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# RADIATION OF ACOUSTIC WAVES FROM A CIRCUMFERENTIAL SLOT ON A CIRCULAR DUCT

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ABSTRACT. Analytical solution is studied for a rigid circular waveguide which has a finite gap on its outer wall. Following the Wiener-Hopf technique, problem is formulated as a Fredholm integral equations. Asymptotic evaluations of integrals are carried out and the field terms are determined explicitly. Finally, the influence of the different parameters (gap length, width of the waveguide, etc.) on the radiation are illustrated numerically.

Keywords: Fourier transform, Wiener-Hopf technique, integral equations, acoustic radiation, coaxial waveguide.

AMS Subject Classification: 78A40, 47A68, 42B10.

## 1. INTRODUCTION

Acoustic wave radiation along duct systems is a basic problem and has been extensively studied in the literature because of its use in many applications, such as noise reduction in exhaust systems, design of aircraft jet and turbofan engines, room ventilators, etc. Therefore, it is essential to investigate more accurate mathematical models for such engineering problems. In order to do that different physical structures including expansion chambers, circumferential slots, acoustically absorbent materials and different techniques including Wiener-Hopf, mode matching, generalized scattering-matrix have been investigated in literature $[1-5]$.

As is well known, using a reactive or a dissipative silencer is an efficient method of reducing noise in duct systems. When a sound wave propagates in cylindrical ducts, sudden area changes on the geometry or coated duct walls with an acoustically absorbent material can help to reduce energy in the transmitted wave. The abrupt changes in cross-sectional area are formed as a reactive silencer and are widely analyzed in literature $[6-9]$. On the other hand, acoustically absorbent lining which was proven to be a very useful method acts as a dissipative silencer since it dissipates acoustic energy into heat energy [10]. Recently, the radiation of acoustic waves from semi-infinite coated pipe, which also acts as a dissipative silencer, is analyzed rigorously [11]. In [12, 13], the effect of non-uniform liners on sound propagation is investigated. These works showed that a

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non-uniform liner was more effective in reducing noise compared to the uniform liner. Then, reflection and transmission matrices in a non-uniformly lined waveguide with a sudden area changes are determined in [14] .

The case in which a circumferential slot exists on the outer wall of a circular duct is studied rigorously in this work. Finite gap on the outer wall can act as a reactive silencer. Our aim is to reveal the effect of a finite gap to the sound propagation. In order to achieve this goal, Wiener-Hopf technique, which is one of the most powerful analytical techniques for analyzing the scattering problems, is used. The boundary-value problem has been stated in the form of a modified Wiener-Hopf equation by considering Fourier transform of the Helmholtz equation, boundary conditions and continuity relations. After classical factorization and decomposition procedures Wiener-Hopf equation is reduced into a pair of Fredholm integral equations and then solved approximately. The effects of the physical parameters to the radiated field and the reflection and transmission coefficients have been shown graphically. Finally, analytical derivations are compared to annular duct with an infinite centerbody and excellent agreement is observed.

The time convention $e^{-iwt}$ is suppressed throughout this paper.

## 2. ANALYSIS

Consider the acoustic plane wave propagation along a rigid circular duct which has a finite large gap on its outer wall as shown in Fig. 1. For analysis purposes, it is convenient

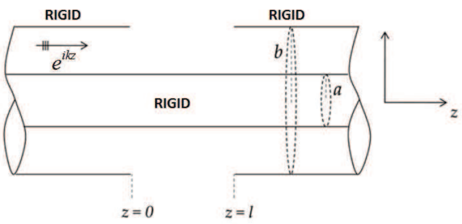


FIGURE 1. The physical structure of the problem.

to express the total field as follows:

$$u_T(\rho,z) = \begin{cases} u_i(\rho,z)+u_1(\rho,z) & , \quad a<\rho<b, \quad z\in(-\infty,\infty) \\ u_2(\rho,z) & , \quad \rho>b, \quad z\in(-\infty,\infty) \end{cases} \tag{1}$$

where $u_i(\rho,z)$ is the incident sound wave given by

$$u_i(\rho,z) = e^{ikz} \tag{2}$$

with $k=w/c$ being the wavenumber, which is assumed to have a small positive imaginary part corresponding to a slightly lossy medium. The lossless case can then be obtained by making Im$(k)$ $\rightarrow$0 at the end of the analysis. In Eq. (1) , $u_1(\rho,z)$ and $u_2(\rho,z)$ are the unknown fields which satisfy the Helmholtz equation

$$\left[\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{\partial}{\partial z^2}+k^2\right]u_j(\rho,z)=0 \quad , \quad j=1,2 \tag{3}$$

is to be determined with the help of the following boundary and continuity conditions:

$$\frac{\partial}{\partial\rho}u_1(a,z)=0 \quad , \quad z\in(-\infty,\infty) \tag{4}$$

$$\frac{\partial}{\partial\rho}u_1(b,z)=0 \quad , \quad z\in(-\infty,0)\cup(l,\infty) \tag{5}$$

$$\frac{\partial}{\partial\rho}u_2(b,z)=0 \quad , \quad z\in(-\infty,0)\cup(l,\infty) \tag{6}$$

$$\frac{\partial}{\partial\rho}u_2(b,z)-\frac{\partial}{\partial\rho}u_1(b,z)=0 \quad , \quad z\in(0,l) \tag{7}$$

$$u_2(b,z)-u_1(b,z)=e^{ikz} \ , \quad z\in(0,l) \tag{8}$$

In order to obtain a unique solution to the problem , one has to take into account the following radiation and edge conditions [15]:

$$\frac{\partial u_2}{\partial r}-iku_2=O(r^{-1/2}), \quad r=\sqrt{\rho^2+z^2}\to\infty \tag{9}$$

$$u_T(b,z)=O(1), \quad z\to 0,l \tag{10}$$

$$\frac{\partial}{\partial\rho}u_T(b,z)=O(z^{-\frac{1}{3}}), \quad z\to 0,l \tag{11}$$

2.1. **Derivation of the Modified Wiener-Hopf Equation.** Since the $u_1(\rho,z)$ satisfies the Helmholtz equation in the range $a<\rho<b$ and $z\in(-\infty,\infty)$, its Fourier transform with respect to z gives

$$\left[\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+K^2(\alpha)\right]F(\rho,\alpha)=0 \tag{12}$$

where

$$F(\rho,\alpha)=F_-(\rho,\alpha)+F_1(\rho,\alpha)+e^{i\alpha l}F_+(\rho,\alpha) \tag{13}$$

and

$$K(\alpha)=\sqrt{k^2-\alpha^2} \tag{14}$$

The square-root function is defined in the complex $\alpha-$plane, cut along $\alpha=k$ to $\alpha=k+i\infty$ and $\alpha=-k$ to $\alpha=-k-i\infty$ , such that $K(0)=k$. $F_-(\rho,\alpha)$, $F_1(\rho,\alpha)$ and $F_+(\rho,\alpha)$ are defined by

$$F_-(\rho,\alpha)=\int_{-\infty}^{0}u_1(\rho,z)e^{i\alpha z}dz \tag{15}$$

$$F_1(\rho,\alpha)=\int_{0}^{l}u_1(\rho,z)e^{i\alpha z}dz \tag{16}$$

$$F_+(\rho,\alpha)=\int_{l}^{\infty}u_1(\rho,z)e^{i\alpha(z-l)}dz \tag{17}$$

Owing to the analytical properties of Fourier integrals, $F_-(\rho,\alpha)$ and $F_+(\rho,\alpha)$ are regular functions in the lower half-plane Im$(\alpha)$ <Im$(k)$ and in the upper half-plane Im$(\alpha)$ >Im$(-k)$ , respectively, while $F_1(\rho,\alpha)$ is an entire function of $\alpha$. The general solution of (12) satisfying the radiation condition reads

$$F(\rho,\alpha)=-A(\alpha)\frac{J_0(K\rho)}{KJ_1(Ka)}-B(\alpha)\frac{Y_0(K\rho)}{KY_1(Ka)} \tag{18}$$

where $A(\alpha)$ and $B(\alpha)$ are unknown spectral coefficients. From equations (4) and (5) we have

$$B(\alpha) = -A(\alpha) \tag{19}$$

and

$$A(\alpha) = F_1^{'}(b,\alpha)\frac{J_1(Ka)Y_1(Ka)}{[J_1(Kb)Y_1(Ka) - J_1(Ka)Y_1(Kb)]} \tag{20}$$

where the dot specifies the derivative with respect to $\rho$. Substituting (19) and (20) into (18), one obtains

$$F(\rho,\alpha) = \frac{F_1^{'}(b,\alpha)[Y_0(K\rho)J_1(Ka) - J_0(K\rho)Y_1(Ka)]}{K[J_1(Kb)Y_1(Ka) - J_1(Ka)Y_1(Kb)]} \tag{21}$$

On the other hand, the field $u_2(\rho,z)$ satisfies the Helmholtz equation in the region $\rho > b$, $z \in (-\infty,\infty)$ whose Fourier transform with respect to z gives

$$\left[\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + K^2(\alpha)\right] G(\rho,\alpha) = 0 \tag{22}$$

with

$$G(\rho,\alpha) = G_-(\rho,\alpha) + G_1(\rho,\alpha) + e^{i\alpha l}G_+(\rho,\alpha) \tag{23}$$

where

$$G_-(\rho,\alpha) = \int_{-\infty}^{0} u_2(\rho,z)e^{i\alpha z}dz \tag{24}$$

$$G_1(\rho,\alpha) = \int_{0}^{l} u_2(\rho,z)e^{i\alpha z}dz \tag{25}$$

$$G_+(\rho,\alpha) = \int_{l}^{\infty} u_2(\rho,z)e^{i\alpha(z-l)}dz \tag{26}$$

Notice that $G_+(\rho,\alpha)$ and $G_-(\rho,\alpha)$ are unknown functions which are regular in the half-planes $\mathrm{Im}(\alpha) > \mathrm{Im}(-k)$ and $\mathrm{Im}(\alpha) < \mathrm{Im}(k)$, respectively, while $G_1(\rho,\alpha)$ is an entire function of $\alpha$. The general solution of (22) is

$$G(\rho,\alpha) = C(\alpha)H_0^{(1)}(K\rho) \tag{27}$$

where $C(\alpha)$ is the unknown coefficient to be determined. According to the boundary condition (6), $C(\alpha)$ is written as follows

$$C(\alpha) = -\frac{G_1^{'}(b,\alpha)}{KH_1^{(1)}(Kb)} \tag{28}$$

Inserting (28) into (27) we obtain

$$G(\rho,\alpha) = -\frac{G_1^{'}(b,\alpha)}{KH_1^{(1)}(Kb)}H_0^{(1)}(K\rho) \tag{29}$$

Now, applying Fourier transform to the continuity relations (7) and (8) and making use of (21) and (29), we obtain the following modified Wiener-Hopf equation

$$-\frac{2M(\alpha)F_1^{'}(b,\alpha)}{\pi b K^2(\alpha)} + S_-(\alpha) + e^{i\alpha l}S_+(\alpha) = \frac{\left(e^{il(\alpha+k)} - 1\right)}{i(\alpha+k)} \tag{30}$$

where

$$S_-(\alpha) = F_-(b,\alpha) - G_-(b,\alpha) \tag{31}$$

$$S_+(\alpha) = F_+(b,\alpha) - G_+(b,\alpha) \tag{32}$$

and

$$N(\alpha) = \frac{H_1^{(1)}(Ka)}{H_1^{(1)}(Kb)[J_1(Kb)Y_1(Ka) - J_1(Ka)Y_1(Kb)]}. \tag{33}$$

2.2. **Solution of the Modified Wiener-Hopf Equation.** The first step to solve the Wiener-Hopf equation is the factorization of the kernel $N(\alpha)$ as:

$$N(\alpha) = N_+(\alpha)N_-(\alpha) \tag{34}$$

where $N_-(\alpha)$ and $N_+(\alpha)$ are regular and nonzero in the half-planes $\mathrm{Im}(\alpha) < \mathrm{Im}(k)$ and $\mathrm{Im}(\alpha) > \mathrm{Im}(-k)$ , respectively, which are given by

$$N_-(\alpha) = N_+(-\alpha) \tag{35}$$

Their explicit expressions are given in [16] as

$$\begin{aligned} N_+(\alpha) \quad = \quad & \sqrt{N(0)}\prod_{m=1}^{\infty}\frac{1}{(1+\alpha/\delta_m)e^{\alpha(b-a)/m\pi}} \\ & \exp\left[\frac{ik(b-a)}{2} + \frac{K(\alpha)(a-b)}{\pi}\log\left(\frac{\alpha+iK(\alpha)}{k}\right) + q_1(\alpha) - q_2(\alpha)\right] \\ & \exp\left\{\frac{\alpha}{\pi i}(b-a)\left[1 - C + \log\left(\frac{2\pi i}{k(b-a)}\right)\right]\right\} \end{aligned} \tag{36}$$

with

$$q_1(\alpha) = \frac{1}{\pi}P\int_0^\infty \left[1 - \frac{2}{\pi x}\frac{1}{J_1^2(x)+Y_1^2(x)}\right]\log\left(1 + \frac{\alpha a}{[(ka)^2 - x^2]^{1/2}}\right)dx \tag{37}$$

$$q_2(\alpha) = \frac{1}{\pi}P\int_0^\infty \left[1 - \frac{2}{\pi x}\frac{1}{J_1^2(x)+Y_1^2(x)}\right]\log\left(1 + \frac{\alpha b}{[(kb)^2 - x^2]^{1/2}}\right)dx \tag{38}$$

Above , the letter $P$ denotes the Cauchy principle value at the singularities $x = ka$ and $x = kb$, $C$ is the Euler's constant given by $C = 0.57721...$ and $\delta_m'$s are the roots of the function

$$J_1(\xi_m b)Y_1(\xi_m a) - J_1(\xi_m a)Y_1(\xi_m b) = 0, \quad m = 1,2,... \tag{39}$$

with

$$\delta_m = \sqrt{k^2 - \xi_m^2}, \quad m = 1,2,... \tag{40}$$

We find from Eq. (36) that the split function $N_\pm(\alpha) = O(\pm\alpha^{-1/2})$ as $|\alpha| \to \infty$.

The second important step is to multiply both sides of Eq. (30) by $\frac{e^{-i\alpha l}(k+\alpha)}{N_+(\alpha)}$ and $(k-\alpha)/N_-(\alpha)$ and decompose the resulting equations in the strip $\mathrm{Im}(-k) < \mathrm{Im}(\alpha) < \mathrm{Im}(k)$ by taking into account the asymptotic behaviour of $N_\pm(\alpha)$. After some manipulations, we arrive at the following pair of simultaneous Fredholm integral equations:

$$\frac{(k+\alpha)}{N_+(\alpha)}U(\alpha) = -\frac{1}{2\pi i}\int_{L_+} e^{-i\tau l}\frac{(k+\tau)L(\tau)}{N_+(\tau)(\tau-\alpha)}d\tau \tag{41}$$

$$\frac{(k-\alpha)}{N_-(\alpha)}L(\alpha)=\frac{1}{2\pi i}\int\limits_{L_-}e^{i\tau l}\frac{(k-\tau)U(\tau)}{N_-(\tau)(\tau-\alpha)}d\tau+\frac{2k}{i(k+\alpha)N_+(k)} \tag{42}$$

where $L_\pm$ are the integration paths as shown in Fig. 2, and

$$U(\alpha)=S_+(\alpha)-\frac{e^{ikl}}{i(\alpha+k)} \tag{43}$$

$$L(\alpha)=S_-(\alpha)+\frac{1}{i(\alpha+k)} \tag{44}$$

Changing the integration variable $\tau$ by -$\tau$ in (41) and replace $\alpha$ by -$\alpha$ in (42) and then

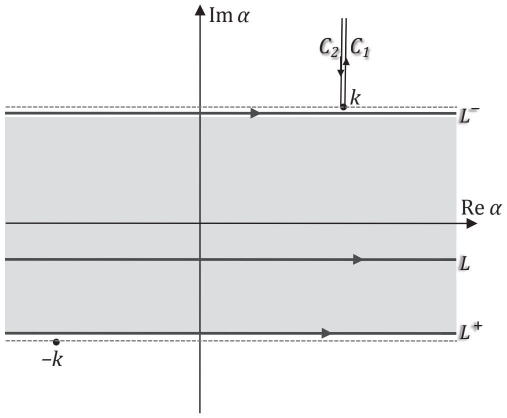


FIGURE 2. Complex $\alpha-$plane.

addition and subtraction of the resulting equations lead to

$$\frac{(k+\alpha)}{N_+(\alpha)}\widetilde{U}(\alpha)=\frac{1}{2\pi i}\int\limits_{L_-}e^{i\tau l}\frac{(k-\tau)}{N_-(\tau)}\frac{\widetilde{U}(\tau)d\tau}{(\tau+\alpha)}+\frac{2k}{i(k-\alpha)N_+(k)} \tag{45}$$

$$\frac{(k+\alpha)}{N_+(\alpha)}\widetilde{L}(\alpha)=-\frac{1}{2\pi i}\int\limits_{L_-}e^{i\tau l}\frac{(k-\tau)}{N_-(\tau)}\frac{\widetilde{L}(\tau)d\tau}{(\tau+\alpha)}-\frac{2k}{i(k-\alpha)N_+(k)} \tag{46}$$

where

$$\widetilde{U}(\alpha)=U(\alpha)+L(-\alpha) \tag{47}$$

$$\widetilde{L}(\alpha)=U(\alpha)-L(-\alpha) \tag{48}$$

These coupled system of integral equations will be solved by applying the method of successive approximations for $kl\gg 1$. In accordance with this approach, $\widetilde{U}(\alpha)$ and $\widetilde{L}(\alpha)$ have the asymptotic representation

$$\widetilde{U}(\alpha)=\widetilde{U}^{(1)}(\alpha)+\widetilde{U}^{(2)}(\alpha)+\widetilde{U}^{(3)}(\alpha)+...$$

$$\widetilde{L}(\alpha)=\widetilde{L}^{(1)}(\alpha)+\widetilde{L}^{(2)}(\alpha)+\widetilde{L}^{(3)}(\alpha)+...$$

where $\overset{\sim}{U}^{(1)}(\alpha)$ and $\overset{\sim}{L}^{(1)}(\alpha)$ are the first-order solutions and obtained by setting the integrals in Eqs. (45) and (46) to zero as

$$\overset{\sim}{U}^{(1)}(\alpha) = \frac{2kN_+(\alpha)}{i(k-\alpha)N_+(k)(k+\alpha)} \tag{49}$$

$$\overset{\sim}{L}^{(1)}(\alpha) = -\frac{2kN_+(\alpha)}{i(k-\alpha)N_+(k)(k+\alpha)} \tag{50}$$

and $\overset{\sim}{U}^{(2)}(\alpha)$ and $\overset{\sim}{L}^{(2)}(\alpha)$ are the second-order solutions determined by replacing the unknown functions appearing in the integrands by their first-order approximations as

$$\overset{\sim}{U}^{(2)}(\alpha) = \overset{\sim}{L}^{(2)}(\alpha) = -\frac{kN_+(\alpha)}{\pi N_+(k)(k+\alpha)} I_1(\alpha) \tag{51}$$

with

$$I_1(\alpha) = \int_{L_-} e^{i\tau l} \frac{[N_+(\tau)]^2}{N(\tau)(k+\tau)} \frac{d\tau}{(\tau+\alpha)} \tag{52}$$

According to Jordan's lemma, the evaluation of $I_1(\alpha)$ is carried out by way of contour deformation onto the branch-cuts $C_1 + C_2$. Using the properties

$$J_1(e^{i\pi}Ka) = -J_1(Ka), \quad Y_1(e^{i\pi}Ka) = -Y_1(Ka) - 2iJ_1(Ka), \quad H_1^{(1)}(e^{i\pi}Ka) = H_1^{(2)}(Ka) \tag{53}$$

and making the substitution $\sqrt{\tau - k} = e^{i\pi/4}\sqrt{t}$ on $C_1$ and $\sqrt{\tau - k} = -e^{i\pi/4}\sqrt{t}$ on $C_2$ gives

$$I_1(\alpha) = \frac{2ie^{ikl}\left[N_+(k)\right]^2}{\pi kb} \beta(b, l; \alpha) \tag{54}$$

where

$$\beta(b, l; \alpha) = \int_0^\infty \frac{e^{-tl}}{K^2\left[J_1^2(Kb) + Y_1^2(Kb)\right]} \frac{1}{[t - i(k+\alpha)]} dt \tag{55}$$

This integral will be solved numerically. The higher-order terms can be derived by following the same procedure and the approximate solution of the modified Wiener-Hopf equation can be written as

$$F_1'(b, \alpha) = \frac{\pi kb}{iN_+(k)N_+(\alpha)} - ie^{i(\alpha+k)l} \frac{(k-\alpha)N_+(k)}{\pi N_-(\alpha)} \beta(b, l; \alpha). \tag{56}$$

## 3. ANALYSIS OF THE FIELDS

The radiated field in the region $\rho > b, -\infty < z < \infty$, namely, $u_2(\rho, z)$ can be obtained by evaluating the following integral

$$u_2(\rho, z) = -\frac{1}{2\pi} \int_L F_1'(b, \alpha) \frac{H_0^{(1)}(K\rho)}{KH_1^{(1)}(Kb)} e^{-i\alpha z} d\alpha \tag{57}$$

where $L$ is a straight line parallel to the real $\alpha-$axis, lying in the strip $\mathrm{Im}(-k) < \mathrm{Im}(\alpha) < \mathrm{Im}(k)$. This integral can be evaluated asymptotically through the saddle point technique. Taking into account the asymptotic expansion of $H_0^{(1)}(K\rho)$ as $K\rho \to \infty$

$$H_0^{(1)}(K\rho) \to \sqrt{\frac{2}{\pi K\rho}} e^{i(K\rho-\pi/4)} \tag{58}$$

and using the change of variables $\alpha = -k\cos\theta$, the radiated field takes the form:

$$u_2(r,\theta) = D(\theta)\frac{e^{ikr}}{r} \tag{59}$$

where

$$D(\theta) = \frac{i}{H_1^{(1)}(kb\sin\theta)\sin\theta}\left\{\frac{ib}{N_+(k)\,N_-(k\cos\theta)} + i\frac{e^{ikl(1-\cos\theta)}}{\pi^2}\frac{(1+\cos\theta)\,N_+(k)}{N_+(k\cos\theta)}\beta(b,l,-k\cos\theta)\right\} \tag{60}$$

where r and $\theta$ are the spherical coordinates defined by

$$\rho = r\sin\theta \tag{61}$$

$$z = r\cos\theta \tag{62}$$

The reflected field in the region $z < 0$, $a < \rho < b$ can be obtained from the inverse Fourier transform of $F_-(\rho,\alpha)$. By using (18), we write

$$u_1(\rho,z) = -\frac{1}{2\pi}\int_L F_1'(b,\alpha)\frac{K[Y_0(K\rho)J_1(Ka) - J_0(K\rho)Y_1(Ka)]}{K^2[J_1(Kb)Y_1(Ka) - J_1(Ka)Y_1(Kb)]}e^{-i\alpha z}d\alpha \tag{63}$$

According to Jordan's lemma, the integral is calculated by closing the contour in the upper half plane and evaluating the residues contributions from the simple poles occuring at the zeros of $K^2[J_1(Kb)Y_1(Ka) - J_1(Ka)Y_1(Kb)]$ lying in the upper half of the $\alpha-$plane, namely at $\alpha = k$ and $\alpha = \delta_m'$s. The reflection coefficient $R$ of the fundamental mode is computed from the contribution of the first pole at $\alpha = k$. The result is

$$R = -\frac{ibF_1'(b,k)}{k(a^2-b^2)} \tag{64}$$

Similarly, the transmission field $T$ of the fundamental mode is obtained by evaluating the integral in (63) for $z > l$. This integral is computed by closing the contour in the lower half of the complex $\alpha-$plane. According to Jordan's lemma and residue theorem, $\alpha = -k$ contribution gives

$$T = -\frac{ibF_1'(b,-k)}{k(a^2-b^2)} \tag{65}$$

## 4. NUMERICAL RESULTS

In order to provide a comparison of the derivations presented, the subproblem which considers radiation from an annular duct with an infinite centerbody is analyzed. For this purpose, annular duct is formed as shown in Fig. 3. Using the Wiener-Hopf technique, the radiated field is expressed as below:

$$u_{rad}(r,\theta) = \frac{kb}{H_1^{(1)}(kb\sin\theta)\sin\theta N_-(k\cos\theta)\,N_+(k)}\frac{e^{ikr}}{r} \tag{66}$$

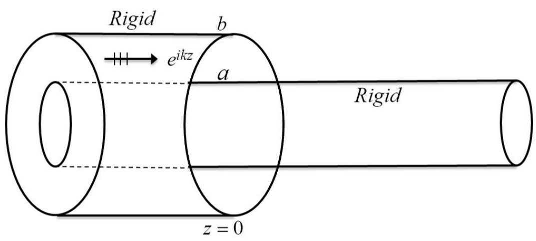


FIGURE 3. Geometry of the subproblem.

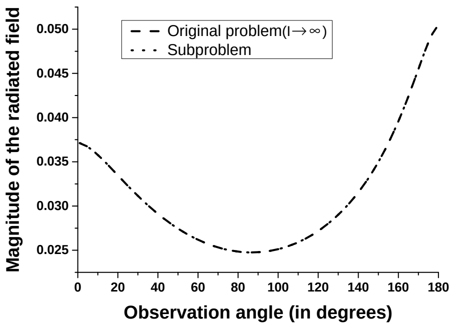


FIGURE 4. Comparison for b/a = 1.5.

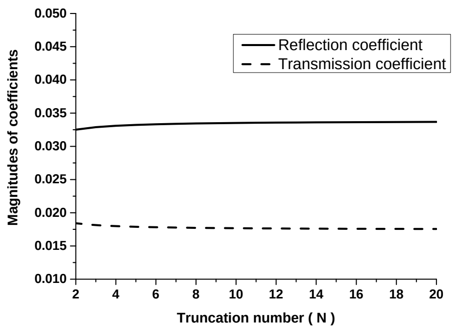


FIGURE 5. Effect of truncation number N on the fields .

Fig. 4 shows that the radiated field solution of the original problem approximates perfectly to the radiated field of the subproblem when $l \to \infty$.

Some graphical results displaying the effects of various parameters such as radii of the walls and gap width on radiated, reflected and transmitted fields are presented. From Fig. 5 we can see that the magnitudes of the reflected and transmitted fields become insensitive to the truncation number for $N > 5$. For this reason, in all numerical computations $N$ is chosen as 20.

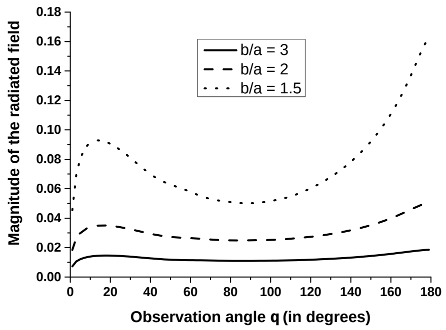


FIGURE 6. Effect of the b/a on the radiated field.

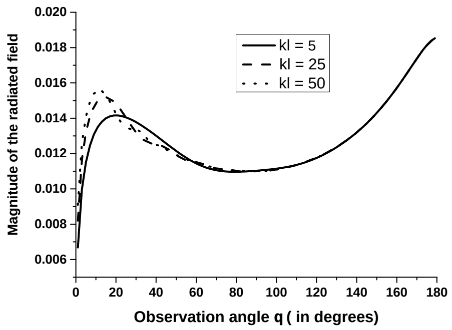


FIGURE 7. Effect of the kl on the radiated field.

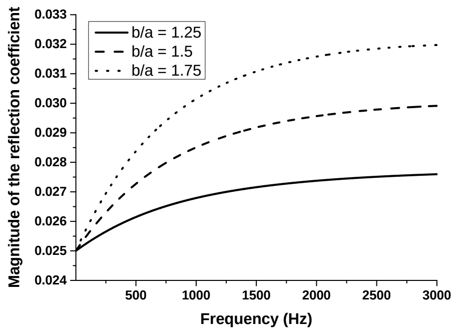


FIGURE 8. Effect of the b/a on the reflection coefficient.

Figures 6 and 7 display the effects of $b/a$ and $kl$ to the radiated field. In Fig. 6, it is noticed that the amplitude of the radiated field is oscillating for increasing $\theta$. On the other hand, it is seen in Fig. 7 that width of the slot has less impact on the radiated field when the observation angle is much more than $50°$.

Variation of the amplitudes of reflected and transmitted fields are presented in Figures 8 and 9. It is deduced that with increasing values of $b/a$, the magnitude of the reflected field increases, while the transmitted field decreases.

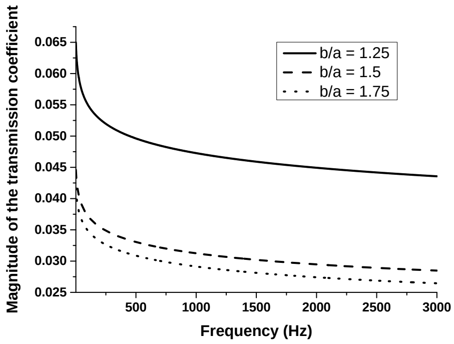


FIGURE 9. Effect of the b/a on the transmission coefficient.

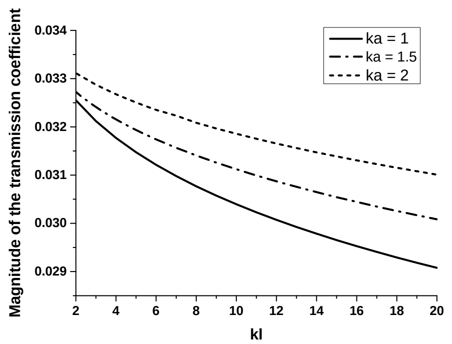


FIGURE 10. Effect of the kl on the transmission coefficient.

Fig. 10 depicts the variation of the amplitude of the transmission coefficient with respect to $kl$ for different values of $ka$. As it can be seen, the amplitude increases as the gap length approaches zero.

## 5. CONCLUSIONS

In this work, radiation of a plane sound wave by a rigid circular duct with a finite gap on its outer wall is considered to reveal the influence of the finite gap on the radiation phenomena. By applying direct Fourier transform, the boundary value problem is formulated as a Wiener-Hopf equation, which is solved via a pair of Fredholm integral equations. At the end of the analysis, the inverse Fourier transform is applied to determine the explicit expressions of the reflection, transmission coefficients and radiated field. Also, some numerical results are presented to show the effects of some physical parameters to the fields. Also, it is possible to extend the method for the problem where a perforated tube is located at the finite gap or a mean flow exists in the duct.

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