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INTERVAL-VALUED INTUITIONISTIC FUZZY SOFT GRAPH

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Abstract. One of the theories designed to deal with uncertainty is the soft set theory. These collections were used due to a lack of membership functions in the fields of decision-making, systems analysis, classification, data mining, medical diagnosis, etc. Fuzzy graphs based on soft sets were developed alongside fuzzy graphs. Studying these graphs, examining the properties and operators on it, give special flexibility in dealing with indeterminate problems. In particular, most of the issues around us are mixed and operations are conveniently used in many combinatorial applications. Therefore, the study of operations have a significant effect on solving problems based on decision-making, medical, etc. In this paper, we introduce the notion of interval-valued intuitionistic fuzzy soft graphs, by combine concepts of interval-valued intuitionistic fuzzy graphs and fuzzy soft graphs. We also present several different types of operations including cartesian product, strong product and composition on interval-valued intuitionistic fuzzy soft graphs and investigate some properties of them.

Keywords: Interval valued intuitionistic fuzzy graph, Fuzzy soft set, Fuzzy soft graph, Interval valued intuitionistic fuzzy soft graph.

AMS Subject Classification: 056C99, 05C76.

1. Introduction

Graphs are one of the mathematical tools that express the connection between different subjects in a tangible way. Examining different topics is not always accompanied by certainty. This problem was partially solved by the introduction of fuzzy sets (FS) by Zadeh [49]. Rosenfeld's definition of fuzzy graphs provided a unique opportunity for researchers to enter the world of indeterminate subjects [35]. The researchers studied different types of fuzzy graphs. Talebi [42] had a study on Kayley fuzzy graph. Al-Masarwah and Ahmad [10, 11, 12, 13] studied BCK/BCI- algebras. Borzooei et al. [17, 18, 19, 20, 21, 33] had many studies on vague graphs. Atanassov [14] introduced the concept of intuitionistic fuzzy set (IFS) as a generalization of FS. Since the first public statement of this

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notion was made in 1983, IFS has become a popular topic of investigation in the FS community [22]. Later, Turksen [46] introduced the concept of IVFS and Atanassov and Gargov [15] introduced the concept of IVIFS, which is a generalization of the IFS. The fundamental characteristic of the IVFS and IVIFS is that the values of its membership and non-membership function are intervals rather than exact values. Mishra and Pal [29] introduced the product of IVIFGs. Akram and Davvaz [3] introduced strong intuitionistic fuzzy graphs (SIFG) and later, Mohamed Ismail and Mohamed Ali [24] studied the strong interval-valued intuitionistic fuzzy graphs. Real-world problems are embedded with uncertainties.

Molodtsov [30] initiated the theory of soft sets which proposes to handle uncertainty more effectively because of the adequacy of the tool of parameterization. Sun et al. [40] examined soft models. Fuzzy soft groups introduced by Aygunoglu [16]. Although the soft set theory is a method of dealing with uncertainties, it has been used in a variety of contexts due to the lack of membership adjustment problems. Maji [26, 27, 28] elaborated on the theory of soft sets, fuzzy soft sets, and intuitionistic fuzzy soft sets and highlighted some of their applications. Some concepts and types of fuzzy soft graph were analyzed by Al-Masarwah and Abu Qamar [8, 9]. Feng et al. [23] had a study on soft semi rings. It was later pointed out that classical soft sets are not appropriate to deal with imprecise and fuzzy parameters, on the basis of this assertion, interval-valued fuzzy soft sets and interval valued intuitionistic fuzzy soft sets were proposed by Yang et al. [47, 48] and Jiang et al. [25], respectively. Thumbakara and George [45] discussed the concept of soft graphs in the specific way. Akram and Nawaz [5] have induced the concepts of soft graphs and vertex-induced soft graphs in a broad spectrum. Measurement theory, game theory, and areas such as decision-making, medical diagnosis, forecasting, and data analysis are some of the applications of this theory. Akram and Shahzadi [7, 38, 39] had studies on intuitionistic fuzzy soft graphs. Abu Qamar and Hassan [1, 2] studied Q-neutrosophic soft set. Sarala and Deepa [36, 37] introduced regular and strong interval-valued intuitionistic fuzzy soft graph.

Moderson and Nair [31] introduced the concept of operations on fuzzy graphs, later this concept was extended by Sunitha and Vijayakumar [41]. Dudek [4] generalized some operations to interval-valued fuzzy graphs. Parvathi et al. defined operations on intuitionistic fuzzy graphs in [32]. Talebi et al. [43, 44] introduced operations on level graphs of bipolar fuzzy graphs and interval-valued intuitionistic fuzzy competition graphs. Product vague graphs was investigated by Rashmanlou [34].

In this paper, we introduce the notions of an interval-valued intuitionistic fuzzy soft graph with some related examples. We also present several different types of operations including cartesian product, strong product, and composition on interval-valued intuitionistic fuzzy soft graphs and investigate some properties of them.

2. Preliminaries

Throught this section, $U = \{x_1, \cdots, x_m\}$, $P(U)$ and $A = \{e_1, \cdots, e_n\}$ denote the initial universe, collection of all subsets of U and the set of parameters, respectively.

Definition 2.1. [30] *If F is a set-valued mapping on E taking values in $P(U)$, then a pair (F, A) is called a soft set over U, i.e. F is a mapping given by $F : A \to P(U)$.*

Definition 2.2. [26] *If $P(U)$ be a collection of all fuzzy subsets of U, then (F, A) is called fuzzy soft set, where $F : A \to P(U)$ is a mapping, called fuzzy approximate function of the fuzzy soft set (F, A).*

Definition 2.3. [5] *A fuzzy soft graph $G = (G^*, F, K, A)$ is a 4-tuple such that*

(a) $G^* = (V, E)$ is a simple graph,
(b) A is a non-empty set of parameters,
(c) (F, A) is a fuzzy soft set over V,
(d) (K, A) is a fuzzy soft set over E,
(e) $(F(a), K(a))$ is a fuzzy (sub)graph of G^* for all $a \in A$.
That is, $K(a)(xy) \leq \min\{F(a)(x), F(a)(y)\}$ for all $a \in A$ and $x, y \in V$. The fuzzy graph $(F(a), K(a))$ is denoted by $H(a)$ for convenience.

Definition 2.4. [15] *An interval valued intuitionistic fuzzy set A over an universe set U is defined as the object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in U\}$, where $\mu_A : U \to D[0, 1]$ and $\nu_A : U \to D[0, 1]$ are functions such that the condition: $\forall x \in U$, $0 \leq \sup \mu_A(x) + \sup \nu_A(x) \leq 1$ is satisfied, where $D[0, 1]$ denotes the set of all closed sub intervals of $[0, 1]$.*
The calss of all interval valued intuitionistic fuzzy sets on U is denoted by $IVIFS(U)$.

Definition 2.5. [38] *An intuitionistic fuzzy soft graph on the set V is given by ordered 4-tuple $G = (G^*, F, K, A)$ such that*
a. A is non-empty set of parameters.
b. (F, A) is an intuitionistic fuzzy soft set over V.
c. (K, A) is an intuitionistic fuzzy soft set over E.
d. $(F(e), K(e))$ is an intuitionistic fuzzy graph, for all $e \in A$. That is

$$\mu_{K(e)}(xy) \leq \min\{\mu_{F(e)}(x), \mu_{F(e)}(y)\}$$
$$\nu_{K(e)}(xy) \leq \max\{\nu_{F(e)}(x), \nu_{F(e)}(y)\}, \ for\ all\ xy \in E$$

such that $\mu_{K(e)}(xy) + \nu_{K(e)}(xy) \leq 1$, $\forall e \in A$. $x, y \in V$.

Definition 2.6. [25] *A pair (F, A) is called an interval valued intuitionistic fuzzy soft set (IVIFSS) over U, where F is a mapping given by $F : A \to IVIFS(U)$. For any parameter $a \in A$, $F(a)$ can be written as*

$$F(a) = \left\{ \left\langle x, \left[\mu^L_{F(a)}(x), \mu^U_{F(a)}(x)\right], \left[\nu^L_{F(a)}(x), \nu^U_{F(a)}(x)\right] \right\rangle \ | \ x \in U \right\}$$

3. Interval-valued intuitionistic fuzzy soft graphs (IVIFSG)

Definition 3.1. *An interval-valued intuitionistic fuzzy soft graph over the set V is given by ordered 4-tuple $G = (G^*, F, K, A)$ such that*
a. A is non-empty set of parameters.
b. (F, A) is an interval-valued intuitionistic fuzzy soft set over V.
c. (K, A) is an interval-valued intuitionistic fuzzy soft set over E.
d. $(F(e), K(e))$ is an interval-valued intuitionistic fuzzy graph for all $e \in A$. That is

$$\mu^L_{k(e)}(xy) \leq \min\{\mu^L_{F(e)}(x), \mu^L_{F(e)}(y)\}$$
$$\mu^U_{k(e)}(xy) \leq \min\{\mu^U_{F(e)}(x), \mu^U_{F(e)}(y)\}$$
$$\nu^L_{k(e)}(xy) \leq \max\{\nu^L_{F(e)}(x), \nu^L_{F(e)}(y)\}$$
$$\nu^U_{k(e)}(xy) \leq \max\{\nu^U_{F(e)}(x), \nu^U_{F(e)}(y)\}, \ for\ all\ xy \in E.$$

Throughout this paper, we denote $G^* = (V, E)$ a crisp graph, $H(e) = (F(e), K(e))$ an interval-valued intuitionistic fuzzy graph and $G = (G^*, F, K, A)$ an interval-valued intuitionistic fuzzy soft graph.

Example 3.1. *Consider a crisp graph $G^* = (V, E)$ such that $V = \{x, y, z, t\}$ and $E = \{xy, yz, xz, xt, yt, zt\}$. Let $A = \{e_1, e_2, e_3\}$ be a parameter set and (F, A) be an interval-valued intuitionistic fuzzy soft set over V with interval-valued intuitionistic fuzzy approximation function $F : A \to P(V)$ defined by*

$$F(e_1) = \{\langle x, [0.4, 0.6], [0.1, 0.3]\rangle, \langle y, [0.5, 0.6], [0.3, 0.4]\rangle, \langle z, [0.2, 0.4], [0.1, 0.2]\rangle, \langle t, [0.4, 0.5].[0.1, 0.2]\rangle\}$$
$$F(e_2) = \{\langle x, [0.5, 0.6], [0.3, 0.4]\rangle, \langle y, [0.6, 0.7], [0.1, 0.2]\rangle, \langle z, [0.5, 0.7], [0.1, 0.3]\rangle, \langle t, [0.4, 0.5].[0.3, 0.4]\rangle\}$$
$$F(e_3) = \{\langle x, [0.3, 0.4], [0.5, 0.6]\rangle, \langle y, [0.3, 0.5], [0.1, 0.2]\rangle, \langle z, [0.2, 0.3], [0.5, 0.6]\rangle, \langle t, [0.4, 0.5].[0.1, 0.2]\rangle\}$$

Let (K, A) be an interval-valued intuitionistic fuzzy soft set over E with interval-valued intuitionistic fuzzy approximation function $K : A \to P(E)$ defined by

$$K(e_1) = \{\langle xy, [0.3, 0.5], [0.2, 0.3]\rangle, \langle yz, [0.2, 0.4], [0.3, 0.4]\rangle, \langle xt, [0.3, 0.5], [0.1, 0.3]\rangle, \langle yt, [0.2, 0.4].[0.3, 0.4]\rangle\}$$
$$K(e_2) = \{\langle xy, [0.4, 0.5], [0.3, 0.4]\rangle, \langle xz, [0.3, 0.4], [0.3, 0.4]\rangle, \langle xt, [0.3, 0.5], [0.2, 0.3]\rangle, \langle zt, [0.3, 0.4].[0.4, 0.5]\rangle\}$$
$$K(e_3) = \{\langle xy, [0.2, 0.3], [0.5, 0.6]\rangle, \langle xz, [0.1, 0.3], [0.5, 0.6]\rangle, \langle xt, [0.2, 0.3], [0.4, 0.5]\rangle, \langle yt, [0.3, 0.5].[0.1, 0.2]\rangle\}$$

Thus interval-valued intuitionistic fuzzy graphs $H(e_1) = (F(e_1), K(e_1))$, $H(e_2) = (F(e_2), K(e_2))$ and $H(e_3) = (F(e_3), K(e_3))$ of G corresponding to the parameters e_1, e_2 and e_3, respectively as shown in Figure 1.

Definition 3.2. *Let $G_1 = (G^*, F_1, K_1, A)$ and $G_2 = (G^*, F_2, K_2, B)$ be two interval-valued intuitionistic fuzzy soft graphs of G^*. Then G_1 is called interval-valued intuitionistic fuzzy soft subgraph of G_2 if*
i. $A \subseteq B$
ii. $H_1(e) = (F_1(e), K_1(e))$ *is an interval-valued intuitionistic fuzzy soft subgraph of* $H_2(e) = (F_2(e), K_2(e))$ *for all* $e \in A$.

Example 3.2. *Consider the interval-valued intuitionistic fuzzy soft graph $G = (G^*, F, K, A)$ as taken in Example 3.1. Let $B = \{e_1, e_2, e_3\}$ be a set of parameters, (F', B) and (K', B) be two interval-valued intuitionistic fuzzy soft sets over V and E, respectively, defined by*

$$F'(e_1) = \{\langle x, [0.1, 0.3], [0.2, 0.3]\rangle, \langle y, [0.2, 0.4], [0.4, 0.6]\rangle, \langle z, [0.3, 0.4], [0.4, 0.5]\rangle, \langle t, [0.2, 0.3], [0.3, 0.4]\rangle\}$$
$$F'(e_2) = \{\langle x, [0.4, 0.5], [0.2, 0.3]\rangle, \langle z, [0.2, 0.3], [0.5, 0.6]\rangle, \langle t, [0.3, 0.4], [0.1, 0.2]\rangle\}$$
$$F'(e_3) = \{\langle x, [0.2, 0.4], [0.1, 0.2]\rangle, \langle y, [0.4, 0.6], [0.2, 0.3]\rangle, \langle z, [0.3, 0.5], [0.2, 0.4]\rangle, \langle t, [0.2, 0.5], [0.1, 0.3]\rangle\}$$

$$K'(e_1) = \{\langle xy, [0.1, 0.3], [0.4, 0.6]\rangle, \langle yz, [0.1, 0.3], [0.4, 0.6]\rangle, \langle xt, [0.1, 0.2], [0.3, 0.4]\rangle\}$$
$$K'(e_2) = \{\langle xz, [0.1, 0.2], [0.5, 0.6]\rangle, \langle xt, [0.2, 0.3], [0.2, 0.3]\rangle, \langle tz, [0.2, 0.3], [0.5, 0.6]\rangle\}$$
$$K'(e_3) = \{\langle xz, [0.1, 0.3], [0.2, 0.3]\rangle, \langle yt, [0.1, 0.4], [0.2, 0.3]\rangle\}$$

It is clearly seen that $H'(e_1) = (F'(e_1), K'(e_1))$, $H'(e_2) = (F'(e_2), K'(e_2))$ and $H'(e_3) = (F'(e_3), K'(e_3))$ are interval-valued intuitionistic fuzzy graphs corresponding to the parameters e_1, e_2 and e_3, respectively.

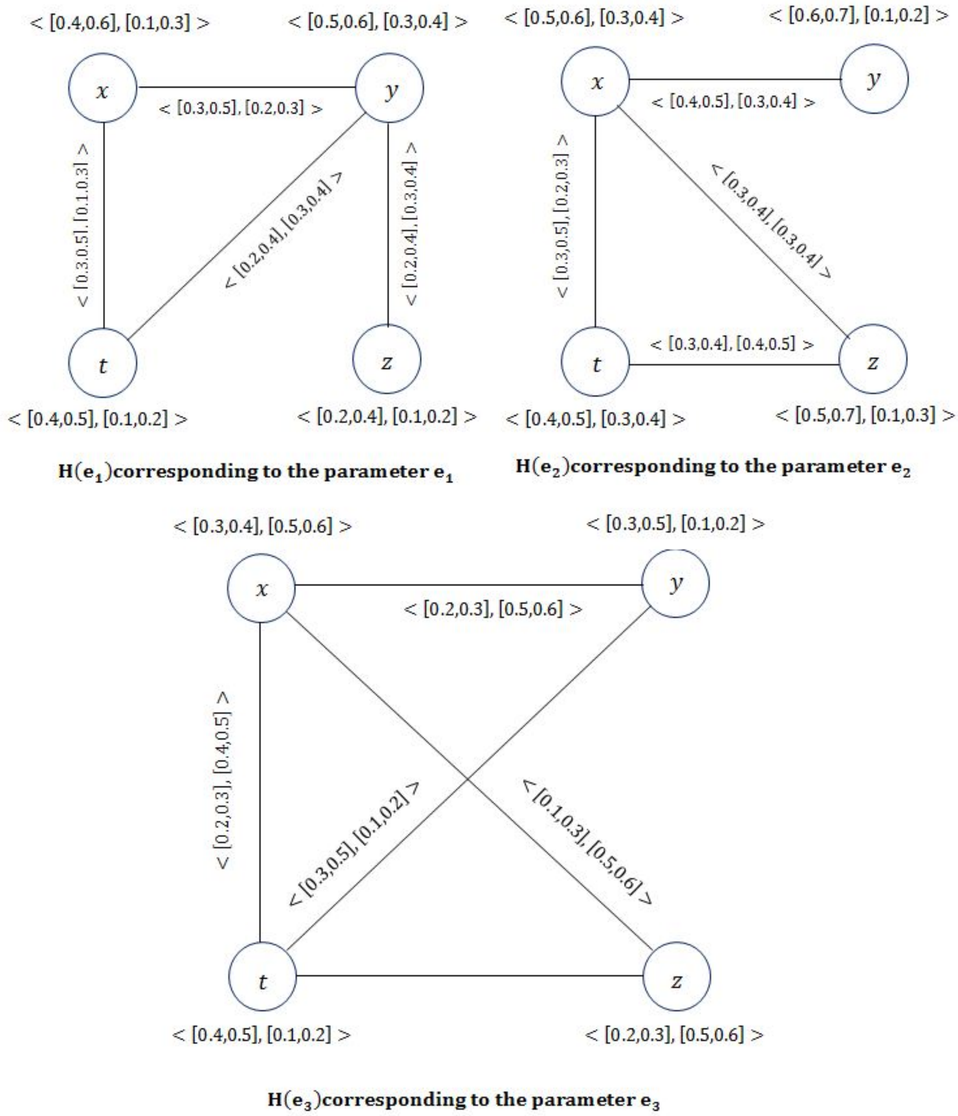

FIGURE 1. Interval-valued intuitionistic fuzzy soft graph $G = \{H(e_1), H(e_2), H(e_3)\}$

Also $G' = (G^*, F', K', B)$ is an interval-valued intuitionistic fuzzy soft graph as shown in Figure 2. Hence G' is an interval-valued intuitionistic fuzzy soft subgraph of G.

Definition 3.3. *An interval-valued intuitionistic fuzzy soft graph $G = (G^*, F, K, A)$ is called complete interval-valued intuitionistic fuzzy soft graph if*

$$\mu^L_{K(e)}(xy) = \min\{\mu^L_{F(e)}(x), \mu^L_{F(e)}(y)\}, \mu^U_{K(e)}(xy) = \min\{\mu^U_{F(e)}(x), \mu^U_{F(e)}(y)\}$$
$$\nu^L_{K(e)}(xy) = \max\{\nu^L_{F(e)}(x), \nu^L_{F(e)}(y)\}, \nu^U_{K(e)}(xy) = \max\{\nu^U_{F(e)}(x), \nu^U_{F(e)}(y)\}$$

for all $e \in A$ and $xy \in f$.

Example 3.3. *Consider a graph $G^* = (V, E)$ such that $V = \{x, y, z, t\}$ and $E = \{xy, yz, xz, xt, yt\}$. Let $A = \{e_1, e_2\}$ be a set of parameters, (F, A) and (K, A) be two*

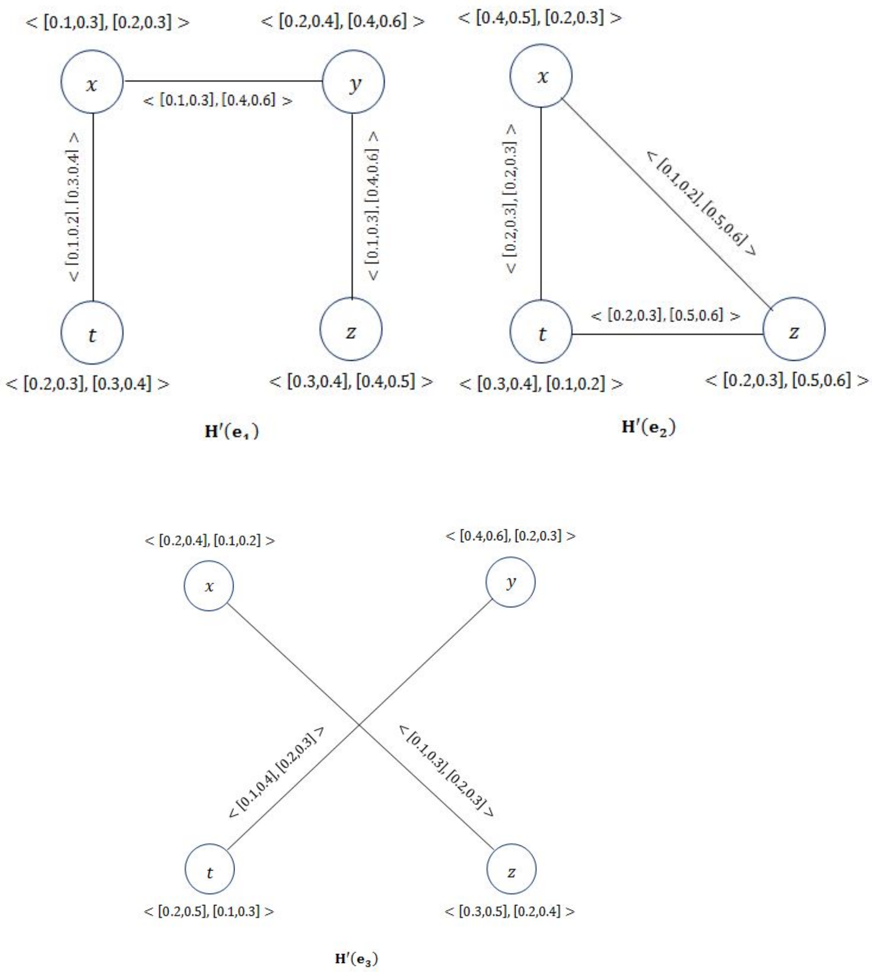

FIGURE 2. Interval-valued intuitionistic fuzzy soft graph G'

interval-valued intuitionistic fuzzy soft sets over V and E, respectively, defined by

$$
\begin{aligned}
&F(e_1) = \{\langle x, [0.2, 0.3], [0.4, 0.5]\rangle, \langle y, [0.1, 0.3], [0.5, 0.6]\rangle, \langle z, [0.3, 0.5], [0.1, 0.2]\rangle\} \\
&F(e_2) = \{\langle x, [0.3, 0.5], [0.2, 0.3]\rangle, \langle y, [0.2, 0.4], [0.1, 0.2]\rangle, \langle z, [0.3, 0.4], [0.2, 0.3]\rangle, \\
&\langle t, [0.4, 0.5], [0.2, 0.3]\rangle\} \\
&K(e_1) = \{\langle xy, [0.1, 0.3], [0.5, 0.6]\rangle, \langle yz, [0.1, 0.3], [0.5, 0.6]\rangle, \langle xz, [0.2, 0.3], [0.4, 0.5]\rangle\} \\
&K(e_2) = \{\langle xy, [0.2, 0.4], [0.2, 0.3]\rangle, \langle yz, [0.2, 0.4], [0.2, 0.3]\rangle, \langle xz, [0.3, 0.4], [0.2, 0.3]\rangle, \\
&\langle xt, [0.3, 0.5], [0.2, 0.3]\rangle, \langle yt, [0.2, 0.4], [0.2, 0.3]\rangle\}
\end{aligned}
$$

It is clearly seen that G is complete IVIFSG of G^* as shown in Figure 3.

Definition 3.4. *Let $G_1 = (G_1^*, F_1, K_1, A)$ and $G_2 = (G_2^*, F_2, K_2, B)$ be two interval-valued intuitionistic fuzzy soft graphs of simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. The cartesian product of G_1 and G_2 is denoted by $G_1 \times G_2 = (G^*, F, K, A \times B)$,*

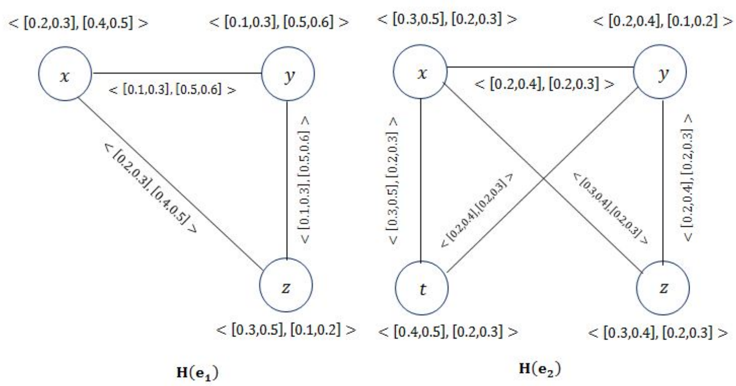

FIGURE 3. Complete IVIFSG G

where $G^* = (V_1 \times V_2, E_1 \times E_2)$, *and is defined by*

$$i.\ \left(\mu^L_{F_1(e_1)} \times \mu^L_{F_2(e_2)}\right)(u_1,u_2) = \min\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{F_2(e_2)}(u_2)\}$$
$$\left(\mu^U_{F_1(e_1)} \times \mu^U_{F_2(e_2)}\right)(u_1,u_2) = \min\{\mu^U_{F_1(e_1)}(u_1), \mu^U_{F_2(e_2)}(u_2)\}$$
$$\left(\nu^L_{F_1(e_1)} \times \nu^L_{F_2(e_2)}\right)(u_1,u_2) = \max\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{F_2(e_2)}(u_2)\}$$
$$\left(\nu^U_{F_1(e_1)} \times \nu^U_{F_2(e_2)}\right)(u_1,u_2) = \max\{\nu^U_{F_1(e_1)}(u_1)\nu^U_{F_2(e_2)}(u_2)\},$$
$$for\ all\ (u_1,u_2) \in V_1 \times V_2.$$

$$ii.\ \left(\mu^L_{K_1(e_1)} \times \mu^L_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \min\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{F_2(e_2)}(u_2v_2)\}$$
$$\left(\mu^U_{K_1(e_1)} \times \mu^U_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \min\{\mu^U_{F_1(e_1)}(u_1), \mu^U_{F_2(e_2)}(u_2v_2)\}$$
$$\left(\nu^L_{K_1(e_1)} \times \nu^L_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \max\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{F_2(e_2)}(u_2v_2)\}$$
$$\left(\nu^U_{K_1(e_1)} \times \nu^U_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \max\{\nu^U_{F_1(e_1)}(u_1), \nu^U_{F_2(e_2)}(u_2v_2)\},$$
$$for\ all\ u_1 \in V_1\ and\ u_2v_2 \in E_2.$$

$$iii.\ \left(\mu^L_{K_1(e_1)} \times \mu^L_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \min\{\mu^L_{K_1(e_1)}(u_1v_1), \mu^L_{F_2(e_2)}(u_2)\}$$
$$\left(\mu^U_{K_1(e_1)} \times \mu^U_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \min\{\mu^U_{K_1(e_1)}(u_1v_1), \mu^U_{F_2(e_2)}(u_2)\}$$
$$\left(\nu^L_{K_1(e_1)} \times \nu^L_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \max\{\nu^L_{K_1(e_1)}(u_1v_1), \nu^L_{F_2(e_2)}(u_2)\}$$
$$\left(\nu^U_{K_1(e_1)} \times \nu^U_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \max\{\nu^U_{K_1(e_1)}(u_1v_1), \nu^U_{F_2(e_2)}(u_2)\},$$
$$for\ all\ u_2 \in V_2\ and\ u_1v_1 \in E_1.$$

for all $e_1 \in A$ *and* $e_2 \in B$.

Example 3.4. *Let* $G^*_1 = (V_1, E_1)$ *and* $G^*_2 = (V_2, E_2)$ *be two graph such that*

$$V_1 = \{x_1, y_1, z_1, t_1\},\ E_1 = \{x_1y_1, z_1t_1\},$$
$$V_2 = \{x_2, y_2, z_2, t_2\},\ E_2 = \{x_2y_2, z_2t_2\}$$

Let $A = \{e_1\}$ be a set of parameters, and let (F_1, A) and (K_1, A) be two IVIFSS over V_1 and E_1, respectively, defined by

$$F_1(e_1) = \{\langle x_1, [0.4, 0.5], [0.1, 0.2]\rangle, \langle y_1, [0.6, 0.7], [0.2, 0.3]\rangle, \langle z_1, [0.3, 0.4], [0.2, 0.4]\rangle,$$
$$\langle t_1, [0.5, 0.6], [01, 0.3]\rangle$$
$$K_1(e_1) = \{\langle x_1y_1, [0.3, 0.4], [0.1, 0.2]\rangle, \langle z_1t_1, [0.2, 0.3], [0.1, 0.4]\rangle\}$$

Now $B = \{e_2\}$ be a set of parameters, and let (F_2, B) and (K_2, B) be two IVIFSS over V_2 and F_2, respectively, defined by

$$F_2(e_2) = \{\langle x_2, [0.2, 0.3], [0.2, 0.3]\rangle, \langle y_2, [0.3, 0.4], [0.2, 0.3]\rangle, \langle z_2, [0.5, 0.6], [0.1, 0.2]\rangle,$$
$$\langle t_2, [0.4, 0.5], [02, 0.3]\rangle$$
$$K_2(e_2) = \{\langle x_2y_2, [0.1, 0.2], [0.2, 0.3]\rangle, \langle z_2t_2, [0.4, 0.5], [0.1, 0.3]\rangle\}$$

Clearly, $H(e_1) = (F_1(e_1), K_1(e_1))$ and $H(e_2) = (F_2(e_2), K_2(e_2))$ are IVIFGs. Hence, $G_1 = (G_1^*, F_1, K_1, A)$ and $G_2 = (G_2^*, F_2, K_2, A)$ are IVIFSGs of G_1^* and G_2^*, respectively, as shown in Figure 4.

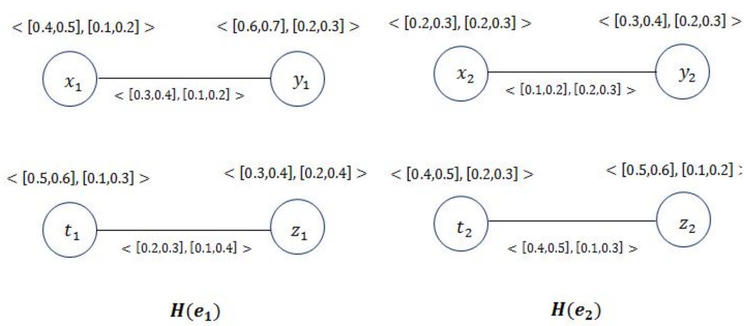

FIGURE 4. IVIFSGs G_1 and G_2

The cartesian product of G_1 and G_2 is as shown in Figure 5.

Theorem 3.1. *If G_1 and G_2 are two interval-valued intuitionistic fuzzy soft graphs, then So is $G_1 \times G_2$.*

Proof. Let $G_1 = (G_1^*, F_1, K_1, A)$ and $G_2 = (G_2^*, F_2, K_2, B)$ be two interval-valued intuitionistic fuzzy soft graphs of simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. From definition 3.4, for all $e_1 \in A$ and $e_2 \in B$, there is three cases.

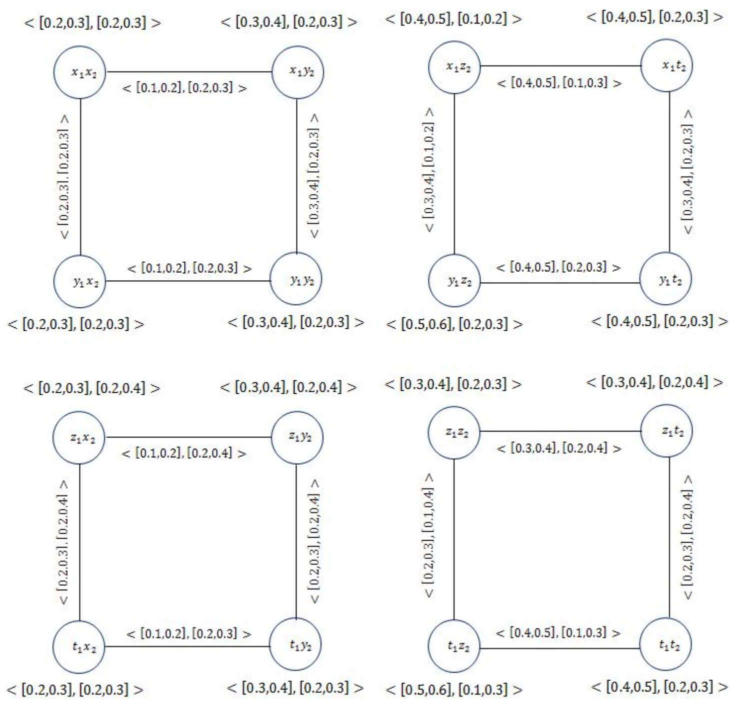

FIGURE 5. Cartesian product of G_1 and G_2

Case (i) If $u_1 \in V_1$ and $u_2 \in V_2$, then

$$\left(\mu^L_{F_1(e_1)} \times \mu^L_{F_2(e_2)}\right)(u_1, u_2) = \min\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{F_2(e_2)}(u_2)\}$$
$$\leq \min\left\{\left(\mu^L_{F_1(e_1)} \times \mu^L_{F_2(e_2)}\right)(u_1), \left(\mu^L_{F_1(e_1)} \times \mu^L_{F_2(e_2)}\right)(u_2)\right\}$$
$$\left(\mu^U_{F_1(e_1)} \times \mu^U_{F_2(e_2)}\right)(u_1, u_2) = \min\{\mu^U_{F_1(e_1)}(u_1), \mu^U_{F_2(e_2)}(u_2)\}$$
$$\leq \min\left\{\left(\mu^U_{F_1(e_1)} \times \mu^U_{F_2(e_2)}\right)(u_1), \left(\mu^U_{F_1(e_1)} \times \mu^U_{F_2(e_2)}\right)(u_2)\right\}$$
$$\left(\nu^L_{F_1(e_1)} \times \nu^L_{F_2(e_2)}\right)(u_1, u_2) = \max\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{F_2(e_2)}(u_2)\}$$
$$\leq \max\left\{\left(\nu^L_{F_1(e_1)} \times \nu^L_{F_2(e_2)}\right)(u_1), \left(\nu^L_{F_1(e_1)} \times \nu^L_{F_2(e_2)}\right)(u_2)\right\}$$
$$\left(\nu^U_{F_1(e_1)} \times \nu^U_{F_2(e_2)}\right)(u_1, u_2) = \max\{\nu^U_{F_1(e_1)}(u_1), \nu^U_{F_2(e_2)}(u_2)\}$$
$$\leq \max\left\{\left(\nu^U_{F_1(e_1)} \times \nu^U_{F_2(e_2)}\right)(u_1), \left(\nu^U_{F_1(e_1)} \times \nu^U_{F_2(e_2)}\right)(u_2)\right\}$$

Case (ii) If $u_1 \in V_1$ and $u_2v_2 \in E_2$, then

$$\left(\mu^L_{K_1(e_1)} \times \mu^L_{K_2(e_2)}\right)((u_1u_2), (u_1v_2)) = \min\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{K_2(e_2)}(u_2v_2)\}$$
$$\leq \min\left\{\mu^L_{F_1(e_1)}(u_1), \min\left\{\mu^L_{F_2(e_2)}(u_2), \mu^L_{F_2(e_2)}(v_2)\right\}\right\}$$
$$= \min\left\{\min\left\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{F_2(e_2)}(u_2)\right\}, \min\left\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{F_2(e_2)}(v_2)\right\}\right\}$$
$$= \min\left\{\left(\mu^L_{F_1(e_1)} \times \mu^L_{F_2(e_2)}\right)(u_1, u_2), \left(\mu^L_{F_1(e_1)} \times \mu^L_{F_2(e_2)}\right)(u_1, v_2)\right\}$$

Similarly, $\left(\mu^U_{K_1(e_1)} \times \mu^U_{K_2(e_2)}\right)((u_1u_2),(u_1v_2)) \leq$
$\min\left\{\left(\mu^U_{F_1(e_1)} \times \mu^U_{F_2(e_2)}\right)(u_1,u_2), \left(\mu^U_{F_1(e_1)} \times \mu^U_{F_2(e_2)}\right)(u_1,v_2)\right\},$

$$\begin{aligned}
&\left(\nu^L_{K_1(e_1)} \times \nu^L_{K_2(e_2)}\right)((u_1u_2),(u_1v_2)) = \max\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{K_2(e_2)}(u_2v_2)\}\\
&\leq \max\left\{\nu^L_{F_1(e_1)}(u_1), \max\left\{\nu^L_{F_2(e_2)}(u_2), \nu^L_{F_2(e_2)}(v_2)\right\}\right\}\\
&= \max\left\{\max\left\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{F_2(e_2)}(u_2)\right\}, \max\left\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{F_2(e_2)}(v_2)\right\}\right\}\\
&= \max\left\{\left(\nu^L_{F_1(e_1)} \times \nu^L_{F_2(e_2)}\right)(u_1,u_2), \left(\nu^L_{F_1(e_1)} \times \nu^L_{F_2(e_2)}\right)(u_1,v_2)\right\}
\end{aligned}$$

Similarly, we can show that,

$$\begin{aligned}
&\left(\nu^U_{K_1(e_1)} \times \nu^U_{K_2(e_2)}\right)((u_1u_2),(u_1v_2)) \leq\\
&\max\left\{\left(\nu^U_{F_1(e_1)} \times \nu^U_{F_2(e_2)}\right)(u_1,u_2), \left(\nu^U_{F_1(e_1)} \times \nu^U_{F_2(e_2)}\right)(u_1,v_2)\right\}.
\end{aligned}$$

Case (iii) If $u_2 \in V_2$ and $u_1v_1 \in E_1$, than

$$\begin{aligned}
&\left(\mu^L_{K_1(e_1)} \times \mu^L_{K_2(e_2)}\right)((u_1u_2),(v_1u_2)) = \min\{\mu^L_{K_1(e_1)}(u_1v_1), \mu^L_{F_2(e_2)}(u_2)\}\\
&\leq \min\left\{\min\left\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{F_2(e_2)}(v_1)\right\}, \mu^L_{F_2(e_2)}(u_2)\right\}\\
&= \min\left\{\min\left\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{F_2(e_2)}(u_2)\right\}, \min\left\{\mu^L_{F_1(e_1)}(v_1), \mu^L_{F_2(e_2)}(u_2)\right\}\right\}\\
&= \min\left\{\left(\mu^L_{F_1(e_1)} \times \mu^L_{F_2(e_2)}\right)(u_1,u_2), \left(\mu^L_{F_1(e_1)} \times \mu^L_{F_2(e_2)}\right)(v_1,u_2)\right\}
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\left(\mu^U_{K_1(e_1)} \times \mu^U_{K_2(e_2)}\right)((u_1u_2),(v_1u_2)) \leq\\
&\min\left\{\left(\mu^U_{F_1(e_1)} \times \mu^U_{F_2(e_2)}\right)(u_1,u_2), \left(\mu^U_{F_1(e_1)} \times \mu^U_{F_2(e_2)}\right)(v_1,u_2)\right\}
\end{aligned}$$

$$\begin{aligned}
&\left(\nu^L_{K_1(e_1)} \times \nu^L_{K_2(e_2)}\right)((u_1u_2),(v_1u_2)) = \max\{\nu^L_{K_1(e_1)}(u_1v_1), \nu^L_{F_2(e_2)}(u_2)\}\\
&\leq \max\left\{\max\left\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{F_2(e_2)}(v_1)\right\}, \nu^L_{F_2(e_2)}(u_2)\right\}\\
&= \max\left\{\max\left\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{F_2(e_2)}(u_2)\right\}, \max\left\{\nu^L_{F_1(e_1)}(v_1), \nu^L_{F_2(e_2)}(u_2)\right\}\right\}\\
&= \max\left\{\left(\nu^L_{F_1(e_1)} \times \nu^L_{F_2(e_2)}\right)(u_1,u_2), \left(\nu^L_{F_1(e_1)} \times \nu^L_{F_2(e_2)}\right)(v_1,u_2)\right\}
\end{aligned}$$

in like manner,

$$\begin{aligned}
&\left(\nu^U_{K_1(e_1)} \times \nu^U_{K_2(e_2)}\right)((u_1u_2),(v_1u_2)) \leq\\
&\max\left\{\left(\nu^U_{F_1(e_1)} \times \nu^U_{F_2(e_2)}\right)(u_1,u_2), \left(\nu^U_{F_1(e_1)} \times \nu^U_{F_2(e_2)}\right)(v_1,u_2)\right\}
\end{aligned}$$

□

Definition 3.5. *Let* $G_1 = (G_1^*, F_1, K_1, A)$ *and* $G_2 = (G_2^*, F_2, K_2, B)$ *be two interval-valued intuitionistic fuzzy soft graphs of simple graphs* $G_1^* = (V_1, E_1)$ *and* $G_2^* = (V_2, E_2)$ *,* *respectively. The strong product of* G_1 *and* G_2 *is defined by* $G_1 \otimes G_2 = (G^*, F, K, A \times B)$ *and is defined by*

$$i\cdot \ \left(\mu^L_{F_1(e_1)} \otimes \mu^L_{F_2(e_2)}\right)(u_1,u_2) = \min\left\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{F_2(e_2)}(u_2)\right\}$$
$$\left(\mu^U_{F_1(e_1)} \otimes \mu^U_{F_2(e_2)}\right)(u_1,u_2) = \min\left\{\mu^U_{F_1(e_1)}(u_1), \mu^U_{F_2(e_2)}(u_2)\right\}$$
$$\left(\nu^L_{F_1(e_1)} \otimes \nu^L_{F_2(e_2)}\right)(u_1,u_2) = \max\left\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{F_2(e_2)}(u_2)\right\}$$
$$\left(\nu^U_{F_1(e_1)} \otimes \nu^U_{F_2(e_2)}\right)(u_1,u_2) = \max\left\{\nu^U_{F_1(e_1)}(u_1), \nu^U_{F_2(e_2)}(u_2)\right\}$$

for all $(u_1,u_2) \in V_1 \times V_2$.

$$ii\cdot \ \left(\mu^L_{K_1(e_1)} \otimes \mu^L_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \min\left\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{K_2(e_2)}(u_2v_2)\right\}$$
$$\left(\mu^U_{K_1(e_1)} \otimes \mu^U_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \min\left\{\mu^U_{F_1(e_1)}(u_1), \mu^U_{K_2(e_2)}(u_2v_2)\right\}$$
$$\left(\nu^L_{K_1(e_1)} \otimes \nu^L_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \max\left\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{K_2(e_2)}(u_2v_2)\right\}$$
$$\left(\nu^U_{K_1(e_1)} \otimes \nu^U_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \max\left\{\nu^U_{F_1(e_1)}(u_1), \nu^U_{K_2(e_2)}(u_2v_2)\right\}$$

for all $u_1 \in V_1$ *and* $u_2v_2 \in E_2$.

$$iii\cdot \ \left(\mu^L_{K_1(e_1)} \otimes \mu^L_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \min\left\{\mu^L_{K_1(e_1)}(u_1v_1), \mu^L_{F_2(e_2)}(u_2)\right\}$$
$$\left(\mu^U_{K_1(e_1)} \otimes \mu^U_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \min\left\{\mu^U_{K_1(e_1)}(u_1v_1), \mu^U_{K_2(e_2)}(u_2)\right\}$$
$$\left(\nu^L_{K_1(e_1)} \otimes \nu^L_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \max\left\{\nu^L_{K_1(e_1)}(u_1v_1), \nu^L_{F_2(e_2)}(u_2)\right\}$$
$$\left(\nu^U_{K_1(e_1)} \otimes \nu^U_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \max\left\{\nu^U_{K_1(e_1)}(u_1v_1), \nu^U_{F_2(e_2)}(u_2)\right\}$$

for all $u_2 \in V_2$ *and* $u_1v_1 \in E_1$

$$iv\cdot \ \left(\mu^L_{K_1(e_1)} \otimes \mu^L_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \min\left\{\mu^L_{K_1(e_1)}(u_1v_1), \mu^L_{K_2(e_2)}(u_2v_2)\right\}$$
$$\left(\mu^U_{K_1(e_1)} \otimes \mu^U_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \min\left\{\mu^U_{K_1(e_1)}(u_1v_1), \mu^U_{K_2(e_2)}(u_2v_2)\right\}$$
$$\left(\nu^L_{K_1(e_1)} \otimes \nu^L_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \max\left\{\nu^L_{K_1(e_1)}(u_1v_1), \nu^L_{K_2(e_2)}(u_2v_2)\right\}$$
$$\left(\nu^U_{K_1(e_1)} \otimes \nu^U_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \max\left\{\nu^U_{K_1(e_1)}(u_1v_1), \nu^U_{K_2(e_2)}(u_2v_2)\right\}$$

for all $u_1v_1 \in E_1$ *and* $u_2v_2 \in E_2$
for all $e_1 \in A$ *and* $e_2 \in B$.

Example 3.5. *Consider* $G_1 = (G_1^*, F_1, K_1, A)$ *and* $G_2 = (G_2^*, F_2, K_2, B)$ *as taken in Example 3.4. The strong product of* G_1 *and* G_2 *is as shown in Figure 6.*

Theorem 3.2. *If* G_1 *and* G_2 *are two intervaal-valued intuitionistic fuzzy soft gaphs, then so is* $G_1 \otimes G_2$.

Proof. By using Definition 3.5, it can be shown in a similar way to proof of Theorem 3.1. □

Definition 3.6. *Let* $G_1 = (G_1^*, F_1, K_1, A)$ *and* $G_2 = (G_2^*, F_2, K_2, B)$ *be two IVIFSGs of simple graphs* $G_1^* = (V_1, E_1)$ *and* $G_2^* = (V_2, E_2)$ *, respectively. The composition of* G_1 *and*

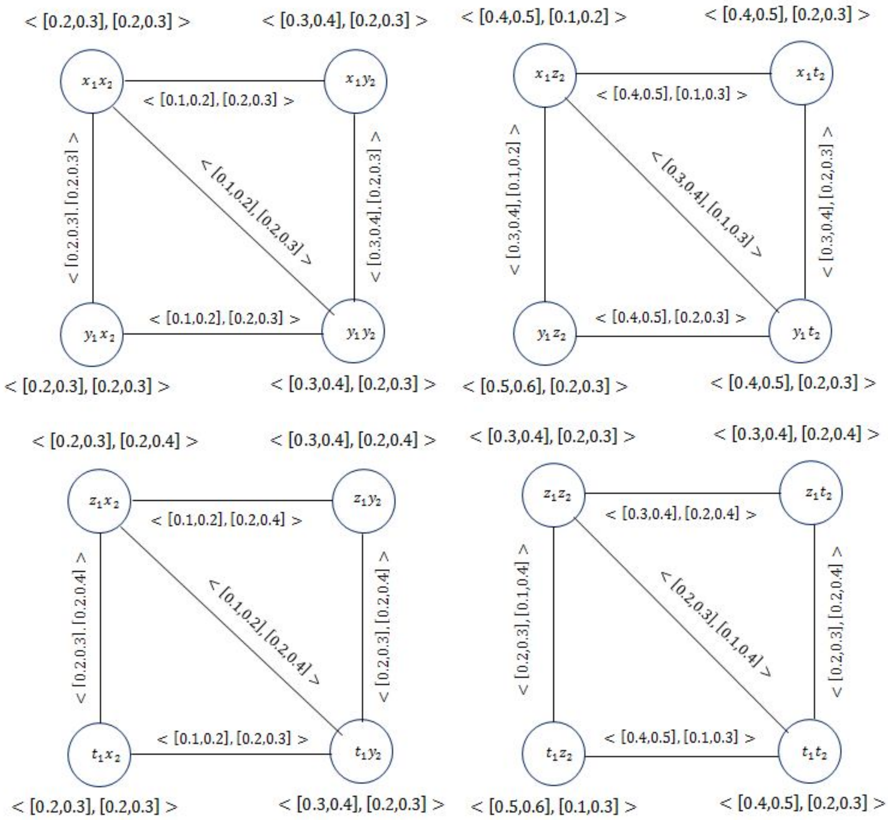

FIGURE 6. Strong product G_1 and G_2

G_2 is defined by $G_1 \circ G_2$ and is defined by

$$
\begin{aligned}
i \cdot \ &\left(\mu^L_{F_1(e_1)} \circ \mu^L_{F_2(e_2)}\right)(u_1, u_2) = \min\left\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{F_2(e_2)}(u_2)\right\}\\
&\left(\mu^U_{F_1(e_1)} \circ \mu^U_{F_2(e_2)}\right)(u_1, u_2) = \min\left\{\mu^U_{F_1(e_1)}(u_1), \mu^U_{F_2(e_2)}(u_2)\right\}\\
&\left(\nu^L_{F_1(e_1)} \circ \nu^L_{F_2(e_2)}\right)(u_1, u_2) = \max\left\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{F_2(e_2)}(u_2)\right\}\\
&\left(\nu^U_{F_1(e_1)} \circ \nu^U_{F_2(e_2)}\right)(u_1, u_2) = \max\left\{\nu^U_{F_1(e_1)}(u_1), \nu^U_{F_2(e_2)}(u_2)\right\}
\end{aligned}
$$

for all $(u_1, u_2) \in V_1 \times V_2$.

$$
\begin{aligned}
ii \cdot \ &\left(\mu^L_{K_1(e_1)} \circ \mu^L_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \min\left\{\mu^L_{F_1(e_1)}(u_1), \mu^L_{K_2(e_2)}(u_2v_2)\right\}\\
&\left(\mu^U_{K_1(e_1)} \circ \mu^U_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \min\left\{\mu^U_{F_1(e_1)}(u_1), \mu^U_{K_2(e_2)}(u_2v_2)\right\}\\
&\left(\nu^L_{K_1(e_1)} \circ \nu^L_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \max\left\{\nu^L_{F_1(e_1)}(u_1), \nu^L_{K_2(e_2)}(u_2v_2)\right\}\\
&\left(\nu^U_{K_1(e_1)} \circ \nu^U_{K_2(e_2)}\right)((u_1u_2)(u_1v_2)) = \max\left\{\nu^U_{F_1(e_1)}(u_1), \nu^U_{K_2(e_2)}(u_2v_2)\right\}
\end{aligned}
$$

for all $u_1 \in V_1$ *and* $u_2v_2 \in E_2$.

$$iii \cdot \ \left(\mu^L_{K_1(e_1)} \circ \mu^L_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \min\left\{\mu^L_{K_1(e_1)}(u_1v_1), \mu^L_{F_2(e_2)}(u_2)\right\}$$
$$\left(\mu^U_{K_1(e_1)} \circ \mu^U_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \min\left\{\mu^U_{K_1(e_1)}(u_1v_1), \mu^U_{K_2(e_2)}(u_2)\right\}$$
$$\left(\nu^L_{K_1(e_1)} \circ \nu^L_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \max\left\{\nu^L_{K_1(e_1)}(u_1v_1), \nu^L_{F_2(e_2)}(u_2)\right\}$$
$$\left(\nu^U_{K_1(e_1)} \circ \nu^U_{K_2(e_2)}\right)((u_1u_2)(v_1u_2)) = \max\left\{\nu^U_{K_1(e_1)}(u_1v_1), \nu^U_{F_2(e_2)}(u_2)\right\}$$

for all $u_2 \in V_2$ *and* $u_1v_1 \in E_1$

$$iv \cdot \ \left(\mu^L_{K_1(e_1)} \circ \mu^L_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \min\left\{\mu^L_{K_1(e_1)}(u_1v_1), \mu^L_{K_2(e_2)}(u_2v_2)\right\}$$
$$\left(\mu^U_{K_1(e_1)} \circ \mu^U_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \min\left\{\mu^U_{K_1(e_1)}(u_1v_1), \mu^U_{K_2(e_2)}(u_2v_2)\right\}$$
$$\left(\nu^L_{K_1(e_1)} \circ \nu^L_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \max\left\{\nu^L_{K_1(e_1)}(u_1v_1), \nu^L_{K_2(e_2)}(u_2v_2)\right\}$$
$$\left(\nu^U_{K_1(e_1)} \circ \nu^U_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \max\left\{\nu^U_{K_1(e_1)}(u_1v_1), \nu^U_{K_2(e_2)}(u_2v_2)\right\}$$

for all $u_1v_1 \in E_1$ *and* $u_2v_2 \in E_2$

$$v \cdot \ \left(\mu^L_{K_1(e_1)} \circ \mu^L_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \min\left\{\mu^L_{F_1(e_1)}(u_2), \mu^L_{F_2(e_2)}(v_2), \mu^L_{K_1(e_1)}(u_1v_1)\right\}$$
$$\left(\mu^U_{K_1(e_1)} \circ \mu^U_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \min\left\{\mu^U_{F_1(e_1)}(u_2), \mu^U_{F_2(e_2)}(v_2), \mu^U_{K_1(e_1)}(u_1v_1)\right\}$$
$$\left(\nu^L_{K_1(e_1)} \circ \nu^L_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \max\left\{\nu^L_{F_1(e_1)}(u_2), \nu^L_{F_2(e_2)}(v_2), \nu^L_{K_1(e_1)}(u_1v_1)\right\}$$
$$\left(\nu^U_{K_1(e_1)} \circ \nu^U_{K_2(e_2)}\right)((u_1u_2)(v_1v_2)) = \max\left\{\nu^U_{F_1(e_1)}(u_2), \nu^U_{F_2(e_2)}(v_2), \nu^U_{K_1(e_1)}(u_1v_1)\right\}$$

for all $u_1v_1 \in E_1$ *and* $u_2v_2 \in E_2$ *such that* $u_2 \neq v_2$
for all $e_1 \in A$ *and* $e_2 \in B$.

Example 3.6. *Consider* $G_1 = (G_1^*, F_1, K_1, A)$ *and* $G_2 = (G_2^*, F_2, K_2, B)$ *as taken in Example 3.4. The composition of* G_1 *and* G_2 *is as shown in Figure 7.*

Theorem 3.3. *If* G_1 *and* G_2 *are two interval-valued intuitionistic fuzzy soft graphs, then so is* $G_1 \circ G_2$.

Proof. By using Definition 3.6, it can shown in a similar way to proof of Theorem 3.1. □

4. Conclusion

Soft set theory has been regarded as an effective mathematical tool to deal with uncertainty. However, it is difficult to be used to represent the fuzziness of problem parameters. In order to handle these types of problem parameters, some fuzzy extensions of soft set theory are presented, yielding fuzzy soft set theory. We applied the concept of interval-valued intuitionistic fuzzy soft sets to graph structures and describe the method of their construction. We also defined Cartesian product, strong product, and composition on interval-valued intuitionistic fuzzy soft graphs and give some of their properties. Subsequent research of such operators could focus on concepts such as lexicographic product, cross product, union, join, and self-complementary.

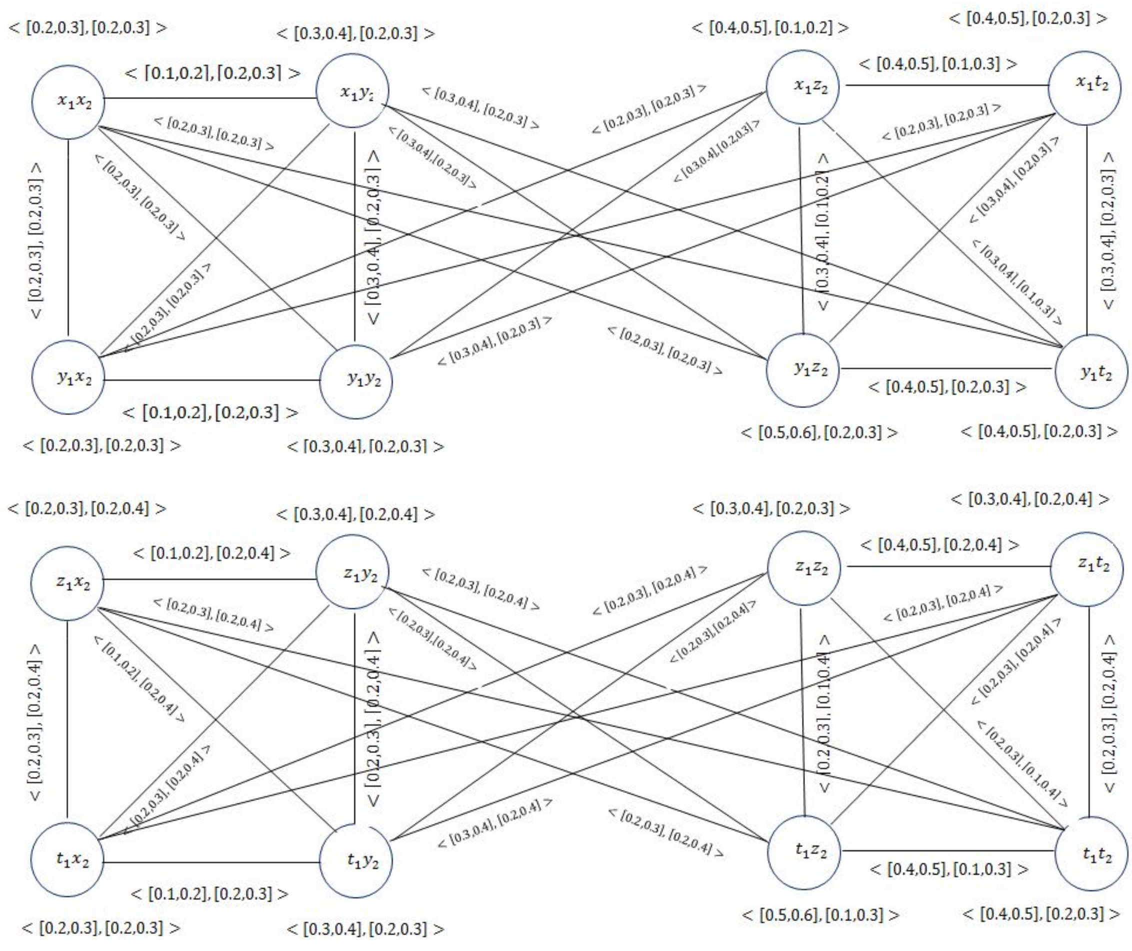

Figure 7. Composition of G_1 and G_2

References

[1] Abu Qamar, M., Hassan, N.,(2018), Q-neutrosophic soft relation and its application in decision making, Entropy 20(3) , 172.

[2] Abu Qamar, M., Hassan, N., (2019), An approach toward a Q-neutrosophic soft set and its application in decision making, Symmetry 11(2) , 139.

[3] Akram, M., Davvaz, B., (2012),, Strong intuitionistic fuzzy graph, Filomat, 26(1), 177-196.

[4] Akram, M., Dudek, W. A., (2011), Interval-valued fuzzy graphs. Comput. Math. Appl., 61, 289-299.

[5] Akram, M., Nawaz, S., (2015), on fuzzy soft graphs, Italian Journal of Pure and Applied Mathematics, 34, 497-514.

[6] Akram, M., Nawaz, S., (2015), Operations on soft graphs, Fuzzy Information and Engineering, 7(4), 423-449.

[7] Akram, M., Shahzadi, S., (2018), Graphs in an intuitionistic fuzzy soft environment, Axioms, 7(2), 20; doi:10.3390/axioms7020020.

[8] Al-Masarwah, A., Abu Qamar, M., (2016), Some New Concepts of Fuzzy Soft Graphs, Fuzzy Information and Engineering 8(4), 427-438.

[9] Al-Masarwah, A., Abu Qamar, M., (2018), Certain types of fuzzy soft graphs, New Mathematics and Natural Computation, 14(2), 145-156.

[10] Al-Masarwah, A., Ahmad, A.G., (2019), m-Polar fuzzy ideals of BCK/BCI-algebras. J. King Saud Univ.-Sci., 31(4) , 1220-1226. DOI:10.1016/j.jksus.2018.10.002

[11] Al-Masarwah, A., Ahmad, A.G., (2019), On (complete) normality of m-pF subalgebras in BCK/BCI-algebras, AIMS Mathematics 4(3) , 740-750.
[12] Al-Masarwah, A., Ahmad, A.G., (2018), On some properties of doubt bipolar fuzzy H-ideals in BCK/BCI-algebras. Eur. J. Pure Appl. Math. 11(3), 652-670
[13] Al-Masarwah, A., Ahmad, A.G., (2018), Novel concepts of doubt bipolar fuzzy H-ideals of BCK/BCI-algebras. Int. J. Innov. Comput. Inf. Control. 14(6), 2025-2041
[14] Atanassov, K., (1986), Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, 87-96.
[15] Atanassov, K., Gargov, G., (1989), Interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 31, 343-349.
[16] Aygunoglu, A., Aygun, H., (2009). , Introduction to fuzzy soft groups, Computers & Mathematics with Applications, 58 (6), 1279-1286.
[17] Borzooei, R.A., Rashmanlou, H., (2015), Domination in vague graphs and its applications, Journal of Intelligent and Fuzzy Systems, 29, 1933-1940.
[18] Borzooei, R.A., Rashmanlou, H., (2015), Degree of vertices in vague graphs, Journal of Applied Mathematics and Informatics, 33(5), 545-557.
[19] Borzooei, R.A. and Rashmanlou, H., (2016), New concepts of vague graphs. International Journal of Machine Learning and Ceybernetics, 8(4), 1081-1092.
[20] Borzooei, R.A., Rashmanlou, H., (2016) , Semi global domination sets in vague graphs with application. J. Intell. Fuzzy Syst., 30(6), 3645-3652.
[21] Borzooei, R.A., Rashmanlou, H., Samanta, S. and Pal, M., (2016), Regularity of vague graphs, Journal of Intelligent and Fuzzy Systems, 30, 3681-3689.
[22] Buckly, J. J., (2006), Fuzzy probability and statistics, Springer.
[23] Feng, F.,. Jun, Y.B., Zhao, X., (2008), Soft semi rings, Computers Mathematics with Applications, 56 (10), 2621-2628.
[24] Ismail, A. M. and Ali, M., (2014), on strong interval-valued intuitionistic fuzzy graph, 4, 161-168.
[25] Jiang, Y., Tang, Y. , Chen, Q., Liu, H. and Tang, J , (2010), Interval-valued intuitionistic fuzzy soft sets and their properties, Computers and Mathematics with Applications, 60 (3), 906-918.
[26] Maji, P. K., Roy, A. R. and Biswas., R., (2001), Fuzzy soft sets, J. Fuzzy Math, 9(3), 589-602.
[27] Maji, P. K., Roy, A. R. and Biswas., R., (2001), Intuitionistic fuzzy soft sets. J. Fuzzy Math., 9, 677-691.
[28] Maji, P. K., Roy, A. R. and Biswas., R., (2004), On intuitionistic fuzzy soft sets. J. Fuzzy Math., 12, 669-683.
[29] Mishra, S. N. and Pal, A., (2013), Product of interval-valued intuitionistic fuzzy graph, Annals of Pure and Applied Mathematics, 4(2), 138-144.
[30] Molodtsov, D., (1999), Soft set theory first results, Comput. Math. Appl, 37(4), 19-31.
[31] Mordeson, J. N. and Nair, P. S., (2000), Fuzzy Graphs and Fuzzy Hyper graphs, Physica-Verlag, Heidelberg, New York, 46.
[32] Parvathi, R., Karunambigai, N. G., Atanassov, K. A., Operations on intuitionistic fuzzy graphs, Fuzzy Systems, FUZZ-IEEE 2009, IEEE International Conference, 2009, 1396-1401.
[33] Rashmanlou, H., Borzooei, R.A., (2015),Vague graphs with application, Journal of Intelligent and Fuzzy Systems, 30, 3291-3299.
[34] Rashmanlou, H., Borzooei, R.A., (2016), Product vague graphs and its applications, Journal of Intelligent and Fuzzy Systems, 30,371-382.
[35] Rosenfeld, A., (1975), Fuzzy graphs, Fuzzy Sets and Their Applications, Academic Press, New York, 77-95.
[36] Sarala, N., Deepa, R., (2020), Regular interval-valued intuitionistic fuzzy soft graph, The International journal of analytical and experimental modal analysis, Volume XII, Issue I, ISSN NO: 0886-9367.
[37] Sarala, N., Deepa, R., (2020), Strong nterval-valued intuitionistic fuzzy soft graph, Journal of Interdisciplinary Cycle Research Volume XII, Issue I, ISSN NO: 0022-1945.
[38] Shahzadi, S., Akram, M., (2017), Intuitionistic fuzzy soft graphs with applications, Journal of Applied Mathematics and Computing, 55(1-2), 369-392.
[39] Shahzadi, S., Akram, M., (2016), Edge regular intuitionistic fuzzy soft graphs, Journal of Intelligent and Fuzzy Systems 31(3),1-15.
[40] Sun, Q.M., Zhang, Z.L. and Liu, J., (2008), Soft sets and soft modules, in: G. Wang, T. Li, J.W. Grzymala-Busse, D. Miao, A. Skowron, Y. Yao (Eds.), Proceedings of The Third International Conference on Rough Sets and nowledge Technology, RSKT 2008, in: Lecture Notes in Computer Science, vol. 5009, Springer, pp. 403-409.

[41] Sunitha, M. S. and Vijayakumar, A., (2002), Complement of a fuzzy graph, Indian J. Pure Appl. Math., 33(9), 1451- 1464.
[42] Talebi, A.A., (2018), Cayley fuzzy graphs on the fuzzy group, Computational and Applied Mathematics, 37.
[43] Talebi, A.A. and Dudek, W.A., (2016), Operations on level graphs of bipolar fuzzy graphs, Bulletin Academiel De Stiinte A Republic Moldova Mathematica, 2 (81), 107-124.
[44] Talebi, A.A., Rashmanlou, H. and Sadati, S.H., (2020), Interval-valued Intuitionistic Fuzzy Competition Graph, J. of Multi-Valued Logic Soft Computing, 34, 335-364.
[45] Thumbakara, R. K., George, B., (2014), Soft graphs, General Mathematics Notes, 21(2), 75-86.
[46] Turksen, B., (1986), Interval-valued intuitionistic fuzzy sets based on normal forms, Fuzzy sets and Systems, 20, 191-210.
[47] Yang X., Lin, T. Y., Yang, J., Li, Y. and. Yu, D, (2009), Combination of interval-valued fuzzy set and soft set, Comp. Math. Appl, 58(3), 521-527.
[48] Yang, X.B., Yu, D.J., Yang, J.Y., and Wu, C., (2007), Generalization of soft set theory: from crisp to fuzzy case, in: B.Y. Cao (Ed.), Proceeding of the Second International Conference on Fuzzy Information and Engineering, in: Advance on Soft Computing, vol. 40, Springer-Verlag, 345-354.
[49] Zadeh, L. A., (1965), Fuzzy sets, Information and Computation, 8, 338-353.

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