TWMS J. App. and Eng. Math. V.12, N.1, 2022, pp. 1-10

MAXIMUM AND MINIMUM DEGREE ENERGIES OF p-SPLITTING AND p-SHADOW GRAPHS

K. S. RAO[1], K. SARAVANAN[1], K. N. PRAKASHA[2], I. N. CANGUL[3], §

Abstract. Let v_i and v_j be two vertices of a graph G. The maximum degree matrix of G is given in [2] by

$$d_{ij} = \begin{cases} \max\{d_i, d_j\} & if\ v_i\ and\ v_j\ are\ adjacent \\ 0 & otherwise. \end{cases}$$

Similarly the (i, j)-th entry of the minimum degree matrix is defined by taking the minimum degree instead of the maximum degree above, [1]. In this paper, we have elucidated a relation between maximum degree energy of $p-$shadow graphs with the maximum degree energy of its underlying graph. Similarly, a relation has been derived for minimum degree energy also. We disprove the results $E_M(S'(G)) = 2E_M(G)$ and $E_m(S'(G)) = 2E_m(G)$ given by Zheng-Qing Chu et al. [3] by giving some counterexamples.

Keywords and Phrases: maximum degree energy, minimum degree energy, splitting graph, shadow graph.

Mathematics Subject Classification: 05C50, 05C35

1. Introduction

The adjacency matrix of a graph G is known to be a $\{0, 1\}$ matrix with the (i, j)-th entry 1 if v_i and v_j are adjacent, and 0 if v_i and v_j are non-adjacent. Gutman introduced the notion of energy of a graph contingent on adjacency matrix which was emanated by the motivation of Hückel molecular orbital approximation, [5]. He defined the energy of a graph as the sum of the absolute values of eigenvalues of the adjacency matrix. Due to the intensive use of the adjacency matrix, many other graph matrices have been introduced which are related to different properties of graph, see e.g. [4, 7, 9]. Distance matrix,

[1] Department of Mathematics, Sri Chandrasekharendra Saraswathi Viswa Mahavidyalaya, Kanchipuram, Tamilnadu, India.
e-mail: raokonda@yahoo.com; ORCID: https://orcid.org/0000-0002-2357-0933.
e-mail: kadirvelsaravanan@gmail.com; ORCID: https://orcid.org/0000-0001-9373-1993
[2] Department of Mathematics, Vidyavardhaka College of Engineering, Mysuru, India.
e-mail: prakashamaths@gmail.com; ORCID: https://orcid.org/0000-0002-6908-4076.
[3] Department of Mathematics, Faculty of Arts and Science, Bursa Uludag University, 16059, Bursa, Turkey.
e-mail: cangul@uludag.edu.tr; ORCID: https://orcid.org/0000-0002-0700-5774; corresponding author.
§ Manuscript received: March 19, 2021; accepted: August 2, 2021.

Randic matrix, Laplacian matrix, partition matrix, sum connectivity matrix, minimum covering matrix etc. are some of such matrices.

Let G be a simple graph with n vertices $\{v_1, v_2, \cdots, v_n\}$ and let d_i be the degree of v_i for $i = 1, 2, 3, \cdots, n$. The maximum degree matrix is defined by Adiga and Smitha in [2] as

$$d_{ij} = \begin{cases} \max\{d_i, d_j\} & if\ v_i\ and\ v_j\ are\ adjacent \\ 0 & otherwise \end{cases}$$

Let $\mu_1, \mu_2, \cdots, \mu_n$ be the maximum degree eigenvalues of $M(G)$. As the maximum degree matrix is a real symmetric matrix with zero trace, these maximum degree eigenvalues are real with sum equal to zero. The maximum degree energy of a graph G is defined similarly to the classical adjacency energy as

$$E_M(G) = \sum_{i=1}^{n} |\mu_i|.$$

It is shown that if the maximum degree energy of a graph is rational, then it must be an even integer [2]. K.Srinivasa Rao, et al. [8] obtained bounds on the maximum degree eigenvalues for a general graph and as well as some frequently used graph classes. Several unicyclic graph classes are defined and their maximum degree eigenvalues and energy are calculated [8].

The minimum degree matrix, [1], is defined similarly to the maximum degree matrix with the change in (i, j)-th entry. Here the (i, j)-th entry is the minimum of the degrees of two adjacent vertices v_i and v_j. Let $\rho_1, \rho_2, \cdots, \rho_n$ be the minimum degree eigenvalues of the minimum degree matrix. The minimum degree energy is defined as

$$E_m(G) = \sum_{i=1}^{n} |\rho_i|.$$

Proposition 1.1. [6] *Let $A \in M^m$, $B \in M^n$. Let λ be an eigenvalue of A corresponding to an eigenvector x and μ be an eigenvalue of B corresponding to an eigenvector y. Then $\lambda\mu$ is an eigenvalue of $A \otimes B$ corresponding to the eigenvector $x \otimes y$.*

2. Maximum and minimum degree energy of splitting graphs

A derived graph is a graph which is obtained from a given graph according to some set of rules. One of the derived graphs is called splitting graph. The splitting graph $S'(G)$ of a graph G is obtained by adding a new vertex u' to each vertex u such that u' is adjacent to every vertex that is adjacent to u in G. See Fig. 1 as an example:

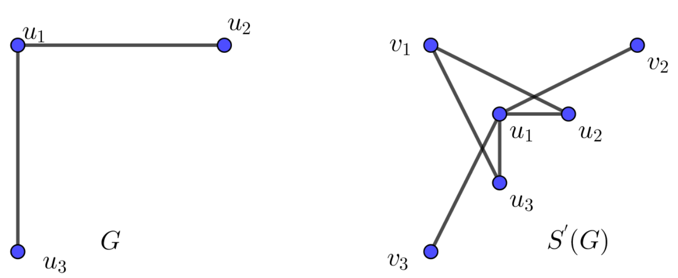


Figure 1

2.1. **Errors.** Zheng-Qing Chu et al., [3], gave the following relations between $E_M(S'(G))$ and $E_M(G)$ and also between $E_m(S'(G))$ and $E_m(G)$. These two results are proven in this paper to be incorrect. We provide some counter-examples which disprove these results. First we recall the erroneous results:

Theorem 2.1. [3] *For a graph G,*

$$E_M(S'(G)) = 2E_M(G).$$

Theorem 2.2. [3] *For a graph G,*

$$E_m(S'(G)) = 2E_m(G).$$

Maximum and minimum degree matrices of G and $S^{'}(G)$ given in Fig. 1 are

$$M(G) = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$$

and

$$m(G) = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

and also

$$M(S^{'}(G)) = \begin{bmatrix} 0 & 4 & 4 & 0 & 4 & 4 \\ 4 & 0 & 0 & 2 & 0 & 0 \\ 4 & 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad m(S^{'}(G)) = \begin{bmatrix} 0 & 2 & 2 & 0 & 1 & 1 \\ 2 & 0 & 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Here $E_M(G) = 5.6569$, $E_m(G) = 2.8284$, $E_M(S'(G)) = 20.3961$ and $E_m(S'(G)) = 10.198$. We can easily observe that $E_M(S'(G)) \neq 2E_M(G)$ and $E_m(S'(G)) \neq 2E_m(G)$. With this example, we can conclude that Theorem 1 and Theorem 2 of [3] are not correct. This is due to the wrong construction of $M(S'(G))$ and $m(S'(G))$ in the proof of Theorem 1 and Theorem 2.

2.2. **Maximum degree energy of p-splitting graphs.** The p-splitting graph $S_p^{'}(G)$ of a graph G is obtained by adding p new vertices, say $\{u_1, u_2, \cdots, u_p\}$, to each vertex u of G such that for $1 \leq i \leq p$, u_i is adjacent to each vertex that is adjacent to u in G.

In this section, we consider an r-regular graph G and obtain the maximum degree energy of a p-splitting graph $S_p^{'}(G)$ in terms of the maximum degree energy of the graph G. Also we obtain the maximum degree energy of p-splitting graphs of some classes of r-regular graphs.

Theorem 2.3. *If G is an r-regular graph, then*

$$E_M({S_p}'(G)) = (p+1)(\sqrt{1+4p})E_M(G).$$

Proof. Let $\{u_1, u_2, \cdots, u_n\}$ be the vertices of an r-regular graph G. Then the maximum degree matrix $M(G)$ is of order n where (i,j)-th entry is given by

$$M(G)(i,j) = \begin{cases} max(d_i, d_j) & \text{if } u_i \text{ and } u_j \text{ are adjacent,} \\ 0 & \text{otherwise.} \end{cases}$$

Let $\{u_i^1, u_i^2, \cdots, u_i^p\}$ be the vertices corresponding to each v_i which are added to G to obtain $S_p^{'}(G)$ such that $N(u_i^1) = N(u_i^2) = \cdots = N(v_i^p) = N(v_i)$ for $i = 1, 2, \cdots, n$. The maximum degree matrix of $S_p^{'}(G)$ is a block matrix of the form

$$M(S_p^{'}(G)) = \begin{bmatrix} (p+1)M(G) & (p+1)M(G) & \cdots & (p+1)M(G) \\ (p+1)M(G) & 0 & \cdots & 0 \\ (p+1)M(G) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ (p+1)M(G) & 0 & \cdots & 0 \end{bmatrix}$$

$$= \begin{bmatrix} p+1 & p+1 & \cdots & p+1 \\ p+1 & 0 & \cdots & 0 \\ p+1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ p+1 & 0 & \cdots & 0 \end{bmatrix} \otimes M(G) = A \otimes M(G)$$

where O is a null matrix and A= $\begin{bmatrix} p+1 & p+1 & \cdots & p+1 \\ p+1 & 0 & \cdots & 0 \\ p+1 & 0 & \cdots & 0 \\ . & . & \cdots & . \\ p+1 & 0 & \cdots & 0 \end{bmatrix}$.

The spectrum of A is $\begin{pmatrix} 0 & \dfrac{(p+1)(1+\sqrt{1+4p})}{2} & \dfrac{(p+1)(1-\sqrt{1+4p})}{2} \\ p-1 & 1 & 1 \end{pmatrix}$.

Hence the spectrum of $M(S_p^{'}(G))$ is

$$\begin{pmatrix} 0\mu_1 & \cdots & 0\mu_n & X\mu_1 & \cdots & X\mu_n & Y\mu_1 & \cdots & Y\mu_n \\ p-1 & \cdots & p-1 & 1 & \cdots & 1 & 1 & \cdots & 1 \end{pmatrix}$$

where $X = \dfrac{(p+1)(1+\sqrt{1+4p})}{2}$ and $Y = \dfrac{(p+1)(1-\sqrt{1+4p})}{2}$. Hence

$$\begin{aligned} E_M(S_p^{'}(G)) &= \sum_{i=1}^{n} \left| (p+1)\left(\frac{1 \pm \sqrt{1+4p}}{2}\right)\mu_i \right| \\ &= (p+1)\sum_{i=1}^{n} |\mu_i| \left(\frac{1+\sqrt{1+4p}}{2} + \frac{\sqrt{1+4p}-1}{2}\right) \\ &= (p+1)(\sqrt{1+4p})E_M(G). \end{aligned}$$

□

Corollary 2.1. *If G is a cycle graph of order $n \geq 3$, then*

$$E_M(S_p^{'}(C_n)) = 4(p+1)\sqrt{1+4p}\sum_{k=0}^{n-1}\left|\cos\frac{2k\pi}{n}\right|.$$

Proof. If G is a cycle graph C_n $(n \geq 3)$, then $E_M(C_n) = 4\sum_{k=0}^{n-1}\left|\cos\dfrac{2k\pi}{n}\right|$, [8]. Since C_n is a 2-regular graph, we have the required result using Theorem 2.3. □

Corollary 2.2. *If G is complete graph of order n, then*

$$E_M(S_p^{'}(K_n)) = 2(p+1)\sqrt{1+4p}(n-1)^2.$$

Proof. If G is the complete graph K_n, then $E_M(K_n) = 2(n-1)^2$, [2]. Since K_n is an $n-1$ regular graph, using Theorem 2.3, we have the required result. □

Corollary 2.3. *If $K_{n,n}$ is a complete bipartite graph, then*

$$E_M(S_p^{'}(K_{n,n})) = 2n^2(p+1)\sqrt{1+4p}.$$

Proof. If G is a complete bipartitie graph, then $E_M(K_{m,n}) = 2\sqrt{mn^3}$ for $m \geq n$, [8]. Therefore $E_M(K_{n,n}) = 2n^2$. Hence by Theorem 2.3, we have the required result. □

Corollary 2.4. *If G is a crown graph on $2n$ vertices, then*

$$E_M(S_p^{'}(G)) = 4(p+1)\sqrt{1+4p}(n-1)^2.$$

Proof. If G is a crown graph on $2n$ vertices then $E_M(G) = 4(n-1)^2$, [8]. Hence by Theorem 2.3, we have the required result. □

2.3. **Minimum Degree Energy of p-Splitting Graphs.** If G is an r-regular graph, then $m(G) = M(G)$. Hence we have the following result:

Theorem 2.4. *If G is an r-regular graph, then*

$$E_m(G) = E_M(G).$$

Theorem 2.5. *If G is an r-regular graph, then*

$$E_m({S_p}'(G)) = \sqrt{(p+1)^2+4p}E_m(G).$$

Proof. Let $\{u_1,\ u_2,\ \cdots,\ u_n\}$ be the vertices of an r-regular graph G and $m(G)$ be the minimum degree matrix. Let $\{u_i^1,\ u_i^2,\ \cdots,\ u_i^p\}$ be the vertices corresponding to each v_i which are added in G to obtain $S_p^{'}(G)$ such that $N(u_i^1) = N(u_i^2) = \cdots = N(v_i^p) = N(v_i)$, $i = 1,\ 2,\ \cdots,\ n$. Then the minimum degree matrix of $S_p^{'}(G)$ is a block matrix of the form

$$\begin{aligned} m(S_p^{'}(G)) &= \begin{bmatrix} (p+1)m(G) & m(G) & \cdots & m(G) \\ m(G) & O & \cdots & O \\ m(G) & O & \cdots & O \\ \vdots & \vdots & \ddots & \vdots \\ m(G) & O & \cdots & O \end{bmatrix} \\ &= \begin{bmatrix} p+1 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix}_{p+1} \otimes m(G) \\ &= A \otimes m(G) \end{aligned}$$

where O is a null matrix and A= $\begin{bmatrix} p+1 & 1 & \cdots & 1 \\ 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdots & \cdot \\ 1 & 0 & \cdots & 0 \end{bmatrix}$. Hence the spectrum of A is

$$\begin{pmatrix} 0 & \frac{p+1+\sqrt{(p+1)^2+4p}}{2} & \frac{p+1-\sqrt{(p+1)^2+4p}}{2} \\ p-1 & 1 & 1 \end{pmatrix}.$$

Now the spectrum of $m(S_p^{'}(G))$ is

$$\begin{pmatrix} 0\mu_1 & \cdots & 0\mu_n & P\mu_1 & \cdots & P\mu_n & Q\mu_1 & \cdots & Q\mu_n \\ p-1 & \cdots & p-1 & 1 & \cdots & 1 & 1 & \cdots & 1 \end{pmatrix}$$

where $P = \frac{p+1+\sqrt{(p+1)^2+4p}}{2}$ and $Q = \frac{p+1-\sqrt{(p+1)^2+4p}}{2}$. Hence

$$\begin{aligned} E_m(S_p^{'}(G)) &= \sum_{i=1}^{n} \left| \frac{p+1 \pm \sqrt{(p+1)^2+4p}}{2} \mu_i \right| \\ &= \sum_{i=1}^{n} |\mu_i| \left(\frac{p+1+\sqrt{(p+1)^2+4p}}{2} + \frac{\sqrt{(p+1)^2+4p}-(p+1)}{2} \right) \\ &= \sqrt{(p+1)^2+4p}\,E_m(G). \end{aligned}$$

□

Corollary 2.5. *If C_n is the cycle graph of order n ($n \geq 3$), then*

$$E_m(S_p^{'}(C_n)) = 4\sqrt{(p+1)^2+4p} \sum_{k=0}^{n-1} \left| \cos \frac{2k\pi}{n} \right|.$$

Corollary 2.6. *If K_n is the complete graph of order n, then*

$$E_m(S_p^{'}(K_n)) = 2(n-1)^2 \sqrt{(p+1)^2+4p}.$$

Corollary 2.7. *If $K_{n,n}$ is complete bipartite graph, then*

$$E_p(S^{'}(K_{n,n})) = 2n^2 \sqrt{(p+1)^2+4p}.$$

Corollary 2.8. *If G is crown graph with $2n$ vertices then*

$$E_p(S^{'}(G)) = 4(n-1)^2 \sqrt{(p+1)^2+4p}.$$

3. Maximum and minimum degree energies of p-shadow graphs

The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say $G^{'}$ and $G^{''}$ and joining each vertex $u^{'}$ in $G^{'}$ to the neighbors of the corresponding $u^{''}$ in $G^{''}$. For example The p-shadow graph $D_p(G)$ of a connected graph G is similarly constructed by taking p copies of G, say $G_1, G_2, \cdots, G_p$ and then joining each vertex u of G_i to the neighbors of the corresponding vertex v in G_j, for $1 \leq i, j \leq p$. For example

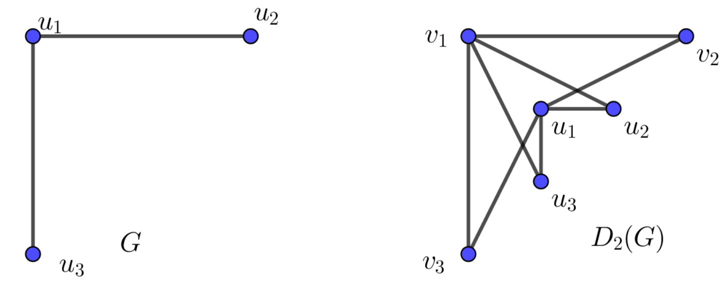
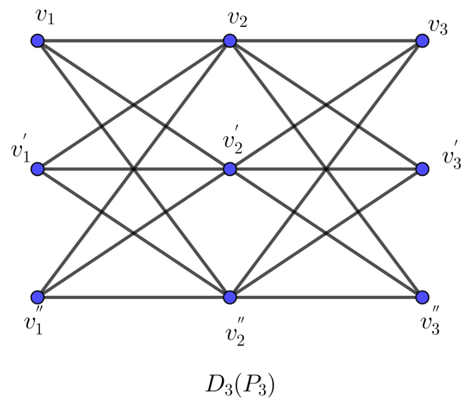


FIGURE 2. The shadow graph of P_3



3.1. **Maximum degree energy of p-shadow graphs.**

Theorem 3.1. *For a graph G, we have*

$$E_M(D_p(G)) = p^2 E_M(G).$$

Proof. Let $\{u_1,\ u_2,\ \cdots,\ u_n\}$ be the set of vertices of a graph G. Then the maximum degree matrix of G is the same with the one in the proof of Theorem 2.3. Consider p-copies of graph G, say $G_1,\ G_2,\ \cdots,\ G_p$ with vertices $u_1^1,\ u_2^2,\ \cdots,\ u_n^p$. To obtain $D_p(G)$, each vertex v in G_j is joined to the neighbors of the corresponding vertex v in G_j, for 1 $\leq i, j \leq p$. Then $M(D_p(G))$ is a block matrix of order np and it is of the form

$$\begin{aligned}
M(D_p(G)) &= \begin{bmatrix} pM(G) & pM(G) & \cdots & pM(G) \\ pM(G) & pM(G) & \cdots & pM(G) \\ pM(G) & pM(G) & \cdots & pM(G) \\ \vdots & \vdots & \ddots & \vdots \\ pM(G) & pM(G) & \cdots & pM(G) \end{bmatrix} \\
&= \begin{bmatrix} p & p & \cdots & p \\ p & p & \cdots & p \\ p & p & \cdots & p \\ \vdots & \vdots & \ddots & \vdots \\ p & p & \cdots & p \end{bmatrix} \otimes M(G) \\
&= pJ_p \otimes M(G).
\end{aligned}$$

Since the spectrum of pJ_p is $\begin{pmatrix} 0 & p^2 \\ p-1 & 1 \end{pmatrix}$, the spectrum of $M(D_p(G))$ is

$$\begin{pmatrix} 0\mu_1 & \cdots & 0\mu_n & p^2\mu_1 & \cdots & p^2\mu_n \\ 0 & \cdots & 0 & 1 & \cdots & 1 \end{pmatrix}.$$

Hence we deduce

$$\begin{aligned} E_M(D_p(G)) &= \sum_{i=1}^{n} |p^2\mu_i| \\ &= p^2 E_M(G). \end{aligned}$$

□

In the following corollaries, we obtain the maximum degree energies of shadow graphs of some classes of graphs:

Corollary 3.1. *If G is a cycle graph of order $n \geq 3$, then*

$$E_M(D_p(C_n)) = 4p^2 \sum_{k=0}^{n-1} \left| \cos\left(\frac{2k\pi}{n}\right) \right|.$$

Corollary 3.2. *If G is a complete graph of order n, then*

$$E_M(D_p(K_n)) = 2p^2(n-1)^2.$$

Corollary 3.3. *If $K_{m,n}$ is the complete bipartite graph with $m \geq n$, then*

$$E_M(D_p(K_{m,n})) = 2p^2\sqrt{mn^3}.$$

Proof. Since $E_M(K_{m,n}) = 2\sqrt{mn^3}$ for $m \geq n$ [8], $E_M(D_p(K_{m,n})) = 2p^2\sqrt{mn^3}$. □

Corollary 3.4. *If G is a path graph on n vertices, then*

$$E_M(D_p(P_n)) = 4p^2 \sum_{k=1}^{n} \left| \cos\frac{k\pi}{n+1} \right|.$$

Proof. If G is a path graph on n vertices, then $E_M(P_n) = 4\sum_{k=1}^{n} \left| \cos\frac{k\pi}{n+1} \right|$, [8]. Therefore $E_M(D_p(P_n)) = 4p^2 \sum_{k=1}^{n} \left| \cos\frac{k\pi}{n+1} \right|$. □

Corollary 3.5. *If G is a crown graph with $2n$ vertices, then*

$$E_M(D_p(G)) = 4p^2(n-1)^2.$$

3.2. **Minimum degree energy of p-shadow graphs.** In this section, we obtain the minimum degee energy of a p-shadow graph in terms of minimum degree energy. Also obtained the minimum degree energy of p-shadow graphs of some classes of regular graphs.

Theorem 3.2. *For a graph G, $E_m(D_p(G)) = p^2 E_m(G)$.*

Proof. Let $m(G)$ be the minimum degree matrix of G. Then, mimimum degree matrix of p-shadow graph of G is block matrix of order pn and it is of the form

$$m(D_p(G)) = \begin{bmatrix} p\cdot m(G) & p\cdot m(G) & \cdots & \cdot m(G) \\ p\cdot m(G) & p\cdot m(G) & \cdots & p\cdot m(G) \\ \vdots & \vdots & \ddots & \vdots \\ p\cdot m(G) & p\cdot m(G) & \cdots & p\cdot m(G) \end{bmatrix} = J_p \otimes m(G).$$

Since the spectrum of pJ_p is

$$\begin{pmatrix} 0 & p^2 \\ p-1 & 1 \end{pmatrix},$$

the spectrum of $m(D_p(G))$ is

$$\begin{pmatrix} 0\rho_1 & \cdots & 0\rho_n & m^2\rho_1 & \cdots & m^2\rho_n \\ 0 & \cdots & 0 & 1 & \cdots & 1 \end{pmatrix}.$$

Hence

$$E_m(D_p(G)) = \sum_{i=1}^{n} |p^2\rho_i| = p^2 E_m(G).$$

□

We now obtain the minimum degree energy of shadow graph of some classes of graphs in the following corollaries:

Corollary 3.6. *If G is a cycle graph of order $n \geq 3$, then*

$$E_m(D_p(C_n)) = 4p^2 \sum_{k=0}^{n-1} \left| \cos \frac{2k\pi}{n} \right|.$$

Corollary 3.7. *If G is a complete graph of order n, then*

$$E_m(D_p(K_n)) = 2p^2(n-1)^2.$$

Corollary 3.8. *If G is crown graph with $2n$ vertices, then*

$$E_m(D_p(G)) = 4p^2(n-1)^2.$$

Corollary 3.9. *If $K_{r,s}$ is the complete bipartite graph with $r \geq s$, then*

$$E_m(D_p(K_{r,s})) = 2p^2\sqrt{rs^3}.$$

References

[1] C. Adiga, C. S. Shivakumar Swamy, Bounds on the Largest of Minimum Degree Eigenvalues of Graphs, International Mathematical Forum, 5, (37), (2010), 1823-1831.

[2] C. Adiga, M. Smitha, On Maximum Degree Energy of a Graph, Int. J. Contemp. Math. Sciences, 4, (8) (2009), 385-396.

[3] Z.-Q. Chu, S. Nazeer, T. J. Zia, I. Ahmed, S. Shahid, Some New Results on Various Graph Energies of the Splitting Graph, Journal of Chemistry, 2019, Article ID 7214047, 12 pages.

[4] G. K. Gok, Some bounds on the Seidel energy of graphs, TWMS J. App. Eng. Math., 9, (4) (2019), 949-956.

[5] I. Gutman, The energy of a graph, Ber. Math.-Statist. Sekt. Forsch. Graz, (100-105): 103, (22), (1978), 10. Steiermaerkisches Mathematisches Symposium (Stift Rein, Graz), (1978).

[6] R. A. Horn, C. R. Johnson, Topics in Matrix Analysis, Cambridge Univ. Press, Cambridge, (1991).

[7] M. H. Nezhaad, M. Ghorbani, Seidel borderenergetic graphs, TWMS J. App. Eng. Math., 10, (2), (2020), 389-399.

[8] K. S. Rao, K. N. Prakasha, K. Saravanan, I. N. Cangul, Maximum Degree Energy, Advanced Studies in Contemporary Mathematics, 31, (1), (2021), 49-66.

[9] E. Sampathkumar, S. V. Roopa, K. A. Vidya, M. A. Sriraj, Partition energy of some trees and their generalized complements, TWMS J. App. and Eng. Math., 10, (2), (2020), 521-531.

K. Srinivasa Rao completed his Ph.D in mathematics at Aacharya Nagarjuna University, M.Phil. at Madurai Kamaraj University, M.Sc. at JNT university, and PGDCS at the University of Hyderbad. His primary research focuses on Chemical Graph Theory and Algebra. Currently, he is working as a professor of mathematics at Sri Chandrasekharendra Saraswathi Viswa Mahavidyalaya, Kanchipuram, India.

K. Saravanan is a research scholar at Sri Chandrasekharendra Saraswathi Viswa Mahavidyalaya (Deemed to be University), Kanchipuram, India. He obtained his bachelor degree and master degree in mathematics at the University of Madras, India. Presently, he is working in the Department of Mathematics, Sri Chandrasekharendra Saraswathi Viswa Mahavidyalaya (Deemed to be University), Kanchipuram, India.

K. N. Prakasha for the photography and short autobiography, see TWMS J. App. and Eng. Math. V.9, N.4, 2019.

Ismail Naci Cangul for the photography and short autobiography, see TWMS J. App. and Eng. Math., V.6, N.2, 2016.