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BIPOLAR INTUITIONISTIC FUZZY $\mathfrak{CK}$ COMPACT SPACES

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Abstract. The objective of this article is to initiate the innovative views on bipolar intuitionistic fuzzy topological space, bipolar intuitionistic fuzzy $\mathfrak{C}$ compact and its preimage, orbit and image followed by some of its special characterizations are studied. The main approaches of its $\mathfrak{CK}$, $\mathfrak{RC}$ compact spaces are established and few significant properties are investigated.

Keywords: Bipolar intuitionistic fuzzy topological space, bipolar intuitionistic fuzzy $\mathfrak{C}$ compact set, bipolar intuitionistic fuzzy $\mathfrak{CK}$ compact space, bipolar intuitionistic fuzzy $\mathfrak{RC}$ compact space.

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1. Introduction

A Cyberneticist, Zadeh [13], established the fuzzy set in 1965 for the first time and was followed by Chang[5] who developed the fuzzy topological spaces in 1968. Currently, this theory works well in uncertainties, ambiguous situations of problems with incomplete information and it is applied in vigorous research disciplines such as medical, spacial and life sciences, engineering, graph theory, artificial intelligence, robotics, computer networking system and decision making problems. After twenty years of gap, the intuitionistic fuzzy sets were discovered and generalized by Atanassov [1]. At the same time, Coker [2,3] introduced the notions of an intuitionistic fuzzy point and intuitionistic fuzzy topological spaces with some relevant concepts. The important applications of IFS have been studied in different fields in pattern recognition and processing of image in computer graphics. The important notion of bipolar fuzzy sets and its membership degree ranges between -1 to 1 was introduced by Zhang in 1994 [14, 15, 8]. The bipolar intuitionistic fuzzy set and strong forms are well developed by Sankar and Ezhilmaran [6]. Recently, bipolar fuzzy topological spaces were initiated by R. Nandhini and D. Amsaveni[10, 7] in 2019. Moreover, it has wide applications in solving real-life problems. In this article, we developed the innovative concept of bipolar intuitionistic fuzzy topological space, bipolar intuitionistic fuzzy image, bipolar intuitionistic fuzzy preimage, bipolar intuitionistic fuzzy orbit

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set and bipolar intuitionistic fuzzy $\mathfrak{C}$ compact set then we investigate on some of its properties. Finally, the intuition of bipolar intuitionistic fuzzy $\mathfrak{CK}$ and $\mathfrak{RC}$ compact spaces are introduced and examined its significant features.

2. Preliminaries

Definition 2.1 (6)**.** *Let X be a non empty set. A bipolar intuitionistic fuzzy set B in X is an object having the form $B = \{(x, \mu^P(x), \mu^N(x), \gamma^P(x), \gamma^N(x) : x \in X\}$, where $\mu^P : X \to [0,1]$, $\mu^N : X \to [-1,0]$, $\gamma^P : X \to [0,1]$, $\gamma^N : X \to [-1,0]$ are the mappings such that $0 \leq \mu^P + \gamma^P \leq 1$. $-1 \leq \mu^N + \gamma^N \leq 0$.*

Definition 2.2 (6)**.** *For any two bipolar intuitionistic fuzzy sets*
$A = (\mu^P_A(x), \mu^N_A(x), \gamma^P_A(x), \gamma^N_A(x))$ *and* $B = (\mu^P_B(x), \mu^N_B(x), \gamma^P_B(x), \gamma^N_B(x))$
$(A \bigcap B)(x) = (\mu^P_A(x) \bigwedge \mu^P_B(x), \mu^N_A(x) \bigvee \mu^N_B(x))$
$(A \bigcup B)(x) = (\mu^P_A(x) \bigvee \mu^P_B(x), \mu^N_A(x) \bigwedge \mu^N_B(x))$
$(A \bigcap B)(x) = (\gamma^P_A(x) \bigvee \gamma^P_B(x), \gamma^N_A(x) \bigwedge \gamma^N_B(x))$
$(A \bigcup B)(x) = (\gamma^P_A(x) \bigwedge \gamma^P_B(x), \gamma^N_A(x) \bigvee \gamma^N_B(x))$

Definition 2.3 (11)**.** *Let (X,T) be an intuitionistic topological space. Then $A = \langle x, A^1, A^2\rangle \in T$ is said to be intuitionistic $\mathcal{C}$-compact set if every $A \subseteq \cup_{i\in\tau} A^c_i$ where A^c_i is an intuitionistic closed set in (X,T). The complement of an intuitionistic $\mathcal{C}$ compact set is an intuitionistic $\mathcal{C}$-cocompact set.*

Definition 2.4 (9)**.** *Let X be a nonempty set and let f:$X \to X$ be any mapping. The fuzzy orbit set of λ under the mapping f is defined as $FO_f(\lambda) = \{\lambda \wedge f(\lambda) \wedge f^2(\lambda) \wedge ...\}$ the intersection of all members of $O_f(\lambda)$*

Definition 2.5 (12)**.** *An IFTS B of (X,T) is said to be IFβ- compact if it is IFβ- compact as a subspace of X.*

3. Bipolar Intuitionistic Fuzzy Topological Space

In this section we introduce, the operations of bipolar intuitionistic fuzzy sets and bipolar intuitionistic fuzzy topological space and discuss some properties on it. Through this article, $A_{B\sim}$ denotes the bipolar intuitionistic fuzzy set(in short $\mathcal{BIF}s$) .

Remark 3.1. *Let $I_+ = [0,1]$ and $I_- = [-1,0]$. Let $I_{+\sim} : \mathfrak{X} \to [0,1]$, $I_{-\sim} : \mathfrak{X} \to [-1,0]$. Then the power set of $\mathcal{BIF}s$ of $\mathfrak{X}$ can be written as $I^{\mathfrak{X}}_{+\sim} \times I^{\mathfrak{X}}_{+\sim} \times I^{\mathfrak{X}}_{-\sim} \times I^{\mathfrak{X}}_{-\sim}$.*

3.1. SOME OPERATIONS ON BIPOLAR INTUITIONISTIC FUZZY SETS.

Definition 3.1. *Let $\mathfrak{X}$ be a non empty fixed set and let $A_{B\sim}, B_{B\sim}$ be any two bipolar intuitionistic fuzzy sets ($\mathcal{BIF}s$) in $\mathfrak{X}$ are defined by*
$A_{B\sim} = \{x,\ \mu^+_{A_{B\sim}}(x), \mu^-_{A_{B\sim}}(x), \gamma^+_{A_{B\sim}}(x), \gamma^-_{A_{B\sim}}(x) : x \in \mathfrak{X}\}$ *and*
$B_{B\sim} = \{x, \mu^+_{B_{B\sim}}(x), \mu^-_{B_{B\sim}}(x), \gamma^+_{B_{B\sim}}(x), \gamma^-_{B_{B\sim}}(x) : x \in \mathfrak{X}\}$. *Then*

(i) $A_{B\sim} \subseteq B_{B\sim}$ *if and only if* $\mu^+_{A_{B\sim}}(x) \leq \mu^+_{B_{B\sim}}(x); \mu^-_{A_{B\sim}}(x) \geq \mu^-_{B_{B\sim}}(x)$;
$\gamma {A_{B\sim}}^+(\xi) \leq \gamma^+_{B_{B\sim}}(x); \gamma {A_{B\sim}}^-(x) \geq \gamma^-_{B_{B\sim}}(x)$;
(ii) $A_{B\sim} = B_{B\sim}$ *iff* $A_{B\sim} \subseteq B_{B\sim}$ *and* $B_{B\sim} \subseteq A_{B\sim}$;
(iii) $\overline{A_{B\sim}} = \{x, 1 - \mu^+_{A_{B\sim}}(x), -1 - \mu^-_{A_{B\sim}}(x), 1 - \gamma^+_{A_{B\sim}}(x), -1 - \gamma^-_{A_{B\sim}}(x)\}$
$= \{\gamma^+_{A_{B\sim}}(x), \gamma^-_{A_{B\sim}}(x), \mu^+_{A_{B\sim}}(x), \mu^-_{A_{B\sim}}(x) : x \in \mathfrak{X}\}$;

(iv) $A_{B\sim} \bigcup B_{B\sim}(x) = \{x, \mu^+_{A_{B\sim}}(x) \bigvee \mu^+_{B_{B\sim}}(x), \mu^-_{A_{B\sim}}(x) \bigwedge \mu^-_{B_{B\sim}}(x)$, $\gamma^+_{A_{B\sim}}(x) \bigwedge \gamma^+_{A_{B\sim}}(x), \gamma^-_{A_{B\sim}}(x) \bigvee \gamma^-_{A_{B\sim}}(x) : x \in \mathfrak{X}\}$;

(v) $A_{B\sim} \bigcap B_{B\sim}(x) = \{x, \mu^+_{A_{B\sim}}(x) \bigwedge \mu^+_{B_{B\sim}}(x), \mu^-_{A_{B\sim}}(x) \bigvee \mu^-_{B_{B\sim}}(x)$, $\gamma^+_{A_{B\sim}}(x) \bigvee \gamma^+_{A_{B\sim}}(x), \gamma^-_{A_{B\sim}}(x) \bigwedge \gamma^-_{A_{B\sim}}(x) : x \in \mathfrak{X}\}$.

Definition 3.2. *Let* $\{A_{B\sim_j}\}$ *be the arbitrary collection of* $\mathcal{BIF}ss$ *in* $\mathfrak{X} : x \in \mathfrak{X}$. *Then*

(i) $\bigcap A_{B_j\sim} = \langle \bigwedge \mu^+_{A_{B_j\sim}}(x), \bigvee \mu^-_{A_{B_j\sim}}(x), \bigvee \nu^+_{A_{B_j\sim}}(x), \bigwedge \nu^+_{A_{B_j\sim}}(x)\rangle$;

(ii) $\bigcup A_{B\sim_j} = \langle \bigvee \mu^+_{A_{B_j\sim}}(x), \bigwedge \mu^-_{A_{B_j\sim}}(x), \bigwedge \nu^+_{A_{B_j\sim}}(x), \bigvee \nu^+_{A_{B_j\sim}}(x)\rangle$;

Definition 3.3. *Let* $0_{B\sim}$ *and* $1_{B\sim}$ *are defined by* $0_{B\sim} = \{\langle x, 0, 0, 1, -1\rangle :$ $x \in \mathfrak{X}\}$; $1_{B\sim} = \{\langle x, 1, -1, 0, 0\rangle : x \in \mathfrak{X}\}$.

Corollary 3.1. *Let* $A_{B\sim}, B_{B\sim}, C_{B\sim}$ *and* $D_{B\sim}$ *be the* $\mathcal{BIF}ss$ *in* $\mathfrak{X}$. *Then*

(i) $A_{B\sim} \subseteq B_{B\sim}$ *and* $C_{B\sim} \subseteq D_{B\sim} \Rightarrow A_{B\sim} \bigcup C_{B\sim} \subseteq B_{B\sim} \bigcup D_{B\sim}$ *and* $A_{B\sim} \bigcap C_{B\sim} \subseteq B_{B\sim} \bigcap D_{B\sim}$,

(ii) $A_{B\sim} \subseteq B_{B\sim}$ *and* $A_{B\sim} \subseteq C_{B\sim} \Rightarrow A_{B\sim} \subseteq B_{B\sim} \bigcap C_{B\sim}$,

(iii) $A_{B\sim} \subseteq C_{B\sim}$ *and* $B_{B\sim} \subseteq C_{B\sim} \Rightarrow A_{B\sim} \bigcup B_{B\sim} \subseteq C_{B\sim}$,

(iv) $A_{B\sim} \subseteq B_{B\sim}$ *and* $B_{B\sim} \subseteq C_{B\sim} \Rightarrow A_{B\sim} \subseteq C_{B\sim}$,

(v) $\overline{A_{B\sim} \bigcup B_{B\sim}} = \overline{A_{B\sim}} \bigcap \overline{B_{B\sim}}$,

(vi) $\overline{A_{B\sim} \bigcap B_{B\sim}} = \overline{A_{B\sim}} \bigcup \overline{B_{B\sim}}$,

(vii) $A_{B\sim} \subseteq B_{B\sim} \Rightarrow \overline{B_{B\sim}} \subseteq \overline{A_{B\sim}}$ *and* $\overline{\overline{A_{B\sim}}} = A_{B\sim}$,

(viii) $\overline{0_{B\sim}} = 1_{B\sim}$,

(ix) $\overline{1_{B\sim}} = 0_{B\sim}$.

Definition 3.4. *Let* $\mathfrak{X}$ *be a non empty fixed set. A bipolar intuitionistic fuzzy topological structure on* $\mathfrak{X}$ *is a collection* $\tau_{\mathfrak{BIF}}$ *of bipolar intuitionistic fuzzy sets in* $\mathfrak{X}$ *satisfying the following three conditions*

1 $0_{B\sim}, 1_{B\sim} \in \tau_{\mathfrak{BIF}}$

2 $A_{B\sim} \bigcap B_{B\sim} \in \tau_{\mathfrak{BIF}}$ *for any* $A_{B\sim}, B_{B\sim} \in \tau_{\mathfrak{BIF}}$

3 $\bigcup A_{B_i\sim} \in \tau_{\mathfrak{BIF}}$ *for arbitrary class of* $\{A_{B_i\sim} : i \in J\} \subseteq \tau_{\mathfrak{BIF}}$.

Then the ordered pair $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *is called a bipolar intuitionistic fuzzy topological structure space (in short* $\mathcal{BIFTS}$*). Every element of* $\tau_{\mathfrak{BIF}}$ *is said to be a* $\tau_{\mathfrak{BIF}}$ *open set in* $\mathfrak{X}$. *The complement of a bipolar intuitionistic fuzzy topological open set (*$\mathcal{BIFO}s$*) is said to be a bipolar intuitionistic fuzzy topological closed set* $\mathcal{BIF}\mathfrak{C}s$ *in* $\mathfrak{X}$.

Example 3.1. *Let* $\mathfrak{X} = \{a, b\}$. *Let* $A_{B\sim} = \langle x, \frac{a}{(0.3,-0.7,0.7,-0.3)} \frac{b}{(0.4,-0.5,0.6,-0.5)}\rangle$, $B_{B\sim} = \langle x, \frac{a}{(0.4,-0.6,0.6,-0.4)} \frac{b}{(0.5,-0.8,0.5,-0.2)}\rangle$, $A_{B\sim} \bigcap B_{B\sim} = \langle x, \frac{a}{(0.3,-0.6,0.7,-0.4)} \frac{b}{(0.4,-0.5,0.6,-0.5)}\rangle$, $A_{B\sim} \bigcup B_{B\sim} = \langle x, \frac{a}{(0.4,-0.7,0.6,-0.3)} \frac{b}{(0.4,-0.8,0.5,-0.2)}\rangle$ *are the* $\mathcal{BIF}s$ *of* $\mathfrak{X}$. *Then the family* $\tau_{\mathfrak{BIF}} = \{0_{B\sim}, 1_{B\sim}, A_{B\sim}, B_{B\sim}, A_{B\sim} \bigcap B_{B\sim}, A_{B\sim} \bigcup B_{B\sim}\}$ *forms a bipolar intuitionistic fuzzy topology (*$\mathcal{BIF}$*topology) on* $\mathfrak{X}$. *Thus the pair (*$\mathfrak{X}$, $\tau_{\mathfrak{BIF}}$*) is said to be a* $\mathcal{BIFTS}$.

Definition 3.5. *Let* $A_{B\sim} = \langle x, \mu^+_{A_{B\sim}}, \nu^-_{A_{B\sim}}, \mu^+_{A_{B\sim}}, \nu^-_{A_{B\sim}}\rangle$ *be a* $\mathcal{BIF}s$ *in* $\mathfrak{X}$ *and* $B_{B\sim} = \{\langle y, \mu^+_{B_{B\sim}}(y), \nu^-_{B_{B\sim}}(y), \mu^+_{B_{B\sim}}(y), \nu^-_{B_{B\sim}}(y)\rangle : y \in \mathfrak{Y}\}$ *be* $\mathcal{BIF}s$ *in* $\mathfrak{Y}$. *Let* $\Phi : \mathfrak{X} \to \mathfrak{Y}$ *be a function. Then*

(i) the preimage of $B_{B\sim}$ *under* Φ *is denoted as* $\Phi^{-1}(B_{B\sim})$ *is the* $\mathcal{BIF}s$ *in* $\mathfrak{X}$, *defined by* $\Phi^{-1}(B_{B\sim}) = \{\langle x, \Phi^{-1}(\mu^+_{B_{B\sim}})(x), \Phi^{-1}(\mu^-_{B_{B\sim}})(x), \Phi^{-1}(\nu^+_{B_{B\sim}})(x),$ $\Phi^{-1}(\nu^-_{B_{B\sim}})(x)\rangle : x \in \mathfrak{X}\}$, *where* $\Phi^{-1}(\mu^+_{B_{B\sim}})(x) = \mu^+_{B_{B\sim}} \circ \Phi(x)$; $\Phi^{-1}(\mu^-_{B_{B\sim}})(x) = \mu^-_{B_{B\sim}} \circ \Phi(x)$; $\Phi^{-1}(\nu^+_{B_{B\sim}})(x) = \nu^+_{B_{B\sim}} \circ \Phi(x)$; $\Phi^{-1}(\nu^-_{B_{B\sim}})(x) = \nu^-_{B_{B\sim}} \circ \Phi(x)$.

(ii) then the image of $A_{B\sim}$ under function Φ, as $\Phi(A_{B\sim})$ is the $\mathcal{BIF}s$ in $\mathfrak{Y}$, defined by $\Phi(A_{B\sim}) = \{\langle y, \Phi(\mu^+_{A_{B\sim}})(y), 1-\Phi(1-\mu^+_{A_{B\sim}})(y), \Phi(\nu^-_{A_{B\sim}})(y),$
$-1-\Phi(-1-\nu^-_{B_{B\sim}})(y)\rangle : y \in \mathfrak{Y}\}$ *where,*

$$\Phi(\mu^+_{A_{B\sim}})(y) = \begin{cases} \sup_{x\in\Phi^{-1}(y)} \mu^+_{B_{B\sim}}(x) & if\ \Phi^{-1}(y) \neq \phi \\ 0\ otherwise \end{cases} \tag{1}$$

$$(1-\Phi(1-\nu^+_{A_{B\sim}}))(y) = \begin{cases} \inf_{x\in\Phi^{-1}(y)} \nu^+_{B_{B\sim}}(x) & if\ \Phi^{-1}(y) \neq \phi \\ 1\ otherwise \end{cases} \tag{2}$$

$$\Phi(\nu^-_{A_{B\sim}})(y) = \begin{cases} \sup_{x\in\Phi^{-1}(y)} \mu^-_{B_{B\sim}}(x) & if\ \Phi^{-1}(y) \neq \phi \\ -1\ otherwise \end{cases} \tag{3}$$

$$(-1-\Phi(-1-\nu^-_{A_{B\sim}}))(y) = \begin{cases} \inf_{x\in\Phi^{-1}(y)} \nu^-_{B_{B\sim}}(x) & if\ \Phi^{-1}(y) \neq \phi \\ 0\ otherwise \end{cases} \tag{4}$$

Corollary 3.2. *Let $A_{B\sim}, A_{B_l\sim}$'s $(l \in M)$ and $B_{B\sim}, B_{B_k\sim}$'s $(k \in L)$ be $\mathcal{BIF}ss$ of $\mathfrak{X}$ and $\mathfrak{Y}$ individually and $\Phi : \mathfrak{X} \to \mathfrak{Y}$. Then*

i) $A_{B_1\sim} \subseteq A_{B_2\sim} (i.e) \mu^+_{A_{B_1\sim}} \leq \mu^+_{A_{B_2\sim}},\ \mu^-_{A_{B_1\sim}} \geq \mu^-_{A_{B_2\sim}}, \nu^+_{A_{B_1\sim}} \geq \nu^+_{A_{B_2\sim}},$
$\nu^-_{A_{B_1\sim}} \leq \nu^-_{A_{B_2\sim}} \Rightarrow \Phi(A_{B_1\sim}) \subseteq \Phi(A_{B_2\sim})\ (i.e)\Phi(\mu^+_{A_{B_1\sim}}) \leq \Phi(\mu^+_{A_{B_2\sim}}),\ \Phi(\mu^-_{A_{B_1\sim}}) \geq$
$\Phi(\mu^-_{A_{B_2\sim}}), \Phi(\nu^+_{A_{B_1\sim}}) \geq \Phi(\nu^+_{A_{B_2\sim}}), \Phi(\nu^-_{A_{B_1\sim}}) \leq \Phi(\nu^-_{A_{B_2\sim}}),$

ii) $B_{B_1\sim} \subseteq B_{B_2\sim} (i.e) \mu^+_{B_{B_1\sim}} \leq \mu^+_{B_{B_2\sim}},\ \mu^-_{B_{B_1\sim}} \geq \mu^-_{B_{B_2\sim}}, \nu^+_{B_{B_1\sim}} \geq \nu^+_{B_{B_2\sim}},$
$\nu^-_{B_{B_1\sim}} \leq \nu^-_{B_{B_2\sim}} \Rightarrow \Phi^{-1}(B_{B_1\sim}) \subseteq \Phi^{-1}(B_{B_2\sim})$
$(i.e)\Phi^{-1}(\mu^+_{A_{B_1\sim}}) \leq \Phi^{-1}(\mu^+_{A_{B_2\sim}}),\ \Phi^{-1}(\mu^-_{A_{B_1\sim}}) \geq \Phi^{-1}(\mu^-_{A_{B_2\sim}}),$
$\Phi^{-1}(\nu^+_{A_{B_1\sim}}) \geq \Phi^{-1}(\nu^+_{A_{B_2\sim}}), \Phi^{-1}(\nu^-_{A_{B_1\sim}}) \leq \Phi^{-1}(\nu^-_{A_{B_2\sim}}),$

iii) $A_{B\sim} \subseteq \Phi^{-1}(\Phi(A_{B\sim}))$ *[If Φ is injective, then $A_{B\sim} = \Phi^{-1}(\Phi(A_{B\sim}))$],*

iv) $\Phi(\Phi^{-1}(B_{B\sim})) \subseteq B_{B\sim}$ *[If Φ is surjective, then $\Phi(\Phi^{-1}(B_{B\sim})) = B_{B\sim}$],*

v) $\Phi^{-1}(\bigcup B_{B_k\sim}) = \bigcup \Phi^{-1}(B_{B_k\sim}) (i.e)(\Phi^{-1}(\bigcup B_{B_k\sim}))$
$= \langle \Phi^{-1}(\bigvee \mu^+_{B_{B_k\sim}})(x),\ \Phi^{-1}(\bigwedge \mu^-_{B_{B_k\sim}})(x), \Phi^{-1}(\bigwedge \nu^+_{B_{B_k\sim}})(x),$
$\Phi^{-1}(\bigvee \nu^+_{B_{B_k\sim}})(x)\rangle;$

vi) $\Phi^{-1}(\bigcap B_{B_k\sim}) = \bigcap \Phi^{-1}(B_{B_k\sim}), (i.e)(\Phi^{-1}(\bigcap B_{B_k\sim}))$
$= \langle \Phi^{-1}(\bigwedge \mu^+_{B_{B_k\sim}})(x),\ \Phi^{-1}(\bigvee \mu^-_{B_{B_k\sim}})(x), \Phi^{-1}(\bigvee \nu^+_{B_{B_k\sim}})(x),$
$\Phi^{-1}(\bigwedge \nu^+_{B_{B_k\sim}})(x)\rangle;$

vii) $\Phi(\bigcup A_{B_l\sim}) = \bigcup \Phi(A_{B_l\sim}), \Phi(\bigcup A_{B_l\sim}) = \bigcup \Phi(A_{B_l\sim})$
(i.e) $\bigcup \Phi(A_{B_l\sim}) = \langle \bigvee \Phi(\mu^+_{A_{B_l\sim}})(x), \bigwedge \Phi(\mu^-_{A_{B_l\sim}})(x),\ \bigwedge(\nu^+_{A_{B_l\sim}})(x),$
$\bigvee \Phi(\nu^+_{A_{B_l\sim}})(x)\rangle,$

viii) $\Phi(\bigcap A_{B_l\sim}) \subseteq \bigcup \Phi(A_{B_l\sim})$, *[If Φ is injective, then*
$\Phi(\bigcap A_{B_l\sim}) = \bigcup \Phi(A_{B_l\sim})$],

ix) $\Phi^{-1}(1_{B\sim}) = 1_{B\sim},$

x) $\Phi^{-1}(0_{B\sim}) = 0_{B\sim},$

xi) $\Phi(1_{B\sim}) = 1_{B\sim}$, *If Φ is surjective,*

xii) $\Phi(0_{B\sim}) = 0_{B\sim},$

xiii) $\overline{\Phi(A_{B\sim})} \subseteq \Phi(\overline{A_{B\sim}})$, *If Φ is surjective,*

xiv) $\Phi^{-1}(\overline{B_{B\sim}}) = \overline{\Phi^{-1}(B_{B\sim})}.$

Definition 3.6. *A collection* $\varsigma = \{B_{B_l\sim} : B_{B_l\sim} \in \tau_{\mathfrak{BIF}}, l \in \Delta\}$ *of all* $\mathcal{BIF}ss$ *is a bipolar intuitionistic fuzzy cover (in short* $\mathcal{BIF}cover$*) of a* $\mathcal{BIF}s$ $A_{B\sim}$ *if and only if* $A_{B\sim} \subseteq \bigcup\{B_{B_l\sim} : B_{B_l\sim} \in \zeta\}$*. It is a bipolar intuitionistic fuzzy open cover (in short* $\mathcal{BIFO}cover$*) of* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *iff every member of* ς *is a* $\mathcal{BIFO}s$*. If* $\exists$ *a subset* $\Delta_1 of \Delta : \bigcup\{B_{B_l\sim} : l \in \Delta_1\} \supseteq A_{B\sim}$*, then* $\{B_{B_l\sim} : l \in \Delta_1\}$ *is called a* $\mathcal{BIF}subcover$*.*

Definition 3.7. *A* $\mathcal{BIFTS}$ *is said to be bipolar intuitionistic fuzzy compact(in short* $\mathcal{BIF}_{\mathbf{C}}$*) if each* $\mathcal{BIFO}cover$ *of* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *by* $\mathcal{BIF}$ *members of* $\tau_{\mathfrak{BIF}}$ *contains a finite* $\mathcal{BIF}subcover$ *(i.e.)* $B_{B_l\sim} \in \tau_{\mathfrak{BIF}}$ *for every* $l \in \Delta$ *and* $\bigcup\{B_{B_l\sim} : l \in \Delta\} = 1_{B\sim}$ *then* $\exists$ *a finite indices* $l_1, l_2, ...l_j \in \Delta : \bigcup_{j\in\Delta}\{B_{B_{l_j}\sim}\} = 1_{B\sim}$*.*

Definition 3.8. *Let* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *be a* $\mathcal{BIFTS}$*. A* $\mathcal{BIF}$ *subset* $B_{B\sim}$ *of a* $\mathcal{BIFTS}$ $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *is said to be* $\mathcal{BIF}_{\mathbf{C}}$ *relative to* $\mathfrak{X}$*, if for every family* $\{A_{B_i\sim} : i \in \Delta\}$ *of* $\mathcal{BIFO}s$ *of* $\mathfrak{X}$: $B_{B\sim} \subseteq \bigcup\{A_{B_i\sim} : i \in \Delta\}$ $\exists$ *a finite subset* Δ_0 *of* $\Delta : B_{B\sim} \subseteq \bigcup\{A_{B_i\sim} : i \in \Delta_0\}$*.*

Definition 3.9. *Let* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *and* $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ *be two* $\mathcal{BIFTS}s$ *. Let* $\Phi : (\mathfrak{X}, \tau_{\mathfrak{BIF}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ *be a mapping . Then* Φ *is called* $\mathcal{BIF}continuous$ *iff* $\Phi^{-1}(A_{B\sim}) \in \mathcal{BIFO}s(\mathfrak{X})$ *for each* $A_{B\sim} \in \mathcal{BIFO}s(\mathfrak{Y})$*.*

Definition 3.10. *Let* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *be a* $\mathcal{BIFTS}$ *and* $A_{B\sim}$ *be any* $\mathcal{BIF}s$ *of* $\mathfrak{X}$*. Then* $\tau_{\mathfrak{BIF}_{1_{B\sim}|A_{B\sim}}} = \{B_{B\sim} \cap A_{B\sim} : B_{B\sim} \in \tau_{\mathfrak{BIF}}\}$ *is a* $\mathcal{BIFTS}$ *and* $(A_{B\sim}, \tau_{\mathfrak{BIF}_{1_{B\sim}|A_{B\sim}}})$ *is called a* $\mathcal{BIF}subspace$ *of* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$*.*

Definition 3.11. *A* $\mathcal{BIF}subset$ $A_{B\sim}$ *of* $\mathcal{BIFTS}$ $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *is* $\mathcal{BIF}_{\mathbf{C}}$ *if* $A_{B\sim}$ *is* $\mathcal{BIF}_{\mathbf{C}}$ *in its subspace topology.*

Proposition 3.1. *Let* $A_{B\sim}$ *be a* $\mathcal{BIF}subset$ *of* $\mathcal{BIFTS}$ $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$*. If* $A_{B\sim}$ *is* $\mathcal{BIF}_{\mathbf{C}}$ *in its subspace topology then, every* $\mathcal{BIFO}cover$ *of* $A_{B\sim}$ *has a finite subcover by* $\mathcal{BIFO}ss$ *in* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$*.*

Proof. Consider that $A_{B\sim}$ is a $\mathcal{BIF}_{\mathbf{C}}$ in the subspace of $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. By definition, then every $\mathcal{BIFO}cover$ of $A_{B\sim}$ by $\mathcal{BIFO}ss$ in $A_{B\sim}$ has a finite sub cover. Assume that $\{D_{B_i\sim} : i \in \Delta\}$ is any $\mathcal{BIFO}cover$ for $A_{B\sim}$ of $\mathfrak{X}$ and $\{D_{B_i\sim} : i \in \Delta\}$ is $\mathcal{BIFO}cover$ of $A_{B\sim}$. This $\Rightarrow A_{B\sim} \subset \bigcup D_{B_i\sim}$
$\Rightarrow A_{B\sim} \bigcap (\bigcup_{i\in\Delta} D_{B_i\sim})$
$\Rightarrow A_{B\sim} \subset \bigcup(A_{B\sim} \bigcap D_{B_i\sim})$ because $D_{B_i\sim}$ is $\mathcal{BIFO}$ in $\mathfrak{X}$ for each i. Therefore, $A_{B\sim} \subset \bigcup_{i\in\Delta}(A_{B\sim} \bigcap D_{B_i\sim})$, this gives $\{A_{B\sim} \bigcap D_{B_i\sim}\}$ is $\mathcal{BIFO}cover$ of $A_{B\sim}$ by $\mathcal{BIFO}ss$ of $A_{B\sim}$. By hypothesis, this $\mathcal{BIFO}cover$ has a finite $\mathcal{BIF}subcover$, $\{A_{B\sim} \bigcap D_{B_{\alpha_i}\sim}\}, i = 1, 2, ...n$. i.e. $A_{B\sim} \subset \bigcup_{i=1}^{n}(A_{B\sim} \bigcap D_{B_{\alpha_i}\sim})$
$= A_{B\sim} \bigcap (\bigcup_{i=1}^{n} D_{B_{\alpha_i}\sim}) \Rightarrow \{D_{B_{\alpha_i}\sim}\}, j = 1, 2, ..n$ is a $\mathcal{BIFO}cover$ for $A_{B\sim}$ by $\mathcal{BIFO}ss$ of $\mathfrak{X}$ and this is a finite subcollection of $\{D_{B_i\sim} : i \in \Delta\}$. □

Proposition 3.2. *Let* $A_{B\sim}$ *be any* $\mathcal{BIF}$ *subset of a* $\mathcal{BIFTS}$ $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$*. If every* $\mathcal{BIFO}cover$ *of* $A_{B\sim}$ *by* $\mathcal{BIFO}ss$ *in* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *has a finite* $\mathcal{BIF}subcover$*, then* $A_{B\sim}$ *is* $\mathcal{BIF}_{\mathbf{C}}$ *in its relative topology.*

Proof. Assume that $\{ H_{\mathfrak{B}_\alpha\sim} : \alpha \in \Delta \}$ is any $\mathcal{BIFO}cover$ of $\mathfrak{X}$ by $\mathcal{BIFO}ss$ in $A_{B\sim}$. Then $A_{B\sim} \subset \bigcup_\alpha H_{\mathfrak{B}_\alpha\sim}$. Because, $H_{\mathfrak{B}_\alpha\sim}$ is $\mathcal{BIFO}$ in $A_{B\sim}$, $H_{\mathfrak{B}_\alpha\sim} = A_{B\sim} \bigcap G_{\mathfrak{B}_\alpha\sim}$ where $G_{\mathfrak{B}_\alpha\sim}$ is $\mathcal{BIFO}$ of $\mathfrak{X}$, for every $\alpha \in \Delta$. So, $A_{B\sim} \subset \bigcup_\alpha H_{\mathfrak{B}_\alpha\sim} \Rightarrow \bigcup_\alpha(A_{B\sim} \bigcap G_{\mathfrak{B}_\alpha\sim})$ $= A_{B\sim} \bigcap (\bigcup_\alpha G_{\mathfrak{B}_\alpha\sim}) \subset \bigcup_\alpha G_{\mathfrak{B}_\alpha\sim}$. Hence, $G_{\mathfrak{B}_\alpha\sim}$ is a $\mathcal{BIFO}cover$ of $A_{B\sim}$ by $\mathcal{BIFO}s$ in $\mathfrak{X}$ and also by the assumption, $G_{\mathfrak{B}_\alpha\sim}$ has a finite $\mathcal{BIF}$ sub cover i.e. $G_{\mathfrak{B}_{\alpha_j}\sim} j = 1, 2, ...n$. (i.e) $A_{B\sim} \subset \bigcup_{l=1}^{n} G_{\mathfrak{B}_{\alpha_l}\sim}$. we have $A_{B\sim} = \bigcup_{l=1}^{n}(A_{B\sim} \bigcap G_{\mathfrak{B}_{\alpha_l}\sim}) = \bigcup_{l=1}^{n} H_{\mathfrak{B}_{\alpha_l}\sim}$. Hence, the family $H_{\mathfrak{B}_{\alpha_l}\sim}$, l=1,2,...n is a $\mathcal{BIFO}cover$ of $A_{B\sim}$ by bipolar intuitionistic fuzzy sets which are $\mathcal{BIFO}$ of $\mathfrak{X}$ and $H_{\mathfrak{B}_{\alpha_l}\sim}$ is a finite subfamily of $H_{\mathfrak{B}_{\alpha}\sim} : \alpha \in \Delta$. Therefore, $A_{B\sim}$ is $\mathcal{BIF}_{\mathbf{C}}$ in its relative topology. □

Proposition 3.3. *A $\mathcal{BIFTS}$ $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ is $\mathcal{BIF}_{\mathbf{C}}$ iff every family of $\mathcal{BIF}\mathfrak{C}$ subsets of $\mathfrak{X}$, having FIP, has a nonempty intersection.*

Proof. Let $\mathfrak{X}$ be $\mathcal{BIF}_{\mathbf{C}}$. Let $A_{B\sim} = \{\zeta_{B_{\alpha_l}\sim} : \alpha_l \in \Delta\}$ be any family of $\mathcal{BIF}\mathfrak{C}$ subsets of $\mathfrak{X}$ having FIP. Suppose $\bigcap\{\zeta_{B_{\alpha_l}\sim} : \alpha_l \in \Delta\} = 0_{B\sim}$. Then $\{\overline{\zeta}_{B_{\alpha_l}\sim} : \alpha_l \in \Delta\}$ is a $\mathcal{BIFO}cover$ of $\mathfrak{X}$ (i.e) $\bigcup_{\alpha_l\in\Delta} \overline{\zeta}_{B_{\alpha_l}\sim} = 1_{B\sim}$. Thus, $\exists$ a finite subset Δ_0 of $\Delta :\Rightarrow \bigcup_{\alpha_l\in\Delta_0} \overline{\zeta}_{B_{\alpha_l}\sim} = 1_{B\sim}$. Then $\bigcap\{\zeta_{B_{\alpha_{l_m}}\sim} : \alpha_{l_m} \in \Delta_0\} = 0_{B\sim}$ for $m = 1, 2, ...q$ which is a $\Rightarrow\Leftarrow$. Thus $A_{B\sim}$ has FIP. Conversely, let $\{\eta_{B_{\alpha_l}\sim} : \alpha_l \in \triangle\}$ be an open cover of $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ by $\mathcal{BIFO}s$. Suppose that for any finite subset Δ_0 of Δ :$\bigcup_{\alpha_l\in\Delta_0} \eta_{\mathfrak{B}\sim} \neq 1_{\mathfrak{B}\sim}$. Then $\bigcap_{\alpha_l\in\Delta_0} \overline{\eta}_{\mathfrak{B}\sim} \neq 0_{\mathfrak{B}\sim\mathfrak{F}}$. Hence, $\{\overline{\eta}_{\mathfrak{B}_{\alpha_l}\sim} : \alpha_l \in \Delta\}$ holds the FIP. Then by hypothesis, $\bigcap_{\alpha_l\in\Delta_0} \overline{\eta}_{\mathfrak{B}\sim} \neq 0_{\mathfrak{B}\sim}$. $\Rightarrow \bigcup_{\alpha_l\in\Delta_0} \eta_{\mathfrak{B}\sim} \neq 1_{\mathfrak{B}\sim}$ and it is a $\Rightarrow\Leftarrow$ for $\{\eta_{\mathfrak{B}_{\alpha_l}\sim} : \alpha_l \in \Delta\}$ is a $\mathcal{BIFO}cover$ of $\mathfrak{X}$. Thus, $\bigcup_{\alpha_l\in\Delta_0} \eta_{\mathfrak{B}\sim} \neq 1_{\mathfrak{B}\sim}$ is wrong, Therefore, $\bigcup_{\alpha_l\in\Delta_0} \eta_{\mathfrak{B}\sim} = 1_{\mathfrak{B}\sim}, \Rightarrow \mathfrak{X}$ is $\mathcal{BIF}_{\mathbf{C}}$. □

Proposition 3.4. *(i) Let $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be two bipolar intuitionistic fuzzy topological spaces ($\mathcal{BIFTS}s$), then a $\mathcal{BIF}$ continuous image of a $\mathcal{BIF}_{\mathbf{C}}\mathcal{S}$ is $\mathcal{BIF}_{\mathbf{C}}$.*
(ii) Let Φ: $(\mathfrak{X}, \tau_{\mathfrak{BIF}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be $\mathcal{BIF}$ continuous map. If $A_{B\sim}$ is $\mathcal{BIF}_{\mathbf{C}}$ relative to $\mathfrak{X}$ hence $\Phi(A_{B\sim})$ is $\mathcal{BIF}_{\mathbf{C}}$ relative to $\mathfrak{Y}$.

Proof. (i) Let $\Phi : \mathfrak{X} \to \mathfrak{Y}$ be $\mathcal{BIF}$ continuous mapping from $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ onto $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$. Assume that $\{\zeta_{B_{\alpha_l}\sim} : \alpha_l \in \triangle\}$ be a $\mathcal{BIFO}$ cover of $\mathfrak{Y}$. Then $\{\Phi^{-1}(\zeta_{B_{\alpha_l}\sim}) : \alpha_l \in \triangle\}$ be a $\mathcal{BIFO}$ cover for $\mathfrak{X}$. Because $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ is a $\mathcal{BIF}_{\mathbf{C}}$ there exists a finite subset Δ_0 of Δ : it has a finite $\mathcal{BIF}$ subfamily $\{\Phi^{-1}(\zeta_{B_{\alpha_1}\sim}), \Phi^{-1}(\zeta_{B_{\alpha_2}\sim}), ...\Phi^{-1}(\zeta_{B_{\alpha_l}\sim}) : \alpha_l \in \Delta_0\}$ covers $\mathfrak{X}$. Since, Φ is surjection and $\{\zeta_{B_{\alpha_1}\sim}, \zeta_{B_{\alpha_2}\sim}, ...\zeta_{B_{\alpha_l}\sim} : \alpha_l \in \Delta_0\}$ is a finite $\mathcal{BIFO}cover$ of $\mathfrak{Y}$ has a finite $\mathcal{BIF}$ subcover and hence $\mathfrak{Y}$ is $\mathcal{BIF}_{\mathbf{C}}$.
(ii) Given that $A_{B\sim}$ be $\mathcal{BIF}$ subset of $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ is $\mathcal{BIF}_{\mathbf{C}}$ relative to $\mathfrak{X}$. To prove that $\Phi(A_{B\sim})$ is $\mathcal{BIF}_{\mathbf{C}}$ relative to $\mathfrak{Y}$. Consider $\{\zeta_{B_{\alpha_l}\sim} : \alpha_l \in \triangle\}$ be a $\mathcal{BIFO}cover$ of $\mathfrak{Y}$, : $\Phi(A_{B\sim}) \subseteq \bigcup_{\alpha_l\in\Delta}\{\zeta_{B_{\alpha_{l_m}}}\sim\}$ by assumption $A_{B\sim}$ is $\mathcal{BIF}_{\mathbf{C}}$ relative to $\mathfrak{X}$ so that there exists a finite $\mathcal{BIF}$subset $\Delta_0 \subseteq \Delta$: $A_{B\sim} \subseteq \bigcup_{\alpha_l\in\Delta_0} \{\Phi^{-1}(\zeta_{B_{\alpha_l}\sim}) : \alpha_l \in \Delta_0\}$ and so $\Phi(A_{B\sim}) \subseteq \bigcup_{\alpha_l\in\Delta_0} \{\zeta_{B_{\alpha_l}\sim} : \alpha_l \in \Delta_0\}$. Thus, $\Phi(A_{B\sim})$ is $\mathcal{BIF}_{\mathbf{C}}$ relative to $\mathfrak{Y}$. □

Definition 3.12. *Let $\mathfrak{X}$ be a non-empty set. Let $\Phi : \mathfrak{X} \to \mathfrak{X}$ be any function and $A_{B\sim}$ be $\mathcal{BIF}s$ of $\mathfrak{X}$. The bipolar intuitionistic fuzzy orbit of $A_{B\sim}$ (in short $\mathcal{BIF}\mathfrak{O}(A_{B\sim})$) under the function Φ is defined as $\mathcal{BIF}\mathfrak{O}(A_{B\sim}) = \{A_{B\sim}, \Phi(A_{B\sim}), \Phi^2(A_{B\sim}), ...\}$.*

Definition 3.13. *Let $\mathfrak{X}$ be a non-empty set. Let $\Phi : \mathfrak{X} \to \mathfrak{X}$ be any function and $A_{B\sim}$ be $\mathcal{BIF}s$ of $\mathfrak{X}$.The bipolar intuitionistic fuzzy orbit set of $A_{B\sim}$ (in short $\mathcal{BIF}\mathfrak{O}(A_{B\sim})$) under the function is defined by $\mathcal{BIF}\mathfrak{O}(A_{B\sim}) = \{A_{B\sim} \bigcap \Phi(A_{B\sim}) \bigcap \Phi^2(A_{B\sim}) \bigcap ......\}$ $= \{x, \mu^+_{A_{B\sim}}(x) \bigwedge \Phi(\mu^+_{A_{B\sim}})(x) \bigwedge \Phi^2(\mu^+_{A_{B\sim}})(x)...,\ \mu^-_{A_{B\sim}}(x) \bigvee \Phi(\mu^-_{A_{B\sim}})(x) \bigvee$ $\Phi^2(\mu^-_{A_{B\sim}})(x).....,\ \gamma^+_{A_{B\sim}}(x) \bigvee \Phi(\gamma^+_{A_{B\sim}})(x) \bigvee \Phi^2((\gamma^+_{A_{B\sim}})(x).....,\ \mu^-_{A_{B\sim}}(x) \bigwedge$ $\Phi(\mu^-_{A_{B\sim}})(x) \bigwedge \Phi^2(\gamma^-_{A_{B\sim}})(x).....\}$ (i.e) the intersection of all members of $\mathcal{BIF}\mathfrak{O}(A_{B\sim})$.*

Example 3.2. *Let $\mathfrak{X} = \{a, b\}$ be any non empty set and consider $A_{B\sim} = \langle x, \frac{a}{(0.3,-0.4,0.7,-0.6)}$ $\frac{b}{(0.4,-0.3,0.6,-0.7)}\rangle$, be a $\mathcal{BIF}s$ of $\mathfrak{X}$. Then the family $\tau_{\mathfrak{BIF}} = \{0_{B\sim}, 1_{B\sim}, A_{B\sim}\}$ is a $\mathcal{BIF}$ topology on $\mathfrak{X}$. Let $\Phi : \mathfrak{X} \to \mathfrak{X}$ is defined by $\Phi(a) = b$ and $\Phi(b) = a$. The $\mathcal{BIF}\mathfrak{O}$ of $A_{B\sim}$ under the function Φ is defined by $\mathcal{BIF}\mathfrak{O}(A_{B\sim})= \{A_{B\sim} \bigcap \Phi(A_{B\sim}) \bigcap \Phi^2(A_{B\sim}) \bigcap ......\}$ $= \langle x, \frac{a}{(0.3,-0.3,0.7,-0.7))} \frac{b}{(0.3,-0.3,0.7,-0.7)}\rangle$*

Definition 3.14. *Let $(\mathfrak{X}, \tau_{\mathfrak{B}\sim\mathfrak{F}})$ be $\mathcal{BIFTS}$. Let $\Phi : \mathfrak{X} \to \mathfrak{X}$ be any function. $\mathcal{BIF}\mathfrak{O}s$ under the function Φ it is in $\mathcal{BIF}$topology $\tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}}$ is said to be bipolar intuitionistic fuzzy orbit open set ($\mathcal{BIF}\mathfrak{O}\mathcal{O}$) under the function Φ and its complement is said to be bipolar intuitionistic fuzzy orbit closed set ($\mathcal{BIF}\mathfrak{O}\mathfrak{C}$)under the function Φ.*

Example 3.3. *Let $\mathfrak{X} = \{a, b\}$ be any non empty set and consider $A_{B\sim}(a) = \langle x, \frac{a}{(0.3,-0.4,0.7,-0.6))} \frac{b}{(0.6,-0.3,0.3,-0.7)}\rangle$, $B_{B\sim}(a) = \langle x, \frac{a}{(0.3,-0.6,0.6,-0.4)} \frac{b}{(0.7,-0.4,0.3,-0.6)}\rangle$, $C_{B\sim}(a) = \langle x, \frac{a}{(0.3,-0.3,0.7,-0.7)} \frac{b}{(0.3,-0.3,0.7,-0.7)}\rangle$, $D_{B\sim}(a) = \langle x, \frac{a}{(0.3,-0.6,0.7,-0.4)} \frac{b}{(0.7,-0.4,0.3,-0.6)}\rangle$ are the $\mathcal{BIF}ss$ of $\mathfrak{X}$. Then the family $\tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}} = \{0_{B\sim}, 1_{B\sim}, A_{B\sim}, B_{B\sim}, C_{B\sim}, D_{B\sim}\}$ is a $\mathcal{BIF}$topology on $\mathfrak{X}$. Let $\Phi : \mathfrak{X} \to \mathfrak{X}$ is defined by $\Phi(a) = b$ and $\Phi(b) = a$, then the ($\mathcal{BIF}\mathfrak{O}\mathcal{O}s$) of $A_{B\sim}$ under the function Φ is defined by ($\mathcal{BIF}\mathfrak{O}\mathcal{O}(A_{B\sim})$)= $\{A_{B\sim} \bigcap \Phi(A_{B\sim}) \bigcap \Phi^2(A_{B\sim}) \bigcap ......\} = D_{B\sim}$. (i.e)($\mathcal{BIF}\mathfrak{O}\mathcal{O}s$)$(A_{B\sim}) = \langle x, \frac{a}{(0.3,-0.6,0.7,-0.4)} \frac{b}{(0.7,-0.4,0.3,-0.6)}\rangle = D_{B\sim}$ under the function Φ.*

Definition 3.15. *Let $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ be $\mathcal{BIFTS}$. Let $A_{B\sim} = \langle x, \mu^+_{A_{B\sim}}, \nu^-_{A_{B\sim}}, \mu^+_{A_{B\sim}}, \nu^-_{A_{B\sim}}\rangle$ be a $\mathcal{BIF}s$ in $\mathfrak{X}$ and $\mathcal{BIF}\mathfrak{O}(A_{B\sim})$. Then the*
(i) bipolar intuitionistic fuzzy kernal of $\mathcal{BIF}\mathfrak{O}(A_{B\sim})$ is denoted and defined by $\mathcal{BIF}Ker(\mathcal{BIF}\mathfrak{O}(A_{B\sim})) = \bigcap\{B_{B\sim} : B_{B\sim}$ is a $\mathcal{BIFO}s$ and $\mathcal{BIF}\mathfrak{O}(A_{B\sim}) \subseteq B_{B\sim}\}$.
(ii) bipolar intuitionistic fuzzy cokernal of $\mathcal{BIF}\mathfrak{O}(A_{B\sim})$ is defined by $\mathcal{BIF}CoKer(\mathcal{BIF}\mathfrak{O}(A_{B\sim})) = \bigcup\{B_{B\sim} : B_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}s$ and $\mathcal{BIF}\mathfrak{O}(A_{B\sim}) \supseteq B_{B\sim}\}$.

Remark 3.2. *Let $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ be $\mathcal{BIFTS}$. Let $A_{B\sim}$ be $\mathcal{BIF}s$ in $\mathfrak{X}$ and its orbit set $\mathcal{BIF}\mathfrak{O}(A_{B\sim})$. Then*
(i) $\overline{\mathcal{BIF}Ker(\mathcal{BIF}\mathfrak{O}(A_{B\sim}))} = \mathcal{BIF}CoKer(\overline{\mathcal{BIF}\mathfrak{O}(A_{B\sim})})$.
(ii) $\overline{\mathcal{BIF}CoKer(\mathcal{BIF}\mathfrak{O}(A_{B\sim}))} = \mathcal{BIF}Ker(\overline{\mathcal{BIF}\mathfrak{O}(A_{B\sim})})$.
(iii)$\mathcal{BIF}Ker(\mathcal{BIF}\mathfrak{O}(0_{B\sim})) = 0_{B\sim}$ and $\mathcal{BIF}CoKer(\mathcal{BIF}\mathfrak{O}(0_{B\sim})) = 0_{B\sim}$.
(iv) $\mathcal{BIF}Ker(\mathcal{BIF}\mathfrak{O}(1_{B\sim})) = 1_{B\sim}$ and $\mathcal{BIF}CoKer(\mathcal{BIF}\mathfrak{O}(1_{B\sim})) = 1_{B\sim}$.
(v) For a $\mathcal{BIF}\mathfrak{O}\mathcal{O}s$ $A_{B\sim}$ then $\mathcal{BIF}Ker(\mathcal{BIF}\mathfrak{O}(A_{B\sim})) = \mathcal{BIF}\mathfrak{O}(A_{B\sim})$.
(iv) For a $\mathcal{BIF}\mathfrak{O}\mathfrak{C}s$ $A_{B\sim}$ then $\mathcal{BIF}CoKer(\mathcal{BIF}\mathfrak{O}(A_{B\sim})) = \mathcal{BIF}\mathfrak{O}(A_{B\sim})$.

Proof. The proof is simple by using above definition. □

4. Bipolar Intuitionistic Fuzzy $\mathfrak{C}\mathfrak{K}$ Compact Spaces

Definition 4.1. *Let $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ be $\mathcal{BIFTS}$. Then $A_{B\sim} = \langle x, \mu^+_{A_{B\sim}}, \nu^-_{A_{B\sim}}, \mu^+_{A_{B\sim}}, \nu^-_{A_{B\sim}}\rangle \in \tau_{\mathfrak{B}\mathfrak{F}}$ is bipolar intuitionistic fuzzy closed compact set (in short $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$) if every $A_{B\sim} \subseteq \bigcup_{i\in J} (A^C_{B_i\sim})$ where $(A^C_{B_i\sim})$ is a $\mathcal{BIF}\mathfrak{C}s$ in $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$. The complement of the $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ is a bipolar intuitionistic fuzzy $\mathfrak{C}$ cocompact set (in short $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$).*

Definition 4.2. *Let $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ be $\mathcal{BIFTS}$ and $A_{B\sim} = \langle x, \mu^+_{A_{B\sim}}, \nu^-_{A_{B\sim}}, \mu^+_{A_{B\sim}}, \nu^-_{A_{B\sim}}\rangle \in \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}}$ is a $\mathcal{BIF}$ $\mathfrak{C}_{\mathbf{C}}s$ of $\mathfrak{X}$. Then*
(i) the bipolar intuitionistic fuzzy $\mathfrak{C}$-compact kernal of $A_{B\sim}$ is defined by $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^\circ(A_{B\sim}) = \bigcup\{B_{B\sim} : B_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ and $B_{B\sim} \subseteq A_{B\sim}\}$;
(ii) the bipolar intuitionistic fuzzy $\mathfrak{C}$-compact cokernal of $A_{B\sim}$ is defined by $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^\neg(A_{B\sim}) = \bigcap\{B_{B\sim} : B_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{X}, \tau'_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ and $A_{B\sim} \subseteq B_{B\sim}\}$.

Remark 4.1. *Let $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ be $\mathcal{BIFTS}$and let $A_{B\sim}$ be a $\mathcal{BIF}s$ of $\mathfrak{X}$. Then*
i) $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^\circ(A_{B\sim}) = A_{B\sim}$ if and only if $A_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$.
ii)$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^\neg(A_{B\sim}) = A_{B\sim}$ if and only if $A_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$.

Definition 4.3. *A $\mathcal{BIFTS}$ $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ is called a bipolar intuitionistic fuzzy $\mathfrak{C}$compact cokernal space is referred as ($\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}\mathfrak{C}\mathfrak{K}\mathcal{S}$) if the $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}\mathfrak{C}\mathfrak{K}$ of every $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$.*

Example 4.1. *Let* $\mathfrak{X} = \{a\}$ *be any non empty set and consider* $A_{B\sim}(a) = \frac{a}{(0.3,-0.3,0.7,-0.7)}$, $B_{B\sim}(a) = \frac{a}{(0.4,-0.2,0.6,-0.8)}$, $C_{B\sim}(a) = \frac{a}{(0.1,-0.2,0.9,-0.8)}$, $D_{B\sim}(a) = \frac{a}{(0.3,-0.2,0.7,-0.8)}$, $E_{B\sim}(a) = \frac{a}{(0.4,-0.3,0.6,-0.7)}$ *are the* $\mathcal{BIF}ss$ *of* $\mathfrak{X}$. *Then the family* $\tau_{\mathfrak{BIF}} = \{0_{B\sim}, 1_{B\sim}, A_{B\sim}, B_{B\sim}, C_{B\sim}, D_{B\sim}, E_{B\sim}\}$ *is a* $\mathcal{BIF}$*topology on* $\mathfrak{X}$. *Thus the pair* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *is a* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}\mathfrak{CKS}$.

Proposition 4.1. *Let* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *be* $\mathcal{BIFTS}$ *and let* $A_{B\sim} = \langle x, \mu^+_{A_{B\sim}}, \nu^-_{A_{B\sim}}, \mu^+_{A_{B\sim}}, \nu^-_{A_{B\sim}}\rangle$ *be a* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ *in* $\mathfrak{X}$. *Then these properties are hold:*
i) $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\overline{A_{B\sim}})$.
ii) $\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(A_{B\sim})} = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\overline{A_{B\sim}})$.

Proof. i) $\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})} = \bigcap\{M_{B\sim} = \langle x, \mu^+_{M_{B\sim}}, \nu^-_{M_{B\sim}}, \mu^+_{M_{B\sim}}, \nu^-_{M_{B\sim}}\rangle : M_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}}), M_{B\sim} \supseteq A_{B\sim}\}$. Applying complements on both sides,
$\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(A_{B\sim})} = \bigcup\{\overline{M_{B\sim}} : \overline{M_{B\sim}}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau'_{\mathfrak{BIF}}), \overline{M_{B\sim}} \subseteq \overline{A_{B\sim}}\} = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\overline{A_{B\sim}})$.
ii)$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\overline{A_{B\sim}}) = \bigcup\{M_{B\sim} = \langle x, \mu^+_{M_{B\sim}}, \nu^-_{M_{B\sim}}, \mu^+_{M_{B\sim}}, \nu^-_{M_{B\sim}}\rangle : M_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}\mathcal{S}$ *in* $(\mathfrak{X}, \tau_{\mathfrak{BIF}}), M_{B\sim} \subseteq A_{B\sim}\}$ Applying complements on both sides,
$\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(A_{B\sim})} = \bigcap\{\overline{M_{B\sim}} : \overline{M_{B\sim}}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{X}, \tau'_{\mathfrak{BIF}}), \overline{M_{B\sim}} \supseteq \overline{A_{B\sim}}\}$
$= \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\overline{A_{B\sim}})$. □

Proposition 4.2. *Let* $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *be* $\mathcal{BIFTS}$. *Then the following equivalent statements are hold.*
i)$(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ *is a* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}\mathfrak{CKS}$.
ii) For every $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ $A_{B\sim}$, *then* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(A_{B\sim})$ *is a* $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$.
iii) For every $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ $A_{B\sim}$, *then*
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})}) = \overline{(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))}$.
iv) For every pair of $\mathcal{BIF}\mathfrak{C}_{\mathcal{C}}ss$ $A_{B\sim}$ *and* $B_{B\sim}$ *with* $\overline{B_{B\sim}} = (\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$ *then* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(B_{B\sim}) = \overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})}$.

Proof. (i) ⇒ (ii) Let $A_{B\sim}$ be any $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ of $\mathfrak{X}$. Then its complement, $\overline{A_{B\sim}}$ is $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Because by assumption, $(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\overline{A_{B\sim}}))$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$, we have $(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\overline{A_{B\sim}})) = (\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(A_{B\sim})}$. Thus, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(A_{B\sim})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Hence, (i) ⇒ (ii).
(ii)⇒ (iii) Let $A_{B\sim}$ be any $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ of $\mathfrak{X}$. Then $\overline{A_{B\sim}}$ is $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{B}\sim\mathfrak{F}})$. Consider $(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\overline{A_{B\sim}}))$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$, now, $(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\overline{A_{B\sim}})) = (\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})}$. Therefore,
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})}) = \overline{(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))}$.
Hence, (ii) ⇒ (iii).
(iii) ⇒ (iv)
Assume (iii), let $A_{B\sim}$ and $B_{B\sim}$be any two $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}ss$ of $\mathfrak{X}$: $\overline{B_{B\sim}} = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})$ and by assumption
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})}) = \overline{(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))}$.
(i.e) $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(B_{B\sim}) = \overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})}$). Thus, (iii) ⇒(iv).
(iv) ⇒ (i)
Let $A_{B\sim}$ and $B_{B\sim}$ be any $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}ss$ of $\mathfrak{X}$: $B_{B\sim} = \overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})}$. By (iv), $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(B_{B\sim}) = \overline{(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))}$.
This gives $\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ of $\mathfrak{X}$ and its complement $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ of $\mathfrak{X}$. Therefore, $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ is $\mathcal{BIF}\mathfrak{CK}_{\mathbf{C}}\mathcal{S}$. Thus, (iv) ⇒ (i).
This ends the proof. □

Proposition 4.3. *Let* $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *be* $\mathcal{BIFTS}$. *Then* $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *is a* $\mathcal{BIF}\mathfrak{C}\mathfrak{K}_{\mathbf{C}}\mathcal{S}$ *if and only if for every* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ $A_{B\sim}$ *and* $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ $B_{B\sim}$: $A_{B\sim} \subseteq B_{B\sim}$, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})$.

Proof. Consider $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ is a $\mathcal{BIF}\mathfrak{C}\mathfrak{K}_{\mathbf{C}}\mathcal{S}$. Let $A_{B\sim}$ be $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ and $B_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ of $\mathfrak{X}$: $A_{B\sim} \subseteq B_{B\sim}$. Since, by above Proposition (ii), $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})$ is a$\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ of $\mathfrak{X}$. Therefore,
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})$.
Since, $A_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ and $A_{B\sim} \subseteq B_{B\sim}$, $A_{B\sim} \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})$.
Now, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\ (\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim}))) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})$,
$\Rightarrow \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})$.
Conversely, consider $B_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ of $\mathfrak{X}$. Then $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ and $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim}) \subseteq B_{B\sim}$. By assumption
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\ (\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim}))) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})$.
Moreover, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\ (\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})))$,
$\Rightarrow \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\ (\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})))$.
Therefore, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})$ is a $\mathcal{BIF}$Cs in $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$. By the above proposition of (ii), we have $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ is a $\mathcal{BIF}\mathfrak{C}\mathfrak{K}_{\mathbf{C}}\mathcal{S}$. □

5. Bipolar Intuitionistic Fuzzy $\mathfrak{C}$ Compact Map

Definition 5.1. *Let* $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$*and* $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *be any two* $\mathcal{BIF}\mathfrak{C}\mathfrak{K}_{\mathbf{C}}\mathcal{S}s$. *A mapping* $\Phi :(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *is said to be a* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ *open mapping if* $\Phi(A_{B\sim})$ $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ *in* $(\mathfrak{Y},\sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$, *for each* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ *in* $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$.

Proposition 5.1. *Let* $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *and* $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *be any two* $\mathcal{BIF}\mathfrak{C}\mathfrak{K}_{\mathbf{C}}\mathcal{S}s$. *Let* $\Phi :(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *be a* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}\mathcal{O}$ *and onto function, for every* $\mathcal{BIF}s$ $A_{B\sim}$ *in* $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$, *hence* $\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim}))$.

Proof. Consider $A_{B\sim}$ is the $\mathcal{BIF}s$ in $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ and $B_{B\sim} = \Phi^{-1}(\overline{A_{B\sim}})$. Then,
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi^{-1}(\overline{A_{B\sim}})) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})$ is $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in
$(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$. Now, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim}) \subseteq B_{B\sim}$.
Thus, $\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim})) \subseteq \Phi(B_{B\sim})$.
(i.e) $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim}))) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi(B_{B\sim}))$.
Because Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}\mathcal{O}$ map, $\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim}))$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in
$(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$. Therefore,
$\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(B_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi(B_{B\sim})) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(A_{B\sim})$.
Thus, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi^{-1}(\overline{A_{B\sim}})) \subseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\overline{A_{B\sim}}))$.
$\Rightarrow$, $\overline{\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi^{-1}(\overline{A_{B\sim}}))} \supseteq \overline{\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\overline{A_{B\sim}}))}$ gives
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Coker^{\urcorner}(\Phi^{-1}(A_{B\sim})) \supseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\overline{A_{B\sim}}))$.
Thus, $\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim}))$. □

Definition 5.2. *Let*$(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$*and* $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *be two* $\mathcal{BIF}\mathfrak{C}\mathfrak{K}_{\mathbf{C}}\mathcal{S}s$. *Let* $\Phi :(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *be a function. Then* Φ *is said to be a* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ *continuous mapping if* $\Phi^{-1}(A_{B\sim})$ *is* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ *in* $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$, *for each* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ *in* $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$.

Remark 5.1. *Let* $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *and* $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *be two* $\mathcal{BIF}\mathfrak{C}\mathfrak{K}_{\mathbf{C}}\mathcal{S}s$. *Let* $\Phi :(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ *be any mapping. If* Φ : *is a* $\mathcal{BIF}\mathfrak{C}_{\mathrm{C}}$ *continuous mapping then,*
$\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})) \supseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim}))$, *for every* $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ $A_{B\sim}$ *in* $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$.

Proof. Let Φ : be a $\mathcal{BIF}\mathfrak{C}_{\mathrm{C}}$ continuous function and consider $A_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$. Assume that $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$. Thus,

$\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Thus,
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))) = \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$.
Because
$A_{B\sim} \subseteq (\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$, $\Phi^{-1}(A_{B\sim}) = \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$.
Therefore,
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$
$= \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$.
(i.e)$\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})) \supseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim}))$. □

Remark 5.2. *Let $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be two $\mathcal{BIF}\mathfrak{CK}_{\mathbf{C}}\mathcal{S}s$ and $\Phi :(\mathfrak{X}, \tau_{\mathfrak{BIF}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ is any mapping. If $\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})) \supseteq$ $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim}))$, for every $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ $A_{B\sim}$ in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ then Φ is $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ continuous mapping.*

Proof. Let us take $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim})) \subseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$ for every $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$. Consider $A_{B\sim}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$. To prove that $\Phi^{-1}(A_{B\sim})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Since $A_{B\sim} = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})$, $\Phi^{-1}(A_{B\sim}) =$ $\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$ and from the given assumption
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim})) \subseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$.
Hence, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim})) \subseteq \Phi^{-1}(A_{B\sim}) \subseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$.
Thus, $\Phi^{-1}(A_{B\sim}) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}(\Phi^{-1}(A_{B\sim}))$, (i.e.) $\Phi^{-1}(A_{B\sim})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$.
This shows that Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ continuous mapping. □

Proposition 5.2. *Let $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be two $\mathcal{BIF}\mathfrak{CK}_{\mathbf{C}}\mathcal{S}s$ and $\Phi :(\mathfrak{X}, \tau_{\mathfrak{BIF}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be a bijection mapping. Then Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathrm{C}}$ continuous mapping if for every $\mathcal{BIF}s$ $A_{B\sim}$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$,*
$\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim}))$.

Proof. Consider Φ be a $\mathcal{BIF}\mathfrak{C}_{\mathrm{C}}$ continuous mapping and $A_{B\sim}$ be a $\mathcal{BIF}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Thus, $\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim})))$ is a $\mathcal{BIF}\mathfrak{C}_{\mathfrak{Co}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. By Remark 5.2,
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(\Phi(A_{B\sim}))) \subseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim})))$.
Because, Φ is a 1-1 map,
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}) \subseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim})))$.
Applying Φ on both sides,
$\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})) \subseteq \Phi(\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim}))))$.
Because Φ is an onto map,
$\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim}))$. □

Proposition 5.3. *Let $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be two $\mathcal{BIF}\mathfrak{CK}_{\mathbf{C}}\mathcal{S}s$ and $\Phi :(\mathfrak{X}, \tau_{\mathfrak{BIF}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be a bijection map. If Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}$ continuous map, hence, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim})) \subseteq \Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$, for every $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ $B_{B\sim}$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$.*

Proof. Let $A_{B\sim} = \langle y, \mu^{+}_{A_{B\sim}}(y), \nu^{-}_{A_{B\sim}}(y), \mu^{+}_{A_{B\sim}}(y), \nu^{-}_{A_{B\sim}}(y)\rangle$ be a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$, in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ and it is clear that $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Since Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}$ continuous map, $\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$. Thus,
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})))$
$= \Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$.
Hence, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim})) \subseteq \Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$. □

Proposition 5.4. *Let $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be two $\mathcal{BIF}\mathfrak{CK}_{\mathbf{C}}\mathcal{S}s$. Let $\Phi : (\mathfrak{X}, \tau_{\mathfrak{BIF}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be a bijection map. If for every $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ $B_{B\sim}$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$, then Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}$ continuous map.*

Proof. Let $B_{B\sim} = \langle y, \mu^{+}_{B_{B\sim}}(y), \nu^{-}_{B_{B\sim}}(y), \mu^{+}_{B_{B\sim}}(y), \nu^{-}_{B_{B\sim}}(y)\rangle$ be a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$.
Then $B_{B\sim} = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(B_{B\sim})$.
By $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim})) \subseteq \Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$,
$= \Phi(A_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim}))$. Thus,
$\Phi(A_{B\sim}) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim}))$ and $\Phi(A_{B\sim})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}$ continuous in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$.
Thus, Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}$ continuous function. □

Definition 5.3. *Let $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be two $\mathcal{BIFTS}s$. A map $\Phi : (\mathfrak{X}, \tau_{\mathfrak{BIF}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ is said to be $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ irresolute map if $\Phi^{-1}(A_{B\sim})$ is $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$, for every $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$.*

Proposition 5.5. *Let $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be two $\mathcal{BIFTS}s$. Let $\Phi : (\mathfrak{X}, \tau_{\mathfrak{BIF}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ irresolute mapping if and only if*
$\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim}))$, for each $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$.

Proof. Let us suppose Φ be the $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ irresolute mapping. Let $A_{B\sim}$ be $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Hence, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim}))$ is the $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$. Assume that, $\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim})))$ is the $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Now, $A_{B\sim} \subseteq \Phi^{-1}(\Phi(A_{B\sim})) \subseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim})))$ and we have $A_{B\sim} \subseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}))$.
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim}))))$
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}) \subseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim})))$.
i.e. $\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(A_{B\sim}))$.

For converse part, suppose that $A_{B\sim}$ is $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$.
Then, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}) = A_{B\sim}$. By assumption,
$\Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim}))) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi(\Phi^{-1}(A_{B\sim})))$
$= \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(A_{B\sim}) = A_{B\sim}$.
Thus, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim})) \subseteq \Phi^{-1}(A_{B\sim})$.
But, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim})) \supseteq \Phi^{-1}(A_{B\sim})$.
Therefore, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(A_{B\sim})) = \Phi^{-1}(A_{B\sim})$. i.e. $\Phi^{-1}(A_{B\sim})$ is the $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Thus, Φ is the $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ irresolute map. □

6. Bipolar Intuitionistic Fuzzy $\mathfrak{RC}$ Compact Spaces

Definition 6.1. *Let $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ be $\mathcal{BIF}\mathfrak{C}\mathfrak{K}_{\mathbf{C}}\mathcal{S}$ and let $A_{B\sim} = \langle\, x, \mu^{+}_{A_{B\sim}}, \mu^{-}_{A_{B\sim}}, \gamma^{+}_{A_{B\sim}}, \gamma^{-}_{A_{B\sim}}\rangle$ be a $\mathcal{BIF}s$ of $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Then $A_{B\sim}$ is called bipolar intuitionistic fuzzy $\mathfrak{RC}$ compact (in short $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$) if*
$A_{B\sim} = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Coker^{\urcorner}(A_{B\sim}))$.

Definition 6.2. *Let $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ be $\mathcal{BIF}\mathfrak{C}\mathfrak{K}_{\mathbf{C}}\mathcal{S}$ and let $A_{B\sim} = \langle x, \mu^{+}_{A_{B\sim}}, \mu^{-}_{A_{B\sim}}, \gamma^{+}_{A_{B\sim}}, \gamma^{-}_{A_{B\sim}}\rangle$ be a $\mathcal{BIF}s$ of $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$. Then $A_{B\sim}$ is said to be bipolar intuitionistic fuzzy $\mathfrak{RC}$ cocompact (in short $\mathcal{BIF}\mathfrak{RC}_{\mathbf{Co}}$) if*
$A_{B\sim} = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(A_{B\sim}))$.

Remark 6.1. *Every $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$.*

Proposition 6.1. *Let $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ be any two $\mathcal{BIFTS}s$. If $\Phi : (\mathfrak{X}, \tau_{\mathfrak{BIF}}) \to (\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ continuous map of $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$ into a $\mathcal{BIF}\mathfrak{C}\mathfrak{K}_{\mathbf{C}}\mathcal{S}$ $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$ and if $B_{B\sim}$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$. Hence $\Phi^{-1}(B_{B\sim})$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$.*

Proof. Given that $B_{B\sim}$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$. By the Remark 6.1, we have $B_{B\sim}$ is $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ in $(\mathfrak{Y}, \sigma_{\mathfrak{BIF}})$. Because, Φ is $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ continuous map, $\Phi^{-1}(B_{B\sim})$ is $(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}})$ in $(\mathfrak{X}, \tau_{\mathfrak{BIF}})$.
i.e. $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}((\Phi^{-1}(B_{B\sim}))) = \Phi^{-1}(B_{B\sim})$ ——(I)

Since, $B_{B\sim}$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ in $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ and it is $\mathcal{BIF}\mathfrak{CK}_{\mathbf{C}}\mathcal{S}$ we have,
$B_{B\sim} = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Coker^{\urcorner}(B_{B\sim}))$
$= \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(B_{B\sim})))$
$= \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(B_{B\sim})$
i.e. $B_{B\sim} = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Coker^{\urcorner}(B_{B\sim})$ —–(II)
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(B_{B\sim})) \subseteq \Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(B_{B\sim}))$. Since, Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ continuous map. Therefore,
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(B_{B\sim}))) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$
$CoKer^{\urcorner}(B_{B\sim})))$. By (II), it gives that
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi^{-1}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(B_{B\sim}))) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi^{-1}(B_{B\sim}))$ –(III) By (I) and (III)
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(B_{B\sim}))) \subseteq \Phi^{-1}(B_{B\sim})$ —–(IV)
Since, $\Phi^{-1}(B_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi^{-1}(B_{B\sim}))$. Then,
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi^{-1}(B_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi^{-1}(B_s sB\sim)))$.
By (I) $\Rightarrow \Phi^{-1}(B_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}CoKer^{\urcorner}(\Phi^{-1}(B_{B\sim})))$ —–(V)
Therefore, from (IV) and (V) $\Phi^{-1}(B_{B\sim}) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Coker^{\urcorner}(\Phi^{-1}(Ker^{\urcorner}_{B\sim})))$.
Thus, $\Phi^{-1}(B_{B\sim})$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ in $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$. ☐

Definition 6.3. *Let $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ be any two $\mathcal{BIFTS}s$. Let $\Phi : (\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}}) \rightarrow (\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$be a mapping. Then Φ is said to be $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ mapping if the image of every $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}s$ in $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$.*

Definition 6.4. *Let $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ be any two $\mathcal{BIFTS}s$. If $\Phi : (\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}}) \rightarrow (\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ is a mapping. Then Φ is said to be $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ mapping if the image of every $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{Co}}s$ in$(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$.*

Proposition 6.2. *For any two bipolar intuitionistic fuzzy topological spaces $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ and $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$and If there is a $\mathcal{BIF}$ continuous bijection mapping $\Phi : (\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}}) \rightarrow (\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ $\mathcal{BIF}\mathfrak{CK}_{\mathbf{C}}\mathcal{S}$ $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ into a space $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$. If $B_{B\sim}$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}s$ of $(\mathfrak{X}, \tau_{\mathfrak{B}\sim\mathfrak{F}})$, hence $\Phi(B_{B\sim})$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ set of $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$.*

Proof. Given that $B_{B\sim}$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}s$ in $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ and because, $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ is a $\mathfrak{CK}_{\mathbf{C}}\mathcal{S}$,
$B_{B\sim} = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(B_{B\sim})) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(B_{B\sim})$,
i.e. $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(B_{B\sim}) = B_{B\sim}$. Since, Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ continuous bijection mapping,
$\Phi(B_{B\sim}) = \Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(B_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi(B_{B\sim}))$. Because, Φ is a bipolar intuitionistic fuzzy continuous mapping,
$\Phi(B_{B\sim}) = \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\Phi(B_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi(B_{B\sim})))$.
i.e. $\Phi(B_{B\sim}) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi(B_{B\sim}))$————(VI)
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}_{B\sim}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi(B_{B\sim})) \subseteq \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi(B_{B\sim})$. Since, Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ bijective mapping, we have Φ is a $\mathcal{BIF}\mathfrak{C}$ co-compact mapping.
Hence, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi(B_{B\sim})) \subseteq \Phi(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(B_{B\sim})) = \Phi(B_{B\sim})$.
Then, $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi(B_{B\sim})) \subseteq \Phi(B_{B\sim})$ ————-(VII).
Thus, we have from (VI) and(VII)
$\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\circ}(\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi(B_{B\sim})) = \Phi(B_{B\sim})$.
Therefore $\Phi(B_{B\sim})$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ set of $(\mathfrak{Y}, \sigma_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$. ☐

Definition 6.5. *Let $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ be a $\mathcal{BIFTS}$. If a collection $\{A_{B\sim_i} = \langle x, \mu^{+}_{A_{B\sim_i}}(x), \mu^{-}_{A_{B\sim_i}}(x), \gamma^{+}_{A_{B\sim_i}}(x), \gamma^{-}_{A_{B\sim_i}}(x)\rangle : i \in \Delta\}$ of $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ of $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$ holds the property that $\bigcup\{A_{B\sim_i} : i \in \Delta\} = 1_{B\sim}$, then it is said to be bipolar intuitionistic fuzzy regular closed compact cover ($\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$cover) of $(\mathfrak{X}, \tau_{\mathfrak{B}\mathcal{I}\mathfrak{F}})$.*

Definition 6.6. *A $\mathcal{BIFTS}$ $(\mathfrak{X},\ \tau_{\mathfrak{BIF}})$ is called $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ space if and only if each $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ cover of $(\mathfrak{X},\ \tau_{\mathfrak{BIF}})$ has a finite sub collection of $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ cokernals of its members cover the space $(\mathfrak{X},\ \tau_{\mathfrak{BIF}})$.*

Proposition 6.3. *For any two $\mathcal{BIFTS}$s $(\mathfrak{X},\ \tau_{\mathfrak{BIF}})$ and $(\mathfrak{Y},\ \sigma_{\mathfrak{BIF}})$and if there is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ mapping $\Phi : (\mathfrak{X},\ \tau_{\mathfrak{BIF}}) \to (\mathfrak{Y},\ \sigma_{\mathfrak{BIF}})$ of a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ space $(\mathfrak{X},\ \tau_{\mathfrak{BIF}})$ onto $\mathcal{BIF}\mathfrak{CK}_{\mathbf{C}}$ space $(\mathfrak{Y},\ \sigma_{\mathfrak{BIF}})$, then $(\mathfrak{Y},\ \sigma_{\mathfrak{BIF}})$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ space.*

Proof. Let $A_{B\sim_i} = \{\langle x, \mu^{+}_{A_{B\sim_i}}(x), \mu^{-}_{A_{B\sim_i}}(x), \gamma^{+}_{A_{B\sim_i}}(x), \gamma^{-}_{A_{B\sim_i}}(x)\rangle : i \in \Delta\}$ be a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ cover of $(\mathfrak{Y},\ \sigma_{\mathfrak{BIF}})$. Since, $(\mathfrak{Y},\ \sigma_{\mathfrak{BIF}})$ is a $\mathcal{BIF}\mathfrak{CK}_{\mathbf{C}}$ space and Φ is a $\mathcal{BIF}\mathfrak{C}_{\mathbf{C}}$ continuous mapping and by Proposition 6.1 we have $\{\Phi^{-1}(A_{B\sim_i}) : i \in \Delta\}$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ cover of $(\mathfrak{X},\ \tau_{\mathfrak{BIF}})$. Since, $(\mathfrak{X},\ \tau_{\mathfrak{BIF}})$ is a $\mathcal{BIF}\mathfrak{RC}_{\mathbf{C}}$ space, $\exists$ a finite sub collection $\Phi^{-1}(A_{B\sim_{i_1}}), \Phi^{-1}(A_{B\sim_{i_{21}}}), ...\Phi^{-1}(A_{B\sim_{i_m}})$:
$\bigcup_{k=1}^{m} \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi^{-1}(A_{B\sim_{i_k}})) = 1_{B\sim\mathfrak{X}}$
Thus, its image $\Phi(\bigcup_{k=1}^{m} \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(\Phi^{-1}(A_{B\sim_{i_k}})) = 1_{B\sim\mathfrak{X}}$
$\subseteq \bigcup_{k=1}^{m} \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(A_{B\sim_{i_k}})$. Hence $\bigcup_{k=1}^{m} \mathcal{BIF}\mathfrak{C}_{\mathbf{C}}Ker^{\urcorner}(A_{B\sim_{i_k}}) = 1_{B\sim\mathfrak{Y}}$. □

7. Conclusions

In this paper, the new concept of bipolar intuitionistic fuzzy topological space was introduced by defining and characterizations of various operations such as bipolar intuitionistic fuzzy sets, bipolar intuitionistic fuzzy image, preimage, bipolar intuitionistic fuzzy continuous map, bipolar intuitionistic fuzzy orbit set, and bipolar intuitionistic fuzzy compact space. Followed by, the bipolar intuitionistic fuzzy $\mathfrak{C}$ compact set, bipolar intuitionistic fuzzy $\mathfrak{CK}$ compact space with certain examples are established and some of its features were also investigated. Finally, the notion of bipolar intuitionistic fuzzy $\mathfrak{C}$ compact map, bipolar intuitionistic fuzzy $\mathfrak{RC}$ compact space are proposed and examined a few of its properties.

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