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DOMINATION NUMBER OF A BIPARTITE SEMIGRAPH WHEN IT IS A CYCLE

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Abstract. Semigraph is a generalization of graph, with two or more vertices on edges which allows multiplicity in every concept of graph when it comes to semigraph. When number of vertices on the edges are restricted to two the semigraph is a graph, so every graph is a semigraph. In this article we deal with the variety of bipartite semigraphs, namely bipartite, s-bipartite and e-bipartite and bounds for their domination number (adjacent domination number and end vertex adjacent domination number) in particular when the semigraph is a cycle and also about possible size of the bipartite sets when the bipartite semigraph is a cycle.

Keywords: bipartite semigraph; Domination number; independent set; cycle

AMS Subject Classification: 05C99.

1. Introduction

In graph theory a connected graph has no cycle is called a tree and trees are the most fundamental graphs both because of their simplicity and the applications they have got in different fields. A graph which has a spanning cycle is called Hamiltonian and checking whether a given graph is Hamiltonian or not is a celebrated NP-complete problem. A graph which does not contain any odd cycle is a bipartite graph and the class of bipartite graphs is a most vibrant class of graphs. A graph on n vertices which contains cycle of all length k, $3 \leq k \leq n$, is called a pancyclicgraph, length of the smallest cycle in a graph, whenever such a cycle exists, is called girth of the graph and is an important graph parameter. Thus, cycle plays an important role in the theory of graphs. Probably, they are next to trees in the order of importance. In case of semigraphs there are many types of cycles possible because of the different types of vertices in it.

One of the most important concept both because of the beauty and applicability, is that of domination in the theory of graphs. Domination number is a widely studied graph parameter.

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In this article, some results on domination number of cycle semigraphs are studied. Some results on the size of the partite set in a bipartite cycle semigraph are also established.

Definitions regarding semigraph and concept of semigraph we have referred to [1] and in particular for types of domination set and its bound we have referred [2] and graph theory from [3]. More about semigraph we refer interested readers to [4, 5, 6, 7, 8].

Definition 1.1. *A semigraph G is a pair (V, E) where V is a nonempty set whose elements are called vertices of G, and E is a set of k-tuples of distinct vertices, called edges of G, for various $k \geq 2$, satisfying the following conditions.*

(1) *Any two edges of G can have at most one vertex in common.*
(2) *Two edges $(a_1, a_2, ..., a_p)$ and $(b_1, b_2, ..., b_q)$ are said to be equal if and only if*
   - *$p = q$ and*
   - *either $a_i = b_i$ for $1 \leq i \leq p$ or $a_i = b_{p-i+1}$ for $1 \leq i \leq p$.*

Note 1.1. *The edges are usually denoted by e with a suffix. By (2) above, edges $e_1 = (u_1, u_2, \ldots, u_k)$ and $e_2 = (u_k, u_{k-1}, \ldots, u_1)$ are same. Let E_i denote the set of vertices on the edge e_i and the cardinality of E_i is called the size of the edge e_i and it is denoted by $|E_i|$.*

Let $G(V, E)$ be a semigraph and let $e = (u_1, u_2, \ldots, u_k)$ be an edge of G. Then u_1 and u_k are called the end vertices and u_i, $2 \leq i \leq k-1$, are called the mid vertices of e. Two vertices v_1 and v_2 of G are adjacent ($v_1 \sim v_k$) if there is an edge containing both of them. An edge is said to be incident on every vertex lying on it. Two edges of G are adjacent if they have a vertex in common. Two vertices on an edge are consecutively adjacent if they are consecutive on the edge containing them.

Like a graph, a semigraph G also has a geometric representation on plane. Vertices of G are represented either by dots or by small circles according as whether they are end vertices or mid vertices of the edge containing them and edges of G by curves passing through all the vertices on them. When a mid vertex v of an edge e_1 is an end vertex of another edge, say e_2, then a small tangent is drawn to the circle representing vertex v where e_2 meets v. Figure 1 in Example 1.1 gives semigraph G and its representation on plane.

Example 1.1. *Consider a nonempty set $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ and collection of subsets of distinct elements of V given by $E(G) = \{(u_4, u_8, u_7, u_6), (u_4, u_5, u_3),$ $(u_1, u_5, u_2, u_6), (u_2, u_3)\}$. Then (V, E) is a semigraph which has a geometric representation as shown in Figure 1.*

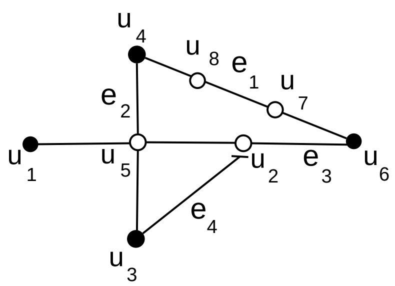

FIGURE 1. A semigraph and its representation on plane with vertex set $V(G) = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8\}$ and edge set $E(G) = \{e_1 = (u_4, u_8, u_7, u_6), e_2 = (u_4, u_5, u_3), e_3 = (u_1, u_5, u_2, u_6), e_4 = (u_2, u_3)\}$

Since an edge of a semigraph can have two or more vertices on it, the concept of degree of a vertices has following variations.

Definition 1.2. *Let $G(V, E)$ be a semigraph and v be a vertex of G.*

(1) *Degree of v, denoted by $\deg v$, is the number of edges having v as an end vertex.*
(2) *Edge degree of v, denoted by $\deg_e v$, is the number of edges containing v.*
(3) *Adjacent degree of v, denoted by $\deg_a v$, is the number of vertices adjacent to v.*
(4) *Consecutive adjacent degree of v, denoted by $\deg_{ca} v$, is the number of vertices which are consecutively adjacent to v.*

Definition 1.3. *The following graphs, associated with a semigraph G, are defined on the same vertex set $V(G)$,*

(1) *End vertex graph G_e : Two vertices are adjacent in G_e if they are end vertices of an edge in the semigraph G. The number of edges in G_e is same as that of the number of edges in G.*
(2) *Adjacency graph G_a : Two vertices are adjacent in G_a if they are adjacent in the semigraph G.*
(3) *Consecutive adjacency graph G_{ca} : Two vertices are adjacent in G_{ca} if they are consecutively adjacent in semigraph G.*

Definition 1.4. *A semigraph G is said to be a zig-zag semigraph if the vertex set $V(G) = \{u_1, u_2, \ldots, u_k, u_{k+1}, u_{k+2}, \ldots, u_{2k-1}\}$ and the edge set $E(G) = \{(u_1, u_2, \ldots, u_k), (u_1, u_{k+1}), (u_2, u_{k+1}), (u_2, u_{k+2}), \ldots, (u_{k-1}, u_{2k-2}), (u_{k-1}, u_{2k-1}), (u_k, u_{2k-1})\}$.*

Definition 1.5. *A $v_0 - v_n$ walk in a semigraph $G = (V, E)$ is sequence of vertices $P : v_0 v_1 v_2 \ldots v_n$ such that any two consecutive vertices in P are adjacent. A $v_0 - v_n$ walk P is a path in which all the vertices are distinct.*

As noted earlier in the introduction, bipartite graphs are one class of important graphs. In bipartite graphs, the vertex set of a graph G is partitioned into two parts, say V_1 and V_2, in such a way that every edge in G has one end vertex in V_1 and other end vertex in V_2. The partite sets are independent. So, before the definition of bipartite semigraphs, independent sets in case of semigraphs have been defined.

Definition 1.6. *Let $G(V, E)$ be a semigraph A non-empty subset S of $V(G)$ is said to be independent if it does not contain all the vertices of any edge of G. If S does contain all the vertices of any edge of G. If S does not contain both the end vertices of any edge then it is called e-independent and S is strongly independent if it contains no two adjacent vertices of G.*

Depending upon the nature of independence of the partite sets, a bipartite semigraph $G(V, E)$ with $V = V_1 \cup V_2$ can be of following types.

(1) *Bipartite Semigraph: G is bipartite if both V_1 and V_2 are independent.*
(2) *e-Bipartite Semigraph: G is e-bipartite if both V_1 and V_2 are e-independent.*
(3) *Strongly Bipartite Semigraph: G is s-bipartite if both V_1 and V_2 are strongly independent.*

Note 1.2. *The only semigraphs which are strongly bipartite are bipartite graphs.*

Note 1.3. *Every e-bipartite semigraph is bipartite but not conversely.*

Proposition 1.1. [1] *A semigraph G is e-bipartite if and only if its end vertex graph G_e is bipartite.*

Proposition 1.2. [1] *Let G be a semigraph which is a cycle having at least one edge of cardinality three. Then G is bipartite.*

The following definition and results on domination in semigraph are taken from the article[2].

Definition 1.7. *Let $G(V, E)$ be a semigraph and V_e be the set of all end vertices of G. A set $D \subseteq V$ is called adjacent dominating set (ad-set) if for every $v \in V - D$ there exists a $u \in D$ such that v is adjacent to u in G. The adjacency domination number $\gamma_a = \gamma_a(G)$ is the minimum cardinality of an ad-set of G. A set $D \subseteq V_e$ is called end vertex adjacent dominating set (ead-set) if (i) D is an ad-set and (ii) Every end vertex $v \in V - D$ is e-adjacent (two vertices are e-adjacent if they are the end vertices of an edge in G) to some end vertex $v \in D$ in G. The end vertex adjacency domination number $\gamma_{ea} = \gamma_{ea}(G)$ is the minimum cardinality of an ead-set of G.*

Proposition 1.3. *Let P_n denote path semigraph and C_n, cycle semigraph with n vertices containing k end vertices and r mid vertices such that $k + r = n$ and let $G_n = P_n$ or C_n. Then,*

$$\gamma_a(G_n) = \gamma_{ea}(G_n) \leq \left\lceil \frac{n}{3} \right\rceil.$$

Note 1.4. *A cycle in a semigraph is a closed path. A cycle semigraph is a semigraph which is a cycle.*

Proposition 1.4. *For any semigraph G with n vertices containing k end vertices and r mid vertices such that $k + r = n$ without isolated vertices, then*

$$\left\lceil \frac{n}{\delta_a + 1} \right\rceil \leq \gamma_a(G), \text{ where } \delta_a \text{ is the maximum adjacency degree of vertices in } G.$$

For domination numbers associated with semigraphs, one may also refer to [8, 9, 10, 11].

2. Main Results

Lemma 2.1. *A cycle semigraph G is s-bipartite if and only if G has even number of vertices.*

Proof. Follows directly from the note 1.2 □

Definition 2.1. *Let $G(V, E)$ be a semigraph and $\phi \neq S \subseteq V$. Then S is called a (consecutively adjacent) ca-independent set if it does not contain two vertices which are consecutively adjacent on some edge of G. The semigraph G is ca-bipartite if the vertex set of G can be partitioned in to $\{V_1, V_2\}$ such that both V_1 and V_2 are ca-independent.*

Lemma 2.2. *A cycle semigraph G is ca-bipartite if and only if G has even number of vertices.*

Proof. Let G be a cycle semigraph which is ca-bipartite. i.e $V(G)$ has a partition $\{V_1, V_2\}$ such that V_i is ca-independent for i=1,2. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $E(G) = \{e_1, e_2, \ldots, e_m\}$. Let $e_1, e_2, \ldots, e_m$ be the edges of the cycle in that order $v_1, v_2, \ldots, v_n$ be the vertices of these edges in that order. Then, suppose $v_1 \in V_1$. Then every even suffixed vertex is in V_2 and every odd suffixed vertices are in V_1. Since v_n is adjacent to v_1, $v_n \in V_2$. Hence n is even set of vertices of a cycle semigraph G.

Coversely, let $\{v_1, v_2, \ldots, v_n\}$, where n is even be the set of vertices of the cycle semigraph G which appears in that order with v_n adjacent to v_1. Define a partition $\{V_1, V_2\}$ of V as follows.V_1 contains all odd suffix vertices and V_2 contains the remaining vertices. Since n is even $v_n \in V_2$ and V_1 and V_2 are ca-independent. Hence G is ca-bipartite. □

The following lemma characterizes e-bipartite cycles.

Lemma 2.3. *A cycle semigraph G is e-bipartite if and only if G has even number of edges.*

Proof. Note that, for a cycle semigraph G the end vertex graph G_e is a cycle graph, number of edges in G_e is same as the number of edges in G. By proposition 1.1, a semigraph G is e-bipartite if and only if its end vertex graph G_e is a bipartite graph. Thus it follow that, a cycle semigraph G is e-bipartite if and only if the corresponding cycle graph G_e is bipartite and the result of this follows from the fact that a bipartite cycle graph has even number of edges. □

Theorem 2.1. *Let $G(V, E)$ be a cycle semigraph. Then there exists a partition $\{V_1, V_2\}$ of $V(G)$ such that the partite sets V_1 and V_2 are both e-independent and ca-independent if and only if G has even number of vertices and every edge is of even size.*

Proof. If a partition $\{V_1, V_2\}$ of $V(G)$ is ca-independent then from Lemma 2.2 G has even number of vertices. If a partition $\{V_1, V_2\}$ of $V(G)$ is e-independent then from Lemma 2.3 G has even number of edges.

The partition $\{V_1, V_2\}$ is ca-independent and e-independent, hence in any edge of G, consecutive vertices should not belong to the same partite set, and both end vertices of the edge should not belong to the same partite set, hence each edge must be of even size. Therefore, G has even number of vertices, even number of edges and all edges are of even size.

Hence a partition $\{V_1, V_2\}$ of $V(G)$ is e-independent and ca-independent if and only if the semigraph G has even number of edges with every edge of even size.

Conversely, let G be a cycle semigraph with $V(G) = \{v_1, v_2, \ldots, v_n\}$ and $E(G) = \{e_1, e_2, \ldots, e_m\}$ where m is even and $|E_i|$ is even for every i, $1 \leq i \leq m$. Let the e_j be given by $e_j = (v_{j_1}, v_{j_2}, \ldots, v_{j_{k_j}})$ where k_j is even and v_{j_1} are end vertices of e_j, $1 \leq j \leq m$. Let $e_1, e_2, \ldots, e_m$ be the edges of the cycle semigraph in that order. Now, consider the bipartition $\{V_1, V_2\}$ defined as follows.

Start with the end vertex v_{1_l} of e_1 in the set V_1. Put v_{1_l} in V_1 when l is odd and v_{1_l} in V_2 when l is even. The other end vertex $v_{1_{k_1}}$ of e_1 will be in V_2 since k_1 is even. Since G is a cycle semigraph, $v_{2_1} = v_{1_{k_1}}$. Put v_{2_r} in V_2 when r is odd and v_{2_r} in V_1 when r is even. Continuing the above above procedure, we observe that every odd suffixed edge, say e_s has its starting vertex i.e v_{s_1} in V_1 and its end vertex $v_{s_{k_s}}$ in V_2 and every even suffixed edge, say e_t has its starting vertex v_{t_1} in V_2 and end vertex $v_{t_{s_t}}$ in V_1. Again, since it is a cycle semigraph, $v_{m_{k_m}} = v_{1_{k_1}}$.

Since both m and k_j for every j, $1 \leq j \leq m$, are even the above defines a bipartition $\{V_1, V_2\}$ of $V(G)$ such that both V_1 and V_2 are e-independent and ca-independent. □

Note 2.1. *Every ca-bipartite semigraph is bipartite semigraph but not conversely.*

Note 2.2. *If a cycle semigraph G of order n, where n is an odd integer, is non bipartite then G has no edge containing a mid-vertex, G is a graph. Since G is a graph $\gamma(G) = \lceil \frac{n}{3} \rceil$,* [12].

Theorem 2.2. *Let G be a bipartite semigraph on n vertices out of which **k** are end vertices and **r** mid vertices with $k + r = n$. Then,*

$$\left\lceil \frac{k}{3} \right\rceil \leq \gamma_a(G) = \gamma_{ea}(G) \leq \left\lceil \frac{k}{2} \right\rceil.$$

Proof. If G be a bipartite semigraph on n vertices out of which **k** are end vertices and **r** mid vertices with $k + r = n$ then G has to be one of the following,

(1) G is a cycle graph with k even ($r = 0$)
(2) G is a cycle semigraph with n vertices with n even and $r > 0$
(3) G is a cycle semigraph with n vertices with k odd and $r \geq 1$.

- Case 1: If G is a cycle graph with k even and $r = 0$.
  Then $\gamma_a(G) = \gamma_{ea}(G) = \gamma(G_e)$ and $\gamma(G_e) = \lceil \frac{k}{3} \rceil$.
- Case 2:
  - If G is a cycle with n even, $r > 0$ and only one edge has mid vertices.
    Then from Case 1, it follows that, $\gamma_a(G) = \lceil \frac{k}{3} \rceil$ (choose D such that it contains an end vertex of the semiedge).
  - If G is a cycle with k even, $r > 0$ and all the edges have mid vertices.
    Then $\gamma_a(G) = \left(\frac{k}{2}\right)$ (as D has to contain every alternative end vertex of G).
- Case 3:
  - If G is a cycle with k odd, $r \geq n$ and only one edge has mid vertices.
    Then from Case 3, it follows that, $\gamma_a(G) = \lceil \frac{k}{3} \rceil$.
  - If G is a cycle with k odd, $r \geq k$ and all the edges have mid vertices.
    Then $\gamma_a(G) = \left(\frac{k-1}{2}\right) + 1 = \left(\frac{k+1}{2}\right)$ (as D has to contain every alternative end vertices of G).

□

Definition 2.2. *The length of a path P is one more than the number of vertices of P at which the path changes from one edge to other.*

The distance between two vertices in a semigraph G is defined as follows. Let u and v be vertices in the semigraph G then the distance between u and v, denoted by $d(u, v)$ is given by,

$$d(u,v) = \begin{cases} 0 & \text{if } u = v \\ \text{length of the shortest path between } u \text{ and } v & \text{if } u \neq v. \end{cases}$$

Example 2.1. *Let P be a path in a semigraph G in Fig. 1. given by $v = v_1(v_1, v_5)v_5$ $(v_5, v_4)v_4(v_4, v_8)v_8 = v$. The P changes edges of G at vertices v_5 and v_4 on it. Hence length of path P is 3.*

Definition 2.3. *Let $G(V, X)$ be a semigraph. A set $D \subseteq V$ is called adjacent dominating set (ad-set) if for every $v \in V - D$ there exists a $u \in D$ such that $d(u, v) = 1$.*

Theorem 2.3. *If G be a bipartite semigraph on n vertices out of which k are end vertices and r mid vertices with $k + r = n$ (no mid-end vertices) then there exists an adjacent dominating set which is independent.*

Proof. Let G be a bipartite cycle semigraph with partition $\{V_1, V_2\}$, to prove that both V_1 and V_2 are ad-sets. Let $v \in V_1$. Since V_1 is independent, every edge passing through v has a vertex $u \in V_2$ on it. Hence, $d(u, v) = 1$, (by definition 2.3) which proves that $V_2 = V - V_1$ is an ad-set. Similarly, V_1 is an ad-set. □

Corollary 2.1. *If G be a bipartite cycle semigraph on n vertices out of which k are end vertices and r mid vertices with $k + r = n$ (no mid-end vertices) with partition $\{V_1, V_2\}$ then,*

$$\min\{|V_1|, |V_2|\} \geq \left\lceil \frac{k}{3} \right\rceil \text{ and } \max\{|V_1|, |V_2|\} \leq \left\lceil \frac{k-1}{3} \right\rceil + \left\lceil \frac{k-2}{3} \right\rceil + r$$

Proof. Let G be a bipartite cycle semigraph with partition $\{V_1, V_2\}$. From Theorem 2.3 both V_1 and V_2 are adjacent dominating sets of G. And from Theorem 2.2, $\lceil \frac{k}{3} \rceil \leq \gamma_a(G)$.

Therefore,

$$\min\{|V_1|, |V_2|\} \geq \left\lceil \frac{k}{3} \right\rceil$$

The number of elements in G, $n = |V_1| + |V_2|$ without loss of generality, $|V_1| \leq n - \lceil \frac{k}{3} \rceil$. For positive integer c and d the following is an established combinatorial result.

$$c = \left\lceil \frac{c}{d} \right\rceil + \left\lceil \frac{c-1}{d} \right\rceil + \ldots + \left\lceil \frac{c-d+1}{d} \right\rceil.$$

Considering $c = n$ and $d = 3$. The above becomes,

$$k = \left\lceil \frac{k}{3} \right\rceil + \left\lceil \frac{k-1}{3} \right\rceil + \left\lceil \frac{k-2}{3} \right\rceil$$

Therefore,

$$k - \left\lceil \frac{k}{3} \right\rceil = \left\lceil \frac{k-1}{3} \right\rceil + \left\lceil \frac{k-2}{3} \right\rceil$$

i.e

$$(r+k) - \left\lceil \frac{k}{3} \right\rceil = r + \left\lceil \frac{k-1}{3} \right\rceil + \left\lceil \frac{k-2}{3} \right\rceil$$

$$n - \left\lceil \frac{k}{3} \right\rceil = r + \left\lceil \frac{k-1}{3} \right\rceil + \left\lceil \frac{k-2}{3} \right\rceil$$

Hence,

$$|V_2| \leq n - \left\lceil \frac{k}{3} \right\rceil = r + \left\lceil \frac{k-1}{3} \right\rceil + \left\lceil \frac{k-2}{3} \right\rceil$$

Therefore,

$$\max\{|V_1|, |V_2|\} \leq \left\lceil \frac{k-1}{3} \right\rceil + \left\lceil \frac{k-2}{3} \right\rceil + r$$

□

Arguing in the similar lines, making use of the result of the Proposition 1.4 that $\left\lceil \frac{n}{\delta_a+1} \right\rceil \leq \gamma_a(G)$, where δ_a is the maximum adjacency degree of vertices in G. We get the following corollary.

Corollary 2.2. *If G be a bipartite semigraph on n vertices out of which k are end vertices and r mid vertices with $k + r = n$ (no mid-end vertices) with partition $\{V_1, V_2\}$ then,*

$$\min\{|V_1|, |V_2|\} \geq \left\lceil \frac{n}{\delta_a + 1} \right\rceil \text{ and } \max\{|V_1|, |V_2|\} \leq \left\lceil \frac{n-1}{\delta_a + 1} \right\rceil + \ldots + \left\lceil \frac{n-\delta_a}{\delta_a + 1} \right\rceil + r,$$

where δ_a is the maximum adjacent degree of vertices in G.

Note 2.3. *As in the case of graphs, any semigraph with either one or two edge is bipartite. In the case of graphs no complete graph with number of vertices greater than or equal to three is bipartite. But in the case of semigraph, for every $n \geq 3$ E_n^c is bipartite. It is easy to note that, for every $k \geq 3$, zig-zag semigraph Z_k^{k-1},is not bipartite.*

3. Conclusion

When a semigraph is a cycle with at least one mid vertex is bipartite, but when it comes to bounds of the domination number of the bipartite semigraph more analysis is needed which gives clear picture about the structure of the semigraph. In this article we have taken the simplest form of a semigraph which is a path in particular a closed path that is a cycle. We have found the bounds of domination number of the semigraph when it is a cycle and also realized that the independent sets of bipartite semigraphs are also the adjacency dominating set of the semigraph.

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