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COMMON FIXED POINT RESULTS FOR W-COMPATIBLE MAPPINGS ALONG WITH (CLR_{ST}) PROPERTY IN FUZZY METRIC SPACES

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Abstract. In this paper, we extend the concepts of common property (E.A) and (CLR_{ST}) property for problems in coupled fixed point theory. Employing these notions, we prove some common fixed point results for the pairs of w-compatible mappings subjected to ϕ – contractions in fuzzy metric spaces and generalize and extend some results present in the literature.

Keywords: Common property (E.A.); (CLR_{ST}) property; metric space; fuzzy metric space; w-compatible mappings; common fixed points

AMS Subject Classification: 47H10, 54H25

1. Introduction and preliminaries

In 1965, the introduction of fuzzy sets by Zadeh [23] proved a turning point in the galaxy of mathematics and logics. The study of the notion of distance for fuzzy situation has been stimulated by various authors in distinct ways. In 1975, the concept of fuzzy metric spaces introduced by Kramosil and Michalek [16] opened an avenue for the further development of analysis in these spaces. Later, George and Veeramani [9] modified the concept of fuzzy metric spaces initiated by Kramosil and Michalek [16] with a view to obtain the Hausdorff topology in such spaces. The present study deals with the fuzzy metric spaces due to George and Veeramani [9].

Guo and Lakshmikantham [7] studied the notion of coupled fixed points and initiated the theory of coupled fixed points. Since then, this notion has attracted the interest of various researchers. Among them, the work of Bhaskar and Lakshmikantham [4] is worth mentioning here, as in their work, they proved some coupled fixed point results in ordered metric spaces. These results were further extended and generalized by Lakshmikantham and Ćirić [17] for a pair of commutative maps in ordered metric spaces.

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Now-a-days, the problems concerning the computation of coupled fixed points are given fuzzy treatment. Sedghi et al. [20] proved a coupled fixed point theorem under contractive condition in fuzzy metric spaces. Hu [11] proved a common fixed point theorem for a pair of compatible mappings under a ϕ-contraction in fuzzy metric spaces. Subsequently, coupled fixed point problems for ϕ-contractions in fuzzy metric spaces were discussed rapidly by various authors.

In 2002, Aamri and El-Moutawakil [1] introduced the concept of (E.A.) property for self mappings which contained the class of non-compatible mappings in metric spaces. On the other hand, Sintunavarat and Kumam [21] introduced a new notion of "common limit in the range" property (or, (CLR) property). Liu et al. [18] extended (E.A.) property to common property (E.A.) for a pair of single- and multivalued maps in metric spaces. Later, the common property (E.A.) was studied by Abbas et al. [2] in fuzzy metric spaces for pairs of self mappings. Chauhan et al. [5] extended the (CLR) property to "joint common limit in the range" property (or, (JCLR) property) of mappings and proved a common fixed point theorem for the pairs of weakly compatible mappings using (JCLR) property in fuzzy metric spaces. On the other hand, Chauhan [6] extended (CLR) property from single pair of self mappings to two pairs of self mappings and introduced "common limit in the range of mappings S and T" property (or, (CLR$_{ST}$) property) in fuzzy metric spaces. Recently, Jain et al. [14] studied the concepts of (E.A.) property and (CLR) property in coupled fixed point theory.

In this paper, we define the concepts of common property (E.A) and (CLR$_{ST}$) property for the problems in coupled fixed point theory in the setup of metric spaces as well as in fuzzy metric spaces and prove a result which generalize the recent works of Hu [11] and Jain et al. [15] and extends the work of Jain et al. [14]. As application, some results in metric space are also established.

Next, we state some allied definitions useful to develop our study.

Definition 1.1 ([23])**.** A fuzzy set A in X is a function with domain X and values in $[0,1]$.

Definition 1.2 ([22])**.** A binary operation $*:[0,1]\times[0,1]\to[0,1]$ is a continuous t-norm if $*$ satisfies the following conditions:

(1) $*$ is associative and commutative,
(2) $*$ is continuous,
(3) $a*1=a$ for all $a\in[0,1]$,
(4) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Some examples of the continuous t-norm are $a*_1 b = ab$ and $a*_2 b=\min\{a,b\}$ for all $a,b\in[0,1]$.

Definition 1.3 ([13])**.** Let $\sup\limits_{0<t<1}\Delta(t,t)=1$. A t-norm Δ is said to be Hadžić type t-norm (in short, H-type t-norm), if the family of functions $\{\Delta^m(t)\}_{m=1}^{\infty}$ is equicontinuous at $t=1$, where

$$\Delta^1(t)=t,\ \Delta^{m+1}(t)=t\Delta(\Delta^m(t)),\ \text{for } t\in[0,1] \text{ and } m=1,2,\ldots.$$

A t-norm Δ is a H-type t-norm iff for any $\lambda\in(0,1)$, there exists $\delta(\lambda)\in(0,1)$ such that $\Delta^m(t)>(1-\lambda)$ for all $m\in N$, when $t>(1-\delta)$.
Clearly, $*_2$ is an example of t-norm of H-type.

Definition 1.4 ([9])**.** The 3-tuple $(X,M,*)$ is called a fuzzy metric space (in the sense of GV), if X is an arbitrary non-empty set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times(0,\infty)$ satisfying the following conditions for each $x,y,z\in X$ and $t,s>0$:

(FM-1) $M(x, y, t) > 0$,
(FM-2) $M(x, y, t) = 1$ if and only if $x = y$,
(FM-3) $M(x, y, t) = M(y, x, t)$,
(FM-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,
(FM-5) $M(x, y, \bullet) : (0, \infty) \to [0, 1]$ is continuous.

Definition 1.5 ([9]). Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}$ in X is said to be

(i) convergent to a point $x \in X$, if $\lim_{n\to\infty} M(x_n, x, t) = 1$, for all $t > 0$; that is, for each $0 < \varepsilon < 1$ and $t > 0$, there exists a positive integer n_0 such that $M(x_n, x, t) > 1 - \varepsilon$ for each $n \geq n_0$;
(ii) Cauchy sequence if for each $0 < \varepsilon < 1$ and $t > 0$, there exists a positive integer n_0 such that $M(x_n, x_m, t) > 1 - \varepsilon$ for each $n, m \geq n_0$;
(iii) a fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Lemma 1.1 ([8]). *Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, \bullet)$ is non-decreasing for all $x, y \in X$.*

Lemma 1.2 ([19]). *Let $(X, M, *)$ be a fuzzy metric space. Then M is a continuous function on $X^2 \times (0, \infty)$.*

Definition 1.6 ([11]). Define $\Phi = \{\phi : R^+ \to R^+\}$, where $R^+ = [0, +\infty)$ and each $\phi \in \Phi$ satisfies the following conditions:

(ϕ-1) ϕ is non-decreasing;
(ϕ-2) ϕ is upper semicontinuous from the right;
(ϕ-3) $\sum_{n=0}^{\infty} \phi^n(t) < +\infty$ for all $t > 0$, where $\phi^{n+1}(t) = \phi(\phi^n(t))$, $n \in N$.

Clearly, if $\phi \in \Phi$, then $\phi(t) < t$ for all $t > 0$.

Definition 1.7 ([4, 7]). *v An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.*

Definition 1.8 ([17]). An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mappings $F : X \times X \to X$ and $g : X \to X$ if $F(x, y) = g(x)$ and $F(y, x) = g(y)$.

Definition 1.9 ([15]). An element $(x, y) \in X \times X$ is called a coupled common fixed point of the mappings $A : X \times X \to X$, $B : X \times X \to X$, $S : X \to X$ and $T : X \to X$ if $A(a, b) = S(a) = a = T(a) = B(a, b)$ and $A(b, a) = S(b) = b = T(b) = B(b, a)$.

Definition 1.10 ([15]). An element $x \in X$ is called a common fixed point of the mappings $A : X \times X \to X$, $B : X \times X \to X$, $S : X \to X$ and $T : X \to X$ if $A(a, a) = B(a, a) = S(a) = T(a) = a$.

Definition 1.11 ([17]). The mappings $F : X \times X \to X$ and $g : X \to X$ are said to be commutative if $F(g(x), g(y)) = g(F(x, y))$ for all $x, y \in X$.

Definition 1.12 ([11]). The mappings $F : X \times X \to X$ and $g : X \to X$ are said to be compatible if

$$\lim_{n\to\infty} M(g(F(x_n, y_n)), F(g(x_n), g(y_n)), t) = 1, \quad \lim_{n\to\infty} M(g(F(y_n, x_n)), F(g(y_n), g(x_n)), t) = 1,$$

for all $t > 0$ whenever $\{x_n\}$ and $\{y_n\}$ are sequences in X, such that $\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x$, $\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y$ for some $x, y \in X$.

Using the pair of compatible mappings, Hu [11] gave the following common fixed point result:

Theorem 1.1 ([11])**.** *Let $(X, M, *)$ be a complete fuzzy metric space, $*$ being continuous t-norm of H-type and $M(x, y, t) \to 1$ as $t \to \infty$. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings and there exists $\phi \in \Phi$ such that*

(1.1) $M(F(x, y), F(u, v), \phi(t)) \geq M(gx, gu, t) * M(gy, gv, t)$, *for all x, y, u, v in X and $t > 0$.*

Suppose that $F(X \times X) \subseteq g(X)$ and g is continuous, F and g are compatible. Then, there exists a unique x in X such that $x = g(x) = F(x, x)$.

Recently, Abbas et al. [3], introduced the concept of w-compatible mappings in order to generalize the concepts of commutative and compatible mappings, following which, some authors established coupled common fixed point results for the similar notion of weakly compatible mappings. Works noted in [12, 14, 15] are some examples in this direction.

Definition 1.13 ([3])**.** The mappings $F : X \times X \to X$ and $g : X \to X$ are said to be w-compatible if $g(F(x, y)) = F(g(x), g(y))$, whenever $g(x) = F(x, y)$ and $g(y) = F(y, x)$.

Definition 1.14 ([12, 14, 15])**.** The mappings $F : X \times X \to X$ and $g : X \to X$ are said to be weakly compatible if $g(F(x, y)) = F(g(x), g(y))$ and $g(F(y, x)) = F(g(y), g(x))$, whenever $g(x) = F(x, y)$ and $g(y) = F(y, x)$.

Interestingly, the concepts of w-compatible mappings and weakly compatible mappings are equivalent.
In order to generalize Theorem 1.1, Jain et al. [15] proved the following result for the pairs of weakly compatible mappings without using the continuity hypothesis of any of the mappings involved.

Theorem 1.2 ([15])**.** *Let $(X, M, *)$ be a fuzzy metric space, $*$ being continuous t-norm of H-type and $M(x, y, t) \to 1$ as $t \to \infty$. Let $A : X \times X \to X$, $B : X \times X \to X$, $S : X \to X$, $T : X \to X$ be four mappings satisfying the following conditions:*

(1.2) $A(X \times X) \subseteq T(X)$, $B(X \times X) \subseteq S(X)$,

(1.3) *there exists $\phi \in \Phi$ such that*
$M(A(x, y), B(u, v), \phi(t)) \geq M(Sx, Tu, t) * M(Sy, Tv, t)$, *for all x, y, u, v in X and $t > 0$,*

(1.4) *the pairs (A, S) and (B, T) are weakly compatible,*

(1.5) *one of the subspaces $A(X \times X)$ or $T(X)$ and one of $B(X \times X)$ or $S(X)$ are complete.*

Then, there exists a unique point a in X such that $A(a, a) = S(a) = a = T(a) = B(a, a)$.

On the other hand, Jain et al. [14] defined the notions of the (E.A.) property and (CLRg) property and used these notions to generalize Theorem 1.1.

Definition 1.15 ([14])**.** Let (X, d) be a metric space. Two mappings $F : X \times X \to X$ and $g : X \to X$ are said to satisfy (E.A.) property if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = x$, $\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = y$ for some x, y in X.

Definition 1.16 ([14])**.** Let (X, d) be a metric space. Two mappings $F : X \times X \to X$ and $g : X \to X$ are said to satisfy (CLRg) property if there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that
$\lim_{n\to\infty} F(x_n, y_n) = \lim_{n\to\infty} g(x_n) = g(p)$, $\lim_{n\to\infty} F(y_n, x_n) = \lim_{n\to\infty} g(y_n) = g(q)$ for some p, q in X.

Jain et al. [14] proved the following result to generalize Theorem 1.1:

Theorem 1.3 ([14])**.** *Let $(X, M, *)$ be a fuzzy metric space, $*$ being continuous t-norm of H-type and $M(x, y, t) \to 1$ as $t \to \infty$. Let $F : X \times X \to X$ and $g : X \to X$ be two mappings and there exists $\phi \in \Phi$ satisfying* (1.1) *with the following conditions:*

(1.6) *the pair (F, g) is weakly compatible,*
(1.7) *the pair (F, g) satisfy (CLRg) property.*

Then F and g have a coupled coincidence point in X. Moreover, there exists a unique point x in X such that $x = F(x, x) = g(x)$.

In the next section, we first define the common property (E.A.) and (CLR$_{ST}$) property for coupled fixed point problems in metric and fuzzy metric spaces and then utilize these new notions to generalize Theorem 1.2 and to extend Theorem 1.3 for the pairs of mappings.

2. Main Results

Recently, Gupta et al. [10] formulated common property (E.A.) in modified intuitionistic fuzzy metric spaces. We first give our metrical as well the fuzzy version of this notion as follows:

Definition 2.1. Let (X, d) be a metric space and $A, B : X \times X \to X$ and $S, T : X \to X$ be the mappings. Then the pairs (A, S) and (B, T) are said to share the common property (E.A.), if there exist sequences $\{x_n\}$, $\{y_n\}$ and $\{p_n\}$, $\{q_n\}$ in X such that
$\lim_{n\to\infty} A(x_n, y_n) = \lim_{n\to\infty} B(p_n, q_n) = \lim_{n\to\infty} T(p_n) = \lim_{n\to\infty} S(x_n) = \alpha$,
$\lim_{n\to\infty} A(y_n, x_n) = \lim_{n\to\infty} B(q_n, p_n) = \lim_{n\to\infty} T(q_n) = \lim_{n\to\infty} S(y_n) = \beta$, for some α, β in X.
We now extend the above notion of common property (E.A.) in fuzzy metric spaces as follows:
"Let $(X, M, *)$ be a FM space and $A, B : X \times X \to X$ and $S, T : X \to X$ be the mappings. Then the pairs (A, S) and (B, T) are said to share the common property (E.A.), if there exists sequences $\{x_n\}$, $\{y_n\}$ and $\{p_n\}$, $\{q_n\}$ in X such that $A(x_n, y_n)$, $B(p_n, q_n)$, $T(p_n)$, $S(x_n)$ converges to α and $A(y_n, x_n)$, $B(q_n, p_n)$, $T(q_n)$, $S(y_n)$ converges to β for some $\alpha, \beta \in X$, in the sense of Definition 1.5".
Clearly, on taking $A = B = F$ and $S = T = g$ in Definition 2.1, we obtain Definition 1.15.

Next, we define our notion of (CLR$_{ST}$) property in coupled fixed point theory.

Definition 2.2. Let (X, d) be a metric space and $A, B : X \times X \to X$ and $S, T : X \to X$ be the mappings. Then the pairs (A, S) and (B, T) are said to satisfy (CLR$_{ST}$) property, if there exist sequences $\{x_n\}$, $\{y_n\}$ and $\{p_n\}$, $\{q_n\}$ in X such that

$$\lim_{n\to\infty} A(x_n, y_n) = \lim_{n\to\infty} B(p_n, q_n) = \lim_{n\to\infty} T(p_n) = \lim_{n\to\infty} S(x_n) = \alpha,$$
$$\lim_{n\to\infty} A(y_n, x_n) = \lim_{n\to\infty} B(q_n, p_n) = \lim_{n\to\infty} T(q_n) = \lim_{n\to\infty} S(y_n) = \beta,$$

for some $\alpha, \beta \in S(X) \cap T(X)$.

We now extend the above notion of (CLR$_{ST}$) property in fuzzy metric spaces as follows:
"Let $(X, M, *)$ be a FM space and $A, B : X \times X \to X$ and $S, T : X \to X$ be the mappings. The pair (A, S) and (B, T) are said to satisfy (CLR$_{ST}$) property, if there exists sequences $\{x_n\}$, $\{y_n\}$ and $\{p_n\}$, $\{q_n\}$ in X such that $A(x_n, y_n)$, $B(p_n, q_n)$, $T(p_n)$, $S(x_n)$ converges to α and $A(y_n, x_n)$, $B(q_n, p_n)$, $T(q_n)$, $S(y_n)$ converges to β for some $\alpha, \beta \in S(X) \cap T(X)$, in the sense of Definition 1.5".
On taking $A = B = F$ and $S = T = g$ in Definition 2.2, we obtain Definition 1.16.
Now, we are ready to give our main result as follows:

Theorem 2.1. *Let $(X, M, *)$ be a fuzzy metric space, $*$ being continuous t-norm of H-type and $M(x, y, t) \to 1$ as $t \to \infty$. Let $A, B : X \times X \to X$ and $S, T : X \to X$ be the mappings satisfying the following conditions:*

(2.1) *there exists $\phi \in \Phi$, such that*
$M(A(x, y), B(u, v), \phi(t)) \geq M(S(x), T(u), t) * M(S(y), T(v), t)$, *for all $x, y, u, v \in X$ and $t > 0$;*

(2.2) *the pairs (A, S) and (B, T) share the (CLR_{ST}) property;*

(2.3) *the pairs (A, S) and (B, T) are w-compatible.*

Then, the mappings A, B, S, T have a unique common fixed point in X.

Proof. Since the pairs (A, S) and (B, T) share the (CLR$_{ST}$) property, there exist sequences $\{x_n\}$, $\{y_n\}$ and $\{p_n\}$, $\{q_n\}$ in X such that

(2.4) $\lim_{n\to\infty} A(x_n, y_n) = \lim_{n\to\infty} B(p_n, q_n) = \lim_{n\to\infty} S(x_n) = \lim_{n\to\infty} T(p_n) = a$,

(2.5) $\lim_{n\to\infty} A(y_n, x_n) = \lim_{n\to\infty} B(q_n, p_n) = \lim_{n\to\infty} S(y_n) = \lim_{n\to\infty} T(q_n) = b$,

for some $a, b \in S(X) \cap T(X)$. Then, there exist some $p, q, r, s \in X$ such that $S(r) = a = T(p)$, $S(s) = b = T(q)$. The proof is divided into following steps:
Step 1. We show that $A(a, b) = S(a)$, $A(b, a) = S(b)$ and $B(a, b) = T(a)$, $B(b, a) = T(b)$. Since $\phi \in \Phi$, we have $\phi(t) < t$ for all $t > 0$. Then, using (2.1), for $t > 0$, we have

$$M(A(x_n, y_n), B(p, q), t) \geq M(A(x_n, y_n), B(p, q), \phi(t)) \geq M(S(x_n), T(p), t) * M(S(y_n), T(q), t).$$

Letting $n \to \infty$ in the last inequality, we obtain for $t > 0$, that
$M(a, B(p, q), t) \geq M(a, T(p), t) * M(b, T(q), t) = M(a, a, t) * M(b, b, t) = 1 * 1 = 1$, that is, $M(a, B(p, q), t) = 1$ and hence, $B(p, q) = a$. Therefore, $B(p, q) = a = T(p)$. Similarly, we can show that $B(q, p) = b = T(q)$.
Again, using (2.1), for $t > 0$, we have
$M(A(r, s), B(p_n, q_n), t) \geq M(A(r, s), B(p_n, q_n), \phi(t)) \geq M(S(r), T(p_n), t) * M(S(s), T(q_n), t)$.
Letting $n \to \infty$ in the last inequality, we obtain for $t > 0$, that
$M(A(r, s), a, t) \geq M(S(r), a, t) * M(S(s), b, t) = M(a, a, t) * M(b, b, t) = 1 * 1 = 1$, that is, $M(a, A(r, s), t) = 1$ and hence, $A(r, s) = a$. Therefore, $A(r, s) = a = S(r)$. Similarly, we can obtain that $A(s, r) = b = S(s)$.
Now, since the pair (B, T) is w-compatible, so that $B(p, q) = a = T(p)$ and $B(q, p) = b = T(q)$ implies that $B(T(p), T(q)) = T(B(p, q))$ and $B(T(q), T(p)) = T(B(q, p))$, that is, $B(a, b) = T(a)$ and $B(b, a) = T(b)$.
Also, since the pair (A, S) is w-compatible, so that $A(r, s) = a = S(r)$ and $A(s, r) = b = S(s)$ implies that $A(a, b) = S(a)$ and $A(b, a) = S(b)$.
Step 2. We next show that $A(a, b) = S(a) = a = T(a) = B(a, b)$ and $A(b, a) = S(b) = b = T(b) = B(b, a)$.
Since $*$ is a t-norm of H-type, for $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta) * (1-\delta) * \dots (1-\delta)}_{p} \geq (1-\varepsilon), \text{ for all } p \in N.$$

Also, since $\lim_{n\to\infty} M(x, y, t) = 1$ for $x, y \in X$, there exists $t_0 > 0$, such that $M(a, T(a), t_0) \geq (1-\delta)$ and $M(b, T(b), t_0) \geq (1-\delta)$. By condition ($\phi$-3), we have $\sum_{n=0}^{\infty} \phi^n(t_0) < +\infty$, then for any $t > 0$, there exists $n_0 \in N$ such that $t > \sum_{k=n_0}^{\infty} \phi^k(t_0)$.

By (2.1), we have

$$\begin{aligned} M(a,T(a),\phi(t_0)) &= M(A(r,s),B(a,b),\phi(t_0)) \geq M(S(r),T(a),t_0) * M(S(s),T(b),t_0) \\ &= M(a,T(a),t_0) * M(b,T(b),t_0). \end{aligned}$$

Similarly, by (2.1), we have

$$\begin{aligned} M(b,T(b),\phi(t_0)) &= M(A(s,r),B(b,a),\phi(t_0)) \geq M(S(s),T(b),t_0) * M(S(r),T(a),t_0) \\ &= M(b,T(b),t_0) * M(a,T(a),t_0). \end{aligned}$$

Now,

$$\begin{aligned} M(a,T(a),\phi^2(t_0)) &= M(a,T(a),\phi(\phi(t_0))) \geq M(a,T(a),\phi(t_0)) * M(b,T(b),\phi(t_0)) \\ &\geq [M(a,T(a),t_0) * M(b,T(b),t_0)] * [M(a,T(a),t_0) * M(b,T(b),t_0)] \\ &= [M(a,T(a),t_0)]^2 * [M(b,T(b),t_0)]^2. \end{aligned}$$

Similarly, we can obtain that

$$M(b,T(b),\phi^2(t_0)) \geq [M(a,T(a),t_0)]^2 * [M(b,T(b),t_0)]^2.$$

Also,

$$\begin{aligned} M(a,T(a),\phi^3(t_0)) &= M(a,T(a),\phi(\phi^2(t_0))) \geq M(a,T(a),\phi^2(t_0)) * M(b,T(b),\phi^2(t_0)) \\ &\geq [M(a,T(a),t_0)]^2 * [M(b,T(b),t_0)]^2 * [M(a,T(a),t_0)]^2 * [M(b,T(b),t_0)]^2 \\ &= [M(a,T(a),t_0)]^4 * [M(b,T(b),t_0)]^4. \end{aligned}$$

Similarly, we can obtain that

$$M(b,T(b),\phi^3(t_0)) \geq [M(a,T(a),t_0)]^4 * [M(b,T(b),t_0)]^4 .$$

Again,

$$\begin{aligned} M(a,T(a),\phi^4(t_0)) &= M(a,T(a),\phi(\phi^3(t_0))) \geq M(a,T(a),\phi^3(t_0)) * M(b,T(b),\phi^3(t_0)) \\ &\geq [M(a,T(a),t_0)]^4 * [M(b,T(b),t_0)]^4 * [M(a,T(a),t_0)]^4 * [M(b,T(b),t_0)]^4 \\ &= [M(a,T(a),t_0)]^8 * [M(b,T(b),t_0)]^8. \end{aligned}$$

And similarly, we can obtain that

$$M(b,T(b),\phi^4(t_0)) \geq [M(a,T(a),t_0)]^8 * [M(b,T(b),t_0)]^8.$$

In general, for $n \geq 1$, we obtain that

$$M(a,T(a),\phi^n(t_0)) \geq [M(a,T(a),t_0)]^{2^{n-1}} * [M(b,T(b),t_0)]^{2^{n-1}}$$

and

$$M(b,T(a),\phi^n(t_0)) \geq [M(a,T(a),t_0)]^{2^{n-1}} * [M(b,T(b),t_0)]^{2^{n-1}}.$$

Then, for $t > 0$, we have

$$\begin{aligned} M(a,T(a),t) &\geq M(a,T(a),\sum_{k=n_0}^{\infty}\phi^k(t_0)) \geq M(a,T(a),\phi^{n_0}(t_0)) \\ &\geq [M(a,T(a),t_0)]^{2^{n_0-1}} * [M(b,T(b),t_0)]^{2^{n_0-1}} \\ &\geq \underbrace{(1-\delta) * (1-\delta) * \cdots * (1-\delta)}_{2^{n_0}} \geq (1-\varepsilon). \end{aligned}$$

Also,

$$\begin{aligned} M(b,T(b),t) &\geq M(b,T(b),\sum_{k=n_0}^{\infty}\phi^k(t_0)) \geq M(b,T(b),\phi^{n_0}(t_0)) \\ &\geq [M(a,T(a),t_0)]^{2^{n_0-1}} * [M(b,T(b),t_0)]^{2^{n_0-1}} \\ &\geq \underbrace{(1-\delta)*(1-\delta)*\cdots*(1-\delta)}_{2^{n_0}} \geq (1-\varepsilon). \end{aligned}$$

Therefore, for $\varepsilon > 0$, we have $M(a,T(a),t) \geq (1-\varepsilon)$ and $M(b,T(b),t) \geq (1-\varepsilon)$ for all $t > 0$, so that we have $T(a) = a$ and $T(b) = b$. Similarly, can obtain that $S(a) = a$ and $S(b) = b$. Therefore, we have $A(a,b) = S(a) = a = T(a) = B(a,b)$ and $A(b,a) = S(b) = b = T(b) = B(b,a)$.

Step 3. We assert that $a = b$.

Since $*$ is a t-norm of H-type, for $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta)*(1-\delta)*\ldots(1-\delta)}_{p} \geq (1-\varepsilon), \quad \text{for all } p \in N.$$

Also, since $\lim_{n\to\infty} M(x,y,t) = 1$ for $x,y \in X$, there exists $t_0 > 0$, such that $M(a,b,t_0) \geq (1-\delta)$. By condition ($\phi$-3), we have $\sum_{n=0}^{\infty}\phi^n(t_0) < +\infty$, then for any $t > 0$, there exists $n_0 \in N$ such that $t > \sum_{k=n_0}^{\infty}\phi^k(t_0)$. By (2.1), we have

$$\begin{aligned} M(a,b,\phi(t_0)) &= M(A(a,b),B(b,a),\phi(t_0)) \geq M(S(a),T(b),t_0) * M(S(b),T(a),t_0) \\ &= M(a,b,t_0) * M(a,b,t_0) = [M(a,b,t_0)]^2. \end{aligned}$$

Also,

$$\begin{aligned} M(a,b,\phi^2(t_0)) &= M(A(a,b),B(b,a),\phi(\phi(t_0))) \geq M(S(a),T(b),\phi(t_0)) * M(S(b),T(a),\phi(t_0)) \\ &= M(a,b,\phi(t_0)) * M(a,b,\phi(t_0)) = [M(a,b,\phi(t_0))]^2 \geq [M(a,b,t_0)]^4 \end{aligned}$$

Again,

$$\begin{aligned} M(a,b,\phi^3(t_0)) &= M(A(a,b),B(b,a),\phi(\phi^2(t_0))) \geq M(S(a),T(b),\phi^2(t_0)) * M(S(b),T(a),\phi^2(t_0)) \\ &= M(a,b,\phi^2(t_0)) * M(a,b,\phi^2(t_0)) = [M(a,b,\phi^2(t_0))]^2 \geq [M(a,b,t_0)]^8 \end{aligned}$$

In general, for $n \geq 1$, we have $M(a,b,\phi^n(t_0)) \geq [M(a,b,t_0)]^{2^n}$.

Now, for $\varepsilon > 0$ and $t > 0$, we have

$$\begin{aligned} M(a,b,t) &\geq M(a,b,\sum_{k=n_0}^{\infty}\phi^k(t_0)) \geq M(a,b,\phi^{n_0}(t_0)) \geq [M(a,b,t_0)]^{2^{n_0}} \\ &\geq \underbrace{(1-\delta)*(1-\delta)*\ldots(1-\delta)}_{2^{n_0}} \geq (1-\varepsilon). \end{aligned}$$

Therefore, we have $a = b$.

Step 4. We finally show the uniqueness of the point a.

Let $\alpha \in X$ such that $A(\alpha,\alpha) = S(\alpha) = \alpha = T(\alpha) = B(\alpha,\alpha)$. We claim that $\alpha = a$.

Since $*$ is a t-norm of H-type, for $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\underbrace{(1-\delta)*(1-\delta)*\ldots(1-\delta)}_{p} \geq (1-\varepsilon), \quad \text{for all } p \in N.$$

Also, since $\lim_{n\to\infty} M(x,y,t) = 1$ for $x, y \in X$, there exists $t_0 > 0$, such that $M(a,\alpha,t_0) \geq (1-\delta)$. By condition ($\phi$-3), we have $\sum_{n=0}^{\infty} \phi^n(t_0) < +\infty$, then for any $t > 0$, there exists $n_0 \in N$ such that $t > \sum_{k=n_0}^{\infty} \phi^k(t_0)$.

By (2.1), we have

$$\begin{aligned} M(a,\alpha,\phi(t_0)) &= M(A(a,a), B(\alpha,\alpha), \phi(t_0)) \geq M(S(a),T(\alpha),t_0) * M(S(a),T(\alpha),t_0) \\ &= M(a,\alpha,t_0) * M(a,\alpha,t_0) = [M(a,\alpha,t_0)]^2. \end{aligned}$$

In general, for $n \geq 1$, we obtain that $M(a,\alpha,\phi^n(t_0)) \geq [M(a,\alpha,t_0)]^{2^n}$.

Now, for $\varepsilon > 0$ and $t > 0$, we have

$$\begin{aligned} M(a,\alpha,t) &\geq M(a,\alpha,\sum_{k=n_0}^{\infty} \phi^k(t_0)) \geq M(a,\alpha,\phi^{n_0}(t_0)) \geq [M(a,\alpha,t_0)]^{2^{n_0}} \\ &\geq \underbrace{(1-\delta) * (1-\delta) * \ldots (1-\delta)}_{2^{n_0}} \geq (1-\varepsilon). \end{aligned}$$

Therefore, we have $a = \alpha$.

This completes the proof of our result. $\square$

Next, we give an example in support of Theorem 2.1 as follows:

Example 2.1. *Let* $X = \{0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots, \frac{1}{n}, \ldots\}$, $a*b = \min\{a,b\}$, $M(x,y,t) = \frac{t}{t+|x-y|}$, *for all* $x, y \in X$ *and* $t > 0$. *Then* $(X, M, *)$ *is a fuzzy metric space. Let us define the functions* $S, T : X \to X$, $A, B : X \times X \to X$, *respectively by* $S(x) = \begin{cases} 0, & \text{if } x = 0 \\ 1, & \text{if } x = \frac{1}{2n+1},\ 1 \\ \frac{1}{2n+1}, & \text{if } x = \frac{1}{2n} \end{cases}$,

$T(x) = \begin{cases} 0, & \text{if } x = 0 \\ \frac{1}{2}, & \text{if } x = \frac{1}{2n+1}, 1 \\ \frac{1}{2n+1}, & \text{if } x = \frac{1}{2n} \end{cases}$, $A(x,y) = B(x,y) = \begin{cases} \frac{1}{(2n+1)^2}, & \text{if } (x,y) = (\frac{1}{2n}, \frac{1}{2n}) \\ 0, & \text{otherwise} \end{cases}$.

Then the pairs (A,S) *and* (B,T) *are* w*-compatible but not compatible. Further, the pairs* (A,S) *and* (B,T) *share the* (CLR_{ST}) *property. Let* $\phi(t) = \frac{t}{2}$. *Then, the inequality*

$$M(A(x,y), B(u,v), \phi(t)) \geq M(S(x),T(u),t) * M(S(y),T(v),t),$$

holds for all $x, y, u, v \in X$. *Therefore, all the conditions of Theorem 2.1 are satisfied, and 0 is the unique common fixed point of the mappings* A, B, S, T.

Remark 2.1. On taking $A = B = F$ and $S = T = g$ in Theorem 2.1, we obtain Theorem 1.3 which is a generalization of Theorem 1.1. Hence, Theorem 2.1 extends Theorem 1.3 for pairs of mappings and generalizes Theorem 1.1. Also, Theorem 2.1 generalizes Theorem 1.2 since in Theorem 2.1, the completeness assumption of the space or the subspaces has been relaxed entirely and also, the containment condition of the range subspaces of mappings into the range subspaces of the other mappings has also been relaxed.

Theorem 2.2. *Let* $(X, M, *)$ *be a fuzzy metric space,* $*$ *being continuous* t*-norm of* H*-type and* $M(x,y,t) \to 1$ *as* $t \to \infty$. *Let* $A, B : X \times X \to X$ *and* $S, T : X \to X$ *be the mappings satisfying the condition* (2.1) *and the following conditions:*

(2.6) $S(X)$ *and* $T(X)$ *are closed subsets of* X;

(2.7) *the pairs* (A,S) *and* (B,T) *share the common property (E.A.);*

(2.8) *the pairs* (A,S) *and* (B,T) *are* w*-compatible.*

Then, the mappings A, B, S, T have a unique common fixed point in X.

Proof. Since the pairs (A,S) and (B,T) share the common property (E.A.), there exist sequences $\{x_n\}$, $\{y_n\}$ and $\{p_n\}$, $\{q_n\}$ in X such that

$$\lim_{n\to\infty} A(x_n,y_n) = \lim_{n\to\infty} B(p_n,q_n) = \lim_{n\to\infty} S(x_n) = \lim_{n\to\infty} T(p_n) = a, \tag{2.9}$$

$$\lim_{n\to\infty} A(y_n,x_n) = \lim_{n\to\infty} B(q_n,p_n) = \lim_{n\to\infty} S(y_n) = \lim_{n\to\infty} T(q_n) = b, \tag{2.10}$$

for some $a,b \in X$. □

Now, by (2.7), since $S(X)$ and $T(X)$ are closed subsets of X, so that using (2.9), we have $a \in S(X)$ and $a \in T(X)$, that is, $a \in S(X)\cap T(X)$. Similarly, using (2.7) and (2.10), it can be shown that $b \in S(X)\cap T(X)$. Hence, it follows that the pairs (A,S) and (B,T) shares the (CLR$_{ST}$) property. Now, applying Theorem 2.1, we obtain that the mappings A,B,S and T have a unique common fixed point in X.

Remark 2.2. Theorem 2.2 also generalize Theorems 1.1 and 1.2.

Lemma 2.1. *Let $(X,M,*)$ be a fuzzy metric space, $*$ being a continuous t-norm of H-type and $M(x,y,t)\to 1$ as $t\to\infty$, for all $x,y\in X$. Let $A,B : X\times X\to X$ and $S,T : X\to X$ be the mappings satisfying the condition* (2.1) *and the following conditions:*

- (2.11) *the pair (A,S) satisfies the (CLR$_S$) property (or the pair (B,T) satisfies the (CLR$_T$) property);*
- (2.12) *$A(X\times X)\subseteq T(X)$ (or $B(X\times X)\subseteq S(X)$);*
- (2.13) *$T(X)$ (or $S(X)$ is a complete subspace of X);*
- (2.14) *$B(p_n,q_n)$ converges for every sequences $\{p_n\}$ and $\{q_n\}$ in X whenever $T(p_n)$, $T(q_n)$ converges (or $A(x_n,y_n)$ converges for every sequences $\{x_n\}$ and $\{y_n\}$ in X whenever $S(x_n)$, $S(y_n)$ converges).*

Then, the pairs (A,S) and (B,T) share the (CLR$_{ST}$) property.

Proof. Without loss of generality, assume that the pairs (A,S) satisfies the (CLR$_S$) property, then there exist the sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} A(x_n,y_n) = \lim_{n\to\infty} S(x_n) = a, \quad \lim_{n\to\infty} A(y_n,x_n) = \lim_{n\to\infty} S(y_n) = b,$$

for some $a,b\in S(X)$.
Now, since $A(X\times X)\subseteq T(X)$ (wherein $T(X)$ is complete), for each $\{x_n\}$ and $\{y_n\}$ in X, there correspond sequences $\{p_n\}$ and $\{q_n\}$ in X such that $A(x_n,y_n) = T(p_n)$ and $A(y_n,x_n) = T(q_n)$. Therefore,

$$\lim_{n\to\infty} A(x_n,y_n) = \lim_{n\to\infty} T(p_n) = a, \quad \lim_{n\to\infty} A(y_n,x_n) = \lim_{n\to\infty} T(q_n) = b,$$

so that $a,b\in T(X)$, therefore, we conclude that $a,b\in S(X)\cap T(X)$. Next, we assert that $B(p_n,q_n)$ converges to a and $B(q_n,p_n)$ converges to b.
Now, since $\phi\in\Phi$, we have $\phi(t)<t$ for all $t>0$. Then, using (2.1), for $t>0$, we have

$$\begin{aligned} M(A(x_n,y_n),B(p_n,q_n),t) &\geq M(A(x_n,y_n),B(p_n,q_n),\phi(t)) \\ &\geq M(S(x_n),T(p_n),t) * M(S(y_n),T(q_n),t), \end{aligned}$$

then, on letting $n\to\infty$ in the last inequality, we obtain that $\lim_{n\to\infty} M(a,B(p_n,q_n),t) = 1$, and hence, we obtain that the sequence $B(p_n,q_n)$ converges to a as $n\to\infty$. Similarly, we can obtain that the sequence $B(q_n,p_n)$ converges to b as $n\to\infty$. Hence, we have

$$\lim_{n\to\infty} A(x_n,y_n) = \lim_{n\to\infty} B(p_n,q_n) = \lim_{n\to\infty} S(x_n) = \lim_{n\to\infty} T(p_n) = a,$$
$$\lim_{n\to\infty} A(y_n,x_n) = \lim_{n\to\infty} B(q_n,p_n) = \lim_{n\to\infty} S(y_n) = \lim_{n\to\infty} T(q_n) = b,$$

for some $a, b \in S(X) \cap T(X)$. Therefore, the pairs (A, S) and (B, T) shares the (CLR$_{ST}$) property. $\square$

Theorem 2.3. *Let $(X, M, *)$ be a fuzzy metric space, $*$ being continuous t-norm of H-type and $M(x, y, t) \to 1$ as $t \to \infty$, for all $x, y \in X$. Let $A, B : X \times X \to X$ and $S, T : X \to X$ be the mappings satisfying the conditions* (2.1), (2.11)-(2.14). *Then, the mappings A, B, S, T have a unique common fixed point in X, if the pairs (A, S) and (B, T) are w-compatible.*

Proof. By using Lemma 2.1, the pairs (A, S) and (B, T) share the (CLR$_{ST}$) property, therefore, there exist sequences $\{x_n\}$, $\{y_n\}$ and $\{p_n\}$, $\{q_n\}$ in X such that

$$\lim_{n\to\infty} A(x_n, y_n) = \lim_{n\to\infty} B(p_n, q_n) = \lim_{n\to\infty} S(x_n) = \lim_{n\to\infty} T(p_n) = a,$$
$$\lim_{n\to\infty} A(y_n, x_n) = \lim_{n\to\infty} B(q_n, p_n) = \lim_{n\to\infty} S(y_n) = \lim_{n\to\infty} T(q_n) = b,$$

for some $a, b \in S(X) \cap T(X)$.
Now, the remaining proof runs on the lines of the proof of Theorem 2.1. $\square$

3. Application in metric space

As an application of the results proved in the Section 2 of this paper, we now establish some corresponding common fixed point results in metric spaces as follows:

Theorem 3.1. *Let (X, d) be a metric space and suppose that $A : X \times X \to X$, $B : X \times X \to X$, $S : X \to X$, $T : X \to X$ be four mappings satisfying the following condition:*

(3.1) $d(A(x, y), B(u, v)) \leq \frac{k}{2}[d(S(x), T(u)) + d(S(y), T(v))]$,

for all $x, y, u, v \in X$, where $0 < k < 1$.
Also, suppose that the pairs (A, S) and (B, T) share (CLR$_{ST}$) property and are w-compatible. Then, there exists a unique point a in X such that $A(a, a) = S(a) = a = T(a) = B(a, a)$.

Proof. For all x, y in X and $t > 0$, define $M(x, y, t) = \frac{t}{t+d(x,y)}$ and $a * b = \min\{a, b\}$. Then $(X, M, *)$ is a fuzzy metric space and $*$ being the Hadžić type t-norm. Further, it is easy to see that $M(x, y, t) = \frac{t}{t+d(x,y)} \to 1$ as $t \to \infty$, for all $x, y \in X$. We next prove that the inequality (3.1) implies (2.1) for $\phi(t) = kt$ with $t > 0$ and $0 < k < 1$. If otherwise, from (2.1), for some $t > 0$ and $x, y, u, v \in X$, we have

$$\frac{t}{t + \frac{1}{k}d(A(x, y), B(u, v))} < \min\{\frac{t}{t + d(S(x), T(u))}, \frac{t}{t + d(S(y), T(v))}\},$$

then, we have

$$\frac{t}{t + \frac{1}{k}d(A(x, y), B(u, v))} < \min\{\frac{t}{t + d(S(x), T(u))}, \frac{t}{t + d(S(y), T(v))}\},$$

which implies that

(3.2) $t + \frac{1}{k}d(A(x, y), B(u, v)) > t + d(S(x), T(u))$,
(3.3) $t + \frac{1}{k}d(A(x, y), B(u, v)) > t + d(S(y), T(v))$.

Combining (3.2) and (3.3), we obtain that

(3.4) $d(A(x, y), B(u, v)) > \frac{k}{2}[d(S(x), T(u)) + d(S(y), T(v))]$,

which is a contradiction to (3.1). Then, the result holds immediately by applying Theorem 2.1. $\square$

Theorem 3.2. *Let (X, d) be a metric space and suppose that $A : X \times X \to X$, $B : X \times X \to X$, $S : X \to X$, $T : X \to X$ be four mappings satisfying the following condition (3.1) for all $x, y, u, v \in X$, where $0 < k < 1$. Also, suppose that the pairs (A, S) and (B, T) share the common property (E.A.) and are w-compatible. If the range spaces of the mappings $S(X)$ and $T(X)$ are closed subsets of X, then, there exists a unique point a in X such that $A(a, a) = S(a) = a = T(a) = B(a, a)$.*

Proof. Proof follows by the application of Theorems 2.2 and 3.1. □

4. Conclusion

The significance of the common property (E.A.) and (CLR$_{ST}$) property is that both the properties relaxes the continuity hypothesis of all the mappings involved and also relaxes the containment condition of the range subspaces of mappings into the range subspaces of the other mappings, which is required for constructing the sequences of joint iterates in fixed point and coupled fixed point results. It has been noticed that the common property (E.A.) replaces the completeness requirement of the space or range subspaces of the involved mappings with a more natural condition of the range subspaces to be closed whereas (CLR$_{ST}$) property ensures that one does not require even this condition also. Consequently, the view point for the existing common fixed point results in coupled fixed point theory has been hereby sharpened and enriched.

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Manish Jain, for the photograph and short biography, see TWMS J. Appl. Eng. Math., V.7, N.1, 2017.

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