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ON THE CHEBYSHEV POLYNOMIAL COEFFICIENT PROBLEM OF BI-BAZILEVIČ FUNCTIONS

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Abstract. A function said to be bi-Bazilevič in the open unit disk $\mathbb{U}$ if both the function and its inverse are Bazilevič there. In this paper, we will study a newly constructed class of bi-Bazilevič functions. Furthermore, we establish Chebyshev polynomial bounds for the coefficients, and get Fekete-Szegö inequality, for the class $\mathcal{B}(\beta, t)$.

Keywords: Chebyshev polynomials, analytic and univalent functions, bi-univalent functions, bi-Bazilevič functions, coefficient bounds, subordination, Fekete-Szegö inequality.

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1. Introduction and Definitions

Let $\mathcal{A}$ indicate an analytic function family, which is normalized under the condition of $f(0) = f'(0) - 1 = 0$ in $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$ and given by the following Taylor-Maclaurin series:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1}$$

Further, by $\mathcal{S}$ we shall denote the class of all functions in $\mathcal{A}$ which are univalent in $\mathbb{U}$. With a view to recalling the principle of subordination between analytic functions, let the functions f and g be analytic in $\mathbb{U}$. Then we say that the function f is subordinate to g if there exists a Schwarz function $w(z)$, analytic in $\mathbb{U}$ with

$$w(0) = 0, \qquad |w(z)| < 1 \qquad (z \in \mathbb{U}),$$

such that

$$f(z) = g(w(z)) \qquad (z \in \mathbb{U}).$$

We denote this subordination by

$$f \prec g \quad \text{or} \quad f(z) \prec g(z) \quad (z \in \mathbb{U}).$$

In particular, if the function g is univalent in $\mathbb{U}$, the above subordination is equivalent to

$$f(0) = g(0), \quad f(\mathbb{U}) \subset g(\mathbb{U}).$$

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The Koebe-One Quarter Theorem [10] ensures that the image of $\mathbb{U}$ under every univalent function $f \in \mathcal{A}$ contains a disc of radius $1/4$. Thus every univalent function f has an inverse f^{-1} satisfying $f^{-1}(f(z)) = z$ and $f\left(f^{-1}(w)\right) = w \quad \left(|w| < r_0(f)\ ,\ r_0(f) \geq \frac{1}{4}\right)$, where

$$f^{-1}(w) = w - a_2 w^2 + \left(2a_2^2 - a_3\right) w^3 - \left(5a_2^3 - 5a_2 a_3 + a_4\right) w^4 + \cdots . \tag{2}$$

A function $f \in \mathcal{A}$ is said to be bi-univalent in $\mathbb{U}$ if both f and f^{-1} are univalent in $\mathbb{U}$. Let Σ denote the class of bi-univalent functions in $\mathbb{U}$ given by (1). For a brief history and interesting examples in the class Σ, see [21] (see also [2], [7], [6], [17], [19]). Furthermore, judging by the remarkable flood of papers on the subject (see, for example, [8], [14], [22], [23], [24], [25], [26], [29], [30], [32], [33]), the pioneering work by Srivastava et al. [21] has apparently revised the study of bi-univalent functions in recent years. Not much is known about the bounds on the general coefficient $|a_n|$. In the literature, there are only a few works determining the general coefficient bounds $|a_n|$ for the analytic bi-univalent functions ([3], [4], [13], [35]). The coefficient estimate problem for each of $|a_n|$ $(n \in \mathbb{N} \backslash \{1, 2\}\,;\ \mathbb{N} = \{1, 2, 3, ...\})$ is still an open problem.

Chebyshev polynomials have become increasingly important in numerical analysis, from both theoretical and practical points of view. There are four kinds of Chebyshev polynomials. The majority of books and research papers dealing with specific orthogonal polynomials of Chebyshev family, contain mainly results of Chebyshev polynomials of first and second kinds $T_n(t)$ and $U_n(t)$ and their numerous uses in different applications, see for example, Doha [11] and Mason [18].

The Chebyshev polynomials of the first and second kinds are well known. In the case of a real variable t on $(-1, 1)$, they are defined by

$$T_n(t) = \cos n\theta,$$

$$U_n(t) = \frac{\sin(n+1)\theta}{\sin\theta},$$

where the subscript n denotes the polynomial degree and where $t = \cos\theta$.

Definition 1.1. *(see [20]) For $0 \leq \beta < 1$ and $f \in \mathcal{A}$, let $\mathcal{B}(\beta)$ denote the class of Bazilevič functions if and only if*

$$\Re\left(\left(\frac{z}{f(z)}\right)^{1-\beta} f'(z)\right) > 0, \qquad (z \in \mathbb{U}).$$

Several authors have discussed various subfamilies of the well-known Bazilevič functions (see, for details, [10]; see also [16], [20], [28]) of type β from various viewpoints such as the perspective of convexity, inclusion theorems, radii of starlikeness and convexity, boundary rotational problems, subordination relationships, and so on. It is interesting to note in this connection that the earlier investigations on the subject do not seem to have addressed the problems involving coefficient inequalities and coefficient bounds for these subfamilies of Bazilevič type functions.

Definition 1.2. *For $f \in \Sigma$ and $t \in \left(\frac{1}{2}, 1\right]$, let $\mathcal{B}(\beta, t)$ denote the class of Bi-Bazilevič functions order t and type β if only if*

$$\left(\frac{z}{f(z)}\right)^{1-\beta} f'(z) \prec H(z,t) = \frac{1}{1 - 2tz + z^2} \qquad (0 \leq \beta < 1,\ z \in \mathbb{U}) \tag{3}$$

and

$$\left(\frac{w}{g(w)}\right)^{1-\beta} g'(w) \prec H(w,t) = \frac{1}{1 - 2tw + w^2} \qquad (0 \leq \beta < 1,\ w \in \mathbb{U}) \tag{4}$$

where $g(w) = f^{-1}(w)$.

Remark 1.1. *We note that for* $\beta = 0$ *the class* $\mathcal{B}(\beta, t)$ *reduces to the class* $\mathcal{B}(t)$.

The class $\mathcal{B}(t)$ is defined as follows:

Definition 1.3. *A function* $f \in \Sigma$ *is said to be in the class* $\mathcal{B}(t)$, *and* $t \in \left(\frac{1}{2}, 1\right]$, *if the following subordinations hold*

$$\frac{zf'(z)}{f(z)} \prec H(z,t) = \frac{1}{1 - 2tz + z^2} \qquad (z \in \mathbb{U})$$

and

$$\frac{wg'(w)}{g(w)} \prec H(w,t) = \frac{1}{1 - 2tw + w^2} \qquad (w \in \mathbb{U})$$

where $g(w) = f^{-1}(w)$.

We note that if $t = \cos\alpha,\ \alpha \in \left(-\frac{\pi}{3}, \frac{\pi}{3}\right)$, then

$$\begin{aligned} H(z,t) &= \frac{1}{1 - 2tz + z^2} \\ &= 1 + \sum_{n=1}^{\infty} \frac{\sin(n+1)\alpha}{\sin\alpha} z^n \quad (z \in \mathbb{U}). \end{aligned}$$

Thus

$$H(z,t) = 1 + 2\cos\alpha z + (3\cos^2\alpha - \sin^2\alpha)z^2 + \cdots \quad (z \in \mathbb{U}).$$

Following see, we write

$$H(z,t) = 1 + U_1(t)z + U_2(t)z^2 + \cdots \quad (z \in \mathbb{U},\ \ t \in (-1,1)),$$

where $U_{n-1} = \dfrac{\sin(n \arccos t)}{\sqrt{1-t^2}}$ $(n \in \mathbb{N})$ are the Chebyshev polynomials of the second kind. Also it is known that

$$U_n(t) = 2tU_{n-1}(t) - U_{n-2}(t),$$

and

$$U_1(t) = 2t,\ U_2(t) = 4t^2 - 1,\ \ U_3(t) = 8t^3 - 4t,\ \ldots \tag{5}$$

The Chebyshev polynomials $T_n(t),\ t \in (-1,1)$, of the first kind have the generating function of the form

$$\sum_{n=0}^{\infty} T_n(t) z^n = \frac{1 - tz}{1 - 2tz + z^2} \quad (z \in \mathbb{U}).$$

However, the Chebyshev polynomials of the first kind $T_n(t)$ and the second kind $U_n(t)$ are well connected by the following relationships

$$\begin{aligned} &\frac{dT_n(t)}{dt} = nU_{n-1}(t), \\ &T_n(t) = U_n(t) - tU_{n-1}(t), \\ &2T_n(t) = U_n(t) - U_{n-2}(t). \end{aligned}$$

Motivated by the earlier work of Dziok et al. [9], we study the Chebyshev polynomial expansions to provide estimates for the initial coefficients of some subclasses of bi-univalent functions (see, for example, [5]). The aim of this paper to discuss a newly constructed class of bi-Bazilevič functions. Furthermore, we establish Chebyshev polynomial bounds for the coefficients and get Fekete-Szegö inequality, for the class $\mathcal{B}(\beta, t)$.

2. Coefficient bounds for the function class $\mathcal{B}(\beta, t)$

We begin this section by finding the estimates on the coefficients $|a_2|$ and $|a_3|$ for functions in the class $\mathcal{B}(\beta, t)$ proposed by Definition 1.2.

Theorem 2.1. *Let the function $f(z)$ given by (1) be in the class $\mathcal{B}(\beta, t)$. Then*

$$|a_2| \leq \frac{2t\sqrt{2t}}{\sqrt{|(\beta+1)^2 - 2\beta(\beta+1)t^2|}}$$

and

$$|a_3| \leq \frac{4t^2}{(\beta+1)^2} + \frac{2t}{\beta+2}.$$

Proof. Let $f \in \mathcal{B}(\beta, t)$ From (3) and (4), we have

$$\left(\frac{z}{f(z)}\right)^{1-\beta} f'(z) = \left(\frac{f(z)}{z}\right)^{\beta} \left(\frac{zf'(z)}{f(z)}\right) = 1 + U_1(t)\phi(z) + U_2(t)\phi^2(z) + \cdots, \tag{6}$$

and

$$\left(\frac{w}{g(w)}\right)^{1-\beta} g'(w) = \left(\frac{g(w)}{w}\right)^{\beta} \left(\frac{wg'(w)}{g(w)}\right) = 1 + U_1(t)\varphi(w) + U_2(t)\varphi^2(w) + \cdots, \tag{7}$$

for some analytic functions ϕ, φ such that $\phi(0) = \varphi(0) = 0$ and $|\phi(z)| < 1, |\varphi(w)| < 1$ for all $z \in \mathbb{U}$. From the equalities (6) and (7), we obtain that

$$\left(\frac{z}{f(z)}\right)^{1-\beta} f'(z) = 1 + U_1(t)c_1 z + \left[U_1(t)c_2 + U_2(t)c_1^2\right] z^2 + \cdots, \tag{8}$$

and

$$\left(\frac{w}{g(w)}\right)^{1-\beta} g'(w) = 1 + U_1(t)d_1 w + \left[U_1(t)d_2 + U_2(t)d_1^2\right] w^2 + \cdots. \tag{9}$$

It is fairly well-known that if $|\phi(z)| = \left|c_1 z + c_2 z^2 + c_3 z^3 + \cdots\right| < 1$ and $|\varphi(w)| = \left|d_1 w + d_2 w^2 + d_3 w^3 + \cdots\right| < 1,\ z, w \in \mathbb{U}$, then

$$|c_j| \leq 1, \quad \forall j \in \mathbb{N}.$$

It follows from (8) and (9) that

$$(\beta+1)a_2 = U_1(t)c_1, \tag{10}$$

$$\frac{(\beta-1)(\beta+2)}{2}a_2^2 + (\beta+2)a_2^2 = U_1(t)c_2 + U_2(t)c_1^2, \tag{11}$$

and

$$-(\beta+1)a_2 = U_1(t)d_1, \tag{12}$$

$$\frac{(\beta+2)(\beta+3)}{2}a_2^2 - (\beta+2)a_2^2 = U_1(t)d_2 + U_2(t)d_1^2. \tag{13}$$

From (10) and (12) we obtain

$$c_1 = -d_1 \tag{14}$$

and

$$2(\beta+2)^2 a_2^2 = U_1^2(t)\left(c_1^2 + d_1^2\right). \tag{15}$$

By adding (11) to (13), we get

$$(\beta^2 + 3\beta + 2)a_2^2 = U_1(t)(c_2 + d_2) + U_2(t)\left(c_1^2 + d_1^2\right). \tag{16}$$

By using (15) in equality (16), we have

$$\left[(\beta^2+3\beta+2)-\frac{2U_2(t)\,(\beta+1)^2}{U_1^2(t)}\right]a_2^2=U_1(t)\,(c_2+d_2)\,. \tag{17}$$

From (5) and (17) we get

$$|a_2|\le\frac{2t\sqrt{2t}}{\sqrt{|(\beta+1)^2-2\beta\,(\beta+1)\,t^2|}}.$$

Next, in order to find the bound on $|a_3|$, by subtracting (13) from (11), we obtain

$$2\,(\beta+2)\,a_3-2\,(\beta+2)\,a_2^2=U_1(t)\,(c_2-d_2)+U_2(t)\left(c_1^2-d_1^2\right). \tag{18}$$

Then, in view of (14) and (15) , we have from (18)

$$a_3=\frac{U_1^2(t)}{2\,(\beta+1)^2}\left(c_1^2+d_1^2\right)+\frac{U_1(t)}{2\,(\beta+2)}\,(c_2-d_2)\,.$$

This completes the proof. □

Corollary 2.1. *Let the function $f(z)$ given by (1) be in the class $\mathcal{B}(t)$. Then*

$$|a_2|\le 2t\sqrt{2t}$$

and

$$|a_3|\le 4t^2+t.$$

3. Fekete-Szegö inequalities for the function class $\mathcal{B}(\beta,t)$

The classical Fekete-Szegö inequality, presented by means of Loewner's method, for the coefficients of $f\in S$ is

$$\left|a_3-\mu a_2^2\right|\le 1+2\exp(-2\mu/(1-\mu))\quad\text{for }\mu\in[0,1)\,.$$

As $\mu\to 1^-$, we have the elementary inequality $\left|a_3-a_2^2\right|\le 1$. Moreover, the coefficient functional

$$\gamma_\mu(f)=a_3-\mu a_2^2$$

on the normalized analytic functions f in the unit disk $\mathbb{U}$ plays an important role in function theory. The problem of maximizing the absolute value of the functional $\gamma_\mu(f)$ is called the Fekete-Szegö problem, see [12]. Many other recent works on the Fekete-Szegö problem include, for example, [1], [15], [27], [31] and [34].

In this section, we aim to provide Fekete-Szegö inequalities for functions in the class $\mathcal{B}(\beta,t)$. These inequalities are given in the following theorem.

Theorem 3.1. *Let f given by (1) be in the class $\mathcal{B}(\beta,t)$ and $\mu\in\mathbb{R}$. Then*

$$\left|a_3-\mu a_2^2\right|\le\begin{cases}\dfrac{2t}{\beta+2}; & \\ \qquad \mathit{for}\ \ |\mu-1|\le\frac{1}{4(\beta+2)}\left|\left(\frac{\beta+1}{t}\right)^2-2\beta\,(\beta+1)\right| & \\[2ex] \dfrac{8\,|1-\mu|\,t^3}{|(\beta+1)^2-2\beta\,(\beta+1)\,t^2|}; & \\ \qquad \mathit{for}\ \ |\mu-1|\ge\frac{1}{4(\beta+2)}\left|\left(\frac{\beta+1}{t}\right)^2-2\beta\,(\beta+1)\right| & \end{cases}.$$

Proof. From (17) and (18), we conclude that

$$\begin{aligned} a_3-\mu a_2^2 &= (1-\mu)\,\frac{U_1^3(t)\,(c_2+d_2)}{(\beta^2+3\beta+2)U_1^2(t)-2\,(\beta+1)^2\,U_2(t)}+\frac{U_1(t)\,(c_2-d_2)}{2(\beta+2)} \\ &= U_1(t)\left[\left(h\,(\mu)+\frac{1}{2\,(\beta+2)}\right)c_2+\left(h\,(\mu)-\frac{1}{2\,(\beta+2)}\right)d_2\right] \end{aligned}$$

where

$$h\,(\mu)=\frac{U_1^2(t)\,(1-\mu)}{(\beta^2+3\beta+2)U_1^2(t)-2\,(\beta+1)^2\,U_2(t)}.$$

Then, in view of (5), we obtain

$$\left|a_3-\mu a_2^2\right|\leq\begin{cases}\dfrac{2t}{\beta+2}; & 0\leq|h\,(\mu)|\leq\dfrac{1}{2\,(\beta+2)}\\[2ex] 4t\,|h\,(\mu)|\,; & |h\,(\mu)|\geq\dfrac{1}{2\,(\beta+2)}.\end{cases}$$

Taking $\mu=1$ we get

Corollary 3.1. *If $f\in\mathcal{B}(\beta,t)$, then*

$$\left|a_3-a_2^2\right|\leq\frac{2t}{\beta+2}.$$

Corollary 3.2. *Let f given by (1) be in the class $\mathcal{B}(t)$. Then*

$$\left|a_3-a_2^2\right|\leq t.$$

□

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Şahsene Altınkaya for the photography and short autobiography, see TWMS J. App. Eng. Math.V.8, N.1a, 2018.

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