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ROUGH POROUS CIRCULAR PLATES LUBRICATED WITH COUPLE STRESS FLUID AND PRESSURE DEPENDENT VISCOSITY

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Abstract. In this paper, the effect of PDV on the couple stress squeeze film lubrication between rough porous circular plates is presented. Keeping the base of Christensen's stochastic theory, modified Reynolds equation is derived. Fluid film pressure, squeeze film time and load carrying capacity are solved using standard perturbation technique. The results are presented graphically for selected physical parameters and found that the squeeze effect is depleted in a porous bearing compared to its non-porous and increasing permeability has an adverse effect on the pressure, load carrying capacity and time of approach.

Keywords: couple stress, porous, squeeze film, pressure dependent viscosity.

AMS Subject Classification: 76D08, 76A05, 35K41, 49L25.

1. Introduction

The squeeze film fluids wonder is seen in a few applications, for example, car motors, machine devices and moving components. The squeeze film emerges when the two rubbing surfaces move towards one another the typical way and produces a positive weight and henceforth supports a load. This is because of the way that a viscous lubricant present between the two surfaces can't be momentarily crushed or squeezed out when the two surfaces move towards one another and this activity gives a impact in direction. Also self lubricating permeability is broadly utilized in industry because of their independent oil supply notwithstanding their minimal effort and different viewpoints worried about oil component. Permeable orientations are broadly utilized in brakes, grips, and so forth because of their independent oil supply and good low grating qualities.

In earlier stages Pinkus and Sternlicht [1], Cameron [6], and Hamrock [8] have studied the squeeze film bearings with Newtonian type of lubrication. With the advancement of present day machine gear, the expanding utilization of fluids containing microstructures, for example, added substances, suspensions, and since quite a while ago polymers; for

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instance, the length of the polymer chain might be a million times the distance across of water particle which has got extraordinary consideration in ongoing year. Because of this exceptional structure of the grease liquid and also the alternate added substances, the Newtonian fluid approximation (which dismisses the span of liquid particles) is anything but not acceptable for engineering approach. Numerous uses of microcontinuum hypotheses have been created by Ariman et al., [1, 2] for depicting the particular conduct of fluids containing substructures, which can interpret, pivot, or even distort autonomously. Among them, the Stokes microcontinuum hypothesis [23] is the least difficult hypothesis that considers couple stresses, for example, the nearness of couple stresses and body couples. A number of studies have applied the Stokes microcontinuum theory to investigate the effect of couple stresses on the performance of different types of fluid-film bearings. The behaviour of the squeeze film along with couple stress between different types of plates were analyzed by Ramanaih [20]. Bujurke and Jayaraman [4] anticipated the attributes in a crush film design with reference to the synovial joint. Lin et al. [14] studied the pure squeeze film behaviour of long partial journal bearings with couple stress fluids under dynamic loading. Naduvinamani et al. [16, 17] have investigated the impacts of couple stresses on the static and dynamic conduct of the squeeze film of porous journal which are narrow in nature.

Reddy et al.,[21], Lu et al., [15], Lin et al.,[12, 13], Hanumagowda [9], all of them have investigated on variation of viscosity over the squeeze film performance of narrow journal bearing, sphere-plate squeeze-film system, wide parallel-plate squeeze-film, parallel circular plates and Circular Step Plates. Recently Hanumagowda et al.,[10], Vasanth et al.,[24], Vasanth et al.,[25] have studied the effects of PDV on porous circular stepped plates, porous annular plates and porous circular plates. They found that the load carrying capacity is increased by the cause of PDV and also the squeeze film time also increases.

In all the above mentioned papers it has been observed that the work is limited only for the smooth surfaces. But practically even the rough surface is very important. Keeping this in mind few researchers have made an attempt to know the effect of PDV on the rough surfaces. Relatively, Bujurke et al.,[5] made an analysis of porous elastic bearings which have rough surfaces for synovial joints. Also some of the studies on roughness and squeeze film is investigated by few researchers [22] and [18]. Recently Hanumagowda et al.,[11] studied the effects of rough surface and PDV between circular plates.

Motivated by these investigations and their applications, in the present article, the authors examine the combined effects of surface roughness on the couple stress squeeze film lubrication between circular stepped plates by considering the pressure dependent viscosity (PDV) variation. The results are analysed for different values of the physical parameters on the pressure, load bearing capacity, surface roughness and squeezing film time. The obtained numerical results for a special case are found to be in good agreement with those of the results available in the literature. Further, the results obtained reveal many interesting behaviours that warrant further study of the equations related to non-Newtonian couple stress fluid phenomena in the presence of pressure dependent viscosity and surface roughness.

2. Mathematical Formulation

Consider a squeezing film flow between two rough circular plates approaching each other with squeezing velocity $v = (dH/dt)$, where H is the film thickness between the two plates. The geometry and coordinates of the problem is shown in Figure 1. The lubricant in the film region is considered to be an incompressible Stokes [23] couple-stress fluid. It is assumed that the fluid inertia, body forces and body couples are negligible and the viscosity μ varies with pressure in the following analysis.

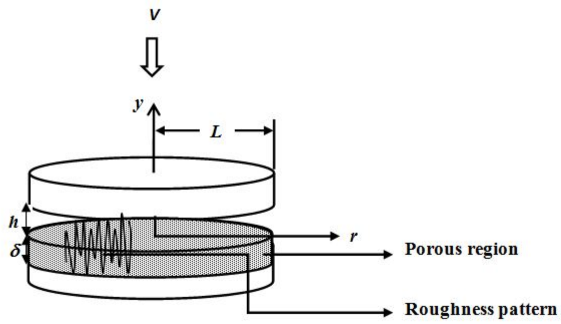

FIGURE 1. Schematic representation of squeeze film rough porous circular system

The basic equations of motion for the couple stress fluid flow with viscosity variation in the film region are given by

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial v}{\partial r} = 0 \tag{1}$$

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \eta\frac{\partial^4 u}{\partial y^4} = \frac{\partial p}{\partial r} \tag{2}$$

$$\frac{\partial p}{\partial y} = 0 \tag{3}$$

In which the components of velocities are u and v considered in r and y direction respectively, p represents the pressure in the region, η is the materialistic constant. The ratio μ/η has the dimension of length squared and hence characterizes the material length of the fluid.

The flow of the couple stress fluid in the porous matrix is governed by the modified form of the Darcy's law for the porous material and is given by

$$u^* = \frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial r} \tag{4}$$

$$v^* = \frac{-k}{\mu(1-\beta)}\frac{\partial p^*}{\partial y} \tag{5}$$

The relevant boundary conditions are: At upper surface y = H:

$$u = 0, \frac{\partial^2 u}{\partial y^2} = 0, v = -\frac{\partial H}{\partial t} \tag{6}$$

At lower surface y = 0:

$$u = 0, \frac{\partial^2 u}{\partial y^2} = 0, v = -v^* \tag{7}$$

Solving the equation (1) with boundary conditions (6) and (7) we obtain

$$u = -\frac{1}{2\mu}\frac{\partial p}{\partial r}\left[y^2 - yH + 2l^2 - \left[1 - \frac{cosh(\frac{2y-H}{2l})}{cosh\frac{H}{2l}}\right]\right] \tag{8}$$

where $l = \sqrt{\eta/\mu_0}$ is the constant parameter

Substituting the expression for u in the continuity equation (3) and integrating using the conditions (6), (7), (4), (5), yields

$$\frac{\partial}{\partial r}\left[\frac{r}{\mu}\left\{H^3-12l^2H+24l^3tanh\left(\frac{h}{2l}\right)+\frac{12\delta k}{(1-\beta)}\right\}\frac{\partial p}{\partial r}\right] \tag{9}$$

The pressure dependent viscosity relation was analysed by Barus [3] and the expression is obtained as

$$\mu=\mu_0 e^{\alpha p} \tag{10}$$

Substituting equation (10) into equation (9), the modified Reynold's equation governing PDV for porous circular plates lubricated with non - Newtonian fluid is expressed as

$$\frac{\partial}{\partial r}\left[A(H,m,\alpha,p)\frac{\partial p}{\partial r}\right]=12r\mu_0\frac{dH}{dt} \tag{11}$$

Where,

$$A(H,m,\alpha,p)=H^3e^{-\alpha p}-12m^2He^{-2\alpha p}+24m^3e^{-2.5\alpha p}\tanh\left(\frac{He^{\alpha p/2}}{2m}\right)+\frac{12k\delta e^{-\alpha p}}{(1-\beta)}$$

and,

$$l=\left(\frac{\eta}{\mu}\right)^{1/2}=\left(\frac{\eta}{\mu_0 e^{\alpha p}}\right)^{1/2}=me^{-\alpha p/2};m=\left(\frac{\eta}{\mu_0}\right)^{1/2}$$

To mathematically model the surface roughness, the fluid film thickness is considered to be made up of two parts

$$H=h+h_s(r,\theta,\xi) \tag{12}$$

Let $f(h_s)$ be the probability density function of the stochastic film thickness h_s. Taking the stochastic average of modified Reynolds equation (11) with respect to $f(h_s)$, the stochastic modified Reynolds equation is obtained in the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{E(A(H,m,\alpha,p))r\frac{\partial E(p)}{\partial r}\right\}=12\mu_0\frac{dH}{dt} \tag{13}$$

where,

$$E(*)=\int_{-\infty}^{\infty}(*)f(h_s)dh_s \tag{14}$$

For most of the lubricating surfaces, the Gaussian distribution for describing the roughness profile heights is valid up to at least three standard deviations. Following Christensen [7], the roughness distribution function is assumed in the form

$$f(h_s)=\begin{cases}\frac{35}{32c^7}(c^2-h_s^2)^3 & -c<h_s<c\\ 0 & elsewhere\end{cases} \tag{15}$$

where $c=3\sigma$ and σ is the standard deviation

In the context of Christensen's stochastic theory for the hydrodynamic lubrication of rough surfaces, two types of one dimensional roughness patterns are considered viz., the radial roughness pattern and the azimuthal roughness pattern.

Radial Roughness

The one dimensional radial roughness pattern has the form of long, narrow ridges and valleys running in the radial direction (i.e. they are straight ridges and valley passing through $z = 0, r = 0$ to form star pattern), in this case the film thickness takes the form

$$H = h + h_s(\theta, \xi) \tag{16}$$

and the average modified Reynolds equation (11) takes the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{E(A(H,m,\alpha,p))r\frac{\partial E(p)}{\partial r}\right\} = 12\mu_0\frac{dH}{dt} \tag{17}$$

Azimuthal Roughness

The one dimensional azimuthal roughness pattern on the bearing surface has the roughness structure in the form of long narrow ridges and valleys running in θ - direction (i.e. they are circular ridges and valleys on the flat plate that are concentric on $z = 0, r = 0$. In this case the film thickness assumes the form

$$H = h + h_s(r, \xi) \tag{18}$$

and the averaged modified Reynolds equation (11) takes the form

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{\frac{1}{E\left(\frac{1}{A(H,m,\alpha,p)}\right)}r\frac{\partial E(p)}{\partial r}\right\} = 12\mu_0\frac{dH}{dt} \tag{19}$$

Equations (17) and (19) together can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left\{g(H,m,\alpha,p)r\frac{\partial E(p)}{\partial r}\right\} = 12\mu_0\frac{dH}{dt} \tag{20}$$

where

$$g(H,m,\alpha,p) = \begin{cases} E\{A(H,m,\alpha,p)\} & \text{for radial roughness} \\ E[1/A(H,m,\alpha,p)]^{-1} & \text{for azimuthal roughness} \end{cases}$$

using non dimensional quantities

$$r^* = \frac{r}{L}, m^* = \frac{m}{h_0}, H^* = \frac{h+h_s}{h_0}, h_s^* = \frac{h_s}{h_0}, h^* = \frac{h}{h_0}, \psi = \frac{k\delta}{h_0^3}$$
$$p^* = \frac{E(p)h_0^3}{\mu_0 L^2(-dH/dt)}, G = \frac{\alpha\mu_0 L^2(-dH/dt)}{h_0^3}$$

The modified Reynold's equation in non dimensional form is obtained as

$$\frac{\partial}{\partial r^*}\left[g^*(H^*, m^*, G, p^*, \psi)\right] = -12r^* \tag{21}$$

where

$$g^*(H^*, m^*, G, p^*, \psi) = \begin{cases} E\{A(H^*, m^*, G, p^*, \psi)\} & \text{for radial roughness} \\ E[1/A(H^*, m^*, G, p^*, \psi)]^{-1} & \text{for azimuthal roughness} \end{cases}$$

$$A^*(H^*, m^*, G, p^*, \psi) = H^{*3}e^{-Gp^*} - 12m^{*2}H^*e^{-2Gp^*} + 24m^{*3}e^{-2.5Gp^*}\tanh\left(\frac{H^*e^{Gp^*/2}}{2m^*}\right) + \frac{12\psi e^{-Gp^*}}{(1-\beta)}$$

The non-dimensional Reynolds equation (20) is highly non-linear. To obtain the first order analytical solution for small values of the viscosity parameter $0 \leq G \leq 1$, a small perturbation method for the film pressure is adopted by putting

$$p^* = p_0^* + Gp_1^* \tag{22}$$

into the Reynolds equation (20) and neglecting second and higher order of G, we get the following two equations responsible for pressure p_0^* and p_1^*

$$\frac{\partial}{\partial r^*}\left\{r^*\frac{dp_0^*}{dr^*}\right\} = \frac{-12r^*}{g_0^*(H^*, m^*, \psi)} \tag{23}$$

$$\frac{\partial}{\partial r^*}\left\{r^*\frac{dp_1^*}{dr^*}\right\} = \frac{g_1^*(H^*, m^*, \psi)}{g_0^*(H^*, m^*, \psi)}\frac{\partial}{\partial r^*}\left\{p_0^* r^*\frac{\partial p_0^*}{\partial r^*}\right\} \tag{24}$$

The solution of p_0^* and p_1^* are obtained by solving equations (23) and (24) as

$$p_o^* = \frac{3(1-r^{*2})}{g_0^*(H^*, m^*, \psi)} \tag{25}$$

$$p_1^* = -\frac{9}{2}\frac{g_1^*(H^*, m^*, \psi)}{g_0^{*3}(H^*, m^*, \psi)}[1-2r^{*2}+r^{*4}] \tag{26}$$

Substituting (25) and (26) in the equation (22) the dimensionless pressure is obtained as

$$p^* = \frac{3(1-r^{*2})}{g_0^*(H^*, m^*, \psi)} - G\frac{9}{2}\frac{g_1^*(H^*, m^*, \psi)}{g_0^{*3}(H^*, m^*, \psi)}[1-2r^{*2}+r^{*4}] \tag{27}$$

The load W^* on the bearing can be obtained using the formula

$$W^* = 2\pi\int_0^1 p^* r^* dr^* \tag{28}$$

After performing the integration, the load on the bearing in dimensionless form can be obtained as

$$W^* = \frac{3\pi}{2}\{\frac{g_0^{*2}(H^*, m^*, \psi) - Gg_1^*(H^*, m^*, \psi)}{g_0^{*3}(H^*, m^*, \psi)}\} \tag{29}$$

The squeeze film time t can be obtained by integrating W^* with respect to h^* with the limits as $h^* = 1$ at $t = 0$ as follows

$$t^* = \frac{3\pi}{2}\int_{h^*}^1\{\frac{g_0^{*2}(H^*, m^*, \psi) - Gg_1^*(H^*, m^*, \psi)}{g_0^{*3}(H^*, m^*, \psi)}\}dh^* \tag{30}$$

Where,

$$g_i(H^*, m^*, \psi) = \begin{cases} E\{A_i(H^*, m^*, \psi)\} & \text{for radial roughness} \\ E[1/A_i(H^*, m^*, \psi)]^{-1} & \text{for azimuthal roughness}\end{cases}$$

$$A_0^*(H^*, m^*, \psi) = H^{*3} - 12m^{*2}H^* + 24m^{*3}\tanh\left(\frac{H^*}{2m^*}\right) + \frac{12\psi}{(1-\beta)}$$

$$A_1^*(H^*, m^*, \psi) = H^{*3} + 6m^{*2}H^*(4 + sech^2(H^*/2m^*)) - 60m^{*3}\tanh\left(\frac{H^*}{2m^*}\right) - \frac{12\psi}{(1-\beta)^2}$$

3. Results and discussions

In the presence of two rough patterns the effects of surface roughness and viscosity pressure dependent on couple stress squeeze film between circular plates with porous along them are analyzed. The impact of couple stresses coming about because of the expansion of different added substances to the Newtonian oil is recognized by the couple pressure parameter K. Based on the Stokes couple stress fluid hypothesis, the new material steady η is in charge of the non-Newtonian property of couple stresses. From the Barus equation for isothermal thickness weight reliance of fluids, the impact of variety of consistency with pressure is appeared by thickness parameter V. The roughness parameter C^* means the impact of surface unpleasantness coming about because of the vast vertical deviations of a smooth surface. On the basis of Christensen stochastic theory for couple stress the effect of PDV and surface roughness over porous circular plates is analysed. The squeeze film characteristics are taken over the dimensionless parameters like P^*, W^* and T^* as the functions of C, G and m^*. The changes in the two types of rough patterns and the viscosity phenomena the numerical outcomes are shown in figures 2 to 10.

Non dimensional Pressure

Figure 2 - 4 demonstrates the variety of dimensionless P^* alongside r^*(radial axes) for various estimations of rough parameter C and PDV G and couple-stress parameter m^* by keeping h^* settled. The graphs are plotted for two rough patterns namely radial and azimuthal roughness. From figure 2 it can be seen that the pressure is decreased for different values of rough parameter C in radial case and also pressure is increased in case of azimuthal case. This is on the grounds that, on account of azimuthal unpleasantness design, the harshness striations are as edges and valleys running in the θ-direction which hinders the stream of fluids, while on account of the radial unpleasantness design, the unpleasantness striations are as edges and valleys running in the r direction by which the fluids can escape very easily. Also from Figure 3 and 4 it can be viewed that the dimensionless pressure increases with increasing values of G and m^*. Also from figures 2, 3 and 4 it can be observed that the pressure will be maximum at $r^* = 0$.

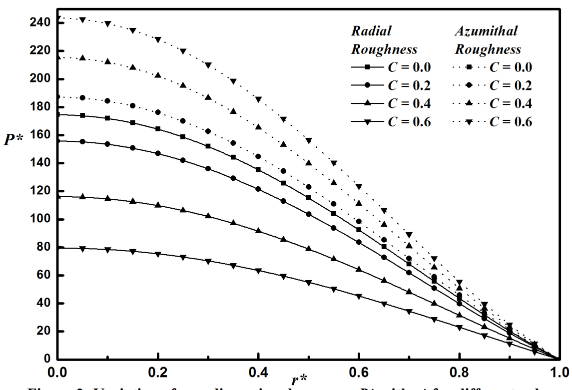

Figure 2: Variation of non-dimensional pressure P^* with r^* for different values of C with $h^* = 0.6$, $G = 0.04$, $m^* = 0.6$, $\psi = 0.001$

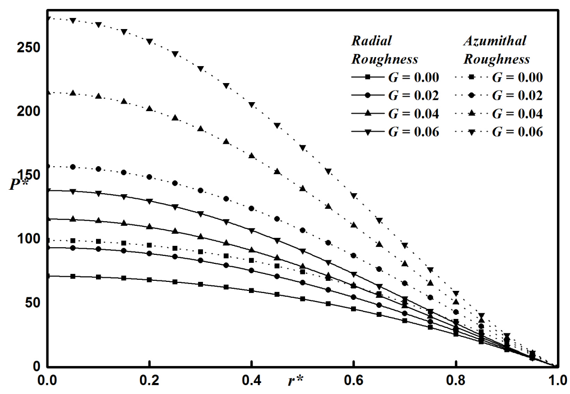

Figure 3: Variation of non-dimensional pressure P^* with r^* for different values of G with $h^* = 0.6$, $C = 0.4$, $m^* = 0.6$, $\psi = 0.001$

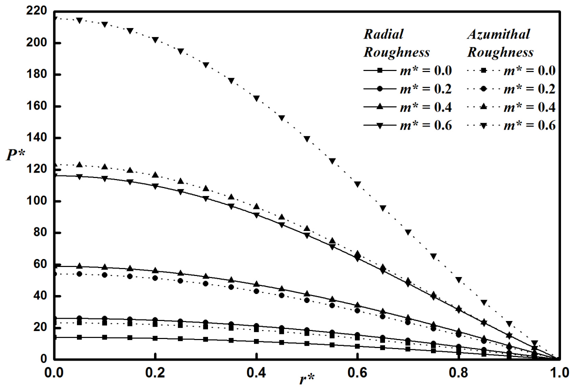

Figure 4: Variation of non-dimensional pressure P^* with r^* for different values of m^* with $h^* = 0.6$, $C = 0.4$, $G = 0.04$, $\psi = 0.001$

Non Dimensional Load carrying capacity

The variation of dimensionless load carrying capacity W^* along with film thickness h^* with different values for C, G and m^* is drawn in figures 5, 6 and 7 for both types of roughness pattern. Figure 6 and 7 are plotted for different values of couplestress parameter and PDV by keeping roughness $C = 0.4$ as fixed. And it is observed that as there is increase in the values of couple stress m^* and pressure dependency G the load carrying capacity increases in both radial and azimuthal rough cases. Figure 5 shows the variation of load with film thickness with $G = 0.04$ and $m^* = 0.6$ by varying the roughness parameter. It is interesting to note that the effect of azimuthal (radial) roughness patterns is to increase (decrease) W^* as compared to the corresponding to smooth case i.e. $(C = 0)$. At $C = 0.03, 0.06, 0.09$ (i.e., with the vertical deviations being large) it is observed that the increase (decrease) in W^* is more pronounced for azimuthal (radial) roughness pattern. The large amount of load is delivered for azimuthal roughness pattern as compared to the radial roughness pattern.

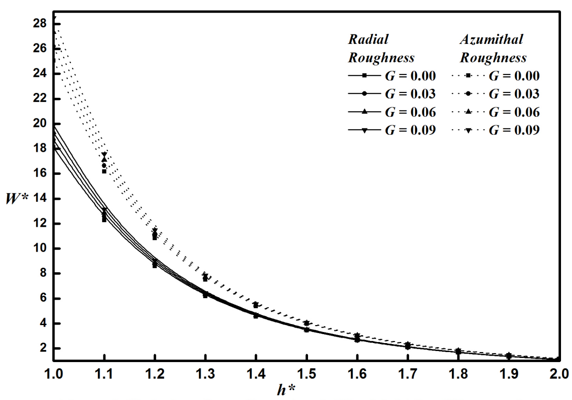

Figure 6: Variation of non-dimensional W^* with h^* for different values of G with , $C = 0.4$, $m^* = 0.6$, $\psi = 0.001$

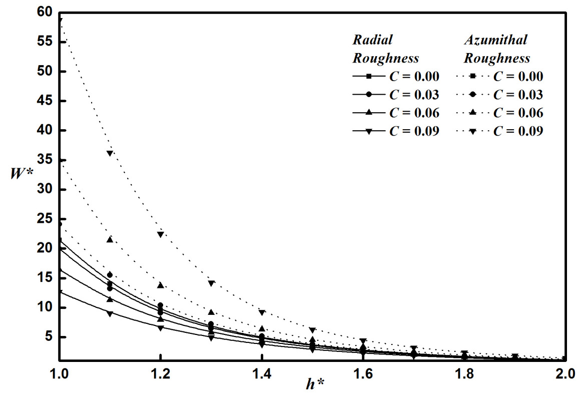

Figure 5: Variation of non-dimensional W^* with h^* for different values of C with, $G = 0.04$, $m^* = 0.6$, $\psi = 0.001$

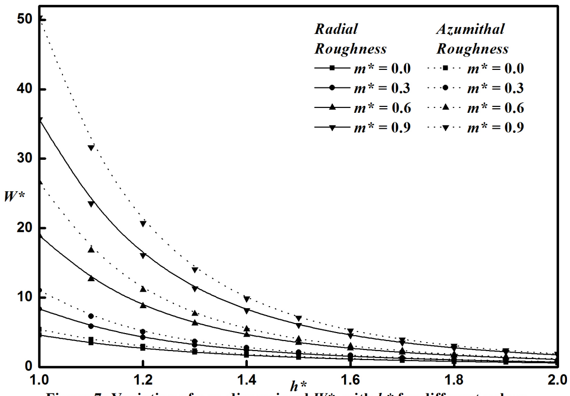

Figure 7: Variation of non-dimensional W^* with h^* for different values of m^* with $C = 0.4$, $G = 0.04$, $\psi = 0.001$

Time height relationship

Figures 8- 10 shows the variation of dimensionless squeeze time along with h_f^* for different values of C, G and m^*. It has been observed that as the value of h_f^* increases the dimensionless squeeze film time T^* decreases. Also it can be viewed that as the values of m^* and G increases the non dimensional squeeze film time T^* increases. Further it can be observed that the dimensionless squeeze film time T^* is more in case of azumithal roughness than radial roughness. It is discovered that because of the nearness of surface roughness, as the height of the vertical deviations of surface expands the time response T^* likewise increments. This expansion is more for azimuthal roughness designs than the radial unpleasantness design.

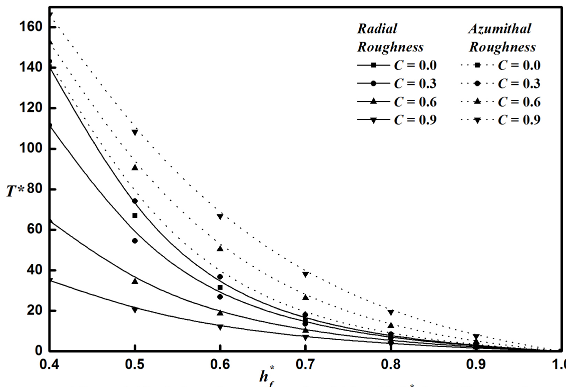

Figure 8: Variation of non-dimensional T^* with h_f^* for different values of C with, $G = 0.04$, $K = 0.6$, $\psi = 0.001$

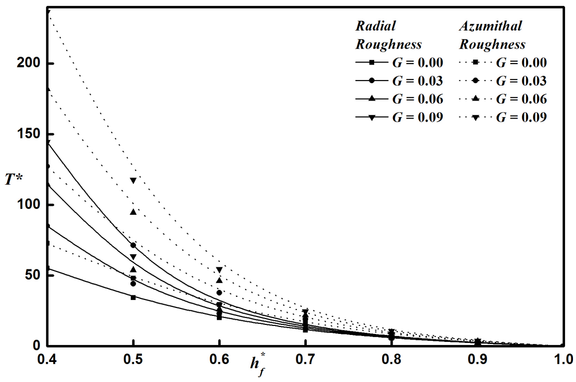

Figure 9: Variation of non-dimensional T^* with h_f^* for different values of G with, $C = 0.4$, $m^* = 0.6$, $\psi = 0.001$

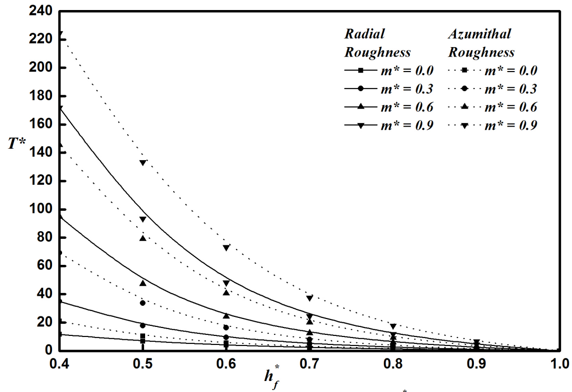

Figure 10: Variation of non-dimensional T^* with h_f^* for different values of K with $C = 0.4$, $G = 0.04$, $\psi = 0.001$

4. Conclusions

On the basis of stokes micro continumm theory the effect of surface roughness and viscosity pressure dependent on couple stress squeeze film between circular plates with porous along them are studied. Based on the theoretical results presented above, the following conclusions can be observed.

(1) The effect of viscosity variation parameter is to enhance the load carrying capacity and response time.
(2) The effect of non-Newtonian couple stress fluid is to increase the load carrying capacity and to lengthen the squeeze film time as compared to the corresponding Newtonian case.
(3) The presence of azimuthal surface roughness pattern on the bearing surface improves the squeeze film characteristics whereas the performance of the squeeze film suffers due to the presence of radial roughness pattern.

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