Early Detection of Rogue Waves Using Compressive Sensing

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We discuss the possible usage of the compressive sampling for the early detection of rogue waves in a chaotic sea state. One of the promising techniques for the early detection of the oceanic rogue waves is to measure the triangular Fourier spectra which begin to appear at the early stages of their development. For the early detection of the rogue waves it is possible to treat such a spectrum as a sparse signal since we would mainly be interested in the high amplitude triangular region located at the central wavenumber. Therefore compressive sampling can be a very efficient tool for the rogue wave early warning systems. Compressed measurements can be acquired by remote sensing techniques such as coherent SAR which measure the ocean surface fluctuation or by insitu techniques such as spectra measuring tools mounted on a ship hull or bottom mounted pressure gauges. By employing a numerical approach we show that triangular Fourier spectra can be sensed by compressed measurements at the early stages of the development of rogue waves in a chaotic sea state. This may lead to development to early warning hardware systems which use the compressive sampling thus the memory requirements for those systems can be greatly reduced.

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I. INTRODUCTION

Rogue waves are commonly defined as the waves with a height more than 2-2.2 times the significant wave height in the wave field [24]. They present a danger to life, marine travel and operations. Their result can be catastrophic and costly [17, 24]. The early detection of rogue waves in the chaotic ocean is a must to ensure the safety of the marine travel and the offshore structures in stormy conditions [17]. This vital problem has been disregarded for a long time and has only been studied for almost a decade [17]. However early detection of rogue waves is an extremely hard and complicated problem due to many processes involved [17]. First of all rogue waves appear in stormy conditions therefore accurate prediction of weather conditions is a must. Secondly rogue waves appear in a length (time) scale that are on the order of their width [4]. Due to the rapidly changing nature of the rogue waves, the reliability of the forecast of the rogue waves currently is not very high and it is hard to expect that it will become more reliable in the near future [4]. One of the promising approaches which addresses this problem is to continuously measure the part of the whole surface spectrum in real time and use the triangular Fourier spectra of the growing rogue waves in early stages of their development in a chaotic wave field before the dangerous peak appears [4].

In this paper, we discuss the possible usage of the compressive sampling for the early detection of the oceanic rogue waves. We use a similar methodology to the one introduced in [4]. Similarly we analyze the emerging triangular rogue wave spectra, however we obtain those spectra via compressed measurements with a significant undersampling ratio. In order to sense the triangular spectra via compressive sampling, we offer a procedure as follows: in the time evolving field we take uniformly spaced undersampled measurements from the ocean surface fluctuations. Then we reconstruct the sparse triangular rogue wave spectra via compressive sampling. In order to filter out the replicas in the spectrum due to uniformly spaced sampling and noisy high wavenumber components, which are not needed for the detection of the emerging rogue waves, we apply the Gaussian filter to the spectrum recovered by compressive sampling. Then we move to the next time step. By implementation of a numerical method we show that using compressive sampling technique we can detect the emergence of the triangular rogue wave spectra using significantly less samples compared to the classical sensing techniques.

Although the processes governed studied in this paper are very complicated, they are still governed by a partial differential equation. Therefore they can be predicted once an initial condition is specified. Thus compared to the completely unpredictable true stochastic processes, the processes described in this study can be named as 'chaotic' [14, 16]. Throughout this paper we use the term 'chaotic' in this setting. The chaotic open ocean presents many difficulties and precisely how the compressive measurements could be accomplished in practice is beyond the scope of this letter. It is possible to use remote sensing techniques such as coherent SAR [6, 7] or insitu techniques such as a device installed on a ship which measure the spectrum of only a part of the whole surface quickly by scanning the water surface bit-by-bit [4]. However as shown in this paper the compressive sampling can significantly reduce the memory requirements and the cost of rogue wave early warning systems which may operate remotely or insitu. This helps avoidance of particular ocean regions when mapping navigation routes efficiently [4]. So by means of smart designs and operations, the safety of the ocean travel and offshore operations can be increased.

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II. REVIEW OF THE NLSE AND ITS ROGUE WAVE SOLUTIONS

Dynamics of weakly nonlinear deep water ocean waves can be described by the nonlinear Schrödinger equation (NLSE) [29]. It has been previously shown that the NLSE can also be used as a model to describe oceanic rogue waves [1, 2, 4, 17]. One of the most widely used forms of the NLSE is given by

$$i\psi_t + \frac{1}{2}\psi_{xx} + |\psi|^2 \psi = 0 \tag{1}$$

where x, t is the spatial and temporal variables, i denotes the imaginary number and ψ is complex amplitude [17]. This notation is mainly used in ocean wave theory however the t and x axes are switched in the optics studies [17]. NLSE is also widely used in other branches of the applied sciences and engineering to describe various phenomena including but not limited to Bose-Einstein condensates, pulse propagation in optical fibers and quantum state of a physical system. Integrability of the NLSE is studied extensively within last forty years and many exact solutions of the NLSE are derived. Some rational soliton solutions of the NLSE are derived as well. One of the most early forms of the rational soliton solution of the NLSE is the Peregrine soliton [26]. It is given by

$$\psi_1 = \left[1 - 4\frac{1+2it}{1+4x^2+4t^2}\right]\exp{[it]} \tag{2}$$

where t is the time and x is the space parameter. It is shown that Peregrine soliton is a first order rational soliton solution of the NLSE and higher order rational solutions also exist [1]. The second order rogue wave which satisfies the NLSE exactly is given as [1]

$$\psi_2 = \left[1 + \frac{G_2 + itH_2}{D_2}\right]\exp{[it]} \tag{3}$$

where

$$G_2 = \frac{3}{8} - 3x^2 - 2x^4 - 9t^2 - 10t^4 - 12x^2t^2 \tag{4}$$

$$H_2 = \frac{15}{4} + 6x^2 - 4x^4 - 2t^2 - 4t^4 - 8x^2t^2 \tag{5}$$

and

$$D_2 = \frac{1}{8}[\frac{3}{4} + 9x^2 + 4x^4 + \frac{16}{3}x^6 + 33t^2 + 36t^4 + \frac{16}{3}t^6 - 24x^2t^2 + 16x^4t^2 + 16x^2t^4] \tag{6}$$

Even higher order rational solutions of the NLSE and a hierarchy of obtaining those rational solutions based on Darboux transformations [25] are given in [1]. Additionally, throughout many simulations it has been confirmed that rogue waves obtained by numerical techniques which solve the NLSE are in the forms of these first (Peregrine) and higher order rational solutions of the NLSE [1, 2, 4]. One of the promising techniques for the early detection of the rogue waves is to use the triangular Fourier spectra at the early stages of the development of the rogue waves [3] therefore the theoretical shape of the Fourier spectra of the rogue waves need to be discussed. The Fourier transform of the Peregrine soliton can analytically be calculated as

$$F(k,t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\psi(t,x)e^{ikx}dx \tag{7}$$

which gives

$$F(k,t) = \sqrt{2\pi}[\frac{1+2it}{\sqrt{1+4t^2}}\exp\left(-\frac{|k|}{\sqrt{2}}\sqrt{1+4t^2}\right) - \delta(k)] \, .\exp{[it]} \tag{8}$$

where k is the wavenumber parameter and δ is the Dirac-delta function [4]. The Fourier spectra of the first, second and higher order rogue waves are compared and discussed in [1] in detail where some analytical expressions and mainly numerical and illustrative results are presented. Similar to the first order rogue wave, the second order rogue wave has roughly a triangular Fourier spectrum [3] when the dirac delta peak due to constant term is ignored. However compared to the Fourier spectrum of the first order rogue wave, the Fourier spectrum of the second order rogue wave exhibits two dips due to increased number of sidebands in the wave profile [3, 17].

III. REVIEW OF THE COMPRESSIVE SAMPLING

After it has been introduced to the scientific community with a seminal paper by [20], compressive sampling (CS) has become a core research area in the last decade. In summary, CS states that a sparse signal can be reconstructed from fewer samples than the samples that Nyquist-Shannon sampling theorem states. Currently it is a common tool in various branches of applied mathematics and engineering and currently many software and hardware systems make use of this efficient signal processing technique. In this section we try to sketch a brief summary of the CS for the oceanographers.

Let η be a K-sparse signal of length N, that is only K out of N elements of the signal are nonzero. η can be represented using a orthonormal basis functions with transformation matrix λ. Typical transformation used in literature are Fourier, discrete cosine or wavelet transforms just to mention few. Therefore one can write $\eta = \lambda\widehat{\eta}$ where $\widehat{\eta}$ is the transformation coefficient vector. Since η is a K-sparse signal one can discard the zero coefficients and obtain $\eta_s = \lambda\widehat{\eta}_s$ where η_s is the signal with non-zero elements only.

The idea underlying in the CS is that a K-sparse signal η of length N can exactly be reconstructed from $M \geq C\mu^2(\mathbf{\Phi}, \lambda)K$ log (N/K) measurements with an overwhelmingly high probability, where C is a positive constant and $\mu^2(\Phi, \lambda)$ is coherence between the sensing basis $\mathbf{\Phi}$ and transform basis λ [20]. Taking M random projections by using the sensing matrix $\mathbf{\Phi}$ one can write $g = \mathbf{\Phi}\eta$. Therefore the problem can be recognized as

$$\min \|\widehat{\eta}\|_{l_1} \quad \text{under constraint} \quad g = \mathbf{\Phi}\lambda\widehat{\eta} \tag{9}$$

where $\|\widehat{\eta}\|_{l_1} = \sum_i |\widehat{\eta}_i|$. So that among all signals which satisfies the given constraints, the l_1 solution of the CS problem can be given as $\eta_{CS} = \lambda\widehat{\eta}$. l_1 minimization is only one of the alternatives which can be used for obtaining the solution of this optimization problem. There are some other algorithms to recover the sparse solutions such as greedy algorithms [19]. A more detailed discussion of the CS can be seen in [20]. It is useful to note that we are using the sparsity property of the Fourier transform of ψ. Therefore we can write $\lambda\psi = \eta$ where η is sparse triangular spectra, λ is the Fourier transformation matrix and $\psi = \widehat{\eta}$ is the sparse surface fluctuation measurement.

IV. COMPRESSIVE SAMPLING OF THE TRIANGULAR ROGUE WAVE SPECTRA

As discussed in [4], the triangular shape of the Fourier spectra can be used for the early detection of the rogue waves since it becomes evident at the early stages of their development. This is true for both the first and the second order rational solitons and also validated for the rogue waves in a chaotic wave field [4, 17]. In the Figure 1, we present the Peregrine soliton at different times with constant level subtracted and its Fourier spectrum obtained from $N = 1024$ classical samples and from $M = 64$ compressed samples, that is with an undersampling ratio of $s = 1024/64 = 16$. The compressive sampling theory states that selection of the M samples must be done randomly, however for the early detection of rogue waves we have the priori knowledge that the triangular spectra will develop around central wavenumber. Therefore we can filter out the high wavenumber components. We take $M = 64$ uniformly spaced samples from the physical ocean surface, ψ. Uniformly spaced sampling results in replicas in the spectra recovered by CS. In order to avoid these replicas and and to suppress the low wavenumber components, which we do not need to capture for the detection of the emerging triangular rogue wave spectra, we use a Gaussian low-pass filter of the form

$$f(k) = \exp\left[-\left(\frac{N}{M}\frac{k}{max(k)}\right)^2\right] \tag{10}$$

We apply this Gaussian filter in the wavenumber domain by simple multiplication. The spectra obtained from $N = 1024$ classical samples and from $M = 64$ compressed samples are plotted in Figure 1. As the Figure 1 confirms, triangular Fourier spectra can be obtained using compressed measurements as well with significantly less samples. As the spectrum becomes more sparse depending on the temporal evolution of the soliton or depending on the selected number of spectral components, the results match with the results obtained using the full spectral components well and can even become same exact value.

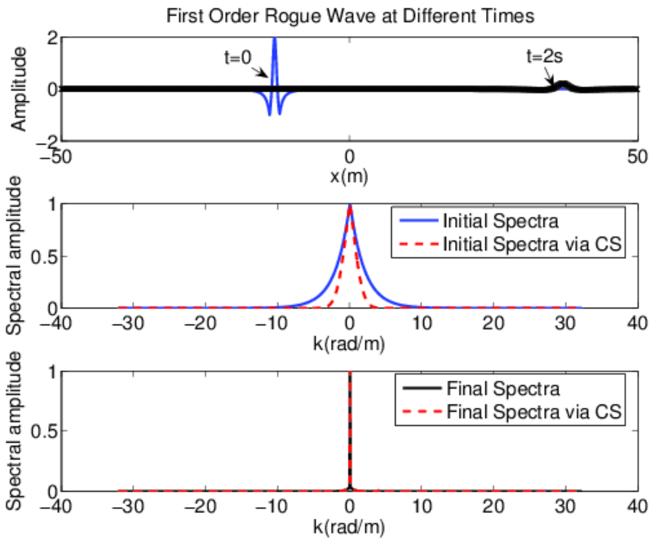

FIG. 1: a) Peregrine soliton at different times b) comparison of its initial Fourier spectra obtained by full (—) vs the compressive sampling techniques (- -) c) comparison of its final Fourier spectra obtained by full (—) vs the compressive sampling techniques (- -).

Additionally many simulations of the chaotic wavefield have revealed that rogue waves with height more that 3 can not be described by first order rogue wave [4, 17]. Therefore second and higher order rational soliton solutions are used to describe the rogue waves with height bigger than 3. It has been shown that second order rational soliton solution of NLSE, with a peak height 5, can be a model for the oceanic rogue waves [4, 17]. It has also been confirmed that the second order rational soliton solution of the NLSE still presumes a triangular Fourier spectra [4, 17]. Therefore we repeat the same analysis for second order rational soliton. In the Figure 2, we present the second order rational rogue wave with constant level subtracted and its Fourier spectrum obtained from $N = 2048$ classical samples and from $M = 128$ compressed samples, that is with an undersampling ratio of $s = 2048/128 = 16$. We select $M = 128$ uniformly spaced samples on the ocean surface envelope, ψ, and reconstruct the Fourier spectra via CS. After reconstruction the Gaussian filter is applied in order to remove spectral replicas due to uniformly spaced sampling and to remove high wavenumber components. As it can be realized the

Figure 2, almost triangular spectra with sidelobes can be reconstructed using both methods where the CS uses significantly less samples.

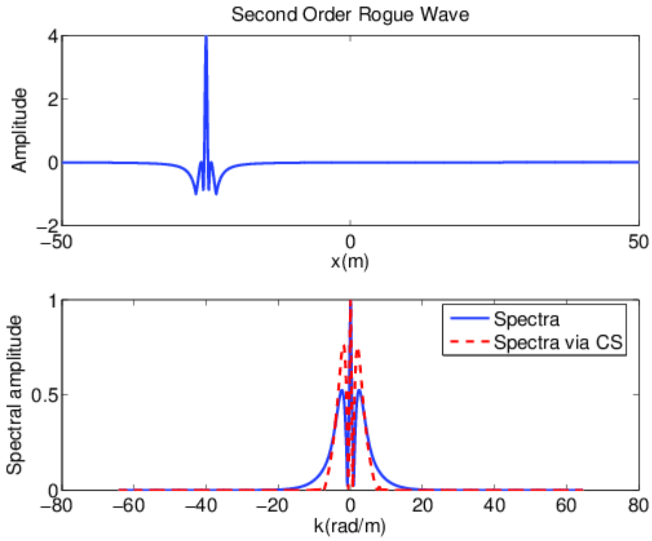

FIG. 2: a) Second order rational soliton b) comparison of its initial Fourier spectra obtained by full (—) vs the compressive sampling techniques (- -).

V. EARLY DETECTION OF THE TRIANGULAR ROGUE WAVE SPECTRA VIA COMPRESSIVE SAMPLING IN A CHAOTIC FIELD

Results presented above are promising for the theoretical spectra calculations however it is necessary to examine how the compressive sampling measurements can be applied to a chaotic sea state with many spectral components. Therefore we use a numerical method for the simulation of chaotic wave field. For this purpose we use a split-step Fourier method (SSFM) which is one of the widely used Fourier spectral methods with efficient time integration. SSFM performs the time integration by time stepping of the exponential function for an equation which includes a first order time derivative [17]. In SSFM, like other spectral techniques [8, 9, 28], the spatial derivatives are calculated by the orthogonal transforms [10, 13, 15]. Some of their applications can be seen in [5, 17, 21–23] and more detailed discussions can be seen in [28]. In the SSFM, the time integration is performed by time stepping of the exponential function for an equation which includes a first order time derivative [17]. SSFM is based on the idea of splitting the governing equation into two parts, the nonlinear and the linear part. For the NLSE, the advance in time due to nonlinear part can be written as [17]

$$i\psi_t = -\left|\psi\right|^2\psi \tag{11}$$

which can be exactly solved as

$$\tilde{\psi}(x, t_0 + \Delta t) = e^{i|\psi(x,t_0)|^2\Delta t}\ \psi(x, t_0) \tag{12}$$

where Δt is the time step. Taking linear part of the NLSE as [17]

$$i\psi_t = -\frac{1}{2}\psi_{xx} \tag{13}$$

Using the Fourier series one can write that

$$\psi(x, t_0 + \Delta t) = F^{-1}\left[e^{-ik^2\Delta t/2}F[\tilde{\psi}(x, t_0 + \Delta t)]\right] \tag{14}$$

where k is the Fourier transform parameter [17]. Therefore combining the expressions in (12) and (14), the complete formulation of the SSFM can be written as

$$\psi(x, t_0 + \Delta t) = F^{-1}\left[e^{-ik^2\Delta t/2}F[e^{i|\psi(x,t_0)|^2\Delta t}\ \psi(x, t_0)]\right] \tag{15}$$

which is used to calculate the surface fluctuations, ψ, starting from the initial conditions. We start the rogue wave simulations using a constant amplitude wave with an additive small chaotic perturbation. Such an initial condition is unstable and it evolves into a full-scale chaotic wave field as shown in the numerical simulations described in [2, 17]. The chaotic wave field with this initial condition evolves into a wave field which exhibits many amplitude peaks, with some of them becoming rogue waves [4]. We use the initial condition

$$\psi(x, t = 0) = 1 + \nu a(x)\exp{[i\theta]} \tag{16}$$

where $a(x)$ is a normalized normally distributed random real function with values in interval $[-1, 1]$, θ is a normally distributed random real function with values in the interval $[0, 2\pi]$ [17]. The number of spectral components are selected as $N = 4096$ in order to make use of the fast Fourier transforms efficiently. The time step is selected as $dt = 0.05$ which does not cause any stability problems. The actual water surface fluctuation for this initial condition would be given by the real part of $|\psi|\exp{[i\omega t]}$ where ω is some carrier wave frequency. A similar but different initial condition is described in [1] and in [4]. Following [4], a value of $\nu = 0.6$ is selected. It is possible to add perturbations with a characteristic length scale L_{pert}, by multiplying the second term in the (16) by a factor of $\exp(i2\pi/L_{pert}x)$. Or one can add perturbations with different length scales using Fourier analysis. However in the present study for illustrative purposes such as scale is not considered.

For a better appreciation of the wave structure and the location of its main peak, the contour plot of the chaotic wave field is shown in the Figure 3. Actual domain of simulations is much longer with an interval of $L = [-500, 500]$ which has been resolved with 4096 spectral components. In the Figure 3, it is observed that the rogue wave with an amplitude close to 5 at $x = 9.52m, t = 42.56s$ has appeared. The shape of this

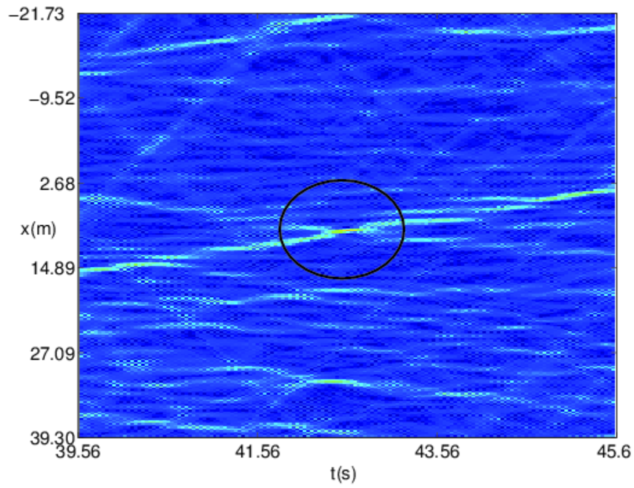

FIG. 3: Contour map of a rogue in the chaotic wave field.

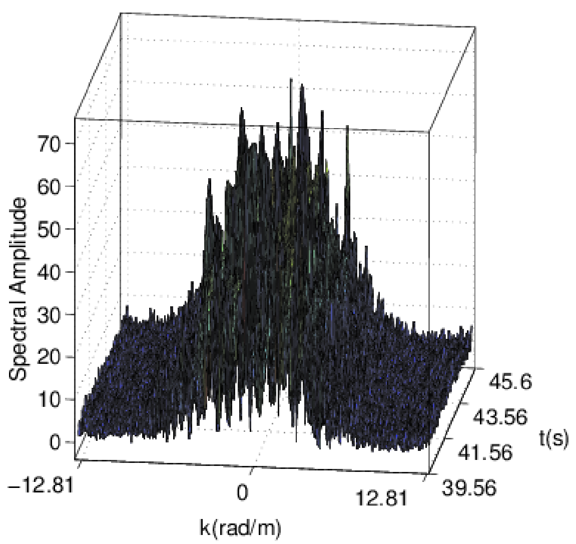

FIG. 4: Triangular Fourier spectrum of the rogue wave in the chaotic wave field obtained classically by $N = 256$ samples.

rogue wave can be described using the second order rational soliton solution of the NLSE as discussed by [17].

It has been previously shown for the similar simulations that, the triangular Fourier spectrum begins to develop in a length (time) scale on the order of the width of the rogue wave itself [4]. In order to show this behavior we plot the spectrum of the chaotic field shown above in the Figure 4, which has been obtained by means of an Fourier transform applied to the chaotic field shown in the Figure 3. However for better presentation we plot only $N = 256$ wavenumber components. The remarkable feature in observing the rogue wave spectrum that it has triangular peak before and after the dangerous peak has appeared [4]. This means that it is possible to detect the rogue wave using the spectrum earlier than it appears in real time [4]. Additionally as the plot confirms, the triangular spectral features are significantly above the noise level which allows for the detection of rogue waves in a chaotic sea state [4].

Although the time scale mentioned above is short for early warning purposes, it is still beneficial for saving lives of people in the marine environment [4]. Currently improving the early warning times is under intense discussion [4] and possibly the early warning times will be enhanced in future.

In this study our main concentration is to sense the emerging triangular rogue wave spectra more efficiently using CS for the similar early warning times. In order to reconstruct triangular spectra via CS, we offer a procedure as follows: in the time evolving numerical model which is represented well by N spectral components we only take M uniformly spaced compressed samples from the amplitude field, ψ, at each time step. Then using the M compressed measurements we solve the l_1 minimization problem of CS summarized in previous section. Then we apply the Gaussian filter given in (10) to the spectrum recovered by CS in order to filter out the replicas in the spectrum due to uniformly spaced sampling and noisy high wavenumber components. Then we move to the next time step.

In Figures 5-6, we plot the rogue wave spectra obtained by the CS techniques with undersampling values of $s = 8, 16$ respectively. That is, we recover the spectrum given in Figures 4 which is well represented by $N = 256$ components using only $M = 32, 16$ components, respectively.

As one can realize from the Figures 5-6, the remarkable feature of the rogue wave, a high peak triangular rogue wave spectra, can be detected using the CS methodology described above. The results plotted on these figures confirm that CS is capable of detecting triangular Fourier spectrum begins to develop in a length (time) scale on the order of the width of the rogue wave itself, similar to the classical sampling with full $N = 256$ components.

The proposed Gaussian filter used to remove the replicas due to uniformly spaced samples and to remove noisy high wavenumber components, which are not needed to detect the emergence of the dangerous peak, works well. However if not only the detection of dangerous peak but also determination of its shape and sidelobes becomes important, it may be necessary to modify this Gaussian filter to capture the correct shape of the rogue wave while the efficiency of the CS based sensing methodology is still used.

The great advantage of the CS based methodology proposed in this paper, is obvious. We can detect the developing Fourier spectra of the emerging rogue waves using CS with significantly less samples compared to the classical sensing theory. This result may lead to development

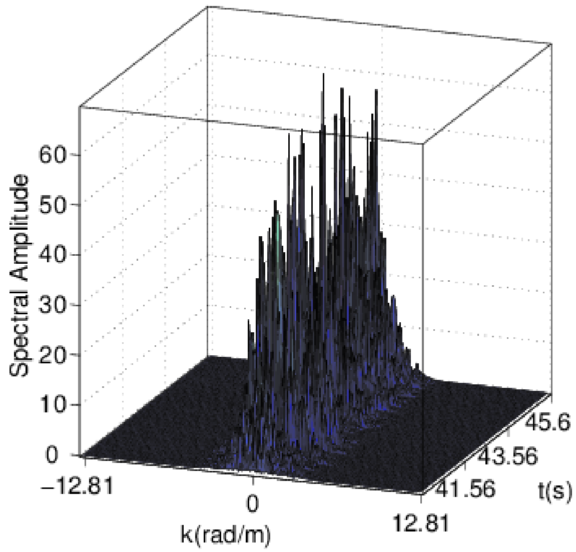

FIG. 5: Triangular Fourier spectrum of the rogue wave in the chaotic wave field obtained by $M = 32$ compressed samples.

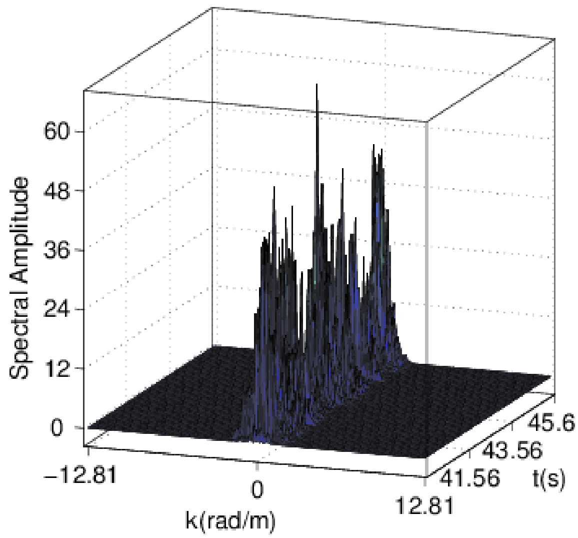

FIG. 6: Triangular Fourier spectrum of the rogue wave in the chaotic wave field obtained by $M = 16$ compressed samples.

of low cost remote and insitu sensing devices with significantly less memory requirements compared to the classical sensing devices. It also can transform the rogue wave early warning technology, rather than directly measuring the spectra it may become easier and more efficient to measure the time series with compressed measurements which can be adapted to measurement systems currently in use. Currently we can not answer how the hardware implementation would be done since it requires many optimizations such as selecting the area for measurement, adjusting the noise and sensitivity of the electronic equipment, estimating the effects of complications due to 2D structure, nonlinearity and dispersion of waves. However the results presented in this paper are promising to enhance the rogue wave early detection technology.

VI. CONCLUSION

In this paper we have discussed the possible usage of the compressive sampling for the early detection of rogue waves emerging in a chaotic sea state. One of the promising techniques for the early detection of rogue waves is to measure the triangular Fourier spectra which begin to appear at the early stages of the development of the oceanic rogue waves. Recognizing that such spectra can be treated as a sparse signal, since we would mainly be interested in the central high amplitude triangular region for the early detection purposes of the rogue waves, the compressive sampling technique can become an efficient tool for the rogue wave early warning systems. Employing a chaotic wave field simulation we have showed that emerging triangular rogue wave spectra can be detected using the compressive sampling technique from significantly less samples compared to the classical sensing methods. These measurements can either be acquired by remote sensing techniques such as coherent SAR or insitu techniques such as spectra measuring tools mounted on a ship hull or bottom mounted pressure gauges. Our results show that the compressive sampling based methodology proposed in this paper can reduce the memory requirements of the early warning hardware systems significantly therefore their efficiency can be enhanced while their cost is reduced.

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