

## TOTAL LEAST SQUARES REGISTRATION of 3D SURFACES

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### ABSTRACT:

Co-registration of point clouds of partially scanned objects is the first step of the 3D modeling workflow. The aim of co-registration is to merge the overlapping point clouds by estimating the spatial transformation parameters. In computer vision and photogrammetry domain one of the most popular methods is the ICP (Iterative Closest Point) algorithm and its variants. There exist the 3D Least Squares (LS) matching methods as well (Gruen and Akca, 2005). The co-registration methods commonly use the least squares (LS) estimation method in which the unknown transformation parameters of the (floating) search surface is functionally related to the observation of the (fixed) template surface. Here, the stochastic properties of the search surfaces are usually omitted. This omission is expected to be minor and does not disturb the solution vector significantly. However, the a posteriori covariance matrix will be affected by the neglected uncertainty of the function values of the search surface. This causes deterioration in the realistic precision estimates. In order to overcome this limitation, we propose a method where the stochastic properties of both the observations and the parameters are considered under an errors-in-variables (EIV) model. The experiments have been carried out using diverse laser scanning data sets and the results of EIV with the ICP and the conventional LS matching methods have been compared.

### 1. INTRODUCTION

3D object modeling plays an important role for many applications from reverse engineering to creating the real-world models for virtual reality, architecture or deformation analysis. In the last decade, laser scanners had an utmost importance for 3D object modeling due to their ability of providing reliable 3D data very fast and directly. Since the range scanners are line-of-sight instruments, in many cases an object has to be scanned from different standpoints to be able to cover the whole object. As a result, separate point clouds, which are in their own local co-ordinate systems uniquely, are obtained. In order to form a 3D model, these point clouds have to be merged in one co-ordinate system. This process is called alignment or registration. Various methods were proposed and the studies in this area are still in progress especially in computer vision discipline including the most popular Iterative Closest Point (ICP) algorithm and its variants. Since the introduction of ICP by Chen and Medioni, (1991) and Besl and McKay, (1992), many variants have been introduced on the basic ICP concept. A detailed review of the ICP variants can be found at Akca, (2010) and Rusinkiewicz, (2001). Despite the popularity of the ICP, there are some disadvantageous aspects of it in terms of accuracy assesment of transformation parameters. ICP based algorithms generally uses closed-form solutions for the estimation of transformation parameters. The closed-form solutions cannot fully consider the statistical accuracy assesment of the estimated parameters. One another powerfull and adaptive method for the registration problem is the 3D least squares surface matching proposed by Gruen and Akca, in (2005). The method is the extension and adaptation of mathematical model of Least Squares 2D image matching for the 3D surface matching problem. The transformation parameters of the search surfaces are estimated with respect to a template

surface. The solution is achieved when the sum of the squares of the 3D spatial (Euclidean) distances between the surfaces are minimized. The parameter estimation is achieved using the Generalized Gauss-Markov model. Akca, (2010). At this model, the points on the template surface are considered as observations, contaminated by random errors, while the search surface points are assumed as error-free.

$$y + e_y = t + R * x \quad (1)$$

With the assumptions

$$e_y \sim N(0, \sigma_0^2 P_y^{-1}) \quad (2)$$

where  $y$  is the template point,  $x$  is the search point,  $e_y$  is the true error vector for template points,  $t$  is the translation vector,  $R$  is the rotation matrix, and  $P$  is the weight matrix. Here, and also in the ICP methods, the stochastic properties of the search surfaces are usually omitted. This omission is expected to be minor and does not disturb the solution vector significantly. However, the a posteriori covariance matrix will be affected by the neglected uncertainty of the function values of  $x$ . This causes deterioration in the realistic precision estimates. More details on this issue can be found in Gruen, (1985), Maas, (2002), Gruen and Akca, (2005), Kraus et al., (2006), and Akca, (2010). These algorithms consider the noise as coming from one measurement only, but in fact both surface measurements are corrupted by noise. To be able to overcome this undesirable situation and obtain more realistic precision estimation values, another approach which takes the stochastic properties of the elements of design matrix into consideration should be applied. The problem can be solved by using a model which is called in the literature as Errors-in-Variables model or the total least squares (TLS). Markovsky and Huffel, (2007) outlines the different solution methods and application areas of EIV model very detailed. Ramos and

Verriest, (1997) proposed to use the total least squares approach for the registration of m-D data. In their study, they use a mixed solution which is the combination of Least squares and Total Least squares methods for the registration of 2D medical images. However, they do not give any information about the precision of the transformation parameters. Akyilmaz, (2007) uses Total Least Squares method for coordinate transformation in Geodetic applications. Since the author uses a closed-form solution method in this study, there is not any information about precision of estimated parameters as well. A mathematical model is given by Neitzel, (2007) where an iterative Gauss-Helmert type of adjustment model with the linearized condition equations is adopted. However, in this method the size of the normal equations to be solved increases dramatically depending on the number of conjugate points, since each point introduces three more Lagrange multipliers into the normal equations. Thus, the larger the number of conjugate points, the greater the normal equations to be solved.

For an optimal solution of the so-called EIV problem, we propose a modified iterative Gauss-Helmert type of adjustment model. In this model, the rotation matrix  $R$  is represented in terms of unit quaternions  $q = [q_0 \ q_1 \ q_2 \ q_3]$ . Moreover, the dimension of the normal equations to be solved is dramatically reduced to the number of unknown transformation parameters which is six for the rigid-body transformation problem. The mathematical model has been implemented in MATLAB. This study mainly aims at comparing the proposed model with LS matching method in terms of the precision of estimated parameters by using diverse laser scanning data sets.

## 2. ERRORS-IN-VARIABLES MODEL

The aim of co-registration process is to transform search surface with respect to the template surface by establishing the correspondences between two overlapping data sets. Assuming the existence of two overlapping 3D data sets  $Q_i$  and  $P_j$  ( $i=1, \dots, N$  and  $j=1, \dots, M$ ), we can find a corresponding point in  $P_j$  for each point in  $Q_i$  by using different error metrics, which  $Q_i$  and  $P_j$  are the template and search surfaces respectively. Once the appropriate correspondences were established between two point data sets the basic procedure is to estimate the transformation parameters using the correspondences. The geometric relationship is established by a six parameters 3D rigid-body transformation. Eq. (3).

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} Tx \\ Ty \\ Tz \end{bmatrix} + R * \begin{bmatrix} x_j \\ y_j \\ z_j \end{bmatrix} \quad (3)$$

In the classical Gauss-Markov model, Eq. (1) represents the observation equation which assumes the template surface elements are observations contaminated by random the errors. In fact, the search surface elements are also erroneous and a true error vector should be added to these elements as well. The observation equations in EIV model are formed as

$$y + e_y = t + R * (x + e_x). \quad (4)$$

If we apply this model to 3D rigid-body transformation, the mathematical model is established as;

$$l + v_x = (A + v_A) * \beta \quad (5)$$

where  $v_x$  is the  $n \times 1$  residual vector of observations and  $v_A$  is an  $n \times m$  error matrix of the corresponding elements of design matrix. The elements of both  $v_x$  and  $v_A$  are independent and conforming the normal distributed with zero mean. Once a minimisation of  $[\tilde{v}_A; \tilde{v}_x]$  is found, then any  $\beta$  satisfying  $(A + \tilde{v}_A) \cdot \beta = l + \tilde{v}_x$  is the solution of the problem by Total Least Squares.

### 2.1 Proposed Modified Gauss-Helmert Model

The generalized total least squares solution of the 3D-similarity transformation by introducing the quaternions as the representation of the rotation matrix\*scale factor ( $S=sR$ ) based on iteratively linearized Gauss-Helmert model has been presented by Akyilmaz, (2010, 2011). However, this model requires the solution of a normal matrix which includes the corresponding terms for transformation parameters as well as the Lagrange multipliers, thus yielding a larger size of system of equations to be solved at each iteration with the increase of the identical points of the transformation problem. Following the idea in Akyilmaz, (2010 and 2011), Kanatani and Niitsuma, (2012) has developed a new computational scheme for 3D-similarity transformation which they call *Modified Iterative Gauss-Helmert* model by reducing the so-called Lagrange multipliers and hence the size of the normal matrix is dramatically reduced. In other words, the unknowns to be solved at each iteration are equal to seven, i.e. the number of transformation parameters. This kind of a reduction provides advantage, especially in terms of computational aspects. We refer to Kanatani and Niitsuma, (2012) for details of the mathematical model. Modified Gauss-Helmert model in Kanatani and Niitsuma, (2012) is a seven parameters similarity transformation. Therefore, in our study, we modified the model by eliminating the scale factor in order to apply 6 parameters rigid-body transformation. For this purpose we normalise the quaternion by using the  $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$  equality. Then the rotation matrix defined by quaternions is obtained as;

$$S = \begin{bmatrix} 2q_2^2 - 2q_3^2 + 1 & 2q_1q_2 - 2q_3N & 2q_2N + 2q_1q_3 \\ 2q_3N + 2q_1q_2 & -2q_1^2 - 2q_3^2 + 1 & 2q_1N \\ 2q_1q_3 - 2q_2N & 2q_1N + 2q_2q_3 & 2q_2^2 + 1 \end{bmatrix} \quad (6)$$

$$N = \sqrt{(-q_1^2 - q_2^2 - q_3^2 + 1)}$$

In so-called model, let  $a_i$  and  $b_i$  are the corresponding pairs ( $i=1, \dots, M$ );  $Q_{xx}[a_i]$  and  $Q_{xx}[b_i]$  are normalized covariance matrices;  $\bar{a}_i$  and  $\bar{b}_i$  are the true positions of  $a_i$  and  $b_i$  respectively. The optimal estimation of the similarity transformation parameters  $R$  (rotation),  $T$  (translation) and  $s$  (scale factor) in the sense of Maximum Likelihood is to minimize the Mahalanobis distance given as follows.

$$J = \frac{1}{2} \sum_{i=1}^M (a_i - \bar{a}_i)^T Q_{xx}^{-1} [a_i] (a_i - \bar{a}_i) + \dots + \frac{1}{2} \sum_{i=1}^M (b_i - \bar{b}_i)^T Q_{xx}^{-1} [b_i] (b_i - \bar{b}_i) \quad (7)$$

and

$$\bar{a}_i = S\bar{b}_i + T \quad (8)$$

Where S is the rotation matrix given in Eq. (6)

Since the model is non-linear, it is linearized by the Taylor Series expansion. Finally, the total error vector is defined as

$$e_i = a_i - Sb_i - T \quad (9)$$

With the weight matrix;

$$W_i = (SQ_{xx}[a_i]S' + Q_{xx}[b_i])^{-1} \quad (10)$$

After modifications, Eq. 7 can be expressed in the following form:

$$J = \frac{1}{2} \sum_1^M (e_i' W_i e_i) \quad (11)$$

Differentiating (6) with respect to  $q_i$ ,  $i = 1, 2, 3$

$$\frac{\partial S}{\partial q_i} = 2Q_i$$

We define a 3x3  $U_i$  matrix as follows

$$U_i = [Q_1 b_i \quad Q_2 b_i \quad Q_3 b_i] \quad (12)$$

After these definitions, parameters are estimated by the solution of following 6-D linear equation.

$$\begin{pmatrix} \sum_1^M U_i^T W_i U_i & \sum_1^M U_i^T W_i \\ \sum_1^M W_i U_i & \sum_1^M W_i \end{pmatrix} \begin{pmatrix} \Delta q \\ \Delta T \end{pmatrix} = \begin{pmatrix} \sum_1^M U_i^T W_i e_i \\ \sum_1^M W_i e_i \end{pmatrix}$$

Since so-called model is non-linear, initial approximations of  $q$  and  $T$  are updated and iteration is repeated until it converges.

## 2.2 Correspondence Search

Correspondence search is the most critical part of all registration algorithms. The success of a registration method depends on how correct correspondences were established between two data sets. False matches cause to uncorrect results. In order to prevent false matches, different type of constraints can be introduced. In our implementation, the correspondence search is guided by using two well-known error metrics. The first one is the point-to-point search which was introduced by Besl and McKay (1992) in their original ICP paper. According to this method, each available point in template surface is matched with the closest point in search surface. Then, the sum of the squared distances between the points in each correspondence pair is minimized. This procedure is very complex in terms of computational aspects and takes the most of the computation time. The procedure has been accelerated by using a kd-tree searcher in our implementation. The second error metric is the point-to-plane algorithm which was introduced by Chen and Medioni (1991). In point-to-plane error metric, the sum of the squared distances between each point in template data and the tangent plane at its corresponding destination point in search data is

minimized. Due to the large search area and heavy mathematical computations like plane parameters, surface normal and Euclidian distance calculation, point-to-plane error metric is much slower than point-to-point version. On the other hand, the researchers have observed significantly better convergence rates with point-to-plane (Rusinkiewicz, 2001). One solution for accelerating the point-to-plane version is to limit the search area of the candidate point at the search surface. Based on the advantageous parts of these two versions, both of them were used together in this study in order to benefit from the advantageous parts of them. The point-to-plane search was accelerated significantly by using a kd-tree nearest neighbor searcher. The coarse match point is found by the point-to-point search; consequently the procedure is followed by the point-to-plane search where the fine matching point is found. The fine matching point is searched inside the 6 neighboring triangles which are fictitiously formed around the coarse matching point. The fine matching point should lie inside of one of those six triangles, and should have the minimum spatial distance to the corresponding template point. Any point satisfying the both conditions are labeled as the fine matching point.

## 2.3 Experimental Results

Two examples are given below in order to show the capability of the proposed method and also to make a comparison between the conventional least squares and total least squares matching. All experiments were executed using home-developed two different programmes created by using MATLAB computing language. Some available functions of MATLAB like kd-tree searcher were used directly instead of re-coding.

In all experiments, the pre-alignment of template and search data sets were done manually by selecting at least 3 common points on two sets. The stopping criteria for both data sets was set as 0.001 mm for translation parameters and 0.001 degrees for rotation parameters.

### 2.3.1 Façade of a historical building

The first experiment is carried out by using 3D overlapping surface patches (Fig. 1) belonging to a historical building. The data was acquired by Leica C-10 time-of-flight type of laser scanner. Average point spacing of original data is 1.5 cm. But data was resampled in order to decrease the number of points and the resampled data has about 3 cm point spacing. These pre-aligned data sets were registered by using the two programs. The numerical results of this test are given in Table 1. According to the obtained results, while a posteriori sigma naught value is slightly smaller for TLS than LS, theoretical precision values at LS are smaller. However, the difference between theoretical precision values is minor.

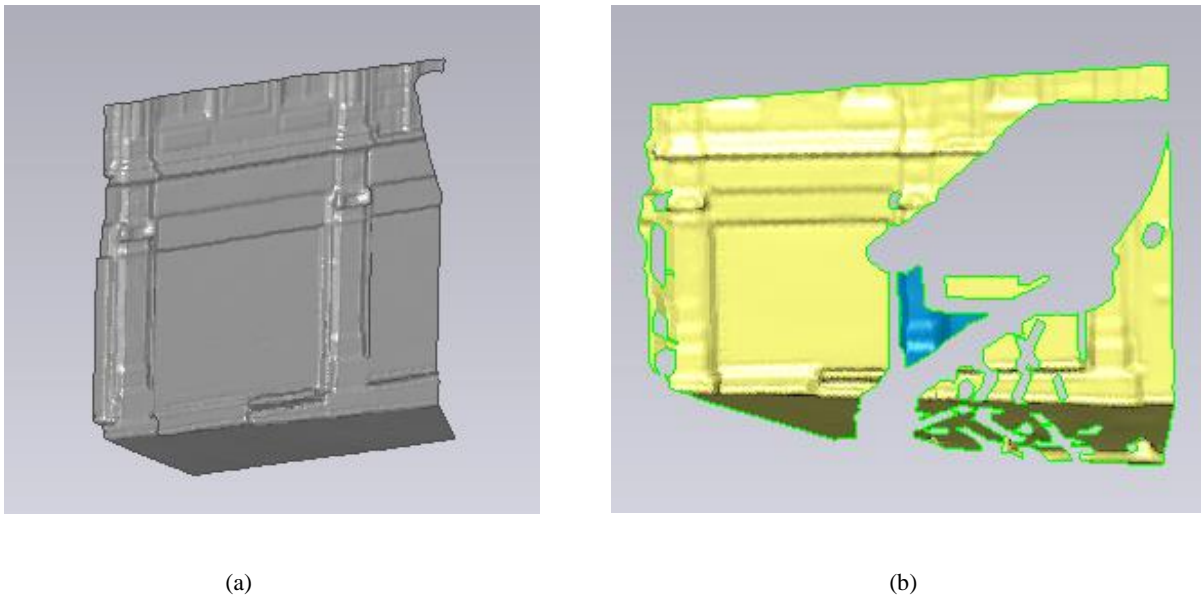


Figure 1. Two overlapping data sets. (a) is the left scan and (b) is the right scan of the facade

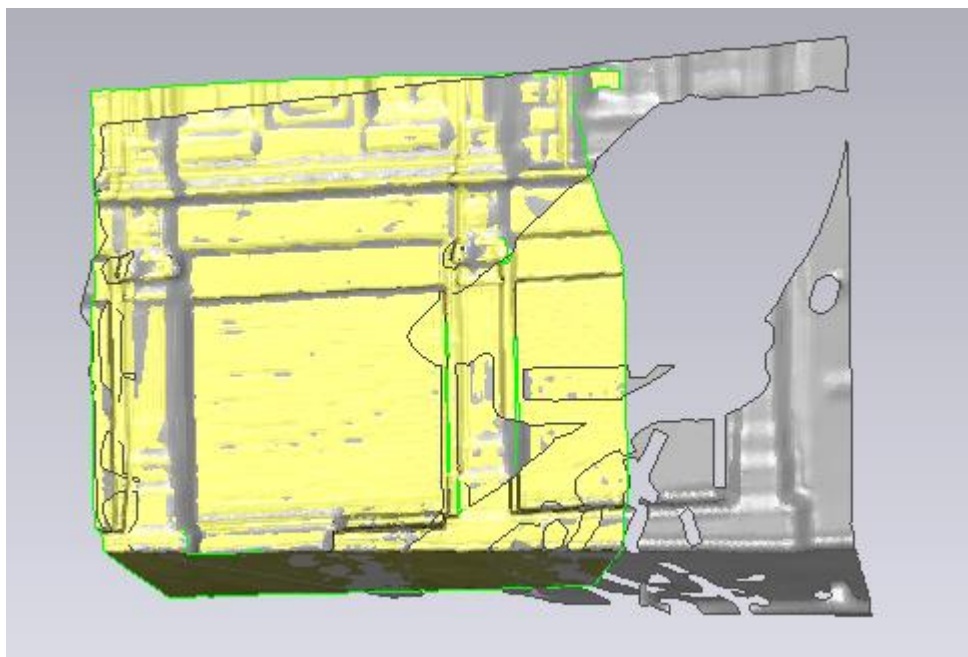


Figure 2. Historical building facade by TLS registration

Model	No. of Matched Points	$\sigma_0$ (m)	$\sigma_{T_x}$ (m)	$\sigma_{T_y}$ (m)	$\sigma_{T_z}$ (m)	$\sigma_\omega$ (deg)	$\sigma_\phi$ (deg)	$\sigma_\kappa$ (deg)
TLS	458	0.005515	0.000392	0.000581	0.000516	0.000034	0.000026	0.000038
LS	544	0.006890	0.000339	0.000500	0.000445	0.000029	0.000023	0.000033

Table 2. Numerical results of 'Historical Building' data

### 2.3.2 Surface patch

The second experiment is the matching of two surface patches (Fig. 3). The data is acquired by a IMAGER 5300 terrestrial laser scanner (Zoeller+Fröchlich). The average point spacing is 1 cm. Obtained numerical results for two different registrations are given in Table 2. In this experiment also, the final sigma naught value at TLS is smaller than LS; and theoretical precisions are smaller for LS matching as it is in the first experiment.

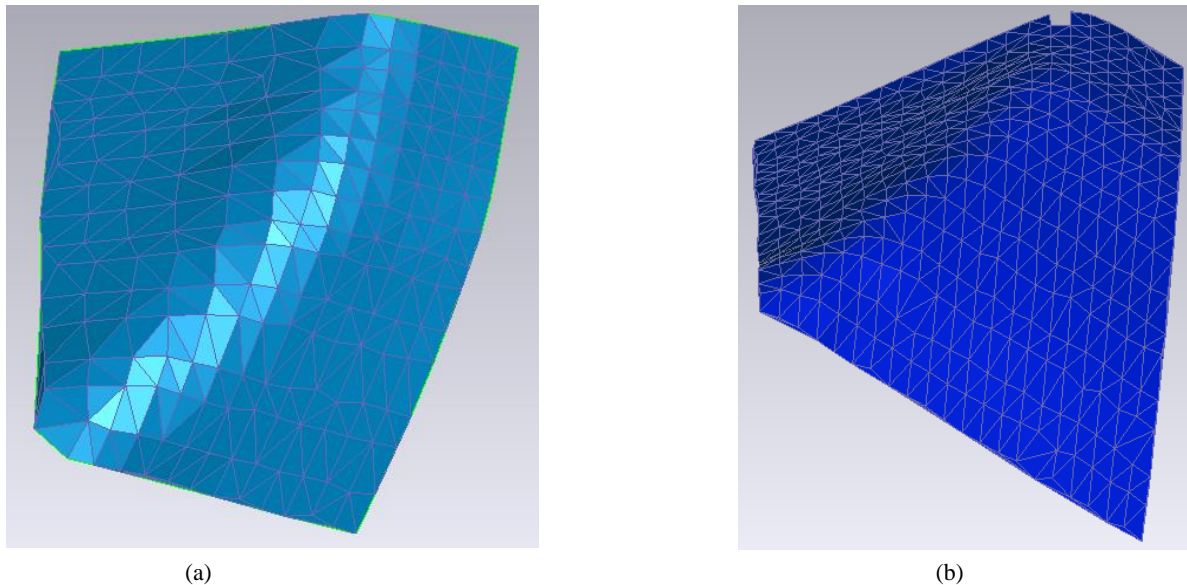


Figure 3. (a) is the template and (b) is the search surface.

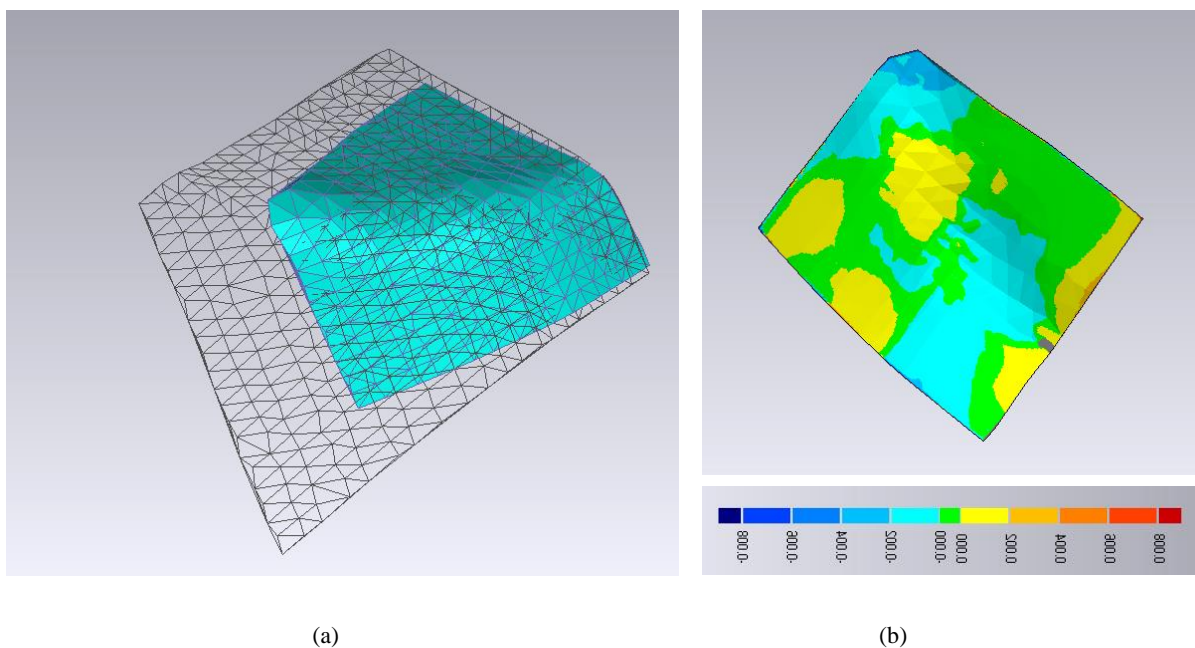


Figure 4. (a) is the registration with TLS and (b) shows the residuals.

Model	No. of Matched Points	$\sigma_0$ (mm)	$\sigma_{T_x}$ (mm)	$\sigma_{T_y}$ (mm)	$\sigma_{T_z}$ (mm)	$\sigma_\omega$ (deg)	$\sigma_\phi$ (deg)	$\sigma_\kappa$ (deg)
TLS	458	0.024763	0.002887	0.002883	0.004862	0.000244	0.000244	0.0001810
LS	544	0.035112	0.002510	0.002511	0.004239	0.000211	0.000210	0.000154

Table 1. Numerical results of 'surface patch' data

### 3. CONCLUSION AND DISCUSSION

The motivation of this study is to investigate the error behaviours of parameter estimation of rigid-body transformation by applying EIV model which considers the both data sets are characterized as erroneous. The omission of the stochastic properties of design matrix in the conventional Gauss-Markov model results in optimistic precision estimates. By taking the stochastic properties of the elements of the design matrix into account, it is possible to have more realistic precision estimates of unknowns with the proposed model. An implementation has been made in MATLAB computing language for the comparison of two mathematical models. The experimental tests show that the proposed method provides the more realistic values but the magnitude of the difference is minor. The final sigma naught values at both experiments are smaller for total least squares registration than conventional least squares. In this kind result, one can expect smaller theoretical precision values for individual estimation parameters for TLS as well. But the results show that these values are smaller at LS registration. This results indicate the higher values of parameters in covariance matrix in TLS estimation due to the contribution of the error vectors of search surface elements. It can obviously be stated that the TLS and LS methods do not differ in solution vector. However, statistical values obtained from TLS is slightly more realistic.

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