TRANSVERSAL HYPERSURFACES OF ALMOST HYPERBOLIC CONTACT MANIFOLDS WITH A QUARTER SYMMETRIC NON METRIC CONNECTION

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ABSTRACT. Transversal hypersurfaces of trans hyperbolic contact manifolds endowed with a quarter symmetric non metric connection are studied. It is proved that transversal hypersurfaces of almost hyperbolic contact manifold with a quarter symmetric non metric connection admits an almost product structure and each transversal hypersurfaces of almost hyperbolic contact metric manifold with a quarter symmetric non metric connection admits an almost product semi-Riemannian structure. The fundamental 2-form on the transversal hypersurfaces of cosymplectic hyperbolic manifold and $(\alpha, 0)$ trans hyberbolic Sasakian manifold with hyperbolic (f, g, u, v, α)-structure are closed. It is also proved that transversal hypersurfaces of trans hyperbolic contact manifold with a quarter symmetric non metric connection admits a product structure. Some properties of transversal hypersurfaces with a quarter symmetric non metric connection are proved.

Keywords : Hypersurfaces, Quarter Symmetric semi-metric connection, Quasi-sasakian manifold, Gass and Weingarten equations.

AMS Subject Classification: 53D12, 53C05.

1. Introduction

In 1975, Golab [23] introduced the notion of quarter-symmetric connection in a Riemannian manifold with affine connection. This was further developed by Yano and Imai [11], Rastogi [24], [25], Mishra and Pandey [15], Mukhopadhyay, Roy and Barua [27], Biswas and De [22], Sengupta and Biswas [9], Singh and Pandey [14] and many other geometers. Almost contact metric manifold with an almost contact metric structure is very well explained by Blair [3]. In [28], S. Tanno gave a classification for connected almost contact metric manifolds whose automorphism groups have the maximum dimension. For such a manifold, the sectional curvature of plane sections containing ξ is a constant, say c. He showed that they can be divided into three classes: (1) Homogenous normal contact Riemannian manifolds with $c > 0$, (2) global Riemannian products of a line or a circle with a Kaehler manifold of constant holomorphic sectional curvature if $c = 0$ and (3) a warped product space $R \times f^{C^n}$ if $c < 0$. It is known that the manifolds of class (1) are characterized by some tensorial relations admitting a contact structure. Kenmotsu [10] characterized the differential geometric properties of the third case by tensor equation

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$(\bar{\nabla}_X \phi)Y = g(\phi X, Y) - \eta(Y)\phi X$. The structure so obtained is now known as Kenmotsu structure. In general, these structures are not Sasakian [10].

Oubina studied a new class of almost contact Riemannian manifold known as trans-Sasakian manifold [6] which generalizes both α -Sasakian [6] and β-Kenmotsu [6] structure.

M. D. Upadhayay studied almost contact hyperbolic (f, g, η, ξ)-structure [13]. Bhatt and Dubey studied on CR-submanifolds of trans hyperbolic contact manifold [12]. B. Y. Chen studied Geometry of submanifolds and its applications. Sci. Univ Tokyo. Tokyo, 1981. [2]. R. Prasad, M. M. Tripathi, J. S. Kim and J-H. Cho., studied some properties of submanifolds of almost contact manifold [17], [18], [19], [20], [21].

2. **Preliminaries**

Let $\bar{M}$ be an $2n + 1$ dimensional manifold with almost hyperbolic contact metric structure (ϕ, ξ, η, g) where ϕ is a tensor field of type $(1, 1)$, ξ is a vector field, η is a 1-form and g is the semi Riemannian metric on $\bar{M}$. Then the following conditions [13] are satisfied

$$\phi^2 X = X + \eta(X)\xi \quad , \quad \phi\xi = 0 \quad \eta \circ \phi = 0 \quad \eta(\xi) = -1 \tag{2.1}$$

$$g(\phi X, \phi Y) = -g(X, Y) - \eta(X)\eta(Y) \tag{2.2}$$

$$g(\phi X, Y) = -g(X, \phi Y) \tag{2.3}$$

for vector fields X,Y on $\bar{M}$. An almost hyperbolic contact metric structure (ϕ, ξ, η, g) on $\bar{M}$ is called trans hyperbolic contact [12] if and only if

$$(\bar{\nabla}_X \phi)Y = \alpha[g(X, Y)\xi - \eta(Y)X] + \beta[g(\phi X, Y)\xi - \eta(Y)\phi X] \tag{2.4}$$

for all smooth vector fields X, Y on $\bar{M}$ and α, β non zero constant, where $\bar{\nabla}$ is the Levi-civita connection with respect to g. From (2.4) it follows that

$$\bar{\nabla}_X \xi = \alpha\phi X + \beta[X + \eta(X)\xi] \tag{2.5}$$

for all smooth vector fields X,Y on $\bar{M}$. On other hand, a quarter symmetric non metric connection $\bar{\nabla}$ on M is defined by

$$\bar{\nabla}_X Y = \bar{\nabla}_X^* Y + \eta(Y)\phi X \tag{2.6}$$

Using (2.1) and (2.6) in (2.4) and (2.5), we get respectively

$$(\bar{\nabla}_X \phi)Y = \alpha[g(X, Y)\xi - \eta(Y)X] + \beta[g(\phi X, Y)\xi - \eta(Y)\phi X]$$
$$-\eta(Y)X - \eta(X)\eta(Y)\xi \tag{2.7}$$

$$\bar{\nabla}_X \xi = (\alpha - 1)\phi X + \beta[X + \eta(X)\xi] \tag{2.8}$$

Let M be a hypersurface of an almost hyperbolic contact manifold $\bar{M}$ equipped with an almost hyperbolic contact structure (ϕ, ξ, η). We assume that the structure vector field ξ never belongs to tangent space of the hypersurface M, such that a hypersurface is called a transversal hypersurface of an almost contact manifold. In this case the structure vector field ξ can be taken as an affine normal to the hypersurface. Vector field X on M and ξ are linearly independent, therefore we may write

$$\phi X = F(X) + \omega(X)\xi \tag{2.9}$$

where F is a $(1, 1)$ tensor field and ω is a 1-form on M.

From (2.9)

$$\phi\xi = F\xi + \omega(\xi)\xi$$

or

$$0 = F\xi + \omega(\xi)\xi$$
$$\phi^2 X = F(\phi X) + \omega(\phi X)\xi \tag{2.10}$$

$$X + \eta(X)\xi = F(FX + \omega(X)\xi) + \omega(FX + \omega(X)\xi)\xi$$

$$X + \eta(X)\xi = F^2 X + (\omega \circ F)(X)\xi \tag{2.11}$$

Taking account of equation (2.11), we get

$$F^2 X = X \tag{2.12}$$

$$F^2 = I \tag{2.13}$$

$$\eta = \omega \circ F$$

Thus we have

Theorem 2.1. *Each transversal hypersurface of an almost hyperbolic contact manifold endowed with a quarter symmetric non metric connection admits an almost product structure and a 1-form ω.*

From (2.12) and (2.13), it follows that

$$\eta = \omega \circ F$$

$$\eta(FX) = (\omega \circ F)FX$$

$$\eta(FX) = \omega(F^2 X)$$

$$(\omega \circ F)X = \omega(X)$$

$$\omega = \eta \circ F \tag{2.14}$$

Now, we assume that $\bar{M}$ admits an almost hyperbolic contact metric structure (ϕ, ξ, η, g) endowed with a quarter symmetric non metric connection. We denote by g the induced metric on M also. Then for all $X, Y \epsilon TM$, we obtain

$$g(FX, FY) = -g(X, Y) - \eta(X)\eta(Y) + \omega(X)\omega(Y) \tag{2.15}$$

We define a new metric G on the transversal hypersurface given by

$$G(X, Y) = g(\phi X, \phi Y) = -g(X, Y) - \eta(X)\eta(Y) \tag{2.16}$$

So,

$$G(FX, FY) = -g(FX, FY) - \eta(FX)\eta(FY)$$

$$= -g(X, Y) - \eta(X)\eta(Y) + \omega(X)\omega(Y) - (\eta \circ F)(X)(\eta \circ F)(Y)$$

$$= -g(X, Y) - \eta(X)\eta(Y) + \omega(X)\omega(Y) - \omega(X)\omega(Y)$$

$$= -g(X, Y) - \eta(X)\eta(Y) = G(X, Y)$$

Then, we get

$$G(FX, FY) = G(X, Y) \tag{2.17}$$

where equation (2.12), (2.14), (2.15) and (2.16) are used. Then G is semi Riemannian metric on M that is (F, G) is an almost product semi-Riemannian structure on the transversal hypersurface M of $\bar{M}$. Thus, we are able to state the following.

Theorem 2.2. *Each transversal hypersurface of an almost hyperbolic contact manifold endowed with a quarter symmetric non metric connection admits an almost product semi-Riemannian structure. We now assume that M is orientable and choose a unit vector field N of M, normal to M. Then Gauss and Weingarten formulae of quarter symmetric non metric connection are given respectively by*

$$\bar{\nabla}_X Y = \nabla_X Y + [h(X,Y) + u(X)\eta(Y)]N, \quad (X, Y \epsilon TM) \tag{2.18}$$

$$\bar{\nabla}_X N = -HX + \lambda f X + u(X)\lambda N \tag{2.19}$$

where $\bar{\nabla}$ and ∇ are respectively the Levi-civita and induced Levi-civita connections in $\bar{M}$, M and h is the second fundamental form related to H by

$$h(X,Y) = g(HX, Y), \tag{2.20}$$

for any vector field X tangent to M, defining

$$\phi X = fX + u(X)N \tag{2.21}$$

$$\phi N = -U, \tag{2.22}$$

$$\xi = V + \lambda N, \tag{2.23}$$

$$\eta(X) = v(X)$$

$$\lambda = \eta(N) = g(\xi, N), \tag{2.24}$$

for $X \epsilon TM$ we get an induced hyperbolic (f, g, u, v, λ)-structure on the transversal hypersurface such that

$$f^2 = I + u \otimes U + v \otimes V \tag{2.25}$$

$$fU = -\lambda V, \quad fV = \lambda U \tag{2.26}$$

$$u \circ f = \lambda v, \quad v \circ f = -\lambda U \tag{2.27}$$

$$u(U) = -1 - \lambda^2, \quad u(V) = v(U) = 0, \quad v(V) = -1 - \lambda^2 \tag{2.28}$$

$$g(fX, fY) = -g(X,Y) - u(X)u(Y) - v(X)v(Y) \tag{2.29}$$

$$g(X, fY) = -g(fX, Y), \quad g(X, U) = u(X), \quad g(X, V) = v(X), \tag{2.30}$$

for all for $X, Y \epsilon TM$ where

$$\lambda = \eta(N) \tag{2.31}$$

Thus, we see that every transversal hypersurface of an almost hyperbolic contact metric manifold endowed with a quarter symmetric non metric connection also admits a hyperbolic (f, g, u, v, λ)-structure. Next we find relation between the induced almost product structure (F, G) and the induced hyperbolic (f, g, u, v, λ)-structure on the transversal hypersurface of an almost hyperbolic contact metric manifold with a quarter symmetric non metric connection. In fact, we have the following

Theorem 2.3. *Let M be a transversal hypersurface of an almost hyperbolic contact metric manifold $\bar{M}$ equipped with almost hyperbolic contact metric structure (ϕ, ξ, η, g) and induced almost product structure (F, G).*

Then we have

$$\lambda \omega = u, \tag{2.32}$$

$$F = f - \frac{1}{\lambda} u \otimes V, \tag{2.33}$$

$$FU = \frac{1}{\lambda} V, \tag{2.34}$$

$$u \circ F = u \circ f = \lambda v, \tag{2.35}$$

$$FV = fV = \lambda U, \tag{2.36}$$

$$u \circ F = \frac{1}{\lambda} u. \tag{2.37}$$

Proof:

$$\phi X = FX + \omega(X)\xi$$

$$\xi = V + \lambda N$$

$$\phi X = FX + \omega(X)V + \lambda\omega(X)N \qquad (2.38)$$
$$\phi X = fX + u(X)N \qquad (2.39)$$

From equation (2.38) and (2.39) we have

$$\lambda\omega X = u(X), \quad \omega(X) = \frac{1}{\lambda}u(X),$$
$$FX = fX - \omega(X)V,$$
$$FX = fX - \frac{1}{\lambda}u(X)V,$$
$$F = f - \frac{1}{\lambda}u \otimes V$$

which is equation (2.33).

$$(u \circ F)(X) = (u \circ f) - \frac{1}{\lambda}u(X)u(V), \quad u(V) = 0$$
$$u \circ F = u \circ f = \lambda v$$

which is equation (2.35).

$$FU = fV - \frac{1}{\lambda}u(v)V$$
$$FU = -\lambda V - \frac{1}{\lambda}(-1-\lambda^2)V,$$
$$FU = \frac{1}{\lambda}V,$$

which is equation (2.34).

$$(u \circ F)(X) = (u \circ f)(X) - \frac{1}{\lambda}u(X)u(V)$$
$$= (u \circ f)(X) - \frac{1}{\lambda}u(X)(-1-\lambda^2)$$
$$= -\lambda u(X) + \frac{1}{\lambda}u(X) + \lambda u(X)$$
$$= \frac{1}{\lambda}u(X),$$
$$u \circ F = \frac{1}{\lambda}u$$
$$FV = fV - \frac{1}{\lambda}u(V)V = fV = \lambda U,$$

which is equation (2.36) here equations (2.26), (2.27), (2.28), (2.29), (2.30), (2.31) are used.

Lemma 2.1. *Let M be a transversal hypersurface with hyperbolic (f, g, u, v, λ)-structure of an almost hyperbolic contact metric manifold $\bar{M}$ endowed with a quarter symmetric non metric connection. Then*

$$(\bar{\nabla}_X\phi)Y = ((\nabla_X f)Y - u(Y)HX + \lambda fXu(Y) + h(X,Y)U + u(X)\eta(Y)U)$$
$$+((\nabla_X u)Y + h(X, fY) + u(X)\eta(fY) + 2u(X)u(Y)\lambda)N \qquad (2.40)$$
$$\bar{\nabla}_X\xi = \nabla_X V - \lambda HX + \lambda^2 fX + [h(X,V) - u(X) + X(\lambda)]N \qquad (2.41)$$
$$(\bar{\nabla}_X\phi)N = -\nabla_X U + fHX - \lambda f^2 X - [h(X,U) - \mu(HX)]N \qquad (2.42)$$
$$(\bar{\nabla}_X\eta)Y = (\nabla_X v)Y + h(X,Y)\lambda + u(X)\eta(Y)\lambda \qquad (2.43)$$

for all $X, Y \epsilon TM$. The proof is straight forward.

3. Transversal hypersurfaces of cosymplectic hyperbolic manifold with a quarter symmetric non metric connection

Trans-Sasakian structures of type $(\alpha, 0)$ are called α-Sasakian and trans-Sasakian structures of type $(0, \beta)$ are called β-Kenmotsu structures. Trans-Sasakian structures of type $(0, 0)$ are called cosymplectic structures.

Theorem 3.1. *Let M be a transversal hypersurfaces with hyperbolic (f, g, u, v, λ)-structure of a hyperbolic cosymplectic manifold $\bar{M}$ endowed with a quarter symmetric non metric connection. Then*

$$(\nabla_X f)Y = u(Y)HX + \lambda f X u(Y) - [h(X, Y) + u(X)\eta(Y)]U \tag{3.1}$$

$$(\nabla_X u)Y = -h(X, fY) - u(X)\eta(fY) - 2u(X)u(Y)\lambda \tag{3.2}$$

$$\nabla_X V = \lambda HX - \lambda^2 fX \tag{3.3}$$

$$h(X, V) = -X(\lambda) + u(X) \tag{3.4}$$

$$\nabla_X U = fHX - \lambda f^2 X \tag{3.5}$$

$$h(X, U) = \mu(HX)$$

$$(\nabla_X v)Y = \lambda HX - \lambda^2 fX \tag{3.6}$$

for all $X, Y \epsilon TM$.

Proof: Using (2.7), (2.20), (2.23) in (2.40), we obtain

$$((\nabla_X f)Y - u(Y)HX + \lambda f X u(Y) + h(X, Y)U + u(X)\eta(Y)U)$$

$$+((\nabla_X u)Y + h(X, fY) + u(X)\eta(fY) + 2u(X)u(Y)\lambda)N = 0$$

Equating tangential and normal parts in the above equation, we get (3.1) and (3.2) respectively. Using (2.8) and (2.23) in (2.41), we have

$$\nabla_X V - \lambda HX + \lambda^2 fX + [h(X, V) - u(X) + X(\lambda)]N = 0$$

Equating tangential and normal parts we get (3.3) and (3.4) respectively. Using (2.7), (2.22) and (2.23) in (2.42)

Using (2.7), (2.22) and (2.23) in (2.43) and equating tangential, we get (3.5). In the last (3.6) follows from (2.43).

Theorem 3.2. *If M be a transversal hypersurface with hyperbolic (f, g, u, v, λ)-structure of a hyperbolic cosymplectic manifold endowed with a quarter symmetric non metric connection, then the 2-form Φ on M is given by*

$$\Phi(X, Y) = g(X, fY)$$

is closed.

Proof: From (3.1) we get

$$(\nabla_X \Phi)(Y, Z) = h(X, Y)u(Z) - h(X, Z)u(Y),$$

$$(\nabla_X \Phi)(Y, Z) + (\nabla_Y \Phi)(Z, X) + (\nabla_Z \Phi)(X, Y) = 0$$

Hence the theorem is proved.

Theorem 3.3. *If M is a transversal hypersurface with almost product semi Riemannian structure (F, G) of a hyperbolic cosymplectic manifold endowed with a quarter symmetric non metric connection. Then the 2-form Ω on M is given by*

$$\Omega(X, Y) = G(X, fY)$$

is closed.

Using (3.1), we calculate the Nijenhuis tensor

$$[F, F] = (\nabla_{FX} F)Y - (\nabla_{FY} F)X - F(\nabla_X F)Y + F(\nabla_Y F)X$$

and find that $[F, F] = 0$.

Therefore, in view of theorem (3.2), we have

Theorem 3.4. *Every transversal hypersurface of a trans hyperbolic cosymplectic manifold endowed with a quarter symmetric non metric connection, admits product structure.*

4. Transversal hypersurfaces of trans hyperbolic sasakian manifolds with a quarter symmetric non metric connection

Theorem 4.1. *Let M be a transversal hypersurface with hyperbolic (f, g, u, v, λ)-structure of a trans hyperbolic Sasakian manifold $\bar{M}$ endowed with a quarter symmetric non metric connection. Then*

$$(\nabla_X f)Y = \alpha[g(X, Y)V - \eta(Y)X] + \beta[g(fX, Y)V - \eta(Y)fX] - \eta(Y)X$$

$$-\eta(X)\eta(Y)V + u(Y)HX - \lambda fXu(Y - h(X, Y)U - u(X)\eta(Y)U \quad (4.1)$$

$$(\nabla_X u)Y = \alpha g(X, Y)\lambda + \beta[g(fX, Y)\lambda - u(X)\eta(Y)] - h(X, fY)$$

$$-\eta(X)\eta(Y)\lambda - \eta(fY)u(X) - 2u(X)u(Y)\lambda \quad (4.2)$$

$$\nabla_X V = (\alpha - 1 - \lambda^2)fX + \beta(X + \eta(X)V + HX\lambda \quad (4.3)$$

$$h(X, V) = \alpha u(X) + \beta\eta(X)\lambda - X(\lambda) \quad (4.4)$$

$$\nabla_X U = \alpha X\lambda - \beta(u(X)V - fX\lambda) + \lambda X + \lambda\eta(X)V + fHX - \lambda f^2 X \quad (4.5)$$

$$h(X, U) = \lambda^2 \eta(X) + \mu(HX) \quad (4.6)$$

for all $X, Y \epsilon TM$.

Proof: Using (2.7), (2.21), (2.23) in (2.40), we obtain

$$\alpha[g(X, Y)V - \eta(Y)X] + \beta[g(fX, Y)V - \eta(Y)fX] - \eta(Y)X - \eta(X)\eta(Y)V$$

$$= (\nabla_X f)Y - u(Y)HX + \lambda fXu(Y) + h(X, Y)U + u(X)\eta(Y)U)$$

Equating tangential and normal parts in the above equation, we get (4.1) and (4.2) respectively. Using (2.8) and (2.23) in (2.41), we have

$$\alpha g(X, Y)\lambda + \beta[g(fX, Y)\lambda - u(X)\eta(Y)] - \eta(X)\eta(Y)\lambda$$

$$= h(X, fY) + \eta(fY)u(X) + (\nabla_X u)Y + 2u(Y)u(Y)\lambda$$

Equating tangential and normal parts we get (4.3) and (4.4) respectively. Using (2.7), (2.22) and (2.23) in (2.42) and equating tangential parts, we get (4.5) in the last (4.6) follows from (2.43).

Theorem 4.2. *If M be a transversal hypersurface with hyperbolic (f, g, u, v, λ) structure of a (α, o) trans hyperbolic Sasakian manifold endowed with a quarter symmetric non metric connection, then the 2-form Φ on M is given by*

$$\Phi(X, Y) = g(X, fY)$$

is closed.

Proof: From (4.1) we get

$$(\nabla_X\Phi)(Y,Z) = -\alpha[g(X,Y)v(Z) - g(X,Z)v(Y)]$$
$$-\beta[g(fX,Y)v(Z) - g(fX,Z)v(Y)]$$
$$+h(X,Y)u(Z) - h(X,Z)u(Y)$$

which gives

$$(\nabla_X\Phi)(Y,Z) + (\nabla_Y\Phi)(Z,X) + (\nabla_Z\Phi)(X,Y)$$
$$= 2\beta(\Phi(X,Y)\eta(Z) + \Phi(Y,Z)\eta(X) + \Phi(Z,X)\eta(Y))$$

If $\beta = 0$, then

$$(\nabla_X\Phi)(Y,Z) + (\nabla_Y\Phi)(Z,X) + (\nabla_Z\Phi)(X,Y) = 0$$

that is $d\Phi = 0$. Hence the theorem is proved.

Theorem 4.3. *If M is a transversal hypersurface with almost product semi Riemannian structure (F,G) of a $(\alpha,0)$ trans hyperbolic Sasakian manifold endowed with a quarter symmetric non metric connection. Then 2-form Ω on M is given by*

$$\Omega(X,Y) = G(X,FY)$$

is closed.

Using (4.1), we calculate the Nijenhuis tensor

$$[F,F] = (\nabla_{FX}F)Y - (\nabla_{FY}F)X - F(\nabla_XF)Y + F(\nabla_YF)X$$

and find that $[F,F] = 0$. Therefore, in view of theorem (4.2), we have

Theorem 4.4. *Every transversal hypersurface of a trans hyperbolic Sasakian manifold endowed with a quarter symmetric non metric connection admits a product structure.*

5. Conclusion

The main objective of this paper is to introduce transversal hypersurfaces of trans hyperbolic contact manifolds endowed with a quarter symmetric non metric connection. Some results regarding transversal hypersurfaces of almost hyperbolic contact manifold with a quarter symmetric non metric connection have been obtained in this context. Further I have tried to show that the fundamental 2-form on the transversal hypersurfaces of cosymplectic hyperbolic manifold and $(\alpha,0)$ trans hyberbolic Sasakian manifold endowed with a quarter symmetric non metric connection with hyperbolic (f,g,u,v,λ)-structure are closed. It is also proved that transversal hypersurfaces of trans hyperbolic contact manifold with a quarter symmetric non metric connection admits a product structure.

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