TWMS J. App. and Eng. Math. V.12, N.2, 2022, pp. 425-434

THE ROLE OF INTERIOR AND CLOSURE OPERATOR IN MEDICAL APPLICATIONS

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Abstract. In this paper, we consider the interior and closure operator as topological tools to apply in divisor cordial labeling. We investigate the properties related to the path with certain examples in a divisor cordial graphic topology. This concept is utilized in human blood circulation path and the results are analyzed.

Keywords: Divisor cordial labeling, path, interior, closure.

AMS Subject Classification: (2010), 54F65.

1. Introduction and Preliminaries

The concept of cordial labeling was first implemented by Cahit I.[9]. S. Varatharajan, Navaneethakrishnan and K.Nagarajan [21] also introduced cordial labeling of graph in 2013. M. Shokry and Reham Emad Aly [19] discussed a medical application using graphs in 2013. Their works derived a new definition for closure and interior and deduced properties for closure. Also they found inequality of any path and open set. In 2018, Shokry Nada and Abd EI fattah Ei Atik and Mohammed Atef [18] announced a relation on a graph that induces a new type of topological structure to the graph. They formed an algorithm to generate the topological structure from different graphs.

The interior and closure tools in topology have many real life applications to Social science, Engineering, Science, Soft computing, DNA computing and Medical transparency by using mathematical concepts like Algebra, Topology, Graph Theory and etc. Therefore, many authors have developed these concepts in the recent years. In 2002, Jiling Cao, Maximilian Ganster , Ivan Reilly [13] established characterizations of extremely disconnected spaces and sg-submaximal spaces by using various kinds of generalized closed sets. In 2019, Soon-Mo Jung and Doyun Nam [20] discussed some properties of interior and closure in general topology and introduced a necessary and sufficient condition for an open subset of a closed subspace of a topological space to be open. In 2019, D. Sasikala, A. Divya [15] implemented a new definition called cordial graphic topological space by using sum cordial graphs and discussed some of its properties. D. Sasikala and A. Divya

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§ Manuscript received: March 4, 2020; accepted: July 13, 2020.

[16] studied divisor cordial graphs on topological space and they derived a new definition called divisor cordial graphic topology by using divisor cordial labeling on graphs and discussed some of the properties of this graphs in 2019. Dr. Arvindan, S. J. D. Sasikala and A. Divya [5] found the relationship between interior, closure of cordial graphs and blood circulation flow from human heart to Kidney in 2020. Present paper has been completed with many enhancements and extensions of the previous papers.[5, 15, 16]

Blood without oxygen (O_2) is pumped into the lungs from the heart. Oxygen (O_2)-rich blood moves from the lungs to the heart through the pulmonary veins. Pulmonary circulation is the movement of blood from the heart to the lungs and back to the heart. This circulation also includes capillary circulation. Oxygen (O_2) we breathe passes through our lungs into our blood through various capillaries within the lungs. Oxygen (O_2)-rich blood moves through our pulmonary veins to the left side of our heart and out of aorta to the rest of our body. Capillaries within the lungs also remove carbon dioxide (CO_2) from blood.

In this paper, we shall integrate some ideas in human blood circulation path by using edge set of divisor cordial graphic topology.

Definition 1.1. [4] *Let X be a topological space, then X is an Alexandroff space if arbitarary intersection of open sets are open.*

Lemma 1.2. [1] *Let (X, τ) be a topological space, then $int(A^c) = [cl(A)]^c$, for all $A \subseteq X$.*

Lemma 1.3. [1] *Let A and B be two subsets of X in a topological space (X, τ). If A is open then $A \cap cl(B) \subseteq cl(A \cap B)$.*

Definition 1.4. [12] *P_n is path of length $n - 1$.*

Definition 1.5. [3] *A mapping $f : V(G) \to \{0, 1\}$ is called binary vertex labeling of G and $f(v)$ is called the label of the vertex v of G under f.*

The induced edge labeling $f^* : E(G) \to \{0, 1\}$ is given by $f^*(e = uv) = |f(u)-f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f, $e_f(0), e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*.

Definition 1.6. [5] *A binary vertex labeling of a graph G is called a cordial labeling if $|v_f(0)-v_f(1)| \leq 1$ and $|e_f(0)-e_f(1)| \leq 1$. A graph G is called cordial if it admits labeling.*

Definition 1.7. [7] *A binary vertex labeling of a graph G with induced edge labeling $f^* : E(G) \to \{0, 1\}$ defined by $f^*(uv) = |f(u) + f(v)|(mod2)$ is called sum cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is called sum cordial if it admits sum cordial labeling.*

Definition 1.8. [6] *Let $G = (V(G), E(G))$ be a simple graph and $f : V(G) \to \{1, 2, 3, ...|V(G)|\}$ be a bijection. For each edge $e = uv$, assign the label 1 if $f(u)|f(v)$ or $f(v)|f(u)$ and the label 0 otherwise. The function f is called a divisor cordial labeling if $|e_f(0) - e_f(1)| \leq 1$. A graph which admits divisor cordial labeling is called a divisor cordial graph.*

Definition 1.9. [13] *Let $G = (V(G), E(G))$ be a simple graph with sum cordial labeling and with out isolated vertex. Define S_{0G} and S_{1G} as follows. $S_{0G} = \{A_v(0)|v \in V\}$ and $S_{1G} = \{A_v(1)|v \in V\}$ such that $A_v(0)$ and $A_v(1)$ is set of all vertices adjacent to v of G having label 0 and 1 respectively. Since G has no isolated vertex, $S_{0G} \cup S_{1G}$ forms a subbasis for a topology τ_{CG} on V is called Cordial graphic topology of G and it is denoted by (V, τ_{CG}).*

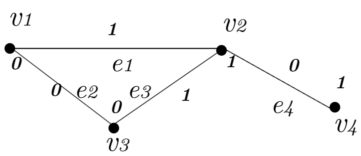

FIGURE 1

Example 1.10. *Let us consider the graph with* $V = \{v_1, v_2, v_3, v_4\}$, $E = \{e_1, e_2, e_3, e_4\}$ *and labeling 0,1.*
Here, $v_f(0) = 2$, $v_f(1) = 2$ *and* $e_f(0) = 2$, $e_f(1) = 2$.
So we have, $|v_f(0) - v_f(1)| \leq 1$ *and* $|e_f(0) - e_f(1)| \leq 1$.
Thus graph admits sum cordial labeling.
$A_{v_1}(0) = \{v_2, v_3\}, A_{v_2}(1) = \{v_1, v_3, v_4\}$,
$A_{v_3}(0) = \{v_1, v_2\}, A_{v_4}(1) = \{v_2\}$
$S_{0G} = \{\{v_2, v_3\}, \{v_1, v_2\}\}$ *and* $S_{1G} = \{\{v_1, v_3, v_4\}, \{v_2\}\}$
Thus $S_{0G} \cup S_{1G} = \{\{v_2, v_3\}, \{v_1, v_2\}, \{v_1, v_3, v_4\}, \{v_2\}\}$
$\tau_{CG} = \{V, \emptyset, \{v_2, v_3\}, \{v_1, v_2\}, \{v_1, v_3, v_4\}, \{v_2\}, \{v_1, v_2, v_3\}, \{v_3\}, \{v_1, v_3\}, \{v_1\}\}$.
Thus the graph admits cordial graphic topology.

Definition 1.11. [14] *Let a graph (simple) with out isolated vertex be denoted by* $G = (V(G), E(G))$, *with divisor cordial labeling.* $S(0)$ *and* $S(1)$ *defined as,* $S(0) = \{A_e(0) | e \in E\}$ *and* $S(1) = \{A_e(1) | e \in E\}$ *such that* $A_e(0)$ *and* $A_e(1)$ *are the set of all edges adjacent to* e *of* G *having label* 0 *and* 1 *respectively. Since* G *has no isolated vertex, Clearly,* $S(0) \cup S(1)$ *forms a subbasis for a topology* τ_{DG} *on* E *is called divisor cordial graphic topology of* G *and it is denoted by* (E, τ_{DG}).

Example 1.12. *Let* $G = (V, E)$ *be a graph with* $V = \{1, 2, 3, 4\}$ *and* $E = \{e_1, e_2, e_3\}$.

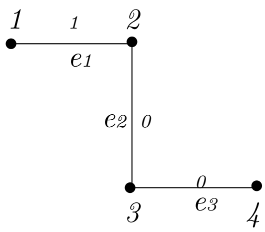

FIGURE 2

From fig. 2, we have
$A_{e_1}(1) = \{e_2\}, A_{e_2}(0) = \{e_1, e_3\}, A_{e_3}(0) = \{e_2\}$,
$S(0) \cup S(1) = \{\{e_1, e_3\}, \{e_2\}\}$,
$\tau_{DG} = \{E, \emptyset, \{e_1, e_3\}, \{e_2\}\}$.
Thus the graph admits divisor cordial graphic topology.

2. Divisor cordial graphic topology on path

Definition 2.1. *Let* $G = (V(G), E(G))$ *be a divisor cordial which admits divisor cordial graphic topology* τ_{DG} *induced by* E *and* p *be the path of* G, *then the interior and closure of* p *has the following form,*

$$int_{DG}(E(p)) = \cup\{U \in \tau_{DG} | U \subseteq E(p)\}$$
$$cl_{DG}(E(p)) = \cap\{F \in \tau^c_{DG} | E(p) \subseteq F\}$$

Example 2.2. *Let $G = (V, E)$ be a graph with $V = \{1, 2, 3, 4\}$ and $E = \{e_1, e_2, e_3, e_4\}$.*

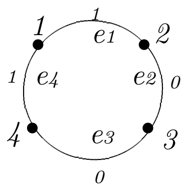

FIGURE 3

Here we have, $A_{e_1}(1) = \{e_2, e_4\}, A_{e_2}(0) = \{e_1, e_3\}$,
$A_{e_3}(0) = \{e_2, e_4\}, A_{e_4}(1) = \{e_1, e_3\}$,
$S(0) \cup S(1) = \{\{e_2, e_4\}, \{e_1, e_3\}\}$,
τ_{DG}=$\{E, \emptyset, \{e_2, e_4\}, \{e_1, e_3\}\}$.
τ^c_{DG}=$\{E, \emptyset, \{e_1, e_3\}, \{e_2, e_4\}\}$.
Let $p = \{v_1e_1v_2e_2v_4\}$, then $E(p) = \{e_1, e_2\}$,
$int_{DG}(E(p)) = \emptyset$ and $cl_{DG}(E(p)) = \{e_1, e_2, e_3, e_4\} = E$

Proposition 2.3. *Let $G = (V(G), E(G))$ be the divisor cordial graphic topology τ_{DG} on E. If p_1 and p_2 are two paths of G, then*

(1) *$int_{DG}[E(p_1)] \subseteq E(p_1) \subseteq cl_{DG}[E(p_1)]$*
(2) *If $E(p_1) \subseteq E(p_2)$ then $int_{DG}[E(p_1)] \subseteq int_{DG}[E(p_2)]$*
(3) *If $E(p_1) \subseteq E(p_2)$ then $cl_{DG}[E(p_1)] \subseteq cl_{DG}[E(p_2)]$*

Proof. (1) The proof is extremely obvious by using the definition of interior and closure of divisor cordial graphic topology.

(2) Let $e \in int_{DG}[E(p_1)]$, $e \in E$ having the label 0 or 1, then there exists the open set $U \in \tau_{DG}$, such that $e \in U \subseteq E(p_1)$.
Since $E(p_1) \subseteq E(p_2)$, thus $e \in U \subseteq E(p_2)$, $U \in \tau_{DG}$ and $e \in int_{DG}[E(p_2)]$. Hence $int_{DG}[E(p_1)] \subseteq int_{DG}[E(p_2)]$.

(3) Let $e \in cl_{DG}[E(p_1)]$, $e \in E$ having the label 0 or 1, then there exists the closed set $F \in \tau^c_{DG}$, such that $e \in F \subseteq E(p_2)$.
Since $E(p_1) \subseteq E(p_2)$ so $e \in F \subseteq E(p_2)$, $F \in \tau^c_{DG}$ and $e \in cl_{DG}[E(p_2)]$.

□

Proposition 2.4. *Let $G = (V(G), E(G))$ be the divisor cordial graphic topology τ_{DG} on E. If p_1 and p_2 are two paths of G, then the given results will be true.*

(1) *$cl_{DG}[E(p_1) \cup E(p_2)] = cl_{DG}[E(p_1)] \cup cl_{DG}[E(p_2)]$*
(2) *$int_{DG}[E(p_1) \cap E(p_2)] = int_{DG}[E(p_1)] \cap int_{DG}[E(p_2)]$*
(3) *$int_{DG}[E(p_1) \cap E(p_2)] \supseteq int_{DG}[E(p_1)] \cup int_{DG}[E(p_2)]$*
(4) *$cl_{DG}[E(p_1) \cup E(p_2)] \subseteq cl_{DG}[E(p_1)] \cap cl_{DG}[E(p_2)]$*

Proof. (1) Let p_1 and p_2 be the paths of graph G and $p_3 = p_1 \cup p_2$, then $e \in p_3$, $e \in E$ having the label 0 or 1. Therefore $E(p_3) = E(p_1) \cup E(p_2)$, then $e \in E(p_3)$, from proposition 2.3, we have $e \in cl_{DG}[E(p_3)]$, which implies that $e \in cl_{DG}[E(p_1) \cup E(p_2)]$.

If $e \in cl_{DG}[E(p_1)] \cup cl_{DG}[E(p_2)]$, then $e \in cl_{DG}[E(p_1)]$ or $e \in cl_{DG}[E(p_2)]$. Applying the closure definition of divisor cordial graphic topology, we have $e \in cl_{DG}[E(p_1) \cup E(p_2)]$

On the other hand, let $e \in p_3$, which means that $e \in E(p_1) \cup E(p_2)$. Therefore we have $e \in cl_{DG}[E(p_1) \cup E(p_2)]$. Since $E(p_1) \cup E(p_2) = E(p_1 \cup p_2) = E(p_3)$, so $e \in E(p_1 \cup p_2)$, which implies that $e \in E(p_1)$ or $e \in E(p_2)$. If $e \in E(p_1)$ then $e \in cl_{DG}[E(p_1)]$ or if $e \in E(p_2)$ then $e \in cl_{DG}[E(p_2)]$, thus $e \in cl_{DG}[E(p_1)] \cup cl_{DG}[E(p_2)]$. Hence, $cl_{DG}[E(p_1) \cup E(p_2)] = cl_{DG}[E(p_1)] \cup cl_{DG}[E(p_2)]$

(2) Let $p_3 = p_1 \cap p_2$, where p_1 and p_2 be the paths of graph G, So that $E(p_3) = E(p_1) \cap E(p_2)$. Since $E(p_1) \cap E(p_2) \subseteq E(p_1)$ and
$E(p_1) \cap E(p_2) \subseteq E(p_2)$, then we have $int_{DG}[E(p_1) \cap E(p_2)] \subseteq int_{DG}[E(p_1)]$ and $int_{DG}[E(p_1) \cap E(p_2)] \subseteq int_{DG}[E(p_2)]$,
$\Rightarrow int_{DG}[E(p_1) \cap E(p_2)] \subseteq int_{DG}[E(p_1)] \cap int_{DG}[E(p_2)]$ ——— (i)
$int_{DG}[E(p_1)] \cap int_{DG}[E(p_2)]] \subseteq int_{DG}[E(p_1) \cap E(p_2)]$.
Since $int_{DG}[E(p_1)]]$ and $int_{DG}[E(p_2)]$ are in τ_{DG}, and using definition of interior of divisor cordial graphic topology we have, $int_{DG}[E(p_1)] \cap int_{DG}[E(p_2)]$ is contained in $E(p_1) \cap E(p_2)$. which gives that,
$int_{DG}[E(p_1)] \cap int_{DG}[E(p_2)] \subseteq int_{DG}[E(p_1) \cap E(p_2)]$ ———(ii)
Form (i)and (ii) we have,
$int_{DG}[E(p_1) \cap E(p_2)] = int_{DG}[E(p_1)] \cap int_{DG}[E(p_2)]$

(3) Since $E(p_1) \subseteq E(p_1)\ E(p_2)$ and $E(p_2) \subseteq E(p_1) \cup E(p_1)$
$\Rightarrow int_{DG}[E(p_1)] \subseteq int_{DG}[E(p_1) \cup E(p_2)]$ and
$int_{DG}[E(p_2)] \subseteq int_{DG}[E(p_1) \cup E(p_2))]$
$\Rightarrow int_{DG}[E(p_1)] \cup int_{DG}[E(p_2)] \subseteq int_{DG}[E(p_1) \cup E(p_2))]$
Hence, $int_{DG}[E(p_1) \cup E(p_2))] \supseteq int_{DG}[E(p_1)] \cup int_{DG}[E(p_2)]$

(4) $E(p_1) \supseteq E(p_1) \cap E(p_2)$ and $E(p_2) \supseteq E(p_1) \cap E(p_2)$, clearly we have
$cl_{DG}[E(p_1)] \supseteq cl_{DG}[E(p_1) \cap E(p_2)]$ and
$cl_{DG}[E(p_2)] \supseteq cl_{DG}[E(p_1) \cap E(p_2))]$
$\Rightarrow cl_{DG}[E(p_1)] \cap cl_{DG}[E(p_2)] \supseteq cl_{DG}[E(p_1) \cap E(p_2))]$
Hence, $cl_{DG}[E(p_1) \cap E(p_2))] \subseteq cl_{DG}[E(p_1)] \cap cl_{DG}[E(p_2)]$

□

Proposition 2.5. *Let $G = (V(G), E(G))$ be the divisor cordial graphic topology τ_{DG} on E. If p_1 and p_2 are two paths of G, then the following results are true.*

(1) $int_{DG}[E(p_1) - E(p_2)] \subseteq int_{DG}[E(p_1)] - int_{DG}[E(p_2)]$
(2) $cl_{DG}[E(p_1) - E(p_2)] \supseteq cl_{DG}[E(p_1)] - cl_{DG}[E(p_2)]$

Proof. (1) Since $E(p_1) - E(p_2) = E(p_1) \cap (E(p_2))^c$
then, $int_{DG}[E(p_1) - E(p_2)] = int_{DG}[E(p_1) \cap (E(p_1))^c]$
$= int_{DG}[E(p_1)] \cap int_{DG}[(E(p_2))^c]$.
Using lemma 1.2, we have,
$int_{DG}[E(p_1) - E(p_2)] = int_{DG}[E(p_1)] \cap [cl(E(p_2))]^c$
$= int_{DG}[E(p_1)] - cl(E(p_2))$
$int_{DG}[E(p_1) - E(p_2)] \subseteq int_{DG}[E(p_1)] - int_{DG}[E(p_2)]$

(2) $cl_{DG}[E(p_1)] - cl_{DG}[E(p_2)] = cl_{DG}[E(p_1] \cap [cl_{DG}(E(p_2)]^c$
$cl_{DG}[E(p_1)] - cl_{DG}[E(p_2)] = cl_{DG}[E(p_1] \cap int_{DG}(E(p_2))^c$
$\subseteq cl_{DG}[E(p_1) \cap int_{DG}(E(p_2))^c]$
$= cl_{DG}[E(p_1) \cap [cl_{DG}(E(p_2))^c]]$
$= cl_{DG}[E(p_1) - cl[E(p_2)]]$

$$cl_{DG}[E(p_1)] - cl_{DG}[E(p_2)] \subseteq cl_{DG}[E(p_1) - E(p_2)]$$

□

3. Connection between divisor cordial labeling and human heart

This section deals with the interior and closure of divisor cordial labeling on blood circulation which plays a vital role in human heart.

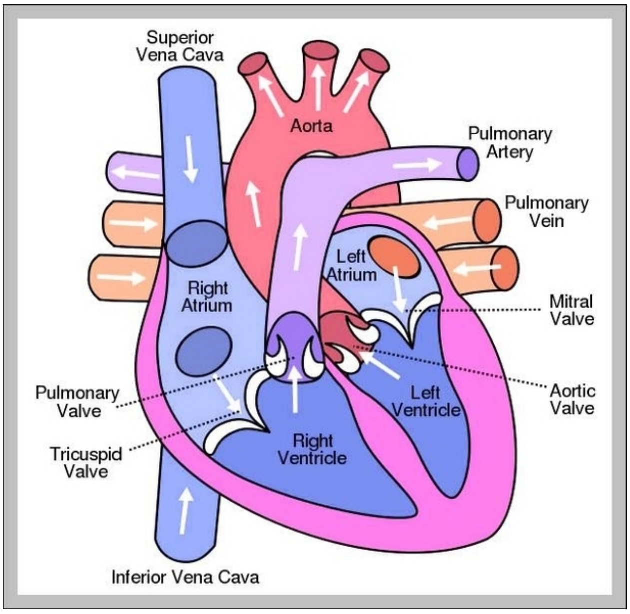

Figure 4. Blood circulation in human heart
[colour figure can be viewed at http://graphdiagram.com/where-is-the-heart-in-the-human-body/]

It is clear that blood should pass through every successive point till it completes its cycle, so we consider the parts of blood flowing path as vertices and edges of mathematical language, because it is easy to generate our divisor cordial graphic topology τ_{DG}. The edges $e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}$ represent blood circulatory way through the human heart.
The line e_1 represents Superior vena cava (v_1) and Right atrium (v_3).
The line e_2 represents Inferior vena cava (v_2) and Right atrium (v_3).
The line e_3 represents Right atrium (v_3) and Right ventricle (v_4).
The line e_4 represents Right ventricle (v_4) and Pulmonary trunk (v_5).
The line e_5 represents Pulmonary trunk (v_5) and Right lung (v_6).
The line e_6 represents Pulmonary trunk (v_5) and Left lung(v_7).
The line e_7 represents Left lung (v_7) and Left atrium (v_8).
The line e_8 represents Right lung (v_6) and Left atrium. (v_8).
The line e_9 represents Left atrium (v_8) and Left ventricle (v_9).
The line e_{10} represents Left Ventricle (v_9) and aorta (v_{10}).
From fig. 6 we have, $e_f(0) = 5; e_f(1) = 5$, the graph admits divisor cordial labeling. Also,
$A_{e_1}(1) = \{e_2\}, A_{e_2}(1) = \{e_1\}, A_{e_3}(1) = \{e_1, e_2\}$,
$A_{e_4}(0) = \{e_3\}, A_{e_5}(1) = \{e_4\}, A_{e_6}(0) = \{e_4\}, A_{e_7}(1) = \{e_6, e_8\}$,
$A_{e_8}(0) = \{e_5, e_7\}, A_{e_9}(0) = \{e_7, e_8\}, A_{e_{10}}(0) = \{e_9\}$.
$S(1) = \{A_{e_1}, A_{e_2}, A_{e_3}, A_{e_5}, A_{e_7}\}$ and $S(0) = \{A_{e_4}, A_{e_6}, A_{e_8}, A_{e_9}, A_{e_{10}}\}$
$S(0) \cup S(1) = \{\{e_2\}, \{e_1\}, \{e_1, e_2\}, \{e_3\}, \{e_4\}, \{e_6, e_8\}, \{e_5, e_7\}, \{e_7, e_8\}, \{e_9\}\}$

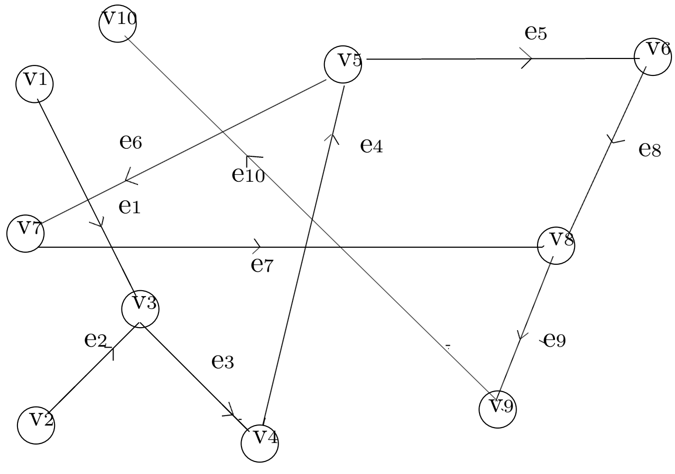

FIGURE 5. Path of blood cirulation with vertices and edges

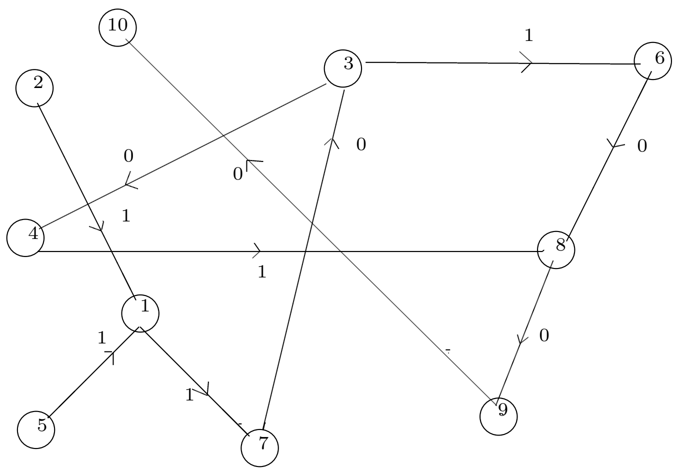

FIGURE 6. Divisor cordial labeling on blood circulation

and $\tau_{DG} = \{E, \emptyset, \{e_2\}, \{e_1\}, \{e_1, e_2, \}, \{e_3\}, \{e_4\}, \{e_6, e_8\}, \{e_5, e_7\},$
$\{e_7, e_8\}, \{e_9\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_2, e_6, e_8\}, \{e_2, e_5, e_7\},$
$\{e_2, e_7, e_8\}, \{e_1, e_3\}, \{e_1, e_4\}, \{e_1, e_6, e_8\}, \{e_1, e_5, e_7\}, \{e_1, e_7, e_8\},$
$\{e_1, e_2, e_3\}, \{e_1, e_2, e_4\}, \{e_1, e_2, e_6, e_8\}, \{e_1, e_2, e_5, e_7\}, \{e_1, e_2, e_7, e_8\},$
$\{e_3, e_4\}, \{e_3, e_6, e_8\}, \{e_3, e_5, e_7\}, \{e_3, e_7, e_8\}, \{e_1, e_6, e_8\}, \{e_4, e_5, e_7\},$
$\{e_4, e_7, e_8\}, \{e_5, e_6, e_7, e_8\}, \{e_6, e_7, e_8\}, \{e_5, e_7, e_8\}, \{e_7\}, \{e_8\}, \{e_2, e_9\},$
$\{e_1, e_2, e_9\}, \{e_3, e_9\}, \{e_4, e_9\}, \{e_6, e_8, e_9\}, \{e_5, e_7, e_9\}, \{e_7, e_8, e_9\},$
$\{e_6, e_8, e_9\}, \{e_4, e_6, e_5, e_7, e_8, e_9\}, etc...\}$

$\tau^c_{DG} = \{E, \emptyset, \{e_1, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}, \{e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\},$
$\{e_3, e_4, e_5, e_6, e_7, e_8, e_9\}, \{e_1, e_2, e_4, e_5, e_6, e_7, e_8, e_9\}, \{e_1, e_2, e_3, e_5, e_6, e_7, e_8, e_9\},$
$\{e_1, e_2, e_3, e_4, e_5, e_7, e_9\}, \{e_1, e_2, e_3, e_4, e_6, e_8, e_9\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_9\},$
$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}, \{e_1, e_4, e_5, e_6, e_7, e_8, e_9\}, \{e_1, e_3, e_5, e_6, e_7, e_8, e_9\},$
$\{e_1, e_3, e_4, e_5, e_7, e_9\}, \{e_1, e_3, e_4, e_6, e_8, e_9\}, \{e_1, e_3, e_4, e_5, e_6, e_9\},$
$\{e_2, e_4, e_5, e_6, e_7, e_8, e_9\}, \{e_2, e_3, e_5, e_6, e_7, e_8, e_9\}, \{e_2, e_3, e_4, e_5, e_7, e_9\},$

$\{e_2, e_3, e_4, e_6, e_8, e_9\}, \{e_2, e_3, e_4, e_5, e_6, e_9\}, \{e_4, e_5, e_6, e_7, e_8, e_9\},$
$\{e_3, e_5, e_6, e_7, e_8, e_9\}, \{e_3, e_4, e_5, e_7, e_9\}, \{e_3, e_4, e_6, e_8, e_9\},$
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$\{e_1, e_2, e_3, e_4, e_5, e_9\}, \{e_1, e_2, e_3, e_4, e_6, e_9\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_8, e_9\},$
$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_9\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}, \{e_1, e_3, e_4, e_5, e_6, e_7, e_8\},$
$\{e_3, e_4, e_5, e_6, e_7, e_8\}, \{e_1, e_2, e_4, e_5, e_6, e_7, e_8\}, \{e_1, e_2, e_3, e_5, e_6, e_7, e_8\},$
$\{e_1, e_2, e_3, e_4, e_5, e_7\}, \{e_1, e_2, e_3, e_4, e_6, e_8, \}, \{e_1, e_2, e_3, e_4, e_5, e_6\}, \{e_1, e_2, e_3\}, etc..\}$

Let us consider the path $p = v_1e_1v_3e_2v_2$, then $E(p) = \{e_1, e_2\}$.
Now apply closure definition of divisor cordial graphic topology to this path which gives, $cl_{DG}[(p)] = \{e_1, e_2, e_3\}$. Medically, once we apply this instance within the heart we will realize it to be true, since blood should passes through every successive point till it completes its cycle. However if blood stops, it will cause several issues as heart failure which happens if the heart cannot pump enough blood to the lungs to pick up oxygen. Left-side heart failure happens if the heart cannot pump enough oxygen-rich blood to the rest of the body.

Let us consider the path $p = v_3e_3v_4e_4v_5e_6v_7$, $E(p) = \{e_3, e_4, e_6\}$.
Then we apply interior definition of divisor cordial graphic topology to this path which gives, $int_{DG}[E(p)] = \{e_3, e_4\}$, here the interior of the path does not consist the edge e_6. So it a contradicts to our medical application. But we can apply this to lung. Usually, one lung will give enough oxygen and take away enough carbon dioxide, unless the opposite lung does not function. But some people have health issues in lungs like pulmonary edema, cancer and tuberculosis for which, it is treated surgically by removing the affected lung, which is known as pneumonectomy.

Conclusion:

The edge e_4 supplies blood into both the right lung (v_6) and the left lung (v_7) by using the edge e_5 and e_6 at the same time. But if the right lung affected by any health issues, then the edge e_4 will not meet the vertex (v_6), so the edge e_4 will circulate blood into the left lung (v_7). If the left lung affected by any health issues, the edge e_4 will not meet the vertex (v_7), so the edge e_4 will circulate blood into the right lung (v_6). It is God's gift to us that, we can live with one lung. Therefore, we conclude that our definiton of interior and closure of divisor cordial labeling can also apply to our blood circulation path.

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