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STANDARD SINGLE VALUED NEUTROSOPHIC METRIC SPACES WITH APPLICATION

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Abstract. In recent years, J. R. Kider and Z.A. Hussain have introduced the notion of standard fuzzy metric space and some basic properties were proved. In this paper, we generalize this notion to the setting of single valued neutrosophic sets. Furthermore, some interesting properties related to this notion are studied such as the continuity property of the mappings defined on standard single valued neutrosophic metric spaces. As an application, we establish the Edelstein fixed point theorem for standard single valued neutrosophic metric spaces.

Keywords: single valued neutrosophic set; metric space; continuous mapping; fixed point property.

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1. Introduction

A metric space is a particular type of topological space where the topology is defined by a distance function. It is one of the important basic areas of research for the mathematicians. In 1906, Maurice Fréchet in his doctoral thesis [9] introduced the concept of a metric space. However, the name is due to Felix Hausdorff. Later on, many researchers developed this concept either by introducing different contractions in different fields or by extending number of variables in it. Metric spaces have applications in many mathematical branches, as well as in topology and its analysis approaches (e.g. [2]), they appear also in differential geometry (e.g. [31]), linear algebra (e.g. [12]), geographic information systems (e.g. [8]), computer science (e.g. [19]) and in coding theory (e.g. [7]).

Recently, Smarandache [23] generalized the concepts of intuitionistic fuzzy sets and other types of sets to the notion of neutrosophic sets. He introduce this concept to know the correct way of dealing with imprecise and indeterminate data. Neutrosophic sets are characterized by three independent components such that truth membership function (T),

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indeterminacy membership function (I) and falsity membership function (F). Many researchers like Abdel-Basset et al. , Broumi et al. [3,4] and others have studied and applied in various fields the neutrosophic sets and its different extensions such as medical diagnosis (e.g. [34,36]), management (e.g. [1]), decision making problems (e.g. [10,35]), image processing (e.g. [5]), educational problem (e.g. [18]), conflict resolution (e.g. [20]) and in social problems (e.g. [21]). In particular, Wang et al. [32] introduced the notion of single valued neutrosophic set as an instance of neutrosophic set which can be used in real scientific and engineering applications. Since its introduction, it has become omnipresent, where the studies whether theoretical or applied on the single valued neutrosophic set have been advancing at an accelerated pace. For more details on single valued neutrosophic set and background, the readers are referred to [13,17] and more others.
From the idea of neutrosophic sets, Kirişci and Şimşek [15] defined the neutrosophic metric spaces (NMS). They investigated some of their properties and proved that Baire category theorem and uniform convergence theorem. Thereafter, Kirişci et al. [16] introduced the neutrosophic contractive mapping and gave a fixed point results in complete neutrosophic metric spaces. Şimşek and Kirişci [26] discussed some fixed point results in (NMS), where they defined new infinite products by continuous t-norms and proved the Banach contraction theorem for (NMS) by these products. Moreover, there are many researchers have created other types of metric spaces in the context of the neutrosophic theory, where the latter also still knows many developments such as Şahin and Kargin [27] introduced the notion of neutrosophic triplet v-generalized metric space (NTVGM) in which that the concept of neutrosophic triplet (NT) is a new theory in neutrosophy, and the v-generalized metric is a specific form of the classical metrics. In this study, they proved that (NTVGM) is different from the classical metric and neutrosophic triplet metric (NTM). In [30], Taş et al. defined the neutrosophic valued metric spaces and the neutrosophic valued g-metric spaces. Furthermore, they determined a mathematical model for clustering the neutrosophic big data sets with the use of g-metric. In this regard, we find that other authors have adopted the same approach, such as Şahin et al. [28,29].

Motivated by the recent relevant developments to this context, in the present paper, we introduce the notion of standard single valued neutrosophic metric space as a generalization of standard fuzzy metric space introduced by Kider and Hussain [14] and many fundamental properties related to this concept are investigated, especially, the continuity property because of its frequent use in many applications such as in fixed point theory. As an application, we extend the Edelstein fixed point theorem [6] to the standard single valued neutrosophic metric spaces. It is worth noting here that the adoption of the new definition of neutrosophic metric space which we called standard single valued neutrosophic metric space, allowed us to obtain many results and demonstrated them smoothly to a large extent. Specifically, during the generalization from the fuzzy case.

This paper is structured as follows. In Section 2, we recall the necessary basic notions and properties of standard fuzzy metric space and single valued neutrosophic sets with some related concepts that will be needed throughout this paper. In Section 3, the notion of standard fuzzy metric space is introduced and some fundamental properties related to this concept are studied. Subsequently, several interesting properties of single valued neutrosophic continuous mappings in standard single valued neutrosophic metric space are investigated. As an application, Section 4 is devoted to explaining a generalization of Edelstein's fixed point theorem in a standard single value neutrosophical metric space. We end with some conclusions and discuss future work in Section 5.

2. Preliminaries

This section contains the basic definitions and properties of single valued neutrosophic sets and some related notions that will be needed throughout this paper.

2.1. **Standard fuzzy metric space.** The standard fuzzy metric space has been mentioned as an example of fuzzy metric in a particular case by George and Veeramani [11] and its definition was introduced with studied extensively by Kider and Hussain [14].

Definition 2.1. [14] *A triple $(X, M, *)$ is said to be standard fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a continuous fuzzy set on X^2 satisfying the following conditions:*

(i) $\mu_M(x, y) > 0$ *for all* $x, y \in X$;
(ii) $\mu_M(x, y) = 1$ *if and only if* $x = y$;
(iii) $\mu_M(x, y) = \mu_M(y, x)$ *for all* $x, y \in X$;
(iv) $\mu_M(x, z) \geq \mu_M(x, y) * \mu_M(y, z)$ *for all* $x, y, z \in X$.

Example 2.1. *Let $X = \mathbb{R}$. Define the t-norm $a * b = ab$, for all $a,\ b \in [0, 1]$ and let M be fuzzy set on X^2 defined as: $\mu(x, y) = \exp^{-|x-y|}$.*
It is clear that all the conditions from (i) to (iv) realized. Therefore, $(X, M, *)$ is a standard fuzzy metric space.

2.2. **Single valued neutrosophic sets.** In 1998, Smarandache [23] defined the concept of a neutrosophic set as a generalization of Atanassov's intuitionistic fuzzy set. Also, he introduced neutrosophic logic, neutrosophic set and its applications in [24, 25]. In particular, Wang et al. [32] introduced the notion of a single valued neutrosophic set.

Definition 2.2. [24] *Let X be a nonempty set. A neutrosophic set (NS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a membership function $\mu_A : X \to]^-0, 1^+[$ and an indeterminacy function $\sigma_A : X \to]^-0, 1^+[$ and a non-membership function $\nu_A : X \to]^-0, 1^+[$ which satisfy the condition:*

$$^-0 \leq \mu_A(x) + \sigma_A(x) + \nu_A(x) \leq 3^+,\ \text{for any } x \in X.$$

Next, we show the notion of single valued neutrosophic set as an instance of neutrosophic set which can be used in real scientific and engineering applications.

Definition 2.3. [32] *Let X be a nonempty set. A single valued neutrosophic set (SVNS, for short) A on X is an object of the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$ characterized by a truth-membership function $\mu_A : X \to [0, 1]$, an indeterminacy-membership function $\sigma_A : X \to [0, 1]$ and a falsity-membership function $\nu_A : X \to [0, 1]$. A continuous single valued neutrosophic set can be defined as $A = \int_X \langle x, \mu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X$.*

The class of single valued neutrosophic sets on X is denoted by $SVN(X)$.
For any two SVNSs A and B on a set X, several operations are defined (see, e.g., [32,33]). Here we will present only those which are related to the present paper.

(i) $A \subseteq B$ if $\mu_A(x) \leq \mu_B(x)$ and $\sigma_A(x) \leq \sigma_B(x)$ and $\nu_A(x) \geq \nu_B(x)$, for all $x \in X$,
(ii) $A = B$ if $\mu_A(x) = \mu_B(x)$ and $\sigma_A(x) = \sigma_B(x)$ and $\nu_A(x) = \nu_B(x)$, for all $x \in X$,
(iii) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x)\rangle \mid x \in X\}$,
(iv) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x), \nu_A(x) \wedge \nu_B(x)\rangle \mid x \in X\}$,
(v) $\overline{A} = \{\langle x, 1 - \nu_A(x), 1 - \sigma_A(x), 1 - \mu_A(x)\rangle \mid x \in X\}$,
(vi) $[A] = \{\langle x, \mu_A(x), \sigma_A(x), 1 - \mu_A(x)\rangle \mid x \in X\}$,
(vii) $\langle A\rangle = \{\langle x, 1 - \nu_A(x), \sigma_A(x), \nu_A(x)\rangle \mid x \in X\}$.

3. Standard single valued neutrosophic metric space

In this section, we generalize the notion of standard fuzzy metric space introduced by Kider and Hussain [14] to the setting of single valued neutrosophic sets. Also, the main properties related to standard single valued neutrosophic metric space are studied.

3.1. Definitions.

Definition 3.1. *A quintuple $(X, M, *, \triangleleft, \diamond)$ is said to be a standard single valued neutrosophic metric space if X is an arbitrary set, $*$, $\triangleleft$ are a continuous t-norms, $\diamond$ is a t-conorm and M is a continuous single valued neutrosophic set on X^2 satisfying the following conditions:*

(i) $\mu_M(x,y) > 0,\ \sigma_M(x,y) > 0$ *and* $\nu_M(x,y) < 1$ *for all* $x, y \in X$*;*
(ii) $\mu_M(x,y) = 1,\ \sigma_M(x,y) = 1$ *and* $\nu_M(x,y) = 0$ *if and only if* $x = y$*;*
(iii) $\mu_M(x,y) = \mu_M(y,x),\ \sigma_M(x,y) = \sigma_M(y,x)$ *and* $\nu_M(x,y) = \nu_M(y,x)$ *for all* $x, y \in X$*;*
(iv) $\mu_M(x,z) \geq \mu_M(x,y) * \mu_M(y,z),\ \sigma_M(x,z) \geq \sigma_M(x,y) \triangleleft \sigma_M(y,z)$ *and* $\nu_M(x,z) \leq \nu_M(x,y) \diamond \nu_M(y,z)$ *for all* $x, y \in X$*.*

Here, $\mu_M(x,y)$, $\sigma_M(x,y)$ and $\nu_M(x,y)$ denotes the degree of nearness, the degree of neutralness, and the degree of non-nearness between x and y, respectively.

Remark 3.1. *If the set X given in the previous definition is a metric space with an ordinary distance d, then $(X, M, *, \triangleleft, \diamond)$ is called a standard single valued neutrosophic metric space induced by (X, d), and the single valued neutrosophic distance M is called a standard if it is induced by the ordinary distance d.*

Example 3.1. *Let (X, d) be an ordinary metric space. Define the t-norms $*$ and $\triangleleft$ by: $x * y = min\{x, y\}$, $x \triangleleft y = min\{x, y\}$, and the t-conorm $\diamond$ by: $x \diamond y = max\{x, y\}$, for all $x,\ y \in [0, 1]$. Define the single valued neutrosophic set M on X^2 as:*
$\mu_M(x,y) = \frac{1}{1+d(x,y)},\ \sigma_M(x,y) = 1 + d(x,y),\ \nu_M(x,y) = \frac{d(x,y)}{1+d(x,y)}$.
Then, $(X, M, *, \triangleleft, \diamond)$ is a standard single valued neutrosophic metric space.

Remark 3.2. *Let $*$ be a t-norm, $\diamond$ be a t-conorm and $r_1, r_2 \in [0, 1]$. If $r_1 > r_2$, then there exist $r_3, r_4 \in]0, 1[$ such that $r_1 * r_3 \geq r_2$ and $r_1 \geq r_2 \diamond r_4$. Moreover, for any $r_5 \in]0, 1[$, there exist $r_6, r_7 \in]0, 1[$ such that $r_6 * r_6 \geq r_5$ and $r_7 \diamond r_7 \leq r_5$.*

Remark 3.3. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. Then, μ_M, σ_M are a monotone mappings, and ν_M is an antitone mapping.*

Next, we introduce the standard single valued neutrosophic distance between an element and a subset of X and the standard single valued neutrosophic distance between two subsets of X.

Definition 3.2. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. For $x \in X$ and A, B are two subsets of X. Then*

(i) *the standard single valued neutrosophic distance between x and A is defined as*
$$\mu_M(x,A) = inf\{\mu_M(x,y) \mid y \in A\},\ \sigma_M(x,A) = inf\{\sigma_M(x,y) \mid y \in A\}$$
$$\text{and } \nu_M(x,A) = sup\{\nu_M(x,y) \mid y \in A\};$$
(ii) *the standard single valued neutrosophic distance between A and B is defined as*
$$\mu_M(A,B) = inf\{\mu_M(x,y) \mid x \in A, y \in B\},\ \sigma_M(A,B) = inf\{\sigma_M(x,y) \mid x \in A, y \in B\}$$
$$\text{and } \nu_M(A,B) = sup\{\nu_M(x,y) \mid x \in A, y \in B\}.$$

Definition 3.3. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. For $x \in X$ and $r \in]0, 1[$, the open ball $\mathcal{B}(x, r)$ with radius r and center x is defined by*

$$\mathcal{B}(x, r) = \{y \in X \mid \mu_M(x, y) > 1 - r,\ \sigma_M(x, y) > 1 - r \text{ and } \nu_M(x, y) < r\}.$$

Definition 3.4. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. A subset A of X is said to be an open set (OS, for short) if for any $x \in A$ there exists $r \in]0, 1[$ such that $\mathcal{B}(x, r) \subseteq A$. The complement of an open set is called a closed set (CS, for short) in X.*

Definition 3.5. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. Then*

(i) *a sequence (x_n) in X is said to be convergent to a point $x \in X$ (i.e., $\lim_{n\to\infty} x_n = x$) if,*

$$\lim_{n\to\infty} \mu_M(x_n, x) = 1,\ \lim_{n\to\infty} \sigma_M(x_n, x) = 1 \text{ and } \lim_{n\to\infty} \nu_M(x_n, x) = 0;$$

(ii) *a sequence (x_n) in X is said to be Cauchy sequence if, for each $k > 0$,*

$$\lim_{n\to\infty} \mu_M(x_{n+k}, x_n) = 1,\ \lim_{n\to\infty} \sigma_M(x_{n+k}, x_n) = 1 \text{ and } \lim_{n\to\infty} \nu_M(x_{n+k}, x_n) = 0.$$

Definition 3.6. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. Then*

(i) *if every Cauchy sequence is convergent, then X is said to be complete.*
(ii) *X is said to be compact if every sequence contains a convergent subsequence.*

3.2. **Fundamental properties of standard single valued neutrosophic metric space.** In this section, we investigate the fundamental properties of standard single valued neutrosophic metric space that will be used in the following section.

Proposition 3.1. *Every open ball in a standard single valued neutrosophic metric space $(X, M, *, \triangleleft, \diamond)$ is an open set.*

Proof. Let $\mathcal{B}(x, r)$ be an open ball with radius r and center x, where $r \in]0, 1[$ and $x \in X$. Suppose that $y \in \mathcal{B}(x, r)$, this implies that

$$\mu_M(x, y) > 1 - r,\ \sigma_M(x, y) > 1 - r \text{ and } \nu_M(x, y) < r.$$

Let $r_0 = \mu_M(x, y)$. Then, there exist $s \in]0, 1[$ such that $r_0 > 1 - s > 1 - r$. Now, for a given r_0 and s such that $r_0 > 1 - s$. Then, Remark 3.2 guarantees that there exist $r_1, r_2, r_3 \in]0, 1[$ such that

$$r_0 * r_1 \geq 1 - s, r_0 \triangleleft r_2 \geq 1 - s \text{ and } (1 - r_0) \diamond (1 - r_3) \leq s.$$

Next, if we put $r_4 = \max\{r_1, r_2, r_3\}$ and consider the open ball $\mathcal{B}(y, 1 - r_4)$, then from the above, we can show that $\mathcal{B}(y, 1 - r_4) \subset \mathcal{B}(x, r)$ as follows:
Let $z \in \mathcal{B}(y, 1 - r_4)$. Then, $\mu_M(y, z) > r_4$, $\sigma_M(y, z) > r_4$ and $\nu_M(y, z) < 1 - r_4$. Furthermore, we obtain

$$\mu_M(x, z) \geq \mu_M(x, y) * \mu_M(y, z) \geq r_0 * r_4 \geq r_0 * r_1 \geq 1 - s > 1 - r,$$

$$\sigma_M(x, z) \geq \sigma_M(x, y) \triangleleft \sigma_M(y, z) \geq r_0 \triangleleft r_4 \geq r_0 \triangleleft r_2 \geq 1 - s > 1 - r$$

$$\text{and } \nu_M(x, z) \leq \nu_M(x, y) \diamond \nu_M(y, z) \leq (1 - r_0) \diamond (1 - r_4) \leq (1 - r_0) \diamond (1 - r_3) \leq s < r.$$

It follows that $z \in \mathcal{B}(x, r)$, and hence $\mathcal{B}(y, 1 - r_4) \subset \mathcal{B}(x, r)$. According to Definition 3.4, then it holds that $\mathcal{B}(x, r)$ is an open set. □

Proposition 3.2. *Let $\mathcal{B}(x, r_1)$ and $\mathcal{B}(x, r_2)$ be two open balls with same center x in a standard fuzzy metric space $(X, M, *, \triangleleft, \diamond)$. Then either $\mathcal{B}(x, r_1) \subseteq \mathcal{B}(x, r_2)$ or $\mathcal{B}(x, r_2) \subseteq \mathcal{B}(x, r_1)$ where r_1, $r_2 \in]0, 1[$.*

Proof. Let $x \in X$ and r_1, $r_2 \in]0, 1[$. If $r_1 = r_2$, then $\mathcal{B}(x, r_1) = \mathcal{B}(x, r_2)$, and hence the result is trivially holds. Next, we assume that $r_1 \neq r_2$. Then, we distinguish two cases: $r_1 < r_2$ and $r_1 > r_2$.

(i) If $r_1 < r_2$ and suppose that $y \in \mathcal{B}(x, r_1)$, then $\mu_M(x, y) > 1 - r_1$, $\sigma_M(x, y) > 1 - r_1$ and $\nu_M(x, y) < r_1$, which implies that $\mu_M(x, y) > 1 - r_2$, $\sigma_M(x, y) > 1 - r_2$ and $\nu_M(x, y) < r_2$. Therefore, $y \in \mathcal{B}(x, r_2)$, and hence $\mathcal{B}(x, r_1) \subseteq \mathcal{B}(x, r_2)$.

(ii) If $r_1 > r_2$, then by applying a similar reasoning, we get $\mathcal{B}(x, r_2) \subseteq \mathcal{B}(x, r_1)$.

□

In the following theorem, we provide the topology induced by the single valued neutrosophic distance.

Theorem 3.1. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. Then it holds that the set*

$$\tau_M = \{A \subseteq X \mid x \in A \textit{ if and only if there exists } r \in]0, 1[\textit{ such that } \mathcal{B}(x, r) \subseteq A\}$$

is a topology on X and it is called the topology induced by the single valued neutrosophic set M.

Proof. (i) It is clear that $\emptyset, X \in \tau_M$.

(ii) Consider $A_1, A_2 \in \tau_M$. Then, on the one hand, $A_1 \cap A_2 \subseteq X$. On the other hand, for any $x \in A_1$, there exists $r_1 \in]0, 1[$, such that $\mathcal{B}(x, r_1) \subseteq A_1$, and, for any $x \in A_2$, there exists $r_2 \in]0, 1[$, such that $\mathcal{B}(x, r_2) \subseteq A_2$. Here, we need to show that for any $x \in A_1 \cap A_2$, there exists $r \in]0, 1[$, such that $\mathcal{B}(x, r) \subseteq A_1 \cap A_2$. Indeed, due to Proposition 3.2, we distinguish two cases:
(a) If $\mathcal{B}(x, r_1) \subseteq \mathcal{B}(x, r_2)$ and take $r = r_1$, then $\mathcal{B}(x, r) \subseteq A_1 \cap A_2$.
(b) If $\mathcal{B}(x, r_2) \subseteq \mathcal{B}(x, r_1)$ and take $r = r_2$, then $\mathcal{B}(x, r) \subseteq A_1 \cap A_2$.
Hence, $A_1 \cap A_2 \in \tau_M$.
For the sake of completeness, we mention here, if we repeat this process for a finite number n ($n \in \mathbb{N}^*$) of sets $A_1, A_2, ..., A_n \in \tau_M$, we deduce that $\bigcap_{1 \leq i \leq n} A_i \in \tau_M$.

(iii) Suppose that $(A_i)_{i \in I} \in \tau_M$ and put $H = \bigcup_{i \in I} A_i$. We will show that $H \in \tau_M$.
Let $x \in H$, then $x \in \bigcup_{i \in I} A_i$. It follows that there exists $i_0 \in I$ such that $x \in A_{i_0}$. The fact that $A_{i_0} \in \tau_M$ then, there exists $r_0 \in]0, 1[$ such that $\mathcal{B}(x, r_0) \subset A_{i_0}$. Hence, $\mathcal{B}(x, r_0) \subset A_{i_0} \subseteq \bigcup_{i \in I} A_i = H$ which implies that $H \in \tau_M$.
Finally, we deduce that τ_M is a topology on X.

□

Remark 3.4. *Let (X, d) be an ordinary metric space and $(X, M, *, \triangleleft, \diamond)$ be the corresponding standard single valued neutrosophic metric space on X. Then, the topology τ_d induced by the metric d and the topology τ_M induced by the single valued neutrosophic set M are coincide. This means that $\tau_d = \tau_M$.*

The following theorem provides a basic property for the standard single valued neutrosophic metric spaces.

Theorem 3.2. *Every standard single valued neutrosophic metric space $(X, M, *, \triangleleft, \diamond)$ is a Hausdorff space.*

Proof. Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space, and choose two distinct points x and y of X. Assume that $r_1 = \mu_M(x, y)$, $r_2 = \sigma_M(x, y)$ and $r_3 = \nu_M(x, y)$ with $r = \max\{r_1, r_2, 1 - r_3\}$. If we take $r_0 \in]r, 1[$, then, from Remark 3.2, it follows that there exist r_4, r_5, r_6 such that

$$r_4 * r_4 \geq r_0, r_5 \triangleleft r_5 \geq r_0 \text{ and } (1 - r_6) \diamond (1 - r_6) \leq 1 - r_0.$$

Next, if we put $r_7 = \max\{r_4, r_5, r_6\}$ and consider the open balls $\mathcal{B}(x, 1-r_7)$ and $\mathcal{B}(y, 1-r_7)$ then, it is clear that $B(x, 1 - r_7) \cap B(y, 1 - r_7) = \emptyset$. Now, if we suppose that there exists $z \in B(x, 1 - r_7) \cap B(y, 1 - r_7)$, then

$$r_1 = \mu_M(x, y) \geq \mu_M(x, z) * \mu_M(z, y) \geq r_7 * r_7 \geq r_4 * r_4 \geq r_0 > r_1,$$

$$r_2 = \sigma_M(x, y) \geq \sigma_M(x, z) \triangleleft \sigma_M(z, y) \geq r_7 \triangleleft r_7 \geq r_5 \triangleleft r_5 \geq r_0 > r_2,$$

and $r_3 = \nu_M(x, y) \leq \nu_M(x, z) \diamond \nu_M(z, y) \leq (1-r_7) \diamond (1-r_7) \leq (1-r_6) \diamond (1-r_6) \leq (1-r_0) < r_3$. which is a contradiction. Hence, $(X, M, *, \triangleleft, \diamond)$ is a Hausdorff space. □

Remark 3.5. *We mention here that a Hausdorff space, also known as a separate space; is a topological space in which any two distinct points always admit disjoint neighborhoods.*

3.3. Continuity property in standard single valued neutrosophic metric space.

In this subsection, we will study some interesting properties of single valued neutrosophic continuous mappings in standard single valued neutrosophic metric space. First, we introduce the notion of single valued neutrosophic continuous mapping.

Definition 3.7. *Let $(X, M, *, \triangleleft, \diamond)$ and $(Y, M', *, \triangleleft, \diamond)$ be two standard single valued neutrosophic metric spaces. The mapping $f : X \to Y$ is a single valued neutrosophic continuous if and only if the inverse of each open set on Y is an open set on X.*

Example 3.2. *Let $(\mathbb{R}_+, M, *, \triangleleft, \diamond)$ and $(\mathbb{R}, M', *, \triangleleft, \diamond)$ be two standard single valued neutrosophic metric spaces. Define the t-norms $x * y = min\{x, y\}$, $x \triangleleft y = min\{x, y\}$ and the t-conorm $x \diamond y = max\{x, y\}$, for all x, $y \in [0, 1]$. Define the single valued neutrosophic sets M and M' on $\mathbb{R}^2_+$ and $\mathbb{R}^2$ respectively as:*

$$\mu_M(x, x') = \frac{1}{e^{|x-x'|}} \ \sigma_M(x, x') = \frac{1}{e^{|x-x'|}}, \text{ and } \nu_M(x, x') = \frac{|x - x'|}{1 + |x - x'|}.$$

$$\mu_{M'}(y, y') = \frac{1}{e^{|y-y'|}} \ \sigma_{M'}(y, y') = \frac{1}{e^{|y-y'|}}, \text{ and } \nu_{M'}(y, y') = \frac{|y - y'|}{1 + |y - y'|}.$$

Then the mapping $f : (\mathbb{R}_+, M, *, \triangleleft, \diamond) \to (\mathbb{R}, M', *, \triangleleft, \diamond)$ defined by $f(x) = x+\alpha$, where $\alpha \in \mathbb{R}$ is a single valued neutrosophic continuous mapping. Indeed, let G be an open set on $(\mathbb{R}, M', *, \triangleleft, \diamond)$, and let $x \in f^{-1}(G)$, then it follows that $f(x) \in G$. Since G is an open set, then $\exists \varepsilon_{f(x)} \in]0, 1[$, where $\mathcal{B}_{\mathbb{R}}(f(x), \varepsilon_{f(x)}) \subseteq G$, and $f^{-1}(\mathcal{B}_{\mathbb{R}}(f(x), \varepsilon_{f(x)})) \subseteq f^{-1}(G)$. For $\varepsilon_x = \varepsilon_{f(x)}$, we will show that $f^{-1}(\mathcal{B}_{\mathbb{R}}(f(x), \varepsilon_{f(x)})) = \mathcal{B}_{\mathbb{R}_+}(x, \varepsilon_x)$. Let $x \in f^{-1}(\mathcal{B}_{\mathbb{R}}(f(x), \varepsilon_{f(x)}))$, it follows that $f(x) \in \mathcal{B}_{\mathbb{R}}(f(x), \varepsilon_{f(x)})$, and

$$\mu_{M'}(f(x), f(x')) = \frac{1}{e^{|f(x)-f(x')|}} > 1 - \varepsilon_{f(x)}, \ \sigma_{M'}(f(x), f(x')) = \frac{1}{e^{|f(x)-f(x')|}} > 1 - \varepsilon_{f(x)}$$

$$\text{and } \nu_{M'}(f(x), f(x')) = \frac{|f(x) - f(x')|}{1 + |f(x) - f(x')|} < \varepsilon_{f(x)}.$$

Then, it holds that

$$\mu_{M'}(f(x), f(x')) = \frac{1}{e^{|(x+\alpha)-(x'+\alpha)|}} > 1-\varepsilon_{f(x)}, \ \sigma_{M'}(f(x), f(x')) = \frac{1}{e^{|(x+\alpha)-(x'+\alpha)|}} > 1-\varepsilon_{f(x)}$$

$$and\ \nu_{M'}(f(x), f(x')) = \frac{|(x+\alpha) - (x'+\alpha)|}{1 + |(x+\alpha) - (x'+\alpha)|} < \varepsilon_{f(x)}.$$

This implies that

$$\mu_{M'}(f(x), f(x')) = \frac{1}{e^{|x-x'|}} > 1 - \varepsilon_{f(x)},\ \sigma_{M'}(f(x), f(x')) = \frac{1}{e^{|x-x'|}} > 1 - \varepsilon_{f(x)}$$

$$and\ \nu_{M'}(f(x), f(x')) = \frac{|x-x'|}{1+|x-x'|} < \varepsilon_{f(x)}.$$

Hence,

$$\mu_M(x, x') > 1 - \varepsilon_x,\ \sigma_M(x, x') > 1 - \varepsilon_x\ and\ \nu_M(x, x') < \varepsilon_x.$$

We get that, $x \in \mathcal{B}_{\mathbb{R}_+}(x, \varepsilon_x)$. In the same way, we obtain the inverse implication. Therefore, $f^{-1}(\mathcal{B}_{\mathbb{R}}(f(x), \varepsilon_{f(x)})) = \mathcal{B}_{\mathbb{R}_+}(x, \varepsilon_x)$. We conclude that $f^{-1}(G)$ is an open set on $(\mathbb{R}_+, M, *, \triangleleft, \diamond)$, which satisfy the condition of the continuity property.

Remark 3.6. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space. Then the single valued neutrosophic identity mapping $Id : X \to X$ is a single valued neutrosophic continuous mapping.*

Remark 3.7. *Let $(X, M, *, \triangleleft, \diamond)$ and $(Y, M, *, \triangleleft, \diamond)$ be two standard single valued neutrosophic metric spaces. The mapping $f : X \to Y$ is a single valued neutrosophic continuous if and only if the inverse of each closed set on Y is a closed set on X.*

The following proposition shows that the continuity property is preserved through the composition of single valued neutrosophic mappings.

Proposition 3.3. *Let $(X, M, *, \triangleleft, \diamond)$, $(Y, M, *, \triangleleft, \diamond)$ and $(Z, M, *, \triangleleft, \diamond)$ be three standard single valued neutrosophic metric spaces and $f : X \to Y$, $g : Y \to Z$ be two continuous mappings. Then the composition $g \circ f : X \to Z$ is a continuous mapping.*

Proof. Let A be an open set of Z. Definition 3.7 guarantees that $g^{-1}(A)$ is an open set of Y and by applying the same definition, we also get that $f^{-1}(g^{-1}(A))$ is an open set of X. Since $(g \circ f)^{-1}(A) = f^{-1}(g^{-1}(A))$, it follows that $g \circ f$ is a continuous mapping. □

4. Edelstein fixed point theorem in standard single valued neutrosophic metric spaces

The aim of the present section, is to give an extension of Edelstein fixed point theorem in a standard single valued neutrosophic metric space. We start with the key results.

Lemma 4.1. *Let $(X, M, *, \triangleleft, \diamond)$ be a standard single valued neutrosophic metric space and let (x_n) and (y_n) be two sequences in X. If $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then the following statements hold:*

(i)

$$\mu_M(x, y) \leq \lim_{n\to\infty} \inf \mu_M(x_n, y_n), \sigma_M(x, y) \leq \lim_{n\to\infty} \inf \sigma_M(x_n, y_n)$$

$$and\ \nu_M(x, y) \geq \lim_{n\to\infty} \sup \nu_M(x_n, y_n)\,. \tag{1}$$

(ii)

$$\mu_M(x, y) \geq \lim_{n\to\infty} \sup \mu_M(x_n, y_n), \sigma_M(x, y) \geq \lim_{n\to\infty} \sup \sigma_M(x_n, y_n)$$

$$and\ \nu_M(x, y) \leq \lim_{n\to\infty} \inf \nu_M(x_n, y_n)\,. \tag{2}$$

Proof. (i) Let (x_n) and (y_n) be two sequences in X. From Definition 3.1 (iv), it follows that

$$\mu_M(x_n, y_n) \geq \mu_M(x_n, x) * \mu_M(x, y) * \mu_M(y, y_n);$$
$$\sigma_M(x_n, y_n) \geq \sigma_M(x_n, x) \triangleleft \sigma_M(x, y) \triangleleft \sigma_M(y, y_n);$$
$$\nu_M(x_n, y_n) \leq \nu_M(x_n, x) \diamond \nu_M(x, y) \diamond \mu_M(y, y_n).$$

Since $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then it holds from Definition 3.5 that $\lim_{n\to\infty} \mu_M(x_n, x) = 1$, $\lim_{n\to\infty} \sigma_M(x_n, x) = 1$ and $\lim_{n\to\infty} \nu_M(x_n, x) = 0$. This implies that,

$$\lim_{n\to\infty} \inf \mu_M(x_n, y_n) \geq 1 * \mu_M(x, y) * 1;$$
$$\lim_{n\to\infty} \inf \sigma_M(x_n, y_n) \geq 1 \triangleleft \sigma_M(x, y) * 1;$$
$$\lim_{n\to\infty} \sup \nu_M(x_n, y_n) \leq 0 \diamond \nu_M(x, y) \diamond 0.$$

Hence,

$$\lim_{n\to\infty} \inf \mu_M(x_n, y_n) \geq \mu_M(x, y);$$
$$\lim_{n\to\infty} \inf \sigma_M(x_n, y_n) \geq \sigma_M(x, y);$$
$$\lim_{n\to\infty} \sup \nu_M(x_n, y_n) \leq \nu_M(x, y).$$

(ii) Follows from Definition 3.5 and (i).

□

In the following theorem, we will establish the Edelstein fixed point theorem for standard single valued neutrosophic metric spaces.

Theorem 4.1 (Edelstein fixed point theorem)**.** *Let $(X, M, *, \triangleleft, \diamond)$ be a compact standard single valued neutrosophic metric space. Let $T : X \to X$ be a mapping satisfying*

$$\mu_M(Tx, Ty) > \mu_M(x, y), \sigma_M(Tx, Ty) > \sigma_M(x, y)$$
$$\text{and } \nu_M(Tx, Ty) < \nu_M(x, y), \tag{3}$$

for all $x \neq y$. Then, T has a unique fixed point.

Proof. Let $x \in X$ and define an iterative sequence (x_n) by $x_{n+1} = Tx_n$, for any $n \in \mathbb{N}$. Let us suppose that $x_n \neq x_{n+1}$ for each n (because, if $x_n = x_{n+1}$ then $x_n = Tx_n$, which means that x_n is a fixed point of T; then the proof is finished). Therefore, $x_n \neq x_k$ where $n \neq k$. For otherwise, i.e., $x_n = x_k$ with $n \neq k$ we obtain

$$\mu_M(x_n, x_{n+1}) = \mu_M(x_k, x_{k+1}) > \mu_M(x_{k-1}, x_k) > ... \geq \mu_M(x_n, x_{n+1}),$$
$$\sigma_M(x_n, x_{n+1}) = \sigma_M(x_k, x_{k+1}) > \sigma_M(x_{k-1}, x_k) > ... \geq \sigma_M(x_n, x_{n+1})$$
$$\text{and } \nu_M(x_n, x_{n+1}) = \nu_M(x_k, x_{k+1}) < \nu_M(x_{k-1}, x_k) < ... \leq \nu_M(x_n, x_{n+1})$$

where $k > n$, a contradiction. Since X is compact, then (x_n) has a convergent subsequence (x_{n_i}). Let $\lim_i x_{n_i} = y$. Also, we suppose that $y, Ty \notin (x_{n_i})$ where $i \in \mathbb{N}$ (if not, choose a subsequence with such a property). Due to the above hypotheses we now have

$$\mu_M(Tx_{n_i}, Ty) > \mu_M(x_{n_i}, y),\ \sigma_M(Tx_{n_i}, Ty) > \sigma_M(x_{n_i}, y)$$

$$\text{and } \nu_M(Tx_{n_i}, Ty) < \nu_M(x_{n_i}, y),$$

for all $i \in \mathbb{N}$. Then, by (1) we obtain

$$\lim \mu_M(Tx_{n_i}, Ty) > \lim \mu_M(x_{n_i}, y) = \mu_M(y, y) = 1,$$

$$\lim \sigma_M(Tx_{n_i}, Ty) > \lim \sigma_M(x_{n_i}, y) = \mu_M(y, y) = 1$$
$$\text{and } \lim \nu_M(Tx_{n_i}, Ty) < \lim \nu_M(x_{n_i}, y) = \nu_M(y, y) = 0.$$

Hence, $\lim Tx_{n_i} = Ty$. Similarly, we get

$$\lim T^2 x_{n_i} = T^2 y\,. \tag{4}$$

Now, recall that $Tx_{n_i} = Ty$ for all $i \in \mathbb{N}$. Notice that

$$\begin{aligned}\mu_M(x_{n_i}, Tx_{n_i}) &\leq \mu_M(Tx_{n_i}, T^2x_{n_i}) \leq ... \leq \mu_M(x_{n_i}, Tx_{n_i})\\ &\leq \mu_M(Tx_{n_i}, T^2x_{n_i}) \leq ... \leq \mu_M(Tx_{n_{i+1}}, Tx_{n_{i+1}})\\ &\leq \mu_M(Tx_{n_{i+1}}, T^2x_{n_{i+1}}) \leq ... \leq 1\,,\end{aligned}$$

$$\begin{aligned}\sigma_M(x_{n_i}, Tx_{n_i}) &\leq \sigma_M(Tx_{n_i}, T^2x_{n_i}) \leq ... \leq \sigma_M(x_{n_i}, Tx_{n_i})\\ &\leq \sigma_M(Tx_{n_i}, T^2x_{n_i}) \leq ... \leq \sigma_M(Tx_{n_{i+1}}, Tx_{n_{i+1}})\\ &\leq \sigma_M(Tx_{n_{i+1}}, T^2x_{n_{i+1}}) \leq ... \leq 1\end{aligned}$$

and

$$\begin{aligned}\nu_M(x_{n_i}, Tx_{n_i}) &\geq \nu_M(Tx_{n_i}, T^2x_{n_i}) \geq ... \geq \nu_M(x_{n_i}, Tx_{n_i})\\ &\geq \nu_M(Tx_{n_i}, T^2x_{n_i}) \geq ... \geq \nu_M(Tx_{n_{i+1}}, Tx_{n_{i+1}})\\ &\geq \nu_M(Tx_{n_{i+1}}, T^2x_{n_{i+1}}) \geq ... \geq 0\,.\end{aligned}$$

Therefore, the sequences formed from $\mu_M(x_{n_i}, Tx_{n_i})$ and $\mu_M(Tx_{n_i}, T^2x_{n_i})$ are convergent to a common limit [22] (we get the same if we replace μ_M by σ_M or ν_M). Thus, by (2), (4) and (1) we obtain

$$\begin{aligned}\mu_M(y, Ty) \geq \limsup \mu_M(x_{n_i}, Tx_{n_i}) &= \limsup \mu_M(Tx_{n_i}, T^2x_{n_i})\\ &\geq \liminf \mu_M(Tx_{n_i}, T^2x_{n_i}) \geq \mu_M(Ty, T^2y),\end{aligned}$$

$$\begin{aligned}\sigma_M(y, Ty) \geq \limsup \sigma_M(x_{n_i}, Tx_{n_i}) &= \limsup \sigma_M(Tx_{n_i}, T^2x_{n_i})\\ &\geq \liminf \sigma_M(Tx_{n_i}, T^2x_{n_i}) \geq \sigma_M(Ty, T^2y),\end{aligned}$$

$$\begin{aligned}\nu_M(y, Ty) \leq \liminf \nu_M(x_{n_i}, Tx_{n_i}) &= \liminf \nu_M(Tx_{n_i}, T^2x_{n_i})\\ &\leq \limsup \nu_M(Tx_{n_i}, T^2x_{n_i}) \leq \nu_M(Ty, T^2y).\end{aligned}$$

Now suppose $y \neq Ty$. Then by (3), we have
$\mu_M(y, Ty) < \mu_M(Ty, T^2y), \sigma_M(y, Ty) < \sigma_M(Ty, T^2y)$ and
$\nu_M(y, Ty) > \nu_M(Ty, T^2y)$, which is a contradiction. (recall that μ_M and σ_M are continuous and non-decreasing, and, ν_M is continuous and non-increasing). Hence, $y = Ty$, i.e., y is a fixed point of T. Uniqueness follows at once from (3). □

5. Conclusion

In this work, we have generalized the notion of standard fuzzy metric space introduced by J.R. Kider and Z.A. Hussain to the setting of single valued neutrosophic sets. Also, the fundamental properties related to the standard single valued neutrosophic metric space have been studied. Moreover, as an application we have been given an extension of the Edelstein fixed point theorem on standard single valued neutrosophic metric space.
Future work is anticipated in multiple directions. We think it makes sense to study some other properties such as the connexity and the convexity of standard single valued neutrosophic metric space. Moreover, we intend to extend this work to other fixed point theorems.

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