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# RESULTS ON THE INVERSE MAJORITY DOMINATION AND MAJORITY INDEPENDENCE NUMBER OF A GRAPH

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Abstract. In this article,the relationship between Inverse Majority Domination number $\gamma_M^{-1}(G)$ and Majority Independence number $\beta_M(G)$ of a graph G is discussed for some classes of graphs. In particular, $\gamma_M^{-1}(G)$ and $\beta_M(G)$ for cubic and cubic bipartite graph are studied with examples. Also characterization theorem for this relation and some results are determined.
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## 1. Introduction

Domination in graph provide numerous applications both in the position or location and protection strategies. This concepts was introduced by Claude Berge in 1958 [1]. In 1962, Ore used the name "Dominating set" and "Domination number". In 1977, Cockayne and Hedetniemi made an interesting and extensive survey of the results known at that time about dominating sets in graphs. The survey paper of Cockayne and Hedetniemi has generated a lot of interest in the study of domination in graphs. Domination has a wide range of application in radio stations, modeling social networks, coding theory, nuclear power plants problems. One of the fastest growing areas in graph theory is the study of domination and related subset problems such as independence, covering, matching and inverse domination.

Let $G = (V(G), E(G))$ be a simple graph with vertex set V(G) of finite order and edge set E(G). Let $v \in V(G)$. The neighborhood of $v$ is the set $N_G(v) = N(v) = \{u \in V(G) : uv \in E(G)\}$. If $S \subseteq V(G)$, then the open neighborhood of S is the set $N_G(s) = N(S) = U_{v\in S} N_G(v)$. The closed neighbourhood of S is $N_G(S) = N[S] = S \cup N(S)$.

A set $S \subseteq V(G)$ of vertices in a graph $G = (V, E)$ is a dominating set if every vertex $v \in V$ is either an element of S or is adjacent to an element of S. A dominating set S is called a minimal dominating set if no proper subset of S is a dominating set. The minimum cardinality of a minimal dominating set is called the domination number and

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the maximum cardinality of a minimal dominating set is called the upper domination number in a graph G. It is denoted by $\gamma(G)$ and $\Gamma(G)$ respectively.

The concept of inverse domination was introduced by V. R. Kulli. If a non-empty subset $D \subseteq V(G)$ is called the minimum dominating set, then if $v - D$ contains a dominating set $D'$,then $D'$ is called the inverse dominating of G with respect to D and $\gamma'(G)$ is the inverse domination number of G.

A set $D \subseteq V(G)$ of vertices in a graph $G = (V, E)$ is called a Majority Dominating set of G [4] if atleast half of the vertices of V(G) are either in S or adjacent to the elements of S. A majority dominating set [4] D is minimal if no proper subset of D is a majority dominating set of a graph G. The minimum cardinality of a minimal majority dominating set of G is called majority domination number of G, is denoted by $\gamma_M(G)$ and the minimum majority dominating set of G is denoted by $\gamma_M - set$. If a vertex $u$ of degree satisfies the condition $d(u) \geq \lceil \frac{p}{2} \rceil - 1$, then the vertex u is called majority dominating vertex of G. All full degree vertices are majority dominating vertices but all majority dominating vertices are not full degree vertices.

A set $S \subseteq V(G)$ of vertices in a graph G is said to be a Majority Independent set [5] if it induces a totally disconnected subgraph with $|N[S]| \geq \lceil \frac{p}{2} \rceil$ and $|p_n[v, S]| > |N[S]| - \lceil \frac{p}{2} \rceil$, for every $v \in S$. If any vertex set S is properly containing S is not majority independent then S is called a maximal majority independent set. The maximum cardinality of a maximal majority independent set of G is called majority independence number of G and it is denoted $\beta_M(G)$.

## 2. Basic Results on $\gamma_M(G)$,$\beta_M(G)$ and $\gamma_M^{-1}(G)$

**Definition 2.1.** [7] *Let G be simple and finite graph with p vertices and q edges and D be a minimum majority dominating set of G. If the set $(V - D)$ contains a majority dominating set say $D'$ then the set $D'$ is called Inverse Majority Dominating set with respect to D. The Inverse Majority Domination number [7] $\gamma_M^{-1}(G)$ of a graph G is the minimum cardinality of a minimal inverse majority dominating set of G.*

**Proposition 2.1.** [4] *For any graph G, $\gamma_M(G) = 1$ if and only if G has atleast one vertex u with degree $d(u) \geq \lceil \frac{p}{2} \rceil - 1$.*

**Proposition 2.2.** [6]

1. *For any graph G, $\beta_M(G) = 1$ if and only if G has all vertices u with degree $d(u) \geq \lceil \frac{p}{2} \rceil - 1$, for all $u \in V(G)$.*
2. *For a cubic bipartite graph G, the majority independence number $\beta_M(G) = \lceil \frac{p}{4} \rceil - 1$.*
3. *For any cubic graph G, $\beta_M(G) = \lceil \frac{p}{8} \rceil$.*
4. *For a Fan graph $G = F_p.\beta_M(G) = \lceil \frac{p}{6} \rceil, p \geq 3$.*
5. *For a star graph $G = K_{(1,p-1)}, \beta_M(G) = \lfloor \frac{(p-2)}{2} \rfloor, p \geq 2$.*

**Proposition 2.3.** [7]

1. *For any graph G, $\gamma_M(G) \leq \gamma_M^{-1}(G)$.*
2. *For a path $P_p, p \geq 2$ and cycle $C_p, p \geq 3$ with p vertices, $\gamma_M^{-1}(G) = \lceil \frac{p}{6} \rceil$.*
3. *For a wheel $W_p, p \geq 5, \gamma_M^{-1}(G) = \lceil \frac{(p-2)}{6} \rceil$.*
4. *For a complete graph $K_p, \gamma_M^{-1}(G) = 1$.*
5. *For a fan graph $F_p, \gamma_M^{-1}(G) = 1$.*
6. *For a star graph $G = K_{(1,p-1)}, \gamma_M^{-1}(G) = \lfloor \frac{(p-2)}{2} \rfloor, p \geq 2$.*

**Proposition 2.4.** [7] *For any graph G,*$\gamma_M^{-1}(G) = 1$ *if and only if G has atleast one majority dominating vertex u in* $(V - D)$*, where D is a minimum majority dominating set of G.*

## 3. Inverse Majority Dominating Set and Majority Independent Set

**Example 3.1.** *Consider the following graph* $G = T_{5k}$*,* $k = 5$ *with* $p = 25$ *vertices. The graph G contains five* $P_5$ *paths which is connected in the middle vertex of each path* $P_5$*. The vertex set is labeled as* $\{y_1, \ldots, y_{10}\}$ *are pendants,* $\{x_1, \ldots, x_{10}\}$ *are two degree vertices and* $(a, b, c, d, e)$ *are middle vertices of each* $P_5$*.*

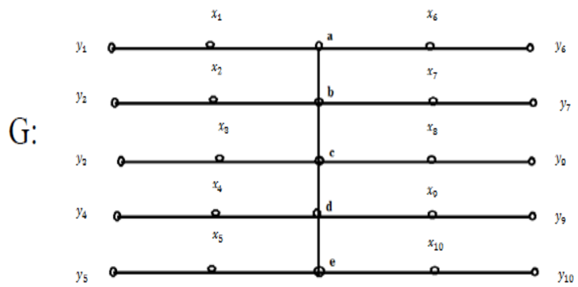


Figure 1

*In G,* $D_1 = \{a, b, c, d\}$ *is a majority dominating set* $\Rightarrow \gamma_M(G) = |D_1| = 4$ $D_2 = \{x_1, x_2, x_3, e\}$ *is a inverse majority dominating set of G with respect to* $D_1 \Rightarrow \gamma_M^{-1}(G) = |D_2| = 4$. $D_3 = \{y_1, y_2, y_3, y_4, y_5, y_6, y_7\}$ *is a majority independent set of G.*$\Rightarrow \beta_M(G) = |D_3| = 7$.
*Hence* $\gamma_M^{-1}(G) < \beta_M(G)$.

**Proposition 3.1.** *For any graph G, it satisfies the following inequalities.*
*i)* $\gamma_M^{-1}(G) \leq \beta_M(G)$ *and*
*ii)* $\gamma_M(G) \leq \gamma_M^{-1}(G) \leq \beta_M(G)$.

**Proof:** i) Let D be a minimum majority dominating set and $D'$ be an inverse majority dominating set with respect to D of a graph G. Since any maximal majority independent set S of G , $S \subseteq (V - D)$ is also a inverse majority dominating set of G. Hence $\gamma_M^{-1}(G) \leq \beta_M(G)$.
ii) By Proposition (2.4), $(i)\gamma_M(G) \leq \gamma_M^{-1}(G)$ and by proposition(3.2)(i) $\gamma_M^{-1}(G) \leq \beta_M(G)$. We obtain the inequality (ii). □

**Proposition 3.2.** *If a graph G has a full degree vertex and others are pendants then* $\gamma_M^{-1}(G) = \beta_M(G)$.

**Proof:** Let G be a graph with p vertices in which u is a full degree vertex and $(p - 1)$ pendants. Therefore, $D = \{u\}$ is $\gamma_M$ set of G.Then $D^{'} = \{u_1, u_2, \ldots, u_{\lceil \frac{p}{2} \rceil - 1}\} \subseteq (V - D)$ is a inverse majority dominating set with $|D'| = \lceil \frac{p}{2} \rceil - 1$. Since all vertices in $D'$ are pendants, $|N[D']| = \lceil \frac{p}{2} \rceil$. It implies that $\gamma_M^{-1}(G) = |D'| = \lceil \frac{p}{2} \rceil - 1$. By proposition(2.3) (5), $\beta_M(G) = \lfloor \frac{(p-1)}{2} \rfloor, p \geq 2$. When p is odd and even $\lceil \frac{p}{2} \rceil - 1 = \lfloor \frac{(p-1)}{2} \rfloor$. $D'$ is also a majority independent set of G and $\beta_M(G) = \lceil \frac{p}{2} \rceil - 1$.
Hence $\gamma_M^{-1}(G) = \beta_M(G)$. □

**Corollary 3.1.** *Let G be a totally disconnected graph with even number of vertices. Then* $\gamma_M^{-1}(G) = \beta_M(G) = \frac{p}{2}$.

**Corollary 3.2.** *Let G be a disconnected graph without isolates. Then*$\gamma_M^{-1}(G) = \beta_M(G)$.

## 4. $\gamma_M^{-1}(G)$ and $\beta_M(G)$ for Cubic Bipartite Graphs

**Definition 4.1.** *Let $G$ be a cubic bipartite graph with a partition of the vertex set $V_1(G)$ and $V_2(G)$ such that $|V_1| + |V_2| = p$. A cubic bipartite graph $G$ with minimum number of vertices is $K_{3,3}$. Also graphs that have an odd number of vertices cannot be a cubic bipartite graph. We now concentrate the cubic bipartite graphs when $p = 6, 8, 10, 12, 14, 16, \ldots$.*

**Theorem 4.1.** *For all cubic and cubic bipartite graph $G$, $\gamma_M^{-1}(G) = \lceil \frac{p}{8} \rceil$.*

**Proof:** Let $V(G) = \{v_1, v_2, \ldots, v_p\}$ be the vertex set of the given graph G. Let D be a majority dominating set of G and $D' = \{v_1, v_2, \ldots, v_t\}$ be the inverse majority dominating set of G with respect to D with $|D'| = t = \gamma_M^{-1}(G)$.Then

$$|N[D']| \geq \lceil \frac{p}{2} \rceil and D' \subseteq (V - D) \tag{1}$$

Then, $|N[D']| \leq \sum_{(i=1)}^{t} d(v_i) + t = 4t = 4\gamma_M^{-1}(G)$
By (1), $\lceil \frac{p}{2} \rceil \leq 4\gamma_M^{-1}(G)$. Therefore,

$$\gamma_M^{-1}(G) \geq \lceil \frac{p}{8} \rceil \tag{2}$$

Suppose $D' = \{u_1, u_2, u_{\lceil \frac{p}{8} \rceil}\}$ is a subset of vertices in $(V - D)$ such that $N[u_i] \cap N[u_j] = \emptyset, i \neq j$ and $|D'| = \lceil \frac{p}{8} \rceil$. Then $|N[D']| = 4\lceil \frac{p}{8} \rceil$.
In all cases of $p = r(mod\ 8)$, when $0 \leq r \leq 7, |N[D']| = 4\lceil \frac{p}{8} \rceil \geq \lceil \frac{p}{2} \rceil$.
And $D' \subseteq V - D$. Therefore $D'$ is a Inverse Majority Dominating set of G.
Hence,

$$\gamma_M^{-1}(G) \leq |D'| = \lceil \frac{p}{8} \rceil \tag{3}$$

from (1) and (2) we obtain $\gamma_M^{-1}(G) = \lceil \frac{p}{8} \rceil$.
The following results are the characterization theorem of an inverse majority dominating sets and majority independent set. □

**Proposition 4.1.** $\gamma_M(G) = \gamma_M^{-1}(G) = \beta_M(G) = 1$ *if and only if the cubic bipartite graph $G$ has all vertices $u \in V(G)$ of degree $d(u) \geq \lceil \frac{p}{2} \rceil - 1$.*

**Proof:** Let G be a cubic bipartite graph with p vertices. If the given graph G has vertices of degree $d(u) \geq \lceil \frac{p}{2} \rceil - 1$, for all $u \in V(G)$, then every vertex is a majority dominating vertex of G.
Therefore, $D = \{u\}$ is a minimal majority dominating set of G and $D' = \{v\} \subseteq V - D$ is a minimal inverse majority dominating set of G, also any one vertex of G forms a majority Independent set of G. Hence the result
For the converse, by the Propositions (2.2), (2.3) and (2.5), we get the condition. □

**Observation 4.1.** *1. For a cubic bipartite graph with $p = 6$ or $8$,*

$$\gamma_M(G) = \gamma_M^{-1}(G) = \beta_M(G) = 1.$$

*2. For a cubic bipartite graph with $p = 10$ or $12$,*

$$\gamma_M(G) = \gamma_M^{-1}(G) = \beta_M(G) = 2.$$

*3. The following graph $G_1$ is a cubic bipartite with $p = 14$,*
*In $G_1; D_1 = \{v_1, v_4\}$ is a Majority Dominating set of $G_1$ and $D_2 = \{v_2, v_6\}$ is a Inverse Majority Dominating set of $G_1$ with respect to $D_1$. $\gamma_M(G_1) = \gamma_M^{-1}(G_1) = 2$. Also,*

$G_1$:

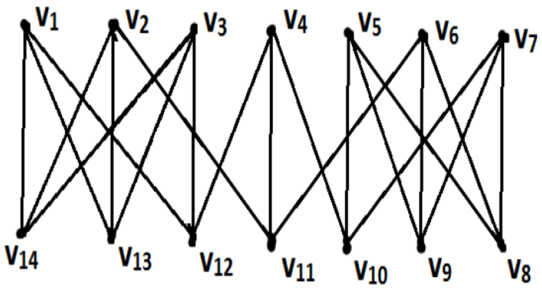


FIGURE 2

*$S = \{v_1, v_2, v_3\}$ is a majority independent set of $G_1$. Hence, $\beta_M(G) = 3$.*
*4. For p = 16, we have*

$$\gamma_M(G) = \gamma_M^{-1}(G) = 2 \quad and \quad \beta_M(G) = 3.$$

*5. For p = 18, We have*

$$\gamma_M(G) = \gamma_M^{-1}(G) = 3 \quad and \quad \beta_M(G) = 4.$$

*6. For p = 20, we have*

$$\gamma_M(G) = \gamma_M^{-1}(G) = 3 \quad and \quad \beta_M(G) = 4.$$

*7. For p = 22 ,we have*

$$\gamma_M(G) = \gamma_M^{-1}(G) = 3 \quad and \quad \beta_M(G) = 4.$$

*8. For p = 24, we have*

$$\gamma_M(G) = \gamma_M^{-1}(G) = 3 \quad and \quad \beta_M(G) = 5.$$

*and so on...*

**Theorem 4.2.** *Let G be an cubic bipartite graph with p vertices. The subsets D and $D'$ and S are the majority domination, inverse majority domination and majority independent sets of G respectively. Then $\gamma_M^{-1}(G) < \beta_M(G)$, when $p \geq 14$ If and only if*
*(i) $|p_n[v, D'| \geq 3$ , for every $v \in D'$ and*
*(ii) $|p_n[v, S| \leq 2$, for every $v \in S$.*

**Proof:** Let D be the majority dominating set of G. Let $\gamma_M^{-1}(G) < \beta_M(G)$. Then the inverse majority domination number $\gamma_M^{-1}(G) = |D'|$ and the majority independence number $\beta_M(G) = |S|$. Also, $D'$ is the minimum inverse majority dominating set with respect to D of G and S is the maximum majority independent set of a cubic bipartite graph G. Let $V_1(G)$ and $V_2(G)$ be the bipartition of V(G). $V_1(G) = \{v_1, v_2, \ldots, v_{p_1}\} V_2(G) = \{u_1, u_2, \ldots, u_{p_2}\}$ with $p = (p_1 + p_2)$. By the theorem (4.2), $\gamma_M^{-1}(G) \leq \lceil \frac{p}{8} \rceil$. Let $D' = \{v_1, v_2, \ldots, v_{\lceil \frac{p}{8} \rceil}\} \subseteq V_1(G)$ such that $d(v_i, v_j) \geq 2$ for any $v_i$ and $v_j \in D'$ and $i \neq j$. Since each vertex has degree $d(v_i) = 3, N(v_i) = (u_i, u_j, u_k) \subseteq V_2(G)$,
For each vertex $v_i \in D', |N(v_i) \cup N(v_j)| \neq \emptyset$, for any $v_i, v_j$ and $i \neq j$. Then there exists atmost one vertex $u$ such that $N(v_i) \cup N(v_j) = \{u\}$.
$|N(v_i) \cup N(v_j)| \geq 1$ and the private neighbour of each vertex $v_i \in D'$ is $\{v_i, u_i, u_j\}$ or $\{v_i, u_i, u_j, u_k\} \Rightarrow |p_n[v_i, D']| \geq 3$, for each $v_i \in D'$. Hence the condition (i) is true, Similarly By proposition (2.3) (2), For any cubic bipartite graph G, $\beta_M(G) \leq \lceil \frac{p}{4} \rceil - 1$.
Let $S = \{v_1, v_2, \ldots, v_t\} \subseteq V_1(G)$ be a majority independent set of G where $t = \lceil \frac{p}{4} \rceil - 1$ such that $d(v_i, v_j) = 2$ for any $i, j$ and $i \neq j$. For every vertex $v_i \in S$,

$N(v_i) \cup N(v_j) = \{(u_i, u_j) or (u_i, u_j, u_k)\} \Rightarrow |N(v_i) \cup N(v_j)| = 2$ or 3, for any $v_i, v_j \in S$. Since $d(v_i) = 3, |p_n[v_i, S]| = 2$ or 1 respectively.
$\Rightarrow |p_n[v, S]| \leq 2$, for all $v_i \in S$. Hence the condition (ii) holds.
Let $D'$ be a Inverse Majority Dominating set of a cubic bipartite graph G. Since $|p_n[v, D']| \geq 3$, for all $v \in D', |N(v_i) \cup N(v_j)| = 1$ or $\emptyset$, for $i \neq j$ and for $v_{(i)}, v_{(j)} \in D'$. Then $|N(v)| = 3 or 4$ and $|p_n[v, D']| = 3$ or 4, for every $v \in D'$ It implies that each vertex $v \in D'$ dominates atleast 3 vertices.
Hence, we get an inverse majority dominating set $D'$ with minimum cardinality for G. The inverse majority dominating number is $\gamma_M^{-1}(G) = |D'|$. Let S be a majority independent set of a cubic bipartite graph of G. Since $|p_n[v_i, S]| \leq 2, |N(v_i) \cup N(v_j)| = 3$ or 2, for any $v_i, v_j \in S$ and $i \neq j$. Then we obtain a majority independent set with maximum cardinality for G. Majority independence number of $G = |S| = \beta_M(G)$. Since $\gamma_M^{-1}(G) \leq \lceil \frac{p}{8} \rceil$ and $\beta_M(G) \leq \lceil \frac{p}{4} \rceil - 1$, we get $\gamma_M^{-1}(G) < \beta_M(G)$. □

**Corollary 4.1.** *Let $G$ be a cubic bipartite graph with $p \leq 13$ vertices and $D$ be majority dominating set of $G$. If $|p_n[v, D']| \geq 3$, for all $v \in D'$ and $|p_n[v, S]| = 3$, for atleast one vertex $v \in S$ then $\gamma_M^{-1}(G) = \beta_M(G)$, where $D'$ and $S$ are the inverse majority dominating set and majority independent set of G.*

**Proof.** Let D be a minimum majority dominating set and S be a maximal majority independent set of G. Since for atleast one vertex $v \in S, |p_n[v, S]| = 3, |N(v_i) \cup N(v_j)| = 1$, for any $v_i$ and $v_j \in S$. Then $|N[S]| \geq \lceil \frac{p}{2} \rceil$ and $|p_n[v, S]| > |N[S]| - \lceil \frac{p}{2} \rceil$, for all $v \in S$. $\Rightarrow$ S is also a minimal majority dominating set which is in $(V - D)$. Hence $\gamma_M^{-1}(G) \geq \beta_M(G)$ By proposition (3.2) (i), for any graph G, $\gamma_M^{-1}(G) \leq \beta_M(G)$. Thus, $\gamma_M^{-1}(G) = \beta_M(G)$. □

## 5. $\gamma_M^{-1}(G)$ and $\beta_M(G)$ for Some Families of Graphs

**Proposition 5.1.** *If $G = K_p$ is a complete graph with $p$ vertices, $\gamma_M(G) = \gamma_M^{-1}(G) = \beta_M(G) = 1$.*

**Proof:** Since the graph G is complete, it is a regular graph of degree $(p-1)$. Each vertex of G is a full degree vertex. The majority dominating set, the Inverse majority dominating set and also majority Independent set are all equal to any one vertex $\{v\}$ of G.
Hence $\gamma_M(G) = \gamma_M^{-1}(G) = \beta_M(G) = 1$ □

**Proposition 5.2.** *For a wheel graph $W_p, p \geq 5, \beta_M(G) = \lceil \frac{(p-2)}{6} \rceil$, if $5 \leq p \leq 18$ and*

$$\beta_M(G) = \left\lceil \frac{(p-3)}{4} \right\rceil, \quad if \quad p \geq 19$$

**Theorem 5.1.** *Let $G = W_p$ be a wheel of $p \geq 5$ vertices. Then*
*(i). $\gamma_M^{-1}(G) = \beta_M(G) = \lceil \frac{(p-2)}{6} \rceil$, if $5 \leq p \leq 18$ and*
*(ii). $\gamma_M^{-1}(G) < \beta_M(G)$, if $p \geq 19$.*

**Proof:** By the proposition (2.4), and the proposition (5.2),

$$\gamma_M^{-1}(G) = \beta_M(G) = \lceil \frac{(p-2)}{6} \rceil, 5 \leq p \leq 18.$$

Thus the result (i) is true. Also when $p \geq 19$, using the above results, We get $\gamma_M^{-1}(G) < \beta_M(G)$. □

**Proposition 5.3.** *For a cycle $C_p, p \geq 3$,*
*(i). $\beta_M(G) = \lceil \frac{p}{6} \rceil, if 3 \leq p \leq 16$ and (ii). $\beta_M(G) = \lceil \frac{(p-4)}{4} \rceil$, if $p \geq 17$.*

**Theorem 5.2.** *Let $G = C_p$, be a cycle $p \geq 3$ vertices Then*
*(i).* $\gamma_M^{-1}(G) = \beta_M(G) = \lceil \frac{p}{6} \rceil, 3 \leq p \leq 16$
*(ii).* $\gamma_M^{-1}(G) < \beta_M(G) = \lceil \frac{(p-4)}{4} \rceil, p \geq 17$

**Proof:** By the proposition (2.4) (2) and the proposition (5.4), we obtain $\gamma_M^{-1}(G) = \beta_M(G) = \lceil \frac{p}{6} \rceil, 3 \leq p \leq 16$. Hence result (i) is true.
Also, when $p \geq 17$,
Using the above results, We get $\gamma_M^{-1}(G) = \lceil \frac{p}{6} \rceil$ and $\beta_M(G) = \rceil \frac{(p-4)}{4} \rceil$.
Hence $\gamma_M^{-1}(G) < \beta_M(G)$. □

**Proposition 5.4.** *For a path $(P_p) p \geq 2, \beta_M(G) = \lceil \frac{p}{4} \rceil$, if $2 \leq p \leq 10$, $\beta_M(G) = \lceil \frac{(p-2)}{6} \rceil$, if $p > 11$.*

**Theorem 5.3.** *Let G be a path of $p \geq 2$, vertices then*
$\gamma_M^{-1}(G) = \beta_M(G) = \lceil \frac{p}{6} \rceil$, *if* $2 \leq p \leq 10$, *and*
$\gamma_M^{-1}(G) < \beta_M(G) = \lceil \frac{(p-2)}{6} \rceil$, *if* $p \geq 11$.

**Proof:** By the proposition (2.4) (2) and the proposition (5.6), we get,
$\gamma_M^{-1}(G) = \beta_M(G) = \lceil \frac{p}{6} \rceil$, if $2 \leq p \leq 10$,
Also, when $p \geq 11, \gamma_M^{-1}(G) < \beta_M(G) = \lceil \frac{(p-2)}{6} \rceil$. □

**Proposition 5.5.** *Let $F_p$, be a fan with $p \geq 4$ vertices. Then*
*(i). When* $p \equiv 1(mod \quad 6), \gamma_M^{-1}(G) < \beta_M(G)$ *and*
*(ii). When* $p \not\equiv 1(mod \quad 6), \gamma_M^{-1}(G) = \beta_M(G)$.

**Proof:** By the proposition (2.3) and propostion (2.4)

$$\gamma_M^{-1}(G) = \lceil \frac{(p-1)}{6} \rceil \quad and \quad \beta_M(G) = \lceil \frac{p}{6} \rceil. \tag{4}$$

(i). When $p \equiv 1(mod \quad 6)$. Since $\lceil \frac{(p-1)}{6} \rceil < \lceil \frac{p}{6} \rceil$, $\gamma_M^{-1}(G) < \beta_M(G)$.
(ii). When $p \not\equiv 1(mod \quad 6)$,Then $p \equiv 0, 2, 3, 4, 5(mod 6)$
$\lceil \frac{(p-1)}{6} \rceil = \lceil \frac{p}{6} \rceil$, if $p = 6r, 6r + 2, 6r + 3, 6r + 4, 6r + 5$
By using (4), We obtain, $\gamma_M^{-1}(G) = \beta_M(G)$. □

**Proposition 5.6.** *If the cubic graph G is a Generalised Petersen $P(n, k)$ graph. Then $\gamma_M^{-1}(G) = \beta_M(G)$.*

**Proof:** Since G is a 3-regular graph and by theorem (2.3) (3), We have $\beta_M(G) = \lceil \frac{p}{8} \rceil$. Also, by theorem (4.2), For a Generlised Petersen graph $P(n, k), \gamma_M^{-1}(G) = \lceil \frac{p}{8} \rceil = \beta_M(G)$.
□

**Proposition 5.7.** *Let $G = K_{(1,p-1)}$ be a star with $p \geq 2$ vertices. Then $\gamma_M^{-1}(G) = \beta_M(G)$.*

**Proof:** By the Proposition (2.4) (6) and Proposition (2.3) (5), We have $\gamma_M^{-1}(G) = \lfloor \frac{(p-1)}{2} \rfloor = \beta_M(G)$. □

**Result 5.1.** *There exists a graph G for which $\gamma_M(G) = \gamma_M^{-1}(G) = 2$ and $\beta_M(G) = 2t = \lceil \frac{p}{2} \rceil - \gamma_M^{-1}(G)$, where $t \geq 3$.*

**Proof:** The graph G is obtained by adding one pendant at each vertex of a complete graph and then add a pendant each time at each vertex of $K_4$. Finally we obtain a new structure with $p = 4 + 4t$, where t is the number of pendants at each time at one vertex of $K_4$.

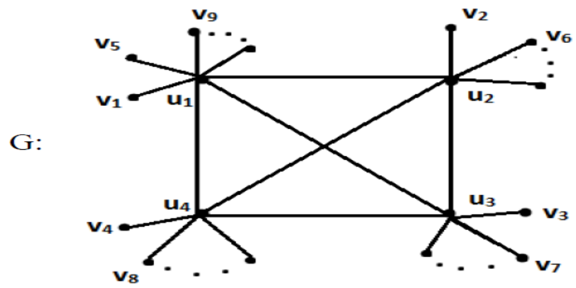


FIGURE 3

Let $|V(G)| = p = 4{+}4t$, when t = 1 then the vertex set $V(G) = \{u_1, u_2, u_3, u_4, v_1, v_2, v_3, v_4\}$ where $(u_1, u_2, u_3, u_4) \subseteq V(K_4)$ and other vertices are pendants. Then $p = 4 + 4 = 8$, and $\gamma_M(G) = |\{u_1\}|$ and $\gamma_M^{-1}(G) = |\{u_2\}|.\gamma_M(G) = \gamma_M^{-1}(G) = 1$ and $\beta_M(G) = |v_1, v_2| = 2$, where $v_1$ and $v_2$ are adjacent to $u_1$ and $u_2$ in G.
when $t = 2, p = 4 + 8 = 12, \gamma_M(G) = \gamma_M^{-1}(G) = 1$ and $\beta_M(G) = |\{v_1, v_2, v_5, v_6\}| = 4 = 2t$, if $t = 2$.
when $t = 3, p = 4 + (4 \times 3) = 16$. In $G_1$, there are 3 pendants at each vertex of $K_4$.
Let $D = \{u_1, u_2\}$ be a majority dominating set of G and $D' = \{u_3, u_4\} \subseteq V - D$ is a inverse majority dominating set of G Therefore $\gamma_M(G) = \gamma_M^{-1}(G)$. Now $S = \{v_1, v_5, v_9, v_2, v_6, v_{10}\}$ such that $|N[S]| = 8$ and $|p_n[v, S]| = 1 > |N[S]| - \lceil \frac{p}{2} \rceil = 0$, for all $u \in S$. Hence $\beta_M(G) = |S| = 6 = 2t$, if t = 3 and so on.
Also,when $t = 3, \lceil \frac{p}{2} \rceil - \gamma_M^{-1}(G) = 8 - 2 = 6 = \beta_M(G)$,
Thus, $\beta_M(G) = 2t = \lceil \frac{p}{2} \rceil - \gamma_M^{-1}(G)$.
In general In this structure, the difference between $\gamma_M^{-1}(G)$ and $\beta_M(G)$ is very large. Hence $\gamma_M^{-1}(G) < \beta_M(G)$ and the difference between these two numbers is very large when p is large and $t \geq 3$. □

## 6. Conclusion

In this article, we have discussed the relation between inverse majority domination number and majority independence number of a graph is discussed. Also some classes of graphs, characterisation theorem for this relation are studied

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