TWMS J. App. and Eng. Math. V.12, N.4, 2022, pp. 1189-1201

LINEAR QUADRATIC REGULATOR APPLIED TO A MAGNETORHEOLOGICAL DAMPER AIMING ATTENUATE VIBRATION IN AN AUTOMOTIVE SUSPENSION

L. G. P. ROEFERO[1], F. R. CHAVARETTE[2*], L. N. MISHRA[3], §

Abstract. Automotive suspension is a mechanical device used in automobiles to attenuate vibrations, which are caused by the undulation of the floor. This device can be modeled by a mass-spring-damper system, and the damper used in most real situations is a viscous medium damper, which dissipates energy passively. In this work, a 1/4 model of automotive suspension was analyzed when the passive shock absorber is exchanged for a magnetorheological shock absorber, whose control current is determined by a law of Optimal Linear Control. Based on the analyzes made in terms of displacement and acceleration, more satisfactory results were obtained in the system containing the AMR, in contrast to the results obtained with the presence of the passive damper.

Keywords: Automotive Suspension, LQR Control, Magnetorheological Damper, Vibration.

AMS Subject Classification: 83-02, 99A00.

1. Introduction

In 1886 the first automobiles powered by the internal combustion engine appeared, whose merit is given to Karl Benz (1844-1929) and Gottlieb Daimler (1834-1900). After a few years, in 1908, the automobile industry was already established in the United States thanks to the founding of Ford and General Motors Corporation [10].

Within this concept of an automotive system, it is necessary that the structural integrity of the designed device is maintained for the longest possible time of operation. Thus, it is common to find in automobiles, several types of vibration filters that protect the physical

[1] Univ. Estadual Paulista, Departamento de Engenharia Mecânica, Faculdade de Engenharia de Ilha Solteira, Av. Brasil Sul, 56, Centro, Ilha Solteira/SP - CEP 15385-000, Sao Paulo, Brasil.
e-mail: luiz.roefero@unesp.br; ORCID: https://orcid.org/0000-0001-9194-8208.

[2] Univ. Estadual Paulista, Departamento de Engenharia, Física e Matemática, Instituto de Química, UNESP, Campus de Araraquara, Rua Prof. Francisco Degni, 55, Quitandinha, Araraquara, SP, CEP 14800-060, Sao Paulo, Brasil.
e-mail: fabio.chavarette@unesp.br; ORCID: https://orcid.org/0000-0002-1203-7586.

[*] Corresponding author.

[3] Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, 632 014, Tamil Nadu, India.
e-mail: lakshminarayanmishra04@gmail.com; ORCID: https://orcid.org/0000-0001-7774-7290.

§ Manuscript received: July 20, 2020; accepted: October 20, 2020.

structure, both from the excitation promoted by the internal combustion engine and from stimuli given by the environment, such as an irregularity in the terrain to be covered [1].

A vibration filter with great importance in the design of a car is the suspension, which acts to preventing external stimulus reaches, abruptly, the rest of the structure. If there were no such mechanism, the entire project would be exposed to a lot of damage caused by very uneven terrain. Thus, automobile suspensions are equipment that seek to promote a certain isolation between the structure of the car and the variations in amplitude of the path traveled. Without this equipment, all vibrations resulting from the excitation of the vehicle's tire would be transferred entirely to it, causing damage to the structure and may even generate undesirable failures [10].

Another primary reason for the use of automobile suspensions is the comfort of its passengers, as vibrations in addition to bothering buyers of an automotive vehicle, can generate serious long-term injuries to occupants of the car [1].

Most automotive suspensions today are composed of passive actuation elements, usually springs and shock absorbers coupled in order to dissipate the energy from the elevations on the ground. Whereas, active control systems have the ability to adapt to different loading conditions and control the vibration modes of a structure, thus minimizing the effects of vibration on the same [12].

The control methods use a reference signal for the controller and an output error, so that adjusting the control signal seeks to reduce this error already mentioned. Some active noise control techniques use this strategy through filters [2].

Active suspensions were already used in Formula 1 in the periods from the late 1980s to the early 1990s, being banned from the competition for promoting an excessive performance gain to the vehicle and not being financially accessible to all teams [4]. But it already has its return predicted by the championship board precisely for the reduction of costs in the development of interconnected passive suspensions.

However, there are other control methods that allow the stabilization of an automotive suspension and are characterized as semi-active methods [5]. Among these types of control methods, magnetorheological dampers have been studied for a long time, which feature the ability to change their physical properties according to the magnetic field generated by a current that is supplied to the system [14].

Another feature that makes the magnetorheological damper widely used is its ability to control systems [5]. This property prevents the structure in which it is applied from performing unstable or chaotic behaviors [7].

This work aims to study the vibration attenuation in a 1/4 model of automotive suspension, using a magnetorheological damper whose performance will be regulated by a law of Optimal Linear Control. Unlike current models in the literature, this work will replace the passive damper element of the suspension by the MR damper and will study this damper as the actuator for the control law.

2. Automotive Suspension 1/4 Model

The studied suspension models are shown in Figure 1 [8].

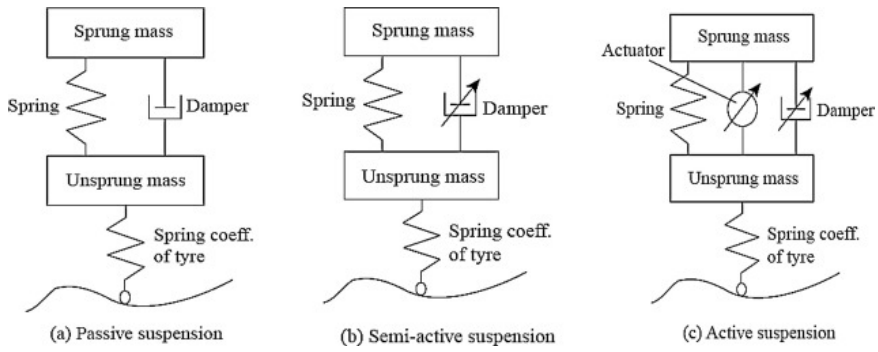

Figure 1. Automotive suspension model

Although the modeling studied to obtain the force that will be performed by the magnetorheological damper is based on an active suspension model, the control mode using the MR damper is called the semi-active method. This classification is given because an active control model is used to change the characteristics of the fluid, present in a shock absorber that performs the control passively.

2.1. Equation of the Passive Model.

Using the principles of Newton's Second Law and the concepts of Action and Reaction of Forces, we have the Equation (1).

$$\begin{gathered} m_s\ddot{z}_s + k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u) = 0 \\ m_u\ddot{z}_u - k_s(z_s - z_u) - c_s(\dot{z}_s - \dot{z}_u) + k_t(z_u - z_r) = 0 \end{gathered} \tag{1}$$

Performing the following variable change to reduce the order of the system:

$$\begin{gathered} x_1 = z_s(t) \\ x_2 = \dot{z}_s(t) \\ x_3 = z_u(t) \\ x_4 = \dot{z}_u(t) \\ w = z_r(t) \end{gathered} \tag{2}$$

We now have the following system:

$$\begin{gathered} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{-k_s(x_1 - x_3) - c_s(x_2 - x_4)}{m_s} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{k_s(x_1 - x_3) + c_s(x_2 - x_4) - k_t(x_3 - w)}{m_u} \end{gathered} \tag{3}$$

So we have a system of four first order linear equations that can be approximated using the 4th Order Runge Kutta Method.

It is possible rewrite the system as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-k_s}{m_s} & \dfrac{-c_s}{m_s} & \dfrac{k_s}{m_s} & \dfrac{c_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & \dfrac{c_s}{m_u} & \dfrac{-k_s-k_t}{m_u} & \dfrac{-c_s}{m_u} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{k_t}{m_u} \end{bmatrix} \cdot w \tag{4}$$

So we have a system of equations in the form:

$$\dot{x} = A \cdot x + B_w \cdot w \tag{5}$$

Where A is said to be a Jacobian matrix.

2.2. **Equation of the Active Model.** The method used in this section is identical to that used in the previous section, but this time there is the force performed by the active control.

As in the previous section, using the principles of Newton's Second Law and the concepts of Action and Reaction of Forces, we arrive at the Equation (6).

$$\begin{gathered} m_s\ddot{z}_s + k_s(z_s - z_u) + F_a = 0 \\ m_u\ddot{z}_u - k_s(z_s - z_u) + k_t(z_u - z_r) - F_a = 0 \end{gathered} \tag{6}$$

Performing the following variable change described in Equation (2) to reduce the order of the system, we now have the following system:

$$\begin{gathered} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{-k_s(x_1 - x_3) - F_a}{m_s} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{k_s(x_1 - x_3) - k_t(x_3 - w) + F_a}{m_u} \end{gathered} \tag{7}$$

So we have a system of four first order linear equations that can be approximated numerically using the 4th Order Runge Kutta Method.

You can rewrite the system as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-k_s}{m_s} & 0 & \dfrac{k_s}{m_s} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & 0 & \dfrac{-k_s-k_t}{m_u} & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{k_t}{m_u} \end{bmatrix} \cdot w + \begin{bmatrix} 0 \\ \dfrac{-1}{m_s} \\ 0 \\ \dfrac{1}{m_u} \end{bmatrix} \cdot F_a \tag{8}$$

So we have a system of equations in the form:

$$\dot{x} = A \cdot x + B_w \cdot w + B \cdot F_a \tag{9}$$

3. Design of Control Problem

A controlled system given by (*symbolic vector arrows has been removed in order to avoid visual pollution*) [11]:

$$\dot{x} = Ax + g(x) + U \tag{10}$$

where $x \in \mathbb{R}^n$ is a state vector, $A \in \mathbb{R}^{nxn}$ is a constant matrix, $g(x)$ is a vector consisting of functions and U is the control vector that can be defined as [11]:

$$U = \tilde{u} + u_t \tag{11}$$

In various engineering problems, the U control law is chosen such that the system is moved from an uncontrolled/chaotic situation to an equilibrium point or to a desired periodic or non-periodic/chaotic orbit [11]. The function vector $\tilde{u}$ is defined as the part that describes the intended trajectory, thus, the portion $\tilde{u}$ of the control vector U that keeps the system in control state in desired trajectory is given by [3]:

$$\tilde{u} = \dot{\tilde{x}} - A\tilde{x} - g(\tilde{x}) \tag{12}$$

The u_t control vector that stabilizes the system around the desired path is defined as:

$$u_t = Bu \tag{13}$$

where $B \in \mathbb{R}^{nxm}$ is a constant matrix.

Defining the deviation from the desired trajectory of the system of Equation (10) as being:

$$y = x - \tilde{x} \tag{14}$$

With Equations (10), (13) and (14), we can write that:

$$\dot{y} = Ay + g(x) - g(\tilde{x}) + Bu \tag{15}$$

The nonlinear portion of the system in Equation (16) is written as:

$$g(x) - g(\tilde{x}) = G(x, \tilde{x})(x - \tilde{x}) \tag{16}$$

where $G(x, \tilde{x})$ is a limited matrix, whose elements are dependent on x and $\tilde{x}$. Substituting Equation (17) in Equation (16), we arrive at:

$$\dot{y} = Ay + G(x, \tilde{x})y + Bu \tag{17}$$

Thus, the theorem stated below is formulated. [11]:

Theorem 3.1. *Assuming the existence of the defined positive R and Q matrices, Q being a symmetric matrix, such that the matrix $\tilde{Q}$ given by:*

$$\tilde{Q} = Q - G^T(x, \tilde{x})P - PG(x, \tilde{x}) \tag{18}$$

is defined positive for the limited G function. The feedback control defined by:

$$y(\infty) = 0 \tag{19}$$

with minimizing the functional

$$\tilde{J} = \int_0^\infty (y^T\tilde{Q}y + u^TRu)dt \tag{20}$$

in which the symmetric matrix P is obtained using Riccati's nonlinear algebraic equation given by [15]*:*

$$PA + A^TP - PBR^{-1}B^TP + Q = 0 \tag{21}$$

where the matrices $Q \in R^{nxn}$ and $R \in R^{nxm}$ are constant and positive defined [3].

3.1. **Application of the Proposed Control.** According to what was developed in section 3, in parallel with Equation (9), we have:

$$\dot{x} = A \cdot x + g(x) + U = A \cdot x + B_w \cdot w + B \cdot F_a \tag{22}$$

Thus, comparing both sides of the equation and using the comparison with Equation (11), we have:

$$\begin{gathered} g(x) = 0 \\ U = \tilde{u} + u_t = B_w \cdot w + B \cdot F_a \end{gathered} \tag{23}$$

Thus:

$$\begin{gathered} u_t = B \cdot u = B \cdot F_a \\ u = F_a \end{gathered} \tag{24}$$

Then we have the information that the force exerted by the damper must equal the value of u. This result associates the Law of Optimal Linear Control with the MR damper.

$$\tilde{u} = B_w \cdot w \tag{25}$$

Then, as the $\tilde{u}$ portion of the control vector describes the intended trajectory, we can say that the intended trajectory for the automotive suspension is intrinsically related to the relief of the w track, as intuitively expected.

Thus, executing the following substitutions, which are in accordance with Equation (14), we have:

$$\begin{gathered} y_1 = x_1 - \tilde{x}_1 \\ y_2 = x_2 - \tilde{x}_2 \\ y_3 = x_3 - \tilde{x}_3 \\ y_4 = x_4 - \tilde{x}_4 \end{gathered} \tag{26}$$

Thus, we arrive at a system in the form:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-k_s}{m_s} & 0 & \dfrac{k_s}{m_s} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{m_u} & 0 & \dfrac{-k_s - k_t}{m_u} & 0 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dfrac{k_t}{m_u} \end{bmatrix} \cdot w + \begin{bmatrix} 0 \\ \dfrac{-1}{m_s} \\ 0 \\ \dfrac{1}{m_u} \end{bmatrix} \cdot F_a \tag{27}$$

Simplifying:

$$\dot{y} = A \cdot y + B_w \cdot w + B \cdot F_a \tag{28}$$

Equation (29) is analogous to Equation (15) and, based on Equations (25) and (26), leads to the conclusion that:

$$G(x, \tilde{x}) = 0 \tag{29}$$

Thus, according to Equation (18), we have to:

$$\tilde{Q} = Q \tag{30}$$

Thus, as we have to $\tilde{Q} = Q$, it is enough to define the positive Q and R matrices for the system to be asymptotically stable [3].

So we have all the information so that the Riccati Equation, shown in Equation (22), can be solved. We have:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_s}{m_s} & 0 & \frac{k_s}{m_s} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & 0 & \frac{-k_s-k_t}{m_u} & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ \frac{-1}{m_s} \\ 0 \\ \frac{1}{m_u} \end{bmatrix}; Q = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & q_4 \end{bmatrix}; R = [r_1] \quad (31)$$

Thus, using the lqr (A, B, Q, R) function of the software free OCTAVE, one can obtain the gain matrix K, given by:

$$K = R^{-1}B^T P \quad (32)$$

Thus, replacing Equation (33) in Equation (19), we have

$$u = -Ky \quad (33)$$

Thus, we have the following equation already controlled:

$$\begin{gathered} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{-k_s(x_1 - x_3) + Ky}{m_s} \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = \frac{k_s(x_1 - x_3) - k_t(x_3 - w) - Ky}{m_u} \end{gathered} \quad (34)$$

On what:

$$K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}; \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} x_1 - \tilde{x}_1 \\ x_2 - \tilde{x}_2 \\ x_3 - \tilde{x}_3 \\ x_4 - \tilde{x}_4 \end{bmatrix} \quad (35)$$

4. Magnetorheological Damper

Fluids called magnetorheologicals have a characteristic that differs from conventional fluids, when this substance is subjected to a magnetic field, the particles present in it start a microscopic alignment [9]. Such alignment results in altering the material characteristics on a macroscopic scale, making it more rigid the larger the [9] field. This field, to which the fluid is subjected, can change its behavior in such a way that the material starts to behave as solid [9].

This phenomenon occurs due to the fact that about 30% of the volume of the material is composed of magnetizable particles and the force applied on the fluid that makes it flow, due to the alignment of these particles, the material will present a greater difficulty in the flow resulting from the increased viscosity [9].

Among the systems that model a magnetorheological damper, the Bouc-Wen model is numerically treatable and has been widely used in the scientific community for faithfully simulating a system with hysteresis [6]. This system can show a wide variation of the same [6].

The model is shown in Figure 2.

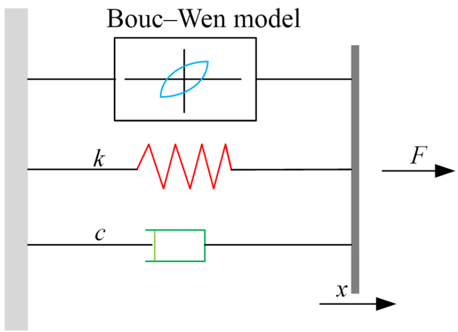

FIGURE 2. Bouc-Wen Model

In this system, the damper's force is given by:

$$F = c_0\dot{x} + k_0(x - x_0) + \alpha z \tag{36}$$

And the variable z is given by:

$$\dot{z} = -\lambda|\dot{x}|z|z|^{n-1} - \gamma\dot{x}|z|^n + \beta\dot{x} \tag{37}$$

- c_0 is the viscous damping coefficient;
- k_0 is elastic stiffness;
- x_0 is the initial displacement;
- x is the dependent variable;
- α is the stiffness coefficient;
- λ, γ, β and n are parameters that depend on the characteristics of the damper;
- f_0 is the initial spring force;
- α and c_0 depend on the electrical voltage applied to the coil that causes the magnetic field

For the calculation of the $alpha$ and c_0 coefficients, the following equations are proposed.

$$\alpha(u) = \alpha_a + \alpha_b u \tag{38}$$

$$c_0(u) = c_{0a} + c_{0b} u \tag{39}$$

$$\dot{u} = -\eta(u - v) \tag{40}$$

It is possible to write, as shown in the literature [13], the force performed by the damper as a function of the current supplied to the damper. The equation is as follows:

$$F = \frac{3.2}{(3e^{-3.4i}) + 1}\dot{x} + k_0 x + \frac{8.5}{(1.28e^{-3.9i}) + 1}z \tag{41}$$

5. Results and discussion

Based on the theory developed in the previous chapter, it is necessary to define the suspension parameters to perform the simulation calculations [9].

TABLE 1. Parameters used in the simulations

m_s	m_u	k_s	k_t	c_s
$372kg$	$45kg$	$40kN/m$	$190kN/m$	$1,3kN \cdot s/m$

The parameters used for the simulation of the Bouc-Wen model. [9].

TABLE 2. Parameters used in MR damper simulations

λ	n	γ	β
408720	2	-360220	$634,82$

And the initial conditions for each simulation, shown in Table 3.

TABLE 3. Initial Conditions

$x_1(0)$	$x_2(0)$	$x_3(0)$	$x_4(0)$
0	0	0	0

Thus, we can define the matrices for the application of the Optimal Linear Control method.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -107,5269 & 0 & 107,5269 & 0 \\ 0 & 0 & 0 & 1 \\ 888,8889 & 0 & -5111,1111 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ -0,0027 \\ 0 \\ 0,0222 \end{bmatrix};$$
$$Q = \begin{bmatrix} 10^9 & 0 & 0 & 0 \\ 0 & 10^7 & 0 & 0 \\ 0 & 0 & 10^6 & 0 \\ 0 & 0 & 0 & 10^6 \end{bmatrix}; \qquad R = [0,25] \tag{42}$$

The $4x4$ controllability matrix, given by:

$$\begin{bmatrix} 0 & -0,0027 & 0 & 2,6785 \\ -0,0027 & 0 & 2,6785 & 0 \\ 0 & 0,0222 & 0 & -115,9697 \\ 0 & 0 & -115,9697 & 0 \end{bmatrix} \tag{43}$$

as all lines are linearly independent, the system is controllable.

We can obtain the gain matrix K:

$$K = \begin{bmatrix} -3,4833 \cdot 10^4 & -9,0281 \cdot 10^3 & 3,4586 \cdot 10^4 & 1,7500 \cdot 10^3 \end{bmatrix}; \tag{44}$$

Defining the desired trajectories as:

$$\begin{matrix} \tilde{x}_1(t) = w(t) \\ \tilde{x}_2(t) = \dot{w}(t) \\ \tilde{x}_3(t) = w(t) \\ \tilde{x}_4(t) = \dot{w}(t) \end{matrix}; \tag{45}$$

We can perform numerical simulations of the model. However, it is still necessary to define which track model will be used. Then, one model of simulation of the track was defined to carry out the analyzes, the model of the track is:

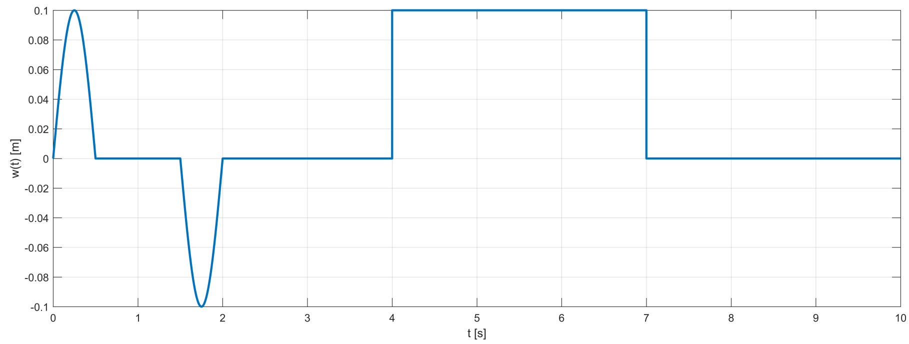

FIGURE 3. Track model

5.1. **Numerical Simulations.** The graphs resulting from the numerical simulation for the analyzed track will be shown below.

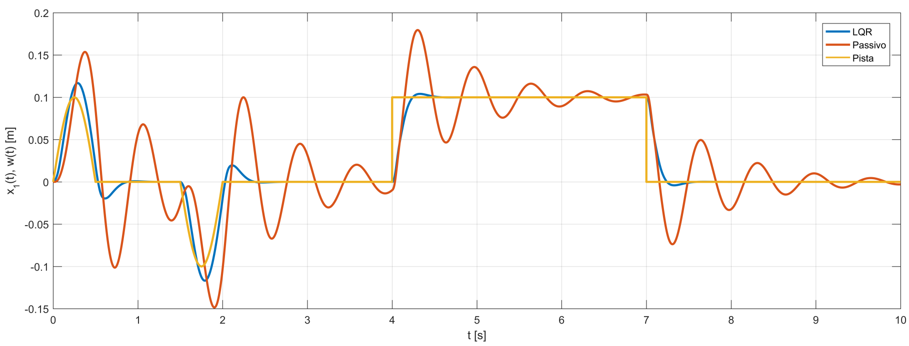

FIGURE 4. Car displacement

Looking at Figure 4, it is possible to notice that the LQR control method makes the displacement of the car not present amplitudes of vibration as large as the passive control. In addition, the control makes the car's displacement converge to the desired trajectory smoothly, without an abrupt conversion.

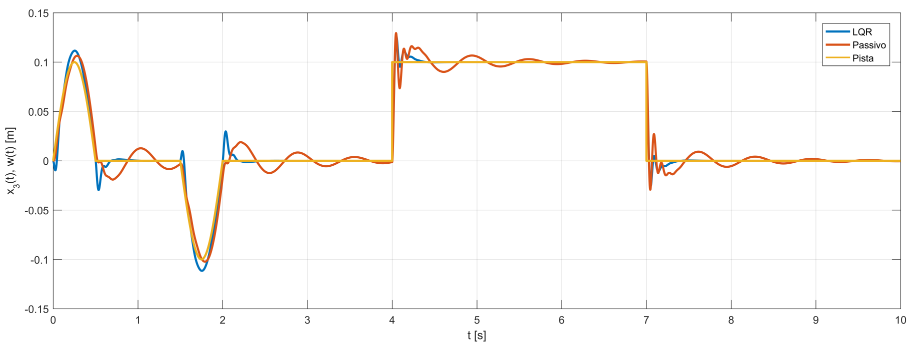

FIGURE 5. Wheel displacement

In Figure 5 it is possible to notice that the wheel converges to the track more quickly with the LQR control model, when compared to the passive model. Thus, it ensures greater control of the vehicle by the driver, preventing the wheel from being too long time without contact with the ground.

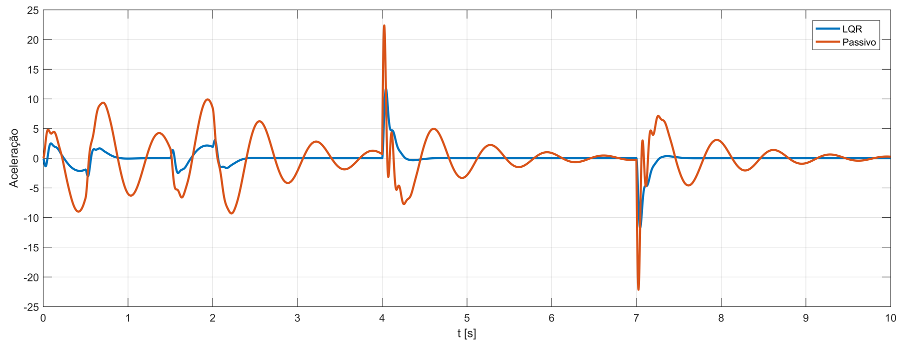

Figure 6. Car acceleration

Based on Figure 6, it is possible to notice that, in all cases of runway simulation, the accelerations resulting from the passive control are superior to those resulting from the Linear Optimal Control. Thus, in addition to taking more time for the car to cease the oscillation caused by the track (shown in the previous sections), passive control causes greater accelerations in the car, causing discomfort to passengers.

Numerically solving Equation (41), it is possible to obtain the current required in the damper for the performance of the force described in the control method.

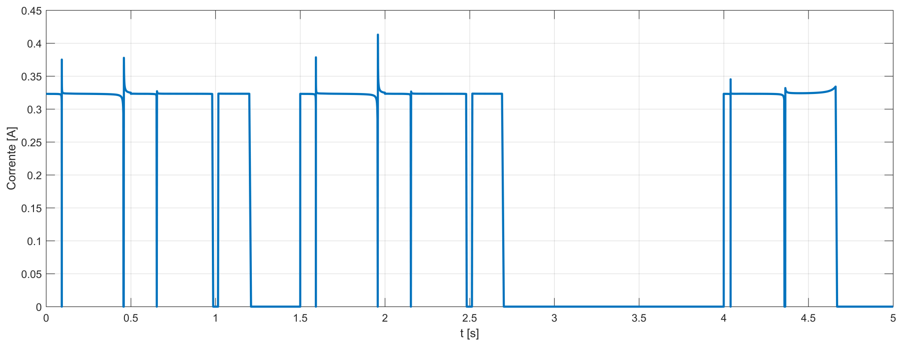

Figure 7. MR damper required current

It can be seen in the Figure 7 that the maximum current required for the performance of the MR damper does not exceed 0.45 amps. Thus, the necessary force for the effectiveness of the control is possible to be performed by the MR shock absorber without the need for very high currents, therefore, it can be supplied by conventional automotive batteries.

6. Conclusions

In this work, we proposed the analysis of the 1/4 model of automotive suspension, with the objective of evaluating the impact of the replacement of the damper conventionally used by a MR damper, whose performance would be governed by a law of Optimal Linear Control. Thus, the model was subjected to a runway model and an analysis was made of how the excitation dissipation occurred to the model studied.

Thus, it was noticed that, with respect to the tire, the Optimal Linear Control shows faster convergence to the desired trajectory on the analyzed track. However, this attenuation is more evident in the behavior of the car, in this case the LQR controller made the displacement of the car converge to the desired trajectory more quickly when compared to the viscous damper, and even so it did not need to perform accelerations higher than the passive controller, avoiding passenger discomfort.

Due to what has been described, it is clear that the use of the control law proposed by Rafikov and Balthazar in [11] is effective in attenuating the vibration of the suspension model used, showing more satisfactory results when compared to the performance of the commonly used passive controller. in the automotive industry.

Acknowledgement. The authors would like extend their gratitude to the Sao Paulo State University (UNESP) and the Complex Systems laboratory (SisPLEXOS) for the physical space provided to carry out the work.

References

[1] Alvarez-Sánchez, E., (2013), A quarter-car suspension system: car body mass estimator and sliding mode control, Procedia Technology, 7, (0), pp. 208-214

[2] Bueno, D. D., (2007), Controle ativo de vibrações e localização ótima de sensores e atuadores piezelétricos.

[3] Chavarette, F. R., (2005), Dinâmica e controle não lineares de um sistema neuronal ideal e não-ideal.

[4] Côrte-Real, E. W., (2002), Dessensibilização da estrutura de controle lqg aplicada ao modelo de uma suspensão ativa utilizando a técnica prabi.

[5] da Cunha, B. S. C. and Chavarette, F. R., (2014), Vibration control applied in a semi-active suspension using magneto rheological damper and optimal linear control design, Applied Mechanics and Materials, 464, pp. 229-234.

[6] Dyke, S. J., (1996), Acceleration feedback control strategies for active and semi-active systems: modeling, algorithm development and experimental verification, University of Notre Dame.

[7] Fiedler-Ferrara, N. and Prado, C. P. C., (1994), Caos: Uma Introdução.

[8] Omar, M., El-Kassaby, M.M. and Abdelghaffar, W., (2017), A universal suspension test rig for electrohydraulic active and passive automotive suspension system, Alexandria Engineering Journal, 56, (4), pp. 359-370.

[9] Paschoal, E. F., (2011), Controle semi-ativo de vibrações usando lógica nebulosa e fluido magnetoreológico.

[10] Picado, R. M., (1998), Controle semi-ativo em suspensões automotivas.

[11] Rafikov, M. and Balthazar, J. M., (2005), Optimal pest control problem in population dynamics, Computational & Applied Mathematics, 24, (1), pp. 65-81.

[12] Roéfero, L. G. P. and Chavarette, F. R., (2018), Controle semi-ativo aplicado a vibrações excessivas de uma suspensão automotiva, Colloquium Exactarum, 10, (3), pp. 16-22.

[13] Tusset, A. M. and Balthazar, J. M., (2013), On the chaotic suppression of both ideal and nonideal duffing based vibrating systems, using a magnetorheological damper, Differential Equations and Dynamical Systems, 21, (1-2), pp. 105-121.

[14] Tusset, A. M., Balthazar, J. M., Chavarette, F. R. and Felix, J. L. P., (2012), On energy transfer phenomena, in a nonlinear ideal and nonideal essential vibrating systems, coupled to a (MR) magnetorheological damper, Nonlinear Dynamics, 69, (4), pp. 1859-1880.

[15] Vicente, J. P. G., Chavarette, F. R. and Roefero, L. G. P., (2019), Chaos control via Mathieu-Van der Pol system and Linear Optimal Control design with a non-ideal excitation and parametric uncertainties, Revista Internacional de Métodos Numéricos para Cálculo y Diseño en Ingeniería, 35 (3).

Luiz Gustavo Pereira Roefero is currently a Mechanical Engineering student at Sao Paulo State University. He has received a scientific initiation scholarship from the Sao Paulo Research Foundation. His interest of researh is in the area of non-linear dynamics and control of mechanical systems.

Fabio Roberto Chaverette holds a degree in Computer Science from School of Engineering of Piracicaba (1996), Msc in Applied Physics from Sao Paulo State University in 2002, and a PhD in Mechanical Engineering from Universidade Estadual de Campinas in 2005. Currently, he is an associate professor in the Department of Engineering, Physics and Mathematics (DEPM) of the Institute of Chemistry (IC), of the Universidade Estadual Paulista "Julio de Mesquita Filho" (UNESP)

Lakshmi Narayan Mishra for the photography and short autobiography, see TWMS J. App. and Eng. Math. V.9, N.4.