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COEFFICIENT ESTIMATES FOR NEW SUBCLASSES OF MEROMORPHIC BI-UNIVALENT FUNCTIONS ASSOCIATED WITH LINEAR OPERATOR

ADNAN GHAZY ALAMOUSH[1], §

Abstract. In this paper, we define a new differential linear operator of meromorphic bi-univalent functions class Σ', and obtain the estimates for the coefficients $|b_0|$ and $|b_1|$. Further we pointed out several new or known consequences of our results.

Keywords: Analytic functions, Univalent functions, Bi-univalent functions, Meromorphic functions, Meromorphic bi-univalent functions, Linear operator, Coefficient estimates.

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1. Introduction

Let $\mathcal{A}$ denote the class of the functions f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$ and satisfy the normalization condition $f(0) = f^{'}(0) - 1 = 0$. Further, by $\mathcal{S}$ we shall denote the class of all functions f in $\mathcal{A}$ which are univalent in $\mathbb{U}$. Some of the important and well-investigated subclasses of the univalent function class $\mathcal{S}$ include (for example) the class $\mathcal{S}^*(\alpha)$ $(0 \leq \alpha < 1)$ of starlike functions of order α in $\mathbb{U}$ and the class $\mathcal{K}(\alpha)$ $(0 \leq \alpha < 1)$ of convex functions of order α

$$\Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \quad \text{and} \quad \Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha, \quad (z \in \mathbb{U})$$

respectively. The well-known Koebe one-quarter theorem asserts that the function $f \in \mathcal{S}$ has an inverse, defined on disc $\mathbb{U}_\rho = \{z \in \mathbb{C} : |z| < \rho\}$, $(\rho \geq \frac{1}{4})$. Thus, the inverse of $f \in \mathcal{S}$ is a univalent analytic function on the disc $\mathbb{U}$. It is well known that every function $f \in \mathcal{S}$ has an inverse f^{-1}, defined by $f^{-1}(f(z)) = z, (z \in \mathbb{U})$ and

$$f^{-1}f(w) = w, \ (|w| < r_0 f(z); \ r_0 f(z) \geq \tfrac{1}{4} \ )$$

[1] Faculty of Science, Taibah University, Saudi Arabia.
e-mail:adnan_omoush@yahoo.com; ORCID: https://orcid.org/0000-0003-3687-9195.
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where

$$g(w) = f^{-1}(w) = w - a_2w^2 + (2a_2^2 - a_3)w^3 - (5a_2^2 - 5a_2a_3 + a_4)w^4 + \dots \ . \qquad (2)$$

Also, we say that a function $f(z) \in \mathcal{A}$ is bi-univalent in $\mathbb{U}$ if both $f(z)$ and $f^{-1}(z)$ are univalent in $\mathbb{U}$, these classes are denoted by Σ. Earlier, Brannan and Taha [12] introduced certain subclasses of bi-univalent function class Σ; namely bi-starlike functions $S^*_\Sigma(\alpha)$ and bi-convex function $K^*_\Sigma(\alpha)$ of order (α) corresponding to the function classes $S^*(\alpha)$ and $K(\alpha)$ respectively.

Many authors investigated bounds for various subclasses bi-univalent function class Σ (see for example ([1],[2],[3],[4],[6],[7],[8],[10],[17],[21])and obtained non-sharp coefficient estimates on the first two coefficients $|a_2|$ and $|a_3|$ of (1). A function f is bi-starlike of Ma-Minda type or bi-convex of Ma-Minda type if both $f(z)$ and $f^{-1}(z)$ are respectively Ma-Minda starlike or convex. These classes are denoted respectively by $S^*_\Sigma(\phi)$ and $K_\Sigma(\phi)$ where $\phi(z)$ is given by

$$\phi(z) = 1 + B_1z + B_2z^2 + B_3z^3 + \dots \ , \ (B_1 > 0, \ z \in \mathbb{U}). \qquad (3)$$

Let Σ' denote the family of all meromorphic univalent functions of the form

$$h(z) = z + b_0 + \sum_{n=1}^{\infty} \frac{b_n}{z^n} \qquad (4)$$

defined on the domain $\mathbb{U}^* = \{z : z \in \mathbb{C} \text{ and } 1 < |z| < \infty\}$. Since $h \in \Sigma'$ is univalent, it has an inverse $h^{-1} = G(z)$ that satisfy $h^{-1}(h(z)) = z, (z \in \mathbb{U}^*)$ and

$$h^{-1}h(w) = w, \ (M < |w| < \infty, \quad M > 0 \ )$$

where

$$G(w) = h^{-1}(w) = w + \sum_{n=0}^{\infty} \frac{B_n}{w^n} \ (M < |w| < \infty, \quad M > 0) \qquad (5)$$

in some neighborhood of $w = \infty$. A simple calculation shows that the function G, is given by

$$G(w) = h^{-1}(w) = w - b_0 - \frac{b_1}{w} - \frac{b_2 + b_0b_1}{w^2} - \frac{b_3 + 2b_0b_2 + b_0^2b_1 + b_1^2}{w^3} + \dots \ . \qquad (6)$$

Analogous to the bi-univalent analytic functions, a function $h \in \Sigma'$ is said to be meromorphic bi-univalent in $\mathbb{U}^*$ if $h^{-1} \in \Sigma'$. We denote by Σ'_b the class of all meromorphic bi-univalent functions in $\mathbb{U}^*$ given by (4). Estimates on the coefficients of meromorphic univalent functions were investigated in the literature. For $h \in \Sigma'_0$, it follows from the area theorem that $|b_1| \le 1$. Schiffer [18] obtained the sharp estimates $|b_2| \le \frac{2}{3}$ for $h \in \Sigma'_0$. Also, Duren [13] gave an elementary proof of the inequality $|b_2| \le \frac{2}{n+1}$ for $h \in \Sigma'$ with $b_k = 0$ for $1 \le k < \frac{n}{2}$. For the coefficients of the inverse of meromorphic univalent functions, Springer [20] used variational methods to prove that

$$|B_3 + \tfrac{1}{2}B_1^2| \le \tfrac{1}{2} \ \text{ and } \ |B_3| \le 1$$

and conjectured that

$$|B_{2n-1}| \le \tfrac{(2n-2)!}{n!(n-1)!} \quad n = 1, 2, 3, \dots \ .$$

Furthermore, Kubota [16] has proved that the Springer conjecture is true for $n = 3, 4, 5$ by an elementary application of Grunsky's inequalities and subsequently, for $G \in \Sigma'_0$ Schober [19] obtained sharp bounds for the coefficients $B_{2n-1}, 1 \le n \le 7$. Recently, Kapoor and Mishra [15] found the coefficient estimates for a class consisting of inverses of meromorphic starlike univalent functions of order α in $\mathbb{U}^*$.

A function h in the class Σ' is said to be meromorphic bi-univalent starlike of order $\alpha(0 \leq \alpha < 1)$ if it satisfies the following inequalities

$$h \in \Sigma_b', \quad \Re\left\{\frac{zh'(z)}{h(z)}\right\} > \alpha \ (z \in \mathbb{U}^*) \quad \text{and} \quad \Re\left\{\frac{wG'(w)}{G(w)}\right\} > \alpha, \quad (w \in U^*),$$

where $G(w) = h^{-1}(w)$ is the inverse of $h(z)$ whose series expansion is given by (6).

We denote by $\Sigma_b'(\alpha)$ the class of all meromorphic bi-univalent starlike functions of order α. Similarly, a function h in the class $\widetilde{\Sigma}_b'(\alpha)$ is said to be meromorphic bi-univalent strongly starlike of order $\alpha(0 < \alpha \leq 1)$ if it satisfies the following conditions

$$h \in \Sigma_b', \quad \left|\arg \frac{zh'(z)}{h(z)}\right| < \frac{\alpha\pi}{2} \ (z \in \mathbb{U}^*) \quad \text{and} \quad \Re\left|\frac{wG'(w)}{G'(w)}\right| < \frac{\alpha\pi}{2}, \quad (w \in \mathbb{U}^*),$$

where $G(w)$ is given by (6). We denote by $\widetilde{\Sigma}_b'$ the class of all meromorphic bi-univalent strongly starlike functions of order α.

For functions $h \in \Sigma'$ in the form (4), we define the following new linear operator $D^0_{\lambda,\mu}h(z) = h(z)$, and when $\lambda = \mu$, also we have $D^k_{\lambda,\mu}h(z) = h(z)$, $(k = 0, 1, 2, ...)$

$$D^1_{\lambda,\mu}h(z) = D_{\lambda,\mu}h(z) = (1 - (\lambda - \mu))h(z) + (\lambda - \mu)zh'(z)$$

$$= z + \sum_{n=0}^{\infty}[1 - (\lambda - \mu)(n - 1)]\frac{b_n}{z^n}, \quad 0 \leq \alpha \leq \lambda < \frac{1}{n+1}$$

and

$$D^2_{\lambda,\mu}h(z) = D[D_{\lambda,\mu}h(z)] = z + \sum_{n=0}^{\infty}[1 - (\lambda - \mu)(n - 1)]^2\frac{b_n}{z^n},$$

hence, it can be easily seen that

$$D^k_{\lambda,\mu}h(z) = D[D^{k-1}_{\lambda,\mu}h(z)] = z + \sum_{n=0}^{\infty}[1 - (\lambda - \mu)(n - 1)]^k\frac{b_n}{z^n}, \tag{7}$$

where $k \in \mathbb{N}_0 = \{0, 1, 2, 3, \cdots\}$, $0 \leq \alpha \leq \lambda < \frac{1}{n+1}$.

Remark 1.1. *Note that if $\mu = 0$, we get the linear operator which is defined by Aziz and Juma [11].*

Motivated by the earlier work of (see ([11], [14])), we define the following new subclasses $\Sigma_b'(k, \lambda, \mu; \beta)$ and $\widetilde{\Sigma}_b'(k, \lambda, \mu; \beta)$ of the function class Σ'.

Definition 1.1. *A function f given by (1.4) is said to be in the class $\Sigma_b'(k, \lambda, \mu; \beta)$ if the following conditions are satisfied:*

$$h \in \Sigma_b', \quad \Re\left\{\frac{z(D^k_{\lambda,\mu}h(z))'}{D^k_{\lambda,\mu}h(z)}\left(\frac{D^k_{\lambda,\mu}h(z)}{z}\right)^{\beta}\right\} > \alpha \ \left(\beta \geq 0, \ 0 \leq \alpha \leq \lambda < \frac{1}{n+1}, \ z \in \mathbb{U}^*\right) \tag{8}$$

and

$$\Re\left\{\frac{w(D^k_{\lambda,\mu}G(w))'}{D^k_{\lambda,\mu}G(w)}\left(\frac{D^k_{\lambda,\mu}G(w)}{w}\right)^{\beta}\right\} > \alpha \ \left(\beta \geq 0, \ 0 \leq \alpha \leq \lambda < \frac{1}{n+1}, \ w \in \mathbb{U}^*\right) \tag{9}$$

for some $\alpha(0 \leq \alpha < 1)$, where G is given by (6).

Definition 1.2. *A function f given by (4) is said to be in the class $\widetilde{\Sigma}_b^{'}(k,\lambda,\mu;\beta)$ if the following conditions are satisfied:*

$$h\in\Sigma_b^{'},\quad \left|\arg\left\{\frac{z(D^k_{\lambda,\mu}h(z))'}{D^k_{\lambda,\mu}h(z)}\left(\frac{D^k_{\lambda,\mu}h(z)}{z}\right)^{\beta}\right\}\right|<\frac{\alpha\pi}{2}\ \left(\beta\geq 0,\ 0\leq\alpha\leq\lambda<\frac{1}{n+1},\ z\in\mathbb{U}^*\right) \tag{10}$$

and

$$\left|\arg\left\{\frac{w(D^k_{\lambda,\mu}h(w))'}{D^k_{\lambda,\mu}G(w)}\left(\frac{D^k_{\lambda,\mu}G(w)}{w}\right)^{\beta}\right\}\right|<\frac{\alpha\pi}{2}\ \left(\beta\geq 0,\ 0\leq\alpha\leq\lambda<\frac{1}{n+1},\ w\in\mathbb{U}^*\right) \tag{11}$$

for some $\alpha(0<\alpha\leq 1)$, where G is given by (6).

Remark 1.2. *We note that, for $k=0,\beta=0$, the classes $\Sigma_b^{'}(k,\lambda,\mu;\beta)$ and $\widetilde{\Sigma}_b^{'}(k,\lambda,\mu;\beta)$ reduce to the classes*

$$\Sigma_b^{'}(0,\lambda,\mu;0)=\Sigma_b^{'},$$
$$\widetilde{\Sigma}_b^{'}(0,\lambda,\mu;0)=\widetilde{\Sigma}_b^{'},$$

respectively, introduced and studied by Halim et al. [14].

In the present investigation, a new subclasses of meromorphic bi-univalent functions are introduced and estimates for the coefficients $|b_0|$ and $|b_1|$ of functions in these subclasses are obtained. Several new consequences of the results are also pointed out.

In order to derive our main results, we shall need the following lemma.

Lemma 1.1. *([10]) If $\phi\in P$, the class of all functions with $\Re(\phi(z))>0$ $(z\in\mathbb{U})$, then*

$$|c_n|\leq 2,\ \textit{for each } k,$$

where

$$\phi(z)=1+c_1z+c_2z^2+\ldots\quad \textit{for}\quad (z\in\mathbb{U}).$$

2. Coefficient Bounds for the Function Classes $\Sigma_b^{'}(k,\lambda,\mu;\beta)$ and $\widetilde{\Sigma}_b^{'}(k,\lambda,\mu;\beta)$

We begin this section by obtaining the coefficients $|b_0|$ and $|b_1|$ for functions in the class $\Sigma_b^{'}(k,\lambda,\mu;\beta)$.

Theorem 2.1. *Let the function $h(z)$ given by (4) be in the class $\Sigma_b^{'}(k,\lambda,\mu;\beta)$. Then*

$$|b_0|\leq\frac{2(1-\alpha)}{(1-\beta)[1-(\lambda-\mu)]^k}. \tag{12}$$

and

$$|b_1|\leq\frac{2(1-\alpha)}{[1-2(\lambda-\mu)]^k}\sqrt{\frac{(1-\alpha)^2}{(1-\beta)^2}+\frac{1}{(2-\beta)^2}}. \tag{13}$$

Proof. It follows from (8) and (9) that

$$\frac{z(D^k_{\lambda,\mu}h(z))'}{D^k_{\lambda,\mu}h(z)}\left(\frac{D^k_{\lambda,\mu}h(z)}{z}\right)^{\beta}=\alpha+(1-\alpha)p(z) \tag{14}$$

and

$$\frac{w(D^k_{\lambda,\mu}G(w))'}{D^k_{\lambda,\mu}G(w)}\left(\frac{D^k_{\lambda,\mu}G(w)}{w}\right)^{\beta}=\alpha+(1-\alpha)q(w), \tag{15}$$

where $p(z)$ and $q(w)$ are functions with positive real part in $\mathbb{U}^*$ and have the following forms:

$$p(z) = 1 + \frac{p_1}{z} + \frac{p_2}{z^2} + \dots \tag{16}$$

and

$$q(w) = 1 + \frac{q_1}{w} + \frac{q_2}{w^2} + \dots \ , \tag{17}$$

respectively. Now, equating coefficients in (14) and (15), we get

$$(\beta - 1)[1 - (\lambda - \mu)]^k b_0 = (1 - \alpha)p_1, \tag{18}$$

$$(\beta - 2)\left[(1 - 2(\lambda - \mu)^k)b_1 + \frac{(\beta - 1)[1 - (\lambda - \mu)^{2k}]}{2} b_0^2\right] = (1 - \alpha)p_2, \tag{19}$$

$$(1 - \beta)[1 - (\lambda - \mu)]^k b_0 = (1 - \alpha)q_1, \tag{20}$$

$$(2 - \beta)\left[(1 - 2(\lambda - \mu)^k)b_1 - \frac{(\beta - 1)[1 - (\lambda - \mu)^{2k}]}{2} b_0^2\right] = (1 - \alpha)q_2. \tag{21}$$

From (18) and (20), we get

$$p_1 = -q_1, \tag{22}$$

$$b_0^2 = \frac{(1 - \alpha)^2(p_1^2 + q_1^2)}{2(1 - \beta)^2[1 - (\lambda - \mu)]^{2k}}. \tag{23}$$

Since $\Re(p(z)) > 0$ in $\mathbb{U}^*$, the function $p(\frac{1}{z}) \in P$ and hence the coefficients p_n and similarly the coefficients q_n of the function q satisfy the inequality in Lemma 1.1, we get

$$|b_0| \leq \frac{2(1 - \alpha)}{(1 - \beta)[1 - (\lambda - \mu)]^k}.$$

This gives the bound on $|b_0|$ as asserted in (12).

Next, in order to find the bound on $|b_1|$, we use (19) and (20), which yields,

$$(1 - \beta)^2(\beta - 2)^2[1 - (\lambda - \mu)]^{4k} b_0^4 - 4(1 - \alpha)^2 p_2 q_2 = 4(2 - \beta)^2[1 - 2(\lambda - \mu)]^{2k} b_1^2. \tag{24}$$

It follows from (23) that

$$b_1^2 = \frac{(1 - \beta)^2[1 - (\lambda - \mu)]^{4k} b_0^4}{4[1 - 2(\lambda - \mu)]^{2k}} - \frac{(1 - \alpha)^2 p_2 q_2}{(2 - \beta)^2[1 - 2(\lambda - \mu)]^{2k}}. \tag{25}$$

Substituting the estimate obtained (24), and applying Lemma 1.1 once again for the coefficients p_2 and q_2, we readily get

$$|b_1| \leq \frac{2(1 - \alpha)}{[1 - 2(\lambda - \mu)]^k}\sqrt{\frac{(1 - \alpha)^2}{(1 - \beta)^2} + \frac{1}{(2 - \beta)^2}}.$$

This completes the proof of Theorem 2.1.

For $\lambda = \mu$ or $k = 0$, we have the following corollary of Theorem 2.1.

Corollary 2.1. *Let the function $h(z)$ given by (4) be in the class $\Sigma_b'(\lambda, \mu; \beta)$. Then*

$$|b_0| \leq \frac{2(1 - \alpha)}{(1 - \beta)}. \tag{26}$$

and

$$|b_1| \leq 2(1 - \alpha)\sqrt{\frac{(1 - \alpha)^2}{(1 - \beta)^2} + \frac{1}{(2 - \beta)^2}}. \tag{27}$$

For $\beta = 0$ in Corollary 2.1, we have the following result.

Corollary 2.2. *(see [11]) Let the function $h(z)$ given by (4) be in the class Σ_b'. Then*

$$|b_0| \leq 2(1-\alpha). \tag{28}$$

and

$$|b_1| \leq (1-\alpha)\sqrt{4\alpha^2 - 8\alpha + 5}. \tag{29}$$

Next, we estimate the coefficients $|b_0|$ and $|b_1|$ for functions in the class $\widetilde{\Sigma}_b'(k, \lambda, \mu; \beta)$

Theorem 2.2. *Let the function $h(z)$ given by (4) be in the class $\widetilde{\Sigma}_b'(k, \lambda, \mu; \beta)$. Then*

$$|b_0| \leq \frac{2\alpha}{(\beta-1)[1-(\lambda-\mu)]^k}. \tag{30}$$

and

$$|b_1| \leq \frac{2\alpha^2}{[1-2(\lambda-\mu)]^k}\sqrt{\frac{1}{(1-\beta)^2} + \frac{1}{(2-\beta)^2}}. \tag{31}$$

Proof. It follows from (10) and (11) that

$$\frac{z(D^k_{\lambda,\mu}h(z))'}{D^k_{\lambda,\mu}h(z)}\left(\frac{D^k_{\lambda,\mu}h(z)}{z}\right)^\beta = [p(z)]^\alpha \tag{32}$$

and

$$\frac{w(D^k_{\lambda,\mu}G(w))'}{D^k_{\lambda,\mu}G(w)}\left(\frac{D^k_{\lambda,\mu}G(w)}{w}\right)^\beta = [q(w)]^\alpha, \tag{33}$$

where $p(z)$ and $q(w)$ have the forms (14) and (15), respectively. Now, equating coefficients in (32) and (33), we get

$$(\beta-1)[1-(\lambda-\mu)]^k b_0 = \alpha\ p_1, \tag{34}$$

$$(\beta-2)\left[(1-2(\lambda-\mu)^k)b_1 + \frac{(\beta-1)[1-(\lambda-\mu)^{2k}]}{2}b_0^2\right] = \frac{1}{2}\left[\alpha(\alpha-1)p_1^2 + 2\alpha p_2\right], \tag{35}$$

$$(1-\beta)[1-(\lambda-\mu)]^k b_0 = \alpha\ q_1, \tag{36}$$

$$(2-\beta)\left[(1-2(\lambda-\mu)^k)b_1 - \frac{(\beta-1)[1-(\lambda-\mu)^{2k}]}{2}b_0^2\right] = \frac{1}{2}\left[\alpha(\alpha-1)q_1^2 + 2\alpha q_2\right]. \tag{37}$$

From (34) and (36), we find that

$$p_1 = -q_1, \tag{38}$$

$$b_0^2 = \frac{\alpha^2(p_1^2+q_1^2)}{2(1-\beta)^2[1-(\lambda-\mu)]^{2k}}. \tag{39}$$

As discussed in the proof of Theorem 2.1, applying Lemma 1.1 for the coefficients p_2 and q_2, we immediately have

$$|b_0| \leq \frac{2\alpha}{(1-\beta)[1-(\lambda-\mu)]^k}.$$

This gives the bound on $|b_0|$ as asserted in (30).

Next, in order to find the bound on $|b_1|$, by using (35) and (37), we get

$$\begin{aligned}&2(2-\beta)^2[1-2(\lambda-\mu)]^{2k}b_1^2 + (1-\beta)^2(\beta-2)^2[1-(\lambda-\mu)]^{4k}\frac{b_0^4}{2}\\ &= \frac{\alpha^2(\alpha-1)^2(p_1^4+q_1^4)}{4} + \alpha^2(p_1^2+q_1^2) + \alpha^2(\alpha-1)(p_1^2p_2+q_1^2q_2)\end{aligned} \tag{40}$$

It follows from (39) and (40) that

$$2(2-\beta)^2[1-2(\lambda-\mu)]^{2k}b_1^2 = \frac{\alpha^2(\alpha-1)^2(p_1^4+q_1^4)}{4} + \alpha^2(p_1^2+q_1^2) + \alpha^2(\alpha-1)(p_1^2p_2+q_1^2q_2) - \frac{(1-\beta)^2(\beta-2)^2\alpha^4}{8(1-\beta)^2[1-2(\lambda-\mu)]^{2k}}(p_1^2+q_1^2)^2.$$

Applying Lemma 1.1 once again for the coefficients p_1, p_2, q_1 and q_2, we readily get

$$|b_1| \leq \frac{2\alpha^2}{[1-2(\lambda-\mu)]^k}\sqrt{\frac{1}{(1-\beta)^2}+\frac{1}{(2-\beta)^2}}.$$

This completes the proof of Theorem 2.2.

For $\lambda=\mu$ or $k=0$, we have the following corollary of Theorem 2.2.

Corollary 2.3. *Let the function $h(z)$ given by (4) be in the class $\Sigma_b'(\lambda,\mu;\beta)$. Then*

$$|b_0| \leq \frac{2\alpha}{(1-\beta)}. \tag{41}$$

and

$$|b_1| \leq 2\alpha^2\sqrt{\frac{1}{(1-\beta)^2}+\frac{1}{(2-\beta)^2}}. \tag{42}$$

For $\beta=0$ in corollary 2.3, we have the following result.

Corollary 2.4. *(see [14]) Let the function $h(z)$ given by (1.4) be in the class Σ_b'. Then*

$$|b_0| \leq 2\alpha. \tag{43}$$

and

$$|b_1| \leq \sqrt{5}\alpha^2. \tag{44}$$

We note that, if $\beta=0$ and $\mu=0$ in Theorem 2.1 and Theorem 2.2, we have the same results due to Aziz and Juma [11].

3. Conclusion

The results here related to meromorphic functions of bi-univalent type. The function is defined by a linear operator and new classes are introduced. Initial coefficient bounds are obtained. These similar results can be obtained for classes defined in ([5],[9]) and other new properties can also be studied.

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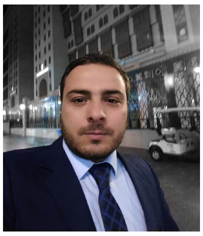

Adnan Ghazy Alamoush received his Ph.D. degree from UKM University, Malaysia. He is an assistant professor in the Faculty of Science, Taibah University, Saudi Arabia. His area of interest include geometric function theory.