	0.0005658	0.022560
β_2	-11.243671	-12.777849	-18.188588	-12.7100000	-12.668521
β_3	0.023257	0.089037	0.155089	0.1381000	0.024468
β_4	0.059897	0.114116	0.106356	0.2248000	0.028745
β_5	0.700986	0.459863	0.133120	0.0617600	0.695790
β_6	-4.168175	-3.886392	-4.766062	-3.5990000	-4.893455
β_7	0.083007	-2.444150	2.350907	12.1100000	-2.170058
β_8	1.116836	-0.459112	-1.331057	-1.6470000	1.137446
β_9	NA	NA	NA	NA	NA
β_{10}	4.293449	3.533321	3.333417	1.3300000	5.223046

Table 4.23: Coefficients of constant term and independent variables of twelveth regression model.

	weight 1	weight 2	weight 3	weight 4	weight 5
R-Squared	0.8434	0.6323	0.1829	0.2064	0.8344
Adjusted R-Squared	0.8413	0.6273	0.1717	0.1955	0.8322
P-Value	0.00	0.00	0.00	0.00	0.00

Table 4.24: Results of twelveth regression model.

4.1.3.3 Regression Model 13

Thirteenth regression model is conducted as multiple linear regression with weighted least square method by using five different weights as explained as above to predict customer demand. In this model, there are eight independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, TT and $Markdown * Year$. The regression equation of this model is as below.

$$Demand = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$

$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * TT + \beta_8 * Markdown * Year \quad (4.16)$$

Results of thirteenth demand prediction equation can be shown as following tables.

Coefficients	weight 1	weight 2	weight 3	weight 4	weight 5
β_0	7.772846	12.200915	20.844418	16.711553	8.498858
β_1	0.139522	0.091263	0.135635	0.167542	0.181949
β_2	-9.580116	-12.167024	-17.462411	-12.412038	-10.934732
β_3	0.024008	0.085801	0.128878	0.110790	0.025224
β_4	0.069195	0.099955	0.084862	0.202871	0.077012
β_5	0.691146	0.457327	0.128840	0.054218	0.680724
β_6	-3.497071	-3.737947	-4.788589	-3.837921	-4.292386
β_7	-0.009568	-0.004873	-0.009018	-0.010351	-0.013063
β_8	3.562613	3.399776	3.473118	1.758199	4.496128

Table 4.25: Coefficients of constant term and independent variables of thirteenth regression models.

	weight 1	weight 2	weight 3	weight 4	weight 5
R-Squared	0.8438	0.6327	0.1852	0.2107	0.8356
Adjusted R-Squared	0.8419	0.6282	0.1753	0.2011	0.8336
P-Value	0.00	0.00	0.00	0.00	0.00

Table 4.26: Results of thirteenth regression model.

4.1.3.4 Regression Model 14

Fourteenth regression model is conducted as multiple linear regression with weighted least square method by using five different weights as explained as above to predict customer demand. In this model, there are eleven independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $FirstWeekofaMonth$, $SecondWeekofaMonth$, $ThirdWeekofaMonth$, $FourthWeekofaMonth$ and $Markdown * Year$. The regression equation of this model is as below.

$$Demand = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$

$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * FirstWeekofaMonth + \beta_8 * SecondWeekofaMonth +$$

$$\beta_9 * ThirdWeekofaMonth + \beta_10 ForthWeekofaMonth + \beta_1 1 * Markdown * Year \tag{4.17}$$

Results of fourteenth demand prediction equation can be shown as following tables.

Coefficients	**weight 1**	**weight 2**	**weight 3**	**weight 4**	**weight 5**
β_0	10.752462	15.265493	22.36760	14.61271	10.396295
β_1	0.016118	0.023376	-0.01109	-0.01493	0.014422
β_2	-10.909820	-14.129804	-16.45722	-6.47703	-12.992445
β_3	0.023613	0.088842	0.18033	0.21025	0.024382
β_4	0.067459	0.134853	0.11883	0.26365	0.032018
β_5	0.706232	0.464708	0.14211	0.06476	0.701335
β_6	-3.791950	-4.590505	-5.23267	-2.36417	-4.592026
β_7	-1.316705	-2.092135	-1.90972	-2.08058	0.792553
β_8	0.685129	-0.071963	-1.24598	-1.52576	3.363219
β_9	-0.855763	-1.062788	-1.35173	-1.47355	1.405782
β_{10}	0.050389	-0.164407	-0.43545	-1.46966	2.911519
β_{11}	3.915596	4.356271	4.06636	0.18363	4.940130

Table 4.27: Coefficients of constant term and independent variables of fourteenth regression model.

	weight 1	weight 2	weight 3	weight 4	weight 5
R-Squared	0.8444	0.6398	0.1926	0.2245	0.8377
Adjusted R-Squared	0.8418	0.6337	0.1791	0.2115	0.835
P-Value	0.00	0.00	0.00	0.00	0.00

Table 4.28: Results of fourteenth regression model.

4.1.3.5 Regression Model 15

Fifteenth regression model is conducted as multiple linear regression with weighted least square method by using five different weights as explained as above to predict customer demand. In this model, there are twelve independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $FirstWeekofaMonth$, $SecondWeekofaMonth$, $ThirdWeekofaMonth$, $FourthWeekofaMonth$, TT and $Markdown * Year$. The regression equation of this model is as below.

$$Demand = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$
$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * FirstWeekofaMonth + \beta_8 * SecondWeekofaMonth +$$
$$\beta_9 * ThirdWeekofaMonth + \beta_10 ForthWeekofaMonth + \beta_11 * TT + \beta_12 * Markdown * Year \tag{4.18}$$

Results of fifteenth demand prediction equation can be shown as following tables.

Coefficients	weight 1	weight 2	weight 3	weight 4	weight 5
β_0	8.629993	12.786447	18.858178	10.662735	8.260085
β_1	0.158104	0.139374	0.158677	0.174611	0.175345
β_2	-9.906617	-12.911286	-15.353537	-5.269548	-12.186897
β_3	0.024042	0.087146	0.155135	0.162126	0.024923
β_4	0.079527	0.122250	0.103444	0.263311	0.087834
β_5	0.690853	0.457525	0.136536	0.056349	0.683543
β_6	-3.568094	-4.254011	-5.039416	-1.962061	-4.522021
β_7	-2.000887	-2.410570	-2.063164	-2.170238	-0.053721
β_8	0.042473	-0.273778	-0.943618	-0.828163	2.556232
β_9	-1.505561	-1.337428	-1.403867	-1.355368	0.609960
β_{10}	-0.524160	-0.392623	-0.534082	-1.635305	2.204330
β_{11}	-0.011198	-0.008273	-0.010852	-0.011800	-0.011956
β_{12}	3.672236	4.047072	3.953327	-0.233304	4.754790

Table 4.29: Coefficients of constant term and independent variables of fifteenth regression model.

	weight 1	weight 2	weight 3	weight 4	weight 5
R-Squared	0.8456	0.6417	0.1972	0.2289	0.8392
Adjusted R-Squared	0.8428	0.6351	0.1824	0.2148	0.8362
P-Value	0.00	0.00	0.00	0.00	0.00

Table 4.30: Results of fifteenth regression model.

4.1.4 Multiple Nonlinear Regression Models with Weighted Least Square Method

In this section, multiple nonlinear regression with weighted least square method is applied to nonlinear regression models that mentioned in multiple nonlinear regression models section. Again same weights that expressed in previous section are used in order to construct weighted nonlinear regression models.

4.1.4.1 Regression Model 16

Sixteenth regression model is conducted as multiple nonlinear regression with weighted least square method by using five different weights as explained as above to predict customer demand. In this model, there are seven independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$ and $Markdown * Year$. The regression equation of this model is as below.

$$\sqrt{Demand} = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory +$$

$$\beta_4 * ProductNumber + \beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * Markdown * Year \tag{4.19}$$

Results of sixteenth demand prediction model can be shown as following tables.

Coefficients	**weight 1**	**weight 2**	**weight 3**	**weight 4**	**weight 5**
β_0	4.866482	4.614312	4.538278	3.429914	5.4203849
β_1	-0.016623	-0.010072	-0.010879	-0.010230	-0.0195738
β_2	-2.741089	-2.641004	-2.014862	-0.609453	-3.2449083
β_3	0.001189	0.007202	0.030086	0.026015	0.0011774
β_4	-0.043962	-0.029577	-0.023132	0.000319	-0.0381426
β_5	0.066551	0.061229	0.038643	0.028421	0.0669102
β_6	-0.808668	-0.623901	-0.262132	0.075163	-1.0709190
β_7	0.948369	0.609236	-0.042807	-0.550144	1.2468194

Table 4.31: Coefficients of constant term and independent variables of sixteenth regression model.

	weight 1	weight 2	weight 3	weight 4	weight 5
R-Squared	0.7045	0.4552	0.2273	0.2317	0.6999
Adjusted R-Squared	0.7014	0.4494	0.2191	0.2236	0.6968
P-Value	0.00	0.00	0.00	0.00	0.00

Table 4.32: Results of sixteenth regression model.

4.1.4.2 Regression Model 17

Seventeenth regression model is conducted as multiple nonlinear regression with weighted least square method by using five different weights as explained as above to predict customer demand. In this model, there are ten independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $Introduction$, $Maturity$, $Decline$ and $Markdown * Year$. The regression equation of this model is as below.

$$\sqrt{Demand} = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$

$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * Introduction + \beta_8 * Maturity +$$

$$\beta_9 * Decline + \beta_1 0 * Markdown * Year \tag{4.20}$$

Results of seventeenth demand prediction equation can be shown as following tables.

Coefficients	weight 1	weight 2	weight 3	weight 4	weight 5
β_0	4.7857875	4.602801	4.582069	3.4320000	5.380621
β_1	-0.0098152	-0.009524	-0.011174	-0.0102400	-0.014051
β_2	-2.9561822	-2.651324	-2.050516	-0.6116000	-3.430283
β_3	0.0010482	0.006999	0.031534	0.0263700	0.001068
β_4	-0.0436412	-0.030058	-0.022489	0.0003322	-0.036327
β_5	0.0654215	0.061403	0.038388	0.0284100	0.065973
β_6	-0.9298929	-0.631678	-0.269764	0.0746200	-1.166548
β_7	0.1483362	0.259884	-0.929813	0.7560000	-0.191042
β_8	0.3454485	0.047202	-0.259534	-0.2889000	0.287235
β_9	NA	NA	NA	NA	NA
β_{10}	1.0747391	0.047202	-0.034850	-0.5496000	1.335771

Table 4.33: Coefficients of constant term and independent variables of seventeenth regression model.

	weight 1	weight 2	weight 3	weight 4	weight 5
R-Squared	0.7099	0.4554	0.2284	0.2318	0.7038
Adjusted R-Squared	0.7059	0.4479	0.2179	0.2213	0.6997
P-Value	0.00	0.00	0.00	0.00	0.00

Table 4.34: Results of seventeenth regression model.

4.1.4.3 Regression Model 18

Eighteenth regression model is conducted as multiple nonlinear regression with weighted least square method by using five different weights as explained as above to predict customer demand. In this model, there are eight independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, TT and $Markdown*Year$. The regression equation of this model is as below.

$$\sqrt{Demand} = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$

$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * TT + \beta_8 * Markdown * Year \quad (4.21)$$

Results of eighteenth demand prediction equation can be shown as following tables.

Coefficients	**weight 1**	**weight 2**	**weight 3**	**weight 4**	**weight 5**
β_0	4.0372060	3.6648004	3.386321	2.098350	4.5423643
β_1	0.0257415	0.0315020	0.051543	0.067157	0.0294051
β_2	-2.4180114	-2.2128525	-1.775065	-0.475342	-2.9967916
β_3	0.0013045	0.0064333	0.021791	0.014283	0.0013058
β_4	-0.0405777	-0.0352842	-0.031162	-0.009833	-0.0232174
β_5	0.0618558	0.0587011	0.036178	0.024896	0.0615586
β_6	-0.7149218	-0.4890397	-0.289352	-0.036805	-1.0161840
β_7	-0.0032725	-0.0029364	-0.003980	-0.004799	-0.0035610
β_8	0.8406870	0.4788121	0.036863	-0.350143	1.1519409

Table 4.35: Coefficients of constant term and independent variables of eighteenth regression models.

	weight 1	**weight 2**	**weight 3**	**weight 4**	**weight 5**
R-Squared	0.7161	0.4671	0.2443	0.2543	0.7141
Adjusted R-Squared	0.7126	0.4606	0.2351	0.2453	0.7106
P-Value	0.00	0.00	0.00	0.00	0.00

Table 4.36: Results of eighteenth regression model.

4.1.4.4 Regression Model 19

Nineteenth regression model is conducted as multiple nonlinear regression with weighted least square method by using five different weights as explained as above to predict customer demand. In this model, there are eleven independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $FirstWeekofaMonth$, $SecondWeekofaMonth$, $ThirdWeekofaM$ $FourthWeekofaMonth$ and $Markdown * Year$. The regression equation of this model is as below.

$$\sqrt{Demand} = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$

$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * FirstWeekofaMonth + \beta_8 * SecondWeekofaMonth +$$

$$\beta_9 * ThirdWeekofaMonth + \beta_1 0 ForthWeekofaMonth + \beta_1 1 * Markdown * Year \tag{4.22}$$

Results of nineteenth demand prediction equation can be shown as following tables.

Coefficients	**weight 1**	**weight 2**	**weight 3**	**weight 4**	**weight 5**
β_0	4.7510740	4.737215	4.583703	2.9488568	5.1691624
β_1	-0.0158819	-0.009631	-0.011266	-0.0122140	-0.0176530
β_2	-2.7636065	-2.809496	-2.046258	-0.0193409	-3.4204055
β_3	0.0011705	0.007196	0.036605	0.0443023	0.0011152
β_4	-0.0432712	-0.027889	-0.023379	-0.0027062	-0.0383890
β_5	0.0665163	0.061705	0.039839	0.0291601	0.0669716
β_6	-0.8053917	-0.708363	-0.509074	0.0005537	-1.1059517
β_7	0.0876089	-0.104308	-0.288154	-0.3846332	0.2928106
β_8	0.2273913	0.052276	-0.329668	-0.4648452	0.5055166
β_9	0.0324578	-0.053551	-0.204934	-0.2716098	0.2560604
β_{10}	0.1685300	0.080949	0.002596	-0.1468471	0.5049126
β_{11}	0.9419936	0.705051	0.282434	-0.3779749	1.2834368

Table 4.37: Coefficients of constant term and independent variables of nineteenth regression model.

	weight 1	weight 2	weight 3	weight 4	weight 5
R-Squared	0.7065	0.4582	0.2378	0.2464	0.7075
Adjusted R-Squared	0.7016	0.4492	0.225	0.2337	0.7025
P-Value	0.00	0.00	0.00	0.00	0.00

Table 4.38: Results of nineteenth regression model.

4.1.4.5 Regression Model 20

Twentieth regression model is conducted as multiple nonlinear regression with weighted least square method by using five different weights as explained as above to predict customer demand. In this model, there are twelve independent variables. These are $SalesWeeks$, $Markdown$, $Invetory$, $ProductNumber$, $PreviousWeekSales$, $Year$, $FirstWeekofaMonth$, $SecondWeekofaMonth$, $ThirdWeekofaM$ $FourthWeekofaMonth$, TT and $Markdown * Year$. The regression equation of this model is as below.

$$\sqrt{Demand} = \beta_0 + \beta_1 * SalesWeek + \beta_2 * Markdown + \beta_3 * Inventory + \beta_4 * ProductNumber +$$

$$\beta_5 * PreviousWeekSales + \beta_6 * Year + \beta_7 * FirstWeekofaMonth + \beta_8 * SecondWeekofaMonth +$$

$$\beta_9 * ThirdWeekofaMonth + \beta_1 0 ForthWeekofaMonth + \beta_1 1 * TT + \beta_1 2 * Markdown * Year \tag{4.23}$$

Results of twentieth demand prediction equation can be shown as following tables.

Coefficients	weight 1	weight 2	weight 3	weight 4	weight 5
β_0	4.1084959	3.7552419	3.287736	1.333173	4.5582568
β_1	0.0271042	0.0363166	0.051425	0.065317	0.0283671
β_2	-2.4598864	-2.3268301	-1.638685	0.474561	-3.1900378
β_3	0.0013006	0.0065241	0.027301	0.024616	0.0012700
β_4	-0.0396176	-0.0328812	-0.029063	-0.002847	-0.0224270
β_5	0.0618602	0.0588599	0.037779	0.025722	0.0618833
β_6	-0.7376195	-0.5750744	-0.437708	0.165032	-1.0859320
β_7	-0.1195272	-0.2304435	-0.344818	-0.421307	0.0507960
β_8	0.0328268	-0.0276647	-0.218011	-0.179502	0.2747373
β_9	-0.1642687	-0.1623386	-0.224188	-0.223271	0.0284742
β_{10}	-0.0054147	-0.0094492	-0.033828	-0.214601	0.3026732
β_{11}	-0.0033901	-0.0032772	-0.004007	-0.004827	-0.0034191
β_{12}	0.8683162	0.5825746	0.240692	-0.548518	1.2304339

Table 4.39: Coefficients of constant term and independent variables of twentieth regression model.

	weight 1	weight 2	weight 3	weight 4	weight 5
R-Squared	0.718	0.4721	0.253	0.2642	0.7193
Adjusted R-Squared	0.7128	0.4624	0.2393	0.2507	0.7142
P-Value	0.00	0.00	0.00	0.00	0.00

Table 4.40: Results of twentieth regression model.

4.2 Analysis of All Regression Models

In order to determine the effectiveness of regression models, validation is made by using k-fold cross validation test which is widely use technique. Test is applied to all regression models to measure performance of each one. After that MAPE, mean absolute percentage error, is calculated for all models. Results are given in the table where following section.

4.2.1 Cross Validation Test

Cross validation is the one of validation technique that evaluates generalizing results of statistical analysis to an independent dataset. It is usually used in order to determine the accuracy of the predictive models. There are two type of cross validation technique, exhaustive cross validation and non-exhausted cross validation. Exhaustive cross validation types are leave-p-out cross validation and leave-one-out cross validation. Non-exhausted cross validation types are k-fold cross validation, holdout cross validation and repeated random sub-sampling validation [12]. In this study, k-fold cross validation technique is used in order to measure performance of demand prediction model.

In k-fold cross validation test, dataset is split subsets in k equal size. Out of the k subsets, one subset which is called test data is stored as verification of the data in order to perform effectiveness of the prediction model and other k-1 subsets which are called training data are used to make regression. Process is made k times. In our study, we chose k is three. We divided our sample data that we used for making regression model in three equal subsets and performed cross validation test. The regression models are conducted by using training data and then regression coefficients are used in test set for estimating demand. After test is made, results are evaluated by calculating MAPE which demonstrates that percentage error between estimated demand and actual value of sales of products. Formulation of MAPE is as follows.

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{\mid Y_t - \hat{Y}_t \mid}{\mid Y_t \mid} \tag{4.24}$$

MAPE should be closer to zero in order to show that error between predicted value and actual value is minimum. MAPE is calculated for each subset. As a result of the test, we took average of each subsets MAPE value. Overall average of MAPE value shows the prediction accuracy of our regression equation. The cross validation test result of our selected demand model is given in the table as below.

4.3 Selecting Demand Model

An overall result of all regression models is given in the table as below. We compared all regression models according to their Adjusted R^2, P-value and MAPE.

Table 4.3 gives overall summary of sixty regression models. We evaluated results of all demand model in order to determine proper regression equation for our study. Results are compared by adjusted R^2, P-value and MAPE that calculated in cross validation test. First of all, P-value is evaluated for entire conducted regression models. P- value of all regression models is smaller than 0.05. It is demonstrated that all regression models are significant. Therefore, P-value is not determinant to find suitable demand equation. Second of all, Adjusted R^2 is compared. Linear least square regression models, nonlinear least square regression models and some weighted linear and nonlinear regression models are higher value than others. Relatively the models which have lower adjusted R^2 is eliminated directly. Finally, we analogize cross validation test result which is MAPE. MAPE should be closer to zero. Therefore, we eliminated models that their MAPE is greater than 1. we only considered relatively smaller value which is more closer to 0.

To sum up, after all determinant is evaluated in order to find more convenient demand equation, we reduced options to two models which are Model 18.1 and Model 20.1. Although adjusted R^2 is a little bit smaller than others, these models have the smallest MAPE value but there is very small difference between selected two regression models. We believe that MAPE is more predictive factor which gives the best fits than adjusted R^2 in our study. Thus, two regression equations which have the smallest MAPE value is selected. Compared results of selected two regression models are given in the table as below.

	Adjusted R^2	P-Value	MAPE
Model 1	0.8327	0.00	1.5512
Model 2	0.8376	0.00	1.7603
Model 3	0.8369	0.00	1.4437
Model 4	0.8327	0.00	1.5549
Model 5	0.8368	0.00	1.4605
Model 6	0.7948	0.00	0.7842
Model 7	0.8181	0.00	0.7847
Model 8	0.8135	0.00	0.7542
Model 9	0.7954	0.00	0.7815
Model 10	0.8135	0.00	0.7549
Model 11.1	0.8412	0.00	1.6203
Model 11.2	0.6281	0.00	4.8602
Model 11.3	0.1729	0.00	7.1928
Model 11.4	0.1979	0.00	10.8239
Model 11.5	0.8319	0.00	1.6366
Model 12.1	0.8413	0.00	1.5746
Model 12.2	0.6273	0.00	4.5751
Model 12.3	0.1717	0.00	5.8005
Model 12.4	0.1955	0.00	13.3333
Model 12.5	0.8322	0.00	1.5626
Model 13.1	0.8419	0.00	1.4456
Model 13.2	0.6282	0.00	4.6982
Model 13.3	0.1753	0.00	6.1113
Model 13.4	0.2011	0.00	15.4754
Model 13.5	0.8336	0.00	1.4052
Model 14.1	0.8418	0.00	1.6167
Model 14.2	0.6337	0.00	4.8589
Model 14.3	0.1791	0.00	8.1989
Model 14.4	0.2115	0.00	15.9116
Model 14.5	0.8350	0.00	1.6824
Model 15.1	0.8428	0.00	1.4216
Model 15.2	0.6351	0.00	4.5914
Model 15.3	0.1824	0.00	7.4366
Model 15.4	0.2148	0.00	28.5441
Model 15.5	0.8362	0.00	1.4647
Model 16.1	0.7014	0.00	0.7701
Model 16.2	0.4494	0.00	0.9376
Model 16.3	0.2191	0.00	2.8485
Model 16.4	0.2236	0.00	2.2699
Model 16.5	0.6968	0.00	0.7763
Model 17.1	0.7059	0.00	0.7702
Model 17.2	0.4479	0.00	0.9321
Model 17.3	0.2179	0.00	1.2725
Model 17.4	0.2213	0.00	2.8513
Model 17.5	0.6997	0.00	0.7738
Model 18.1	0.7126	0.00	0.7439
Model 18.2	0.4606	0.00	0.8819
Model 18.3	0.2351	0.00	1.2576
Model 18.4	0.2453	0.00	3.2121
Model 18.5	0.7106	0.00	0.7446
Model 19.1	0.7016	0.00	0.7701
Model 19.2	0.4492	0.00	0.9331
Model 19.3	0.2250	0.00	1.6553
Model 19.4	0.2337	0.00	3.6270
Model 19.5	0.7025	0.00	0.7775
Model 20.1	0.7128	0.00	0.7438
Model 20.2	0.4624	0.00	0.8726
Model 20.3	0.2393	0.00	1.3932
Model 20.4	0.2507	0.00	6.3399
Model 20.5	0.7142	0.00	0.7487

Table 4.41: Overall result summary of all regression models.

	Adjusted R^2	P-Value	MAPE	Weights
Model 18.1	0.7126	0.00	0.7439	w1
Model 20.1	0.7128	0.00	0.7438	w1

Table 4.42: Comparision of selected two regression models.

As can be seen in the table 4.42 , there is extremely small difference between two chosen models. Although first model has little smaller adjusted R^2 and little higher MAPE, we chose Model 18.1. Because second model has four extra cathegoric variables which are $FirstWeekofaMonth$, $SecondWeekofaMonth$, $ThirdWeekofaMonth$ and $FourthWeekofaMonth$. Fewer variables in regression equation make simplify our study for markdown optimization model. Hence first model is selected as a constraint for optimization model. Selected demand model equation and results are given as following.

$$\sqrt{Demand} = 4.0372060 + 0.0257415 * SalesWeek - 2.4180114 * Markdown + 0.0013045 * Inventory$$

$$0.0405777 * ProductNumber + 0.0618558 * PreviousWeekSales - 0.7149218 * Year -$$

$$0.0032725 * TT + 0.8406870 * Markdown * Year, Weight = w1 \tag{4.25}$$

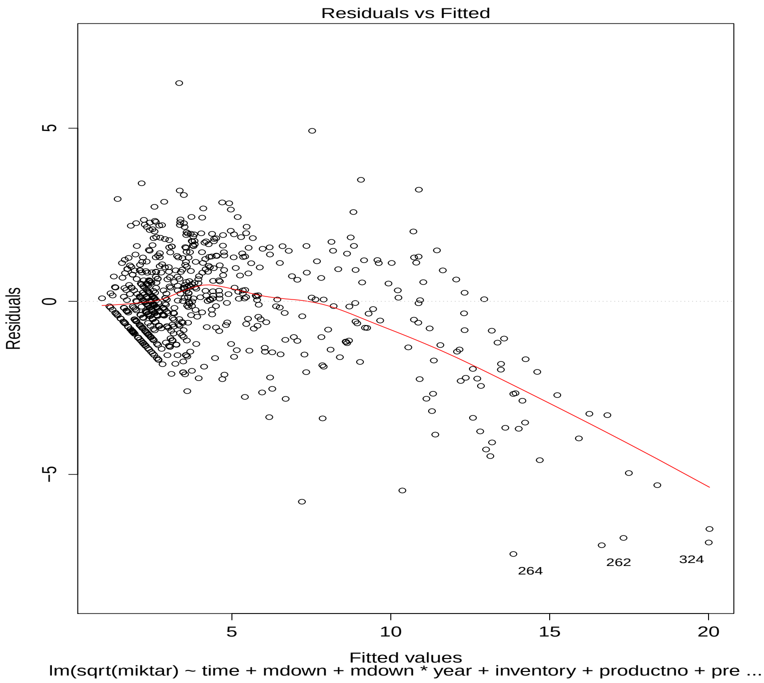

Figure 4.2: Residuals vs Fitted Plot of selected demand model

Figure 4.2 shows that the residual vs fitted plot. This plot provides to understand accuracy among the actual values and predicted values. Also, it provides that detecting the nonlinearity, outliers, variance etc [13]. As can be seen from the plot, there is nonlinearity between residuals and fitted values and can be seen the outliers.

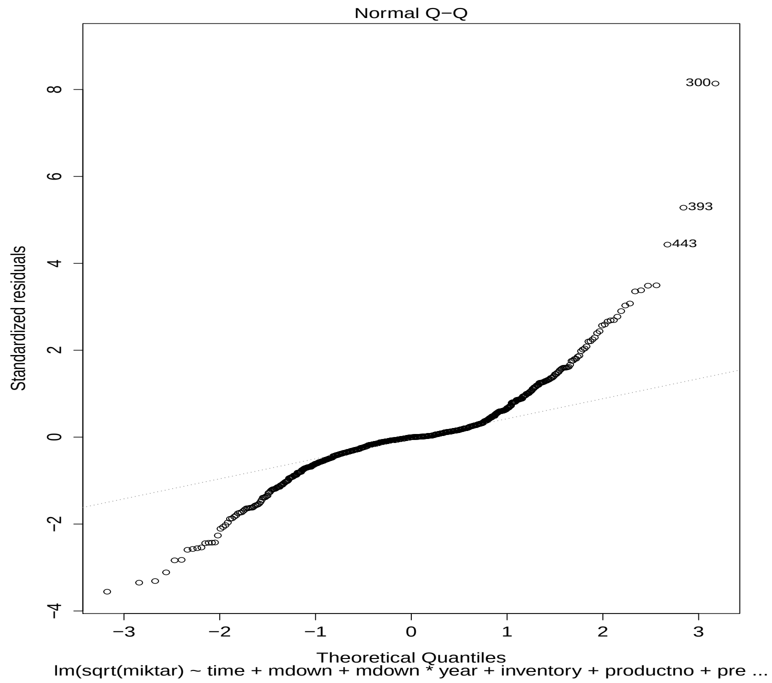

Figure 4.3: Normal Quantile-Quantile Plot of selected demand model

Figure 4.3 demonstrates that the Normal Quantile-Quantile plot. This plot shows whether the data comes from a Normal Distribution [14]. For the reason that using Normal Q-Q plot is to assume that dependent variable of our demand model comes from the normal distribution. As can be seen in the graph the data points are on the middle of the line. This means that there o lot of extreme points at the beginning and end of the line. Therefore, we can say that our dependent variable is Normally distributed with the extreme points.

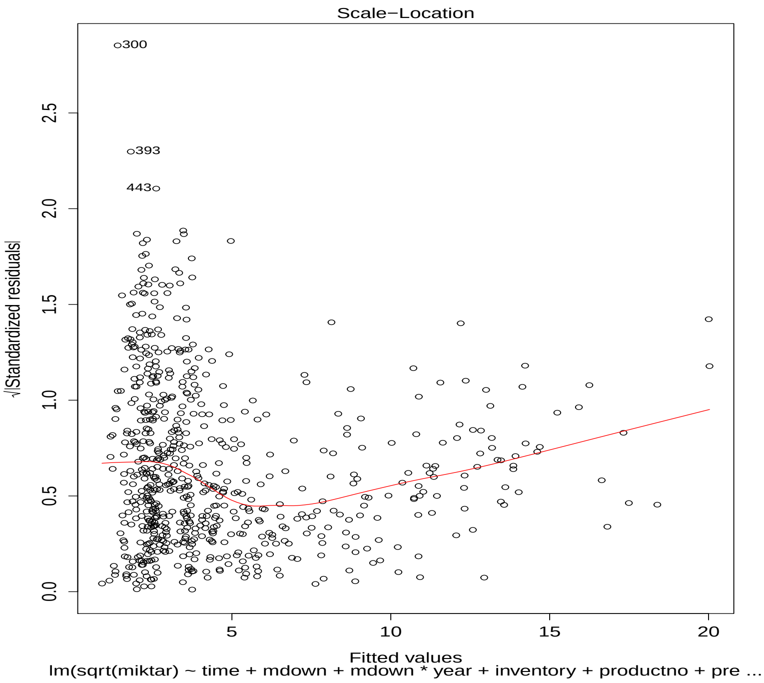

Figure 4.4: Scale-Location Plot of selected demand model

Figure 4.4 shows Scale-Location Plot. This plot provides us to control whether heteroscedasticity of residuals is exist. It is assumed that there is a homoscedasticity in which residuals are equally located on a horizontal line. In our graph, residuals are not located equally on the red line at the beginning in which there is an accumulation. But residuals are getting spreading.

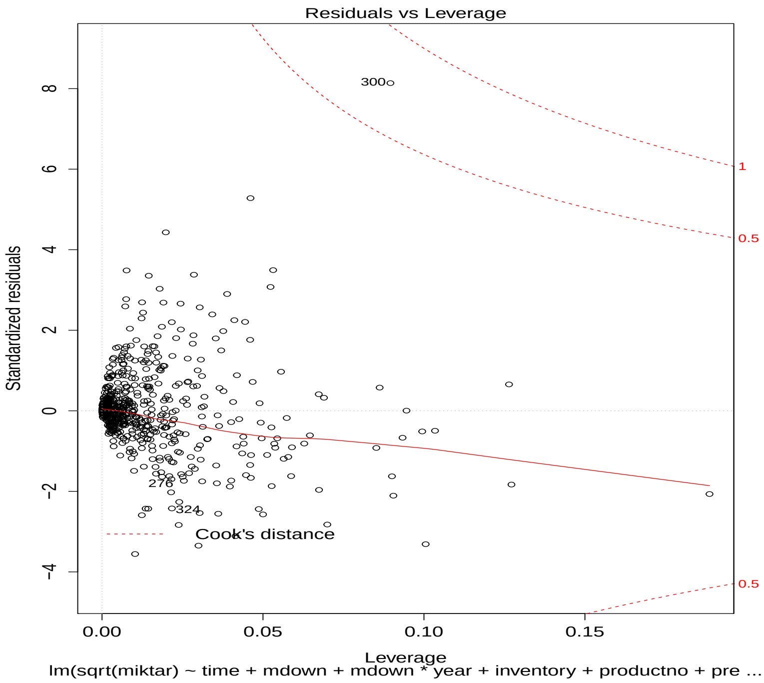

Figure 4.5: Residual vs Leverage Plot of selected demand model

Figure 4.5 demonstrates the Residual vs Leverage Plot. This plot provides whether is there any influential cases of outliers. Although there are extreme points in the data, in some cases, they may not affected to regression line [15]. The good case is no outliers with high Cook's distances to not the affect regression line. In our graph, the values place at the lower right corner and there some outliner which is close the Cook's distance. Therefore data can be influential to determine regression line.

Chapter 5

Markdown Optimization Model

In recent years, markdown optimization becomes important for fashion retailers. Fashion sector grows increasingly. Therefore, there is a big competition between retailers. Hereby, markdown plan plays a significant role for any retailer in order to handle tough competition in the market. Markdown plan is an application of discount at a specific time period during selling season. Discounts should be non-decreasing. It cannot be less than previous time period that applied any discount. The well implemented markdown optimization plan provides many advantages to retailers such as increasing sales, inventory control, improving total revenue etc. Consequently, the aim of the markdown optimization is to provide benefit to retailers.

5.1 Mathematical Model Development

In this section, A markdown optimization model is developed for making a markdown plan. The aim of the optimization model is to maximize the total revenue of the apparel retailer.

The selected regression equation is used as constraint in mathematical model in order to determine customer demand. In our model, inventory level is not known. Therefore we run optimization model for different inventory level. Mathematical model that developed for markdown optimization is as follows.

$$max \sum_{t=T} \sum_{p=P} m_{t,p} * Sales_{t,p} - \sum_{p=P} I_{f_p} * h \tag{5.1}$$

subject to

$$D_{t,p} = (\beta_0 + \beta_1 * w_{t,p} + \beta_2 * m_{t,p} + \beta_3 * I_{t,p} + \beta_4 * pno_{t,p} + \\ \beta_5 * D_{t-1,p} + \beta_6 * y_{t,p} + \beta_7 * tt_{t,p} + \beta_8 * m_{t,p} * y_{t,p})^2 \tag{5.2}$$

$$I_{t,p} = I_{t-1,p} - Sales_{t,p} \tag{5.3}$$

$$m_{t,p} \leq m_{t-1,p} \tag{5.4}$$

$$Sales_{t,p} \leq D_{t,p} \tag{5.5}$$

$$I_{t,p}, D_{t,p}, Sales_{t,p}, m_{t,p} \geq 0 \tag{5.6}$$

Objective function (5.1) maximizes total revenue of a store of the apparel retail company where $m_{t,p}$ represents the markdowns as decision variable for all weeks t and all products p, $Sales_{t,p}$ represents sales of all products p for all weeks t as decision variable, I_{f_p} shows that remaining inventory level at last week as parameter and h is the holding cost as parameter. First constraint (5.2) is the demand equation which is conducted as weighted nonlinear least square estimation method where regression coefficients and independent variables in regression equation are parameters except $m_{t,p}$ and $I_{t,p}$. These are decision variables. Second constraint (5.3) represents definition of inventory by subtracting previous week demand from previous week inventory. Third constraint (5.4) ensures that previous week markdown cannot be smaller than current week markdown. Fourth constraint (5.5) ensures that sales have to be equal or less than demand. Finally last constraint (5.6) shows that all decision variables should be equal or greater than zero.

5.2 Static Approach

Static approach is the one time solution method for the planning year. The mathematical model is solved in GAMS environment by using NLP solver which is CONOPT. The whole sample dataset including entire selling years of every product is used in order to estimate demand model but, optimization model is run for only last selling years of all product. We made prediction for the last selling years of original data.

Mathematical model is run for one year which is 52 weeks for all products in the sample dataset. Regression coefficients, $\beta_0, \beta_1, ...\beta_8$, selling weeks, $w_{t,p}$, product numbers, $pno_{t,p}$, selling year, $y_{t,p}$ and $tt_{t,p}$ are used as parameters and the variables of optimization model are markdown, $m_{t,p}$, inventory, $I_{t,p}$, demands, $D_{t,p}$ and sales, $Sales_{t,p}$. As inventory level is not known, the optimization model is solved for different inventory level. Therefore, initial inventory is selected randomly as 1500, 2000 and 2400 for every product. The tabular and graphical results of static approach are given as below.

weeks	products	m	i	d	s	m	i	d	s	m	i	d	s
week1	product1	1	1500	4.08	4.08	1	2000	7.13	7.13	1	2400	10.19	10.19
week1	product2	1	1500	2.98	2.98	1	2000	5.66	5.66	1	2400	8.41	8.41
week1	product3	1	1500	3.75	3.75	1	2000	6.71	6.71	1	2400	9.68	9.68
week1	product4	1	1500	3.6	3.6	1	2000	6.5	6.5	1	2400	9.43	9.43
week1	product5	1	1500	3	3	1	2000	5.68	5.68	1	2400	8.44	8.44
week1	product6	1	1500	3.3	3.3	1	2000	6.09	6.09	1	2400	8.94	8.94
week1	product7	1	1500	3.61	3.61	1	2000	6.52	6.52	1	2400	9.46	9.46
week1	product8	1	1500	3.46	3.46	1	2000	6.31	6.31	1	2400	9.21	9.21
week1	product9	1	1500	3.31	3.31	1	2000	6.11	6.11	1	2400	8.96	8.96
week1	product10	1	1500	3.17	3.17	1	2000	5.91	5.91	1	2400	8.72	8.72
week2	product1	1	1495.92	6.01	6.01	1	1992.87	10.82	10.82	1	2389.81	15.97	15.97
week2	product2	1	1497.02	4.38	4.38	1	1994.34	8.46	8.46	1	2391.59	12.93	12.93
week2	product3	1	1496.25	5.53	5.53	1	1993.29	10.13	10.13	1	2390.32	15.09	15.09
week2	product4	1	1496.4	5.3	5.3	1	1993.5	9.79	9.79	1	2390.57	14.66	14.66
week2	product5	1	1497	4.4	4.4	1	1994.32	8.49	8.49	1	2391.56	12.97	12.97
week2	product6	1	1496.7	4.85	4.85	1	1993.91	9.14	9.14	1	2391.06	13.82	13.82
week2	product7	1	1496.39	5.32	5.32	1	1993.48	9.82	9.82	1	2390.54	14.7	14.7
week2	product8	1	1496.54	5.09	5.09	1	1993.69	9.49	9.49	1	2390.79	14.28	14.28
week2	product9	1	1496.69	4.87	4.87	1	1993.89	9.17	9.17	1	2391.04	13.86	13.86
week2	product10	1	1496.83	4.65	4.65	1	1994.09	8.86	8.86	1	2391.28	13.45	13.45
…	…	…	…	…	…	…	…	…	…	…	…	…	…
week52	product1	1	289.14	3.4	3.4	1	208.35	2.9	2.9	0.8	0	1.26	1.26
week52	product2	1	482.72	3.2	3.2	1	507.12	3.36	3.36	0.79	0	1.35	1.35
week52	product3	1	345.57	3.36	3.36	1	303.13	3.09	3.09	0.86	0	1.2	1.2
week52	product4	1	372.99	3.34	3.34	1	346.14	3.17	3.17	0.89	0	1.16	1.16
week52	product5	1	480.22	3.21	3.21	1	503.58	3.36	3.36	1	100.87	1.35	1.35
week52	product6	1	426.31	3.28	3.28	1	426.11	3.28	3.28	0.95	0	1.1	1.1
week52	product7	1	370.29	3.34	3.34	1	341.97	3.16	3.16	0.13	0	0.21	0.21
week52	product8	1	397.24	3.31	3.31	1	383.03	3.22	3.22	0.96	0	1.13	1.13
week52	product9	1	423.71	3.28	3.28	1	422.29	3.27	3.27	0.97	0	1.1	1.1
week52	product10	1	449.69	3.25	3.25	1	460.04	3.32	3.32	0.99	0	1.07	1.07

Table 5.1: Results of static approach for the initial inventory level: 1500, 2000 and 2400.

Overall results of static approach are given in Table ??. Respectively selling weeks, product number, markdown, inventory, demand and sales can be shown in the table for three different initial inventory level: 1500, 2000 and 2400. For the initial inventory 1500 and 2000 markdown levels are 1 for each product for the entire selling year. In other words, there is no discount during the selling year. As expected, there is remaining inventory at the end of the year for each product. However, for initial inventory 2400 clearance prices are observed starting from the middle of the season through the end of the season four nine products. Markdowns prices are increased the sales of the products. Therefore, there is no on hand inventory at the end for nine products.

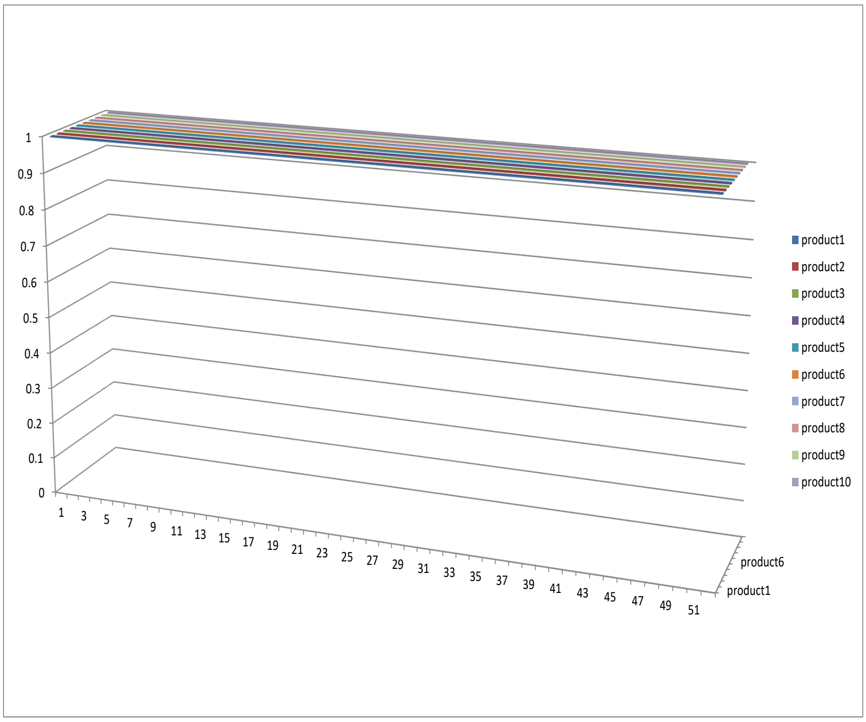

Figure 5.1: Markdown result for inventory level 1500

Figure 5.1 shows that markdown level for each product with the initial inventory 1500. As can be seen, no discount is applied during the selling year for ten products. The initial inventory level may be caused this situation. The mathematical model may no need to markdown prices when the initial inventory is low, because it is easy to supply the holding cost for low inventory level.

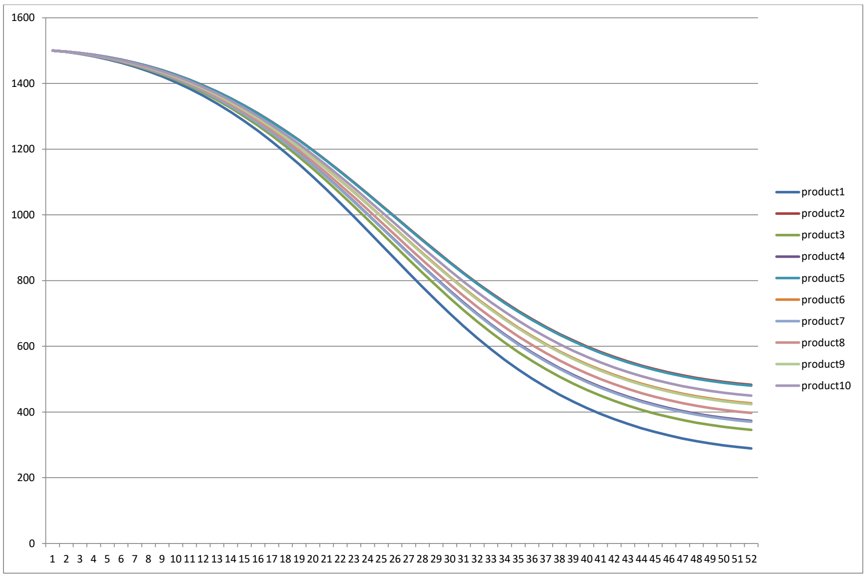

Figure 5.2: Inventory result for inventory level 1500

Figure 5.2 demonstrates inventory level for each product with the initial inventory 1500. Products are sold during the selling year but there is on hand inventory for each product at the end of the year. The remaining inventory of each product are 289.14, 482.72, 345.57, 372.99, 480.22, 426.31, 370.29, 397.24, 426.71 and 449.69 respectively. The reason of this situation can be making no discount for each product. Customers may not willing to buy much the products when their prices is not marked down.

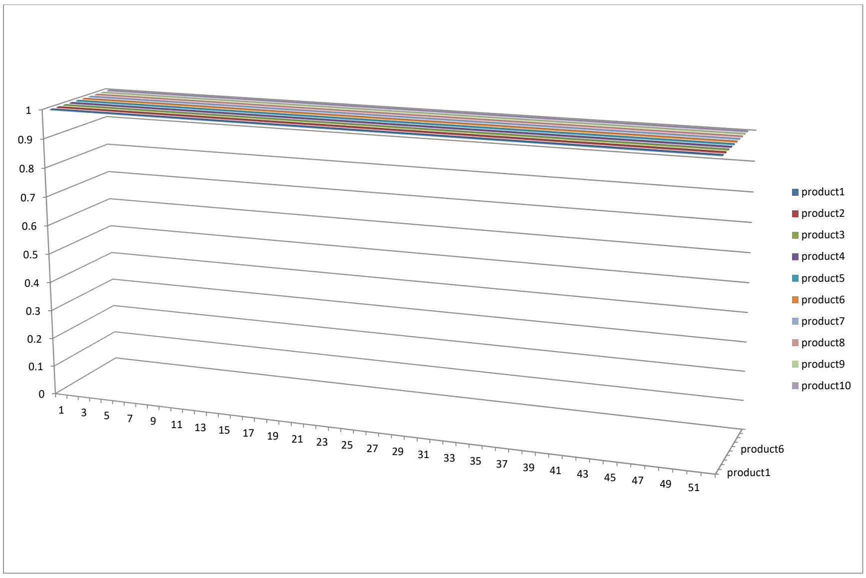

Figure 5.3: Markdown result for inventory level 2000

Figure 5.3 shows that markdown level for each product for the initial inventory 2000. As can be seen, no discount is applied during the selling year for ten products. The initial inventory level may be caused this situation. The mathematical model may no need to markdown prices when the initial inventory is low, because it is easy to supply the holding cost for low inventory level.

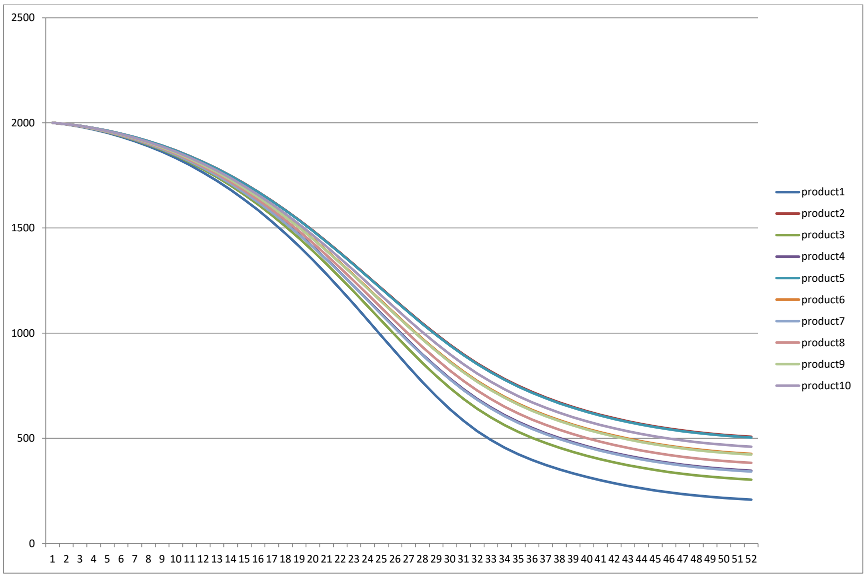

Figure 5.4: Inventory result for inventory level 2000

Figure 5.4 demonstrates inventory level for each product with the initial inventory 2000. Products are sold during the selling year but there is on hand inventory for each product at the end of the year. The remaining inventories of each product are 208.35, 507.12, 303.13, 346.14, 503.58, 426.11, 341.97, 383.03, 422.29 and 460.04 respectively. The reason of this situation can be making no discount for each product. Customers may not willing to buy much the products when their prices is not marked down.

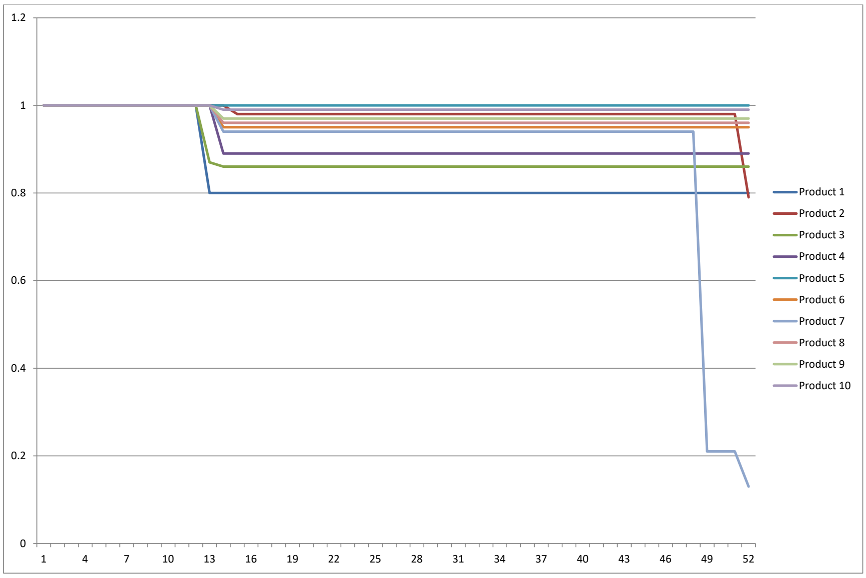

Figure 5.5: Markdown result for inventory level 2400

Figure 5.5 shows that markdown level for each product for the initial inventory 2400. The selling prices are marked down beginning from the middle of the year to the end of the year. Thus we can conclude that the product prices are marked down when the inventory level is increased.

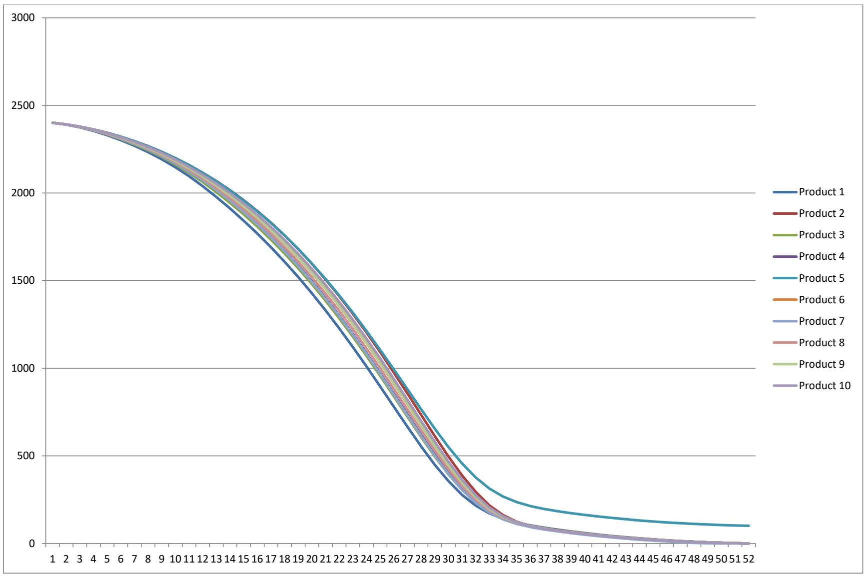

Figure 5.6: Inventory result for inventory level 2400

Figure 5.6 demonstrates inventory level for each product with the initial inventory 2400. The products are completely sold during the selling year except product 5. There is no discount for it in the entire selling year. The remaining inventory for product 5 is 100.87. Other products have discount. Therefore there is no on hand inventory for them.

According to overall results, we observed that markdown level is connected to initial inventory for the static approach. When the inventory level is getting increase, markdown prices are starting. Customers are willing to buy the products when their prices are fallen. Therefore the all products sells with the discounted prices until no inventory of products are remained. To conclude that there is a strong relationship between markdown prices and inventory level.

5.3 Dynamic Approach

Even the markdown optimization model is solved by using static approach; we know that our problem is dynamic problem. Because, demand changes in every week and inventory level should be updated according to demand. For this reason, the solution approach of markdown pricing is dynamic programming [4]. Therefore, we developed a approximate dynamic programming solution for one product that is randomly selected from the sample dataset. The selected product has two selling years. We predicted its last selling which is sold 26 weeks year by using this method. Hence, mathematical model is run for its last selling year which is 26 weeks. As in deterministic approach, regression coefficients, $\beta_0, \beta_1, ...\beta_8$, selling weeks, $w_{t,p}$, product numbers, $pno_{t,p}$, selling year, $y_{t,p}$ and $tt_{t,p}$ are used as parameters and the variables of optimization model are markdown, $m_{t,p}$, inventory, $I_{t,p}$, demands, $D_{t,p}$ and sales, $Sales_{t,p}$. The initial inventory is considered as 2400.

In order to use approximate dynamic programming method, an interface is developed in Excel VBA. The basic concept of the interface is solving optimization model for every week by updating inventory level due to customer demand. We connected R Programming and GAMS to Excel VBA and all parameters are given in Excel sheet. The developed interface can be shown as below.

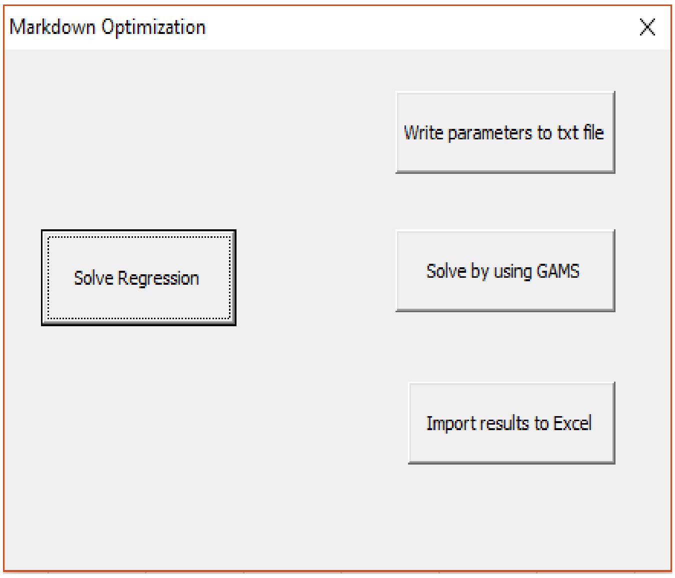

Figure 5.7: The interface for approximate dynamic programming

Figure 5.7 shows that the first button, *Solve Regression*, is activates R programming to solve regression and writes the regression's coefficients to txt file. The second button, *Write parameters to txt file*, writes all parameters that are given in Excel sheet to txt file. The third button, *Solve by using GAMS*, activates GAMS to solve optimization model by using parameters in txt file that occurred by clicking the first and the second button. The forth button, *Import results to Excel*, imports optimization model results from GAMS to Excel sheet.

The result of approximate dynamic programming for one product can be shown as below.

weeks	markdown	inventory	demand	sales
1	1	2400.00	11.58	11.58
2	1	2388.42	19.03	19.03
3	1	2369.39	26.53	26.53
4	1	2342.86	34.71	34.71
5	1	2308.16	43.59	43.59
6	1	2264.57	53.05	53.05
7	1	2211.51	62.91	62.91
8	1	2148.60	72.91	72.91
9	1	2075.69	82.79	82.79
10	1	1992.90	92.22	92.22
11	1	1900.68	100.87	100.87
12	1	1799.81	108.35	108.35
13	1	1691.46	114.31	114.31
14	1	1577.15	118.40	118.40
15	1	1458.75	120.35	120.35
16	1	1338.40	119.97	119.97
17	1	1218.43	117.22	117.22
18	1	1101.21	112.21	112.21
19	1	988.99	105.22	105.22
20	1	883.77	96.66	96.66
21	1	787.11	87.02	87.02
22	1	700.09	76.83	76.83
23	1	623.27	66.54	66.54
24	1	556.73	56.53	56.53
25	1	500.21	47.06	47.06
26	1	453.15	38.31	38.31
⋮	⋮	⋮	⋮	⋮
14	1	1893.74	61.55	61.55
15	1	1832.19	94.44	94.44
16	1	1737.75	113.55	113.55
17	1	1624.20	123.63	123.63
18	1	1500.57	126.81	126.81
19	1	1373.76	124.38	124.38
20	1	1249.38	117.50	117.50
21	1	1131.88	107.37	107.37
22	1	1024.51	95.25	95.25
23	1	929.26	82.30	82.30
24	1	846.95	69.48	69.48
25	1	777.47	57.42	57.42
26	1	720.05	46.47	46.47
⋮	⋮	⋮	⋮	⋮
24	1	1325.46	41.58	41.58
25	1	1283.88	54.00	54.00
26	1	1229.88	53.63	53.63
25	1	1283.88	37.29	37.29
26	1	1246.59	46.84	46.84
26	1	1283.88	37.29	37.29

Table 5.2: Results of approximate dynamic programming approach for the inventory level 2400.

Table 5.2 demonstrates that the result of approximate dynamic approach for single product with the initial inventory 2400 during its 26 week of the selling year. Behind the idea of the approximate dynamic programming, in the first iteration, the model is run from the week 1 to the week 26. In the second iteration, the model is run from the week 2 to the week 26. In the third iteration, the model is run from the week 3 to the week 26. It is continue until the week 26 by updating inventory level according to up-to-date customer demand. In the table as above, some of the iteration can be shown. First week of each iteration is generated the whole selling year for the single product.

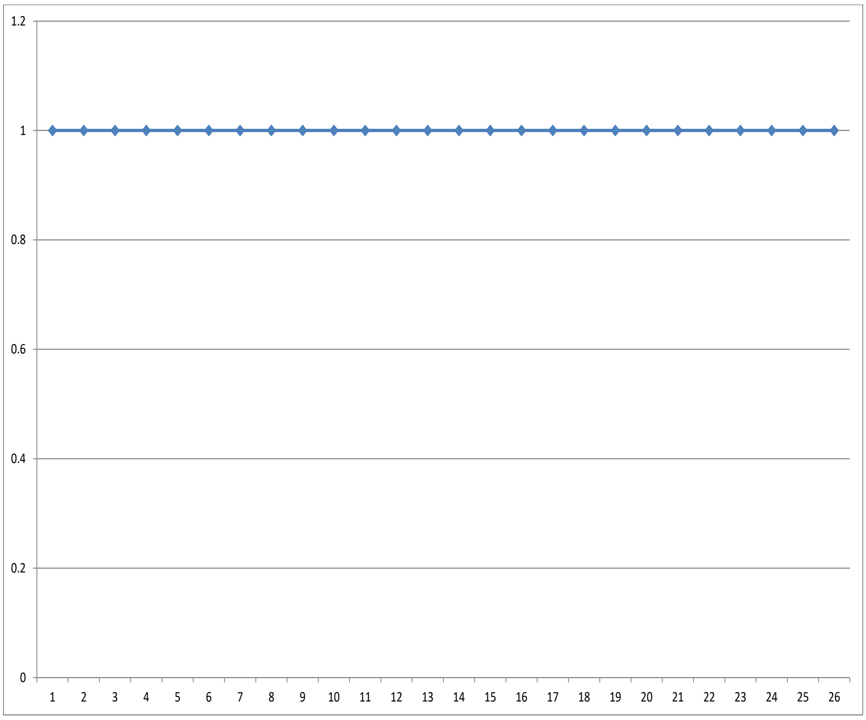

Figure 5.8: Markdown for single product

Figure 5.8 shows that markdown level which is found by using approximate dynamic programming for the single product. Markdown level is 1 for each week. This means that there is no discount for the single product during the selling year.

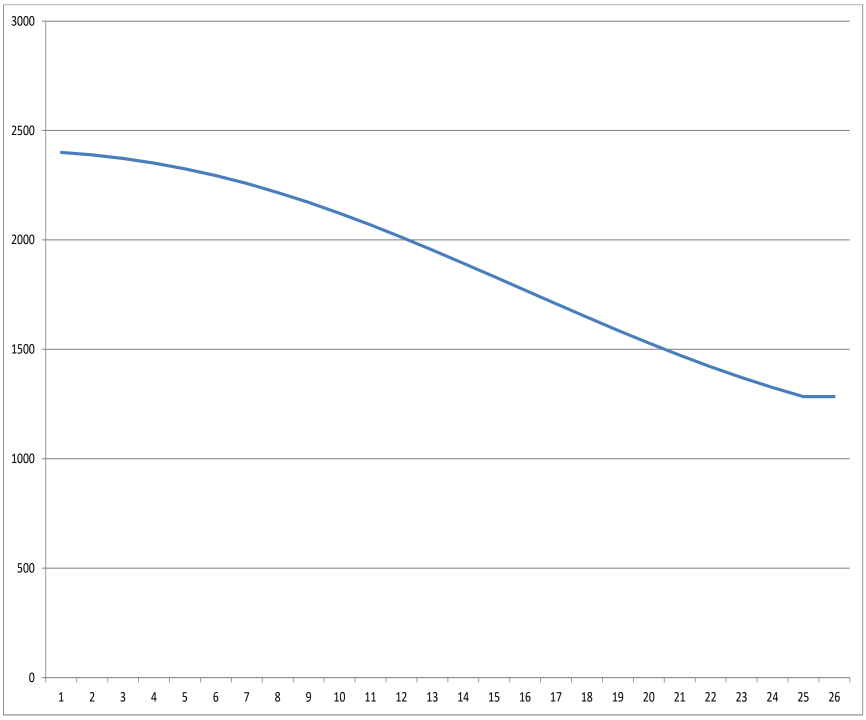

Figure 5.9: Inventory level for single product

Figure 5.9 demonstrates that the inventory level for single product. The remaining inventory is 1283.88 for the single product. It is not sold much due to discount level during the selling year.

The observation of dynamic approach is that there is no discount for the single product in regular selling year and depending on a markdown level, there is on hand inventory at the end of year. This approach will be developed and extended for the whole products in the dataset for our future researches.

Chapter 6

Conclusion

We studied markdown optimization in apparel retail sector. Markdown optimization is a discount application for products at the selling horizon. In order to determine optimum discount level for revenue maximization of the retailer, the dataset is obtained from the retailer. By using this dataset, the demand prediction model is determined. This demand model is used in markdown optimization model.

The aim of this study is to maximize the total revenue of the retailer by determining optimum discount for products at a specific time in the selling year considering the remaining inventory level.

The dataset is received from a company in apparel retail sector. The dataset contains date of sale, product identity number, brand identity number, selling years, selling seasons, group identity number, product number, collection identity number, collection name, gender information, product name, sales quantity, sales price and list price for every product of all product groups between 2011 and 2015. Within the huge dataset, ten products are selected randomly in order to generate a sample dataset. We think that the sample dataset represents the whole data due to different aspects of each product, i.e. belonging different collections, different gender, different season etc. Also each product in the sample dataset has different cycle time in a range two years to four years.

In order to predict customer demand, approximately sixty regression models are constructed with various combinations of independent variables in the sample dataset by using different methods which are multiple linear regression, multiple nonlinear regression, multiple linear regression with weighted least square method and multiple nonlinear regression with weighted least square method. After regression equations are conducted, cross validation test is applied to each demand prediction model in order to determine the effectiveness of models. All models are evaluated according to their adjusted R^2 value, p-value and MAPE. Within all regression models, one model is selected which gives best fits. Although, it's adjusted R^2 is not higher than others, selected demand model has the smallest MAPE. We believe that cross validation test result is more reliable to select demand model rather than it's adjusted R^2. In this study area, generally nonlinear regression models are used. However, unlike the literature, we estimated customer demand by using weighted least square estimation method. The weighted nonlinear regression model is outperformed all other regression models for our data. In this study, we showed that the benefit of using weighted least square method instead of least square methods.

After demand model is determined, an optimization model is developed for markdown prices with the aim that maximizing total revenue of the apparel retailer. Demand equation is used as a constraint in the mathematical model. Mathematical model is constructed as nonlinear programming model in GAMS environment. Therefore, NLP solver which is CONOPT is used to solve the model.

We considered two approaches in order to solve our model: static approach and dynamic approach. The static approach is one time solution method for the planning year. In the static approach, the model is run for one selling year for all products in the sample dataset. As we expected, the results showed that price markdowns are started at the mid-season through the end of the season mostly and no inventory is left at the end of the season for every product excluding one product for the proper initial inventory level. We conclude that if there is any markdown in the selling year, sales of products increases. Therefore, the inventory level is affected by sales and because of that at the end of the year, there is no remaining inventory. However, instead of the static approach, the dynamic approach might be more proper for our study. The dynamic approach is an iterative solution method for the planning year. In approximate dynamic programming, optimization model is solved by updating inventory level in every week for one product which is randomly selected in the sample dataset. This approach is proceeded in Microsoft Excel VBA. The result of approximate dynamic programming approach showed that there is no markdown for the selected product and there is on hand inventory at the end of the season.

When we compared two solution approaches, dynamic approach should give better results, but static approach has given better for proper initial inventory level. We believe that this situation is a consequence of running approximate dynamic programming for only one product. It might be better when we run this approach for all products in the sample dataset.

For future researches, we will be developing the approximate dynamic programming approach for markdown optimization model. Currently, this approach is performed for the randomly selected one product. We will run this approach for all products in the dataset for the future study.

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