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STABILITY AND DATA DEPENDENCE RESULTS FOR ZAMFIRESCU MULTI-VALUED MAPPINGS

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Abstract. In this paper, we prove some stability and data dependence results for the class of multivalued Zamfirescu operators. Our results generalize and improve several existing results in literature. It is worth mentioning here that our results are new even for single valued mappings.

Keywords: Fixed point, Almost stability, Zamfirescu operators and Data dependence.

AMS Subject Classification: 54H25, 47H10.

1. Introduction

To approximate the fixed points of a nonlinear operator is one of the most commonly used techniques for solving differential/integral equations. There exists several procedures for approximating fixed points. Picard [1] iteration, Krasnoselski iteration, Mann [2] iteration and Ishikawa [3] iteration are the basic processes. The utility of a numerical iteration procedure depends on its numerically stability. A fixed point iteration is numerically stable if small errors (due to approximation, rounding errors etc.) during calculations, will produce small changes on the approximate value of the fixed point calculated by means of this method. In 1967, Ostrowski [5] first introduced the concept of stability of iteration procedures and proved Picard iteration for contraction mappings is stable.

In 1987, Harder [4] studied the concept of stability which utilized by many authors for different classes of mappings (for example cf. [6, 7, 8, 9]). In 2000, Osilike [10] introduced a slightly weaker form of stability called almost stability and proved some results. In 2003, Berinde [11] introduced sharper concept of almost stability and showed some almost stable fixed point iteration procedures which are also summably almost stable with respect to some classes of contractive mappings.

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2. Preliminaries

Let E be a metric space and S a self map on E with $F(S) = \{y \in E : Sy = y\} \neq \emptyset$. Consider a fixed point iteration procedure, i.e., for any $z_0 \in E$ define a sequence $\{z_n\}$ as

$$z_{n+1} = f(S, z_n), \ n = 0, 1, 2, ..., \tag{1}$$

where f is some function. Suppose $\{z_n\}$ converges strongly to some $z \in F(S)$. In concrete applications, when computing $\{z_n\}$ we usually follow the following procedure:
1. Choose the initial approximation $z_0 \in E$.
2. Compute $z_1 = f(S, z_0)$. But, due to various errors that occur during the calculations (rounding errors, numerical approximations of functions, derivatives, integrals etc.), we do not obtain the exact value of z_1 but a different one (say w_1) which is however closed enough to z_1 i.e., $w_1 \approx z_1$
3. Consequently, while computing $z_2 = f(S, z_1)$ we actually obtain z_2 as $z_2 = f(S, w_1)$. So, again, instead of the theoretical value of z_2 we find another value w_2, w_2 being closed enough to z_2 i.e., $w_2 \approx z_2$ and so on. In this way, instead of the theoretical sequence $\{z_n\}$ defined by the iteration (1), we obtain practically an approximate sequence $\{w_n\}$. Recall that a given fixed point iteration method is numerically stable if and only if for w_n close enough to z_n at each stage, the approximate sequence $\{w_n\}$ still converges to the fixed point z of S.

In 1988, Harder and Hicks [6] introduced the concept of stability which runs as follows:

Definition 2.1. *Let E be a metric space, $S : E \to E$ and $z_0 \in E$. Assume that the iteration process (1) converges to a fixed point z of S. Let $\{w_n\}$ be an arbitrary sequence in E and define*

$$\sigma_n = d(w_{n+1}, f(S, w_n)), n = 0, 1, 2, ... \tag{2}$$

The fixed point procedure (1) is said to be S-stable or stable with respect to S if

$$\lim_{n\to\infty} \sigma_n = 0 \Rightarrow \lim_{n\to\infty} w_n = z.$$

By utilizing this notion, Harder and Hicks [6, 7] verified the stability of various iteration procedures (Picard, Mann and Kirk's iterations) with respect to several classes of contractive type operators. In 2000, Osilike [10] introduced a weaker concept of stability, called almost stability and he also investigated the stability of Ishikawa iteration with respect to some classes of pseudocontractive operators.

Definition 2.2. *Let E be a metric space and $S : E \to E$. Assume that the iteration procedure (1) converges to a fixed point z of S. Let $\{w_n\}$ be an arbitrary sequence in E and let σ_n defined by (2). The fixed point procedure (1) is said to be almost S-stable or almost stable with respect to S if*

$$\sum_{n=0}^{\infty} \sigma_n < \infty \Rightarrow \lim_{n\to\infty} w_n = z.$$

Recently, Berinde [11] introduced a sharper concept of almost stability and showed some almost stable fixed point iteration procedures which are also summably almost stable with respect to some classes of contractive operators.

Definition 2.3. *Let E be a metric space and $S : E \to E$. Assume that the iteration procedure (1) converges to a fixed point z of S. Let $\{w_n\}$ be an arbitrary sequence in E*

and let σ_n defined by (2). The fixed point procedure (1) is said to be summably almost S-stable or summably almost stable with respect to S if

$$\sum_{n=0}^{\infty} \sigma_n < \infty \Rightarrow \sum_{n=0}^{\infty} d(w_n, z) < \infty.$$

It is worth mentioning here that any almost stable iteration procedure is also summably almost stable, since

$$\sum_{n=0}^{\infty} d(w_n, z) < \infty \Rightarrow \lim_{n\to\infty} w_n = z.$$

but the converse need not be true in general. For illustrative examples, one may referred to [11].

To make our paper self contained, we state following known but important definition and lemma. Let E be a metric space and K be a nonempty subset of E. Let $CB(K)$ be the family of nonempty closed bounded subsets of K. A subset K of E is called proximinal if for each $x \in E$, there exists an element $k \in K$ such that

$$d(x, k) = dist(x, K) = \inf\{d(x, y) : y \in K\}.$$

We shall denote by $PB(K)$, the family of nonempty bounded proximinal subsets of K. The Hausdorff metric H on $CB(K)$ is defined as

$$H(A, B) = max\left\{\sup_{x\in A} d(x, B), \sup_{y\in B} d(y, A)\right\} \text{ for } A,\ B \in CB(K).$$

In 1972, Zamfirescu [12] extended contraction mapping to the class of maps which may not be continuous, called Zamfirescu mappings and utilized them to extend the Banach contraction principle. Now, we present the definition of multi-valued Zamfirescu mapping.

Definition 2.4. *Let E be a metric space and $S : E \to CB(E)$, a multi-valued mapping. Then S is said to be a multi-valued Zamfirescu mapping if there exist real numbers α, β and γ satisfying $0 \le \alpha < 1$, $0 \le \beta < \frac{1}{2}$ and $0 \le \gamma < \frac{1}{2}$ such that for each $x, y \in E$ at least one of the followings is true:*

$(\tilde{z_1})$ $H(Sx, Sy) \le \alpha d(x, y)$
$(\tilde{z_2})$ $H(Sx, Sy) \le \beta[d(x, Sx) + d(y, Sy)]$
$(\tilde{z_3})H(Sx, Sy) \le \gamma[d(x, Sy) + d(y, Sx)]$.

The following lemma plays a key role in proving the main results which is actually the generalized ratio test for positive series (cf. [13] and references cited therein).

Lemma 2.1. *Let $\{\alpha_n\}$, $\{\beta_n\}$ be sequences of nonnegative numbers and $0 < c < 1$ such that*

$$\alpha_{n+1} \le c\alpha_n + \beta_n \ for\ all\ n \ge 0.$$

(i) If $\lim_{n\to\infty} \beta_n = 0$, then $\lim_{n\to\infty} \alpha_n = 0$.
(ii) If $\sum_{n=0}^{\infty} \beta_n < \infty$, then $\sum_{n=0}^{\infty} \alpha_n < \infty$.

The following lemma due to Song and Cho [14] is very cruicial for our main result:

Lemma 2.2. *Let $S : K \to P(K)$ be a multivalued mapping and $P_S(x) = \{y \in Sx : \|x - y\| = d(x, Sx)\}$. Then the following are equivalent.*
(i) $x \in F(S)$,

(ii) $P_S(x) = \{x\}$,
(iii) $x \in F(P_S)$. *Moreover,* $F(S) = F(P_S)$.

In this paper, we utilize the concept of summably almost stable due to Berinde [11] to prove some stability and data dependence results for Zamfirescu multivalued operators. Hence our results generalize and improve the corresponding results of Harder and Hicks [6], Berinde [11], Oslike [10] and Singh et al. [15].

3. Main Results

Theorem 3.1. *Let E be a metric space and $S : E \to CB(E)$ a multi-valued Zamfirescu mapping with $Sz = \{z\}$ for every $z \in F(S)$. Let $z_0 \in E$ and $z_{n+1} = a_n \in Sz_n, n \geq 0$. Then $\{z_n\}$ converges to z and is summable almost stable with respect to S.*

Proof. Let $\{w_n\}$ be any given sequence in E and S a multi-valued Zamfirescu mapping such that

$$f(S, w_n) = b_n \in Sw_n \; for \; all \; n \geq 0.$$

Now, we have the following three cases:
Case I. When S satisfies $(\tilde{z_1})$, i.e.

$$H(Sx, Sy) \leq \alpha \; d(x, y) \text{ for some } \alpha \in [0, 1).$$

Now,

$$\begin{aligned} d(w_{n+1}, z) &\leq d(w_{n+1}, b_n) + d(b_n, z) \\ &= \sigma_n + d(b_n, Sz) \\ &\leq \sigma_n + H(Sw_n, Sz) \\ &\leq \sigma_n + \alpha d(w_n, z). \end{aligned} \tag{3.1}$$

Case II. When S satisfies $(\tilde{z_2})$, i.e. $H(Sx, Sy) \leq \beta[d(x, Sx) + d(y, Sy)]$ for some β with $0 \leq \beta < 1/2$.
Now,

$$\begin{aligned} d(w_{n+1}, z) &\leq d(w_{n+1}, b_n) + d(b_n, z) \\ &= d(w_{n+1}, b_n) + d(b_n, Sz) \\ &\leq d(w_{n+1}, b_n) + H(Sw_n, Sz) \\ &\leq d(w_{n+1}, b_n) + \beta[d(w_n, Sw_n) + d(z, Sz)] \\ &\leq d(w_{n+1}, b_n) + \beta[d(w_n, z) + d(z, w_{n+1}) + d(w_{n+1}, b_n) + d(b_n, Sw_n)] \\ &\leq \frac{1+\beta}{1-\beta}\sigma_n + \frac{\beta}{1-\beta} d(w_n, z). \end{aligned} \tag{3.2}$$

Case III. When S satisfies $(\tilde{z_3})$, i.e. $H(Sx, Sy) \leq \gamma[d(x, Sy) + d(y, Sx)]$ for some γ with $0 \leq \gamma < 1/2$.
Now,

$$\begin{aligned} d(w_{n+1}, z) &\leq d(w_{n+1}, b_n) + d(b_n, z) \\ &= d(w_{n+1}, b_n) + d(b_n, Sz) \\ &\leq d(w_{n+1}, b_n) + H(Sw_n, Sz) \\ &\leq d(w_{n+1}, b_n) + \gamma[d(w_n, Sz) + d(z, Sw_n)] \\ &\leq d(w_{n+1}, b_n) + \gamma[d(w_n, z) + d(z, Sz) + d(z, w_{n+1}) + d(w_{n+1}, b_n) + d(b_n, Sw_n)] \\ &\leq \frac{1+\gamma}{1-\gamma}\sigma_n + \frac{\gamma}{1-\gamma} d(w_n, z). \end{aligned} \tag{3.3}$$

Hence, in view of (3.1), (3.2) and (3.3) with Lemma 2.1, $\{z_n\}$ is summably almost stable with respect to S. To show $\{z_n\}$ converges to z, letting $w_n = z_n$ we get $\sigma_n = d(z_{n+1}, Sz_n) = 0$ and by choosing $a = \max\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\}$, we have

$$d(z_{n+1}, z) \leq ad(z_n, z)$$

yielding thereby

$$d(z_n, z) \leq a^n d(z_0, z) \to 0 \text{ as } n \to \infty.$$

Thus $\{z_n\}$ converges to fixed point z. □

We proved Theorem 3.1 with the aid of end point condition, i.e. for a multivalued mapping S, $Sz = \{z\}$ for all fixed point z of S which is a strong condition in practical. To remove this, we consider Picard projection iteration, i.e.

$$z_{n+1} = a_n \in P_S z_n \text{ for all } n \geq 0.$$

Theorem 3.2. *Let E be a metric space and $S : E \to PB(E)$ a multi-valued mapping such that P_S is Zamfirescu multi-valued mapping. Let $z_0 \in E$ and $z_{n+1} = a_n \in P_S z_n, n \geq 0$. Then $\{z_n\}$ converges to z and is summable almost stable with respect to S.*

Proof. Let $\{w_n\}$ be any given sequence in E and S a multi-valued mapping such that P_S is Zamfirescu multivalued mapping and define

$$f(P_S, w_n) = b_n \in P_S w_n \text{ for all } n \geq 0,$$

then we have the following three cases:
Case I. When P_S satisfies $(\tilde{z_1})$, i.e.

$$H(P_S x, P_S y) \leq \alpha\ d(x, y) \text{ for some } \alpha \in [0, 1).$$

For any fixed point z of S, in view of Lemma 2.2, we have $z \in P_S z = \{z\}$.

$$\begin{aligned} d(w_{n+1}, z) &\leq d(w_{n+1}, b_n) + d(b_n, z) \\ &= \sigma_n + d(b_n, P_S z) \\ &\leq \sigma_n + H(P_S w_n, P_S z) \\ &\leq \sigma_n + \alpha d(w_n, z) \end{aligned} \tag{3.4}$$

Case II. When S satisfies $(\tilde{z_2})$, i.e. $H(P_S x, P_S y) \leq \beta[d(x, P_S x) + d(y, P_S y)]$ for some β with $0 \leq \beta < 1/2$.
Now,

$$\begin{aligned} d(w_{n+1}, z) &\leq d(w_{n+1}, b_n) + d(b_n, z) \\ &= d(w_{n+1}, b_n) + d(b_n, P_S z) \\ &\leq d(w_{n+1}, b_n) + H(P_S w_n, P_S z) \\ &\leq d(w_{n+1}, b_n) + \beta[d(w_n, P_S w_n) + d(z, P_S z)] \\ &\leq d(w_{n+1}, b_n) + \beta[d(w_n, z) + d(z, w_{n+1}) + d(w_{n+1}, b_n) + d(b_n, P_S w_n)] \\ &\leq \frac{1+\beta}{1-\beta}\sigma_n + \frac{\beta}{1-\beta} d(w_n, z). \end{aligned} \tag{3.5}$$

Case III. When S satisfies $(\tilde{z_3})$, i.e. $H(P_S x, P_S y) \leq \gamma[d(x, P_S y) + d(y, P_S x)]$ for some γ with $0 \leq \gamma < 1/2$.

Now,

$$\begin{aligned}
d(w_{n+1}, z) &\leq d(w_{n+1}, b_n) + d(b_n, z) \\
&= d(w_{n+1}, b_n) + d(b_n, P_S z) \\
&\leq d(w_{n+1}, b_n) + H(P_S w_n, P_S z) \\
&\leq d(w_{n+1}, b_n) + \gamma[d(w_n, P_S z) + d(z, P_S w_n)] \\
&\leq d(w_{n+1}, b_n) + \gamma[d(w_n, z) + d(z, P_S z) + d(z, w_{n+1}) + d(w_{n+1}, b_n) + d(b_n, P_S w_n)] \\
&\leq \frac{1+\gamma}{1-\gamma}\sigma_n + \frac{\gamma}{1-\gamma} d(w_n, z). \qquad (3.6)
\end{aligned}$$

Hence, in view of (3.4), (3.5) and (3.6) with Lemma 2.1, $\{z_n\}$ is summably almost stable with respect to S. To show $\{z_n\}$ converges to z, letting $w_n = z_n$ we get $\sigma_n = d(z_{n+1}, a_n) = 0$ and so by choosing $a = \max\{\alpha, \frac{\beta}{1-\beta}, \frac{\gamma}{1-\gamma}\}$, we have

$$d(z_{n+1}, z) \leq a d(z_n, z)$$

yielding thereby

$$d(z_n, z) \leq a^n d(z_0, z) \to 0 \ as \ n \to \infty.$$

Thus $\{z_n\}$ converges to fixed point z.

□

Now, we prove following data dependence result for two multi-valued Zamfirescu operators.

Theorem 3.3. *Let E be a metric space and $S, \tilde{S} : E \to CB(E)$ two multi-valued Zamfirescu operators with the fixed points z and $\tilde{z}$ respectively. If for every $x \in E$ following holds*

$$H(Sx, \tilde{S}x) \leq \sigma,$$

then

$$d(z, \tilde{z}) \leq \frac{\sigma}{1-a}$$

Proof. Let $S, \tilde{S} : E \to CB(E)$ be two multi-valued Zamfirescu operators such that $z_{n+1} = a_n \in Sz_n, n \geq 0$ and $\tilde{z}_{n+1} = \tilde{a}_n \in \tilde{S}\tilde{z}_n, n \geq 0$ with $z_n \to z$ and $\tilde{z}_n \to \tilde{z}$. Now, consider

$$\begin{aligned}
d(z_{n+1}, \tilde{z}_{n+1}) &= d(a_n, \tilde{a}_n) \\
&\leq d(a_n, \tilde{S}z_n) + d(\tilde{S}z_n, \tilde{a_n}) \\
&\leq H(Sz_n, \tilde{S}z_n) + H(\tilde{S}z_n, \tilde{S}\tilde{z_n}) \\
&\leq \sigma + H(\tilde{S}z_n, \tilde{S}\tilde{z_n}). \qquad (3.7)
\end{aligned}$$

Now, we have following three cases:

Case I: When $\tilde{S}$ satisfies $(\tilde{z}_1)$, then from equation (3.7), we have

$$d(z_{n+1}, \tilde{z}_{n+1}) \leq \sigma + \alpha d(z_n, \tilde{z_n}).$$

On letting $n \to \infty$, we have $d(z, \tilde{z}) \leq \frac{\sigma}{1-\alpha}$. (3.8)

Case II: When $\tilde{S}$ satisfies $(\tilde{z}_2)$, then from equation (3.7), we have

$$\begin{aligned}
d(z_{n+1}, \tilde{z}_{n+1}) &\leq \sigma + \beta[d(z_n, \tilde{S}z_n) + d(\tilde{z_n}, \tilde{S}\tilde{z_n}) \\
&\leq \sigma + \beta d(z_n, Sz_n) + \beta d(Sz_n, \tilde{S}z_n) + \beta d(\tilde{z_n}, \tilde{S}\tilde{z_n}) \\
&\leq (1+\beta)\sigma + \beta d(z_n, Sz_n) + \beta d(\tilde{z_n}, \tilde{S}\tilde{z_n}). \qquad (3.9)
\end{aligned}$$

On letting $n \to \infty$, we have $d(z, \tilde{z}) \leq (1+\beta)\sigma$.

Case III. When S satisfies $(\tilde{z_3})$, then from equation (3.7), we have

$$\begin{aligned} d(z_{n+1}, \tilde{z}_{n+1}) &\leq \sigma + \gamma[d(z_n, \tilde{S}\tilde{z}_n) + d(\tilde{z_n}, \tilde{S}z_n)] \\ &\leq \sigma + \gamma d(z_n, \tilde{z}_n) + \gamma d(\tilde{z}_n, \tilde{S}\tilde{z}_n) + \gamma d(\tilde{z}_n, z_n) + \gamma d(z_n, Sz_n) + \gamma d(Sz_n, \tilde{S}z_n). \end{aligned}$$

On letting $n \to \infty$, we have

$$d(z, \tilde{z}) \leq \frac{1+\gamma}{1-2\gamma}\sigma. \tag{3.10}$$

On choosing $a \in (0,1)$ such that

$$\frac{1}{1-a} = max\left\{\frac{1}{1-\alpha}, (1+\beta), \frac{1+\gamma}{1-2\gamma}\right\},$$

and by equations (3.8), (3.9) and (3.10), we get

$$d(z, \tilde{z}) \leq \frac{\sigma}{1-a}.$$

□

4. Conclusion

We obtained interesting and important stability and data dependence results for class of multi-valued Zamfirescu operators.

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