# FRACTIONAL INTEGRALS FOR THE PRODUCT OF SRIVASTAVA'S POLYNOMIAL AND $(p, q)$–EXTENDED HYPERGEOMETRIC FUNCTION

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ABSTRACT. The main object of this paper is to present certain new image formulas for the product of general class of polynomial and $(p, q)$–extended Gauss's hypergeometric function by applying the Saigo-Maeda fractional integral operators involving Appell's function $F_3$. Certain interesting special cases of our main results are also considered.

Keywords: $(p, q)$–extended Beta function, $(p, q)$–extended Gauss's hypergeometric function, general class of polynomial, generalized fractional integral operators.

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## 1. INTRODUCTION AND PRELIMINARIES

Extensions, generalizations and unifications of Euler's Beta together with related higher transcendent hypergeometric type special functions were investigated recently by several authors, consult for instance (see, *e.g.*, [1], [2], [6], [20]) and for a very recent work (see also, [9], [10]). In particular, Chaudhry *et al.* [1, p. 20, Equation (1.7)] presented the following extension of the Beta function as:

$$\mathrm{B}(x, y; p) = \int_0^1 t^{x-1} (1-t)^{y-1} \mathrm{e}^{-\frac{p}{t(1-t)}} \mathrm{d}t, \qquad (\Re(p) > 0); \tag{1}$$

where for $p = 0$, $\min\{\Re(x), \Re(y)\} > 0$. They obtained related connections of $\mathrm{B}(x, y; p)$ with Macdonald (or modified Bessel function of the second kind), error and Whittaker

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functions. Further, Chaudhry *et al.* [2] used $B(x, y; p)$ to extend the Gaussian hypergeometric function in the following manner

$$F_p(a, b; c; z) = \sum_{n \geq 0}^{\infty} (a)_n \frac{B(b+n,\, c-b\,;\, p)}{B(b,\, c-b)} \frac{z^n}{n!} \tag{2}$$

$$(p \geq 0;\, For\, p = 0,\, |z| < 1\,;\, \Re(c) > \Re(b) > 0\,).$$

Recently, Choi *et al.* [3] introduce *further* extension of $B(x, y; p)$ and $F_p(a, b; c; z)$ in the following manner:

$$B(x,\, y\,;\, p,\, q) := \int_0^1 t^{x-1}(1-t)^{y-1} \, \exp\left(-\frac{p}{t} - \frac{q}{1-t}\right) dt \tag{3}$$

$$\big( \min\{\Re(x), \Re(y)\} > 0;\, \min\{\Re(p), \Re(q)\} \geq 0 \big)$$

and

$$F_{p,q}\,(a,\, b;\, c;\, z) := {}_2F_1\,(a,\, b;\, c;\, z\, p, q) = \sum_{n=0}^{\infty} (a)_n \frac{B(b+n,\, c-b\,;\, p, q)}{B(b,\, c-b)} \frac{z^n}{n!} \tag{4}$$

$$\big(p \geq 0,\, q \geq 0;\, For\, p = 0 = q,\, |z| < 1;\, \Re(c) > \Re(b) > 0\big).$$

The more general definitions of (3) and (4) have already discussed in [18]. Also, for our present investigation, we need the concept of Hadamard product (or convolution) of two analytic functions. It can help us in decomposing a newly emerged function into two known functions. If, in particular, one of the power series defines an entire function, then the Hadamard product series defines an entire function, too. Let

$$f(z) := \sum_{n=0}^{\infty} a_n z^n\, (|z| < R_f) \text{ and } g(z) := \sum_{n=0}^{\infty} b_n z^n\, (|z| < R_g)$$

be two given power series whose radii of convergence are given by $R_f$ and $R_g$, respectively. Then their Hadamard product is a power series defined by

$$(f * g)(z) := \sum_{n=0}^{\infty} a_n\, b_n z^n = (g * f)(z) \quad (|z| < R), \tag{5}$$

whose radius of convergence $R$ is

$$\frac{1}{R} = \limsup_{n \to \infty} (|a_n\, b_n|)^{\frac{1}{n}} \leq \left(\limsup_{n \to \infty} (|a_n|)^{\frac{1}{n}}\right) \left(\limsup_{n \to \infty} (|b_n|)^{\frac{1}{n}}\right) = \frac{1}{R_f \cdot R_g}$$

and so $R \geq R_f \cdot R_g$ (see [12]).

The general class of polynomials defined by Srivastava in the following manner [16, p. 1, Eq.(1)]:

$$S_w^u[x] = \sum_{s=0}^{[w/u]} \frac{(-w)_{u\,s}}{s!} A_{w,s}\, x^s \; w = 0, 1, 2, ... \tag{6}$$

where $u$ is an arbitrary positive integer and the coefficients $A_{w,s}(w, s) \geq 0$ are arbitrary constants, real or complex. The polynomial family $S_w^u[x]$ gives a number of known polynomials as its special cases on suitably specializing the coefficient $A_{w,s}$.

Recently, Parmar and Purohit [11] investigated certain fractional integral formulas involving Saigo operaters for the extended hypergeometric functions $F_{p,q}(z)$ given by Choi *et al.* [3]. Motivated by the above work, here we aim to establish certain new image formulas for the product of general class of polynomial and $(p, q)$–extended Gauss's hypergeometric function by applying the Saigo-Maeda fractional integral operators involving Appell's function $F_3$. Certain interesting special cases of our main results are also considered.

## 2. Fractional Integral approach

Fractional integral operators involving the various special functions have been actively investigated in various mathematical tools (see, *e.g.*, [5]). We recall here the Saigo and Maeda generalized fractional integral operators involving Appell function $F_3$ in the kernel [15, p. 293]:

$$\left(I_{0,+}^{\mu,\mu',\nu,\nu',\delta}f\right)(x) = \frac{x^{-\mu}}{\Gamma(\delta)}\int_0^x (x-t)^{\delta-1}\,t^{-\mu'}F_3\left(\mu,\mu',\nu,\nu';\delta;1-\frac{t}{x},1-\frac{x}{t}\right)f(t)dt \quad (7)$$

and

$$\left(I_{0,-}^{\mu,\mu',\nu,\nu',\delta}f\right)(x) = \frac{x^{-\mu'}}{\Gamma(\delta)}\int_x^\infty (t-x)^{\delta-1}\,t^{-\mu}F_3\left(\mu,\mu',\nu,\nu';\delta;1-\frac{x}{t},1-\frac{t}{x}\right)f(t)dt\,, \quad (8)$$

respectively. For the definition of the Appell function $F_3(.)$ the interested reader may refer to the monograph by Srivastava and Karlson [17] (see Erdélyi *et al.* [4] and Prudnikov *et al.*[13]). We begin by stating some image formulas regarding (7) and (8) which may be known formulas and are given in the following lemma [14].

**Lemma 2.1.** *Let* $\mu,\mu',\nu,\nu',\delta \in \mathrm{C}$ *and* $x > 0$. *Then*

**(a)** *If* $\Re(\rho) > max\,\{0,\Re(\mu+\mu'+\nu-\delta),\Re(\mu'-\nu')\}$ *and* $\Re(\delta) > 0$, *then*

$$\left(I_{0,+}^{\mu,\mu',\nu,\nu',\delta}x^{\rho-1}\right)(x) = x^{\rho-\mu-\mu'+\delta-1}\Gamma\left[\begin{array}{c} \rho,\,\rho+\delta-\mu-\mu'-\nu,\,\rho+\nu'-\mu' \\ \rho+\nu',\,\rho+\delta-\mu-\mu',\,\rho+\delta-\mu'-\nu \end{array}\right]. \quad (9)$$

**(b)** *If* $\Re(\rho) < 1 + min\,\{\Re(-\nu),\Re(\mu+\mu'-\delta),\Re(\mu+\nu'-\delta)\}$ *and* $\Re(\delta) > 0$, *then*

$$\left(I_{0,-}^{\mu,\mu',\nu,\nu',\delta}x^{\rho-1}\right)(x) = x^{\rho-\mu-\mu'+\delta-1}\Gamma\left[\begin{array}{c} 1-\rho-\nu,\,1-\rho-\delta+\mu+\mu',\,1-\rho+\mu+\nu'-\delta \\ 1-\rho,\,1-\rho+\mu+\mu'+\nu'-\delta,\,1-\rho+\mu-\nu \end{array}\right]. \quad (10)$$

The symbol occurring in (9) and (10) is given by

$$\Gamma\left[\begin{array}{ccc} a, & b, & c \\ d, & e, & f \end{array}\right] = \frac{\Gamma(a)\,\Gamma(b)\Gamma(c)}{\Gamma(d)\,\Gamma(e)\Gamma(f)}.$$

Now the composition formulas of generalized fractional integrals (9) and (10) involving the product of the general class of polynomial and generalized Gauss hypergeometric type functions $F_{p,q}(a,b;c;z)$ are given in Theorems 2.1 and 2.2.

**Theorem 2.1.** *Let* $\mu,\mu',\nu,\nu',\delta,\rho \in \mathrm{C}$ *be such that* $\min\{\Re(p),\Re(q)\} > 0$, $\Re(\delta) > 0$ *and* $\Re(\rho+s) > \max[0,\Re(\mu+\mu'+\nu-\delta),\Re(\mu'-\nu')]$. *Then for* $x > 0$

$$\left(I_{0+}^{\mu,\mu',\nu,\nu',\delta}\left[t^{\rho-1}S_w^u[\sigma t]\,F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};et\right]\right]\right)(x)] = x^{\rho-\mu-\mu'+\delta-1}\sum_{s=0}^{[w/u]}\frac{(-w)_{u\,s}}{s!}A_{w,s}(\sigma x)^s$$

$$\times \frac{\Gamma(\rho+s)\Gamma(\rho+\delta-\mu-\mu'-\nu+s)\Gamma(\rho+\nu'-\mu'+s)}{\Gamma(\rho+\delta-\mu-\mu'+s)\Gamma(\rho+\delta-\mu'-\nu+s)\Gamma(\rho+\nu'+s)}$$

$$\times\,F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};ex\right]*{}_4F_3\left[\begin{array}{c} 1,\rho+s,\rho+\delta-\mu-\mu'-\nu+s,\rho+\nu'-\mu'+s; \\ \rho+\delta-\mu-\mu'+s,\rho+\delta-\mu'-\nu+s,\rho+\nu'+s; \end{array}ex\right], \quad (11)$$

*where* $*$ *denotes the Hadamard product in* (5) *and whose left-sided hypergeometric fractional integral is assumed to be convergent.*

*Proof.* Let $\mathcal{L}$ be the left-hand side of (11). Applying (4) and (6) to (7) and changing the order of integration and summation, which is valid under the given conditions here, and using (9), we find

$$\mathcal{L} = \sum_{s=0}^{[w/u]} \sum_{n=0}^{\infty} \frac{(-w)_{u\,s}(\sigma)^s}{s!} A_{w,s}\,(a)_n \frac{\mathrm{B}_{p,q}(b+n,\,c-b)}{\mathrm{B}(b,\,c-b)} \frac{e^n}{n!} \left( I_{0+}^{\mu,\mu',\nu,\nu',\delta}\, t^{\rho+n+s-1} \right)(x)$$

$$= x^{\rho-\mu-\mu'+\delta-1} \sum_{s=0}^{[w/u]} \sum_{n=0}^{\infty} \frac{(-w)_{u\,s}(\sigma x)^s}{s!} A_{w,s}\,(a)_n \frac{\mathrm{B}_{p,q}(b+n,\,c-b)}{\mathrm{B}(b,\,c-b)} \qquad (12)$$

$$\times \frac{\Gamma(\rho+n+s)\Gamma(\rho+\delta-\mu-\mu'-\nu+n+s)\Gamma(\rho+\nu'-\mu'+n+s)}{\Gamma(\rho+\delta-\mu-\mu'+n+s)\Gamma(\rho+\delta-\mu'-\nu+n+s)\Gamma(\rho+\nu'+n+s)} \frac{(ex)^n}{n!},$$

Expressing the last summation in (12) in terms of the Hadamard product (5) with the functions (4) and generalized hypergeometric function, we obtain the right-hand side of (11).    $\square$

**Theorem 2.2.** *Let* $\mu, \mu', \nu, \nu', \delta, \rho \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(\delta) > 0$ *and* $\Re(\rho) < 1 + \min[\Re(-\nu), \Re(\mu+\mu'-\delta), \Re(\mu-\nu'-\delta)]$. *Then for* $x > 0$

$$\left( I_{-}^{\mu,\mu',\nu,\nu',\delta} \left[ t^{\rho-1} S_w^u[\sigma t]\, F_{p,q} \begin{bmatrix} a,b \\ c \end{bmatrix}; \frac{e}{t} \end{bmatrix} \right] \right)(x) = x^{\rho-\mu-\mu'+\delta-1} \sum_{s=0}^{[w/u]} \frac{(-w)_{u\,s}}{s!} A_{w,s}(\sigma x)^s$$

$$\times \frac{\Gamma(1+\mu+\mu'-\delta-\rho-s)\Gamma(1+\mu+\nu'-\delta-\rho-s)\Gamma(1-\nu-\rho-s)}{\Gamma(1-\rho-s)\Gamma(1+\mu+\mu'+\nu'-\delta-\rho-s)\Gamma(1+\mu-\nu-\rho-s)} \times F_{p,q} \begin{bmatrix} a,b \\ c \end{bmatrix}; \frac{e}{x} \end{bmatrix}$$

$$*\,_4F_3 \begin{bmatrix} 1, 1+\mu+\mu'-\delta-\rho-s, 1+\mu+\nu'-\eta-\rho-s, 1-\nu-\rho-s; & \frac{e}{x} \\ 1-\rho-s, 1+\mu+\mu'+\nu'-\delta-\rho-s, 1+\mu-\nu-\rho-s; & \frac{e}{x} \end{bmatrix},$$
(13)

*where* $*$ *denotes the Hadamard product in* (5) *and whose right-sided hypergeometric fractional integral is assumed to be convergent.*

*Proof.* Applying a similar argument as in the proof of 2.1 by using (4) and (6) to (8), and using (9), we obtain the right-hand side of (11).    $\square$

If we set $\mu' = \nu' = 0$, $\nu = -\eta$, $\mu = \mu+\nu$, $\delta = \mu$ in the operators (7) and (8), then we arrive at Saigo hypergeometric fractional integral operators (see, [5], Mathai *et al.* [8, p. 104]): For $\Re(\mu) > 0$,

$$(I_{0+}^{\mu,\nu,\eta} f(t))(x) = \frac{x^{-\mu-\nu}}{\Gamma(\mu)} \int_0^x (x-t)^{\mu-1}\,_2F_1 \left( \mu+\nu, -\eta; \mu; 1-\frac{t}{x} \right) f(t)\, dt \qquad (14)$$

and

$$(I_{-}^{\mu,\nu,\eta} f(t))(x) = \frac{1}{\Gamma(\mu)} \int_x^\infty (t-x)^{\mu-1} t^{-\mu-\nu}\,_2F_1 \left( \mu+\nu, -\eta; \mu; 1-\frac{x}{t} \right) f(t)\, dt. \qquad (15)$$

**Corollary 2.1.** *Let* $\mu, \nu, \eta, \rho \in \mathbb{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(\mu) > 0$ *and* $\Re(\rho) > \max\{0, \Re(\nu-\eta)\}$. *Then for* $x > 0$

$$\left( I_{0+}^{\mu,\nu,\eta} \left[ t^{\rho-1} S_w^u[\sigma t]\, F_{p,q} \begin{bmatrix} a,b \\ c \end{bmatrix}; et \end{bmatrix} \right] \right)(x) = x^{\rho-\nu-1} \sum_{s=0}^{[w/u]} \frac{(-w)_{u\,s}}{s!} A_{w,s}(\sigma x)^s$$

$$\times \frac{\Gamma(\rho+s)\Gamma(\rho-\nu+\eta+s)}{\Gamma(\rho+\mu+\eta+s)\Gamma(\rho-\nu+s)}$$

$$\times F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};ex\right] * {}_3F_2\left[\begin{array}{c} 1,\rho+s,\rho-\nu+\eta+s; \\ \rho-\nu+s,\rho+\mu+\eta+s; \end{array} ex\right]. \tag{16}$$

**Corollary 2.2.** *Let* $\mu,\nu,\eta,\rho \in C$ *be such that* $\min\{\Re(p),\Re(q)\} > 0$, $\Re(\mu) > 0$ *and* $\Re(\rho) < 1 + \min\{\Re(\eta),\Re(\nu)\}$. *Then for* $x > 0$

$$\left(I_-^{\mu,\nu,\eta}\left[t^{\rho-1}\,S_w^u[\sigma t]\,F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{t}\right]\right]\right)(x) = x^{\rho-\nu-1}\sum_{s=0}^{[w/u]}\frac{(-w)_{u\,s}}{s!}A_{w,s}(\sigma x)^s$$

$$\times \frac{\Gamma(1-\rho+\nu-s)\Gamma(1-\rho+\eta-s)}{\Gamma(1-\rho-s)\Gamma(1-\rho-\eta+\nu+\mu-s)}$$

$$\times F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{x}\right] * {}_3F_2\left[\begin{array}{c} 1,1-\rho+\nu-s,1-\rho+\eta-s; \\ 1-\rho-s,1-\rho+\mu+\nu-\eta-s; \end{array}\frac{e}{x}\right]. \tag{17}$$

The operator $I_{0+}^{\mu,\nu,\eta}(\cdot)$ contains both the Riemann-Liouville $I_{0+}^\mu(\cdot)$ and the Erdélyi-Kober $I_{\eta,\mu}^+(\cdot)$ fractional integral operators by means of the following relationships:

$$(I_{0+}^\mu f(t))(x) = (I_{0+}^{\mu,-\mu,\eta}f(t))(x) = \frac{1}{\Gamma(\mu)}\int_0^x (x-t)^{\mu-1}f(t)\,dt \tag{18}$$

and

$$(I_{\eta,\mu}^+ f(t))(x) = (I_{0+}^{\mu,0,\eta}f(t))(x) = \frac{x^{-\mu-\eta}}{\Gamma(\mu)}\int_0^x (x-t)^{\mu-1}t^\eta f(t)\,dt. \tag{19}$$

It is noted that the operator (15) unifies the Weyl type and the Erdélyi-Kober fractional operators as follows:

$$(I_-^\mu f(t))(x) = (I_-^{\mu,-\mu,\eta}f(t))(x) = \frac{1}{\Gamma(\mu)}\int_x^\infty (t-x)^{\mu-1}f(t)\,dt \tag{20}$$

and

$$(K_{\eta,\mu}^- f(t))(x) = (I_-^{\mu,0,\eta}f(t))(x) = \frac{x^\eta}{\Gamma(\mu)}\int_x^\infty (t-x)^{\mu-1}t^{-\mu-\eta}f(t)\,dt. \tag{21}$$

**Corollary 2.3.** *Let* $\mu,\eta,\rho \in C$ *be such that* $\min\{\Re(p),\Re(q)\} > 0$, $\Re(\mu) > 0$ *and* $\Re(\rho+s) > \Re(-\eta)$. *Then for* $x > 0$

$$\left(I_{\eta,\mu}^+\left[t^{\rho-1}\,S_w^u[\sigma t]\,F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};et\right]\right]\right)(x) = x^{\rho-1}\sum_{s=0}^{[w/u]}\frac{(-w)_{u\,s}}{s!}A_{w,s}(\sigma x)^s$$

$$\times \frac{\Gamma(\rho+\eta+s)}{\Gamma(\rho+\mu+s)}F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};ex\right] * {}_2F_1\left[\begin{array}{c} 1,\rho+\eta+s; \\ \rho+\mu+\eta+s; \end{array}ex\right]. \tag{22}$$

**Corollary 2.4.** *Let* $\mu,\eta,\rho \in C$ *be such that* $\min\{\Re(p),\Re(q)\} > 0$, $\Re(\mu) > 0$ *and* $\Re(\rho+s) < 1 + \Re(\eta)$. *Then for* $x > 0$

$$\left(K_{\eta,\mu}^-\left[t^{\rho-1}\,S_w^u[\sigma t]\,F_{p,q}\left[\begin{array}{c} a,b \\ c \end{array};\frac{e}{t}\right]\right]\right)(x) = x^{\rho-1}\sum_{s=0}^{[w/u]}\frac{(-w)_{u\,s}}{s!}A_{w,s}(\sigma x)^s$$

$$\times \frac{\Gamma(1-\rho+\eta-s)}{\Gamma(1-\rho-\eta+\mu-s)} F_{p,q} \left[ \begin{array}{c} a,b \\ c \end{array} ; \frac{e}{x} \right] * {}_2F_1 \left[ \begin{array}{c} 1, 1-\rho+\eta-s; \\ 1-\rho+\mu-\eta-s; \end{array} \frac{e}{x} \right]. \qquad (23)$$

Further, replacing $\nu$ by $-\mu$ in Corollary 2.1 and 2.2 and making use of the relations (18) and (20) gives the other Riemann-Liouville and Weyl fractional integrals of the extended hypergeometric function in (4) given by the following Corollaries.

**Corollary 2.5.** *Let* $\mu, \rho \in \mathrm{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$ *and* $\Re(\mu) > 0$. *Then for* $x > 0$

$$\left( I_{0+}^{\mu} \left[ t^{\rho-1} S_w^u[\sigma t] F_{p,q} \left[ \begin{array}{c} a,b \\ c \end{array} ; et \right] \right] \right)(x) = x^{\rho+\mu-1} \sum_{s=0}^{[w/u]} \frac{(-w)_{us}}{s!} A_{w,s} (\sigma x)^s$$

$$\times \frac{\Gamma(\rho+\mu+\eta+s)}{\Gamma(\rho+\mu+s)} F_{p,q} \left[ \begin{array}{c} a,b \\ c \end{array} ; ex \right] * {}_2F_1 \left[ \begin{array}{c} 1, \rho+s; \\ \rho+\mu+s; \end{array} ex \right]. \qquad (24)$$

**Corollary 2.6.** *Let* $\mu, \rho \in \mathrm{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$ *and* $\Re(\mu) > 0$. *Then for* $x > 0$

$$\left( I_{-}^{\mu} \left[ t^{\rho-1} S_w^u[\sigma t] F_{p,q} \left[ \begin{array}{c} a,b \\ c \end{array} ; \frac{e}{t} \right] \right] \right)(x) = x^{\rho+\mu-1} \sum_{s=0}^{[w/u]} \frac{(-w)_{us}}{s!} A_{w,s} (\sigma x)^s$$

$$\times \frac{\Gamma(1-\rho-\mu-s)}{\Gamma(1-\rho-s)} \frac{\Gamma(1-\rho+\eta-s)}{\Gamma(1-\rho-\eta-s)}$$

$$F_{p,q} \left[ \begin{array}{c} a,b \\ c \end{array} ; \frac{e}{x} \right] * {}_3F_2 \left[ \begin{array}{c} 1, 1-\rho-\mu-s, 1-\rho+\eta-s; \\ 1-\rho-s, 1-\rho-\eta-s; \end{array} \frac{e}{x} \right]. \qquad (25)$$

### 3. Concluding remark and observations

We conclude this paper by emphasizing that on giving suitable special values to the coefficient $A_{w,s}$, the general class of polynomial gives many known classical orthogonal polynomial as its particular cases. In Particular, if we set $w = 0$, $A_{0,0} = 1$ then $S_w^u = 1$ in (11) and (13), we obtain new results asserted in Corollary 3.1 and 3.2.

**Corollary 3.1.** *Let* $\mu, \mu', \nu, \nu', \delta, \rho \in \mathrm{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(\delta) > 0$ *and* $\Re(\rho+s) > \max[0, \Re(\mu+\mu'+\nu-\delta), \Re(\mu'-\nu')]$. *Then for* $x > 0$

$$\left( I_{0+}^{\mu,\mu',\nu,\nu',\delta} \left[ t^{\rho-1} F_{p,q} \left[ \begin{array}{c} a,b \\ c \end{array} ; et \right] \right] \right)(x)]$$

$$= x^{\rho-\mu-\mu'+\delta-1} \frac{\Gamma(\rho+s)\Gamma(\rho+\delta-\mu-\mu'-\nu+s)\Gamma(\rho+\nu'-\mu'+s)}{\Gamma(\rho+\delta-\mu-\mu'+s)\Gamma(\rho+\delta-\mu'-\nu+s)\Gamma(\rho+\nu'+s)}$$

$$\times F_{p,q} \left[ \begin{array}{c} a,b \\ c \end{array} ; ex \right] * {}_4F_3 \left[ \begin{array}{c} 1, \rho+s, \rho+\delta-\mu-\mu'-\nu+s, \rho+\nu'-\mu'+s; \\ \rho+\delta-\mu-\mu'+s, \rho+\delta-\mu'-\nu+s, \rho+\nu'+s; \end{array} ex \right], \qquad (26)$$

**Corollary 3.2.** *Let* $\mu, \mu', \nu, \nu', \delta, \rho \in \mathrm{C}$ *be such that* $\min\{\Re(p), \Re(q)\} > 0$, $\Re(\delta) > 0$ *and* $\Re(\rho) < 1 + \min[\Re(-\nu), \Re(\mu+\mu'-\delta), \Re(\mu-\nu'-\delta)]$. *Then for* $x > 0$

$$\left( I_{-}^{\mu,\mu',\nu,\nu',\delta} \left[ t^{\rho-1} S_w^u[\sigma t] F_{p,q} \left[ \begin{array}{c} a,b \\ c \end{array} ; \frac{e}{t} \right] \right] \right)(x)$$

$$= x^{\rho-\mu-\mu'+\delta-1} \frac{\Gamma(1+\mu+\mu'-\delta-\rho-s)\Gamma(1+\mu+\nu'-\delta-\rho-s)\Gamma(1-\nu-\rho-s)}{\Gamma(1-\rho-s)\Gamma(1+\mu+\mu'+\nu'-\delta-\rho-s)\Gamma(1+\mu-\nu-\rho-s)}$$

$$\times F_{p,q} \left[ \begin{array}{cc} a,b & ; \dfrac{e}{x} \\ c & \end{array} \right] * {}_4F_3 \left[ \begin{array}{c} 1, 1+\mu+\mu'-\delta-\rho-s, 1+\mu+\nu'-\eta-\rho-s, 1-\nu-\rho-s; \\ 1-\rho-s, 1+\mu+\mu'+\nu'-\delta-\rho-s, 1+\mu-\nu-\rho-s; \end{array} \dfrac{e}{x} \right],$$

$$(27)$$

Also, it is interesting to observe that if we set $w = 0$, $A_{0,0} = 1$ and $S_w^u = 1$, the results obtained in Corollaries 2.1 to 2.6, yield corresponding results given in [11]. Further, the polynomial family $S_w^u[x]$ gives a number of known polynomials as its special cases on suitably specializing the coefficient $A_{w,s}$. These include Hermite, Leguerre, Jacobi, the Konhauser polynomials and so on. If we set $u = 2$ and $A_{w,s} = (-1)^s$, then the general class of polynomials reduce to

$$S_2^w[x] \longrightarrow x^{u/2} H_w \left( \frac{1}{2\sqrt{x}} \right) \tag{28}$$

where $H_w(x)$ denotes the well known Hermite polynomials, and defined by

$$H_w(x) = \sum_{s=0}^{[w/2]} (-1)^s \frac{w}{s!(w-2s)!} (2x)^{w-2s}. \tag{29}$$

We left the results for interested readers.

**Conflict of Interests**

The authors declare that there is no conflict of interests regarding the publication of this paper.

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