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IDENTIFYING STRUCTURAL ISOMORPHISM BETWEEN TWO KINEMATIC CHAINS VIA NANO TOPOLOGY

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Abstract. This research paper aims to analyse the structural equivalence of two undirected graphs via nano topology. Nano homeomorphism has been applied to identify the similarity between two graphs. We have proposed a new notion to define the neighbourhood of a vertex of an undirected graph and have defined the approximations through the neighbourhood and found out the nano topologies induced by the vertices of the graph and discussed the nano homeomorphisms and nano isomorphisms. We have checked the structural equivalence of the graphs via nano isomorphism. Also we have proposed an algorithm and have checked the structural equivalence of two kinematic chains by using nano isomorphism.

Keywords: Nano topology, neighbourhoods, Nano Homeomorphism, Nano isomorphism, Kinematic chains

AMS Subject Classification: 54A05, 54A10, 54B05

1. Introduction

Isomorphism plays a major role in identifying the structural equivalence of two graphs. It is necessary to prevent the duplication of structures before applying practically. In this paper we have approached the concept of isomorphism between two undirected graphs, through nano topology induced by the vertices of the given graphs. The concept of nano topology was introduced by Lellis Thivagar [3]. The first step on nano topological graph theory was done by Lellis Thivagar et.al [4]. They had discussed nano topological graph structures for directed graphs and studied isomorphism. Two nano topological spaces are said to be topologically equivalent, if there exists a nano homeomorphism [1] between them.

In this paper, we have discussed the graphical isomorphism for undirected graphs through nano homeomorphism and have checked whether two undirected graphs have

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similar pattern of connections. Also we have formalised the structural equivalence of two kinematic chains.

A kinematic chain is an assembly of rigid bodies connected by joints. A kinematic chain is one with constrained motion, which means that a definite motion of any link produces unique motion of all other links. Isomorphism identification in kinematic chains is one of the most important and challenging task in mathematical problems in the field of mechanical structural synthesis.

We have modelled the links and the connections of the kinematic chains as an undirected graphs and derived the nano topologies induced by the vertices of the graphs. We have checked the topological equivalence of the graphs through nano homeomorphisms, from which concluded that whether the pattern of the given two kinematics chains are similar or not.

2. Preliminaries

In this section, we have brought some basic definitions and results related to graph theory and nano topology which are used in the sequel.

Definition 2.1. [5] *A graph $G(V, E)$ is a triple consisting of*

(i) *Vertex set $V(G)$*
(ii) *An edge set $E(G)$*
(iii) *A relation between an edge and pair of vertices.*

Definition 2.2. [5]

(i) *An edge between the vertices u and v can be represented by (u, v) or (v, u).*
(ii) *A graph G is said to be a directed graph if the edges are the ordered pair of elements of $V(G)$. (That is if the edges are directed.)*
(iii) *A graph G is said to be an undirected graph if the edge set E is the unordered pairs of distinct elements of $V(G)$.*

Definition 2.3. [5] *A graph is said to be finite, if the vertex set $V(G)$ and the edge set $E(G)$ are finite.*

Definition 2.4. [5] *An edge whose end points are same is called a self loop. The edges having same pair of end points are called parallel edges. A graph without self loops and parallel edges is called as a simple graph. Two vertices are said to be adjacent vertices, if they are the end points of an edge. The number of edges incident on a vertex v is called the degree of the vertex v and it is denoted by $d(v)$.*

Definition 2.5. [5] *A subgraph H of a graph $G(V, E)$is a graph such that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ and the assignment of end points to edges in H is same as that of G.*

Throughout this paper, with graph we mean undirected simple graph.

Definition 2.6. [2] *In an universe U, define an equivalence relation R, and a set $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$ and the approximations are defined as*

(i) $L_R(X) = \cup\{R(X) : R(X) \subseteq X, x \in U\}$
(ii) $U_R(X) = \cup\{R(X) : R(X) \cap X \neq \phi, x \in U\}$.
(iii) $B_R(X) = \overline{R}(X) - \underline{R}(X)$.

The set $\tau_R(X)$ satisfies the axioms of a topology.

(i) *U and $\phi \in \tau_R(X)$.*

(ii) *The union of the elements of any sub-collection of* $\tau_R(X)$ *is in* $\tau_R(X)$.
(iii) *The intersection of the elements of any finite sub collection of* $\tau_R(X)$ *is in* $\tau_R(X)$.
Here $\tau_R(X)$ *is termed to be a nano topology on* U *with respect to* X. $(U, \tau_R(X))$ *is termed to be the nano topological space. Elements of* $(U, \tau_R(X))$ *are known as nano open sets in* U *and the complement of nano open sets are called a nano closed sets. The basis of* $\tau_R(X)$ *is the set* $B = \{U, L_R(X), U_R(X)\}$.

Definition 2.7. [2] *A function* $f : (U, \tau R(X)) \longrightarrow (V, \tau R(Y))$ *is said to be a nano open function if for every nano open set* A *of* U, *the image* $f(A)$ *is a nano open set in* V.

Definition 2.8. [2] *A function defined between two nano topological spaces* $f : (U, \tau R(X)) \longrightarrow (V, \tau R(Y))$ *is termed to be a nano continuous function if the the inverse image* $f^{-1}(P)$ *is a nano open set in* $(U, \tau R(X)$ *for every nano open set* P *in* $(V, \tau R(Y))$.

Definition 2.9. [2] *A function* $f : (U, \tau R(X)) \longrightarrow (V, \tau R(Y))$ *is said to be a nano homeomorphism if* f *satisfies the following.*
(i) f *is* $1-1$ *and onto*
(ii) f *is nano continuous*
(iii) f *is a nano open map*

Definition 2.10. [4] *Let* $G(V, E)$ *and* $G'(V', E')$ *be any two graphs.* G *and* G' *are said to be isomorphic graphs, if there exists a homeomorphism* $\phi : (V(G), \tau_N(V(H))) \longrightarrow (V(G'), \tau_N f[(V(H)]))$ *for every subgraph* H *of* G, *where* $\tau_N(V(H)$ *and* $\tau_N f[(V(H)]$ *are the nano topologies induced by the vertices of the respective subgraphs* H *of* G *and* G'.

3. Neighbourhood and Approximations

This section deals with utilization of the definitions of Lower and Upper approximations of the vertices of an undirected graph $G(V, E)$ and the nano topology induced by the vertices of the subgraph H of G.

Theorem 3.1. *Let* $G(V, E)$ *be a graph with the vertex set* $V(G)$. *The neighbourhood of a vertex* $v \in V(G)$ *is defined as* $N(v) = \{v\} \cup \{u : (u, v) or (v, u) \in E(G)\}$. *That is* $N(v)$= *Union of* v *and the set of all adjacent vertices of* v.

Theorem 3.2. *Consider* $G(V, E)$ *to be a graph,* H *a subgraph of* G *and* $N(v)$ *the neighbourhood* $v \in V$. *Then*
(i) *The lower approximation* $R_L[V(H)] = \bigcup_{v \in V(G)} \{v : N(v) \subseteq V(H)\}$.
(ii) *The upper approximation* $R_U[V(H)] = \bigcup_{v \in V(H)} \{v : N(v) \cap V(H) \neq \phi\}$.
(iii) *The boundary region* $R_B[V(H)] = R_U[V(H)] - R_L[V(H)]$.
The nano topology induced by the vertices of $V(H)$ *is given by* $\tau_N[V(H)] = \{V(G), \phi, R_L[V(H)], R_U[V(H)], R_B[V(H)]\}$. *The space* $(V(G), \tau_N[V(H)]$ *may be referred as the nano topological space induced by the graph.*

Example 3.1. *This example explains the construction of nano topology induced by the vertices of the given graph.*

In the following graph (See Figure 1), $V(G) = \{1, 2, 3, 4, 5, 6\}$ and the neighbourhood of the vertices are $N(1) = \{1, 2, 4, 6\}, N(2) = \{1, 2, 3\}, N(3) = \{2, 3, 4\}, N(4) = \{1, 3, 4, 5\}, N(5) = \{4, 5, 6\}, N(6) = \{1, 5, 6\}$. Consider the subgraph $V(H) = \{1, 2, 3\}$, then $R_L[V(H)] = \{2\}$,
$R_U[V(H)] = \{1, 2, 3, 4, 5, 6\} = U, R_B[V(H)] = \{1, 3, 4, 5, 6\}$. The nano topology induced by the vertices of the subgraph H is $\tau[V(H)] = \{U, \phi, \{2\}, \{1, 3, 4, 5, 6\}\}$.

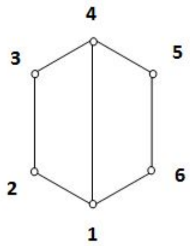

FIGURE 1

4. IDENTIFYING THE STRUCTURAL EQUIVALENCE IN KINEMATIC CHAINS USING NANO TOPOLOGY

Theorem 4.1. *Two graphs G and G' are said to be isomorphic, if G' can be obtained by relabelling the vertices of G. More conveniently, if there is a bijection $f : V(G) \longrightarrow V(G')$ with the property that the vertices u and v of G are adjacent if and only if $f(u)$ and $f(v)$ are adjacent in G' for all u and v in $V(G)$.*
In this work we have derived the nano topologies induced by the vertices of all subgraphs of G and G' and let them be $\tau_N[V(H)]$ and $\tau_N[f(V(H))]$ and defined a function $f : (V(G), \tau_N(V(H))) \longrightarrow (V(G'), \tau_N f[(V(H)]))$. If there is a homeomorphism between the nano topologies for all subgraphs of G and G', then the two graphs are topologically equivalent.
Kinematic chain is an assembly of rigid bodies connected by joints to provide constrained or desired motion, that is the mathematical model for a machine system. The rigid bodies or links are constrained by their connections to other links. Mathematical model of the connections or joints between the two links are termed to be Kinematic pairs. A Kinematic diagram is schematic of the mechanical system that shows the Kinematic chain.
The isomorphism detection in Kinematic chain is a necessary task to avoid the duplication of chains. Omission of the duplication is mechanically more useful.
Representing the Kinematic chains as undirected graphs and by proving the topological equivalence of the graphs we have checked the isomorphic nature of the Chains.

ALGORITHM :

Step 1 : *Given the suggested two Kinematic Chains K1 and K2.*

Step 2 : *Represent the Kinematic chains as graphs by assuming their links as the vertices and the connection between the links as the edges. Let the graphs be $G(V, E)$ and $G'(V'.E')$.*

Step 3 : *Derive the neighbourhoods, approximations and the nano topologies induced by the vertices of all sub graphs of G and G' by using Definition 3.2. Let them be $\tau_N[V(H)]$ and $\tau_N[f(V(H))]$.*

Step 4 : *Define a function $f : (V(G), \tau_N(V(H))) \longrightarrow (V(G'), \tau_N f[(V(H)]))$ and check the nano homeomorphism for all subgraphs by using Definition 2.11*

Step 5 : *If f is homeomorphic for all the subgraphs, then the graphs G and G' are isomorphic and hence the structure of the given two Kinematic chains K1 and K2 are same. That is the proposed Kinematic chains are similar, otherwise K1 and K2 are distinct.*

Example 4.1. *Checking the structural equivalence of two Kinematic chains given below, by using the above proposed Algorithm.*
Step 1. *Let the two Kinematic chains be K1 and K2.*

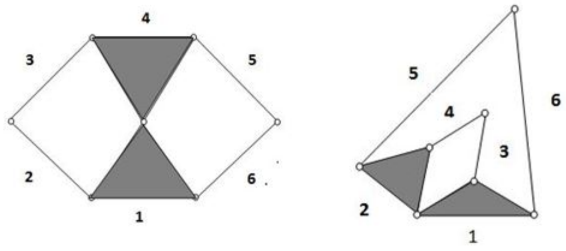

FIGURE 2. LINKS K1, K2

Step 2. *Graphs of K1 and K2 referred as $G1$and $G2$.*

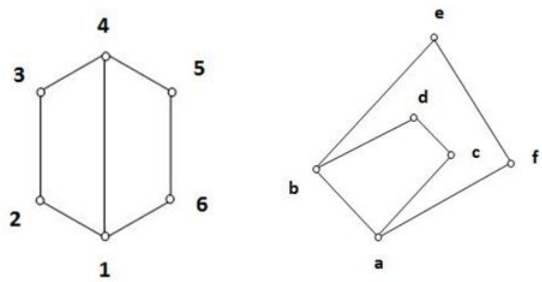

FIGURE 3. GRAPHS G1(for K1), G2(for K2)

Step 3. *Neighbourhoods,approximations and the nano topologies for all the subgraphs.*

Neighbourhoods of Graph G1: *Referring the graph G1 in Figure.3 and applying Definition 3.2, let* $V(G) = \{1, 2, 3, 4, 5, 6\}$ *and the neighbourhood of the vertices are* $N(1) = \{1, 2, 4, 6\}, N(2) = \{1, 2, 3\}, N(3) = \{2, 3, 4\}, N(4) = \{1, 3, 4, 5\}, N(5) = \{4, 5, 6\}, N(6) = \{1, 5, 6\}$.
TABLE.1 and TABLE.2 gives the nano topology induced by the vertices of G1 and G2.

Step 4. *Define a function* $f : (V(G1), \tau_N[V(H)])) \longrightarrow (V(G2), \tau_N f[V(H)]))$ *by* $f(1) = a, f(2) = c, f(3) = d, f(4) = b, f(5) = e, f(6) = f$. *TABLE.3 explains the structural equivalence of kinematic chains K1 and K2.*

Inference from TABLE.3

(i) *Distinct elements of* $V(G1)$ *have distinct images in* $V(G2)$. *Therefore* f *is 1-1 and since each element of* $V(G2)$ *has a pre image in* $V(G1)$, f *is onto.*
(ii) *The image of all nano open sets of* $(V(G1), \tau_N[V(H)])$ *are all nano open sets. Thus* f *is nano open map.*
(iii) *For every nano open set* P *of* $(V(G2), \tau_N f[V(H)])$ *is nano open in* $(V(G1), \tau_N[V(H)])$. *Therefore* f *is nano continuous.*

TABLE 1. Nano topology induced by the vertices of the graph G1

Subgraph $V(H)$	$R_L[V(H)]$	$R_U[V(H)]$	$R_B[V(H)]$	$\tau_N(V(H))$
$\{1\}$	ϕ	$\{1,2,4,6\}$	$\{1,2,4,6\}$	$\{U,\phi,\{1,2,4,6\}\}$
$\{2\}$	ϕ	$\{1,2,3\}$	$\{1,2,3\}$	$\{U,\phi,\{1,2,3\}\}$
$\{3\}$	ϕ	$\{2,3,4\}$	$\{2,3,4\}$	$\{U,\phi,\{2,3,4\}\}$
$\{4\}$	ϕ	$\{1,3,4,5\}$	$\{1,3,4,5\}$	$\{U,\phi,\{1,3,4,5\}\}$
$\{5\}$	ϕ	$\{4,5,6\}$	$\{4,5,6\}$	$\{U,\phi,\{4,5,6\}\}$
$\{6\}$	ϕ	$\{1,5,6\}$	$\{1,5,6\}$	$\{U,\phi,\{1,5,6\}\}$
$\{1,2\}$	ϕ	$\{1,2,3,4,6\}$	$\{1,2,3,4,6\}$	$\{U,\phi,\{1,2,3,4,6\}\}$
...	...	...	...	...
$\{1,2,3\}$	$\{2\}$	$\{1,2,3,4,6\}$	$\{1,3,4,6\}$	$\{U,\phi,\{2\},\{1,3,4,6\},\{1,2,3,4,6\}\}$
$\{1,2,3,4\}$	$\{2,3\}$	U	$\{1,4,5,6\}$	$\{U,\phi,\{2,3\},\{1,4,5,6\}\}$
$\{1,3,4,5\}$	$\{4\}$	U	$\{1,2,3,5,6\}$	$\{U,\phi,\{4\},\{1,2,3,5,6\}\}$
$\{1,3,4,6\}$	ϕ	U	U	$\{U,\phi\}$
$\{1,4,5,6\}$	$\{5,6\}$	U	$\{1,2,3,4\}$	$\{U,\phi,\{5,6\},\{1,2,3,4\}\}$
$\{2,3,4,5\}$	$\{3\}$	U	$\{1,2,4,5,6\}$	$\{U,\phi,\{3\},\{1,2,4,5,6\}\}$
...	...	...	...	...
$\{1,2,4,5,6\}$	$\{1,5,6\}$	U	$\{2,3,4\}$	$\{U,\phi,\{1,5,6\},\{2,3,4\}\}$
U	U	U	ϕ	$\{U,\phi\}$
ϕ	ϕ	ϕ	ϕ	$\{U,\phi\}$

TABLE 2. Nano topology induced by the vertices of the graph G2

$f[V(H)]$	$R_Lf[V(H)]$	$R_Uf[V(H)]$	$R_Bf[V(H)]$	$\tau_Nf[(V(H))]$
$\{a\}$	ϕ	$\{a,b,c,f\}$	$\{a,b,c,f\}$	$\{U,\phi,\{a,b,c,f\}\}$
$\{b\}$	ϕ	$\{a,b,d,e\}$	$\{a,b,d,e\}$	$\{U,\phi,\{a,b,d,e\}\}$
$\{c\}$	ϕ	$\{a,c,d\}$	$\{a,c,d\}$	$\{U,\phi,\{a,c,d\}\}$
$\{d\}$	ϕ	$\{b,c,d\}$	$\{b,c,d\}$	$\{U,\phi,\{b,c,d\}\}$
$\{e\}$	ϕ	$\{b,e,f\}$	$\{b,e,f\}$	$\{U,\phi,\{b,e,f\}\}$
$\{f\}$	ϕ	$\{a,e,f\}$	$\{a,e,f\}$	$\{U,\phi,\{a,e,f\}\}$
$\{a,b\}$	ϕ	U	U	$\{U,\phi\}\}$
...	...	...	...	...
$\{a,e,f\}$	$\{f\}$	$\{a,b,c,e,f\}$	$\{a,b,c,e\}$	$\{U,\phi,\{f\},\{a,b,c,e\},\{a,b,c,e,f\}\}$
$\{a,d,f\}$	ϕ	U	U	$\{U,\phi\}$
$\{a,e,f\}$	$\{f\}$	$\{a,b,c,e,f\}$	$\{a,b,c,e\}$	$\{U,\phi,\{f\},\{a,b,c,e\},\{a,b,c,e,f\}\}$
$\{b,c,d\}$	$\{d\}$	$\{a,b,c,d,e\}$	$\{a,b,c,e\}$	$\{U,\phi,\{f\},\{a,b,c,e\},\{a,b,c,d,e\}\}$
...	...	...	...	...
$\{a,b,c,d\}$	$\{c,d\}$	U	$\{a,b,e,f\}$	$\{U,\phi,\{c,d\},\{a,b,e,f\}\}$
$\{a,b,e,f\}$	$\{e,f\}$	U	$\{a,b,c,d\}$	$\{U,\phi,\{e,f\},\{a,b,c,d\}\}$
$\{a,b,c,d,f\}$	$\{a,c,d\}$	U	$\{b,e,f\}$	$\{U,\phi,\{a,c,d\},\{b,e,f\}\}$
U	U	U	ϕ	$\{U,\phi\}$
ϕ	ϕ	ϕ	ϕ	$\{U,\phi\}$

Step 5. *TABLE.3 shows that the function defined in step 4 is nano homeomorphic for all possible subsets $V(H)$ of the vertex set $V(G1)$. Therefore f is isomorphic.*

Step 6. *Since the function $f:(V(G1),\tau_N[V(H)])\longrightarrow(V(G2),\tau_Nf[V(H)])$ is isomorphic, the graphs G1 and G2 are topologically equivalent. Therefore the concerned kinematic*

Table 3. Structural equivalence of kinematic chains

$V(H)$	$\tau_N(V(H))$	$f[V(H)]$	$\tau_N f[V(H)]$
$\{1\}$	$\{U, \phi, \{1, 2, 4, 6\}\}$	$\{a\}$	$\{U, \phi, \{a, b, c, f\}\}$
$\{2\}$	$\{U, \phi, \{1, 2, 3\}\}$	$\{c\}$	$\{U, \phi, \{a, c, d\}\}$
$\{3\}$	$\{U, \phi, \{2, 3, 4\}\}$	$\{d\}$	$\{U, \phi, \{b, c, d\}\}$
$\{4\}$	$\{U, \phi, \{1, 3, 4, 5\}\}$	$\{b\}$	$\{U, \phi, \{a, b, d, e\}\}$
$\{5\}$	$\{U, \phi, \{4, 5, 6\}\}$	$\{e\}$	$\{U, \phi, \{b, e, f\}\}$
$\{6\}$	$\{U, \phi, \{1, 5, 6\}\}$	$\{f\}$	$\{U, \phi, \{a, e, f\}\}$
$\{1, 2\}$	$\{U, \phi, \{1, 2, 3, 4, 6\}\}$	$\{a, c\}$	$\{U, \phi, \{a, b, c, d, f\}\}$
...	...	...	...
$\{1, 2, 3\}$	$\{U, \phi, \{2\}, \{1, 3, 4, 6\}, \{1, 2, 3, 4, 6\}\}$	$\{a, c, d\}$	$\{U, \phi, \{c\}, \{a, b, d, f\}, \{a, b, c, d, f\}\}$
$\{1, 2, 3, 4\}$	$\{U, \phi, \{2, 3\}, \{1, 4, 5, 6\}\}$	$\{a, b, c, d\}$	$\{U, \phi, \{c, d\}, \{a, b, e, f\}\}$
$\{1, 2, 3, 4, 5\}$	$\{U, \phi, \{2, 3, 4\}, \{1, 5, 6\}\}$	$\{a, b, c, d, e\}$	$\{U, \phi, \{b, c, d\}, \{a, e, f\}\}$
$\{2, 3, 4, 5, 6\}$	$\{U, \phi, \{3, 5\}, \{1, 2, 4, 6\}\}$	$\{b, c, d, e, f\}$	$\{U, \phi, \{d, e\}, \{a, b, c, f\}\}$
...	...	...	...
U	$\{U, \phi\}$	U	$\{U, \phi\}$
ϕ	$\{U, \phi\}$	ϕ	$\{U, \phi\}$

chains K1 and K2 are structurally equivalent.

Computer Algorithm to derive the nano topologies

Manual work on deriving the power set of a set and the respective nano topologies is possible, only for the graphs with minimum number of vertices. In this paper we have tried to do this for a graph with 6 vertices. When the graph contains more vertices, then the manual work is tedious.

In this section we have proposed a computer algorithm, which gives the power set (2^n subsets), of the vertex set with any number(n) vertices and derives the nano topologies induced by all subsets of the vertex set. We can utilize those nano topologies and define the related function for checking the topological isomorphism between the graphs. Therefore with the support of the proposed algorithm, checking of structural equivalence of the kinematic chains with any number of links the connections is possible.
The input of the algorithm is the neighbourhood of the vertices.

(1) **Subsets construction:** Set of node numbers will be given as input to combinations functions.
(2) Given out list of subsets for n nodes, 2^n subsets.
(3) Initialise list-of-nodes $N_1, N_2, ------, N_n$.
(4) In list-of-nodes, list - $V(H)$
(5) // for
$L[V(H)] = \bigcup_{v \in V(G)} \{v : N(v) \subseteq V(H)\}$
for
node in list-of-nodes:
if node N_i is subset (subset $V(H)$)
add that in a list
else
Print "Null"
// gives $L[V(H)]$
// for

$U[V(H)] = \bigcup_{v \in V(H)} \{v : N(v) \cap V(H) \neq \phi\}$
for
Subset in $V(H)$
for each element in subset:
if list-of-nodes $[i-1].intersection$
append that to a list
find union of all sets of list gives $U[V(H)]$
// for
$B[V(H)] = U[V(H)] - L[V(H)]$, find the difference of two lists/sets $U[V(H)]$ and $L[V(H)]$
List $\{U, \phi, L[V(H)], U[V(H)], B[V(H)]\}$ in $\tau_R(U)$

5. Conclusions

In this paper we have discussed nano homeomorphisms and nano isomorphisms. We have discussed topological graph theory. We have proposed a new definition to find the neighbourhoods of the vertices of an undirected graph, by using the neighbourhoods we find the approximations and derived the nano topologies induced by the vertices of the graph. Moreover we have proposed an algorithm to identify the structural equivalence of two undirected graphs. By using the algorithm we have proved that the structure of the two kinematic chains are identical. Also a computer coding is given, which is very much useful for constructing the nano topologies of a graph with more number of vertices.

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