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A STUDY ON IRREGULARITY IN VAGUE GRAPHS WITH APPLICATION IN SOCIAL RELATIONS

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Abstract. Considering all physical, biological and social systems, fuzzy graph models serves the elemental processes of all natural and artificial structures. As the indeterminate information is an essential real-life problems, which are mostly uncertain, modeling those problems based on fuzzy graph is highly demanding for an expert. Vague graph can manage the uncertainty relevant to the inconsistent and indeterminate information of all real-world problems, in which fuzzy graphs possibly will not succeed into bringing about satisfactory results. Also, vague graphs are so useful tool to examine many issues such as networking, social systems, geometry, biology, clustering, and traffic plan. Hence, in this paper, we introduce strongly edge irregular vague graphs and strongly edge totally irregular vague graphs. A comparative study between strongly edge irregular vague graphs and strongly edge totally irregular vague graphs is done. Finally, we represent an application of irregular vague influence graph to show the importance of irregularity in vague graphs.

Keywords: Vague set, vague graphs, strongly edge irregular, strongly edge totally irregular, social relation.

AMS Subject Classification:05C99, 03E72

1. Introduction

Many real-world situations can accessibly be explained by means of a diagram consisting of a set of points together with lines joining certain pairs of these points. Notice that in such diagrams one is mainly interested in whether or not two given points are joined by a line; the manner in which they are joined is immaterial. A mathematical abstraction of situations of this type gives rise to the concept of a graph. To exemplify the objects and the connection between them, the graph nodes and edges are being employed accordingly. Fuzzy graphs are intended to demonstrate the connection structure among objects so that the concrete object existence (node) and the relationship between two objects (edge) are matters of degree. Fuzzy graph models are advantageous mathematical tools for addressing the combinatorial problems in several fields integrating research, algebra, computing, environmental science, and topology. In 1965, Zadeh [31] proposed fuzzy set

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theory as a model for the exemplification of uncertainty and vagueness in real-world systems. By defining the VS notion through changing the value of an element in a set with a sub-interval of $[0, 1]$, Gau and Buehrer [7] introduced the VS theory. More probabilities are illustrated by vague sets compared to fuzzy sets. A vague set is more effective for explaining the false membership degree existence. Many events in the real world provided the incentive for introducing the fuzzy graphs. Kauffman [8] described fuzzy graphs based on Zadeh's fuzzy relation [32]. Borzooie and Rashmanlou [1, 2, 3, 4, 5, 6] analyzed several concepts of vague graph. Samanta et al. [9, 10, 21, 22, 23, 24, 25, 26, 27, 28] defined fuzzy competition graphs, fuzzy coloring of fuzzy graphs, telecommunication system based on fuzzy graphs, and some results on bipolar fuzzy graphs. Ramakrishna [19] presented the vague graph concepts and examined the properties. Rashmanlou et al. [14, 15, 16, 17, 18] advanced new concepts in vague graphs. Shoaib et al. [29, 30] investigated complex vague graphs, and new concepts in intuitionistic fuzzy graphs. A VG is a generalized structure of a fuzzy graph that provides more exactness, adaptability and compatibility to a system when matched with systems run on fuzzy graphs. Also, a vague graph is able to concentrate on determining the uncertainty coupled with the inconsistent and indeterminate information of any real-world problem, where fuzzy graphs may not lead to adequate the results. Gani and Radha [11] introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. Gani and Latha [12] defined the concept of irregular fuzzy graphs, neighborly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008. Radha and Kumaravel investigated the concept of edge degree, total edge degree and edge regular fuzzy graphs and discussed about the degree of an edge in some fuzzy graphs [13]. In this paper, we represent new concepts of vague graphs such as strongly edge irregular vague graphs and strongly edge totally irregular vague graphs. A comparative study between strongly edge irregular vague graphs and strongly edge totally irregular vague graphs is done. Finally, we give an application of irregular vague influence graph to show the importance of irregularity in vague graphs.

2. Preliminaries

In this section, to consider the stage for our analysis, and to facilitate the following of our discussion, a brief overview of some of the basic definitions is introduced. A graph denotes a pair $G^* = (V, E)$ satisfying $E \subseteq V \times V$. The elements of V and E are the nodes and edges of the graph G^*, correspondingly.

An fuzzy graph has the form of $\xi = (\gamma, \nu)$, where $\gamma : V \to [0, 1]$ and $\nu : V \times V \to [0, 1]$ as is defined as $\nu(uv) \leq \gamma(u) \wedge \gamma(v)$, $\forall u, v \in V$, and ν is a symmetric fuzzy relation on γ and $\wedge$ denotes the minimum.

Definition 2.1. [7] *A vague set A in an ordinary finite non-empty set X, is a pair (t_A, f_A), where $t_a : X \to [0, 1]$, and $f_A : X \to [0, 1]$ are true and false membership functions, respectively such that for all $u \in X$, $0 \leq t_A(u) + f_A(u) \leq 1$.*

Definition 2.2. [19] *A vague graph is defined to be a pair $G = (A, B)$, where $A = (t_A, f_A)$ is a vague set on V and $B = (t_B, f_B)$ is a vague set on $E \subseteq V \times V$ such that for each $uv \in E$, $t_B(uv) \leq t_A(u) \wedge t_A(v)$, $f_B(uv) \geq f_B(u) \vee f_B(v)$. A vague graph G is called strong vague graph if $t_B(uv) = t_A(u) \wedge t_A(v)$, and $f_B(uv) = f_B(u) \vee f_B(v)$, for each edge $uv \in E$.*

Example 2.1. *Consider the vague graph G as shown in Figure 1. By a simple calculation, it is easy to see that G is a vague graph.*

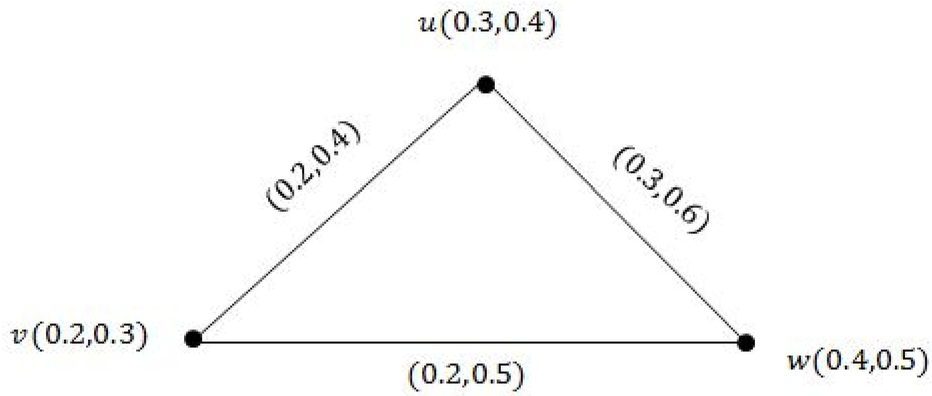

Figure 1. Vague graph G

Definition 2.3. [3] *(i) Let* $G = (A, B)$ *be a vague graph. The degree of a vertex* u *is* $d_G(u) = (d^t_G(u), d^f_G(u))$ *where* $d^t_G(u) = \sum_{u \neq v} t_B(uv)$ *and* $d^f_G(u) = \sum_{u \neq v} f_B(uv)$, *for* $uv \in E$.
(ii) The total degree of a vertex u *is defined as* $td(u) = (td^t(u), td^f(u))$ *where* $td^t(u) = \sum t_B(uv) + t_A(u) = d^t_G(u) + t_A(u)$ *and* $td^f(u) = \sum f_B(uv) + f_A(u) = d^f_G(u) + f_A(u)$, $uv \in E$.

Definition 2.4. [3] *Let* $G = (A, B)$ *be a vague graph. The degree of an edge* uv *is defined as* $d_G(uv) = (d^t_G(uv), d^f_G(uv))$, *where* $d^t_G(uv) = d^t_G(u) + d^t_G(v) - 2t_B(uv)$, $d^f_G(uv) = d^f_G(u) + d^f_G(v) - 2f_B(uv)$. *The minimum degree of an edge is* $\delta_E(G) = \wedge\{d_G(uv) : uv \in E\}$. *The maximum degree of an edge is* $\Delta_E(G) = \vee\{d_G(uv) : uv \in E\}$.

Definition 2.5. [3] *Let* $G = (A, B)$ *be a vague graph. The total degree of an edge* uv *is defined as* $td_G(uv) = (td^t_G(uv), td^f_G(uv))$, *where* $td^t_G(uv) = d^t_G(u) + d^t_G(v) - t_B(uv)$, $td^f_G(uv) = d^f_G(u) + d^f_G(v) - f_B(uv)$. *Note that the degree of an edge* uv *in the underlying edge is defined as* $d_G(uv) = d_G(u) + d_G(v) - 2$.

Definition 2.6. [4] *Let* $G = (A, B)$ *be a vague graph.*
(i) The order of G *is defined to be* $O(G) = \big(O_t(G), O_f(G)\big)$, *where* $O_t(G) = \sum_{u \in V} t_A(u)$ *and* $O_f(G) = \sum_{u \in V} f_A(u)$.
(ii) The size of G *is defined to be* $S(G) = \big(S_t(G), S_f(G)\big)$, *where* $S_t(G) = \sum_{u \neq v} t_B(uv)$ *and* $S_f(G) = \sum_{u \neq v} f_B(uv)$.

Definition 2.7. [4] *A vague graph* G *is said to be:*
(i) (M_1, M_2)*-regular if* $d_G(v_i) = (M_1, M_2)$, *for all* $v_i \in V$ *and also* G *is said to be regular vague graph of degree* (M_1, M_2).
(ii) (M_1, M_2)*-totally regular if* $td_G(v_i) = (M_1, M_2)$, *for all* $v_i \in V$.

3. Strongly edge irregular vague graphs

In this section, strongly edge irregular vague graphs and strongly edge totally irregular vague graphs are introduced.

Definition 3.1. *Let* $G = (A, B)$ *be a vague graph. Then,*
(i) G *is said to be a irregular vague graph is there exists a vertex which is adjacent to a*

vertices with distinct degrees.
(ii) *G is said to be a totally irregular vague graph is there exists a vertex which is adjacent to a vertices with distinct total degrees.*
(iii) *G is said to be a strongly irregular vague graph if every pair of vertices have distinct degree.*
(iv) *G is said to be a highly irregular vague graph if every vertex in G is adjacent to the vertices having distinct degrees.*

Definition 3.2. *Let $G = (A, B)$ be a connected vague graph. Then*
(i) *G is said to be a neighborly edge irregular vague graph if every pair of adjacent edges have distinct degrees.*
(ii) *G is said to be a neighborly edge totally irregular vague graph if every pair of adjacent edges having distinct total degrees.*
(iii) *G is said to be a strongly edge irregular vague graph if every pair of edges having distinct degrees (or) no two edges have same degree.*
(iv) *is said to be a strongly edge totally irregular vague graph if every pair of edges having distinct total degrees or no two edges have same total degree.*

Example 3.1. *Consider the vague graph $G = (A, B)$ as Figure 2. We have:*

$$d_G(u) = (0.4, 0.9),\ d_G(v) = (0.3, 1.2),\ d_G(w) = (0.3, 1.3)$$
$$d_G(x) = (0.3, 1.1),\ d_G(y) = (0.5, 1.9)$$

Degrees of the edges are calculated as follows:

$$d_G(uv) = (0.5, 1.1),\ d_G(vw) = (0.2, 1.1),\ d_G(wx) = (0.4, 1.2)$$
$$d_G(xy) = (0.4, 2),\ d_G(yu) = (0.3, 2).$$

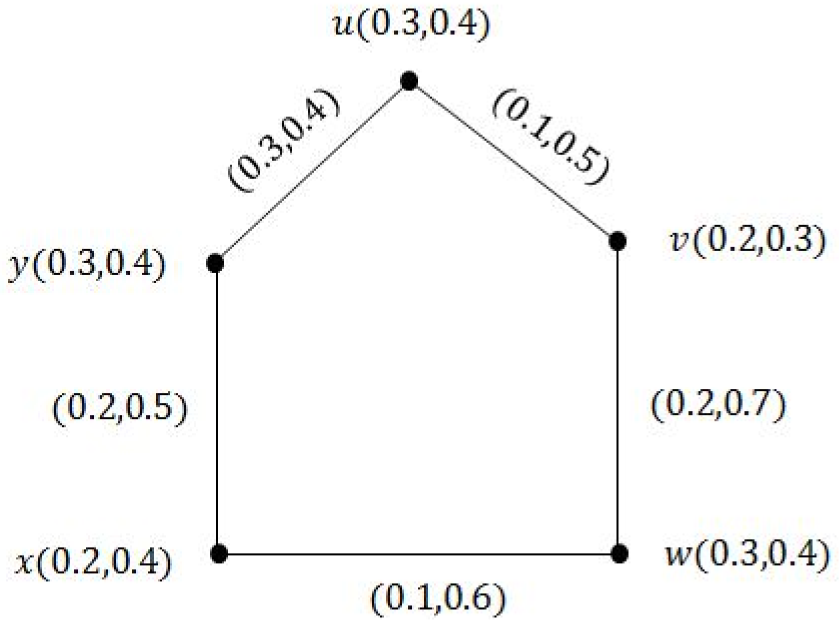

FIGURE 2. Vague graph G

It is clear that every pair of edges having distinct degrees, so, G is strongly edge irregular vague graph. Total degrees of the edges are as follows:

$$\begin{aligned}
td^t_G(uv) &= d^t_G(uv) + t_B(uv) = 0.5 + 0.1 = 0.6,\\
td^f_G(uv) &= d^f_G(uv) + f_B(uv) = 1.1 + 0.5 = 1.6,\\
td^t_G(vw) &= d^t_G(vw) + t_B(vw) = 0.2 + 0.2 = 0.4,\\
td^f_G(vw) &= d^f_G(vw) + f_B(vw) = 1.1 + 0.7 = 1.8,\\
td^t_G(wx) &= d^t_G(wx) + t_B(wx) = 0.4 + 0.1 = 0.5,\\
td^f_G(wx) &= d^f_G(wx) + f_B(wx) = 1.2 + 0.6 = 1.8,\\
td^t_G(xy) &= d^t_G(xy) + t_B(xy) = 0.4 + 0.2 = 0.6,\\
td^f_G(xy) &= d^f_G(xy) + f_B(xy) = 2 + 0.5 = 2.5,\\
td^t_G(yu) &= d^t_G(yu) + t_B(yu) = 0.3 + 0.3 = 0.6,\\
td^f_G(yu) &= d^f_G(yu) + f_B(yu) = 2 + 0.4 = 2.4.
\end{aligned}$$

It is obvious that every pair of edges in G having distinct total degrees, hence, G is strongly edge totally irregular vague graph. Therefore, G is both strongly edge irregular vague graph and strongly edge totally irregular vague graph.

Example 3.2. *Strongly edge irregular vague graphs need not be strongly edge totally irregular vague graphs. From Figure 3 we have:*

$$\begin{aligned}
&d_G(u) = (0.5, 1),\ d_G(v) = (0.4, 1.3),\ d_G(w) = (0.3, 1.1),\\
&d_G(uv) = (0.3, 1.1),\ d_G(vw) = (0.5, 1),\ d_G(wu) = (0.4, 1.3),\\
&td_G(uv) = (0.6, 1.7),\ td_G(vw) = (0.6, 1.7),\ td_G(wu) = (0.6, 1.7)
\end{aligned}$$

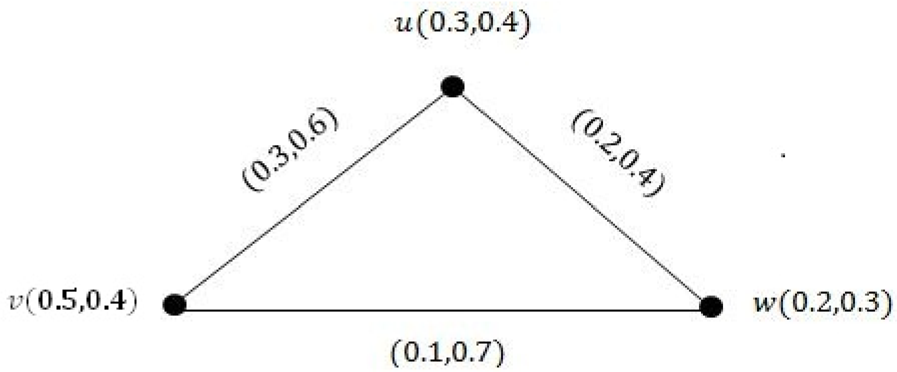

FIGURE 3. Vague graph G

We note that G is strongly edge irregular vague graph, since every pair of edges having distinct degrees. Also, G is not strongly edge totally irregular vague graph, since all the edges having same total degree.

Hence, strongly edge irregular vague graph need not be strongly edge totally irregular vague graph.

Theorem 3.1. *Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$ and (t_B, f_B) is a constant function. If G is strongly edge irregular vague graph, then G is strongly edge totally irregular vague graph.*

Proof. Assume that (t_B, f_B) is a constant function.Let $(t_B(uv), f_B(uv)) = (c_1, c_2)$, for all $uv \in E$, where (c_1, c_2) is constant. Let uv and xy be any pair of edges in E. Suppose that G is strongly edge irregular vague graph. Then $d_G(uv) \neq d_G(xy)$, where uv and xy are any pair of edges in E. So, $d_G(uv) \neq d_G(xy)$, $d^t_G(uv) + c_1 \neq d^t_G(xy) + c_1$, $d^f_G(uv) + c_2 \neq d^f_G + c_2(xy)$, $d^t_G(uv) + t_B(uv) \neq d^t_G(xy) + t_B(xy)$, $d^f_G(uv) + f_B(uv) \neq d^f_G(xy) + f_B(xy)$, $td^t_G(uv) \neq td^t_G(xy)$, $td^f_G(uv) \neq td^f_G(xy)$, where uv and xy are any pair of edges in E.

Hence, G is strongly edge totally irregular vague graph. □

Theorem 3.2. *Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$ and (t_B, f_B) is a constant function. If G is strongly edge totally irregular vague graph, then G is strongly edge irregular vague graph.*

Proof. Assume that (t_B, f_B) is a constant function.Let $(t_B(uv), f_B(uv)) = (c_1, c_2)$, for all $uv \in E$, where (c_1, c_2) is constant. Let uv and xy be any pair of edges in E. Suppose that G is strongly edge totally irregular vague graph. Then $td^t_G(uv) \neq td^f_G(xy)$, $td^f_G(uv) \neq td^f_G(xy)$, where uv and xy are any pair of edges in E. $td^t_G(uv) \neq td^f_G(xy)$, $td^f_G(uv) \neq td^f_G(xy)$. Hence, $d^t_G(uv) + t_B(uv) \neq d^t_G(xy) + t_B(xy)$ and $d^f_G(uv) + f_B(uv) \neq d^f_G(xy) + f_B(xy)$. Therefore, $d^t_G(uv) + c_1 \neq d^t_G(xy) + c_1$, $d^f_G(uv) + c_2 \neq d^f_G + c_2(xy)$, where uv and xy are any pair of edges in E. So, G is strongly edge irregular vague graph. □

Remark 3.1. *Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$. If G is both strongly edge irregular vague graph and strongly edge totally irregular vague graph, then (t_B, f_B) need not be a constant function.*

Example 3.3. *In Example 3.1, G is both strongly edge irregular vague graph and strongly edge totally irregular vague graph. But (t_B, f_B) is not constant function.*

Theorem 3.3. *Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$. If G is strongly edge irregular vague graph, then G is neighborly edge irregular vague graph.*

Proof. Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$. Let us assume that G is strongly edge irregular vague graph. Then every pair of edges in G have distinct degrees. Hence, every pair of adjacent edges have distinct degrees. So, G is neighborly edge irregular vague graph. □

Theorem 3.4. *Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$. If G is strongly edge totally irregular vague graph, then G is neighborly edge totally irregular vague graph.*

Proof. Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$. Let us assume that G is strongly edge totally irregular vague graph, then every pair of edges in G have distinct total degrees. So, every pair of adjacent edges have distinct degrees. Hence, G is neighborly edge total irregular vague graph. □

Remark 3.2. *Converse of the above Theorems 3.3 and 3.4 need not be true.*

Example 3.4. *Consider vague graph $G = (A, B)$ on $G^* = (V, E)$ that ia a path on four vertices.*

From Figure 4, $d_G(u) = (0.1, 0.5)$, $d_G(v) = (0.2, 1)$, $d_G(w) = (0.2, 1)$, $d_G(x) = (0.1, 0.5)$, $d_G(uv) = (0.1, 0.5)$, $d_G(vw) = (0.2, 1)$, $d_G(wx) = (0.1, 0.5)$. Here, $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Hence, G is neighborly edge irregular vague graph. But G is not

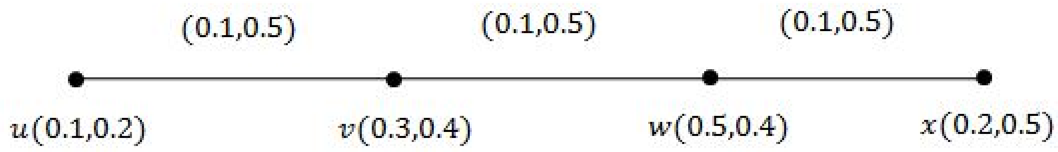

FIGURE 4. Vague graph G

strongly edge irregular vague graph, since $d_G(uv) = d_G(wx)$. Also, $td_G(uv) = (0.2, 1)$, $td_G(vw) = (0.3, 1.5)$, $td_G(wx) = (0.2, 1)$. Note that $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence, G is neighborly edge totally irregular vague graph. But G is not strongly edge totally irregular vague graph, since $td_G(uv) = td_G(wx)$.

Theorem 3.5. *Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$ and (t_B, f_B) is a constant function. If G is strongly edge irregular vague graph, then G is an irregular vague graph.*

Proof. Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$. Assume that (t_B, f_B) is a constant function, let $(t_B(uv), f_B(uv)) = (c_1, c_2)$, for all $uv \in E$, where (c_1, c_2) is constant. Let us suppose that G is strongly edge irregular vague graph. Then every pair of edges having distinct degrees. Let uv and vw are adjacent edges in G having distinct degrees. Then $d^t_G(uv) \neq d^t_G(vw)$ and $d^f_G(uv) \neq d^f_G(vw)$. So, $d^t_G(u) + d^t_G(v) - 2t_B(uv) \neq d^t_G(v) + d^t_G(w) - 2t_B(vw)$ and $d^f_G(u) + d^f_G(v) - 2f_B(uv) \neq d^f_G(v) + d^f_G(w) - 2f_B(vw)$. Hence, $d^t_G(u) + d^t_G(v) - 2c_1 \neq d^t_G(v) + d^t_G(w) - 2c_1$ and $d^f_G(u) + d^f_G(v) - 2c_2 \neq d^f_G(v) + d^f_G(w) - 2c_2$. Therefore, $d^t_G(u) + d^t_G(v) \neq d^t_G(v) + d^t_G(w)$ and $d^f_G(u) + d^f_G(v) \neq d^f_G(v) + d^f_G(w)$, that we get $d_G(u) \neq d_G(w)$. So, there exists a vertex v which is adjacent to a vertices u and w having distinct degrees. Hence, G is an irregular vague graph.

□

Theorem 3.6. *Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$ and (t_B, f_B) is a constant function. If G is strongly edge irregular vague graph, then G is highly irregular vague graph.*

Proof. Let $G = (A, B)$ be a connected vague graph. Assume that (t_B, f_B) is a constant function. Let $(t_B(uv), f_B(uv)) = (c_1, c_2)$, for all $uv \in E$, where (c_1, c_2) is constant. Let v be any vertex adjacent with u, w, and x. Then uv, vw and vx are adjacent edges in G. Let us suppose that G is strongly edge irregular vague graph. Then every pair of edges having distinct degrees. So, every pair of adjacent edges in G have distinct degrees Hence, $d^t_G(uv) \neq d^t_G(vw) \neq d^t_G(vx)$. So, and $d^t_G(u) + d^t_G(v) - 2t_B(uv) \neq d^t_G(v) + d^t_G(w) - 2t_B(vw) \neq d^t_G(v) + d^t_G(x) - 2t_B(vx)$. Hence, $d^t_G(u) + d^t_G(v) - 2c_1 \neq d^t_G(v) + d^t_G(w) - 2c_1 \neq d^t_G(v) + d^t_G(x) - 2c_1$. Therefore, $d^t_G(u) + d^t_G(v) \neq d^t_G(v) + d^t_G(w) \neq d^t_G(v) + d^t_G(x)$. So, $d^t_G(u) \neq d^t_G(w) \neq d^t_G(x)$. Similarly, we can show $d^f_G(u) \neq d^f_G(w) \neq d^f_G(x)$. The vertex v is adjacent to the vertices with distinct degrees. Hence, G is highly irregular vague graph. □

Theorem 3.7. *Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$ and (t_B, f_B) is a constant function. If G is strongly edge totally irregular vague graph, then G is highly irregular vague graph.*

Proof. It is similar to the above theorem. □

Theorem 3.8. *Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$, a path on $2m$ $(m > 1)$ vertices. If the membership value of the edges $e_1, e_2, \cdots, e_{2m-1}$ are respectively $(c_1, t_1), (c_2, t_2), (c_3, t_3), \cdots, (c_{2m-1}, t_{2m-1})$ such that $c_1 < c_2 < c_3 < \cdots < c_{2m-1}$ and $t_1 > t_2 > t_3 \cdots > t_{2m-1}$. Then G is both strongly edge irregular vague graph and strongly edge totally irregular vague graph.*

Proof. Let $G = (A, B)$ be a connected vague graph on $G^* = (V, E)$ and $e_1, e_2, \cdots, e_{2m-1}$ be the edges of a path G^* in that order. Let the membership values of the edges $e_1, e_2, \cdots, e_{2m-1}$ are $(c_1, t_1), (c_2, t_2), (c_3, t_3), \cdots, (c_{2m-1}, t_{2m-1})$ respectively such that: $c_1 < c_2 < c_3 < \cdots < c_{2m-1}$ and $t_1 > t_2 > t_3 \cdots > t_{2m-1}$. Then, $d^t_G(e_i) = c_{i-1} + c_i$, for $i = 2, 3, 4, 5, \cdots, 2m-1$, $d^t_G(v_1) = c_1$, $d^t_G(v_{2m}) = c_{2m-1}$, $d_G(e_i) = c_{i-1} + c_{i+1}$, for $i = 2, 3, \cdots, 2m-2$. $d_G(e_1) = c_2$, $d_G(e_{2m-1}) = c_{2m-2}$. Hence, G is strongly edge irregular vague graph since $td^t_G(e_i) = c_{i-1} + c_{i+1} + c_i$, for $i = 2, 3, 4, 5, \cdots, 2m-2$, $td^t_G(e_1) = c_2 + c_1 + c_i$, $td^t_G(e_{2m-1}) = c_{2m-2} + c_{2m-1}$, G is strongly edge totally irregular vague graph. □

4. Application Example of vague influence graph in Social Relation

Graph models find wide application in many areas of mathematics, computer science, and the natural and social sciences. Often these models need to incorporate more structure than simply the adjacencies between vertices. In studies of group behavior, it is observed that certain people can influence thinking of others. A directed graph, called an influence graph, can be used to model this behavior. Each person of a group is represented by a vertex. There is a directed edge from vertex x to vertex y, when the person represented by vertex x influence the person represented by vertex y. This graph does not contain loops and it does not contain multiple directed edges. We now explore vague influence graph model to find out the influential person within a social group. In influence graph, the vertex (node) represents a power (authority) of a person and the edge represents the influence of a person on another person in the social group. Consider a vague influence graph of a social group. In Fig. 5, vague influence graph, the degree of power of a person is defined in terms of its trueness and falseness. The node of the vague influence graph shows the authority a person possesses in the group; for example, $Jafar$ has 20% authority in the group, but he does not have 50% power, and 30% power is not decided, whereas the edges show the influence of a person on another in a group; for example $Jafar$ can influence $Kamran$ 10%, but he can not convince him 50%, and remaining 40% is hesitation part. The degree of a vertex and edge in a vague influence graph is also characterized by an interval $[t_A(x), 1 - f_A(x)]$. It is worth mentioning here that interval-valued fuzzy sets are not vague sets. In vague sets both are independently proposed by the decision maker. Thus a vague influence graph can be interpreted in the form of interval-valued membership. The node of the vague influence graph shows the likelihood of power a person possesses in the group; for example, $Jafar$ possesses $t_A = 20\%$ to $1 - f_A = 50\%$ power, whereas the edges show the interval of influence a person has on another person in a social group. $Jafar$ has $t_A = 10\%$ to $1 - f_A = 50\%$ influence on $Kamran$ and $Kamran$ has $t_A = 10\%$ to $1 - f_A = 20\%$ influence on $Emran$.

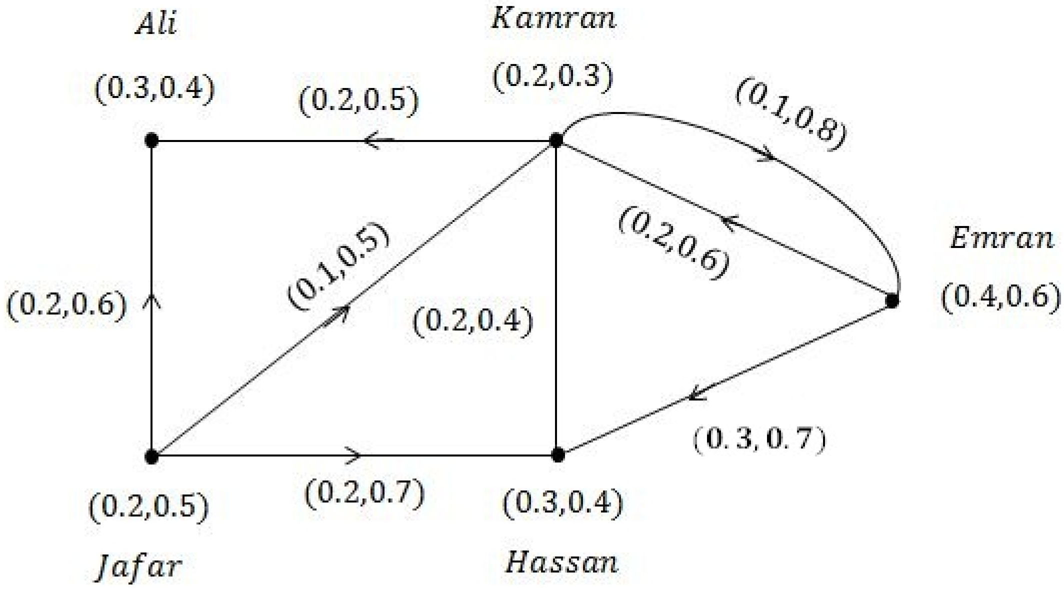

FIGURE 5. Vague influence graph

5. Conclusion

Considering the precision, elasticity, and compatibility in a system, vague models outweigh the other FGs. The VG concept generally has a large variety of applications in different areas such as computer science, operation research, topology, and natural networks. In this paper, we introduced new kinds of irregularity in vague graphs such as strongly edge irregular and strongly edge totally irregular vague graphs. A comparative study between strongly edge irregular vague graphs and strongly edge totally irregular vague graphs is done. Finally an application of irregular vague influence graph in social relations has given. In our future work, we will introduce vague graph structure and study new kinds of irregularity on it.

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