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CONGRUENT DOMINATING SETS IN A GRAPH - A NEW CONCEPT

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Abstract. A dominating set $D \subseteq V(G)$ is said to be a congruent dominating set of G if

$$\sum_{v \in V(G)} d(v) \equiv 0 \left(\bmod \sum_{v \in D} d(v) \right).$$

The minimum cardinality of a minimal congruent dominating set of G is called the congruent domination number of G which is denoted by $\gamma_{cd}(G)$. Some characterizations are established and congruent domination number for various graphs have been investigated.

Keywords: Dominating Set, Domination Number, Congruence, Congruent Dominating Set, Congruent Domination Number.

AMS Subject Classification: 05C07, 05C69, 11A07.

1. Introduction

Many variants of dominating models are available in the existing literature. A comprehensive bibliography of references on the concept of domination can be found in Hedetniemi and Laskar [6, 7], Cockayne and Hedetniemi[3] as well as Walikar *et al.* [9]. For any undefined term and notation in graph theory we rely upon West [10], while the term related to theory of domination and number theory are used in the sense of Haynes *et al.* [4, 5] and Burton [2], respectively.

We consider a finite, connected and undirected graph $G = (V(G), E(G))$ without loops and multiple edges. A set $D \subseteq V(G)$ is called a dominating set if every vertex $v \in V(G)$ is either in D or is adjacent to atleast one element of D. That is $N[D] = V(G)$. A dominating set D is said to be a minimal dominating set if no proper subset D' of D is a dominating set of G. The minimum cardinality of minimal dominating set is called the domination number $\gamma(G)$.

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A set $S \subseteq V(G)$ is called independent set if no two vertices in S are adjacent. A set $S \subseteq V(G)$ of vertices in a graph G is called an independent dominating set if it is both independent and dominating.

The present work is aimed to introduce a new type of dominating set in graphs which has been termed as congruent dominating set. Some new characterizations are derived and congruent domination numbers of some standard graph families have been investigated.

2. Main Results

Definition 2.1. A dominating set $D \subseteq V(G)$ is said to be a *congruent dominating set* of G if

$$\sum_{v \in V(G)} d(v) \equiv 0 \left(\bmod \sum_{v \in D} d(v) \right).$$

Definition 2.2. *A congruent dominating set $D \subseteq V(G)$ is said to be a* minimal congruent dominating set *if no proper subset D' of D is a congruent dominating set.*

Definition 2.3. The minimum cardinality of a minimal congruent dominating set of G is called the congruent domination number of G which is denoted by $\gamma_{cd}(G)$.

As

$$\sum_{v \in V(G)} d(v) \equiv 0 \left(\bmod \sum_{v \in D} d(v) \right)$$

is always true when $D = V(G)$, we have $\gamma_{cd}(G) \leqslant n$.

Hence, $1 \leqslant \gamma_{cd}(G) \leqslant n$. Both the bounds are sharp.

The graphs $K_{1,n}$ and C_p when p is a prime attains the above bounds. That is, $\gamma_{cd}(K_{1,n}) = \gamma(K_{1,n}) = 1$ and $\gamma_{cd}(C_p) = p$ when p is a prime.

Since every congruent dominating set is a dominating set, we have, $\gamma(G) \leqslant \gamma_{cd}(G)$.

Thus, $1 \leqslant \gamma(G) \leqslant \gamma_{cd}(G) \leqslant n$.

Definition 2.4. A dominating set $D \subseteq V(G)$ of graph G is called *independent congruent dominating set* if it is independent set as well as congruent dominating set.

Definition 2.5. An independent congruent dominating set D is said to be maximal independent congruent dominating set if no proper superset of D is an independent congruent dominating set.

Proposition 2.1. [8] *Every connected graph G of order $n \geqslant 2$ has a dominating set D whose complement $V(G) - D$ is also a dominating set.*

Lemma 2.1. *If $a \equiv 0 \pmod{m}$ and $a \equiv 0 \pmod{n}$ with $m + n = a$ then $m = n = \dfrac{a}{2}$, where $a, n, m \in \mathbb{N}$.*

Proof. Here $m + n = a$. So without loss of generality we may assume that $m \leqslant \dfrac{a}{2}$ and $n \geqslant \dfrac{a}{2}$.

If $m < \dfrac{a}{2}$ then $n > \dfrac{a}{2}$.

But $n|a$, which is a contradiction as $n > \dfrac{a}{2}$.

Therefore neither $m < \dfrac{a}{2}$ nor $n > \dfrac{a}{2}$.

Hence, $m = n = \dfrac{a}{2}$.

$\square$

Theorem 2.1. *A connected graph G of order $n \geqslant 2$ has a congruent dominating set D whose complement $V(G) - D$ is also a congruent dominating set if and only if*

$$\sum_{v \in D} d(v) = \sum_{v \in V(G)-D} d(v).$$

Proof. First assume that D and $V(G) - D$ both are congruent dominating sets.

Then,

$$\sum_{v \in V(G)} d(v) \equiv 0 \left(\bmod \sum_{v \in D} d(v) \right) \text{ and } \sum_{v \in V(G)} d(v) \equiv 0 \left(\bmod \sum_{v \in V(G)-D} d(v) \right) \text{ with}$$

$$\sum_{v \in D} d(v) + \sum_{v \in V(G)-D} d(v) = \sum_{v \in V(G)} d(v).$$

Thus from Lemma 2.1,

$$\sum_{v \in D} d(v) = \sum_{v \in V(G)-D} d(v).$$

Conversly suppose that D and $V(G) - D$ both are dominating sets of G and

$$\sum_{v \in D} d(v) = \sum_{v \in V(G)-D} d(v).$$

Then,

$$\sum_{v \in V(G)} d(v) = 2 \sum_{v \in D} d(v) = 2 \sum_{v \in V(G)-D} d(v).$$

Thus,

$$\sum_{v \in V(G)} d(v) \equiv 0 \left(\bmod \sum_{v \in D} d(v) \right) \text{ and } \sum_{v \in V(G)} d(v) \equiv 0 \left(\bmod \sum_{v \in V(G)-D} d(v) \right).$$

Hence, D and $V(G) - D$ both are congruent dominating sets of graph G.

$\square$

Theorem 2.2. *Let G be an Euler graph with*

$$\sum_{v \in V(G)} d(v) = 2p$$

where p is prime and $G \neq C_3$. Then $\gamma_{cd}(G) = n$.

Proof. Let G be an Euler graph.

Let $D = \{v_1, v_2, ...v_k\}$ be a congruent dominating set with

$$\sum_{v \in D} d(v) = m$$

Here m is even as $d(v)$ is even, for all $v \in D$.

Since D is congruent dominating set, we have, $m|2p$ then $m \in \{1, 2, p, 2p\}$.

But $m \neq 1$ as m is even similarly $m \neq 2$ as there is exactly one Euler graph which is cycle C_3 with $\gamma(C_3) = 1$, $m = 2$ and $G \neq C_3$.

Hence, either $m = p$ or $m = 2p$.

But $m = p$ is not possible as G is an Euler graph and $m > 2$ is even while p is prime . Hence, $m = 2p$ and so,

$$\sum_{v \in D} d(v) = m = 2p.$$

Then,

$$\sum_{v \in V(G)} d(v) \equiv 0 \left(\bmod \sum_{v \in D} d(v) \right).$$

By construction of D we say that it is minimal congruent dominating set with minimum cardinality.

Hence, $\gamma_{cd}(G) = n$.

□

Proposition 2.2. [1] *Every maximal independent set in a graph G is a minimal dominating set of G.*

Theorem 2.3. *Every maximal independent congruent dominating set is a minimal congruent dominating set.*

Proof. Let $D \subseteq V(G)$ be a maximal independent congruent dominating set of graph G. Then D is a maximal independent dominating set as every congruent dominating set is also a dominating set. Since every maximal independent dominating set is a minimal dominating set, we get D is a minimal dominating set. Thus, D is a minimal dominating set as well as congruent dominating set. Hence, D is a minimal congruent dominating set. □

3. Congruent Domination Number of Some Standard Graphs

Theorem 3.1. *For the cycle C_n,*

$$\gamma_{cd}(C_n) = \begin{cases} \dfrac{n}{3} & ; \text{if } n \equiv 0(\text{mod} 3) \\ \dfrac{n}{2} & ; \text{if } n \equiv 0(\text{mod} 2) \text{ and } n \not\equiv 0(\text{mod} 3) \\ n & ; \text{otherwise.} \end{cases}$$

Proof. Let $V(C_n) = \{v_1, v_2, ..., v_n\}$ be the set of vertices of cycle C_n.

Obviously $d(v) = 2$, for all $v \in V(C_n)$ and so

$$\sum_{v \in V(C_n)} d(v) = 2n.$$

Now $\gamma(C_n) = \left\lceil \dfrac{n}{3} \right\rceil$ and so $\left\lceil \dfrac{n}{3} \right\rceil \leqslant \gamma_{cd}(C_n)$.

Case:1 If $n \equiv 0(\text{mod} 3)$

Consider $D = \left\{ v_{3k+1} / 0 \leqslant k \leqslant \left(\dfrac{n}{3} - 1 \right) \right\}$ then $|D| = \dfrac{n}{3}$ and D is a minimal dominating set with minimum cardinality as $\gamma(C_n) = \dfrac{n}{3}$ for $n \equiv 0(\text{mod } 3)$.

Now

$$\sum_{v \in D} d(v) = 2\left(\frac{n}{3}\right) \text{ and } \sum_{v \in V(C_n)} d(v) = 2n.$$

Thus,

$$\sum_{v\in V(C_n)} d(v) \equiv 0 \left(\text{mod} \sum_{v\in D} d(v)\right).$$

Since D is minimal dominating set with minimum cardinality it is also minimal congruent dominating set with minimum cardinality.

Case:2 If $n \equiv 0(\text{mod}2)$ and $n \not\equiv 0(\text{mod}3)$

Consider a subset $D \subseteq V(C_n)$ of vertices as follows:

$D = \left\{v_{2k+1}/0 \leqslant k \leqslant \left(\frac{n}{2}-1\right)\right\}$ with $|D| = \frac{n}{2}$. Then D is a dominating set and

$$\sum_{v\in D} d(v) = 2\left(\frac{n}{2}\right) = n.$$

Thus,

$$\sum_{v\in V(C_n)} d(v) \equiv 0 \left(\text{mod} \sum_{v\in D} d(v)\right).$$

Therefore D is congruent dominating set.

Select $i \in \mathbb{Q}^+$ such that $i|2n$ and $\left(\frac{2n}{i}\right)$ is an integer.

Now consider the set of vertices $S \subseteq V(C_n)$ such that S is a dominating set and $|S| < |D|$ with

$$\sum_{v\in S} d(v) = \frac{2n}{i}.$$

Then, $\left(\frac{2n}{i}\right)|2n$.

But for each $i > 2$, $|S| < \left\lceil\frac{n}{3}\right\rceil$ and $\gamma(C_n) = \left\lceil\frac{n}{3}\right\rceil$.

Therefore no such congruent dominating set S exists such that $|S| < \frac{n}{2}$.

This implies that D is a minimal congruent dominating set with minimum cardinality.

Case:3 If $n \not\equiv 0(\text{mod}2)$ and $n \not\equiv 0(\text{mod}3)$

Consider $D = V(C_n)$ with $|D| = n = |V(C_n)|$. Then

$$\sum_{v\in V(C_n)} d(v) \equiv 0 \left(\text{mod} \sum_{v\in D} d(v)\right).$$

Therefore D is a congruent dominating set.

Now $|D| = n = |V(C_n)|$ so only one possibility remain to check that whether D is minimal dominating set or not.

Select $i \in \mathbb{Q}^+$ such that $i|2n$ and $\left(\frac{2n}{i}\right)$ is an integer.

Now consider the set of vertices $S \subseteq V(C_n)$ such that S is a dominating set and $|S| < |D|$ with

$$\sum_{v\in S} d(v) = \frac{2n}{i}.$$

Then, $\left(\frac{2n}{i}\right)|2n$.

But for each $i > 1$, $|S| < \left\lceil\frac{n}{3}\right\rceil$ and $\gamma(C_n) = \left\lceil\frac{n}{3}\right\rceil$.

Therefore no such congruent dominating set S exists such that $|S| < n$.

This implies that D is a minimal congruent dominating set with minimum cardinality. Hence,

$$\gamma_{cd}(C_n) = \begin{cases} \dfrac{n}{3} & ;\text{if } n \equiv 0(\text{mod}3) \\ \dfrac{n}{2} & ;\text{if } n \equiv 0(\text{mod}2) \text{ and } n \not\equiv 0(\text{mod}3) \\ n & ;\text{otherwise.} \end{cases}$$

□

Example 3.1. Congruent dominating set of cycle C_8 is shown by solid vertices in Figure 1.

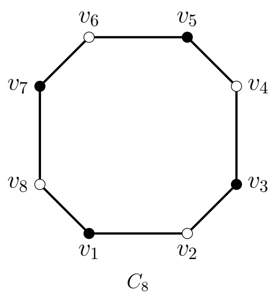

FIGURE 1. $\gamma_{cd}(C_8) = 4$.

Theorem 3.2. *For the path P_n, $\gamma_{cd}(P_n) = \left\lfloor \dfrac{n}{2} \right\rfloor$; $n \geqslant 2$.*

Proof. Let $V(P_n) = \{v_1, v_2, ..., v_n\}$ be the set of vertices of path P_n with $d(v_1) = d(v_n) = 1$ and $d(v_i) = 2$, for all $2 \leqslant i \leqslant n-1$.

Then,

$$\sum_{v \in V(P_n)} d(v) = 2(n-2) + 2 = 2(n-1).$$

Consider the set of vertices $D \subset V(P_n)$ as follows:
$D = \left\{ v_{2k}/1 \leqslant k \leqslant \left\lfloor \dfrac{n}{2} \right\rfloor \right\}$ with $|D| = \left\lfloor \dfrac{n}{2} \right\rfloor$. Then D is a dominating set.

Moreover D is also a minimal dominating set as for any $u \in D$, u will not be dominated by $D - \{u\}$.

Now if n is even then there is exactly $\left(\dfrac{n}{2} - 1\right)$ vertices of degree 2 and one vertex of degree 1 and if n is odd then there is exactly $\left(\dfrac{n-1}{2}\right)$ vertices of degree 2 and so

$$\sum_{v \in D} d(v) = n - 1.$$

Hence,

$$\sum_{v \in V(P_n)} d(v) \equiv 0 \left(\text{mod} \sum_{v \in D} d(v) \right).$$

Then D is a minimal congruent dominating set as it is congruent dominating set as well as minimal dominating set.

Now $\gamma(P_n) = \left\lceil \dfrac{n}{3} \right\rceil$ and so $\left\lceil \dfrac{n}{3} \right\rceil \leqslant \gamma_{cd}(P_n)$.

Select $i \in \mathbb{Q}^+$ such that $i|2(n-1)$ and $\left(\dfrac{2(n-1)}{i}\right)$ is an integer.

Now consider the set of vertices $S \subseteq V(P_n)$ such that S is a dominating set and $|S| < |D|$ with

$$\sum_{v \in S} d(v) = \frac{2(n-1)}{i}.$$

Then, $\left(\dfrac{2(n-1)}{i}\right) |2(n-1)$.

But for each $i > 2$, $|S| < \left\lceil \dfrac{n}{3} \right\rceil$ and $\gamma(P_n) = \left\lceil \dfrac{n}{3} \right\rceil$.

Thus, no such congruent dominating set S exists such that $|S| < \left\lfloor \dfrac{n}{2} \right\rfloor$.

This implies that D is a minimal congruent dominating set with minimum cardinality.

Hence, $\gamma_{cd}(P_n) = \left\lfloor \dfrac{n}{2} \right\rfloor$. □

Example 3.2. Congruent dominating set for path P_9 is shown by solid vertices in Figure 2.

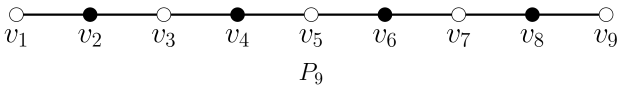

FIGURE 2. $\gamma_{cd}(P_9) = 4$.

Theorem 3.3. *For the complete graph K_n, $\gamma_{cd}(K_n) = 1$; $n \geqslant 2$.*

Proof. Let $V(K_n) = \{v_1, v_2, ..., v_n\}$ be the set of vertices of K_n.

Then,

$$\sum_{v \in V(K_n)} d(v) = n(n-1).$$

Consider the set $D \subseteq V(K_n)$ of vertices as follows:

$D = \{v_i\}$; $v_i \in V(K_n)$ and $1 \leqslant i \leqslant n$ then $|D| = 1$ and D is a minimal dominating set with minimum cardinality as at least one vertex is essential to dominate all the vertices of any graph.

Now

$$\sum_{v \in D} d(v) = n - 1.$$

Thus,

$$\sum_{v \in V(K_n)} d(v) \equiv 0 \left(\bmod \sum_{v \in D} d(v) \right).$$

Hence, D is congruent dominating set. Moreover D is minimal congruent dominating set with minimum cardinality as it is a minimal dominating set with minimum cardinality as well as congruent dominating set.

Hence, $\gamma_{cd}(K_n) = 1$; $n \geqslant 2$.

□

Example 3.3. Congruent dominating set for complete graph K_5 is shown by solid vertices in Figure 3.

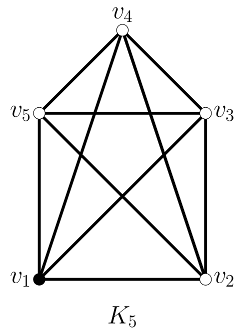

FIGURE 3. $\gamma_{cd}(K_5) = 1$.

Theorem 3.4. *For the complete bipartite graph $K_{m,n}$ with $2 \leqslant m \leqslant n$,*

$$\gamma_{cd}(K_{m,n}) = \begin{cases} 2 & ;\text{if } n = m \text{ or } n = m(m-1) \\ m & ;\text{otherwise.} \end{cases}$$

Proof. Let $V(K_{m,n}) = V_1 \cup V_2$ be the set of vertices of $K_{m,n}$, where $V_1 = \{v_1, v_2, ..., v_m\}$ and $V_2 = \{u_1, u_2, ..., u_n\}$.

Here $d(v_i) = n$ and $d(u_j) = m$, for all $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$ and so,

$$\sum_{v \in V(K_{m,n})} d(v) = 2mn.$$

Case:1 If $n = m$ or $n = m(m-1)$.

Consider $D = \{v_i, u_j\}$;where, $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$ then $|D| = 2$ and D is a minimal dominating set with minimum cardinality as $\gamma(K_{m,n}) = 2$.

Subcase:1 If $n = m$.

Note that

$$\sum_{v \in D} d(v) = m + n = 2m.$$

Therefore

$$\sum_{v \in V(K_{m,n})} d(v) \equiv 0 \left(\text{mod } \sum_{v \in D} d(v) \right).$$

This implies that D is a congruent dominating set.

Since D is a minimal dominating set with minimum cardinality,it is also minimal congruent dominating set with minimum cardinality.

Subcase 2: If $n = m(m-1)$

Here

$$\sum_{v \in D} d(v) = m + n = m + m(m-1) = m^2 \text{ and } \sum_{v \in V(K_{m,n})} d(v) = 2mn = 2m^2$$

Thus,

$$\sum_{v \in V(K_{m,n})} d(v) \equiv 0 \left(\text{mod } \sum_{v \in D} d(v) \right).$$

This implies that D is a congruent dominating set.

Since D is a minimal dominating set with minimum cardinality, it is also minimal congruent dominating set with minimum cardinality.

Case:2 If $n \neq m$ and $n \neq m(m-1)$.

Consider $D = V_1$ then $|D| = m$ and D is a minimal dominating set as every vertex in D is an isolate in the subgraph induced by D.

Here

$$\sum_{v \in D} d(v) = mn.$$

Thus,

$$\sum_{v \in V(K_{m,n})} d(v) \equiv 0 \left(\bmod \sum_{v \in D} d(v) \right).$$

Hence, D is congruent dominating set.

Moreover D is a minimal congruent dominating set as it is minimal dominating set.

Select $i \in \mathbb{Q}^+$ such that $i|2mn$ and $\left(\dfrac{2nm}{i}\right)$ is an integer.

Now consider the set of vertices $S \subseteq V(K_{m,n})$ such that S is a dominating set and $|S| < |D|$ with

$$\sum_{v \in S} d(v) = \frac{2mn}{i}.$$

Then, $\left(\dfrac{2mn}{i}\right)|2mn$.

But for each $i > 2$, $|S| < 2$ and $\gamma(K_{m,n}) = 2$.

Therefore no such congruent dominating set S exists such that $|S| < m$.

This implies that D is a minimal congruent dominating set with minimum cadinality.

Hence,

$$\gamma_{cd}(K_{m,n}) = \begin{cases} 2 & ;\text{if } n = m \text{ or } n = m(m-1) \\ m & ;\text{otherwise.} \end{cases}$$

□

Example 3.4. Congruent dominating set for complete bipartite graph $K_{3,4}$ is shown by solid vertices in Figure 4.

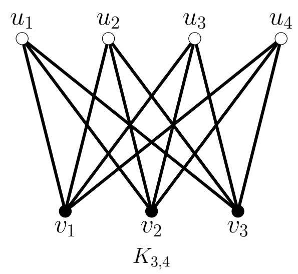

FIGURE 4. $\gamma_{cd}(K_{3,4}) = 3$.

Theorem 3.5. *For the star graph $K_{1,n}$, $\gamma_{cd}(K_{1,n}) = 1$; $n \geqslant 1$.*

Proof. Let $V(K_{1,n}) = \{u, v_1, v_2, ..., v_n\}$ be the vertices of $K_{1,n}$, where u is a apex vertex with degree n and $v_1, v_2, ..., v_n$ are the vertices of degree 1.

Then,

$$\sum_{v \in V(K_{1,n})} d(v) = 2n.$$

Consider $D = \{u\}$ be the set of singleton vertex then $|D| = 1$ and D is a minimal dominating set with minimum cardinality as $\gamma(K_{1,n}) = 1$.

Here

$$\sum_{v \in D} d(v) = n.$$

Thus,

$$\sum_{v \in V(K_{1,n})} d(v) \equiv 0 \left(\bmod \sum_{v \in D} d(v) \right).$$

Therefore D is a congruent dominating set.

Since D is a minimal dominating set with minimum cardinality, it is also a minimal congruent dominating set with minimum cardinality.

Hence, $\gamma_{cd}(K_{1,n}) = 1$; $n \geqslant 1$.

□

Example 3.5. Congruent dominating set for star graph $K_{1,8}$ is shown by solid vertices in Figure 5.

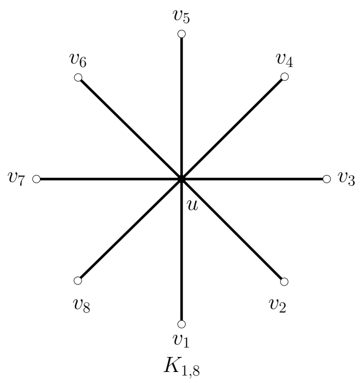

FIGURE 5. $\gamma_{cd}(K_{1,8}) = 1$.

4. Conclusion

A new type of dominating sets in graph has been introduced and it is named as 'Congruent Dominating Set'. This work also demonstrate the fact that how the basic concept of degree of a vertex is related to congruence relation. The present work is a frontier between theory of graphs and number theory.

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