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ON LEAP ZAGREB INDICES OF SOME CYCLE RELATED GRAPHS

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Abstract. Let $G = (V, E)$ represent a graph. The 2-degree of a vertex v in G is the number of vertices which are at distance two from v and denoted by $d_2(v)$. The leap Zagreb indices are the recently introduced distance-based topological indices defined using 2-degrees of vertices in G. In this paper, we compute the leap Zagreb indices of some cycle related graphs, namely book graph, cycle with parallel chords, cycle with parallel P_k chords, shell graph and shell type graphs.

Keywords: Zagreb indices, Leap Zagreb indices, cycle, shell graph.

AMS Subject Classification: 05C07, 05C35, 05C40.

1. Introduction

Topological indices in chemical graph theory can be widely classified into two types: degree-based and distance-based. Recently, Naji et al.[5] introduced a novel graph invariant called leap Zagreb indices of a graph which fall under the second category. These indices are analogous to the oldest and celebrated molecular descriptors namely Zagreb indices. The definitions of these new graph invariants are as follows: Let $G = (V, E)$ denote a simple, connected graph with p vertices and q edges. For graph theoretic terminologies not defined here, we refer to [1].

First leap Zagreb index: $LM_1(G) = \sum\limits_{v \in V(G)} d_2(v)^2$
Second leap Zagreb index: $LM_2(G) = \sum\limits_{uv \in E(G)} d_2(u)d_2(v)$
Third leap Zagreb index: $LM_3(G) = \sum\limits_{v \in V(G)} d_2(v)deg(v)$ where $d_2(v)$ is the 2-degree of

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v, defined as the number of vertices which are at distance two from v in G and $deg(v)$ is the degree of v in G, that is, the number of edges incident at v in G.

Following the seminel paper [5], A.M.Naji and N.D.Soner [6] studied the first leap Zagreb index of some graph operations. Shao et al.[8] presented interesting results on leap Zagreb indices of trees and unicyclic graphs. Shiladhar et al. [9] computed leap Zagreb indices of windmill graphs. Basavanagoud et al. [2, 3] discussed leap Zagreb indices of some nanostructures. S.Swathi et al. [10] obtained results for third leap Zagreb index of some generalized graph structures.

2. Leap Zagreb indices of cycle related graphs

In this section we compute Leap Zagreb indices of some cycle related graphs. We refer [4] for notions about special types of graphs discussed here and also [7] for some cycle related graphs.

2.1. **Book graph with r-gonal pages.** A book graph $\mathcal{B} = B(n, r)$ with r-gonal pages is a graph constructed using n copies of a cycle of length r that share exactly one common edge.

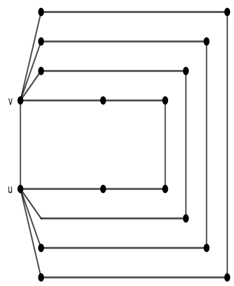

Figure 1. Book graph $B(4, 6)$ with 4 pages

Table 1. Vertex partition of $\mathcal{B}$

Vertex	**2-degree**	**degree**	**Number of vertices**
$x = u$	$2n$	$n + 1$	1
$x = v$	$2n$	$n + 1$	1
$x \in N_{\mathcal{B}}(u) \setminus \{v\}$	$n + 1$	2	n
$x \in N_{\mathcal{B}}(v) \setminus \{u\}$	$n + 1$	2	n
$x \in V(\mathcal{B}) \setminus \{N_{\mathcal{B}}(u) \cup N_{\mathcal{B}}(v)\}$	2	2	$n(r - 4)$

Theorem 2.1. $LM_1(\mathcal{B}) = 2n^3 + 12n^2 - 14n + 4nr$

Proof. By the vertex partition of $\mathcal{B}$ we get

$$\begin{aligned}
LM_1(\mathcal{B}) &= \sum_{x\in V(\mathcal{B})} \left[d_2(x)^2\right] \\
&= d_2(u)^2 + d_2(v)^2 + \sum_{x\in N_{\mathcal{B}}(u)\setminus\{v\}} d_2(x)^2 \\
&+ \sum_{x\in N_{\mathcal{B}}(v)\setminus\{u\}} d_2(x)^2 + \sum_{x\in V(\mathcal{B})\setminus\{N_{\mathcal{B}}(u)\cup N_{\mathcal{B}}(v)\}} d_2(x)^2 \\
&= 4n^2 + 4n^2 + n(n+1)^2 + n(n+1)^2 + 4n(r-4) \\
&= 2n^3 + 12n^2 - 14n + 4nr.
\end{aligned}$$

□

Theorem 2.2. $LM_3(\mathcal{B}) = 8n^2 + 4nr - 8n$.

Proof. $LM_3(\mathcal{B}) = \sum_{x\in V(\mathcal{B})} d_2(x)deg(x)$
Using the vertex partition of $\mathcal{B}$ we have

$$\begin{aligned}
LM_3(\mathcal{B}) &= d_2(u)deg(u) + d_2(v)deg(v) + \sum_{x\in N_{\mathcal{B}(u)\setminus\{v\}}} d_2(x)deg(x) \\
&+ \sum_{x\in N_{\mathcal{B}(v)\setminus\{u\}}} d_2(x)deg(x) + \sum_{x\in V(\mathcal{B})\setminus\{N_{\mathcal{B}}(u)\cup N_{\mathcal{B}}(v)\}} d_2(x)deg(x) \\
&= 2(2n)(n+1) + 2n(n+1) + 2n(n+1) + 4n(r-4) \\
&= 8n(n+1) + 4nr - 16n \\
&= 8n^2 + 4nr - 8n.
\end{aligned}$$

□

TABLE 2. Edge partition of $\mathcal{B}$

Edge xy	$(d_2(x), d_2(y))$	Number of edges
$x = u,\ y = v$	$(2n, 2n)$	1
$x = u, y \neq v$	$(2n, n+1)$	n
$x = v,\ y \neq u$	$(2n, n+1)$	n
$x \in N(u) \setminus \{v\},\ y \neq u$	$(n+1, 2)$	n
$x \in N(v) \setminus \{u\},\ y \neq v$	$(n+1, 2)$	n
$x, y \notin N[u] \cup N[v]$	$(2, 2)$	$n(r-5)$

Theorem 2.3. $LM_2(\mathcal{B}) = 4n^3 + 12n^2 + 4nr - 16n$.

Proof. From the edge partition of $\mathcal{B}$ we get

$$\begin{aligned}
LM_2(\mathcal{B}) &= \sum_{xy\in E(\mathcal{B})} d_2(x)d_2(y)\\
&= d_2(u)d_2(v) + \sum_{xy,x=u,y\neq v} d_2(x)d_2(y) + \sum_{xy,x=v,y\neq u} d_2(x)d_2(y)\\
&+ \sum_{xy,x\in N(u)\setminus\{v\},y\neq u} d_2(x)d_2(y) + \sum_{xy,x\in N(v)\setminus\{u\},y\neq v} d_2(x)d_2(y)\\
&+ \sum_{xy,x,y\notin N[u]\cup N[v]} d_2(x)d_2(y)\\
&= 4n^2 + n(2n)(n+1) + n(2n)(n+1) + 2n(n+1) + 2n(n+1) + 4n(r-5)\\
&= 4n^3 + 12n^2 + 4nr - 16n.
\end{aligned}$$

□

2.2. **Cycle with parallel chords.** A cycle with parallel chords $C(n,k)$ is a graph obtained from a cycle C_n of order n as follows:
Let $v_1, v_2, \cdots, v_n$ be vertices of C_n. Join v_2 with v_n, v_3 with $v_{n-1}, \cdots, v_\alpha$ with v_β where $\alpha = \lfloor \frac{n}{2} \rfloor$ and $\beta = \lfloor \frac{n}{2} \rfloor + 2$ if n is even and $\beta = \lfloor \frac{n}{2} \rfloor + 3$ if n is odd.

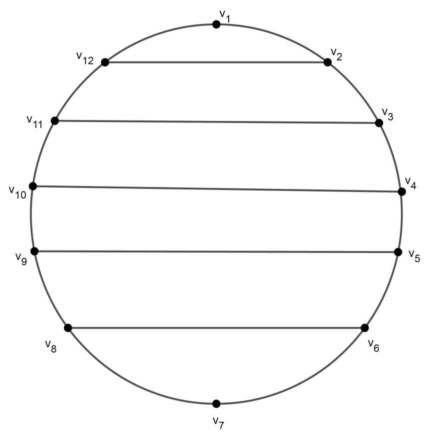

FIGURE 2. Cycle $C(12,5)$ with 5 parallel chords

Note that the number of chords in $C(n,k)$ is $k = \lfloor \frac{n}{2} \rfloor - 1$.

Theorem 2.4. $LM_1[C(n,k)] = \begin{cases} 16n - 72, & if\ n\ is\ even \\ 16n - 74, & if\ n\ is\ odd. \end{cases}$

Proof. **Case 1:** n is even.

TABLE 3. Vertex partition of $C(n,k)$ for n is even

2-degree of x	Number of vertices
2	6
4	$n-6$

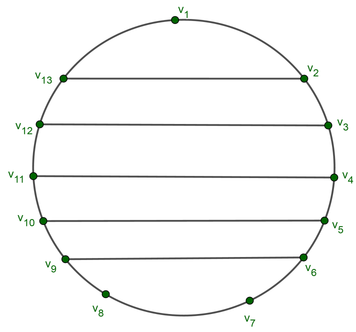

FIGURE 3. Cycle $C(13, 5)$ with 5 parallel chords

From the above vertex partition of $C(n, k)$, it is clear that $LM_1[C(n, k)] = \sum_{x \in V[C(n,k)]} d_2(x)^2 = 16n - 72$.
Case 2: n is odd

TABLE 4. Vertex partition of $C(n, k)$ for n is odd

2-degree of x	Number of vertices
2	5
3	2
4	$n - 7$

In view of the above vertex partition of $C(n, k)$ with respect to 2-degree, we get $LM_1[C(n, k)] = \sum_{x \in V[C(n,k)]} d_2(x)^2 = 16n - 74$.

□

Theorem 2.5. $LM_3[C(n, k)] = \begin{cases} 12n - 40, \; if \; n \; is \; even \\ 12n - 42, \; if \; n \; is \; odd \end{cases}$

Proof. **Case 1:** n is even.

TABLE 5. Vertex partition of $C(n, k)$ based on 2-degree and degree

2-degree and degree of $x \in V[C(n, k)]$	Number of vertices
$(2, 2)$	2
$(2, 3)$	4
$(4, 3)$	$n - 6$

$LM_3[C(n,k)] = \sum\limits_{x\in V[C(n,k)]} d_2(x)deg(x) = 12n - 40$ which follows trivially from the above vertex partition of $C(n,k)$.
Case 2: n is odd

TABLE 6. Vertex partition of $C(n,k)$ based on 2-degree and degree

2-degree and degree of x	Number of vertices
$(2,2)$	3
$(2,3)$	2
$(3,3)$	2
$(4,3)$	$n-7$

$LM_3[C(n,k)] = 12n - 42$ which follows from the above vertex partition of $C(n,k)$. □

Theorem 2.6. $LM_2[C(n,k)] = \begin{cases} 16n + 16\lfloor\frac{n}{2}\rfloor - 80, \ if\ n\ is\ evem \\ 16n + 16\lfloor\frac{n}{2}\rfloor - 115, \ if\ n\ is\ odd. \end{cases}$

Proof. **Case 1:** n is even.

TABLE 7. Edge partition of $C(n,k)$

Edge xy with $(d_2(x), d_2(y))$	Number of edges
$(2,2)$	6
$(2,4)$	4
$(4,4)$	$n + \lfloor\frac{n}{2}\rfloor - 11$

$$\begin{aligned} LM_2[C(n,k)] &= \sum_{xy\in E[C(n,k)]} d_2(x)d_2(y) \\ &= 6(4) + 4(8) + 16(n + \lfloor\frac{n}{2}\rfloor - 11) \\ &= 16n + 16\lfloor\frac{n}{2}\rfloor - 80. \end{aligned}$$

Case 2: n is odd.

From the edge partition of $C(n,k)$ we have

$$\begin{aligned} LM_2[C(n,k)] &= 4(4) + 2(6) + 2(8) + 1(9) + 2(12) + 16(n + \lfloor\frac{n}{2}\rfloor - 12) \\ &= 16(n + \lfloor\frac{n}{2}\rfloor) - 115. \end{aligned}$$

□

TABLE 8. Edge partition of $C(n, k)$

Edge xy with $(d_2(x), d_2(y))$	Number of edges
$(2, 2)$	4
$(2, 3)$	2
$(2, 4)$	2
$(3, 3)$	1
$(3, 4)$	2
$(4, 4)$	$n + \lfloor \frac{n}{2} \rfloor - 12$

2.3. **Cycle with parallel P_k-chords.** A cycle with parallel P_k-chords, denoted by $C(n, P_k)$ is a graph obtained from a cycle $C_n : v_1, v_2, \cdots, v_n$, $n \geq 6$, by adding disjoint paths P_k of order k between the pairs of vertices (v_2, v_n), (v_3, v_{n-1}), $\cdots$, (v_α, v_β) of C_n where $\alpha = \lfloor \frac{n}{2} \rfloor$, $\beta = \lfloor \frac{n}{2} \rfloor + 2$ if n is even and $\beta = \lfloor \frac{n}{2} \rfloor + 3$ if n is odd.

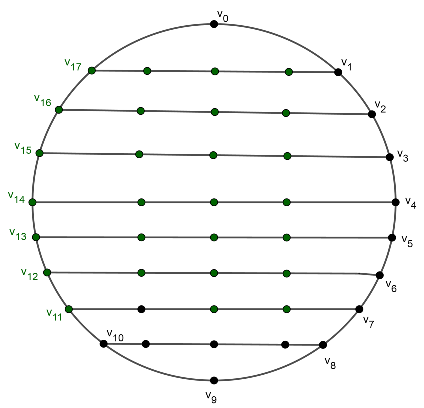

FIGURE 4. Cycle $C(18, P_5)$ with parallel P_5 chords

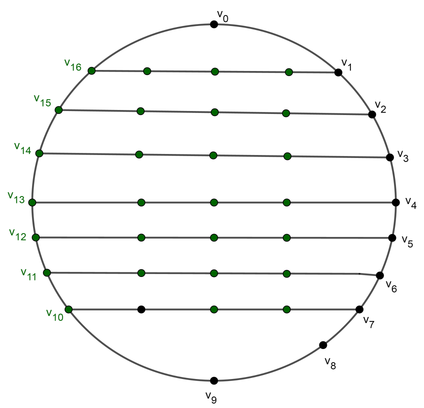

FIGURE 5. Cycle $C(17, P_5)$ with parallel P_5 chords

TABLE 9. Vertex partition of $C(n, P_k)$ with respect to 2-degree

$d_2(x)$	Number of vertices
2	$(\lfloor \frac{n}{2} \rfloor - 1)(k-4)$
3	$2(\lfloor \frac{n}{2} \rfloor - 1)$
4	6
5	$n-6$

Theorem 2.7. $LM_1[C(n,P_k)] = \begin{cases} 4k(\lfloor \frac{n}{2} \rfloor - 1) + 2\lfloor \frac{n}{2} \rfloor + 25n - 72, \; if\ n\ is\ even \\ 4k(\lfloor \frac{n}{2} \rfloor - 1) + 2\lfloor \frac{n}{2} \rfloor + 25n - 79, \; if\ n\ is\ odd. \end{cases}$

Proof. **Case 1:** n is even.

$$\begin{aligned} LM_1[C(n,P_k)] &= \sum_{x \in V[C(n,P_k)]} d_2(x)^2 \\ &= 4(\lfloor \frac{n}{2} \rfloor - 1)(k-4) + 18(\lfloor \frac{n}{2} \rfloor - 1) + 16(6) + 25(n-6) \\ &= 4k(\lfloor \frac{n}{2} \rfloor - 1) + 2\lfloor \frac{n}{2} \rfloor + 25n - 72. \end{aligned}$$

Case 2: n is odd.

TABLE 10. Vertex partition of $C(n, P_k)$

$d_2(x)$	Number of vertices
2	$(\lfloor \frac{n}{2} \rfloor - 1)(k-4)$
3	$2(\lfloor \frac{n}{2} \rfloor - 1) + 2 = 2\lfloor \frac{n}{2} \rfloor$
4	5
5	$n-7$

$$\begin{aligned} LM_1[C(n,P_k)] &= \sum_{x \in V[C(n,P_k)]} d_2(x)^2 \\ &= 4(\lfloor \frac{n}{2} - 1)(k-4) + 18\lfloor \frac{n}{2} \rfloor + 16(5) + 25(n-7) \\ &= 4k(\lfloor \frac{n}{2} \rfloor - 1)) + 2\lfloor \frac{n}{2} \rfloor + 25n - 79. \end{aligned}$$

□

Theorem 2.8. $LM_3[C(n,P_k)] = \begin{cases} 4k(\lfloor \frac{n}{2} \rfloor - 1) - 4\lfloor \frac{n}{2} \rfloor + 15n - 22, \; if\ n\ is\ even \\ 4k(\lfloor \frac{n}{2} \rfloor - 1) - 4\lfloor \frac{n}{2} \rfloor + 15n - 33, \; if\ n\ is\ odd \end{cases}$

Proof. **Case 1:** n is even.

TABLE 11. Vertex partition of $C(n, P_k)$ with respect to deg and 2-deg

Vertex x with $(deg(x), d_2(x))$	Number of vertices
$(2, 2)$	$(\lfloor \frac{n}{2} \rfloor - 1)(k - 4)$
$(2, 3)$	$2(\lfloor \frac{n}{2} \rfloor - 1)$
$(2, 4)$	2
$(3, 4)$	4
$(3, 5)$	$n - 6$

$$\begin{aligned} LM_3[C(n, P_k)] &= \sum_{x \in V[C(n,P_k)]} d_2(x)deg(x) \\ &= 4(\lfloor \frac{n}{2} \rfloor - 1)(k - 4) + 12(\lfloor \frac{n}{2} \rfloor - 1) + 16 + 48 + 15(n - 6) \\ &= 4k(\lfloor \frac{n}{2} \rfloor - 1) - 4\lfloor \frac{n}{2} \rfloor + 15n - 22. \end{aligned}$$

Case 2: n is odd.

TABLE 12. Vertex partition of $C(n, P_k)$ when n is odd

Vertex x with $(deg(x), d_2(x))$	Number of vertices
$(2, 2)$	$(\lfloor \frac{n}{2} \rfloor - 1)(k - 4)$
$(2, 3)$	$2(\lfloor \frac{n}{2} \rfloor - 1) + 2 = 2\lfloor \frac{n}{2} \rfloor$
$(2, 4)$	1
$(3, 4)$	4
$(3, 5)$	$n - 7$

$$\begin{aligned} LM_3[C(n, P_k)] &= \sum_{x \in V[C(n,P_k)]} d_2(x)deg(x) \\ &= (4k - 16)(\lfloor \frac{n}{2} \rfloor - 1) + 12\lfloor \frac{n}{2} \rfloor + 8 + 48 + 15(n - 7) \\ &= 4k(\lfloor \frac{n}{2} \rfloor - 1) - 4\lfloor \frac{n}{2} \rfloor + 15n - 33 \end{aligned}$$

□

Theorem 2.9. $LM_2[C(n, P_k)] = \begin{cases} 4k(\lfloor \frac{n}{2} \rfloor - 1) + 22\lfloor \frac{n}{2} \rfloor + 25n - 90, if\ n\ is\ even \\ 4k(\lfloor \frac{n}{2} \rfloor - 1) + 22\lfloor \frac{n}{2} \rfloor - 114 + 25n,\ if\ n\ is\ odd. \end{cases}$

Proof. **Case 1:** n is even.
Edge partition of $C(n, P_k)$ with respect to 2-degree is as follows:

TABLE 13. Edge partition of $C(n, P_k)$

Edge xy with $(d_2(x), d_2(y))$	Number of edges
$(2,3)$	$2(\lfloor\frac{n}{2}\rfloor - 1)$
$(2,2)$	$(k-5)(\lfloor\frac{n}{2}\rfloor - 1)$
$(3,4)$	4
$(3,5)$	$2(\lfloor\frac{n}{2}\rfloor - 3)$
$(4,4)$	4
$(4,5)$	4
$(5,5)$	$n-8$

$$
\begin{aligned}
LM_2[C(n,P_k)] &= \sum_{xy\in E[C(n,P_k)]} d_2(x)d_2(y) \\
&= 12(\lfloor\frac{n}{2}\rfloor - 1) + 4(k-5)(\lfloor\frac{n}{2}\rfloor - 1)) \\
&\quad +4(12) + 30(\lfloor\frac{n}{2}\rfloor - 3) + 64 + 80 + 25(n-8) \\
&= 4k(\lfloor\frac{n}{2}\rfloor - 1) + 22\lfloor\frac{n}{2}\rfloor + 25n - 90.
\end{aligned}
$$

Case 2: n is odd

TABLE 14. Edge partition of $C(n, P_k)$

Edge xy with $(d_2(x), d_2(y))$	Number of edges
$(2,2)$	$(k-5)(\lfloor\frac{n}{2}\rfloor - 1)$
$(2,3)$	$2(\lfloor\frac{n}{2}\rfloor - 1))$
$(3,3)$	1
$(3,4)$	6
$(3,5)$	$2(\lfloor\frac{n}{2}\rfloor - 3))$
$(4,4)$	2
$(4,5)$	4
$(5,5)$	$n-9$

$$
\begin{aligned}
LM_2[C(n,P_k)] &= 4(k-5)(\lfloor\frac{n}{2}\rfloor - 1) + 12(\lfloor\frac{n}{2}\rfloor - 1) + 9 + 72 \\
&\quad +30(\lfloor\frac{n}{2}\rfloor - 3) + 32 + 80 + 25(n-9) \\
&= 4k(\lfloor\frac{n}{2}\rfloor - 1) + 22\lfloor\frac{n}{2}\rfloor + 25n - 114.
\end{aligned}
$$

□

2.4. **Shell graph.** The shell graph $S(n,k)$ represents a cycle C_n with k chords sharing a common end vertex called an apex.

Theorem 2.10. *For a shell graph $S(n, n-3)$,*

(i) $LM_1[S(n,n-3)] = n^3 - 9n^2 + 28n - 30$

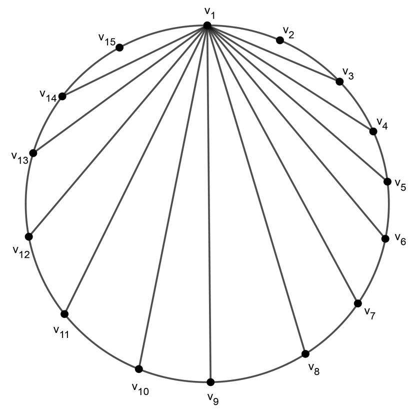

FIGURE 6. Shell graph $S(15, 12)$ with apex v_1

(ii) $LM_3[S(n, n-3)] = 3n^2 - 17n + 24$
(iii) $LM_2[S(n, n-3)] = n^3 - 10n^2 + 34n - 40.$

Proof. First, we observe the following vertex partition of the shell graph $S(n, n-3)$ with respect to 2-degree as well as the degree of every vertex in $S(n, n-3)$.

TABLE 15. Vertex partition of $S(n, n-3)$

2-degree of x	degree of x	Number of vertices
0	$n-1$	1
$n-4$	3	$n-3$
$n-3$	2	2

$$\begin{aligned} LM_1[S(n, n-3)] &= \sum_{x \in V[S(n,n-3)]} d_2(x)^2 \\ &= 2(n-3)^2 + (n-3)(n-4)^2 \\ &= n^3 - 9n^2 + 28n - 30. \end{aligned}$$

$$\begin{aligned} LM_3[S(n, n-3)] &= \sum_{x \in V[S(n,n-3)]} d_2(x)deg(x) \\ &= 0(n-1) + 4(n-3) + 3(n-3)(n-4) \\ &= 3n^2 - 17n + 24. \end{aligned}$$

In order to obtain the result for the second leap Zagreb index, we form the following edge partition table.

TABLE 16. Edge partition of $S(n, n-3)$

Edge xy with $(d_2(x), d_2(y))$	Number of edges
$(0, n-3)$	2
$(0, n-4)$	$n-3$
$(n-3, n-4)$	2
$(n-4, n-4)$	$n-4$

$$\begin{aligned} LM_2[S(n, n-3)] &= \sum_{xy \in E[S(n,n-3)]} d_2(x)d_2(y) \\ &= 2(0) + (n-3)(0) + 2(n-3)(n-4) + (n-4)(n-4)^2 \\ &= n^3 - 10n^2 + 34n - 40. \end{aligned}$$

□

2.5. **Shell type graph** $Sh(n, k)$**.** $Sh(n, k)$ is a shell type graph formed using a cycle C_n of order n in which $n-3$ copies of a path P_k of order $k \geq 4$ share a common end vertex called apex.

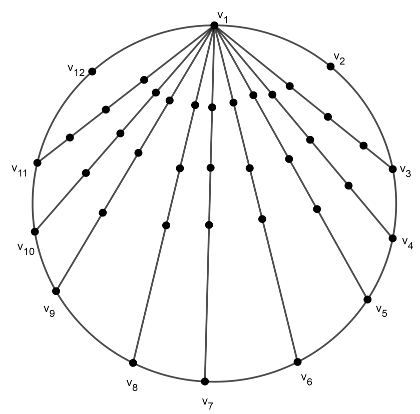

FIGURE 7. Shell type graph $Sh(12, 5)$ with apex v_1

Observe that if we replace the chords of the shell graph $S(n, n-3)$ by $n-3$ copies of a path of order k, we get the shell type graph $Sh(n, k)$.

Theorem 2.11. *For a shell type graph $Sh(n, k)$,*

(i) $LM_1[Sh(n, k)] = n^3 - 2n^2 + 23n + 4nk - 12k - 74$
(ii) $LM_3[Sh(n, k)] = 3n^2 - n + 4nk - 12k - 14$
(iii) $LM_2[Sh(n, k)] = n^3 - n^2 + 39n + 4nk - 12k - 122.$

Proof. First we observe the following vertex and edge partitions of $Sh(n, k)$.

TABLE 17. Vertex partition of $Sh(n,k)$ with respect to 2-degree

2-degree of x	Number of vertices
n	2
$n-1$	$n-2$
5	$n-5$
4	2
3	$n-3$
2	$(n-3)(k-4)$

TABLE 18. Vertex partition of $Sh(n,k)$ with respect to 2-degree and degree

Vertex x with $(d_2(x), deg(x))$	Number of vertices
$(n,2)$	2
$(n-1,n-1)$	1
$(n-1,2)$	$n-3$
$(5,3)$	$n-5$
$(4,3)$	2
$(3,2)$	$n-3$
$(2,2)$	$(n-3)(k-5)$

TABLE 19. Edge partition of $Sh(n,k)$

Edge xy with $(d_2(x), d_2(y))$	Number of edges
$(n-1,n)$	2
$(n-1,n-1)$	$n-3$
$(n,4)$	2
$(n-1,2)$	$n-3$
$(4,5)$	2
$(5,5)$	$n-6$
$(3,4)$	2
$(3,5)$	$n-5$
$(2,3)$	$n-3$
$(2,2)$	$(n-3)(k-3)$

(i) Using Table 17 we get

$$\begin{aligned} LM_1[Sh(n,k)] &= \sum_{x\in V[Sh(n,k)]} d_2(x)^2 \\ &= 2n^2 + (n-2)(n-1)^2 + 25(n-5) + 2(16) + 9(n-3) + 4(n-3)(k-4) \\ &= n^3 - 2n^2 + 23n + 4nk - 12k - 74. \end{aligned}$$

(ii) From Table 19 we have

$$\begin{aligned} LM_3[Sh(n,k)] &= \sum_{x \in V[Sh(n,k)]} d_2(x) deg(x) \\ &= 2(2n) + (n-1)^2 + 2(n-1)(n-3) + 15(n-5) + 24 \\ &\quad + 6(n-3) + 4(n-3)(k-4) \\ &= 3n^2 - n + 4nk - 12k - 14. \end{aligned}$$

(iii) From Table 18 we have

$$\begin{aligned} LM_2[Sh(n,k)] &= \sum_{xy \in E[Sh(n,k)]} d_2(x) d_2(y) \\ &= 2n(n-1) + (n-3)(n-1)^2 + 8n + 2(n-1)(n-3) + 40 + 25(n-6) + 24 \\ &\quad + 15(n-5) + 6(n-3) + 4(n-3)(k-3) \\ &= n^3 - n^2 + 39n + 4nk - 12k - 122. \end{aligned}$$

□

3. Conclusion

The distance-based topological indices called Leap Zagreb indices have been studied for some cycle related graphs and shell type graphs. These new topological indices need to be explored for several chemical compounds as well as a variety of graph operations. Thereby, paving the way for new insights in future.

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