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# THE CLASSES OF BIPOLAR SINGLE VALUED NEUTROSOPHIC GRAPHS

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Abstract. The bipolar single valued neutrosophic graph is the generalization of fuzzy, intuitionistic, bipolar, bipolar intuitionistic graphs. The concept of antipodal bipolar single valued neutrosophic graph (ABSVNG), eccentric BSVNG, self centered BSVNG and self median BSVNG of the given BSVNG are introduced here. We also investigated isomorphism properties of antipodal BSVNGs, eccentric BSVNGs and self sentered BSVNGs. The edge connectivity plays important role in computer network problems and path problems. In this paper, we introduce special types of bipolar single valued neutrosophic (BSVN) bridges, BSVN-Cut vertices, BSVN-Cycles and BSVN-Trees in BSVNG and introduced some of their properties.

Keywords: Antipodal BSVNG, eccentric BSVNG, self centered BSVNG, self median BSVNG, BSVN-Cycles, BSVN-Trees, BSVN-Bridges,BSVN-Cut-vertices and BSVN-Levels.

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## 1. Introduction

The concept of neutrosophic set theory is a generalization of the theory of fuzzy set [11], intuitionistic fuzzy sets [4], interval-valued fuzzy sets [3] and interval-valued intuitionistic fuzzy sets [5]. The concept of neutrosophic set is characterized by a truth-membership degree (T), an indeterminacy-membership degree (I) and a falsity-membership degree (f) independently, which are within the real standard or nonstandard unit interval $]^{-}0, 1^{+}[$. Therefore, if their range is restrained within the real standard unit interval $[0, 1]$. Nevertheless, NSs are hard to be apply in practical problems since the values of the functions of truth, indeterminacy and falsity lie in $]^{-}0, 1^{+}[$. The single valued neutrosophic set was introduced for the first time by Smarandache in his 1998 book. The single valued neutrosophic sets as subclass of neutrosophic sets in which the value of truth-membership, indeterminacy-membership and falsity-membership degrees are intervals of numbers instead of the real numbers. Later on, Wang et al. [16] studied some properties related to single valued neutrosophic sets.

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The bipolar single valued neutrosophic graphs were introduced by Broumi, Talea, Bakali and Smarandache [7]. Also Recently in [9, 10, 8] proposed some algorithms dealt with shortest path problem in a network (graph) where edge weights are characterized by a neutrosophic numbers including single valued neutrosophic numbers, bipolar neutrosophic numbers and interval valued neutrosophic numbers. The concept of neutrosophic hypergraphs, its regularity and totally regularity discussed by Malik, Hassan, Broumi and Smarandache in [1, 2].

P. K. Singh [12] has discussed adequate analysis of uncertainty and vagueness in medical data set using the properties of three-way fuzzy concept lattice. This study provided a precise representation of medical diagnoses problems using the vertices and edges of neutrosophic graph. Further to refine the knowledge three-way fuzzy concepts generation and their hierarchical order visualization in the concept lattice is proposed using neutrosophic graph and component-wise Godel resituated lattice. One application of the proposed method is also discussed to analyze the multi-criteria decision making process.

Ulucay et al. [14] defined the concept of neutrosophic soft expert graph and have established a link between graphs and neutrosophic soft expert sets and studies some basic operations of neutrosophic soft experts graphs such as union, intersection and complement. Similar to the fuzzy graphs, which have a common property that each edge must have a membership value less than or equal to the minimum membership of the nodes it connects.

The BSVNGs have also many applications in path problems, networks and computer science. The concept of antipodal fuzzy graphs introduced by Gani and Malarvizhi [15]. The self centered intuitionistic fuzzy graphs were introduced by Karunambigai, the complete intuitionistic fuzzy graph to be a self centered intuitionistic fuzzy graph and its properties discussed, also the necessary and sufficient condition to be a self centered intuitionistic fuzzy graph were discussed in [17]. M.S. Sunitha and A. Vijayakumar [13] gives the definition of complement of a fuzzy graph for understand and utilize in general concept of fuzzy graphs with respect to complement properties. In this paper, we introduce new classes of BSVNGs, antipodal BSVNGs, eccentric BSVNGs, self centered and self median BSVNGs, BSVN-Bridges, BSVN-Cycles, BSVN-Trees, BSVN-Firm and BSVN-Blocks on the basis of weight of edge connectivity.

## 2. Preliminary

**Definition 2.1.** [16] *Let $X$ be a crisp set, the single valued neutrosophic set (SVNS) $Z$ is characterized by three membership functions $T_Z(x), I_Z(x)$ and $F_Z(x)$ which are truth, indeterminacy and falsity membership functions,* $\forall x \in X$

$$T_Z(x), I_Z(x), F_Z(x) \in [0,1].$$

**Definition 2.2.** [2] *Let $X$ be a crisp set, the bipolar single valued neutrosophic set (BSVNS) $Z$ is characterized by membership functions $T_Z^+(x)$, $I_Z^+(x)$, $F_Z^+(x)$, $T_Z^-(x)$, $I_Z^-(x)$, and $F_Z^-(x)$. That is $\forall x \in X$*

$$T_Z^+(x), I_Z^+(x), F_Z^+(x) \in [0,1],$$

$$T_Z^-(x), I_Z^-(x), F_Z^-(x) \in [-1,0].$$

*A bipolar single valued neutrosophic graph (BSVNG) is a pair $G = (Y, Z)$ of $G^*$, where $Y$ is BSVNS on $V$ and $Z$ is BSVNS on $E$ such that*

$$T_Z^+(\beta\gamma) \leq \min(T_Y^+(\beta), T_Y^+(\gamma)),\ I_Z^+(\beta\gamma) \geq \max(I_Y^+(\beta), I_Y^+(\gamma)),$$

$$I_Z^-(\beta\gamma) \leq \min(I_Y^-(\beta), I_Y^-(\gamma)),\ F_Z^-(\beta\gamma) \leq \min(F_Y^-(\beta), F_Y^-(\gamma)),$$

$$F_Z^+(\beta\gamma) \geq \max(F_Y^+(\beta), F_Y^+(\gamma)),\ T_Z^-(\beta\gamma) \geq \max(T_Y^-(\beta), T_Y^-(\gamma)),$$

*where*

$$0 \leq T_Z^+(\beta\gamma) + I_Z^+(\beta\gamma) + F_Z^+(\beta\gamma) \leq 3$$
$$-3 \leq T_Z^-(\beta\gamma) + I_Z^-(\beta\gamma) + F_Z^-(\beta\gamma) \leq 0$$

$\forall\ \beta, \gamma \in V$. *In this case $D$ is bipolar single valued neutrosophic relation (BSVNR) on $C$. The BSVNG $G = (Y, Z)$ is complete (strong) BSVNG, if*

$$T_Z^+(\beta\gamma) = \min(T_Y^+(\beta), T_Y^+(\gamma)),\ I_Z^+(\beta\gamma) = \max(I_Y^+(\beta), I_Y^+(\gamma)),$$
$$I_Z^-(\beta\gamma) = \min(I_Y^-(\beta), I_Y^-(\gamma)),\ F_Z^-(\beta\gamma) = \min(F_Y^-(\beta), F_Y^-(\gamma)),$$
$$F_Z^+(\beta\gamma) = \max(F_Y^+(\beta), F_Y^+(\gamma)),\ T_Z^-(\beta\gamma) = \max(T_Y^-(\beta), T_Y^-(\gamma)),$$

$\forall\ \beta, \gamma \in V(\forall\ \beta\gamma \in E)$. *The order of $G$, which is denoted by $O(G)$, is defined by*

$$O(G) = (O_T^+(G), O_I^+(G), O_F^+(G), O_T^-(G), O_I^-(G), O_F^-(G))$$

*where,*

$$O_T^+(G) = \sum_{\alpha \in V} T_A^+(\alpha),\ O_I^+(G) = \sum_{\alpha \in V} I_A^+(\alpha),\ O_F^+(G) = \sum_{\alpha \in V} F_A^+(\alpha),$$
$$O_T^-(G) = \sum_{\alpha \in V} T_A^-(\alpha),\ O_I^-(G) = \sum_{\alpha \in V} I_A^-(\alpha),\ O_F^-(G) = \sum_{\alpha \in V} F_A^-(\alpha).$$

*The size of $G$, which is denoted by $S(G)$, is defined by*

$$S(G) = (S_T^+(G), S_I^+(G), S_F^+(G), S_T^-(G), S_I^-(G), S_F^-(G))$$

*where*

$$S_T^+(G) = \sum_{\beta\gamma \in E} T_B^+(\beta\gamma),\ S_T^-(G) = \sum_{\beta\gamma \in E} T_B^-(\beta\gamma),$$
$$S_I^+(G) = \sum_{\beta\gamma \in E} I_B^+(\beta\gamma),\ S_I^-(G) = \sum_{\beta\gamma \in E} I_B^-(\beta\gamma),$$
$$S_F^+(G) = \sum_{\beta\gamma \in E} F_B^+(\beta\gamma),\ S_F^-(G) = \sum_{\beta\gamma \in E} F_B^-(\beta\gamma).$$

*The degree of a vertex $\beta$ in $G$, which is denoted by $d_G(\beta)$, is defined by*

$$d_G(\beta) = (d_T^+(\beta), d_I^+(\beta), d_F^+(\beta), d_T^-(\beta), d_I^-(\beta), d_F^-(\beta))$$

*where*

$$d_T^+(\beta) = \sum_{\beta\gamma \in E} T_B^+(\beta\gamma),\ d_T^-(\beta) = \sum_{\beta\gamma \in E} T_B^-(\beta\gamma),$$
$$d_I^+(\beta) = \sum_{\beta\gamma \in E} I_B^+(\beta\gamma),\ d_I^-(\beta) = \sum_{\beta\gamma \in E} I_B^-(\beta\gamma),$$
$$d_F^+(\beta) = \sum_{\beta\gamma \in E} F_B^+(\beta\gamma),\ d_F^-(\beta) = \sum_{\beta\gamma \in E} F_B^-(\beta\gamma).$$

**Definition 2.3.** [7] *The BSVNG $G = (Y, Z)$ is complete (strong) BSVNG, whenever*

$$T_Z^+(\beta, \gamma) = \min(T_Y^+(\beta), T_Y^+(\gamma)),\ I_Z^+(\beta, \gamma) = \max(I_Y^+(\beta), I_Y^+(\gamma)),$$
$$I_Z^-(\beta, \gamma) = \min(I_Y^-(\beta), I_Y^-(\gamma)),\ F_Z^-(\beta, \gamma) = \min(F_Y^-(\beta), F_Y^-(\gamma)),$$
$$F_Z^+(\beta, \gamma) = \max(F_Y^+(\beta), F_Y^+(\gamma)),\ T_Z^-(\beta, \gamma) = \max(T_Y^-(\beta), T_Y^-(\gamma)),$$

$\forall\ \beta, \gamma \in V$. ($\forall\ (\beta, \gamma) \in E$).

**Definition 2.4.** [7] *A path $P$ in a BSVNG $G = (A, B)$ is $P : v_1, v_2, v_3, \ldots, v_n$ such that $T_B^+(v_i, v_{i+1}) > 0, I_B^+(v_i, v_{i+1}) > 0, F_B^+(v_i, v_{i+1}) > 0, T_B^-(v_i, v_{i+1}) < 0, I_B^-(v_i, v_{i+1}) < 0, F_B^-(v_i, v_{i+1}) < 0$ for $1 \leq i \leq n$. The BSVNG to be a connected, if there is path between every two vertices, else $G$ is disconnected.*

## 3. Special classes of BSVNGs

Let $G$ denotes BSVNG and $G^* = (V, E)$ denotes underlying crisp graph. In this section we discuss the antipodal, eccentric, self centered and self median bipolar single valued neutrosophic graphs.

**Definition 3.1.** *The bipolar single valued neutrosophic subgraph of BSVNG $G = (C, D)$ of $G^* = (V, E)$ is a BSVNG $H = (C', D')$ on a $H^* = (V', E')$, such that*
**(1)** $C' = C$, *that is* $\forall\ x \in V' \subseteq V$ *with*

$$T_{C'}^+(x) = T_C^+(x),\ I_{C'}^+(x) = I_C^+(x),\ F_{C'}^+(x) = F_C^+(x),$$
$$T_{C'}^-(x) = T_C^-(x),\ I_{C'}^-(x) = I_C^-(x),\ F_{C'}^-(x) = F_C^-(x).$$

**(2)** $D' = D$, *that is* $\forall\ (\beta, \gamma) \in E$ *in the edge set* $E' \subseteq E$ *with*

$$T_{D'}^+(\beta, \gamma) = T_D^+(\beta, \gamma),\ I_{D'}^+(\beta, \gamma) = I_D^+(\beta, \gamma),\ F_{D'}^+(\beta, \gamma) = F_D^+(\beta, \gamma),$$
$$T_{D'}^-(\beta, \gamma) = T_D^-(\beta, \gamma),\ I_{D'}^-(\beta, \gamma) = I_D^-(\beta, \gamma),\ F_{D'}^-(\beta, \gamma) = F_D^-(\beta, \gamma).$$

**Definition 3.2.** *Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two BSVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then the homomorphism $\chi : V_1 \to V_2$ is a mapping from $V_1$ into $V_2$ satisfying following conditions*

$$T_{C_1}^+(p) \leq T_{C_2}^+(\chi(p)),\ I_{C_1}^+(p) \geq I_{C_2}^+(\chi(p)),\ F_{C_1}^+(p) \geq F_{C_2}^+(\chi(p)),$$
$$T_{C_1}^-(p) \geq T_{C_2}^-(\chi(p)),\ I_{C_1}^-(p) \leq I_{C_2}^-(\chi(p)),\ F_{C_1}^-(p) \leq F_{C_2}^-(\chi(p)),$$

$\forall\ p \in V_1$.

$$T_{D_1}^+(p, q) \leq T_{D_2}^+(\chi(p), \chi(q)),\ I_{D_1}^+(p, q) \geq I_{D_2}^+(\chi(p), \chi(q)),\ F_{D_1}^+(p, q), \geq F_{D_2}^+(\chi(p), \chi(q)),$$
$$T_{D_1}^-(p, q) \geq T_{D_2}^-(\chi(p), \chi(q)),\ I_{D_1}^-(p, q) \leq I_{D_2}^-(\chi(p), \chi(q)),\ F_{D_1}^-(p, q) \leq F_{D_2}^-(\chi(p), \chi(q)),$$

$\forall\ (p, q) \in E_1$.

**Definition 3.3.** *Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two BSVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then the weak isomorphism $\upsilon : V_1 \to V_2$ is a bijective homomorphism from $V_1$ into $V_2$ satisfying following conditions*

$$T_{C_1}^+(p) = T_{C_2}^+(\upsilon(p)),\ I_{C_1}^+(p) = I_{C_2}^+(\upsilon(p)),\ F_{C_1}^+(p) = F_{C_2}^+(\upsilon(p)),$$
$$T_{C_1}^-(p) = T_{C_2}^-(\upsilon(p)),\ I_{C_1}^-(p) = I_{C_2}^-(\upsilon(p)),\ F_{C_1}^-(p) = F_{C_2}^-(\upsilon(p)),$$

$\forall\ p \in V_1$.

**Remark 3.1.** *The weak isomorphism between two BSVNGs preserves the orders.*

**Remark 3.2.** *The weak isomorphism between BSVNGs is a partial order relation.*

**Definition 3.4.** *Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two BSVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then the co-weak isomorphism $\kappa : V_1 \to V_2$ is a bijective homomorphism from $V_1$ into $V_2$ satisfying following conditions*

$$T_{D_1}^+(p, q) = T_{D_2}^+(\kappa(p), \kappa(q)),\ I_{D_1}^+(p, q) = I_{D_2}^+(\kappa(p), \kappa(q)),\ F_{D_1}^+(p, q), = F_{D_2}^+(\kappa(p), \kappa(q)),$$
$$T_{D_1}^-(p, q) = T_{D_2}^-(\kappa(p), \kappa(q)),\ I_{D_1}^-(p, q) = I_{D_2}^-(\kappa(p), \kappa(q)),\ F_{D_1}^-(p, q) = F_{D_2}^-(\kappa(p), \kappa(q)),$$

$\forall\ (p, q) \in E_1$.

**Remark 3.3.** *The co-weak isomorphism between two BSVNGs preserves the sizes.*

**Remark 3.4.** *The co-weak isomorphism between BSVNGs is a partial order relation.*

**Definition 3.5.** *Let $G_1 = (C_1, D_1)$ and $G_2 = (C_2, D_2)$ be two BSVNGs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Then an isomorphism $\psi : V_1 \to V_2$ is a bijective homomorphism from $V_1$ into $V_2$ satisfying following conditions*

$$T^+_{C_1}(p) = T^+_{C_2}(\psi(p)),\ I^+_{C_1}(p) = I^+_{C_2}(\psi(p)),\ F^+_{C_1}(p) = F^+_{C_2}(\psi(p)),$$

$$T^-_{C_1}(p) = T^-_{C_2}(\psi(p)),\ I^-_{C_1}(p) = I^-_{C_2}(\psi(p)),\ F^-_{C_1}(p) = F^-_{C_2}(\psi(p)),$$

$\forall\ p \in V_1$.

$$T^+_{D_1}(p,q) = T^+_{D_2}(\psi(p),\psi(q)),\ I^+_{D_1}(p,q) = I^+_{D_2}(\psi(p),\psi(q)),\ F^+_{D_1}(p,q), = F^+_{D_2}(\psi(p),\psi(q)),$$

$$T^-_{D_1}(p,q) = T^-_{D_2}(\psi(p),\psi(q)),\ I^-_{D_1}(p,q) = I^-_{D_2}(\psi(p),\psi(q)),\ F^-_{D_1}(p,q) = F^-_{D_2}(\psi(p),\psi(q)),$$

$\forall\ (p,q) \in E_1$.

**Remark 3.5.** *The isomorphism between two BSVNGs is an equivalence relation.*

**Remark 3.6.** *The isomorphism between two BSVNGs preserves the orders and sizes.*

**Remark 3.7.** *The isomorphism between two BSVNGs preserves the degrees of their vertices.*

**Definition 3.6.** *The strength of connectedness between $x$ and $y$ in $V$ of BSVNG $G = (A, B)$, which is denoted by $S^\infty_B(x,y)$, is defined by*

$$S^\infty_B(x,y) = (T^{\infty+}_B(x,y), I^{\infty+}_B(x,y), F^{\infty+}_B(x,y), T^{\infty-}_B(x,y), I^{\infty-}_B(x,y), F^{\infty-}_B(x,y))$$

*where*

$$T^{\infty+}_B(x,y) = \sup\{T^{k+}_B(x,y) : k = 1,2,\ldots,n\},$$

$$T^{\infty+}_B(x,y) = \sup\{T^+_B(x,v_1) \wedge \ldots \wedge T^+_B(v_{k-1},y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1,2,\ldots,n\}.$$

$$I^{\infty+}_B(x,y) = \inf\{I^{k+}_B(x,y) : k = 1,2,\ldots,n\},$$

$$I^{\infty+}_B(x,y) = \inf\{I^+_B(x,v_1) \vee \ldots \vee I^+_B(v_{k-1},y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1,2,\ldots,n\}.$$

$$F^{\infty+}_B(x,y) = \inf\{F^{k+}_B(x,y) : k = 1,2,\ldots,n\},$$

$$F^{\infty+}_B(x,y) = \inf\{F^+_B(x,v_1) \vee \ldots \vee F^+_B(v_{k-1},y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1,2,\ldots,n\}.$$

$$T^{\infty-}_B(x,y) = \inf\{T^{k-}_B(x,y) : k = 1,2,\ldots,n\},$$

$$T^{\infty-}_B(x,y) = \inf\{T^-_B(x,v_1) \vee \ldots \vee T^-_B(v_{k-1},y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1,2,\ldots,n\}.$$

$$I^{\infty-}_B(x,y) = \sup\{I^{k-}_B(x,y) : k = 1,2,\ldots,n\},$$

$$I^{\infty-}_B(x,y) = \sup\{I^-_B(x,v_1) \wedge \ldots \wedge I^-_B(v_{k-1},y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1,2,\ldots,n\}.$$

$$F^{\infty-}_B(x,y) = \sup\{F^{k-}_B(x,y) : k = 1,2,\ldots,n\},$$

$$F^{\infty-}_B(x,y) = \sup\{F^-_B(x,v_1) \wedge \ldots \wedge F^-_B(v_{k-1},y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1,2,\ldots,n\}.$$

*where $T^{\infty+}_B(x,y), I^{\infty+}_B(x,y), F^{\infty+}_B(x,y), T^{\infty-}_B(x,y), I^{\infty-}_B(x,y)$ and $F^{\infty-}_B(x,y)$ are positive and negative truth, indeterminacy and falsity connectedness between $x$ and $y$ in $V$, respectively.*

**Definition 3.7.** *Let $G=(A,B)$ be a BSVNG of $G^*$, the length of path $Q:v_1,v_2,\dots,v_n$, which is denoted by l(Q), is defined by*

$$l(Q)=(l_T^+(Q),l_I^+(Q),l_F^+(Q),l_T^-(Q),l_I^-(Q),l_F^-(Q))$$

*where*

$$l_T^+(Q)=\sum_{i=1}^{n-1}\frac{1}{T_B^+(v_i,v_{i+1})},\ l_I^+(Q)=\sum_{i=1}^{n-1}\frac{1}{I_B^+(v_i,v_{i+1})},\ l_F^+(Q)=\sum_{i=1}^{n-1}\frac{1}{F_B^+(v_i,v_{i+1})},$$
$$l_T^-(Q)=\sum_{i=1}^{n-1}\frac{1}{T_B^-(v_i,v_{i+1})},\ l_I^-(Q)=\sum_{i=1}^{n-1}\frac{1}{I_B^-(v_i,v_{i+1})},\ l_F^-(Q)=\sum_{i=1}^{n-1}\frac{1}{F_B^-(v_i,v_{i+1})}.$$

*The $l_T^+(Q),l_I^+(Q)$ and $l_F^+(Q)$ are called the positive $T$-Length, positive $I$-Length and positive $F$-Length of path $Q$, respectively and $l_T^-(Q),l_I^-(Q)$ and $l_F^-(Q)$ are called the negative $T$-Length, negative $I$-Length and negative $F$-Length of path $Q$, respectively. The distance between two vertices $\alpha$ and $\beta$ which is denoted by $\delta(\alpha,\beta)$, is defined by*

$$\delta(\alpha,\beta)=(\delta_T^+(\alpha,\beta),\delta_I^+(\alpha,\beta),\delta_F^+(\alpha,\beta),\delta_T^-(\alpha,\beta),\delta_I^-(\alpha,\beta),\delta_F^-(\alpha,\beta))$$

*where $\delta_T^+(\alpha,\beta)$, $\delta_I^+(\alpha,\beta)$, $\delta_F^+(\alpha,\beta)$, $\delta_T^-(\alpha,\beta)$, $\delta_I^-(\alpha,\beta)$ and $\delta_F^-(\alpha,\beta)$ are called the positive $T$-Distance, positive $I$-Distance, positive $F$-Distance, negative $T$-Distance, negative $I$-Distance and negative $F$-Distance of any path $\alpha-\beta$ which are*

$$\delta_T^+(\alpha,\beta)=\min(l_T^+(Q)),\ \delta_I^+(\alpha,\beta)=\min(l_I^+(Q)),\ \delta_F^+(\alpha,\beta)=\min(l_F^+(Q)),$$
$$\delta_T^-(\alpha,\beta)=\max(l_T^-(Q)),\ \delta_I^-(\alpha,\beta)=\max(l_I^-(Q)),\ \delta_F^-(\alpha,\beta)=\max(l_F^-(Q)).$$

*The eccentricity of vertex $v_i\in V$, which is denoted by $e(v_i)$, is defined by*

$$e(v_i)=(e_T^+(v_i),e_I^+(v_i),e_F^+(v_i),e_T^-(v_i),e_I^-(v_i),e_F^-(v_i))$$

*where*

$$e_T^+(v_i)=\max\{\delta_T^+(v_i,v_j):v_j\in V,v_i\neq v_j\},\ e_T^-(v_i)=\min\{\delta_T^-(v_i,v_j):v_j\in V,v_i\neq v_j\},$$
$$e_I^+(v_i)=\min\{\delta_T^+(v_i,v_j):v_j\in V,v_i\neq v_j\},\ e_I^-(v_i)=\max\{\delta_T^-(v_i,v_j):v_j\in V,v_i\neq v_j\},$$
$$e_F^+(v_i)=\min\{\delta_T^+(v_i,v_j):v_j\in V,v_i\neq v_j\},\ e_F^-(v_i)=\max\{\delta_T^-(v_i,v_j):v_j\in V,v_i\neq v_j\},$$

*where $e_T^+(v_i),e_I^+(v_i)$, $e_F^+(v_i)$, $e_T^-(v_i),e_I^-(v_i)$ and $e_F^-(v_i)$ are called the positive $T$-Eccentricity, positive $I$-Eccentricity, positive $F$-Eccentricity, negative $T$-Eccentricity, negative $I$-Eccentricity and negative $F$-Eccentricity of vertex $v_i$, respectively. The radius of $G$, which is denoted by $r(G)$, is defined by*

$$r(G)=(r_T^+(G),r_I^+(G),r_F^+(G),r_T^-(G),r_I^-(G),r_F^-(G))$$

*where*

$$r_T^+(G)=\min\{e_T^+(v_i):v_i\in V\},\ r_I^+(G)=\min\{e_I^+(v_i):v_i\in V\},$$
$$r_F^+(G)=\min\{e_F^+(v_i):v_i\in V\},\ r_T^-(G)=\max\{e_T^-(v_i):v_i\in V\},$$
$$r_I^-(G)=\max\{e_I^-(v_i):v_i\in V\},\ r_F^-(G)=\max\{e_F^-(v_i):v_i\in V\},$$

*where $r_T^+(G),r_I^+(G)$, $r_F^+(G)$, $r_T^-(G),r_I^-(G)$, and $r_F^-(G)$ are called the positive $T$-Radius, positive $I$-Radius, positive $F$-Radius, negative $T$-Radius, negative $I$-Radius and negative $F$-Radius of graph $G$, respectively. The diameter of $G$, which is denoted by $d(G)$, is defined by*

$$d(G)=(d_T^+(G),d_I^+(G),d_F^+(G),d_T^-(G),d_I^-(G),d_F^-(G))$$

*where*

$$d_T^+(G)=\max\{e_T^+(v_i):v_i\in V\},\ d_I^+(G)=\max\{e_I^+(v_i):v_i\in V\},$$
$$d_F^+(G)=\max\{e_F^+(v_i):v_i\in V\},\ d_T^-(G)=\min\{e_T^-(v_i):v_i\in V\},$$

$$d_I^-(G) = \min\{e_I^-(v_i) : v_i \in V\},\ d_F^-(G) = \min\{e_F^-(v_i) : v_i \in V\},$$

*where* $d_T^+(G), d_I^+(G)$, $d_F^+(G)$, $d_T^-(G), d_I^-(G)$, *and* $d_F^-(G)$ *are the positive T-Diameter, positive I-Diameter, positive F-Diameter, negative T-Diameter, negative I-Diameter and negative F-Diameter of graph G, respectively.*

**Definition 3.8.** *An antipodal bipolar single valued neutrosophic graph (ABSVNG)* $A(G) = (Q, R)$ *of a BSVNG* $G = (A, B)$ *is the BSVNG in which*
(**a**) $Q = A$ *on V.* (**b**) *If* $\delta(p, q) = d(G)$ *then*
(**i**) *If* $(p, q) \in E$ *then* $R = B$ *on E.*
(**ii**) *If* $(p, q) \notin E$ *then*

$$T_R^+(p,q) = \min(T_A^+(p), T_A^+(q)),\ I_R^+(p,q) = \max(I_A^+(p), I_A^+(q)),$$
$$I_R^-(p,q) = \min(I_A^-(p), I_A^-(q)),\ F_R^-(p,q) = \min(F_A^-(p), F_A^-(q)),$$
$$F_R^+(p,q) = \max(F_A^+(p), F_A^+(q)),\ T_R^-(p,q) = \max(T_A^-(p), T_A^-(q)).$$

**Example 3.1.** *Consider the BSVNG* $G = (A, B)$ *of* $G^*$, *the BSVNSs A and B over* $V = \{\xi, \eta, \zeta\}$ *and* $E = \{(\xi, \eta), (\eta, \zeta)(\zeta, \xi)\}$ *are defined in Table. 1.*

| $A$ | $T_A^+$ | $I_A^+$ | $F_A^+$ | $T_A^-$ | $I_A^-$ | $F_A^-$ | $B$ | $T_B^+$ | $I_B^+$ | $F_B^+$ | $T_B^-$ | $I_B^-$ | $F_B^-$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\xi$ | 1/5 | 1/4 | 1/3 | −1/4 | −1/5 | −1/6 | $(\xi,\eta)$ | 1/7 | 1/2 | 1/3 | −1/4 | −1/5 | −1/6 |
| $\eta$ | 1/7 | 1/2 | 1/5 | −1/4 | −1/5 | −1/6 | $(\eta,\zeta)$ | 1/7 | 1/2 | 1/5 | −1/4 | −1/5 | −1/6 |
| $\zeta$ | 1/4 | 1/6 | 1/8 | −1/4 | −1/5 | −1/6 | $(\zeta,\xi)$ | 1/5 | 1/4 | 1/3 | −1/4 | −1/5 | −1/6 |

Table 1. BSVNSs of BSVNG

*Then by calculation*

$$\delta(\xi,\eta) = (7,2,3,-4,-5,-6), \delta(\xi,\zeta) = (5,4,3,-4,-5,-6), \delta(\eta,\zeta) = (7,2,5,-4,-5,-6),$$

$e(\xi) = (7,2,3,-4,-5,-6)$, $e(\eta) = (7,2,3,-4,-5,-6)$, $e(\zeta) = (7,2,3,-4,-5,-6)$, $d(G) = (7,2,3,-4,-5,-6) = \delta(\xi,\eta)$. *Also BSVNSs of ABSVNG are defined in Table. 2.*

| $Q$ | $T_Q^+$ | $I_Q^+$ | $F_Q^+$ | $T_Q^-$ | $I_Q^-$ | $F_Q^-$ | $R$ | $T_R^+$ | $I_R^+$ | $F_R^+$ | $T_R^-$ | $I_R^-$ | $F_R^-$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\xi$ | 1/5 | 1/4 | 1/3 | −1/4 | −1/5 | −1/6 | $(\xi,\eta)$ | 1/7 | 1/2 | 1/3 | −1/4 | −1/5 | −1/6 |
| $\eta$ | 1/7 | 1/2 | 1/5 | −1/4 | −1/5 | −1/6 | $(\eta,\zeta)$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $\zeta$ | 1/4 | 1/6 | 1/8 | −1/4 | −1/5 | −1/6 | $(\zeta,\xi)$ | 0 | 0 | 0 | 0 | 0 | 0 |

Table 2. BSVNSs of ABSVNG

**Definition 3.9.** *An eccentric BSVNG* $G_e = (P, Q)$ *of a BSVNG* $G = (A, B)$ *is the BSVNG in which,*
(**a**) $P = A$ *on V.* (**b**) *If*

$$\delta_T^+(p,q) = \min(e_T^+(p), e_T^+(q)),\ \delta_I^+(p,q) = \max(e_I^+(p), e_I^+(q)),$$
$$\delta_I^-(p,q) = \min(e_I^-(p), e_I^-(q)),\ \delta_F^-(p,q) = \min(e_F^-(p), e_F^-(q)),$$
$$\delta_F^+(p,q) = \max(e_F^+(p), e_F^+(q)),\ \delta_T^-(p,q) = \max(e_T^-(p), e_T^-(q)),$$

*then*
(**i**) *If* $(p, q) \in E$ *then* $Q = B$ *on E.*
(**ii**) *If* $(p, q) \notin E$ *then*

$$T_Q^+(p,q) = \min(T_A^+(p), T_A^+(q)),\ I_Q^+(p,q) = \max(I_A^+(p), I_A^+(q)),$$

$$I_Q^-(p,q) = \min(I_A^-(p), I_A^-(q)),\ F_Q^-(p,q) = \min(F_A^-(p), F_A^-(q)),$$
$$F_Q^+(p,q) = \max(F_A^+(p), F_A^+(q)),\ T_Q^-(p,q) = \max(T_A^-(p), T_A^-(q)).$$

(**c**) *Otherwise* $Q = O = (0,0,0,0,0,0)$.

**Example 3.2.** *Consider the BSVNG $G = (A, B)$ of $G^*$, which is given in Example. 3.1. Then by calculation*

$\delta(\xi,\eta) = (7,2,3,-4,-5,-6), \delta(\xi,\zeta) = (5,4,3,-4,-5,-6), \delta(\eta,\zeta) = (7,2,5,-4,-5,-6),$

$e(\xi) = (7,2,3,-4,-5,-6), e(\eta) = (7,2,3,-4,-5,-6), e(\zeta) = (7,2,3,-4,-5,-6), \delta_T^-(\xi,\eta) = -4 = \max(e_T^-(\xi), e_T^-(\eta)), \delta_F^-(\eta,\zeta) = -6 = \max(e_F^-(\eta), e_F^-(\zeta)),$

$$\delta_I^-(\xi,\eta) = -5 = \min(e_I^-(\xi), e_I^-(\eta)),\ \delta_T^-(\xi,\eta) = -4 = \max(e_T^-(\xi), e_T^-(\eta)),$$
$$\delta_F^-(\xi,\eta) = -6 = \min(e_F^-(\xi), e_F^-(\eta)),\ \delta_T^-(\xi,\zeta) = -4 = \max(e_T^-(\xi), e_T^-(\zeta)),$$
$$\delta_I^-(\xi,\zeta) = -5 = \min(e_I^-(\xi), e_I^-(\zeta)),\ \delta_F^-(\xi,\zeta) = -6 = \min(e_F^-(\xi), e_F^-(\zeta)),$$
$$\delta_T^-(\eta,\zeta) = -4 = \max(e_T^-(\eta), e_T^-(\zeta)),\ \delta_I^-(\eta,\zeta) = -5 = \max(e_I^-(\eta), e_I^-(\zeta)),$$
$$\delta_T^+(\xi,\eta) = 7 = \min(e_T^+(\xi), e_T^+(\eta)),\ \delta_I^+(\xi,\eta) = 2 = \max(e_I^+(\xi), e_I^+(\eta)),$$
$$\delta_F^+(\xi,\eta) = 3 = \max(e_F^+(\xi), e_F^+(\eta)),\ \delta_T^+(\eta,\zeta) = 7 = \min(e_T^+(\eta), e_T^+(\zeta)).$$

*The BSVNSs of EBSVNG are defined in Table. 3.*

| $P$ | $T_P^+$ | $I_P^+$ | $F_P^+$ | $T_P^-$ | $I_P^-$ | $F_P^-$ | $Q$ | $T_Q^+$ | $I_Q^+$ | $F_Q^+$ | $T_Q^-$ | $I_Q^-$ | $F_Q^-$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\xi$ | 1/5 | 1/4 | 1/3 | −1/4 | −1/5 | −1/6 | $(\xi,\eta)$ | 1/7 | 1/2 | 1/3 | −1/4 | −1/5 | −1/6 |
| $\eta$ | 1/7 | 1/2 | 1/5 | −1/4 | −1/5 | −1/6 | $(\eta,\zeta)$ | 1/7 | 1/2 | 0 | −1/4 | −1/5 | −1/6 |
| $\zeta$ | 1/4 | 1/6 | 1/8 | −1/4 | −1/5 | −1/6 | $(\zeta,\xi)$ | 0 | 0 | 1/3 | −1/4 | −1/5 | −1/6 |

Table 3. BSVNSs of EBSVNG

**Proposition 3.1.** *The ABSVNG of the BSVNG is the generalization of antipodal bipolar fuzzy graph of bipolar fuzzy graph and antipodal intuitionistic bipolar fuzzy graph of intuitionistic bipolar fuzzy graph.*

**Proposition 3.2.** *The EBSVNG of BSVNG is the generalization of eccentric bipolar fuzzy graph of bipolar fuzzy graph and eccentric intuitionistic bipolar fuzzy graph of intuitionistic bipolar fuzzy graph.*

**Proposition 3.3.** *$A(G)$ is a BSVN subgraph of $G_e$.*

**Definition 3.10.** *The connected BSVNG $G = (A, B)$ is distance regular BSVNG whenever*

$$\delta(x,y) = k = (k_1,k_2,k_3,k_4,k_5,k_6)$$

$\forall\ x, y \in V$.

**Proposition 3.4.** *If $G = (A, B)$ is distance regular BSVNG, then $G$ is BSVN-spanning subgraph of $A(G)$, such that $A(G)$ is same as $G_e$.*

**Theorem 3.1.** *For the complete BSVNG $G = (A, B)$ where $A$ be constant BSVNS, then $G$ and $A(G)$ are isomorphic.*

**Theorem 3.2.** *The ABSVNG $A(G) = (Q, R)$ of a BSVNG $G = (A, B)$ is spanning subgraph of $G$.*

*Proof.* Since by the definition of $A(G)$ $(i)$ $Q = A$ on $V$. $(ii)$ If $\delta(\xi, \eta) = d(G)$ then (a) If $\xi$ and $\eta$ are adjacent in $G$ then $R = B$ on $E$. (b) If $\xi$ and $\eta$ are not adjacent in $G$, then by definition of $A(G)$

$$T_R^+(\xi,\eta) = \min(T_A^+(\xi), T_A^+(\eta)),\ I_R^+(\xi,\eta) = \max(I_A^+(\xi), I_A^+(\eta)),$$
$$I_R^-(\xi,\eta) = \min(I_A^-(\xi), I_A^-(\eta)),\ F_R^-(\xi,\eta) = \min(F_A^-(\xi), F_A^-(\eta)),$$
$$F_R^+(\xi,\eta) = \max(F_A^+(\xi), F_A^+(\eta)),\ T_R^-(\xi,\eta) = \max(T_A^-(\xi), T_A^-(\eta)).$$

this completes the result. □

**Theorem 3.3.** *If $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ are isomorphic BSVNGs, then so $A(G_1)$ and $A(G_2)$.*

*Proof.* By hypothesis there is an isomorphism $f$ between them preserves the weights of edges. Hence if vertex $\alpha$ has maximum positive $T$-eccentricity, minimum positive $I$-eccentricity, positive minimum $F$-eccentricity minimum negative $T$-eccentricity, maximum negative $I$-eccentricity and maximum negative $F$-eccentricity in $G_1$. Then $f(\alpha)$ has maximum positive $T$-eccentricity, minimum positive $I$-eccentricity, minimum positive $F$-eccentricity minimum negative $T$-eccentricity, maximum negative $I$-eccentricity and maximum negative $F$-eccentricity in $G_2$, so $G_1$ and $G_2$ will have same diameter. If distance between $\alpha$ and $\beta$ is $k = (k_1, k_2, k_3, k_4, k_5, k_6)$ in $G_1$ then $f(\alpha)$ and $f(\beta)$ will also have their distance as $k$ in $G_2$, since $f$ is a bijective function between $A(G_1)$ and $A(G_2)$ with $Q_1(\alpha) = A_1(\alpha) = A_2(\alpha) = Q_2(\alpha)$ $\forall \alpha \in V_1$ and $(i)$ If $(\alpha, \beta) \in E_1$ then $R_1 = B_1$. $(ii)$ If $(\alpha, \beta) \notin E_1$ then

$$T_{R_1}^+(\alpha,\beta) = \min(T_{A_1}^+(\alpha), T_{A_1}^+(\beta)),\ I_{R_1}^+(\alpha,\beta) = \max(I_{A_1}^+(\alpha), I_{A_1}^+(\beta)),$$
$$I_{R_1}^-(\alpha,\beta) = \min(I_{A_1}^-(\alpha), I_{A_1}^-(\beta)),\ F_{R_1}^-(\alpha,\beta) = \min(F_{A_1}^-(\alpha), F_{A_1}^-(\beta)),$$
$$F_{R_1}^+(\alpha,\beta) = \max(F_{A_1}^+(\alpha), F_{A_1}^+(\beta)),\ T_{R_1}^-(\alpha,\beta) = \max(T_{A_1}^-(\alpha), T_{A_1}^-(\beta)).$$

Since $f : G_1 \to G_2$ is an isomorphism if $(\alpha, \beta) \in E_1$ then $R_1(\alpha, \beta) = B_2(f(\alpha), f(\beta))$, if $(\alpha, \beta) \notin E_1$ then

$$T_{R_1}^+(\alpha,\beta) = \min(f(\alpha), f(\beta)),\ I_{R_1}^+(\alpha,\beta) = \max(f(\alpha), f(\beta)),$$
$$I_{R_1}^-(\alpha,\beta) = \min(f(\alpha), f(\beta)),\ F_{R_1}^-(\alpha,\beta) = \min(f(\alpha), f(\beta)),$$
$$F_{R_1}^+(\alpha,\beta) = \max(f(\alpha), f(\beta)),\ T_{R_1}^-(\alpha,\beta) = \max(f(\alpha), f(\beta)).$$

Therefore we conclude that $R_1(\alpha, \beta) = R_2(f(\alpha), f(\beta))$. □

**Theorem 3.4.** *Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be two connected BSVNGs, If $G_1$ and $G_2$ are co-weak isomorphic, then $A(G_1)$ is homomorphic to $A(G_2)$.*

*Proof.* As $G_1$ and $G_2$ be co-weak isomorphic BSVNGs, then there exist a bijection $f : G_1 \to G_2$ satisfying the conditions $T_{A_1}^+(\alpha) \leq T_{A_2}^+(f(\alpha)), I_{A_1}^+(\alpha) \geq I_{A_2}^+(f(\alpha)), F_{A_1}^+(\alpha) \geq F_{A_2}^+(f(\alpha)), T_{A_1}^-(\alpha) \geq T_{A_2}^-(f(\alpha)), I_{A_1}^-(\alpha) \leq I_{A_2}^-(f(\alpha)), F_{A_1}^-(\alpha) \leq F_{A_2}^-(f(\alpha))$ $\forall \alpha \in V_1$ and $B_1^+(\alpha, \beta) = B_2^+(f(\alpha), f(\beta))$ $\forall (\alpha, \beta) \in E_1$, so the distance and diameters will preserved. Let $d(G_1) = d(G_2) = k = (k_1, k_2, k_3, k_4, k_5, k_6)$ if $u, v \in V_1$ distance $k$ in $G_1$ then $(u, v) \in E(A(G_1))$, so $f(u), f(v) \in V_2$ distance $k$ in $G_2$ then $(f(u), f(v)) \in E(A(G_2))$. If $(u, v) \in E(G_1)$ then $R_1(u, v) = B_1(u, v) = B_2(f(u), f(v)) = R_2(f(u), f(v))$. If $(u, v) \notin E(G_1)$ then

$$T_{R_1}^+(u,v) = \min(T_{A_1}^+(u), T_{A_1}^+(v)) \leq \min(T_{A_2}^+(f(u)), T_{A_2}^+(f(v))) = T_{R_2}^+(f(u), f(v)),$$

similarly others can be proved, therefore

$$T_{R_1}^+(u,v) \leq T_{R_2}^+(f(u), f(v)),\ I_{R_1}^+(u,v) \geq I_{R_2}^+(f(u), f(v)),\ F_{R_1}^+(u,v) \geq F_{R_2}^+(f(u), f(v)),$$

$T^-_{R_1}(u,v) \geq T^-_{R_2}(f(u),f(v)),\ I^-_{R_1}(u,v) \leq I^-_{R_2}(f(u),f(v)),\ F^-_{R_1}(u,v) \leq F^-_{R_2}(f(u),f(v))$.
Hence $A(G_1)$ is homomorphic to $A(G_2)$. □

**Theorem 3.5.** *Let $G_1 = (A_1, B_1)$ and $G_2 = (A_2, B_2)$ be complete BSVNGs, if $G_1$ is co-weak isomorphic to $G_2$ then $A(G_1)$ is co-weak isomorphic to $A(G_2)$.*

*Proof.* Straight forward as Theorem 3.4. is proved. □

Next we introduce the self centered and self median bipolar single valued neutrosophic graphs.

**Definition 3.11.** *Let $G = (A, B)$ be a BSVNG, a vertex $v_i \in V$ is said to be a central vertex if $r(G) = e(v_i)$. The set of all central vertices of a BSVNG G is denoted by $C(G)$, G is said to be a self centered bipolar single valued neutrosophic graph (SCBSVNG) if $r(G) = e(v_i)\ \forall v_i \in V$.*

**Example 3.3.** *Consider the BSVNG $G = (A, B)$ of $G^*$, which is given in Example 3.1 is also a self centered BSVNG.*

**Definition 3.12.** *A path cover of a BSVNG $G = (A, B)$ is the set $Q$ of paths so that every vertex of $G$ is incident to some path of $Q$.*

**Theorem 3.6.** *Every complete BSVNG $G = (A, B)$ is a self centered BSVNG and*

$$r(G) = \left(\frac{1}{T^+_{Ai}}, \frac{1}{I^+_{Ai}}, \frac{1}{F^+_{Ai}}, \frac{1}{T^-_{Ai}}, \frac{1}{I^-_{Ai}}, \frac{1}{F^-_{Ai}}\right)$$

*where $T^+_{Ai}, I^-_{Ai}$ and $F^-_{Ai}$ are minimal, $T^-_{Ai}$, $I^+_{Ai}$ and $F^+_{Ai}$ are maximal.*

*Proof.* Let $v_i \in V$ such that $T^+_{Ai}$ is least truth membership of vertex value in $G$.
**Case(i)** $\forall\ v_i - v_j$ paths $P$ having $n$ length in $G\ \forall v_j \in V$.
for $n = 1$ trivially holds, if $n > 1$, the positive $T$-strength of one edge $T^+_{Ai}$ and therefore positive $T$-length of a $v_i - v_j$ path will exceed $\frac{1}{T^+_{Ai}}$, thus positive $T$-length of path $P = l^+_T(P) > \frac{1}{T^+_{Ai}}$, hence $\delta^+_T(v_i, v_j) = \min(l^+_T(P)) = \frac{1}{T^+_{Ai}}\ \forall v_j \in V$.
**Case(ii)** Whenever $v_k \neq v_i \in V$, consider all $v_k - v_j$ paths $Q$ having $n$ length in $G$ $\forall v_j \in V$.
**Subcase(i)** Whenever $n = 1$, $T^+_B(v_k, v_j) = \min(T^+_{Ak}, T^+_{Aj}) \geq T^+_{Ai}$ since $T^+_{Ai}$ is minimal, hence positive $T$-length of $Q = l^+_T(Q) = \frac{1}{T^+_B(v_k,v_j)} \leq \frac{1}{T^+_{Ai}}$.
**Subcase(ii)** Whenever $n = 2$ then $l^+_T(Q) = \frac{1}{T^+_B(v_k,v_{k+1})} + \frac{1}{T^+_B(v_{k+1},v_j)} \leq \frac{2}{T^+_{Ai}}$ since $T^+_{Ai}$ is minimal.
**Subcase(iii)** Whenever $n > 2$ then $l^+_T(Q) \leq \frac{n}{T^+_{Ai}}$ since $T^+_{Ai}$ is minimal, hence we have $\delta^+_T(v_k, v_j) = \min(l^+_T(Q)) \leq \frac{1}{T^+_{Ai}}\ \forall v_k, v_j \in V$. Thus we have $e^+_T(v_i) = \max(\delta^+_T(v_i, v_j)) = \frac{1}{T^+_{Ai}}$ $\forall v_i \in V$. Next $r^+_T(G) = \min(e^+_T(v_i)) = \frac{1}{T^+_{Ai}}$, hence $r^+_T(G) = \frac{1}{T^+_{Ai}}$ where $T^+_A(v_i)$ is minimal. Similarly others can be proved. Hence $G$ is self centered BSVNG. □

**Remark 3.8.** *In general converse part does not hold of Theorem 3.6.*

**Example 3.4.** *Consider a BSVNG $G = (A, B)$ of $G^* = (V, E)$, where $A$ and $B$ be BSVNSs of $V = \{\alpha, \beta, \gamma, \xi\}$ and $E = \{(\alpha, \beta), (\beta, \gamma), (\gamma, \xi), (\xi, \alpha)\}$ respectively, which are defined in Table. 4*

$$\delta(\alpha, \gamma) = (11, 6, 4, -10, -6, -7),\ \delta(\beta, \xi) = (11, 6, 4, -10, -6, -7),$$
$$\delta(\alpha, \beta) = (6, 3, 2, -6, -4, -5),\ \delta(\alpha, \xi) = (5, 3, 2, -4, -2, -2),$$

$$\delta(\beta,\gamma) = (5,3,2,-4,-2,-2),\ \delta(\gamma,\xi) = (6,3,2,-6,-4,-5),$$
$$e(\alpha) = (11,3,2,-10,-2,-2),\ e(\beta) = (11,3,2,-10,-2,-2),$$
$$e(\gamma) = (11,3,2,-10,-2,-2),\ e(\xi) = (11,3,2,-10,-2,-2),$$

*here $r(G) = e(G)$. Thus $G$ is self centered BSVNG but $G$ is not complete BSVNG.*

| $A$ | $T_A^+$ | $I_A^+$ | $F_A^+$ | $T_A^-$ | $I_A^-$ | $F_A^-$ | $B$ | $T_B^+$ | $I_B^+$ | $F_B^+$ | $T_B^-$ | $I_B^-$ | $F_B^-$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\alpha$ | 1/5 | 1/3 | 1/3 | −1/5 | −1/5 | −1/5 | $(\alpha,\beta)$ | 1/6 | 1/3 | 1/2 | −1/6 | −1/4 | −1/5 |
| $\beta$ | 1/5 | 1/5 | 1/5 | −1/3 | −1/7 | −1/6 | $(\beta,\gamma)$ | 1/5 | 1/3 | 1/2 | −1/4 | −1/2 | −1/2 |
| $\gamma$ | 1/3 | 1/6 | 1/6 | −1/2 | −1/2 | −1/2 | $(\gamma,\xi)$ | 1/6 | 1/3 | 1/2 | −1/6 | −1/4 | −1/5 |
| $\xi$ | 1/4 | 1/4 | 1/4 | −1/6 | −1/3 | −1/3 | $(\xi,\alpha)$ | 1/5 | 1/3 | 1/2 | −1/4 | −1/2 | −1/2 |

Table 4. BSVNSs of SCBSVNG

**Remark 3.9.** *A BSVNG $G = (A, B)$ is self centered BSVNG if and only if $d(G) = r(G)$.*

**Remark 3.10.** *Let BSVNG $G$ with path covers $P_1, P_2, P_3, P_4, P_5$ and $P_6$, then $G$ is self centered BSVNG if and only if*

$$\delta_T^+(v_i,v_j) = d_T^+(G)\ \forall (v_i,v_j) \in P_1,\ \delta_T^-(v_i,v_j) = d_T^-(G)\ \forall (v_i,v_j) \in P_4,$$
$$\delta_I^+(v_i,v_j) = d_I^+(G)\ \forall (v_i,v_j) \in P_2,\ \delta_I^-(v_i,v_j) = d_I^-(G)\ \forall (v_i,v_j) \in P_5,$$
$$\delta_F^+(v_i,v_j) = d_F^+(G)\ \forall (v_i,v_j) \in P_3,\ \delta_F^-(v_i,v_j) = d_F^-(G)\ \forall (v_i,v_j) \in P_6.$$

**Theorem 3.7.** *Let $H = (A', B')$ be a connected self centered BSVNG, then there exists a connected BSVNG $G = (A, B)$ for which $< C(G) >$ and $H$ isomorphic and $2r(G) = d(G)$*

**Definition 3.13.** *The status in $G$ of vertex $\xi$, which is denoted by $S(\xi)$, is defined by*

$$S(\xi) = (S_T^+(\xi), S_I^+(\xi), S_F^+(\xi), S_T^-(\xi), S_I^-(\xi), S_F^-(\xi))$$

*where*

$$S_T^+(\xi) = \sum_{\eta\in V} \delta_T^+(\xi,\eta),\ S_I^+(\xi) = \sum_{\eta\in V} \delta_I^+(\xi,\eta),\ S_F^+(\xi) = \sum_{\eta\in V} \delta_F^+(\xi,\eta),$$
$$S_T^-(\xi) = \sum_{\eta\in V} \delta_T^-(\xi,\eta),\ S_I^-(\xi) = \sum_{\eta\in V} \delta_I^-(\xi,\eta),\ S_F^-(\xi) = \sum_{\eta\in V} \delta_F^-(\xi,\eta),$$

*where $S_T^+(\xi), S_I^+(\xi), S_F^+(\xi), S_T^-(\xi), S_I^-(\xi)$ and $S_F^-(\xi)$ are positive $T$-status, positive $I$-status, positive $F$-status, negative $T$-status, negative $I$-status and negative $F$-status of the vertex $\xi$, respectively. The connected BSVNG $G = (C, D)$ is called self median if every vertex has the same status.*

**Example 3.5.** *Consider the BSVNG $G = (A, B)$ of $G^* = (V, E)$, where BSVNSs $A$ and $B$ be BSVNSs of $V = \{\alpha, \beta, \gamma, \xi\}$ and $E = \{(\alpha, \beta), (\beta, \gamma), (\gamma, \xi), (\xi, \alpha), (\alpha, \gamma)\}$ respectively, which are defined in Table. 5. Then,*

$$\delta(\alpha,\gamma) = (5,3,4,-5,-2,-2),\ \delta(\beta,\gamma) = (5,4,5,-10,-3,-2),$$
$$\delta(\beta,\xi) = (13,5,7,-10,-5,-4),\ \delta(\gamma,\xi) = (8,2,3,-5,-2,-2),$$
$$S(\alpha) = (5,3,4,-5,-2,-2),\ (\beta) = (18,9,12,-20,-8,-6),$$
$$S(\gamma) = (13,6,8,-15,-5,-6),\ S(\xi) = (21,7,10,-15,-7,-6).$$

*Thus $G$ is not self median BSVNG.*

| $A$ | $T_A^+$ | $I_A^+$ | $F_A^+$ | $T_A^-$ | $I_A^-$ | $F_A^-$ |
|---|---|---|---|---|---|---|
| $\alpha$ | 1/3 | 1/4 | 1/5 | −1/3 | −1/4 | −1/5 |
| $\beta$ | 1/2 | 1/5 | 1/6 | −1/2 | −1/5 | −1/2 |
| $\gamma$ | 1/4 | 1/6 | 1/7 | −1/2 | −1/2 | −1/3 |
| $\xi$ | 1/7 | 1/3 | 1/4 | −1/3 | −1/6 | −1/5 |
| $B$ | $T_B^+$ | $I_B^+$ | $F_B^+$ | $T_B^-$ | $I_B^-$ | $F_B^-$ |
| $(\alpha,\beta)$ | 1/4 | 1/3 | 1/4 | −1/5 | −1/3 | −1/2 |
| $(\beta,\gamma)$ | 1/5 | 1/4 | 1/5 | −1/10 | −1/3 | −1/2 |
| $(\gamma,\xi)$ | 1/8 | 1/2 | 1/3 | −1/5 | −1/2 | −1/2 |
| $(\xi,\alpha)$ | 1/9 | 1/2 | 1/3 | −1/5 | −1/3 | −1/5 |
| $(\alpha,\gamma)$ | 1/5 | 1/3 | 1/4 | −1/5 | −1/2 | −1/2 |

Table 5. BSVNSs of BSVNG

**Remark 3.11.** *Let $G = (C, D)$ be a connected BSVNG of $G^* = (W, Y)$, which is even cycle then $G$ is self median BSVNG, if alternative edges have same positive and negative truth, indeterminacy and falsity membership values.*

**Example 3.6.** *The BSVNG $G$ of $G^*$, which is given in Example 3.4 is also self median BSVNG.*

## 4. Bipolar Single Valued Neutrosophic Trees

**Definition 4.1.** *The partial BSVN subgraph of BSVNG $G = (C, D)$ on a crisp graph $G^* = (V, E)$ is a BSVNG $H = (C', D')$, such that*
**(1)** *$C' \subseteq C$, that is for all $p \in V$*

$$T^+_{C'}(p) = T^+_C(p),\ I^+_{C'}(p) = I^+_C(p),\ F^+_{C'}(p) = F^+_C(p)$$

$$T^-_{C'}(p) = T^-_C(p),\ I^-_{C'}(p) = I^-_C(p),\ F^-_{C'}(p) = F^-_C(p).$$

**(2)** *$D' \subseteq D$, that is for all $pq \in E$*

$$T^+_{D'}(pq) = T^+_D(pq),\ I^+_{D'}(pq) = I^+_D(pq),\ F^+_{D'}(pq) = F^+_D(pq)$$

$$T^-_{D'}(pq) = T^-_D(pq),\ I^-_{D'}(pq) = I^-_D(pq),\ F^-_{D'}(pq) = F^-_D(pq).$$

*The BSVN subgraph of BSVNG $G = (C, D)$ of crisp graph $G^* = (V, E)$ is a BSVNG $H = (C', D')$ on a $H^* = (V', E')$, such that*
**(1)** *$C' = C$, that is for all $p \in V'$*

$$T^+_{C'}(p) = T^+_C(p),\ I^+_{C'}(p) = I^+_C(p),\ F^+_{C'}(p) = F^+_C(p)$$

$$T^-_{C'}(p) = T^-_C(p),\ I^-_{C'}(p) = I^-_C(p),\ F^-_{C'}(p) = F^-_C(p).$$

**(2)** *$D' = D$, that is for all $pq \in E$ in the edge set $E'$*

$$T^+_{D'}(pq) = T^+_D(pq),\ I^+_{D'}(pq) = I^+_D(pq),\ F^+_{D'}(pq) = F^+_D(pq)$$

$$T^-_{D'}(pq) = T^-_D(pq),\ I^-_{D'}(pq) = I^-_D(pq),\ F^-_{D'}(pq) = F^-_D(pq).$$

**Definition 4.2.** *Let $C$ be a BSVNS on $X$, the support of $C$, which is denoted by $supp(C)$, defined by $supp(C) = supp(T_C^+) \cup supp(I_C^+) \cup supp(F_C^+) \cup (T_C^-) \cup supp(I_C^-) \cup supp(F_C^-)$, where*

$$supp(T_C^+) = \{x : x \in X,\ T_C^+(x) > 0\},\ supp(I_C^+) = \{x : x \in X,\ I_C^+(x) > 0\},$$
$$supp(F_C^+) = \{x : x \in X,\ F_C^+(x) > 0\},\ supp(T_C^-) = \{x : x \in X,\ T_C^-(x) < 0\},$$
$$supp(I_C^-) = \{x : x \in X,\ I_C^-(x) < 0\},\ supp(F_C^-) = \{x : x \in X,\ F_C^-(x) < 0\}.$$

*We call $supp(T_C^+)$, $supp(I_C^+)$ and $supp(F_C^+)$ truth support, indeterminacy support and falsity support, respectively. And $supp(T_C^-)$, $supp(I_C^-)$ and $supp(F_C^-)$ are called the negative truth support, indeterminacy support and falsity support, respectively. Let $C$ be a BSVNS on $X$, the $(\xi, \eta, \zeta, \alpha, \beta, \gamma)$-level subset of $C$, which is denoted by $C^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ and defined by $C^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)} = C^\xi \cup C^\eta \cup C^\zeta \cup C^\alpha \cup C^\beta \cup C^\gamma$, where*

$$C^\xi = \{x : x \in X,\ T_C(x) \geq \xi\},\ C^\eta = \{x : x \in X,\ I_C(x) \leq \eta\},$$
$$C^\zeta = \{x : x \in X,\ F_C(x) \leq \zeta\},\ C^\alpha = \{x : x \in X,\ T_C(x) \leq \alpha\},$$
$$C^\beta = \{x : x \in X,\ I_C(x) \geq \beta\},\ C^\gamma = \{x : x \in X,\ F_C(x) \geq \gamma\}.$$

*The height of $C$, which is denoted by $h(C)$, defined by $h(C) = (h_T^+(C), h_I^+(C), h_F^+(C), h_T^-(C), h_I^-(C), h_F^-(C))$, where*

$$h_T^+(C) = \sup\{T_C^+(x) : x \in X\},\ h_I^+(C) = \inf\{I_C^+(x) : x \in X\},$$
$$h_F^+(C) = \inf\{F_C^+(x) : x \in X\},\ h_T^-(C) = \inf\{T_C^-(x) : x \in X\},$$
$$h_I^-(C) = \sup\{I_C^-(x) : x \in X\},\ h_F^-(C) = \sup\{F_C^-(x) : x \in X\}.$$

*The BSVNS $C$ is normal if there is $p \in X$ such that $C(p) = (1, 0, 0, -1, 0, 0)$. The depth of $C$, which is denoted by $d(C)$, defined by $d(C) = (d_T^+(C), d_I^+(C), d_F^+(C), d_T^-(C), d_I^-(C), d_F^-(C))$, where*

$$d_T^+(C) = \inf\{T_C^+(x) : x \in X\},\ d_I^+(C) = \sup\{I_C^+(x) : x \in X\},$$
$$d_F^+(C) = \sup\{F_C^+(x) : x \in X\},\ d_T^-(C) = \sup\{T_C^-(x) : x \in X\},$$
$$d_I^-(C) = \inf\{I_C^-(x) : x \in X\},\ d_F^-(C) = \inf\{F_C^-(x) : x \in X\}.$$

*The crisp graph of a BSVNG $G = (A, B)$ is $G^* = (A^*, B^*)$, where $A^* = supp(A)$ and $B^* = supp(B)$. Let $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)} = (A^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}, B^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)})$ where $\xi, \eta, \zeta \in [0, 1]$ and $\alpha, \beta, \gamma \in [-1, 0]$. $A^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)} = \{x : x \in V,\ T_A^+(x) \geq \xi,\ I_A^+(x) \leq \eta,\ F_A^+(x) \leq \zeta, T_A^-(x) \leq \alpha,\ I_A^-(x) \geq \beta,\ F_A^-(x) \geq \gamma\}$ is the $(\xi, \eta, \zeta, \alpha, \beta, \gamma)$-level subset of $A$ and $B^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)} = \{xy : xy \in E,\ T_B^+(xy) \geq \xi,\ I_B^+(xy) \leq \eta,\ F_B^+(xy) \leq \zeta, T_B^-(xy) \leq \alpha,\ I_B^-(xy) \geq \beta,\ F_B^-(xy) \geq \gamma\}$ is the $(\xi, \eta, \zeta, \alpha, \beta, \gamma)$-level subset of $B$. Note that $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a crisp graph.*

**Definition 4.3.** *Let $G = (A, B)$ be a BSVNG on the crisp graph $G^* = (V, E)$, the positive $T$-strength of connectedness between $x$ and $y$ in $V$ is*

$$T_B^{\infty+}(xy) = \sup\{T_B^{k+}(xy) : k = 1, \ldots, n\},$$
$$T_B^{\infty+}(xy) = \sup\{T_B^+(xv_1) \wedge \ldots \wedge T_B^+(v_{k-1}y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1, \ldots, n\}$$

*the positive $I$-strength of connectedness between $x$ and $y$ in $V$ is*

$$I_B^{\infty+}(xy) = \inf\{I_B^{k+}(xy) : k = 1, \ldots, n\},$$
$$I_B^{\infty+}(xy) = \inf\{I_B^+(xv_1) \vee \ldots \vee I_B^+(v_{k-1}y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1, \ldots, n\}$$

*and the positive $F$-strength of connectedness between $x$ and $y$ in $V$ is*

$$F_B^{\infty+}(xy) = \inf\{F_B^{k+}(xy) : k = 1, \ldots, n\},$$

$$F_B^{\infty+}(xy) = \inf\{F_B^+(xv_1) \vee \ldots \vee F_B^+(v_{k-1}y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1, \ldots, n\}$$

*The negative $T$-strength of connectedness between $x$ and $y$ in $V$ is*

$$T_B^{\infty-}(xy) = \inf\{T_B^{k-}(xy) : k = 1, \ldots, n\},$$

$$T_B^{\infty-}(xy) = \inf\{T_B^-(xv_1) \vee \ldots \vee T_B^-(v_{k-1}y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1, \ldots, n\}$$

*the negative $I$-strength of connectedness between $x$ and $y$ in $V$ is*

$$I_B^{\infty-}(xy) = \sup\{I_B^{k-}(xy) : k = 1, \ldots, n\},$$

$$I_B^{\infty-}(xy) = \sup\{I_B^-(xv_1) \wedge \ldots \wedge I_B^-(v_{k-1}y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1, \ldots, n\}$$

*and the negative $F$-strength of connectedness between $x$ and $y$ in $V$ is*

$$F_B^{\infty-}(xy) = \sup\{F_B^{k-}(xy) : k = 1, \ldots, n\},$$

$$F_B^{\infty-}(xy) = \sup\{F_B^-(xv_1) \wedge \ldots \vee F_B^-(v_{k-1}y) : x, v_1, \ldots, v_{k-1}, y \in V, k = 1, \ldots, n\}.$$

*The positive $T$-strength, $I$-strength and $F$-strength between $x$ and $y$ in $G$ is denoted by $T_G^{\infty+}(xy)$, $I_G^{\infty+}(xy)$ and $F_G^{\infty+}(xy)$ respectively. The negative $T$-strength, $I$-strength and $F$-strength between $x$ and $y$ in $G$ is denoted by $T_G^{\infty-}(xy)$, $I_G^{\infty-}(xy)$ and $F_G^{\infty-}(xy)$ respectively. Next $T_B^{'\infty+}(xy)$, $I_B^{'\infty+}(xy)$ and $F_B^{'\infty+}(xy)$ denote $T_{G-\{xy\}}^{\infty+}(xy)$, $I_{G-\{xy\}}^{a}\infty+(xy)$ and $F_{G-\{xy\}}^{\infty+}(xy)$, respectively. Also $T_B^{'\infty-}(xy)$, $I_B^{'\infty-}(xy)$ and $F_B^{'\infty-}(xy)$ denote $T_{G-\{xy\}}^{\infty-}(xy)$, $I_{G-\{xy\}}^{\infty-}(xy)$ and $F_{G-\{xy\}}^{\infty-}(xy)$, respectively. Here $G-\{xy\}$ is obtained from $G$ by removing the edge $xy$.*

**Definition 4.4.** *A bridge in BSVNG $G = (A, B)$ is said to be positive (negative) $T$-bridge, if removing the edge $xy$ decreases (increases) the positive $T$-strength of connectivity of some two vertices. A bridge in $G$ is said to be positive (negative) $I$-bridge, if removing the edge $xy$ increases (decreases) the $I$-strength of connectedness of two vertices. A bridge in $G$ is said to be positive (negative) $F$-bridge, if by removing the edge $xy$ increases (decreases) the $F$-strength of connectedness of some two vertices. A bridge in BSVNG $G$ is said to be BSVN-Bridge $xy$ if it is positive and negative $T$-bridge, $I$-bridge and $F$-bridge.*

**Definition 4.5.** *Let $G = (A, B)$ be a BSVNG on the crisp graph $G^* = (V, E)$,*
(**i**) *$xy \in E$ is called bridge if $xy$ is bridge of $G^* = (A^*, B^*)$.*
(**ii**) *$xy \in E$ is called BSVN-Bridge if*

$$T_B^{'\infty+}(uv) < T_B^{\infty+}(uv),\ I_B^{'\infty+}(uv) > I_B^{\infty+}(uv),\ F_B^{'\infty+}(uv) > F_B^{\infty+}(uv)$$

$$T_B^{'\infty-}(uv) > T_B^{\infty-}(uv),\ I_B^{'\infty-}(uv) < I_B^{\infty-}(uv),\ F_B^{'\infty-}(uv) < F_B^{\infty-}(uv)$$

*for some $uv \in E$, where $T_B^{'+}$, $I_B^{'+}$, $F_B^{'+}$, $T_B^{'-}$, $I_B^{'-}$ and $F_B^{'-}$, are $T_B^+$, $I_B^+$,$F_B^+$, $T_B^-$, $I_B^-$ and $F_B^-$, which are restricted to $V \times V - \{xy, yx\}$.*
(**iii**) *$xy \in E$ is called a weak BSVN-Bridge if there exist $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (O, h(B)]$ such that $xy$ is bridge of $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$, where $O = (0, 0, 0, 0, 0, 0)$.*
(**iv**) *$xy \in E$ is called partial SVN bridge if $xy$ is bridge for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (d(B), h(B)]$ $\cup \{h(B)\}$.*
(**v**) *$xy \in E$ is called full BSVN-Bridge if $xy$ is bridge for $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma)$ $\in (O, h(B)]$, where $0 = (0, 0, 0, 0, 0, 0)$.*

**Remark 4.1.** *Let $xy$ be a bridge in $G^*$ then $xy$ is BSVN-Bridge if and only if*

$$T_B^+(xy) > T_B^{'\infty+}(xy),\ I_B^+(xy) < I_B^{'\infty+}(xy),\ F_B^+(xy) < F_B^{'\infty+}(xy)$$

$$T_B^-(xy) < T_B^{'\infty-}(xy),\ I_B^-(xy) > I_B^{'\infty-}(xy),\ F_B^-(xy) > F_B^{'\infty-}(xy)$$

**Remark 4.2.** *An edge $xy$ is BSVN-Bridge if and only if $xy$ is not weakest bridge of any cycle.*

**Proposition 4.1.** *An edge $xy$ is BSVN-Bridge if and only if $xy$ is bridge for $G^*$ and*

$$T_B^+(xy) = h(T_B^+),\ I_B^+(xy) = h(I_B^+),\ F_B^+(xy) = h(F_B^+)$$
$$T_B^-(xy) = h(T_B^-),\ I_B^-(xy) = h(I_B^-),\ F_B^-(xy) = h(F_B^-)$$

*Proof.* Suppose that $xy$ is full bridge then $xy$ is bridge of $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ for all $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O, h(B)] = (0, h(T_B)] \times (0, h(I_B)] \times (0, h(F_B)]$. Hence $xy \in B^{h(B)}$ and so

$$T_B^+(xy) = h(T_B^+),\ I_B^+(xy) = h(I_B^+),\ F_B^+(xy) = h(F_B^+)$$
$$T_B^-(xy) = h(T_B^-),\ I_B^-(xy) = h(I_B^-),\ F_B^-(xy) = h(F_B^-)$$

since $xy$ is bridge for $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ for all $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O, h(B)]$. It follows that $xy$ is bridge for $G^*$, since $V = A^{d(B)}$ and $E = B^{d(B)}$.
Conversely: Suppose $xy$ is bridge for $G^*$ and

$$T_B^+(xy) = h(T_B^+),\ I_B^+(xy) = h(I_B^+),\ F_B^+(xy) = h(F_B^+)$$
$$T_B^-(xy) = h(T_B^-),\ I_B^-(xy) = h(I_B^-),\ F_B^-(xy) = h(F_B^-)$$

Then $xy \in B^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ for all $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O, h(B)]$, thus since $xy$ is bridge for $G^*$, $xy$ is bridge for $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ for all $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O, h(B)]$, since each $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is subgraph of $G^*$. Hence $xy$ is a full BSVN-Bridge. □

**Proposition 4.2.** *If an arc $xy$ is not in the cycle of crisp graph $G^*$, then the following conditions are equivalent.*
(**i**) $T_B^+(xy) = h(T_B^+),\ I_B^+(xy) = h(I_B^+),\ F_B^+(xy) = h(F_B^+), T_B^-(xy) = h(T_B^-),\ I_B^-(xy) = h(I_B^-), F_B^-(xy) = h(F_B^-)$.
(**ii**) *$xy$ is partial BSVN-Bridge.*
(**iii**) *$xy$ is full BSVN-Bridge.*

*Proof.* Since $xy$ is not contained in a cycle of $G^*$ and $xy$ is bridge of $G^*$. Hence by proposition 4.1, $(i) \Leftrightarrow (iii)$ obvious $(iii) \Leftrightarrow (ii)$. Next suppose that $(ii)$ holds, then $xy$ is bridge for $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ for all $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (d(B), h(B)]$ and so $xy \in B^{h(B)}$. Thus $T_B^+(xy) = h(T_B^+),\ I_B^+(xy) = h(I_B^+),\ F_B^+(xy) = h(F_B^+),\ T_B^-(xy) = h(T_B^-),\ I_B^-(xy) = h(I_B^-),\ F_B^-(xy) = h(F_B^-)$. Thus $(i)$ is true. □

**Remark 4.3.** *If $xy$ is a bridge, then $xy$ is weak BSVN-Bridge and BSVN-Bridge.*

**Proposition 4.3.** *An arc $xy$ is BSVN-Bridge if and only if $xy$ is weak BSVN-Bridge.*

*Proof.* Suppose that $xy$ is a weak BSVN-Bridge, then there exists $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O, h(B)]$ such that $xy$ is bridge for $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$. Hence by removing $xy$ it disconnects $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$, thus any path from $x$ to $y$ in $G$ has an edge $uv$ with $T_B^+(uv) < \xi,\ I_B^+(uv) > \eta,\ F_B^+(uv) > \zeta$, $T_B^-(uv) > \alpha,\ I_B^-(uv) < \beta,\ F_B^-(uv) < \gamma$. Hence by removal of arc $xy$ implies that

$$T_B^{'\infty+}(xy) < \xi \le T_B^{\infty+}(xy),\ I_B^{'\infty+}(xy) > \eta \ge I_B^{\infty+}(xy),\ F_B^{'\infty+}(xy) > \zeta \ge F_B^{\infty+}(xy)$$
$$T_B^{'\infty-}(xy) > \alpha \ge T_B^{\infty-}(xy),\ I_B^{'\infty-}(xy) < \beta \le I_B^{\infty-}(xy),\ F_B^{'\infty-}(xy) < \gamma \le F_B^{\infty-}(xy).$$

Hence $xy$ is BSVN-Bridge.
Conversely: Suppose that $xy$ is BSVN-Bridge, then there is an arc $uv$ such that by removing of $xy$ implies that

$$T_B^{'\infty+}(uv) < T_B^{\infty+}(uv),\ I_B^{'\infty+}(uv) > I_B^{\infty+}(uv),\ F_B^{'\infty+}(uv) > F_B^{\infty+}(uv)$$

$$T_B^{'\infty-}(uv) > T_B^{\infty-}(uv),\ I_B^{'\infty-}(uv) < I_B^{\infty-}(uv),\ F_B^{'\infty-}(uv) < F_B^{\infty-}(uv)$$

Hence $xy$ is on every strongest path joining $u$ and $v$ and in fact $T_B^+(uv) \geq,\ I_B^+(uv) \leq,$ $F_B^+(uv) \leq,\ T_B^-(uv) \leq,\ I_B^-(uv) \geq,\ F_B^-(uv) \geq$ this value. Thus there does not exist a path other than $xy$ connecting $x$ and $y$ in $G^{(T_B^+(xy), I_B^+(xy), F_B^+(xy), T_B^-(xy), I_B^-(xy), F_B^-(xy))}$, else this other path without $xy$ would be of strength $\geq T_B^+(xy),\ \leq I_B^+(xy),\ \leq F_B^+(xy),\ \leq T_B^-(xy),$ $\geq I_B^-(xy)$ and $\geq F_B^-(xy)$. Also it would be part of a path connecting $u$ and $v$ of strongest length, contrary to fact that $xy$ is on every such path. Hence $xy$ is on every such path. Hence $xy$ is a bridge of $G^{(T_B^+(xy), I_B^+(xy), F_B^+(xy), T_B^-(xy), I_B^-(xy), F_B^-(xy))}$ and

$$0 < T_B^+(xy) \leq h(T_B^+),\ 0 < I_B^+(xy) \leq h(I_B^+),\ 0 < F_B^+(xy) \leq h(F_B^+)$$

$$0 > T_B^-(xy) \geq h(T_B^-),\ 0 > I_B^-(xy) \geq h(I_B^-),\ 0 > F_B^-(xy) \geq h(F_B^-)$$

Thus $(T_B^+(xy), I_B^+(xy), F_B^+(xy), T_B^-(xy), I_B^-(xy), F_B^-(xy))$ are the desired $(\xi, \eta, \zeta, \alpha, \beta, \gamma)$. □

**Definition 4.6.** *Let $x \in V$,*
(**i**) *The vertex $x$ is called a cut vertex, if $x$ is a cut vertex of $G^* = (A^*, B^*)$.*
(**ii**) *The vertex $x \in V$ is called BSVN-Cut vertex if $T_B^{'\infty+}(uv) < T_B^{\infty+}(uv),\ I_B^{'\infty+}(uv)$ $> I_B^{\infty+}(uv),\ F_B^{'\infty+}(uv) > F_B^{\infty+}(uv)\ T_B^{'\infty-}(uv) > T_B^{\infty-}(uv),\ I_B^{'\infty-}(uv) < I_B^{\infty-}(uv),$ $F_B^{'\infty-}(uv), F_B^{\infty-}(uv)$ for some $u, v \in V$, where $T_B^{'}, I_B^{'}$ and $F_B^{'}$ are $T_B, I_B$ and $F_B$ restricted to $V \times V - \{xz, zx : z \in V\}$.*
(**iii**) *The vertex $x \in V$ is called a partial BSVN-Cut vertex if $x$ is a cut vertex for $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**iv**) *The vertex $x \in V$ is called a weak BSVN-Cut vertex if there exists $(\xi, \eta, \zeta, \alpha, \beta, \gamma)$ $\in (O, h(B)]$ such that $x$ is a cut vertex of $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$.*
(**v**) *The vertex $x \in V$ is called a full BSVN-Cut vertex if $x$ is a cut vertex for $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ if there exists $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (O, h(B)]$.*

**Remark 4.4.** *Let $G$ be a BSVNG such that $G^*$ is a cycle, then a vertex is BSVN cut vertex of $G$ if and only if it is a same vertex of two BSVN bridges.*

**Remark 4.5.** *If $z \in V$ is a same vertex of at least two BSVN bridges, then $z$ is a BSVN cut vertex.*

**Remark 4.6.** *If $G$ is a complete BSVNG, then $T_B^{\infty+}(uv) = T_B^+(uv),\ I_B^{\infty+}(uv) = I_B^+(uv),$ $F_B^{\infty+}(uv) = F_B^+(uv),\ T_B^{\infty-}(uv) = T_B^-(uv),\ I_B^{\infty-}(uv) = I_B^-(uv)$ and $F_B^{\infty-}(uv) = F_B^-(uv)$.*

**Remark 4.7.** *The complete BSVNG has no BSVN-Cut vertex.*

**Definition 4.7.** (**i**) *The BSVNG $G$ is called a block if $G^*$ is a block.*
(**ii**) *The BSVNG $G$ is called a block if it has no BSVN-Cut vertices.*
(**iii**) *The BSVNG $G$ is called a weak block if there exists $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (O, h(B)]$, such that $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a block.*
(**iv**) *The BSVNG $G$ is called a partial BSVN-Block if $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a block for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**v**) *The BSVNG $G$ is called a full BSVN-Block if $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is block for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma)$ $\in (O, h(B)]$.*

**Definition 4.8.** *The connected BSVNG $G$ is said to be a firm if*

$$\min\{T_A^+(x) : x \in V\} \geq \max\{T_B^+(xy) : xy \in E\},$$

$$\max\{I_A^+(x) : x \in V\} \leq \min\{I_B^+(xy) : xy \in E\},$$

$$\max\{F_A^+(x) : x \in V\} \leq \min\{F_B^+(xy) : xy \in E\},$$
$$\max\{T_A^-(x) : x \in V\} \leq \min\{T_B^-(xy) : xy \in E\},$$
$$\min\{I_A^-(x) : x \in V\} \geq \max\{I_B^-(xy) : xy \in E\},$$
$$\min\{F_A^-(x) : x \in V\} \geq \max\{F_B^-(xy) : xy \in E\}.$$

**Definition 4.9.** *Let $G$ be a connected BSVNG, then*
(**i**) *The BSVNG $G$ is said to be a cycle whenever $G^*$ is a cycle.*
(**ii**) *The BSVNG $G$ is said to be a BSVN-Cycle whenever $G^*$ is a cycle and there is a unique $pq \in E$ such that*

$$T_B^+(pq) = \min\{T_B^+(uv) : uv \in E\},\ I_B^+(pq) = \max\{I_B^+(uv) : uv \in E\},$$
$$F_B^+(pq) = \max\{F_B^+(uv) : uv \in E\},\ T_B^-(pq) = \max\{T_B^-(uv) : uv \in E\},$$
$$I_B^-(pq) = \min\{I_B^-(uv) : uv \in E\},\ F_B^-(pq) = \min\{F_B^-(uv) : uv \in E\}.$$

(**iii**) *The BSVNG $G$ is said to be a weak BSVN-Cycle if there exists $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (O, h(B)]$ such that $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a cycle.*
(**iv**) *The BSVNG $G$ is called a partial BSVN-Cycle if $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a cycle for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**v**) *The BSVNG $G$ is called a full BSVN-Cycle if $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is cycle for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma)$ $\in (O, h(B)]$.*

**Remark 4.8.** *The BSVN-Cycle $G$ is partial BSVN-Cycle if and only if $G$ is a full BSVN cycle.*

**Remark 4.9.** *The BSVNG $G$ is a full BSVN-Cycle if and only if $B$ is constant on $E$. and $G$ is a cycle.*

**Definition 4.10.** *A connected BSVNG $G = (A, B)$ is said to be a BSVN-Tree if it has a BSVN spanning subgraph $H = (A, C)$ which is a tree, where for all edges $xy$ not in $H$ satisfying $T_B^+(xy) < T_C^{\infty+}(xy)$, $I_B^+(xy) > I_C^{\infty+}(xy)$, $F_B^+(xy) > F_C^{\infty+}(xy)$, $T_B^-(xy) > T_C^{\infty-}(xy)$, $I_B^-(xy) < I_C^{\infty-}(xy)$, $F_B^-(xy) < F_C^{\infty-}(xy)$.*

**Definition 4.11.** (**i**) *The BSVNG $G$ is called a forest if $G^*$ is a forest.*
(**ii**) *The BSVNG $G = (A, B)$ is said to be a BSVN-Forest if $G$ has a BSVN spanning subgraph forest $H = (A, C)$, where all arcs $uv \in E - W$, satisfying $T_B^+(uv) < T_C^{\infty+}(uv)$, $I_B^+(uv) > I_C^{\infty+}(uv)$, $F_B^+(uv) > F_C^{\infty+}(uv)$, $T_B^-(uv) > T_C^{\infty-}(uv)$, $I_B^-(uv) < I_C^{\infty-}(uv)$, $F_B^-(uv) < F_C^{\infty-}(uv)$.*
(**iii**) *The BSVNG $G$ is called a weak BSVN-Forest if for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (O, h(B)]$ such that $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a forest.*
(**iv**) *The BSVNG $G$ is called a partial BSVN-Forest if $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a forest for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**v**) *The BSVNG $G$ is called a full BSVN-Forest if $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is forest for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma)$ $\in (O, h(B)]$.*

**Proposition 4.4.** *The BSVNG $G$ is full BSVN-Forest if and only if $G$ is forest.*

*Proof.* Suppose that $G$ is a full BSVN forest, then $G^*$ is a forest.
Conversely: Suppose that $G$ is forest, then $G^*$ is a forest and so must be $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ for all $(\xi, \eta, \zeta, \alpha, \beta, \gamma) \in (O, h(B)]$, since each $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a subgraph of $G^*$. □

**Proposition 4.5.** *The BSVNG $G$ is weak BSVN-Forest if and only if $G$ does not contain a cycle whose edges are of strength $h(B)$.*

*Proof.* Suppose that $G$ contains a cycle whose edges are of strength $h(B)$, then $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ for $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O, h(B)]$ that contains this cycle and so is not a forest, thus $G$ is not a weak BSVN-Forest.
Conversely: Suppose $G$ does not contain a cycle whose edges are of strength $h(B)$, then $G^{h(B)}$ does not contain a cycle and so it is forest. □

**Remark 4.10.** *If $G$ is a BSVN-Forest, then $G$ is a weak BSVN-Forest.*

**Theorem 4.1.** *Let $G$ be a forest and $B$ is a constant on $E$ if and only if $G$ is a full BSVN-Forest, $G^*$ and $G^{h(B)}$ have the same number of connected components, and $G$ is a firm.*

*Proof.* Suppose that $G$ is a forest and $B$ is constant on $E$, then for all $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O, h(B)]$, then $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)} = G^*$ and so $G$ is full BSVN-Forest also $G^*$ and $G^{h(B)}$ have the same number of connected components, clearly $G$ is a firm, since $B$ is constant on $E$. Converse part is obvious. □

**Corollary 4.1.** *The BSVNG $G$ is a tree and $B$ is constant on $E$ if and only if $G$ is a full BSVN-Tree and $G$ is a firm.*

**Definition 4.12.** (**i**) *The BSVNG $G$ is called a tree if $G^*$ is a tree.*
(**ii**) *The BSVNG $G = (A, B)$ is said to be a BSVN-Tree if it has a BSVN spanning subgraph $H = (A, C)$ which is a tree, where for all edges $uv \in E - W$, satisfying $T^+_B(uv) < T^{\infty+}_C(uv)$, $I^+_B(uv) > I^{\infty+}_C(uv)$, $F^+_B(uv) > F^{\infty+}_C(uv)$, $T^-_B(uv) > T^{\infty-}_C(uv)$, $I^-_B(uv) < I^{\infty-}_C(uv)$, $F^-_B(uv) < F^{\infty-}_C(uv)$.*
(**iii**) *The BSVNG $G$ is called a weak BSVN-Tree if for all $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O, h(B)]$ such that $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a tree.*
(**iv**) *The BSVNG $G$ is called a partial BSVN-Tree if $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a tree for all $(\xi,\eta,\zeta,\alpha,\beta,$ $\gamma) \in (d(B), h(B)] \cup \{h(B)\}$.*
(**v**) *The BSVNG $G$ is called a full BSVN-Tree if $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is tree for all $(\xi,\eta,\zeta,\alpha,\beta,\gamma)$ $\in (O, h(B)]$.*

**Remark 4.11.** *If $G$ is a BSVN-Tree, then $G$ is not complete BSVNG.*

**Remark 4.12.** *If $G$ is a BSVN-Tree, then arcs of spanning subgraph $H$ are the BSVN-Bridges of $G$.*

**Remark 4.13.** *If $G$ is a BSVN-Tree, then internal vertices of spanning subgraph $H$ are the BSVN-Cut vertices of $G$.*

**Remark 4.14.** *If $G$ is a BSVN-Tree, then $xy$ is BSVN-Bridge if and only if $T^{\infty+}_B(xy) = T^+_B(xy)$, $I^{\infty+}_B(xy) = I^+_B(xy)$, $F^{\infty+}_B(xy) = F^+_B(xy)$, $T^{\infty-}_B(xy) = T^-_B(xy)$, $I^{\infty-}_B(xy) = I^-_B(xy)$, $F^{\infty-}_B(xy) = F^-_B(xy)$.*

**Remark 4.15.** *The BSVNG $G$ is a BSVN-Tree if and only if there is a unique maximum spanning tree of $G$.*

**Remark 4.16.** *Let $G$ be a firm, if $G$ is a weak BSVN-Tree, then $G$ is a BSVN-Tree.*

**Definition 4.13.** (**i**) *The BSVNG $G$ is called a connected if $G^*$ is a connected.*
(**ii**) *The BSVNG $G = (A, B)$ is said to be a BSVN connected if $G$ is BSVN-Block.*
(**iii**) *The BSVNG $G$ is called a weak BSVN connected if there exists $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O, h(B)]$ such that $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a connected.*
(**iv**) *The BSVNG $G$ is called a partial BSVN connected if $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a connected for all $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (d(B), h(B)] \cup \{h(B)\}$.*

(**v**) *The BSVNG G is called a full BSVN connected if* $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ *is tree for all* $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O,h(B)]$.

**Proposition 4.6.** *If G is connected then G is weakly connected.*

*Proof.* Since $G$ is connected implies that $G^*$ is connected. Now $G^* = G^{h(B)}$ and so $G$ is weak connected. □

**Proposition 4.7.** *If G is firm and weak connected then G is connected.*

*Proof.* If $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is connected for some $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O,h(B)]$, then $G^*$ is connected, since $G$ is firm. □

**Proposition 4.8.** (**i**) *If G is a weak BSVN-Tree, then G is weak connected and G is a weak BSVN-Forest, conversely if there are* $(\xi_1,\eta_1,\zeta_1,\alpha_1,\beta_1,\gamma_1),(\xi_2,\eta_2,\zeta_2,\alpha_2,\beta_2,\gamma_2) \in (0,h(B)]$, *with* $\xi_1 < \xi_2$, $\eta_1 < \eta_2$ *and* $\zeta_1 < \zeta_2$ *such that* $G^{(\xi_1,\eta_1,\zeta_1)}$ *is a forest and* $G^{(\xi_2,\eta_2,\zeta_2)}$ *is connected, then G is weak SVN tree.*
(**ii**) *The SVNG G is a tree if and only if G is a forest and G is connected.*
(**iii**) *The SVNG G is partial SVN tree if and only if G is a partial SVN forest and G is partially connected SVNG.*
(**iv**) *The SVNG G is full SVN tree if and only if G is a full SVN forest and G is fully connected SVNG.*

*Proof.* **(i)** If $G^{(\xi,\eta,\zeta)}$ is a tree for some $(\xi,\eta,\zeta) \in (0,h(B)]$, then $G^{(\xi,\eta,\zeta)}$ is connected and is a forest. For converse, note that $G^{(\xi_2,\eta_2,\zeta_2)}$ must also be a forest, since also $G^{(\xi_2,\eta_2,\zeta_2)}$ is connected, $G^{(\xi_2,\eta_2,\zeta_2)}$ is a tree.
**(ii)**, **(iii)** and **(iv)** are obvious. □

**Proposition 4.9.** *The BSVNG G is firm if and only if* $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ *is firm for all* $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O,h(B)]$.

*Proof.* Suppose $G$ is firm, let $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O,h(B)]$, for $xy \in T^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ then

$$\xi \le T_B^+(xy) \le \min\{T_A^+(x) : x \in V\} \le \min\{T_A^+(x) : x \in T_A^{\xi+}\}$$
$$\eta \ge I_B^+(xy) \ge \max\{I_A^+(x) : x \in V\} \ge \max\{I_A^+(x) : x \in I_A^{\eta+}\}$$
$$\zeta \ge F_B^+(xy) \ge \max\{F_A^+(x) : x \in V\} \ge \max\{F_A^+(x) : x \in F_A^{\zeta+}\}$$
$$\alpha \ge T_B^-(xy) \ge \max\{T_A^-(x) : x \in V\} \ge \max\{T_A^-(x) : x \in T_A^{\xi-}\}$$
$$\beta \le I_B^-(xy) \le \min\{I_A^-(x) : x \in V\} \le \min\{I_A^-(x) : x \in I_A^{\eta-}\}$$
$$\gamma \le F_B^-(xy) \le \min\{F_A^-(x) : x \in V\} \le \min\{F_A^-(x) : x \in F_A^{\zeta-}\}$$

therefore

$$\max\{T_B^+(xy) : xy \in T_B^{\xi+}\} \le \min\{T_A^+(x) : x \in T_A^{\xi+}\}$$
$$\min\{I_B^+(xy) : xy \in I_B^{\eta+}\} \le \max\{I_A^+(x) : x \in I_A^{\eta+}\}$$
$$\min\{F_B^+(xy) : xy \in F_B^{\zeta+}\} \le \max\{F_A^+(x) : x \in F_A^{\zeta+}\}$$
$$\min\{T_B^-(xy) : xy \in T_B^{\xi-}\} \ge \max\{T_A^-(x) : x \in T_A^{\xi-}\}$$
$$\max\{I_B^-(xy) : xy \in I_B^{\eta-}\} \ge \min\{I_A^-(x) : x \in I_A^{\eta-}\}$$
$$\max\{F_B^-(xy) : xy \in F_B^{\zeta-}\} \ge \min\{F_A^-(x) : x \in F_A^{\zeta-}\}$$

thus we conclude that $B^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)*} = B^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$, $A^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)*} = A^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ and $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a firm.

Conversely: Suppose that $G^{(\xi,\eta,\zeta,\alpha,\beta,\gamma)}$ is a firm for all $(\xi,\eta,\zeta,\alpha,\beta,\gamma) \in (O, h(B)]$. Let

$$\min\{T_A^+(x) : x \in V\} = \xi_0 > 0, \ \max\{I_A^+(x) : x \in V\} = \eta_0 > 0$$
$$\max\{F_A^+(x) : x \in V\} = \zeta_0 > 0, \ \max\{T_A^-(x) : x \in V\} = \alpha_0 < 0$$
$$\min\{I_A^-(x) : x \in V\} = \beta_0 < 0, \ \min\{F_A^-(x) : x \in V\} = \gamma_0 < 0$$

next

$$\max\{T_B^+(xy) : xy \in T_B^{\xi_0+}\} \le \xi_0, \ \min\{I_B^+(xy) : xy \in I_B^{\eta_0+}\} \ge \eta_0$$
$$\min\{F_B^+(xy) : xy \in F_B^{\zeta_0+}\} \ge \zeta_0, \ \min\{T_B^-(xy) : xy \in T_B^{\alpha_0-}\} \ge \alpha_0$$
$$\max\{I_B^-(xy) : xy \in I_B^{\beta_0-}\} \le \beta_0, \ \max\{F_B^-(xy) : xy \in F_B^{\gamma_0-}\} \le \gamma_0$$

since $G^{(\xi_0,\eta_0,\zeta_0,\alpha_0,\beta_0,\gamma_0)}$ is firm and $V = A^{(\xi_0,\eta_0,\zeta_0,\alpha_0,\beta_0,\gamma_0)} = A^{(\xi_0,\eta_0,\zeta_0,\alpha_0,\beta_0,\gamma_0)*}$. Let $xy \in E - B^{(\xi_0,\eta_0,\zeta_0,\alpha_0,\beta_0,\gamma_0)*}$, then $T_B^+(xy) < \xi_0$, $I_B^+(xy) > \eta_0$, $F_B^+(xy) > \zeta_0$, $T_B^-(xy) > \alpha_0$, $I_B^-(xy) < \beta_0$ and $F_B^-(xy) > \gamma_0$. Thus

$$\max\{T_B^+(xy) : xy \in E\} \le \xi_0 = \min\{T_A^+(x) : x \in V\}$$
$$\min\{T_B^-(xy) : xy \in E\} \ge \xi_0 = \min\{T_A^-(x) : x \in V\}$$

$$\min\{I_B^+(xy) : xy \in E\} \ge \eta_0 = \max\{I_A^+(x) : x \in V\}$$
$$\max\{I_B^-(xy) : xy \in E\} \le \eta_0 = \max\{I_A^-(x) : x \in V\}$$

$$\min\{F_B^+(xy) : xy \in E\} \ge \zeta_0 = \max\{F_A^+(x) : x \in V\}$$
$$\max\{F_B^-(xy) : xy \in E\} \le \zeta_0 = \max\{F_A^-(x) : x \in V\}$$

Hence $G$ is firm. □

## 5. Conclusions

The special classes of BSVNGs, antipodal BSVNGs, eccentric BSVNGs, self centered BSVNGs and self median BSVNGs of BSVNGs were introduced here. We investigated isomorphism properties on antipodal BSVNGs, eccentric BSVNGs and self centered BSVNGs. The neutrosophic graphs have many applications in path problems, networks and computer science. The edge connectivity in BSVNG is basic concept to understand the connections of connectedness between two systems of computers. The BSVN-Bridges, BSVN-Cycles, BSVN-Trees, BSVN-Cut vertices and BSVN-Levels are introduced here, also the BSVN-Blocks and BSVN-Firms are introduced with its properties and criteria to prove the BSVNG to be firm or Block. In our future research, we will focus on antipodal, eccentric, self centered and self median interval valued neutrosophic graphs of IVNGs.

## References

[1] Malik, M. A., Hassan, A., Broumi, S. and Smarandache, F., (2016), Regular single valued neutrosophic hypergraphs, Neutrosophic Sets and Systems, 13, (1), pp. 18-23.

[2] Malik, M. A., Hassan, A., Broumi, S. and Smarandache, F., (2016), Regular bipolar single valued neutrosophic hypergraphs, Neutrosophic Sets and Systems, 13, (1), pp. 84-89.

[3] Turksen, I., (1986), Interval valued fuzzy sets based on normal forms, Fuzzy Sets and Systems, 20, (1), pp. 191-210.

[4] Atanassov, K., (1986), Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, (1), pp. 87-96.

[5] Atanassov, K. and Gargov, G., (1989), Interval valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 31, (1), pp. 343-349.

[6] Karunambigai, M. G. and Kalaivani, O. K., (2011), Self centered intuitionistic fuzzy graphs, World Applied Science Journal, 14, (12), pp. 1928-1936.

[7] Broumi, S., Talea, M., Bakali, A., Smarandache, F. and Vladareanu, L., (2016), Bipolar single valued neutrosophic graphs, Journal of New Theory, 6, (1), pp. 2149-1402.

[8] Broumi, S., Talea, M., Bakali, A., Smarandache, F. and Vladareanu, L., (2016), Computation of shortest path problem in a network with SV-Trapezoidal neutrosophic numbers, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, pp. 417-422.

[9] Broumi, S., Talea, M., Bakali, A., Smarandache, F., (2016), Applying Dijkstra algorithm for solving neutrosophic shortest path problem, Proceedings of the 2016 International Conference on Advanced Mechatronic Systems, Melbourne, Australia, pp.412-416.

[10] Broumi, S., Talea, M., Bakali, A., Smarandache, F. and Ali, M., (2016), Shortest path problem under bipolar neutrosphic setting, Applied Mechanics and Materials, 859(1), pp. 59-66.

[11] Zadeh, L., (1965), Fuzzy sets. Inform and Control, 8, (1), pp. 338-353

[12] Singh, P. K., (2016), Three way fuzzy concept lattice representation using neutrosophic set, International Journal of Machine Learning and Cybernetics, 5, (1) pp. 1-11.

[13] Sunitha, M. S. and Vijayakumar. A, (2002), Complement Of Fuzzy Graph, Indian journal of Pure Applied Math, 4, (1), pp. 1451-1464.

[14] Fathi, S., Elchawalby, H. and Salama, A. A., (2016), A neutrosophic graph similarity measures, chapter in book, New Trends in Neutrosophic Theory and Applications, 10, (3), pp. 223-230.

[15] Gani, A. N. and Malarvizhi, J., (2014), On antipodal fuzzy graph, Applied Mathematical Sciences, 43, (4), pp. 2145-2155.

[16] Wang, H., Smarandache, F., Zhang, Y. and Sunderraman, R., (2010), Single valued neutrosophic Sets, Multisspace and Multistructure, 4, (1), pp. 410-413.

[17] Karunambigai, M. G. and Kalaivani, O. K., (2011), Self centered intuitionistic fuzzy graphs, World Applied Science Journal, 14, (12), pp. 1928-1936.

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