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EXACT SOLITARY WAVE SOLUTIONS TO THE FRACTIONAL GERDJIKOV-IVANOV EQUATION WITH CONFORMABLE DERIVATIVE

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Abstract. Searching for exact analytic solutions to the partial differential equations is one of the most challenging problems in mathematical physics. The main contribution in this paper is to consider the fractional Gerdjikov-Ivanov equation with conformable derivative, and obtaining new exact solitary solutions with the aid of conformable derivative and Kudryashov method. Then, we get new soliton solutions for fGI equation. This equation plays a significant role in non-linear fiber optics. It also has many important applications in photonic crystal fibers. To this end, firstly, we obtain some novel optical solutions of the equation via a newly proposed analytical method called generalized exponential rational function method. In order to understand the dynamic behavior of these solutions, several graphs are plotted. To the best of our knowledge, these two techniques have never been tested for the equation in the literature. The findings of this article may have a high significance application while handling the other non-linear PDEs.

Keywords: Kudryashov method, fractional Gerdjikov-Ivanov equation, conformable derivative.

AMS Subject Classification: 35C08; 35Q55; 83C15.

1. Introduction

Non-linear Schrödinger equations (NLSE) are often studied from different points of view. In recent years a great variety of analytical and numerical methods have been proposed for solving these equations. The most studied NLSE equation is that which has a cubic non-linearity. Nonlinear evolution equations (NEEs) which describe many physical phenomena are often illustrated by nonlinear partial differential equations. So, the exact solutions of NLPDE are explored in detail in order to understand the physical structure of natural phenomena that are described by such equations. Searching for explicit, exact solutions of NLPDE by many different methods is the main goal of this active research area. Some of these methods, the Riccati Equation method [1], Hirota's bilinear operators [2], Hirota's dependent variable transformation [3], the Jacobi elliptic function expansion [4], the homogeneous balance method [5], the tanh-function expansion [6], first integral

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method [7,8], the sub-equation method [9], the exp-function method [10], the Backlund transformation, and similarity reduction [11-15] are used to obtain the exact solutions of NLPDE. In the present paper, we will explore an NLSE that has a quintic non-linearity, namely the perturbed Gerdjikov-Ivanov (pGI) equation. The main contribution in this study is to consider the Fractional Gerdjikov-Ivanov equation with conformable derivative, as follows

$$iD_t^{\alpha}q + aD_x^{2\alpha}q + b\left|q\right|^4 q = i\left[cq^2 D_x^{\alpha}\bar{q} + \lambda_1 D_x^{\alpha}q + \lambda_2 D_x^{\alpha}\left(\left|q\right|^2 q\right) + \theta D_x^{\alpha}\left|q\right|^2 q\right], \tag{1}$$

The parameters a, b are the coefficients. In light of previous work, we will apply the generalized exponential rational function method (GERFM) to retrieve some new analytical optical solutions of the fractional pGI equation with the conformable derivative [17]. This new definition of derivative is based on the basic limit definition of the derivative that has been successfully tackled in solving many different problems [18–25].

The paper is organized as follows. In section 2, a global solution of the fGI equation is constructed by a complex Fractional Gerdjikov-Ivanov equation. In section4, we study the finite (or infinite) time blowup solution for the fGI equation, some implicit self-similar solutions are constructed.

2. Optical solutions of the fractional Gerdjikov-Ivanov equation

The main assumption is to taking the stationary soliton solution form of

$$\begin{aligned} &q\left(x,t\right) = Q\left(\xi\right)e^{i\phi(x,t)},\\ &\xi = \left(\tfrac{1}{\alpha}\right)x^{\alpha} - \left(\tfrac{v}{\alpha}\right)t^{\alpha},\\ &\phi = \left(-\tfrac{k}{\alpha}\right)x^{\alpha} + \left(\tfrac{\omega}{\alpha}\right)t^{\alpha}, \end{aligned} \tag{2}$$

where $\nu, k,$and ω are the phase component, the frequency of solitons, and the wavenumber, respectively. Substituting the stationary soliton solution form (2) into Equation (1), we arrive at a complex equation whose imaginary part is as follows

$$\left(v + \lambda_1 + 2ak\right) + \left(c + 3\lambda_2 + 2\theta\right)Q^2 = 0 \tag{3}$$

So we will have

$$v = -\lambda_1 - 2ak, \qquad \theta = -\frac{1}{2}\left(c + 3\lambda_2\right) \tag{4}$$

From the real part, the following formula is also extracted

$$aQ'' - \left(\omega + ak^2 + \lambda_1 k\right)Q + \left(c - \lambda_2\right)kQ^3 + bQ^5 = 0. \tag{5}$$

Thus, in the following, we focus our attention on deriving solutions of Equation (5). Now balancing between two terms of $Q^5$and Q''in Equation (11) suggests$M = \frac{1}{2}$. If we want to get a closed-form solution, we need to define a new variable of$Q\left(\xi\right) = \Re^2\left(\xi\right)$. This substitution leads us to

$$a\left(2\Re\Re'' - \left(\Re'\right)^2\right) - 4\left(\omega + ak^2 + \lambda_1 k\right)\Re^2 + 4\left(c - \lambda_2\right)k\Re^3 + 4b\Re^2 = 0. \tag{6}$$

to solve (6), we assume that the solution $\phi\left(\xi\right)$ of the nonlinear Eq. (6) can be presented as

$$\phi\left(\xi\right) = \sum_{i=0}^{M} A_i \Upsilon^i\left(\xi\right), \tag{7}$$

And Υ satisfied in following Riccati equation

$$\Upsilon'(\xi) = \Upsilon^2(\xi) - \Upsilon(\xi), \tag{8}$$

Eq. (8) gives the solution, as follows:

$$\Upsilon(\xi) = \frac{1}{1+e^{\xi}}, \tag{9}$$

Substituting Eqs (7)-(9) into Eq. (6) and collecting all terms with the same order of Υ^j together, we convert the left-hand side of Eq. (6) into a polynomial inΥ^j. Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations forA_0, A_1, A_2 . By solving these algebraic equations, we obtain several case of variables solutions [15-16].

Remark: This Riccati equation (8) also admits the following exact solutions:

$$\phi_1(\xi) = \frac{1}{2}\left(1 - \tanh\left[\frac{\xi}{2} - \frac{\varepsilon \ln \xi_0}{2}\right]\right), \quad \xi_0 > 0, \tag{10}$$

$$\phi_2(\xi) = \frac{1}{2}\left(1 - \coth\left[\frac{\xi}{2} - \frac{\varepsilon \ln \xi_0}{2}\right]\right), \quad \xi_0 < 0, \tag{11}$$

***Stage 3**:* By substituting the obtained solutions in stage 2 into Eq. (6) along with general solutions of Eq. (8), finally generates new exact solutions for the nonlinear PDE (1).

3. Results

Now, the homogeneous balance in Equation (6) suggests$N = 1$. Setting $N = 1$ along with Equation (8), one gets By Kudryashov's method, the solution of (6) is assumed as

$$\Re\left(\xi\right) = A_1 \Upsilon\left(\xi\right) + A_0, \tag{12}$$

where $A_1$and$A_0$are constants. Substituting (12) into (6) and comparing the coefficients of alike powers of $\Upsilon\left(\xi\right)$provides algebraic system of equations. After solving the system, the A_i, $i = 0, 1$are obtained and produces following new sets of solution for (6).

Case-1

$$\begin{array}{l} A_1 = \sqrt{-\frac{a}{b}}, A_0 = \frac{3}{4}\sqrt{-\frac{a}{b}}, k = \frac{1}{2}\frac{\sqrt{-ab}}{c-\lambda_1}, \\ \omega = \sqrt{\frac{1}{2}\left(-\frac{a}{b}\right)\frac{ab-3\left(\sqrt{-ab}(\lambda_2-c)+\lambda_1 b\right)}{\lambda_2-c}} \end{array} \tag{13}$$

So, the general solution of Eq. (6) as follows

$$\Re\left(\xi\right) = \sqrt{-\frac{a}{b}}\frac{1}{1+e^{\left(\frac{1}{\alpha}\right)x^{\alpha}-\left(\frac{v}{\alpha}\right)t^{\alpha}}} + \frac{3}{4}\sqrt{-\frac{a}{b}}, \tag{14}$$

and

$$Q\left(\xi\right) = \left[\sqrt{-\frac{a}{b}}\frac{1}{1+e^{\left(\frac{1}{\alpha}\right)x^{\alpha}-\left(\frac{v}{\alpha}\right)t^{\alpha}}} + \frac{3}{4}\sqrt{-\frac{a}{b}}\right]^2 \tag{15}$$

So, complete solution of Eq. (1) as follow

$$\begin{array}{l} q_1\left(x,t\right) = \left[\sqrt{-\frac{a}{b}}\frac{1}{1+e^{\left(\frac{1}{\alpha}\right)x^{\alpha}-\left(\frac{v}{\alpha}\right)t^{\alpha}}} + \frac{3}{4}\sqrt{-\frac{a}{b}}\right]^2 \times \\ e^{i\left(\left(-\frac{\frac{1}{2}\frac{\sqrt{-ab}}{c-\lambda_1}}{\alpha}\right)x^{\alpha} + \left(\frac{\sqrt{\frac{1}{2}\left(-\frac{a}{b}\right)\frac{ab-3\left(\sqrt{-ab}(\lambda_2-c)+\lambda_1 b\right)}{\lambda_2-c}}}{\alpha}\right)t^{\alpha}\right)}, \end{array} \tag{16}$$

Case 2:

$$\begin{aligned}&A_1=-\tfrac{16}{5}\sqrt{\tfrac{a}{b}},A_0=-\sqrt{\tfrac{a}{b}},k=-\tfrac{3}{8}\tfrac{\sqrt{ab}}{c-\lambda_1}\\&\omega=-\tfrac{16}{3}\sqrt{\tfrac{1}{2}\left(\tfrac{a}{b}\right)\tfrac{ab-3\left(\sqrt{ab}(\lambda_2-c)+\lambda_1 b\right)}{\lambda_2-c}}\end{aligned}\tag{17}$$

From case 2 we have

$$\Re(\xi)=-\frac{16}{5}\sqrt{\frac{a}{b}}\frac{1}{1+e^{\left(\frac{1}{\alpha}\right)x^\alpha-\left(\frac{v}{\alpha}\right)t^\alpha}}-\sqrt{\frac{a}{b}},$$

So

$$Q(\xi)=\left[-\frac{16}{5}\sqrt{\frac{a}{b}}\frac{1}{1+e^{\left(\frac{1}{\alpha}\right)x^\alpha-\left(\frac{v}{\alpha}\right)t^\alpha}}-\sqrt{\frac{a}{b}}\right]^2\tag{18}$$

and

$$\begin{aligned}&q_2(x,t)=\left[-\tfrac{16}{5}\sqrt{\tfrac{a}{b}}\tfrac{1}{1+e^{\left(\frac{1}{\alpha}\right)x^\alpha-\left(\frac{v}{\alpha}\right)t^\alpha}}-\sqrt{\tfrac{a}{b}}\right]^2\times\\&e^{i\left(\left(-\frac{\frac{1}{2}\frac{\sqrt{ab}}{c-\lambda_1}}{\alpha}\right)x^\alpha+\left(\frac{\sqrt{\frac{1}{2}\left(\frac{a}{b}\right)\frac{ab-3\left(\sqrt{ab}(\lambda_2-c)+\lambda_1 b\right)}{\lambda_2-c}}}{\alpha}\right)\right)t^\alpha},\end{aligned}\tag{19}$$

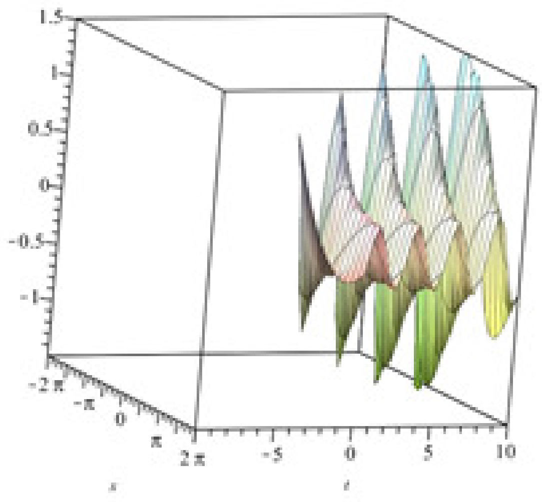

Figure 1 | Dynamic behaviors of $q_1(x,t)$ for $a=-4, b=2, c=2,\ \lambda_1=0.1,\ \lambda_2=0.1,$ $\alpha=0.95$.

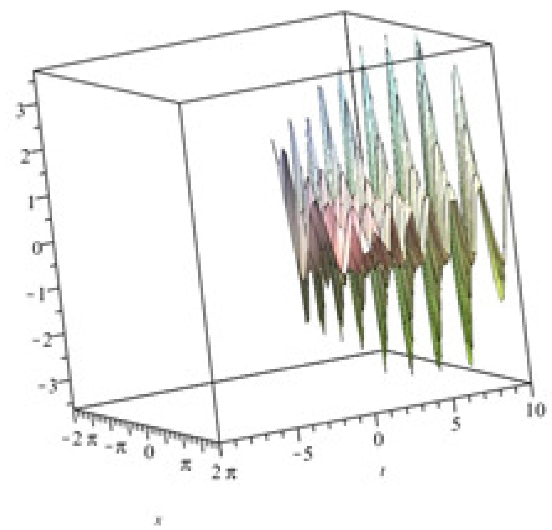

Figure 2 | Dynamic behaviors of $q_1(x,t)$ for $a = 0.2, b = -0.5, c = 0.5$, $\lambda_1 = 3$, $\lambda_2 = 3$, $\alpha = 0.97$.

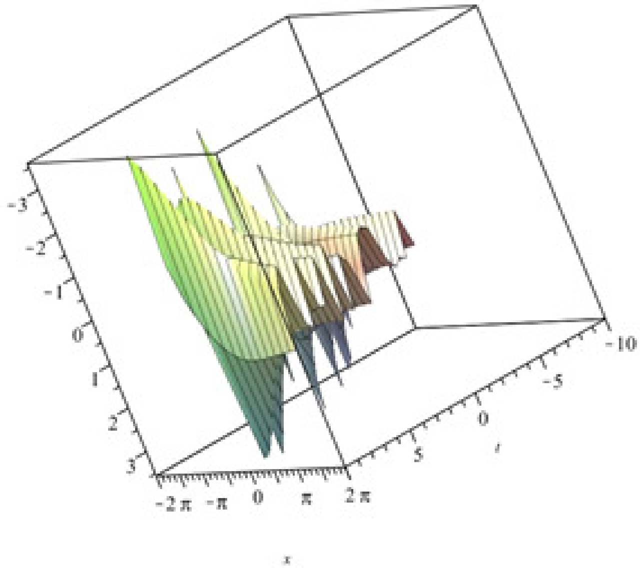

Figure 3 | Dynamic behaviors of $q_1(x,t)$ for $a = 1, b = -3, c = 0.5$, $\lambda_1 = 1$, $\lambda_2 = 4$, $\alpha = 0.9$.

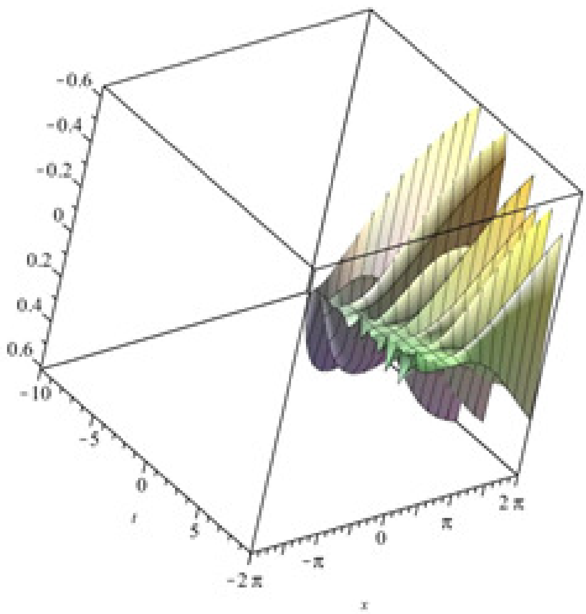

Figure 4 | Dynamic behaviors of $q_2(x,t)$ for $a = -4, b = 2, c = 2,\ \lambda_1 = 0.1,\ \lambda_2 = 0.1,$ $\alpha = 0.95$.

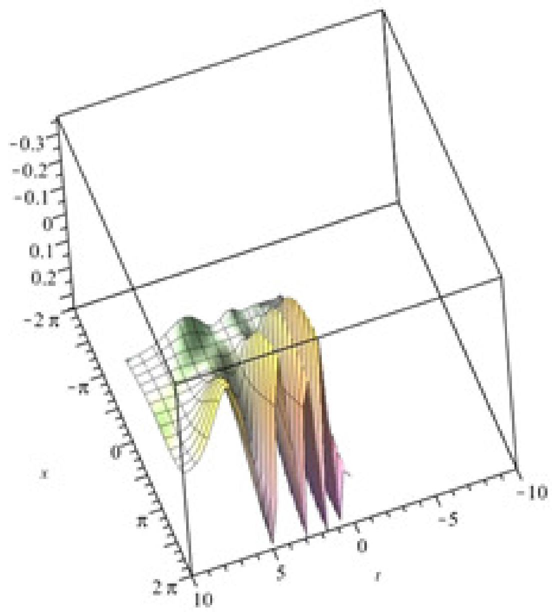

FIGURE 5 | Dynamic behaviors of $q_2(x,t)$ for $a = 0.2, b = -0.5, c = 0.5,\ \lambda_1 = 3,\ \lambda_2 = 3,$ $\alpha = 0.97$.

4. Conclusions

Finding the exact solution to such equations is one of the most challenging problems in mathematics. There is also no specific way of solving many of these equations. In these cases, we must resort to the approximate analytical methods due to the limitations of exact solver methods. According to what stated above, new approaches to solving PDE equations are of great importance and application. The solutions obtained in this paper have not been reported in the old research. The main objective of this paper is to employ a well-known technique called Kudryashov method to solve the perturbed Gerdjikov-Ivanov equation with the comfortable derivative. One of the outstanding features of the model considered in this article is the use of the definition of the comfortable derivative in the structure of the model. This definition is one of the most interesting definitions for a derivative that has many ideal features for a derivative.

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