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COMPARISON BETWEEN NUMERICAL METHODS FOR GENERALIZED ZAKHAROV SYSTEM

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Abstract. In this paper, two numerical methods has been applied to numerically solve the generalized Zakharov system (GZS). The spectral collocation method, which based on two dimensional Legendre polynomials (LCM) and the well-known differential transform method (DTM). Both of the proposed methods have high accuracy and have been successfully compared with Adomian decomposition method.

Keywords: Differential transform method; Legendre collocation method; Generalized Zakharov system.

AMS Subject Classification: 83-02, 99A00.

1. Introduction

The Zakharov system, which plays an important role in plasma physics, is a couple of nonlinear partial differential equations, presented by Vladimir Zakharov[17]. In a general form, it describes interactions between high frequency and low frequency waves. The most important example involves interactions between the Langmuir and ion-acoustic waves in plasma. Other physical applications can be found in [18, 19]. The Zakharov system describes Langmuir waves propagation in ionized plasma as described in [3], consists of a complex field $\psi(x,t)$ representing the envelope of the high frequency electric field and the plasma density measured from its equilibrium value symbolized by a real field $w(x,t)$. The real constant coefficient β can be a positive or negative number, and in the case of β vanishing, the system is minimized to the classical Zakharov system of plasma physics.

$$i\ \partial_t \psi(x,t) + \partial_{xx}\psi(x,t) - 2\beta|\psi(x,t)|^2\psi(x,t) + 2\psi(x,t)w(x,t) = 0,$$
$$\partial_{tt} w(x,t) - \partial_{xx} w(x,t) + \partial_{xx}(|\psi(x,t)|^2) = 0.$$

Up to now, there are many methods have been proposed to solve this kind of systems. For example, Glassey[12] presented finite difference scheme for the ZS in one-dimension, Chang et al.[8] presented a difference scheme for the generalized ZS, Bao et al.[5] construct time splitting spectral discretizatons method to solve the generalized ZS in one dimension, Wang

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[15] proposed F-expansion method to find wave solutions of the generalized ZS and Su C.[14] present numerical comparison for several methods to get approximate solutions for Zakharov system ZS in the subsonic limit regime. Spectral methods are a highly accurate and efficient schemes compared with local methods. Based on the test functions choice, we have three kinds of spectral methods, tau, Galerkin, and collocation methods. The idea in these methods is to present an expression to the solution as a finite combination based on orthogonal polynomials. Among these types of spectral methods, collocation method [1, 2, 9] has become a common method to solve differential equations. Also it is very useful to provide high accurate solutions to variable coefficient, and nonlinear problems. The Legendre collocation method proposed in [4], and the well-known differential transform method DTM [7, 13, 10, 11], will be applied to get an approximate solution for ZS, DTM is one of the approximate methods which can be easily applied to many linear and nonlinear problems, based on the Taylor series expansion, a certain transformation rules converts the problem to a set of algebraic equations and the solution of these algebraic equations represents the required solution of the problem. The paper is organized as follows. In section 2 and 3, theoretical aspects of the method are discussed. In section 4, examples with analytical solutions will be given to compare errors of the suggested method. Finally, conclusions are given in section 5.

2. Legendre collocation method

The well-known Legendre polynomials are defined on the interval [-1, 1] and can be determined with the recurrence formula

$$P_{k+1}\left(x\right)=\frac{2k+1}{k+1}xP_{k}\left(x\right)-\frac{k}{k+1}P_{k-1}\left(x\right),k=1,2,..$$

where$P_0(x)=1$ and $P_1(x)=x$, and the orthogonality relation is

$$\int_{-1}^{1}P_i(x)P_j(x)dx=\frac{2}{2j+1}\delta_{ij}$$

any function of two variables $u(x,t)$ which is infinitely differential in [-1,1]×[0,1] may be expressed in terms of the double Legendre polynomials as

$$u_{NM}(x,t)=\sum_{j=0}^{N}\sum_{i=0}^{M}a_{ij}P_i\left(t\right)P_j\left(x\right)=\phi(t)^T A\ \phi(x)$$

with Legendre vector

$$\phi(x)=[P_0(x)P_1(x)\ldots\ldots P_N(x)]$$

and Legendre coefficients matrix

$$A=(a_{ij}),0\leq i\leq M,0\leq j\leq N,$$

$$a_{ij}=\frac{(2i+1)}{2}\frac{(2j+1)}{2}\int_{-1}^{1}\int_{-1}^{1}u(x,t)P_i(t)P_j(x)dtdx$$

Now, we will extend Legendre collocation method to numerically solve for GZS in complex form:

$$i\psi_t(x,t)+\psi_{xx}(x,t)-2\beta|\psi(x,t)|^2\psi(x,t)+2\psi(x,t)w(x,t)=0, \quad (1)$$

$$w_{tt}(x,t)-w_{xx}(x,t)+(|\psi(x,t)|^2)_{xx}=0. \quad (2)$$

with boundary and initial conditions

$$\begin{aligned} \psi(x,0) &= \psi_1(x), \\ w(x,0) &= w_1(x), \\ w_t(x,0) &= w_2(x). \end{aligned} \tag{3}$$

$$\begin{aligned} \psi(-1,t) = \psi_2(t), \psi(1,t) = \psi_3(t), \\ w(-1,t) = w_3(t), w(1,t) = w_4(t). \end{aligned} \tag{4}$$

the complex ZS (1) and (2) may be written as a system of three partial differential equations by splitting the complex function to real and imaginary parts as follows:

$$\psi(x,t) = u(x,t) + iv(x,t) \tag{5}$$

$$\begin{aligned} u_t(x,t) + v_{xx}(x,t) - 2\beta(u^2v + v^3) + 2v(x,t)w(x,t) = 0, \\ -v_t(x,t) + u_{xx}(x,t) - 2\beta(v^2u + u^3) + 2u(x,t)w(x,t) = 0, \\ w_{tt}(x,t) - w_{xx}(x,t) + (u^2 + v^2)_{xx} = 0. \end{aligned} \tag{6}$$

where *u(x,t)*and *v(x,t)* are real functions, also the boundary and initial condition(3) and (4) will be:

$$\begin{aligned} u(x,0) = f_1(x), \quad v(x,0) = f_2(x), \\ w(x,0) = w_1(x), \quad w_t(x,0) = w_2(x) \\ u(-1,t) = g_1(t), \quad u(1,t) = g_2(t), \\ v(-1,t) = g_3(t), \quad v(1,t) = g_4(t) \\ w(-1,t) = w_3(t), \quad w(1,t) = w_4(t). \end{aligned} \tag{7}$$

Now, we use Legendre polynomials to approximate *u(x, t)*,*v(x, t)* and *w(x, t)* as:

$$\begin{aligned} u_{NM}(x,t) &= \sum_{j=0}^{N}\sum_{i=0}^{M} a_{ij}P_i(t)P_j(x) = \phi(t)^T A\phi(x), \\ v_{NM}(x,t) &= \sum_{j=0}^{N}\sum_{i=0}^{M} b_{ij}P_i(t)P_j(x) = \phi(t)^T B\phi(x), \\ w_{NM}(x,t) &= \sum_{j=0}^{N}\sum_{i=0}^{M} c_{ij}P_i(t)P_j(x) = \phi(t)^T C\phi(x). \end{aligned} \tag{8}$$

where A, B and C are unknown (N+1) × (M+1) matrices and the Legendre vector given by

$$\phi(x) = [P_0(x)P_1(x)P_2(x)\ldots P_N(x)]$$

and

$$\phi(t) = [P_0(t)P_1(t)P_2(t)\ldots P_M(t)].$$

The first derivative of the vector $\phi(x)$ as expressed by [6]

$$\frac{d}{dx}\phi(x) = D^{\phi}(x) = D^{(1)}\phi(x) \tag{9}$$

where D is the (M+ 1)×(M+ 1) operational matrix of derivative given by $D^{(1)} = d_{ij}$ where

$$d_{ij} = \begin{cases} 2j+1\ , j = i-k, \begin{cases} k = 1,3,..,m \text{ if m is odd} \\ k = 1,3,..,m-1 \text{ if m is even.} \end{cases} \\ 0, \qquad \text{otherwise.} \end{cases}$$

from (9) we can find the k-th derivative as follows:

$$\frac{d^k}{dx^k}\phi(x) = D^{(k)}\phi(x) = (D^{(1)})^k\phi(x) \tag{10}$$

using (10) and (8), we can write

$$\begin{array}{c}\frac{\partial}{\partial t}u(x,t)=\phi(t)^T D_{(1)}^T A\phi(x),\ \frac{\partial}{\partial x}u(x,t)=\phi(t)^T AD_{(1)}\phi(x),\\ \frac{\partial}{\partial t}v(x,t)=\phi(t)^T D_{(1)}^T B\phi(x),\ \frac{\partial}{\partial x}v(x,t)=\phi(t)^T BD_{(1)}\phi(x),\\ \frac{\partial}{\partial t}w(x,t)=\phi(t)^T D_{(1)}^T C\phi(x),\ \frac{\partial}{\partial x}w(x,t)=\phi(t)^T CD_{(1)}\phi(x),\\ \frac{\partial^2}{\partial x^2}u(x,t)=\phi(t)^T AD_{(2)}\phi(x),\ \frac{\partial^2}{\partial x^2}v(x,t)=\phi(t)^T BD_{(2)}\phi(x),\\ \frac{\partial^2}{\partial x^2}w(x,t)=\phi(t)^T CD_{(2)}\phi(x),\ \frac{\partial^2}{\partial t^2}w(x,t)=\phi(t)^T D_{(2)}^T C\phi(x).\end{array} \quad (11)$$

By substituting (11) and (8) in GZS (6) and its initial and boundary conditions(7), we get

$$\begin{array}{c}\phi(t)^T D_{(1)}^T A\phi(x)+\phi(t)^T BD_{(2)}\phi(x)-2\beta(\{\phi(t)^T A\phi(x)\}\{\phi(t)^T A\phi(x)\}\{(\phi(t)^T B\phi(x)\})\\ -2\beta(\{\phi(t)^T B\phi(x)\}\{\phi(t)^T B\phi(x)\}\{(\phi(t)^T B\phi(x)\})+2\{\phi(t)^T B\phi(x)\}\{(\phi(t)^T C\phi(x)\}=0,\end{array} \quad (12)$$

$$\begin{array}{c}-\phi(t)^T D_{(1)}^T B\phi(x)+\phi(t)^T AD_{(2)}\phi(x)-2\beta(\{\phi(t)^T A\phi(x)\}\{\phi(t)^T A\phi(x)\}\{(\phi(t)^T A\phi(x)\})\\ -2\beta(\{\phi(t)^T B\phi(x)\}\{\phi(t)^T B\phi(x)\}\{(\phi(t)^T A\phi(x)\})+2\{\phi(t)^T A\phi(x)\}\{(\phi(t)^T C\phi(x)\}=0,\end{array} \quad (13)$$

$$\begin{array}{c}\phi(t)^T D_{(2)}^T C\phi(x)-\phi(t)^T CD_{(2)}\phi(x)+2\{\phi(t)^T AD\phi(x)\}\{(\phi(t)^T AD_{(2)}\phi(x)\}\\ +2\{\phi(t)^T BD\phi(x)\}\{(\phi(t)^T BD_{(2)}\phi(x)\}+2\{\phi(t)^T A\phi(x)\}\{(\phi(t)^T AD\phi(x)\}\\ +2\{\phi(t)^T B\phi(x)\}\{(\phi(t)^T BD\phi(x)\}=0.\end{array} \quad (14)$$

with initial and boundary conditions,

$$\begin{array}{c}\phi(0)^T A\phi(x)=f_1(x),\ \phi(0)^T B\phi(x)=f_2(x),\\ \phi(0)^T C\phi(x)=w_1(x),\ \phi(0)^T D_{(1)}^T C\phi(x)=w_2(x)\\ \phi(t)^T A\phi(-1)=g_1(t),\ \phi(t)^T A\phi(1)=g_2(t),\\ \phi(t)^T B\phi(-1)=g_3(t),\ \phi(t)^T B\phi(1)=g_4(t),\\ \phi(t)^T C\phi(-1)=w_3(t),\ \phi(t)^T C\phi(1)=w_4(t).\end{array} \quad (15)$$

We can collocate(12-14) at suitable points to get $3(N+1)\times(M+1)$ system of nonlinear algebraic equations in the unknown coefficients $a_{ij}, b_{ij} and c_{ij}$ the collocation points (x_j, t_i) where $t_i,\ i=0,1,2,3,\dots,M$ are the roots of $P_M(t)$, and $x_j,\ j=0,1,2,3,\dots,N-1$, are the roots of $P_{N-1}(x)$.
Throughout this paper, we use the Mathematica package to construct and solve the nonlinear algebraic system to get the coefficients a_{ij}, b_{ij} and c_{ij}.

3. Differential transform method

Consider a function *w(x, y)* is analytic and differentiated continuously with respect to y, then

$$W(k,h)=\frac{1}{k!h!}\left(\frac{\partial^{k+h}w(x,y)}{\partial^k x\partial^k y}\right)_{\substack{x=x_0\\ y=y_0}}$$

W(k, h) is the transformed function and $w(x,y)$ represent the original function. The differential inverse transform of *W(k, h)* is defined as follows:

$$w(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}W(k,h)(x-x_0)^k(y-y_0)^h. \quad (16)$$

Combining (16) and (17)

$$w(x,y)=\sum_{k=0}^{\infty}\sum_{h=0}^{\infty}\frac{1}{k!h!}\left(\frac{\partial^{k+h}w(x,y)}{\partial^k x\partial^k y}\right)(x-x_0)^k(y-y_0)^h. \quad (17)$$

Original Function	Transformed Function
$w(x,y) = u(x,y) \pm v(x,y)$	$W(k,h) = U(k,h) \pm V(k,h)$
$w(x,y) = \lambda u(x,y)$	$W(k,h) = \lambda u(k,h)$
$w(x,y) = \frac{\partial u(x,y)}{\partial x}$	$W(k,h) = (k+1)U(k+1,h)$
$w(x,y) = \frac{\partial u(x,y)}{\partial y}$	$W(k,h) = (h+1)U(k,h+1)$
$w(x,y) = \frac{\partial^{r+s}u(x,y)}{\partial^r x \partial^s y}$	$W(k,h) = (k+1)(k+2).....(k+r)$ $(h+1)(h+2)...(h+s)U(k+r,h+s)$
$w(x,y) = u(x,y)\,v(x,y)$	$W(k,h) = \sum_{r=0}^{k}\sum_{s=0}^{h} U(r,h-s)\,V(k-r,h)$

for $x_0 = 0$ and $y_0 = 0$ equation (3.3) can be written as a finite series

$$w(x,y) = \sum_{k=0}^{n}\sum_{h=0}^{m} W(k,h)x^k y^h. \tag{18}$$

Theorems that are frequently used in the transformation procedure are introduced in the following table.
Now,we will start applying DTM to get an approximate solution for generalized ZS, the differential transform for *u(x, t)*,*v(x, t)* and *w(x, t)* and equation (6) will be:

$$\begin{aligned} u(x,y) &= \textstyle\sum_{k=0}^{n}\sum_{h=0}^{m} U(k,h)x^k y^h, \\ v(x,y) &= \textstyle\sum_{k=0}^{n}\sum_{h=0}^{m} V(k,h)x^k y^h, \\ w(x,y) &= \textstyle\sum_{k=0}^{n}\sum_{h=0}^{m} W(k,h)x^k y^h. \end{aligned} \tag{19}$$

$$\begin{aligned} (h+1)U[k,h+1] &= -(k+1)(k+2)V[k+2,h] + 2\sum_{r=0}^{k}\sum_{l=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} U[r,h-s-p]V[l,s]U[k-r+l,p] \\ &+2\sum_{r=0}^{k}\sum_{l=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} U[r,h-s-p]U[l,s]U[k-r+l,p] - 2\sum_{r=0}^{k}\sum_{s=0}^{h} V[r,h-s]W[k-r,s], \end{aligned} \tag{20}$$

$$\begin{aligned} (h+1)V[k,h+1] &= (k+1)(k+2)U[k+2,h] - 2\sum_{r=0}^{k}\sum_{l=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} V[r,h-s-p]V[l,s]V[k-r+l,p] \\ &+2\sum_{r=0}^{k}\sum_{l=0}^{k-r}\sum_{s=0}^{h}\sum_{p=0}^{h-s} V[r,h-s-p]U[l,s]V[k-r+l,p] + 2\sum_{r=0}^{k}\sum_{s=0}^{h} U[r,h-s]W[k-r,s], \end{aligned} \tag{21}$$

$$\begin{aligned} (h+1)(h+2)W[k,h+2] &= (k+1)(k+2)W[k+2,h] - 2\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)(k-r+2)U[k-r+2,s]U[r,h-s] \\ &-2\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)(k-r+2)V[k-r+2,s]V[r,h-s] - 2\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)(r+1)V[k-r+1,s]V[r+1,h-s] \\ &-2\sum_{r=0}^{k}\sum_{s=0}^{h}(k-r+1)(r+1)U[k-r+1,s]U[r+1,h-s] \end{aligned} \tag{22}$$

Using the differential transform of initial conditions (7) to start the recurrence relations using (20),(21) and (22) and consequently substituting all the getting values of U[k,h], V[k,h] and W[k,h] into equation (19) to get the approximate solutions for the three functions *u(x,t)* , *v(x,t)* and *w(x,t)*.

4. Implementation of the methods

In this part, Legendre collocation method and DTM will be applied for solving generalized Zakharov system with two different values for β .

TABLE 1. Error comparison for LCM, DTM and mADM [16] for GZS Ex.4.1

		Legendre collocation method n=8		Differential Transform method n=7		mADM n=7 [16]	
	t	$\|e_w^n\|_{L_\infty}$	$\|e_{\psi^2}^n\|_{L_\infty}$	$\|e_w^n\|_{L_\infty}$	$\|e_{\psi^2}^n\|_{L_\infty}$	$\|e_w^n\|_{L_\infty}$	$\|e_{\psi^2}^n\|_{L_\infty}$
x= 0.1	0.1	-6.0495E-12	5.1247E-13	1.3878E-15	-1.0743E-14	7.526E-11	2.0304E-12
	0.2	1.8057E-11	8.0842E-12	4.2133E-14	-7.1403E-13	9.6091E-11	1.7588E-10
	0.3	3.2905E-11	4.9447E-11	-4.0523E-15	-9.8200E-12	3.7703E-11	2.1360E-09
	0.4	6.3849E-12	1.8376E-10	-2.6129E-12	-6.9295E-11	9.9827E-11	1.2273E-08
	0.5	-1.0182E-10	4.9424E-10	-1.9376E-11	-3.2347E-10	1.8164E-10	4.7108E-08
x=0.2	0.1	-7.0745E-12	8.1548E-18	8.4377E-15	3.4104E-20	0.0000E+00	4.6803E-16
	0.2	3.5119E-11	1.3121E-11	1.2140E-13	-8.1162E-13	1.0417E-10	1.2937E-10
	0.3	5.7004E-11	8.3411E-11	5.5866E-13	-6.7999E-12	6.6652E-11	1.8295E-09
	0.4	1.0361E-11	2.8741E-10	5.6372E-13	-4.1195E-11	3.7328E-11	1.1077E-08
	0.5	-1.4598E-10	6.9663E-10	-5.7294E-12	-2.1316E-10	6.5701E-11	4.3627E-08
x=0.3	0.1	-3.3251E-12	-1.6791E-12	1.4932E-14	9.4220E-14	2.4740E-11	2.7984E-12
	0.2	3.4449E-11	9.5256E-12	1.5848E-13	-5.7661E-13	2.4740E-11	7.0697E-11
	0.3	4.6981E-11	8.8567E-11	8.2118E-13	-4.2938E-12	9.6091E-11	1.4621E-09
	0.4	-8.0760E-12	3.2414E-10	2.9541E-12	-2.2890E-11	3.7703E-11	9.6930E-09
	0.5	-1.4916E-10	7.7649E-10	7.8891E-12	-1.5765E-10	9.9827E-11	3.9695E-08
x=0.4	0.1	3.3212E-12	-3.3389E-12	1.4322E-14	3.8038E-13	4.1664E-11	6.3623E-12
	0.2	9.6308E-12	1.2079E-15	1.0819E-13	2.8605E-18	0.0000E+00	1.3363E-13
	0.3	1.0165E-12	6.7426E-11	7.0655E-13	-2.2981E-12	1.0417E-10	1.0340E-09
	0.4	-2.8003E-11	2.9401E-10	4.6122E-12	-1.3061E-11	6.6652E-11	8.1202E-09
	0.5	-4.7826E-11	7.3308E-10	2.2005E-11	-1.4384E-10	3.7328E-11	3.5314E-08
x=0.5	0.1	7.3328E-12	-3.0355E-12	6.1062E-16	1.5582E-12	9.6091E-11	1.0692E-11
	0.2	-3.1373E-11	-8.8479E-12	-4.9127E-14	1.0292E-12	2.4740E-11	8.3107E-11
	0.3	-4.8566E-11	3.1111E-11	2.3531E-13	-1.0438E-12	7.5260E-11	5.4526E-10
	0.4	1.6656E-11	2.1129E-10	5.6073E-12	-9.3120E-12	3.9089E-12	6.3593E-09
	0.5	2.5620E-10	5.8542E-10	3.5885E-11	-1.4929E-10	3.7703E-11	3.0485E-08

Example 4.1
Consider the GZS (1) and (2) with β=1 and the following initial conditions:

$$\begin{aligned} \psi(x,0) &= \frac{\sqrt{3}}{40} tanh(\frac{x}{20}) e^{ix}, \\ w(x,0) &= \frac{1}{3} - \frac{1}{1600} tanh^2(\frac{x}{20}) \\ w_t(x,0) &= \frac{1}{8000} tanh(\frac{x}{20}) sech^2(\frac{x}{20}). \end{aligned} \tag{23}$$

and boundary conditions

$$\begin{aligned} \psi(1,t) &= \frac{\sqrt{3}}{40}\ tanh(\frac{1}{20} - \frac{t}{10})\ e^{i(1-\frac{203}{600}t)} \\ \psi(-1,t) &= \frac{\sqrt{3}}{40}\ tanh(\frac{1}{20} + \frac{t}{10})\ e^{i(-1-\frac{203}{600}t)} \\ w(1,t) &= \frac{1}{3} - \frac{1}{1600}\ tanh^2(\frac{1}{20} - \frac{t}{10}) \\ w(-1,t) &= \frac{1}{3} - \frac{1}{1600}\ tanh^2(\frac{1}{20} + \frac{t}{10}) \end{aligned} \tag{24}$$

with exact solution as given in [16]

$$\begin{aligned} \psi(x,t) &= \frac{\sqrt{3}}{40}\ tanh(\frac{1}{20}(2t-x)) e^{i(x-\frac{203}{600}t)}, \\ w(x,t) &= \frac{1}{3} - \frac{1}{1600} tanh^2(\frac{1}{20}(2t-x)). \end{aligned} \tag{25}$$

Table 1 shows the error comparison for $|\psi|^2$ and w using DTM, LCM and modified Adomian decomposition method [16], the reason why we compare $|\psi|^2$ instead $|\psi|$is that we usually study the square of the module of the high-frequency electric field in plasma. The calculated errors in Table 1 indicate a very good approximation with the actual solution and the error grows higher as the x- distance value increases, and the DTM error better than mADM with three digits and reaches 4 digits at some points,also LCM is better than mADM with two digits.

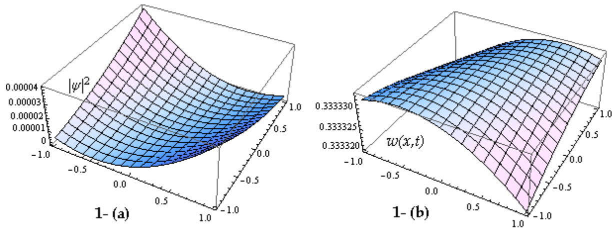

FIGURE 1.
(1a-1b) shows example 4.1 exact solution for $|\psi|^2$ and $w(x,t)$ with β=1.

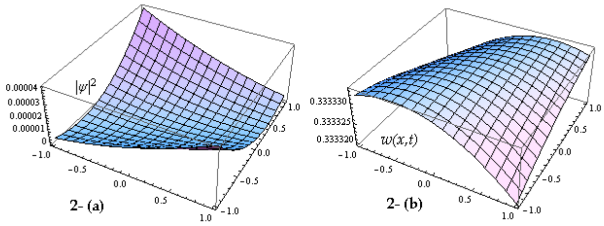

FIGURE 2.
(2a-2b) shows example 4.1 approximate solution using Legendre collocation method with n=m=8 and β=1, the numerical estimations for $|\psi|^2$ and $w(x,t)$ are found to be quite accurate

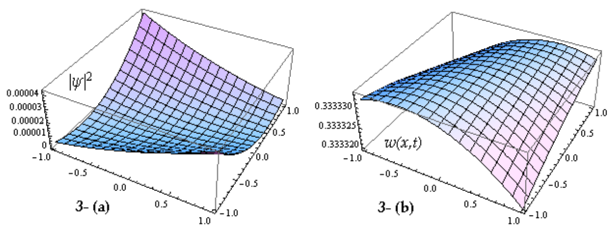

FIGURE 3.
(3a-3b) shows example 4.1 approximate solution using Differential Transform method with n=m=7 and β=1, the numerical estimations for $|\psi|^2$ and $w(x,t)$ are found to be quite accurate

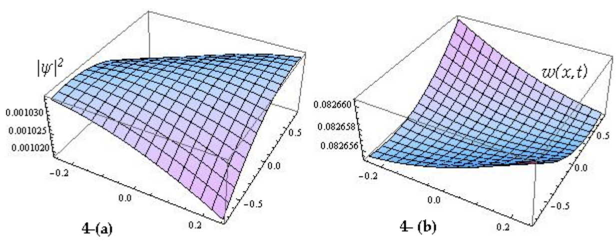

FIGURE 4.
(4a-4b) shows example 4.2 exact solution for $|\psi|^2$ and $w(x,t)$ with β=-10.

Table 2. Error comparison for LCM, DTM and mADM [16] for GZS Ex.4.2

		Legendre collocation method n=8		Differential Transform method n=7		mADM n=7 [16]	
	t	$\|e^n_w\|_{L_\infty}$	$\|e^n_{\psi^2}\|_{L_\infty}$	$\|e^n_w\|_{L_\infty}$	$\|e^n_{\psi^2}\|_{L_\infty}$	$\|e^n_w\|_{L_\infty}$	$\|e^n_{\psi^2}\|_{L_\infty}$
x = 0.1	0.1	-2.4344E-10	-4.8707E-11	4.2143E-08	-4.7719E-06	1.5900E-11	2.4000E-10
	0.2	-4.0441E-10	-8.7202E-10	3.4631E-07	-9.0056E-06	2.4000E-12	4.1300E-09
	0.3	-3.2613E-10	-3.1754E-09	1.1570E-06	-1.1561E-05	3.2100E-11	2.2896E-08
	0.4	2.2725E-10	-7.1455E-09	2.6129E-06	-1.1598E-05	4.6000E-12	8.0653E-08
	0.5	1.6778E-09	-1.1986E-08	4.6522E-06	-8.1612E-06	4.6700E-11	2.2127E-07
x=0.2	0.1	-3.8439E-10	-1.4192E-10	4.9226E-08	-4.9408E-06	1.6300E-11	2.3700E-10
	0.2	-5.4840E-10	-1.6873E-09	3.9414E-07	-9.4018E-06	3.2000E-11	4.0760E-09
	0.3	-1.7712E-10	-5.8162E-09	1.2731E-06	-1.2116E-05	4.0100E-11	2.2635E-08
	0.4	1.0485E-09	-1.2748E-08	2.7441E-06	-1.2134E-05	4.0300E-11	7.9769E-08
	0.5	3.5158E-09	-2.0978E-08	4.5553E-06	-8.3681E-06	3.1800E-11	2.1884E-07
x=0.3	0.1	-3.1078E-10	-2.6425E-10	5.1679E-08	-4.8083E-06	1.5900E-11	2.3200E-10
	0.2	-3.1216E-10	-2.4649E-09	4.0507E-07	-9.2317E-06	1.5900E-11	4.0190E-09
	0.3	3.8443E-10	-8.1358E-09	1.2722E-06	-1.1975E-05	2.4000E-12	2.2359E-08
	0.4	2.1139E-09	-1.7464E-08	2.6326E-06	-1.2057E-05	3.2100E-11	7.8838E-08
	0.5	5.1443E-09	-2.8237E-08	4.0850E-06	-8.3476E-06	4.6000E-12	2.1630E-07
x=0.4	0.1	1.5067E-11	-3.7544E-10	4.8984E-08	-4.3614E-06	1.8000E-11	2.2600E-10
	0.2	3.0666E-10	-3.0450E-09	3.7708E-07	-8.4925E-06	4.3700E-11	3.9580E-09
	0.3	1.1880E-09	-9.7715E-09	1.1559E-06	-1.1148E-05	1.8000E-11	2.2070E-08
	0.4	2.9206E-09	-2.0695E-08	2.3051E-06	-1.1340E-05	9.9000E-12	7.7866E-08
	0.5	5.6101E-09	-3.2998E-08	3.3438E-06	-7.8976E-06	9.7000E-12	2.1365E-07
x=0.5	0.1	4.7814E-10	-4.2329E-10	4.1638E-08	-3.6154E-06	2.4000E-12	2.1900E-10
	0.2	1.0185E-09	-3.2240E-09	3.1601E-07	-7.2351E-06	1.5900E-11	3.8940E-09
	0.3	1.6927E-09	-1.0302E-08	9.4971E-07	-9.7082E-06	1.5900E-11	2.1767E-08
	0.4	2.6152E-09	-2.1819E-08	1.8341E-06	-1.0056E-05	5.2400E-11	7.6851E-08
	0.5	3.7701E-09	-3.4589E-08	2.4936E-06	-7.0445E-06	2.7900E-11	2.1089E-07

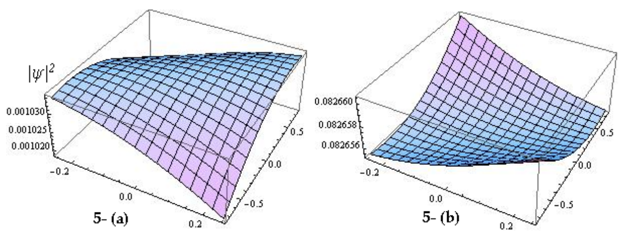

Figure 5.
(5a-5b) shows example 4.2 approximate solution using Legendre collocation method with n=m=8 and β=-10, the numerical estimations for $|\psi|^2$ and $w(x,t)$ are found to be quite accurate

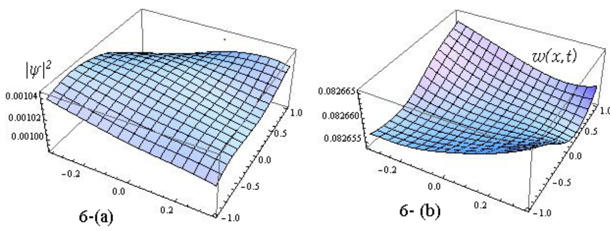

Figure 6.
(6a-6b) shows example 4.2 approximate solution using Differential Transform method with n=m=7 and β=-10, the numerical estimations for $|\psi|^2$ and $w(x,t)$ are found to be quite accurate

Example 4.2
Consider the GZS (1) and (2) with β=-10 and the following initial conditions:

$$\begin{aligned} \psi(x,0) &= \frac{1}{10}\sqrt{\frac{3}{29}}sech(\frac{x}{10})\ e^{ix}, \\ w(x,0) &= \frac{83}{1000} - \frac{1}{2900}sech^2(\frac{x}{10}) \\ w_t(x,0) &= -\frac{1}{7250}tanh(\frac{x}{10})sech^2(\frac{x}{10}) \end{aligned} \tag{26}$$

and boundary conditions

$$\begin{aligned} \psi(1,t) &= \tfrac{1}{10}\sqrt{\tfrac{3}{29}}sech(\tfrac{1}{10}-\tfrac{t}{5})\ e^{i}(1-\tfrac{103}{125}t) \\ \psi(-1,t) &= \tfrac{1}{10}\sqrt{\tfrac{3}{29}}sech(\tfrac{1}{10}+\tfrac{t}{5})\ e^{i}(-1-\tfrac{103}{125}t) \\ w(1,t) &= \tfrac{83}{1000}-\tfrac{1}{2900}sech^2(\tfrac{1}{10}-\tfrac{t}{5}) \\ w(-1,t) &= \tfrac{83}{1000}-\tfrac{1}{2900}sech^2(\tfrac{1}{10}+\tfrac{t}{5}) \end{aligned} \tag{27}$$

with exact solution

$$\begin{aligned} \psi(x,t) &= \tfrac{1}{10}\sqrt{\tfrac{3}{29}}\ sech(\tfrac{1}{10}(2t-x))\ e^{i(x-\frac{103}{125}t)}, \\ w(x,t) &= \tfrac{83}{1000}-\tfrac{1}{2900}sech^2(\tfrac{1}{10}(2t-x)). \end{aligned} \tag{28}$$

Table 2 shows the error comparison for $|\psi|^2$ and w using DTM, LCM and and modified Adomian decomposition method [16],
The calculated errors in Table 2 indicate an effective approximation with the actual solution and the error grows higher as the x- distance value increases, and the LCM gave a better error than DTM and mADM.

5. Summary and Conclusion

Application of the Legendre collocation method and Differential transform method are effective than Adomain decomposition technique to investigate numerical solutions of nonlinear complex system problems. In our first caseβ =1, the results shows that both of them is a powerful technique for finding approximate solutions with better accuracy than mADM for GZS reaches 3 and 4 digits at some points. Forβ = -10 numerical results indicate that LCM for the square of the module of the high-frequency electric field in plasma perform better accuracy than DTM and mADM.

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