TWMS J. App. and Eng. Math. V.10, N.3, 2020, pp. 669-676

GLOBAL EXISTENCE AND NONEXISTENCE OF SOLUTIONS FOR A KLEIN-GORDON EQUATION WITH EXPONENTIAL TYPE NONLINEAR TERM

H. TASKESEN, §

ABSTRACT. In this paper, the global existence and nonexistence of solutions for a Klein-Gordon equation, appearing in a variety of physical situations, with exponential type source term and supercritical initial energy ($E(0) > d$) are investigated in a bounded domain . In the framework of potential well, a functional including both of initial data is defined and by sign invariance of this functional the global existence of weak solutions in the case of high initial energy is proved. Moreover, under some conditions imposed on initial displacement and initial velocity a finite time blow up result is provided which extends a result given in the literature.

Keywords: Klein-Gordon equation; Exponential nonlinearity; High initial energy level; Global solution; Blow-up.

AMS Subject Classification: 35A01;35D30;35G25

1. INTRODUCTION

This paper presents the global existence and nonexistence of solutions for the initial-boundary value problem of a nonlinear Klein-Gordon equation

$$u_{tt} - \Delta u + u = f(u), x \in \Omega, \quad t > 0, \tag{1.1}$$

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x) \quad x \in \Omega, \tag{1.2}$$

$$u = 0, \quad x \in \partial\Omega, \quad t \geq 0, \tag{1.3}$$

where $f(u) = e^u - 1 + \alpha u$, $-2 < \alpha < -1$, $\Omega \subset R^2$ is a bounded domain.

The Klein-Gordon equation is a relativistic wave equation in quantum mechanics for a particle with zero spin [4]. It arises in a variety of physical situations, e.g., in modelling of dislocations in crystals, the propagation of waves in ferromagnetic materials, laser pulses in two state media [23]. The equation can be reduced to the form of a Schrödinger equation by two coupled differential equations. Eq. (1.1) has been studied from various aspects with some additional terms [1, 2, 3, 7, 8, 9, 12, 13, 14, 15, 17, 20, 22, 24, 27]. Most of the papers mentioned above studied Eq. (1.1) with polynomial nonlinearity. The exponential nonlinearity was only considered in the papers of Saanouni and Mahouachi [15, 13, 14].

Van Yuzuncu Yil University, Faculty of Science, Department of Mathematics, Van, Turkey.
e-mail: kayaalphatice@gmail.com; ORCID: http://orcid.org/0000-0003-1058-0507.

§ Manuscript received: December 5, 2018; accepted: September 3, 2019.

TWMS Journal of Applied and Engineering Mathematics, Vol.10, No.3 © Işık University, Department of Mathematics, 2020; all rights reserved.

Let us comment on the studies mentioned above. The global existence and blow-up of solutions are established for problem (1.1)-(1.3) with a nonlinear term of polynomial growth in [2]. The author proved the blow-up of solutions for $E(0) \leq 0$. In [22], a blow up result was given for Eq. (1.1) with arbitrary positive energy ($E(0) > 0$) and polynomial nonlinearity. Kutev et. al. [9] improved the results of [22] for Eq. (1.1) with combined power-type nonlinearities. In [24] and [27], the global existence, asymptotic behavior and blow-up of solutions for Eq. (1.1) with different types of damping terms and polynomial nonlinearity are studied. A finite time blow-up phenomenon is also exhibited for arbitrary positive energy in [27]. Taskesen and Polat [17] treated Eq. (1.1) with a damping term, polynomial nonlinearity and arbitrary positive energy. They obtained a global existence result by using a modified potential well method. Aloui et. al. [1] proved the exponential decay of the total energy for Eq. (1.1) with a damping term and defocusing nonlinearity of arbitrary growth. In [13] and [14], global well-posedness and linearization of Eq. (1.1) was studied with exponential type nonlinearities. A finite time blow up result was given in [15] for arbitrary positive energy. In this study, we fix our attention on two questions:

– Are there any global solutions when $E(0) > 0$, i.e., in the case of high initial energy?
– Can we extend the blow up result obtained in [15]?

To the best of our knowledge, the question of global existence for problem (1.1)-(1.3) with both the exponential source term and high initial energy has not been treated previously. The argument we use in this paper, the potential well method, was first introduced in [16], and recently was developed by Kutev et. al. [10] to establish global existence of a Boussinesq-type equation in the case of high initial energy. Later, the modified method of Kutev et. al. was applied to prove global existence of some evolution equations in [18, 19, 21, 26].

Finite time blow-up of solutions for parabolic equations with arbitrary positive energy was considered in some papers [5, 25] by comparison principle. Due to the lack of comparison principle for hyperbolic equations, it cannot be applied to wave equation. The shortcoming of comparison principle was overcame by Gazzola and Squassina [6] for a semilinear wave equation of the form

$$u_{tt} - \Delta u - \omega \Delta u_t + \mu u_t = |u|^{p-2} u$$

by imposing some conditions on initial data when $\omega = 0$ and $\mu \geq 0$. Then the blow-up of solutions for arbitrary positive energy was considered in many papers for different type of evolution equations. The problem of blow up of solutions on a bounded domain for (1.1)-(1.3) was considered in the paper of Saanouni [15] under the following conditions:

$$I(0) < 0, J_\varepsilon(0) < 0, \int_\Omega u_0 u_1 dx > 0 \text{ and } |u_0|^2 \geq \frac{2}{\varepsilon}\left((2+\varepsilon) E(0) + |b| |\Omega|\right), \qquad (1.4)$$

where I is the Nehari functional, $E(0) > 0$, $b = \inf_{x \in R}(xf(x) - (2+\varepsilon) F(x))$ and F is the primitive of f. The second aim of this paper is to prove the blow-up of solutions by imposing more general conditions than (1.4). The conditions depend both u_0 and u_1 as in the global existence.

The paper is organized as follows. Section 2 contains some preliminaries and a local existence result, proved in [13]. The sign invariance of the new functional and global existence of solutions are proved in Section 3. In Section 4, a blow-up result is given.

Throughout the study, $\|f\|_p$, $\|f\|$ and $\|f\|_\infty$ will be used instead of norms of $L^p(\Omega)$, $L^2(\Omega)$ and $L^\infty(\Omega)$, respectively. We also use the following abbreviations:

$C^k = C^k(\Omega)$, $H^m = W^{m,2}$, $H_0^m = W_0^{m,2}$, $(\cdot,\cdot)$ denotes the L^2inner product.

2. Preliminaries

We start this section by introducing the energy related to problem (1.1)-(1.3). Moreover, we present a result concerned with the local existence of solutions for problem (1.1)-(1.3) which is proved by Mahouachi [13]. We also give a standard result about blow up of solutions that is due to Levine [11].

Let us start by defining the energy functional

$$E(t) = \frac{1}{2}\left[\|u_t(t)\|^2 + \|u(t)\|^2 + \|\nabla u(t)\|^2\right] - \int_\Omega F(u)\,dx, \tag{2.1}$$

where $F(u) = \int\limits_0^u f(s)\,ds = e^u - 1 - u + \frac{\alpha}{2}u^2$ is the primitive of the nonlinear term $f(u)$,

Then we define the Nehari functional

$$I(u) = \|u(t)\|^2 + \|\nabla u(t)\|^2 - \int_\Omega u f(u)\,dx, \tag{2.2}$$

and

$$J(u) = \frac{1}{2}\left(\|u(t)\|^2 + \|\nabla u(t)\|^2\right) - \int_\Omega F(u)\,dx,$$

$$d = \inf_{u\in\mathcal{N}} J(u),$$

where $\mathcal{N} = \left\{u \in H_0^1(\Omega) \mid I(u) = 0, \nabla u \neq 0\right\}$, d and $J(u)$ describes the depth of potential well and the potential energy, respectively.

In the following we give a well known condition, generally used in elliptic problems, imposed on the source term. The following condition is also known as Ambrosetti–Rabinowitz condition.

Lemma 2.1. *For any $-1 < \alpha < -2$, there exists $\eta > 2$ such that*

$$sf(s) \geq \eta F(s) \geq 0, \quad \text{for any } s \in R. \tag{2.3}$$

Proposition 2.1. [13] *Let $u_0 \in H_0^1$, $u_1 \in L^2$, and $f \in C^1$ satisfies the following condition*

$$(H)\quad \begin{cases} f(0) = f'(0) = 0, \\ \forall\beta > 0, \exists R_\beta > 0 \text{ s.t. } |f(u) - f(v)|^2 \leq R_\beta |u-v|^2\left(e^{\beta u^2} - 1 + e^{\beta v^2} - 1\right). \end{cases}$$

Then problem (1.1)-(1.3) possesses a unique maximal solution $u \in C\left([0,T^*), H_0^1(\Omega)\right) \cap C^1\left([0,T^*), L^2(\Omega)\right)$. Moreover, the energy related with problem (1.1)-(1.3)

$$E(t) = \frac{1}{2}\left[\|u_t(t)\|^2 + \|u(t)\|^2 + \|\nabla u(t)\|^2\right] - \int_\Omega F(u)\,dx = E(0)$$

is conserved.

Lemma 2.2. [11] *Assume that the following inequality holds for a positive twice differentiable function $\Psi(t)$ on $t \geq t_1 \geq 0$*

$$\Psi''(t)\Psi(t) - (1+\upsilon)\left(\Psi'(t)\right)^2 \geq 0$$

for some $\upsilon > 0$. If $\Psi(t_1) > 0$ and $\Psi'(t_1) > 0$, then $\Psi(t)$ tends to infinity as

$$t \to t_2 \leq t_1 + \frac{\Psi(t_1)}{\upsilon\Psi'(t_1)}.$$

3. Existence of Global Solutions

This section is devoted to the global existence of weak solutions of problem (1.1)-(1.3). We prove firstly the sign invariance of a functional, which includes both u_0 and u_1. The sign invariance of this functional plays a key role in the proof of main theorem about existence of global solution.

$$\begin{aligned} K(u,t) &= \|u(t)\|^2 + \|\nabla u(t)\|^2 - \int_\Omega u f(u)\,dx - \|u_t\|^2 \\ &= I(u) - \|u_t\|^2 . \end{aligned} \tag{3.1}$$

Theorem 3.1. *Let $u_0 \in H_0^1(\Omega)$, $u_1 \in L^2(\Omega)$, and $E(0) > 0$. Assume that*

$$(u_0, u_1) + \frac{1}{2}\|u_0\|^2 \le -\frac{\eta}{\eta+2}E(0). \tag{3.2}$$

Then $K(u,t)$ is positive for every $t \in [0,\infty)$, provided that $K(u,0)$ is positive.

Proof. We carry out the proof by modifying a blow up technique [11]. For this purpose, we define

$$\theta(t) = \|u\|^2 .$$

Then we have

$$\theta'(t) = 2(u_t, u),$$

$$\begin{aligned} \theta''(t) &= 2\|u_t\|^2 + 2(u_{tt}, u) \\ &= -2\|u(t)\|^2 - 2\|\nabla u(t)\|^2 + 2\int_\Omega u f(u)\,dx + 2\|u_t\|^2 \\ &= -2K(u,t). \end{aligned}$$

Suppose that there exists a time $t' > 0$ such that $K(u,t') = 0$, and t' is the first time with this property. $\theta''(t) < 0$ results in $\theta'(t)$ is strictly decreasing on $[0,t')$. It follows from (3.2) that $\theta'(0) < 0$ and therefore $\theta'(t) < 0$ in $[0,t']$. On account of this, we conclude that $\theta(t)$ is strictly decreasing on $[0,t']$. By (3.2) for all $t \in [0,t')$, we have

$$\begin{aligned} \theta(t) &< \|u_0\|^2 \\ &< -2(u_0,u_1) - \frac{2\eta}{\eta+2}E(0). \end{aligned}$$

From the continuity of θ in t, we get

$$\theta(t') < -2(u_0,u_1) - \frac{2\eta}{\eta+2}E(0).$$

By the conservation of energy, Lemma 2.1 and $K(u,t')=0$, we have

$$\begin{aligned}
E(0) &= \frac{1}{2}\left[\|u_t(t')\|^2+\|u(t')\|^2_{H_0^1}\right]-\int_\Omega F(u(t',x))\,dx \\
&\geq \frac{1}{2}\left[\|u_t(t')\|^2+\|u(t')\|^2_{H_0^1}\right]-\frac{1}{\eta}\int_\Omega uf(u(t',x))\,dx \\
&= \frac{1}{2}\|u_t(t')\|^2+\left(\frac{1}{2}-\frac{1}{\eta}\right)\|u(t')\|^2_{H_0^1}+\frac{1}{\eta}I(u(t')) \\
&= \left(\frac{1}{2}+\frac{1}{\eta}\right)\|u_t(t')\|^2+\left(\frac{1}{2}-\frac{1}{\eta}\right)\|u(t')\|^2_{H_0^1} \\
&\geq \left(\frac{1}{2}+\frac{1}{\eta}\right)\|u_t(t')\|^2. \qquad (3.3)
\end{aligned}$$

Then we obtain

$$\begin{aligned}
E(0) &\geq \left(\frac{1}{2}+\frac{1}{\eta}\right)\|u_t(t')\|^2 \\
&= \frac{\eta+2}{2\eta}\left[\|u_t(t')+u(t')\|^2-2(u_t(t'),u(t'))-\|u(t')\|^2\right].
\end{aligned}$$

Since $\theta(t)$ and $\theta'(t)$ are strictly decreasing, we get

$$E(0)\geq\frac{\eta+2}{\eta}\left[-(u_0,u_1)-\frac{1}{2}\|u_0\|^2\right].$$

This contradicts with (3.2), hence the proof of the theorem is completed. □

Theorem 3.2. *Let $u_0\in H_0^1(\Omega)$, $u_1\in L^2(\Omega)$. Assume that condition (3.2) holds, $E(0)>0$ and $K(u,0)>0$. Then, for problem (1.1)-(1.3) u is a global weak solution .*

Proof. Since $K(u,t)$ is invariant under the flow of (1.1)-(1.3), we have $I(u)>0$ for every $t>0$. By the energy identity and Lemma 2.1, we have

$$\begin{aligned}
E(0) &\geq \frac{1}{2}\left[\|u_t\|^2+\|u\|^2_{H_0^1}\right]-\frac{1}{\eta}\int_\Omega uf(u)\,dx \\
&= \frac{1}{2}\|u_t\|^2+\left(\frac{1}{2}-\frac{1}{\eta}\right)\|u\|^2_{H_0^1}+\frac{1}{\eta}I(u) \\
&\geq \frac{1}{2}\|u_t\|^2+\left(\frac{1}{2}-\frac{1}{\eta}\right)\|u\|^2_{H_0^1}.
\end{aligned}$$

This yields the boundedness of $\|u\|_{H_0^1}$ and $\|u_t\|_{L^2}$ for every $t>0$. The combination of the local existence theorem given in Section 2 and the above estimate yield existence of global solution. Thus we complete the proof. □

4. Nonexistence (Blow-up) of Solutions

In this section, we improve the finite time blow-up result given by Saanouni [15].

Theorem 4.1. *Let $u_0\in H_0^1(\Omega)$, $u_1\in L^2(\Omega)$. Assume that the following statements are satisfied.*

$$\|u_0\|^2>0, \qquad (4.1)$$

$$(u_0,u_1)\geq 0, \qquad (4.2)$$

$$(u_0,u_1)-\frac{1}{2}\|u_0\|^2>\eta E(0)>0. \qquad (4.3)$$

Then the weak solution u of (1.1)-(1.3) blows up in finite time.

Proof. The theorem can be proved arguing by contradiction. Let us suppose that the solution u of problem (1.1)-(1.3) is global, and define

$$M(t) = (u, u_t) - \frac{1}{2}\|u\|^2 - \eta E(0).$$

Then direct computations, the energy identity and (2.3) yield

$$\begin{aligned} M'(t) &= (u, u_{tt}) + \|u_t\|^2 - (u, u_t) \\ &= -\|u(t)\|^2 - \|\nabla u(t)\|^2 + \int_\Omega u f(u)\, dx + \|u_t\|^2 - (u, u_t) \\ &\geq \left(\frac{\eta+2}{2}\right)\|u_t\|^2 + \left(\frac{\eta-2}{2}\right)\|u(t)\|_{H^1}^2 - \eta E(0) - (u, u_t) \\ &\geq \left(\frac{\eta+2}{2}\right)\|u_t\|^2 + \left(\frac{\eta-2}{2}\right)\|u(t)\|^2 - \frac{1}{2}\|u(t)\|^2 - \eta E(0) - (u, u_t). \end{aligned} \tag{4.4}$$

On the other hand, making use of Cauchy inequality, we obtain

$$\left(\frac{\eta+2}{2}\right)\|u_t\|^2 + \left(\frac{\eta-2}{2}\right)\|u(t)\|^2 \geq \sqrt{(\eta+2)(\eta-2)}\,|(u, u_t)|.$$

Employing the above inequality and Sobolev inequality in (4.4), we have

$$\begin{aligned} M'(t) &\geq \sqrt{(\eta+2)(\eta-2)}\,(u, u_t) - \frac{1}{2}\|u\|^2 - \eta E(0) - (u, u_t) \\ &= \left(\sqrt{(\eta+2)(\eta-2)} - 1\right)(u, u_t) - \frac{1}{2}\|u\|^2 - \eta E(0) \\ &\geq (u, u_t) - \frac{1}{2}\|u\|^2 - \eta E(0) \\ &= M(t) \end{aligned}$$

From the above inequality we infer

$$M(t) \geq M(0)\, e^t, \quad t \geq 0.$$

Thus, by the assumption (4.3), we get

$$\lim_{t\to+\infty} M(t) = +\infty.$$

By means of Cauchy inequality, we have

$$(\eta+2)\|u_t\|^2 + (\eta-2)\left(\|u(t)\|^2 + \|\nabla u(t)\|^2\right) - \eta E(0) \to \infty \text{ as } t \to \infty.$$

So, there exist $0 < \zeta < 1$, $\varepsilon > 0$, $t_0 > 0$ such that the following assertions hold:
i) $\zeta(\eta+2) > 4 + \varepsilon$, $\eta > 2 + \frac{\varepsilon}{2}$,
ii) $(\eta+2)\|u_t\|^2 + (\eta-2)\left(\|u(t)\|^2 + \|\nabla u(t)\|^2\right) - 2\eta E(0)$
$\geq \zeta\left[(\eta+2)\|u_t\|^2 + (\eta-2)\left(\|u(t)\|^2 + \|\nabla u(t)\|^2\right)\right]$, $\forall t > t_0$.
We previously defined

$$\theta(t) = \|u(t)\|^2.$$

Then

$$\theta'(t) = 2(u, u_t).$$

By virtue of the above assertions for $t_0 < t < T$, we have

$$\begin{aligned}\theta''(t) &= 2(u, u_{tt}) + 2\|u_t\|^2 \\ &\geq (\eta+2)\|u_t\|^2 + (\eta-2)\left(\|u(t)\|^2 + \|\nabla u(t)\|^2\right) - 2\eta E(0) > M'(t) \\ &> \zeta\left[(\eta+2)\|u_t\|^2 + (\eta-2)\left(\|u(t)\|^2 + \|\nabla u(t)\|^2\right)\right] \\ &> (4+\varepsilon)\|u_t\|^2.\end{aligned}$$

The following inequality can be easily verified by simple calculations

$$\left[\theta'(t)\right]^2 \leq 4\theta(t)\|u_t(t)\|^2, \quad t_0 < t < T.$$

So

$$\theta(t)\theta''(t) - \left(1 + \frac{\varepsilon}{4}\right)\left[\theta'(t)\right]^2 > 0.$$

By Lemma 2.3 we conclude that for sufficiently large $T > t_0$,

$$\lim_{t \uparrow T} \theta(t) = +\infty.$$

We get a contradiction. Thus the proof is completed. □

5. Conclusions

We investigated the existence of global solution and blow up of solution for the Klein-Gordon equation with exponential source term. The proofs are provided for high inital energy case by a modified potential well method. Although the blow up of solution was given before with arbitrary initial energy, the present article improves the results mentioned in the introduction.

References

[1] Aloui, L., Ibrahim, S., Nakanishi, K. (2010), Exponential energy decay for damped Klein–Gordon equation with nonlinearities of arbitrary growth, Communications in Partial Differential Equations, 36(5) pp. 797-818.

[2] Cazenave, T., (1985), Uniform estimates for solutions of nonlinear Klein-Gordon equations. J. Funct. Anal., 60, pp. 36-55.

[3] Changxing, M., Bo, Z., Daoyuan, F., (2004), Global well-posedness for the Klein-Gordon equation below the energy norm. J. Partial Diff. Eqs., 17, pp. 97–121.

[4] Feshbach, H., Villars, F., (1958), Elementary relativistic wave mechanics of spin 0 and spin 1/2 particles. Rev. Mod. Phys. 30(1), pp. 24-45.

[5] Gazzola, F., Weth, T., (2005), Finite time blow-up and global solutions for semilinear parabolic equations with initial data at high energy level. Differential Integral Equations, 18, pp. 961–990.

[6] Gazzola, F., Squassina, M., (2006), Global solutions and finite time blow up for damped semilinear wave equations. Ann. Inst. H. Poincar e Anal. Non Lineaire, 23, 185–207.

[7] Komech, A.I., Kopylova, E.A. (2010), Weighted energy decay for 3D Klein–Gordon equation. J. Differential Equations, 248, pp. 501–520.

[8] Korpusov, M.O., (2012), Blow-up of a positive energy solution of model wave equations in nonlinear dynamics. Theoretical and Mathematical Physics, 171(1), pp. 421–434.

[9] Kutev, N., Kolkovska, N., Dimova, M., (2016), Sign-preserving functionals and blow-up to Klein–Gordon equation with arbitrary high energy. Applicable Analysis, 95 (4), pp. 860-873.

[10] Kutev, N., Kolkovska, N., Dimova, M., (2013), Global existence of Cauchy problem for Boussinesq paradigm equation. Comput. Math. Appl., 65, pp. 500-511.

[11] Levine, H.A., (1974), Instability and nonexistence of global solutions to nonlinear wave equations of the form $Pu = u_{tt} + Fu$. Transactions of the American Mathematical Society, 192, pp. 1–21.

[12] Liu, Y., Ohta, M., Todorova, G., (2007), Strong instability of solitary waves for nonlinear Klein–Gordon equations and generalized Boussinesq equations. Ann. I. H. Poincaré, AN 24, pp. 539–548.

[13] Mahouachi, O., Saanouni, T., (2011), Well and ill-posedness issues for a class of 2D wave equation with exponential growth. J. Partial Differ. Equ., 24(4), pp. 361–384.
[14] Mahouachi, O., Saanouni, T., (2010), Global well posedness and linearization of a semilinear wave equation with exponential growth. Georgian Math. J., 17, pp. 543–562.
[15] Saanouni, T., (2015), A blowing up wave equation with exponential type nonlinearity and arbitrary positive energy. J. Math. Anal. Appl., 421, pp. 444–452.
[16] Sattinger, D.H., (1968), On global solution of nonlinear hyperbolic equations. Arch. Rational Mech. Anal., 30, pp. 148-172.
[17] Polat, N., Taskesen, H., (2014), On the existence of global solutions for a nonlinear Klein-Gordon equation. Filomat, 28(5), pp. 1073-1079.
[18] Taskesen, H., Polat, N., (2015), Existence results for a nonlinear Timoshenko equation with high initial energy,AIP Conference Proceedings, 1676, 020023; doi: 10.1063/1.4930449.
[19] Taskesen, H., Polat, N., Ertaş, A. (2012), On global solutions for the Cauchy problem of a Boussinesq-type equation. Abst. Appl. Anal,. 2012, 10 pages, Doi:10.1155/2012/535031.
[20] Wang, B., (1999), Scattering of solutions for critical and subcritical nonlinear Klein-Gordon equation in H^s. Discrete and Continuous Dynamical Systems, 5(4), pp. 753-763.
[21] Wang, S., Su, X., (2016), Global existence and nonexistence of the initial-boundary value problem for the dissipative Boussinesq equation. Nonlinear Anal., 134, pp. 164-188.
[22] Wang, Y., (2008), A sufficient condition for finite time blow up of the nonlinear Klein-Gordon equations with arbitrary positive initial energy. Proc. Am. Math. Soc., 136, pp. 3477–3482 .
[23] Whitham, G.B., (1974), Linear and nonlinear waves. John Wiley, New York.
[24] Xu, R., (2010), Global existence, blow up and asymptotic behaviour of solutions for nonlinear Klein–Gordon equation with dissipative term. Math. Meth. Appl. Sci., 33, pp. 831–844.
[25] Xu, R., Su, J. (2013), Global existence and finite time bow-up for a class of semilinear pseudo-parabolic equations. J. Funct. Anal., 264, pp. 2732–2763.
[26] Xu, R., Yang, Y., Liu, B., Shen, J., Huang, S., (2015), Global existence and blow-up of solutions for the multidimensional sith-order "good" Boussinesq equation. Z. Angew. Math. Phys., 66, pp. 955-976.
[27] Xu, R., Ding, Y., (2013), Global solutions and finite time blow up for damped Klein-Gordon equation. Acta Mathematica Scientia, 33B(3), pp. 643–652.

Hatice Taskesen is an assistant professor in the Department of Mathematics at Van Yuzuncu Yil University. She received her Ph.D. degree from Dicle University in 2012. Her research interests include partial differential equations (PDEs) and stochastic PDEs , especially the behavior of solutions such as existence and uniqueness, decay, and blow-up for initial/initial-boundary value problem of PDEs and stochastic PDEs.