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GEZGİN İLETİŞİM İÇİN UZAY-ZAMAN KODLAMALI ÇOK-TAŞIYICILI TÜMLEŞİK SİSTEM TASARIMI

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ÖNSÖZ

Bu sonuç raporu, TÜBİTAK Elektrik-Elektronik ve Enformasyon Araştırma Grubu tarafından Ağustos 2000-Ağustos 2002 tarihleri arasında alınan destek ile IŞIK Universitesi, Istanbul Teknik Üniversitesi ve İstanbul Üniversitesi Elektrik-Elektronik Mühendisliği Bölümlerinde yürütülen EEEAG100E006 sayılı "Gezgin İletişim İçin Uzay-Zaman Kodlamalı Çok-Taşıyıcılı Tümleşik Sistem Tasarımı" başlıklı proje kapsamında yapılan çalışmaları içermektedir.

TÜBİTAK Elektrik-Elektronik ve Enformatik Araştırma Grubuna, projeye verdiği destek nedeniyle ve IŞIK Üniversitesine, bu projenin gerçekleşmesinde büyük katkısının olduğuna inandığımız,uluslararası konferanslara katılım için sağlamış olduğu destek nedeniyle teşekkür ederiz.

İçindekiler

Şekiller

ÖZ

Günümüzde önemi hızla artan gezgin ve telsiz (mobil/wireless) iletişim sistemleri (uydu iletişimi, gezgin radyo, "indoor" iletişim) iletişim kanalının neden olduğu toplamsal Gauss gürültüsüne ek olarak sönümlemeden (fading) ve faz seyirmesinden de büyük ölçüde etkilenmektedir. Bu tür sistemler için, gerek kullanıcı sayısının artması ve gerekse gezgin anten boyutları ve uydunun ışınım sınırlamaları nedeniyle band ve güç verimliliği yüksek iletişim tekniklerinin geliştirilmesi gerekmektedir. Gezgin iletişim sistemlerinde kanalın neden olduğu sönümleme etkisini azaltmanın diğer bir yolu da çeşitleme (diversity) yöntemlerinden yararlanmaktır. Çeşitleme sisteme değişik şekillerde, örneğin frekans ya da zamanda konabilir. Çeşitlemenin doğal olarak varolduğu durumlarda ise, örnğin çok yollu yansımada, yapılacak şey uygun bir alıcı ile varolan çeşitlemeden yararlanmaktır. Bu amaçla son bir kaç yıl içinde, kafes kodlamalı modülasyon tekniğinin bir genellemesi gibi düşünebileceğimiz, "uzay-zaman kodlama (space-time coding)" adıyla yeni bir teknik ortaya atılmıştır. Kodlamanın, modülasyonun ve çeşitlemenin optimum bir şekilde birleştirildiği bu yöntemle tasarlanmış gezgin iletişim sistemlerinin başarımlarında büyük iyileşmeler sağladığı görülmüştür.

Diğer taraftan geniş bandlı gezgin iletişim sistemlerinde, özellikle frekans seçici kanallar üzerinden iletimde yüksek başarımlarından dolayı, "çok taşıyıcılı (multicarrier)" sistemler günümüzde yaygın olarak kullanılmaya başlamıştır. Bu sistemlerin OFDM diye adlandırılan versiyonu, tüm iletişim kanalını belli sayıda alt-kanallara bölerek bilgiyi birbirine dik (orthogonal) seçilmiş alt-taşıyıcı frekanslarla iletme ilkesine dayanmaktadır. Bu şekilde frekans seçici bir kanalın çok-yollu ve sönümleme etkileri en aza indirilmiş olmaktadır. Ancak, OFDM sistemlerinin gerek taşıyıcı frekans ve gerekse faz kaymalarına çok duyarlı olduğu bilinmektedir. Bu nedenle OFDM sistemlerinde frekans ve faz eşzamanlama (senkronizasyonu) probleminin bu duyarlığı da aşacak şekilde çözülmesi gerekir.

Projenin amacı uzay-zaman kodlama tekniği ile OFDM tekniğinin birleştirildiği ve sürekli faz modülasyonunun kullanıldığı yeni bir "uzay-zaman kodlamalı OFDM tümleşik, geniş bandlı gezgin iletişim sisteminin" verici ve alıcı kısımlarının tasarlanması ve böyle bir sistem için gerekli eşzamanlama algoritmalarının geliştirilmesidir. Daha sonra tasarlanan sistemin hata başarımları gerek analitik yöntemlerle ve gerekse benzetim yoluyla incelenecektir. Başarım analizlerinde bit hata olasılıklarının üst sınırlarının belirlenmesi amaçlanacak ve diğer sistemlerle karşılaştırılması yoluna gidilecektir.

Anahtar Kelimeler: Gezgin iletişim, Uzay zaman kodlaması, OFDM, taşıyıcı eşzamanlaması, kanal kestirimi.

ABSTRACT

Mobile/wireless communication systems are largely effected by the fading and phase shift as well as the additive Gaussian noise caused by the transmission channel. These kinds of systems require development of power and band efficient transmission techniques for some reasons such as the increase in the number of users, dimensions of the mobile receiver antenna or the radiation limitations of the satellite. In mobile communication systems one way to reduce the effects of fading is to make use of diversity techniques, which can be adapted to the system in various forms such as frequency or time diversity. Sometimes diversity may also exist naturally, in such situations it is convenient to make use of this diversity via an appropriate receiver structure. For this purpose, space-time coding, a generalization of trellis coded modulation technique, is recently developed. A substantial amount of improvements have been observed in the performance of mobile communication systems that utilize this technique having optimal combination of coding, diversity and modulation.

On the other hand in wide band transmission systems, especially in the presence of the frequency selective channels, multicarrier systems are being employed extensively because of their high performance. A version of multicarrier systems that is called OFDM divides the transmission channel into a number of sub-channels and transmits information from these sub-channels over orthogonal sub-carriers. This helps to minimize the multi path and fading effects of the frequency selective channel. However, OFDM systems are very sensitive to frequency and phase shifts. So in these systems frequency and phase synchronization issue must be dealt in a way to overcome this sensitivity as well.

The objective of this project is to design receiver and transmitter structures and develop synchronization algorithms for a new "space-time coded OFDM integrated wide band mobile communication system" that utilizes a combination of space-time coding and OFDM with continuous phase modulation. The performance of these developed structures will be examined by analytical means and computer simulations. The performance analysis will be presented by means of the upper limets of the bit error rate(BER) and the BER performance will be compared with other systems.

Key Words: Mobile communications, space-time coding, OFDM, carrier synchronization, channel estimation.

Bölüm I

GİRİŞ

1.1 Konu

Bu projede, günümüzde çok önem kazanan gezgin-telsiz (mobil/wireless) iletişim alanında son yıllarda yaygın olarak kullanılmaya başlanan uzay-zaman kodlama ve OFDM tekniklerinin birleştirildiği ve sürekli faz modülasyonunun kullanıldığı, yeni bir "uzay-zaman kodlamalı OFDM tümleşik geniş bandlı gezgin iletişim sistemi"nin verici ve alıcı kısımlarınının tasarlanması ve böyle bir sistem için gerekli eşzamanlama ve kanal kestirim algoritmalarının geliştirilmesi öngörülmektedir.

1.2 Literatür Özeti

Günümüzde önemi hızla artan gezgin ve telsiz iletişim sistemleri (uydu iletişimi, gezgin radyo, "indoor" iletişim), iletişim kanalının neden olduğu toplamsal Gauss gürültüsüne ek olarak sönümlemeden(fading) ve faz seyirmesinden de büyük ölçüde etkilenmektedir. Bu tür sistemler için, gerek kullanıcı sayısının artması ve gerekse gezgin anten boyutları veya uydunun ışınım sınırlamaları nedeniyle band ve güç verimliliği yüksek iletişim tekniklerinin geliştirilmesi gerekmektedir. Bu amaçla, kafes kodlamalı modülasyon (Trellis Coded Modulation(TCM)) yöntemi ile kısmi yanıtlı (Partial Response) sinyal işleme tekniği, daha önceden, grubumuz tarafından tümleştirilerek QPR-TCM (Quadrature Partial Response TCM) adını verdiğimiz yeni bir kodlu modülasyon yapısı önerilmiş, bu yapıyı içeren bir sistemin başarım analizi toplamsal beyaz Gauss gürültüsü ve Rician fading etkileri altında analitik olarak incelenmiştir. Daha sonra, bu başarımın iyileştirilmesine yönelik yeni kod tasarım ölçütü elde edilmiş ve referans sistemlere göre üstünlüğü ortaya konmuştur [1,2]. Kısmi yanıtlı iletişimin M-PSK modülasyon teknikleriyle tümleştirilmesi [3] ve M-PSK TCM in çeşitli sönümleme etkileri altında başarım analizleri de yine grubumuzca gerçekleştirilmiştir [4,6,17,18,19,21,23]. İletişim band verimliliğini artırmada genellikle sürekli faz modülasyonu tekniklerinden yararlanılmaktadır. Bu konuda grubumuz tarafından yapılan çalışmalarda yeni bir takım yapılar üretilmiştir [5,7,15,16,20,22,24,25]. Gezgin iletişim sistemlerinde kanalın neden olduğu sönümleme etkisini azaltmanın diğer bir yolu da çeşitleme (diversity) yöntemlerinden yararlanmaktır. Çeşitleme, sisteme değişik şekillerde, örneğin frekans, uzay ya da zamanda konabilir. Çeşitlemenin doğal olarak varolduğu durumlarda ise, örneğin çok yollu yansımada, yapılacak şey uygun bir alıcı ile varolan çeşitlemeden yararlanmaktır. Bu amaçla son yıllarda, kafes kodlamalı modülasyon tekniğinin bir genellemesi gibi düşünebileceğimiz, "uzay-zaman kodlama (space-time coding)" adıyla yeni bir teknik ortaya atılmış ve kodlamanın, modülasyonun ve çeşitlemenin optimum bir şekilde birleştirildiği bu yöntemle tasarlanmış gezgin iletişim sistemlerinin başarımlarında büyük iyileşmeler sağlandığı görülmüştür [8-13]. Diğer taraftan geniş bandlı gezgin iletişim sistemlerinde, özellikle frekans seçici kanallar üzerinden iletimde yüksek başarımlarından dolayı, "çok taşıyıcılı (multicarrier)" sistemler günümüzde yaygın olarak kullanılmaya başlamıştır. Bu sistemlerin OFDM (Orthogonal Frequency Division Multiplexing) diye adlandırılan versiyonu, tüm iletişim kanalını belli sayıda alt-kanallara bölerek bilgiyi birbirine dik(orthogonal) olacak biçimde seçilmiş alt-taşıyıcı frekanslarla iletme ilkesine dayanmaktadır. OFDM sistemlerinde alt taşıyıcıların spektrumlarının birbirleriyle örtüşmelerinin iletişim açısından bir problem oluşturmaması nedeniyle, sistemin band genişliği randımanı açısından büyük bir üstünlük sağlanmış olunmaktadır. Öte yandan OFDM, tek taşıyıcılı sistemlere nazaran oldukça uzun bir sinyalleşme peryoduna sahip olduğundan, özellikle frekans seçici kanalllar üzerinden iletişimlerde, çok-yollu ve sönümleme etkilerini enaza indirmekte ve daha iyi bir başarıma sahip olmaktadır. Ancak, OFDM sistemlerinin gerek taşıyıcı frekans ve gerekse faz kaymalarına çok duyarlı olduğu bilinmektedir. Bu nedenle OFDM sistemlerinde frekans ve faz eşzamanlama (senkronizasyon) probleminin bu duyarlığı da aşacak şekilde çözülmesi gerekir. Panayırcı ve Texas A&M Üniversitesinden bir araştırma gurubu, OFDM sistemlerini de frekans ve faz eşzamanlaması için En Büyük Olabilirlik (Maximum Likelihood) yöntemine dayanan hızlı ve iteratif yapıda yeni algoritmalar önermiş ve bu algoritmaların özelllikle frekans seçici kanallar için de çok iyi sonuçlar verdiğini göstermişlerdir [14,26-29].

1.3 Amaç

Proje grubunun yukarıda literatür özetinde belirtilen araştırmalarda edindiği bilgi ve deneyim birikiminden yararlanarak projede şunlar amaçlanmaktadır:

- Uzay-zaman kodlama tekniği ile OFDM tekniğini birleştirerek ve sürekli faz modülasyonunu da kullanarak, yeni bir tümleşik geniş bandlı "Uzay-zaman Kodlamalı OFDM Gezgin İletişim Sistemi'nin verici ve alıcı kısımlarının tasarlanması
- Böyle bir sistemin özellikle frekans seçici kanallar üzerinden iletişim yapması durumunda başarım analizinin analitik yöntemlerle ve bilgisayar benzetimleri ile gerçekleştirilmesi

- Frekans seçici ve sönümlemeli kanallarda, uzay- zaman kodlama tekniğinin sağladığı çeşitlemenin olumlu etkileri ile OFDM tekniğinin getirdiği sistemin frekans seçici etkilere bağışıklığının incelenmesi ve klasik sistemlerle karşılaştırılması

- OFDM sistemi için önceden geliştirilen frekans ve faz eşzamanlama algoritmalarının nasıl bir modifikasyonla yeni sisteme uydurulacağının araştırılması ve yeni eşzamanlama algoritmalarının geliştirilmesi.

Böylece, hem çeşitleme sağlayan ve hem de kanalın frekans seçiciliğine duyarlı olmayan yeni bir tümleşik iletişim sisteminin mimarisi bu projede ortaya konacak ve başarımı ayrıntılı olarak incelenecektir.

1.4 Yöntem

Projede gerçekleştirmeyi düşündüğümüz tümleşik iletişim sisteminin verici ve alıcı bölümlerinin blok şemaları Şekil 1.a ve b de gösterilmiştir. Şekil 1. a daki sistemin verici bölümünün blok şemasından görüleceği gibi, kaynaktan üretilen bitler bir konvolusyonel kodlayıcı ile kodlanmakta ve kodlayıcı çıkışındaki kod sözcükleri iki ayrı sinyal eşleme(signal mapping) kuralı ile bir sayısal modülasyon türündeki simgelere, (örneğin M-PSK veya M-QAM) karşı düşürülmektedir. Daha sonra bu simgeler OFDM tekniği ile kendi verici antenleri aracılığı ile alıcıya iletilmektedir. Şekil 1.b deki alıcı blok şemasından da görüleceği gibi, iki verici anteninden gelen sinyaller bir alıcı anten vasıtasıyla toplanmakta, demodülatörle temelbanda (baseband) indirilmekte, daha sonra OFDM alıcısı ve bunu izleyen uygun biçimde tasarlanmış bir Viterbi kod çözücüsü ile çözülerek iletilen bitler alıcıda yüksek bir doğrulukla elde edilmektedir.

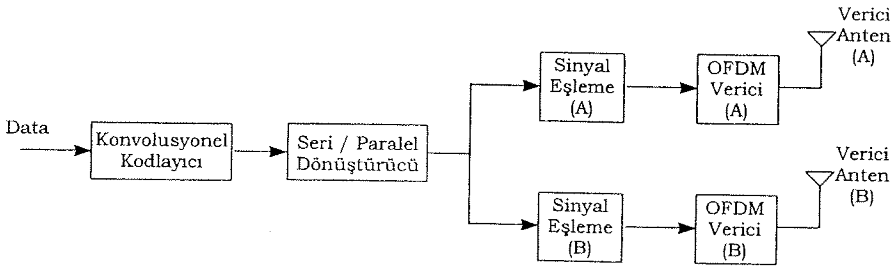
Bu sistem yapısı temel alınarak projede:

- Tümleşik uzay-zaman kodlamalı OFDM sisteminin tasarım ölçütlerinin araştırılması ve bilgisayar arama algoritmaları (computer search algorithms) yardımı ile eniyi sistem yapısının bulunmasına çalışılmakta

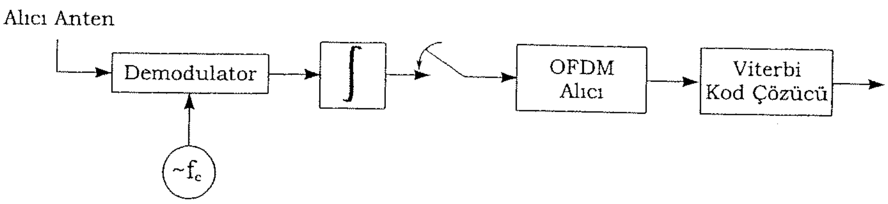
- Bit hata olasılığı üst sınır analiz yöntemlerinden yararlanılarak tasarlanmış tüm sistemin başarım analizlerinin analitik olarak elde edilmesi üzerine çalışmalar yapılmakta.

- Tasarlanan tüm sistemin hata başarımı bilgisayar benzetimi ile gerçekleştirilmektedir.

- "Expectation-Maximization(EM)" optimizasyon yöntemi ile tasarlanan sistem için taşıyıcı frekans ve faz eşzamanlamasının enbüyük-olabilirlik (Maximum likelihood) tekniği ile gerçekleştirilmesi üzerine çalışmalar yapılmaktadır. Özellikle OFDM gibi çok taşıyıcılı bir sisteminin sönümlemeli kanallar üzerinden iletişim yapılması durumunda, tüm alt-taşıyıcı frekans ve fazlar farklı kaymalara maruz kalacaktır. Bu durumda eniyi olabilirlik tekniği ile kayan frekans ve fazların kestirimlerinin (estimation) yapılabilmesi matematiksel olarak pek olanaklı gözükmemektedir. Bu durumlarda EM optimizasyon yönteminin çok iyi sonuçlar verdiği [14,26-29] çalışmalarda gösterilmiştir.



Şekil 1.4.1: Tümleşik sistemin verici blok şeması



Şekil 1.4.2: Tümleşik sistemin alıcı blok şeması

1.5 Katkılar

Sönümlemeli kanallar üzerinden gezgin iletişimde, yüksek başarımla çalışan uzay-zaman kodlamalı OFDM sistemine yönelik çalışmalardan üretilen bilimsel sonuçlar ve tasarım teknikleri hem uluslararası ve hem de ulusal düzeydeki konferanslara sunulup, yüksek saygınlığa sahip dergilerde yayınlanmıştır.

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Bölüm II
PROJEDE GERÇEKLEŞTİRİLEN ÇALIŞMALARIN ÖZETLERİ

2.1 Giriş

Bu bölümün temel amacı, uzay-zaman kodlamalı OFDM tümleşik bir sistemin tasarımı için projede gerçekleştilen tüm çalışmaların bir özetinin verilmesidir. Bu çerçevede, hemen izleyen Bölüm 2.2 de uzay zaman kodlamalı OFDM tümleşik sistemi üzerinde Minimum Faz Kaydırmalı Anahtarlama (Minimum Shift Keying(MSK))tekniği ile iletişim yapan bir sitemin tasarımı ele alınmakta ve bu çalışma Bölüm 2.3 de çoklu MSK modülasyonlu sistemlere genişletilmektedir. Bölüm 2.4 ise birden fazla alıcı anten kullanan dik uzay-zaman kodlarının telsiz kanallarda, hata başarımının arttırılması amacıyla kullanılan, güç kontrol yapılarının incelenerek sönümlemeli kanallarda yüksek hata başarımına sahip bir iletişim sistemi önerilmektedir. Bölüm 2.5 te ilintili frekans-seçici sönümlemeye sahip kanallar üzerinden iletişim için dik uzay-zaman kodlamalı OFDM kullanan tümleşik iletişim sistemi tasarımları ele alınmaktadır. Bölüm 2.6 te telsiz kanallarda verici çeşitlemesi kullanan iletişim sistemi için dizi yeni bir dizi kestirim yöntemi önerilmekte ve Bölüm 2.7 da da uzay-zaman kodlamalı OFDM tümleşik sistemleri için yeni kanal kestirim algoritmaları teklif edilmektedir. Son olarak Bölüm 2.8 de bu sistemler için yeni taşıyıcı frekans ve faz eşzamanlama algoritmaları sunulmaktadır.

2.2 MSK-OFDM Tümleşik Sistem Tasarımı

Bu çalışmada, çoklu kafes kodlu MSK(Minimum shift keying) modülasyonu OFDM iletim tekniği ile birleştirilerek özellikle sönümlemeli kanallar için yüksek başarıma sahip yeni bir iletişim sistemi önerilmektedir. Sistemin verici kısmında bilgi dizisi, katlamalı kodlayıcı yardımıyla kodlanarak çoklu kafes kodlu bir MSK sinyalini oluşturmak üzere önceden belirlenmiş kanal simgeleri ile eşleştirilmek- tedir. OFDM sisteminin en belirgin özelliği olan dik taşıyıcıları gerçekleştirmek için uygulaması kolay ve maliyeti düşük olan hızlı Fourier (Fast Fourier Transform, FFT) ve ters hızlı Fourier dönüştürücülerden (Inverse Fast Fourier Transform) yararlanılmaktadır. Kodlanmış ve eşlenmiş simge dizisini alt-taşıyıcılara ötelemek amacı ile N 'lik IFFT bloğu kullanılmaktadır. Burada, N sayısı alt-kanal sayısını göstermekte olup, Fourier işlemlerinin hızlı ve etkin yapılabilmesi için N değerinin 2'nin bir üssü olarak seçilmesi gerekir. IFFT işlemine uygun biçime getirilmek amacıyla kodlanmış sinyal dizisi bir seri- paralele dönüştürücü yardımıyla N blok uzunluğunda paralel bir diziye dönüştürülür. IFFT bloğu çıkışı yeniden seri biçime dönüştürülerek simgeler arka arkaya kanala gönderilir. Alıcı tarafta ise peşpeşe alınan her N kanal simgesi paralele dönüştürülüp FFT si alınır. Daha sonra, FFT çıkışı paralelden seriye dönüştürülerek çıkışta çoklu kafes kodlanmış MSK sinyal dizisinin kanalın sönümleme ve gürültü etkileriyle bozulmuş biçimi elde edilir. Alıcının son adımında ise TCM kod çözücü kafes kodların en büyük olabilirlikli çözümünü gerçekleştiren Viterbi algoritması kullanılarak iletilen bilginin yeniden elde edilmesine çalışılır. Geniş bandlı iletişimde en kötü durumlardan birisi de iletişim kanalının frekans seçici olmasıdır. OFDM tekniği böyle bir kanalı çok sayıda (N adet) birbiriyle örtüşebilen alt-bandlara bölerek kullanmayı sağlarken, frekans-seçici kanalı frekans-seçici olmayan yani düzgün sönümlemeye (flat fading) sahip N alt-banddan oluşmasını sağlar. Böylece, her bir bandtan iletilen sinyaller alıcıda daha kolay çözülebilir. Viterbi algoritmasını kullanmayı kolaylaştıran başka bir etken ise kanal girişindeki IFFT ve kanal çıkışındaki FFT işlemleri nedeniyle sönümlemenin alt-kanallar üzerindeki etkisinin, $\rho_k, 0 < k < N-1$ sönümleme katsayılarının FFT çıkışında kodlanmış simgeleri çarpması biçiminde oluşmasıdır:

$$r_k = \rho_k C_k + n_k, 0 < k < N-1.$$

Burada, r_k, N 'lik bir iletim çerçevesi içersinde k. zaman diliminde kod çözücü girişine gelen örnektir. C_k bu çerçevede k. alt-kanaldan iletilmiş olan çoklu kafes kodlu MSK simgesini gösterirken, n_k sıfır ortalamalı, boyut başına $N_0/2$ varyanslı istatistiksel bağımsız Gauss dağılımlı gürültü örneğidir. ρ_k ise kanalın k. alt-kanalının bu çerçeve süresince geçerli sönümleme katsayısıdır ve kompleks Gauss dağılımı ile modellenebilir. Böylece, iletimde problemlere neden olan frekans-seçici kanal , alıcı tarafından bakıldığında zaman-seçici kanala dönüştürülmüş olup sisteme oldukça büyük bir çeşitleme (diversity) eklemiştir.

Çalışmada, çeşitli çoklu MSK kafes kodların OFDM sistemlerde hata başarımları bilgisayar benzetimleri yardımıyla incelenmiş ve elde edilen sonuçlar "MSK modülasyonlu OFDM sistemleri" adlı bir bildiri ile 25-27 Nisan 2001 tarihleri arasında Kuzey Kıbrıs Türk Cumhuriyeti'nde düzenlenmiş olan 9. Sinyal İşleme ve Uygulamaları Kurultayı'nda sunulmuştur.

2.3 Uzay-Zaman Kodlamalı Çoklu-MSK Sistem Tasarımı

Proje amacı doğrultusundaki diğer bir çalışma ise, MSK modülasyonunun band verimliliği ile uzay-zaman kodlarının güç verimliliğini biraraya getiren çoklu MSK modülasyonlu bir uzay-zaman kodlamalı sistemin alıcı performansının analitik yöntemlerle incelenmesi ve elde edilen başarım sonuçlarının bilgisayar benzetim sonuçları ile karşılaştırılmasından oluşmaktadır.

Minimum kaydırmalı anahtarlama (Minimum Shift Keying, MSK) modülasyonu, sürekli faz modülasyonunun özel bir biçimi olarak sabit zarf, band verimliliği gibi özellikleri nedeniyle band ve/veya güç sınırlı iletişim ortamları için oldukça uygun bir modülasyon tekniğidir. Yapısında barındırdığı doğal kodlamaya ek olarak band verimliliğinden bir miktar özveride bulunularak güç verimliliğinin kodlama işlemi yardımıyla daha da artırılabiliyor olması, bu modülasyon tekniğini söz konusu iletişim ortamları için daha da çekici duruma getirmektedir. Son yıllarda yaygın olarak incelenen ve sistemin kodlama kazancını artıran bu tür yöntemler genellikle kafes kodlamalı modülasyon (trellis coded modulation, TCM) tekniğine dayanır. MSK modülasyonu, toplamsal beyaz Gauss gürültülü kanalların yanısıra özellikle gezgin iletişim sistemlerinde karşılaşılan sönümlemeli (fading) kanallar için de çok uygun bir modülasyon tekniğidir. Kafes kodlamalı sistemler için Gauss gürültülü kanallarda, özellikle yüksek sinyal-gürültü oranlarında, hata başarım ölçütü serbest Öklid uzaklığı olmasına karşın, sönümlemeli kanallarda, yüksek sinyal-gürültü oranlarında en önemli hata başarım ölçütü etkin kod uzunluğu (effective code length, ECL), ikincil olarak da çarpımsal uzaklıktır (product distance, d_p^2). İyi bir kod tasarımında, etkin kod uzunluğunun ve çarpımsal uzaklığının olabildiğince büyük yapılmasına çalışılır. MSK modülasyonunda etkin kod uzunluğu ve çarpımsal uzaklığı artırmanın bir yolu da çoklu kafes kodlaması (multiple trellis coding) kullanılmasıdır. Birden çok kafes adımının birleştirilmesi sonucunda oluşturulan yeni kafeste her dala birden fazla simge eşleştirilerek kodun etkin kod uzunluğu ve çarpımsal uzaklığı oldukça artırılır. Sönümlemeli kanallarda hata başarımını artırmanın bir yolu da çeşitleme (diversity) tekniğinden yararlanmaktır. Çeşitleme, tüm kanallarda birden derin sönümleme olasılığının küçük olacağı varsayımı altında, aynı bilgiyi birden fazla bağımsız kanaldan iletmeye dayanır. Bu bağımsız kanallar frekansta, zamanda ve/veya uzayda (farklı anten) çeşitleme yoluyla yaratılır. Çeşitleme türleri birlikte veya ayrı ayrı bir iletişim sisteminde kullanılabilir. Zaman çeşitlemesinde, gönderilen sinyalin en az zayıflamış bir kopyası farklı iletim anlarından birinde elde edilirken frekans çeşitlemesinde bu kopya aynı zaman aralığında farklı frekans bölgelerinden birinden elde edilir. Uzay ya da anten çeşitlemesi ise birden çok verici ve/veya alıcı anten kullanılarak hata başarımının artırılmasına dayanır.

Alıcı anten sayısını artırarak çeşitleme sağlama literatürde yeterince incelenmiş bir konu olmasına karşın çeşitleme için verici anten sayısını artırmak ve bu antenleri kod tasarımı sırasında birlikte göz önüne alarak her biri için farklı kodlayıcılar geliştirmek yeni bir konudur. Bu yeni tekniğe uzay-zaman kodlaması denmektedir. Bu çalışmada, uzay-zaman kodlama tekniği MSK modülasyonuna uygulanmakta, iki verici ve bir alıcı anten için iki, dört ve sekiz durumlu uzay-zaman kodlamalı çoklu MSK sistemler önerilmektedir. Bu sistemlerin tasarımlarında, düzgün ve yavaş sönümlemeli kanallarda uzay-zaman kodlarının tasarım ölçütlerini oluşturan rank ve determinant ölçütlerinin eniyileştirilmesi yoluna gidilmiş ve bu amaçla geliştirilen bir kod arama algoritmasından yararlanılmıştır. Önerilen kodların hata başarımları geliştirilen bir bilgisayar benzetim programı yardımıyla incelenmiş, tek verici anten kullanılması ve her iki verici antende aynı MSK kafes kodunun kullanılması durumlarına olan üstünlükleri Rayleigh sönümlemeli kanallar için ortaya konulmuştur. Bu çalışma bu yıl 25-27 Nisan 2001 tarihleri arasında Kuzey Kıbrıs Türk Cumhuriyeti'nde düzenlenmiş olan *9. Sinyal İşleme ve Uygulamaları Kurultayı*'nda "Uzay-zaman kodlamalı çoklu MSK modülasyonu" adlı bildiriyle sunulmuştur. Bu konuyla ilgili olarak elde edilen uzay-zaman kodlamalı çoklu MSK yapılarının hızlı Rayleigh sönümlemeli kanallar üzerinde hata başarımını incelemek amacıyla, bundan önce kafes kodlamalı yapıların yol çiftleri hatasını ve bit olasılıklarını kestirmek amacıyla ortaya atılan teknikler, uzay-zaman kodlamalı çoklu yapılara genelleştirilmiş ve daha önce elde ettiğimiz uzay-zaman kodlamalı çoklu MSK yapılarına uygulanmıştır. Çalışmanın bu bölümünün sonuçlarını da içeren genişletilmiş biçimi 25-29 Haziran 2001 tarihleri arasında Rethymno, Yunanistan'da düzenlenmiş olan *8th International Conference on Advances in Communications and Control (COMCON)*'da "Space-time coded multiple MSK" adlı bildiriyle sunulmuştur.

2.4 Dik Uzay-Zaman Kodlarında Güç Kontrolu

Bu çalışmada literatürde bir alıcı antenli dik uzay-zaman kodları için ortaya atılan güç kontrol yapısı birden fazla alıcı anten kullanan dik uzay-zaman kodlamalı iletişim sistemleri için genelleştirilerek sönümlemeli kanallarda yüksek hata başarımına sahip bir iletişim sistemi önerilmiştir. Kanal kazançlarının alıcıda hatalı kestirilmesi durumunda yüksek başarım sağlayan iki ve üç alıcı antenli iletişim yapıları tasarlanarak güç kontrolü uygulanmadığı duruma göre olan kazançları bilgisayar benzetimleri yardımıyla sunulmuştur. Gezgin kanallar üzerinden bilginin hızlı ve güvenli iletimini engelleyecek çeşitli etkenler bulunmaktadır. Band genişliği ve iletim gücü sınırlamaları, kullanılacak iletişim sisteminin tasarımında önemli ölçütler olarak ortaya çıkmakta iken kanaldaki toplamsal gürültü ve özellikle de çok-yollu iletişim yapısının ortaya çıkardığı sönümleme etkisi hata başarımını oldukça kötüleştirmektedir. Giderek artan gezgin birimlerin hareketliliği sonucunda önemli boyutlara varan sönümleme etkisi iletişim kalitesini düşüren başlıca etkendir. Sönümlemenin gezgin iletişim sistemleri üzerindeki etkisini azaltmanın en iyi yolu çeşitleme tekniklerinden yararlanmaktır. Uzay, zaman ve frekans çeşitlemesini de içeren çeşitleme tekniklerinin amacı, iletim ortamında bağımsız kanallar ortaya çıkararak aynı bilgiye ilişkin çeşitli sinyallerin alıcıya ulaşmasını sağlamaktır. Böylece, kullanılan bağımsız kanallardan biri üzerinden iletilen bilgi derin sönümleme etkisi sonucunda alıcıya çok zayıflamış olarak ulaşsa bile bir diğer kanaldan alıcıya ulaşabilecek daha az zayıflamış kopya alıcının hata başarımını arttıracaktır. Uzay çeşitlemesi tekniğinde, alıcı ve/veya vericide birden fazla anten kullanılmaktadır. Uzayda oluşturulan bu kanalların bağımsızlığını sağlamak amacıyla, kullanılan antenler birbirlerinden yeterince uzağa yerleştirilirler. Alıcı tarafta birden fazla anten kullanımı ve bu yapıyla birlikte kullanılan sinyal işleme tabanlı çeşitleme/birleştirme teknikleri literatürde yoğun şekilde işlemiş ve yüksek hata başarımına sahip çeşitli yapılar geliştirilmiştir. Ancak, günümüzde kullanılan hücresel telefon şebekeleri gibi iletişim sistemlerinde, alıcı tarafta (gezgin birimde) birden fazla anten kullanmak, gezgin birimin boyutlarını ve maliyetini arttıracaktır. Sisteme kayıtlı her gezgin birimde böyle bir boyut ve maliyet artışı yerine çoklu anten çeşitlemesini verici tarafta (baz istasyonda) kullanmak aynı hata başarımını sağlayabileceği gibi diğer sisteme oranla oldukça düşük maliyetlidir. Verici anten çeşitlemesi son yıllarda giderek artan bir öneme sahip olmuş ve dikkatleri üzerine çekmiştir. Çok verici/çok alıcı antenli yapılarla kanal sığasının arttığının gösterilmesi ile başlayan bu süreç, Tarokh *et al* [1]'in uzay-zaman kodlaması tekniğini ortaya atmaları ile literatürde önemli bir yere oturmuştur. Tarokh *et al* [1] çalışmalarında birden çok verici/alıcı anten kullanılması durumunda elde edilecek iletişim sisteminin hata olasılığı üst sınırı ifadelerini elde ederek kod tasarım ölçütlerini vermişlerdir. Bu ölçütlere dayanılarak tasarlanan iletişim sistemleri tam çeşitleme kazancı ve yüksek kodlama kazancına sahip olabilmektedirler. Öte yandan, Alamouti [2], karmaşıklığı az olan dik verici çeşitlemesini (OTD) ortaya atmıştır. İki verici, M alıcı anten kullanan bu yapı tam çeşitleme kazancı sağlamaktadır. Alıcıda kanal kazançlarının kestiriminin hatalı yapılması durumunda Alamouti'nin [2] önerdiği yapının başarımı oldukça düşmektedir. Alıcıda gerçekleştirilen birleştirme işlemi sonucunda iki antenden iletilmiş kanal simgelerinin birbirinden ayrılmasını sağlayan diklik özelliği kanal kestirim hatası nedeniyle kaybolmakta ve simgelerarası girişime neden olmaktadır. Ortaya çıkan simgelerarası girişim nedeniyle iletişim sistemi yüksek sinyal-gürültü oranlarında bile yüksek hata miktarıyla çalışmaktadır. Bu sorunu gidermek amacıyla Fan *et al*[3] iki verici, bir alıcı anten kullanan dik uzay-zaman kodları için bir güç kontrol yapısı önermiştir. Vericide kullanılacak iletim enerjisinin daha iyi kanal kazancına sahip verici antene yoğunlaştırılması ile gerçekleştirilen bu yapı oldukça yüksek başarıma sahiptir. Çok alıcı antenli dik uzay-zaman kodlarına ait simgeler arası girişimi bastırmak amacıyla ihtiyaç duyulan güç kontrol yapısı oldukça karmaşıktır. Bunun nedeni, istatistiksel bağımsız sönümlemeden etkilenen alıcı antenlerin birine ilk antene ilişkin kanal kazancı daha yüksek görünürken; diğerine ikinci antene ilişkin kanal kazancı daha yüksek görünebilir.

Bu kararsız durumu da göz önüne alabilmek amacıyla 2 bitlik bir geri besleme kanalı kullanılarak kontrol yapısının durum sayısı dörde çıkarılmıştır. Bu çalışmada, iki ve üç alıcı antenli dik uzay-zaman kodları için hata başarımını arttıran bir güç kontrol tekniği ortaya atılmıştır. Hazırlanan bilgisayar benzetimleri yardımıyla elde edilen yapıların çeşitli kanal kestirim hatası değerleri için hata başarımları elde edilmiş ve referans yapılara göre olan üstünlükleri ortaya konmuştur. Bu çalışmanın sonuçları, bu yıl 13-15 Haziran 2002 tarihleri arasında Pamukkale, Denizli' de düzenlenmiş olan 10. Sinyal İşleme ve Uygulamaları Kurultayı'nda "Çok alıcı antenli dik uzay-zaman kodları için güç kontrolu" adlı bildiriyle sunulmuştur. Konferans kitabında yayınlanan bu çalışmanın bir kopyası ilişikte görülebilir. Aynı zamanda 15-18 Eylül 2002 tarihleri arasında Lizbon, Portekiz' de düzenlenen olan 13th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC-2002) konferansında "Power control for orthogonal space-time coding with multiple receive antennas" adlı bildiriyle sunulmuştur.

2.5 İlintili Frekans-Seçici Sönümlemeli Kanallar İçin Dik Uzay-Zaman Kodlamalı OFDM Tümleşik İletişim Sistem Tasarımı

Bu çalışmada OFDM kullanan iletişim sistemlerinde uzay, zaman ve frekans çeşitleme teknikleri birlikte uygulanarak kanaldaki ilintili sönümleme ve toplamsal beyaz Gauss gürültüsüne karşın yüksek hata başarımına sahip bir tümleşik iletişim sistemi önerilmektedir. Bu yapıda uzay ve zaman çeşitlemesini sağlamak amacıyla son zamanlarda uygulamaları sıklıkla karşımıza çıkan dik uzay-zaman kodları kullanılmıştır. Alamouti tarafından ortaya atılan bu yapı iki verici, M alıcı anten kullanarak tam çeşitleme kazancı sağlamaktadır. Genişbandlı iletişim sistemlerinde, özellikle frekans seçici kanallar üzerinden iletimde yüksek başarımlarından dolayı, çok-taşıyıcılı sistemler günümüzde yaygın olarak kullanılmaktadır. Bu sistemlerin OFDM olarak adlandırılan biçimi, genişbandlı iletişim kanalını belli sayıda alt kanallara bölerek bilgiyi birbirine dik seçilmiş alt taşıyıcı frekanslarda iletme ilkesine dayanmaktadır. Geleneksel frekans bölmeli çoğullamalı sistemler ile karşılaştırıldığında, dik alt kanalların örtüşmesine izin verildiğinden band verimliliği açısından bir üstünlük sağlamaktadır. İlintili sönümlemeye sahip alt kanallar üzerinde çeşitleme sağlamak amacıyla sistemin girişine kafes kodlamalı modülatör eklenmiştir. Böylece, kafes kodlamalı modülasyonun içinde barındırdığı zaman çeşitlemesinden de yararlanılmaktadır. Ancak, kullanılan IFFT işlemi nedeniyle kafes kodlamalı modülasyonun sahip olduğu bu özellik tasarlanan iletişim sisteminde frekans boyutunda çeşitleme sağlayarak ilintili sönümlemeye karşın yüksek hata başarımı sağlamaktadır. Önerilen tümleşik yapıda, bir çerçevede $2N$ ikili simgeden oluşan bilgi dizisi kafes kodlamalı modülatör ile kodlanarak kodlanmış simge dizisini oluşturmaktadır. Dağıtıcı, girişine verilen kanal simgelerini iki verici antene ait OFDM çerçevelerine dağıtmaktadır. Kodlanmış simgeler tek indisli OFDM çerçevelerine yerleştirilirken, eşlenikleri Alamouti'nin önerdiği dik çeşitlemeli iletim yapısını koruyacak biçimde çift indisli OFDM çerçevelerine yerleştirilir. Her bir anten için oluşturulan bu çerçeveler IFFT bloğundan geçirilerek iki verici anten üzerinden kanala iletilirler. Alıcı tarafta FFT bloğunu takip eden birleştirici bloğu, dik uzay-zaman kodlarının çözülmesi için gerekli birleştirme işlemini yaparak elde ettiği kestirim örneklerini kod çözme işlemini gerçekleştirmesi için Viterbi algoritması bloğuna iletir. Burada, alınan örnekler kullanılarak iletilmiş olan bilgi dizisine ilişkin optimum kestirim metriği kullanılarak gerçekleştirilir.

Bu çalışmada, frekans-seçici kanallar üzerinde yüksek hata başarımına sahip dik uzay-zaman kodlamalı OFDM sistemi önerilmiştir. OFDM alt kanallarına ilişkin sönümleme etkilerinin ilintili olduğu durumda önerilen tümleşik iletişim sistemine ait bilgisayar benzetimleri yapılmış ve çeşitli sinyal-gürültü oranları için hata başarımları elde edilerek ilintisiz sönümleme durumuyla karşılaştırılmıştır. Bu çalışmanın sonuçları, bu yıl 13-15 Haziran 2002 tarihleri arasında Pamukkale, Denizli' de düzenlenmiş olan *10. Sinyal İşleme ve Uygulamaları Kurultayı*' nda "İlintili sönümlemeli kanallarda dik uzay-zaman kodlamalı OFDM" adlı bildiriyle sunulmuştur.

2.6 Telsiz Kanallarda Verici Çeşitlemesi Kullanan İletişim Sistemi İçin Dizi Kestirimi

Bu çalışmada EM (Expectation-Maximization) algoritması kullanılarak ilintili sönümlemeli telsiz kanallarda çalışan verici çeşitlemeli iletişim sistemleri için dizi kestirimi ele alınmıştır. İki verici anten kullanan yapı için önerilen yöntem incelenmiş ve gerekli karar metrikleri analitik hesaplar yardımıyla ortaya konarak hazırlanan bilgisayar benzetimleri yardımıyla hata başarımı elde edilmiştir. Verici çeşitlemesi, çok-yollu telsiz kanallarda ortaya çıkan sönümleme etkisi ile başa çıkmanın etkin bir yoludur. Telsiz kanallar üzerinden yüksek veri hızlı iletişime olanak veren uzay-zaman kodlamalı sistemler yakın zamanda Tarokh *et al*[1] tarafından ortaya atılmıştır. Uzay-zaman kodlarının alıcıda çözülmesi için ideal kanal durum bilgisine gereksinim duyulurken pratikte kanalın ideal olarak kestirimi oldukça zordur. Tarokh *et al* 'in çalışmasında kanal durum bilgisinin alıcıda ideal olarak kestirilebildiği varsayılmıştır. Bu çalışmanın sonrasında, Alamouti [2] iki verici anten kullanan bir verici çeşitlemeli yapı öne sürmüştür. Daha sonraları ikiden çok verici anten durumlarına da genişletilen bu yapı tam çeşitleme kazancına sahiptir. Kanal durum bilgisinin ideal olarak bilindiği varsayımı altında bu yapının alıcısı yapının içerisinde barındırdığı diklik nedeniyle en büyük olabilirlikli kod çözme metriğini ikiye bölerek her bir antene ilişkin kanal simgesinin ayrı ayrı çözülmesini sağlamaktadır. Öte yandan, kanal durum bilgisinin alıcıda ideal olarak bilinmediği durumda kaybolan diklik nedeniyle bu yapının hata başarımı oldukça kötüleşmektedir. Bu çalışmada, kanal durum bilgisinin ideal kestirilememesi durumunda hata başarımını iyileştirmek amacıyla EM algoritmasından yararlanılması önerilmektedir. EM algoritması iteratif kod çözme gerçekleştirerek hem kanal durum bilgisini hem de iletilen kanal simgelerini kestirebilmektedir. EM algoritmasının iteratif olarak en büyük olabilirlikli karara yakınsaması için başlangıçta kanalın durumunu bilmelidir. İlk anda gerçekleştirilecek bu kanal kestirimi ne kadar başarılı ise EM algoritması o kadar çabuk (az iterasyonla) yakınsayacaktır. İletişim sisteminin bu aşamasında kanalın kestirimi PSAM (Pilot Symbol Assisted Modulation) tekniği ile gerçekleştirilmektedir. Kanaldan iletilecek simgelerin arasına alıcı tarafta değerleri önceden bilinen pilot simgeler eklenerek kanalın bu simgelerin iletildiği aralıklardaki kazançları kestirilebilir. Daha sonra, bilgi taşıyan simgelere ilişkin kanal kazançlarını kestirmek amacıyla pilot simgelere ilişkin kazançlara Lagrange aradeğerleme işlemi uygulanmaktadır. EM algoritması, bir sonraki adımda, PSAM ile kestirilmiş kanal kazanç değerlerini kullanarak en büyük olabilirlikli karar metriğini maksimize edecek şekilde iteratif yöntemle çalışmaktadır. Bu çalışmanın sonuçları *AEU* de 2003 yılında yayınlanacaktır.

2.7 Uzay-Zaman Kodlamalı OFDM Tümleşik Sistemi İçin Yeni Kanal Kestirim Algoritmaları

Özellikle, uzay-zaman kodlanmış ve OFDM sinyaller tarafından uyarılmış sönümlemelı (fading) kanalların kestirimi, telsiz iletişim sistemlerinin alıcılarının tasarımlarında büyük önem taşımaktadır. Bu tür süstemlerin alıcılarında demodülasyon ve sezim (detection) işlemleri ancak kanal parametrelerinin bilindiği varsayılarak gerçekleşebilmektedir. Bu nedenle alıcıda demodülasyon ve bunu izleyen sayısal sinyalin sezimi işlemine başlamadan önce kanal katsayılarının bir şekilde kestirilmesi gerekmektedir. Bunu izleyen altbölümlerde kanal kestirimi için yapılan araştırmalar özetlenmekte ve elde edilen bir takım enteresan ve yeni sonuçlar vurgulanmaktadır.

2.7.1 Uzay-Zaman Kodlanmış Sinyaller Tarafından Uyarılmış Kanalların Gözü-Kapalı Kestirimi

Sınırlı radyo spektrumunu mümkün olduğu kadar verimli kullanmasını sağlayacak fiziksel katmanla ilgili yeni tekniklerin geliştirilmesi gerekmektedir. Bu amaca yönelik olarak kapasitenin önemli oranda artmasını sağlayan çeşitleme tekniklerinin kullanılması önerilmiştir. Çeşitleme teknikleri çokyollu iletimin neden olduğu sönümleme etkisini azaltmakla beraber karışım toleransını geliştirmekte ve dolayısıyla sistem kapasitesini arttırmaktadır. Mobil iletişim sistemlerinin taşınabilir birimlerinde en fazla bir yada iki antenin yerleştirilmesi mümkün olduğundan, baz istasyonlarında anten dizilimi kullanılarak gerçekleştirilen verici çeşitlemesine ilgi hızla artmıştır. Ayrıca verici çeşitlemesi ile kanal kodlamasını birleştiren bir yaklaşımla gerçekleştirilen bir uzay-zaman kodlama yöntemi önerilmiştir. Uzay-zaman kodlaması yönteminin sönümlemeli kanallarda frekans bandı verimli bir şekilde kullanılırken aynı zamanda önemli oranda sistem kapasite kazancı sağladığı gösterilmiştir.

Uzay-zaman kodlamasının kullanıldığı sistemlerde çeşitleme kazancının sağlanabilmesi için kanal parametrelerinin bilinmesi veya kestirilmesi gerekmektedir. Dolayısıyla uzay-zaman kodlamalı sistemin uygulanabilir olması için karşılaşılan önemli bir problem kanal kestirim yönteminin geliştirilmesidir. Bu çalışmada uzay-zaman kodlanmış işaretlerin iletildiği çokyollu telsiz iletişim senaryosu göz önüne alınarak, sönümlemeli kanal katsayıları matrisi ile iletilen işaretlerin ortak kestirimi için gözü kapalı(blind) bir yöntem önerilmiştir. Bu yöntem kodlanan işaretlerin bağımsız, özdeş dağılımlı olasılıksal diziler olduğu varsayımı ile elde edilen koşulsuz olabilirlik işlevininin enküçültmesine dayanmaktadır. Bu amaçla uzay-zaman kodlanmış işaretlerin sonlu alfabeden değerler alma özelliğinden faydalanılır. Ancak elde edilen koşulsuz olabilirlik işlevinin doğrudan enküçültmesinin hesaplama karmaşıklığı oldukça fazladır. Bundan dolayı, gözü kapalı kestirim yaklaşımı Markov zinciri modeli temel alınarak koşulsuz en büyük olabilirlik çatısı içerisinde geliştirilmiştir. Önerilen yöntem sönümleme kanal katsayıları matrisi ile iletilen işaretlerin ortak en büyük olabilirlik kestirimlerini elde ettiğinden, en büyük olabilirlik kestirimcilerinin birçok özelliğini sağlamaktadır. Ayrıca, önerilen kanal kestirim yönteminin başarımı çeşitli kanal örnekleri üzerinden bilgisayar simulasyonları denenerek elde edilen sonuçlar grafikler halinde özetlenmiştir.

Bu çalışma, 25-27 Nisan 2001 tarihlerinde Kuzey Kıbrıs Türk Cumhuriyeti'nde düzenlenmiş olan 9. Sinyal İşleme ve Uygulamaları Kurultayı nda "Uzay-Zaman Kodlanmış Sinyaller tarafından Uyarılmış Kanalların Gözü-Kapalı Kestirimi" adlı bildiriyle sunulmuştur. Konferans kitabında yayınlanan bu çalışmanın bir kopyası ilişikte görülebilir.

Baum-Welch algoritması ile koşulsuz en büyük olabilirlik işlevinin enküçültmesine dayalı yaklaşımından elde edilen sonuçlar kısmen, 28 Nisan-2 Mayıs 2002 yılında tarihlerinde New

York, USA de yapılan IEEE International Conference on Communications (ICC-2002) konferansında, "Blind Channel Estimation for Space-Time Coding Systems with Baum-Welch Algorithm" adıyla sunulmuştur. Konferans kitabında yayınlanan bu çalışmanın bir kopyası ilişikte görülebilir.

Ayrıca, sönümlemeli kanal katsayıları matrisi ile iletilen işaretlerin ortak kestirimi için önerilen gözü kapalı(blind) koşulsuz en büyük olabilirlik yaklaşımına karşılık koşullu en büyük olabilirlik yöntemi de önerilmiş ve başarım analizi yapılmıştır.

Koşulsuz en büyük olabilirlik yaklaşımına ait sonuçlar, 7-11 Kasım 2001 tarihinde Bursa-Türkiye'de düzenlenen "Second International Conference on Electrical and Electronics Engineering" konferansında "Blind Maximum Likelihood Channel Estimation for Space-Time Coding Systems" adlı bildiriyle sunulmuştur. Konferans kitabında yayınlanan bu çalışmanın bir kopyası ilişikte görülebilir. Bu çalışmaların sonuçları: "EURASIP JOURNAL ON APPLIED SIGNAL PROCESSING" dergisinin yayınladığı "Special Issue on: Space Time Coding and Its Applications-Part II" adlı özel baskısında "Maximum Likelihood Blind Channel Estimation for Space-Time Coding Systems" başlığıyla Mayıs 2002 tarihinde yayınlanmıştır.

Yukarıdaki paragrafta özetlenen çalışmalar daha sonra genişletilerek sürdürülmüş ve teklif edilen kestirim algoritmaları ile kestirimcilerin başarımlarına ait Cramer-Rao alt sınırları analitik olarak elde edilmiştir. Önerilen koşullu ve koşulsuz en büyük olabilirlik yaklaşımları karşılaştırılmış ve birbirleriyle olan üstünlükleri tartışılmıştır.

2.7.2 OFDM Sistemler tarafından Uyarılmış Zamanla-Değişen Kanallar için EM-Tabanlı Eğitim Verilerine(Non-Data-Aided) Gereksinim Duymayan Kanal Kestirim Algoritması

Bu çalışmada M-PSK sinyal ile iletişim yapan OFDM sistemleri için EM (Expectation-Maximization) yöntemine dayanan, hesaplama yönünden çok hızlı, bir MAP(Maximum a-posteriori) kanal kestirim algoritması geliştirilmektedir. İletilen M-PSK verileri üzerinden istatistiksel bir ortalama alınarak, kestirim algoritmasının eğitim verilerine gereksinim duymayacak biçimde tasarlanması gerçekleştirilmektedir (Non-data-aided). Ayrık, çok-yollu sönümlemeli kanalı belirleyen, ilintili(correlated) ve çok sayıda kanal parametreleri, Karhunen-Loeve dik açılımından yararlanılarak ilintisiz(uncorrelated) ve az sayıdaki kanal parametrelerine dönüştürülmekte ve bu parametreler de yukarıda belirtilen hızlı algoritma ile kestirilmektedir. Geliştirilen algoritma daha sonra QPSK sinyalleri ile modüle edilmiş OFDM sistemlerine uygulanmış ve kanal parametre kestirimi için kesin analitik sonuçlar elde edilmiştir.

Bu çalışma, Non-Data Aided EM-Based Channel Estimation for OFDM Systems with Time-Varying Fading Channels adlı bildiriyle 26-28 Eylül 2002 tarihleri arasında Almanya da yapılan 2001 Third International Workshop on Multi-carrier Spread-Spectrum(MC-SS 2001) and Related Topics te sunulmuştur.

Önerilen EM-Tabanlı eğitim verilerine gereksinim duymayan kanal kestirimcisinin başarım analizi, Cramer-Rao analitik sınırlarının çıkarılmasıyla genişletilmiş ve elde edilen sonuçlar K. Fazel ve S. Kaiser'in editörlüğünü yaptığı *Kluwer Academic Publishers* tarafından 2002 yılında basılan "Multi-Carrier Spread Spectrum & Related Topics" kitapda "Non-data aided EM-based channel estimation for OFDM systems with time-varying fading channels" başlığıyla kitap bölümü olarak yer almıştır.

Önerilen EM (Expectation-Maximization) yöntemine dayanan MAP(Maximum a-posteriori) kanal kestirim algoritması sadece altkanallar arasındaki ilintiden faydalanacak şekilde yeniden geliştirilmiştir. Bu çalışmadan elde edilen sonuçlar 5-7 Kasım 2001 tarihlerinde Antalya, Türkiye'de düzenlenen The Sixteenth International Symposium on Computer and Information Sciences konferansında "Maximum A Posteriori Multipath Fading Channel Estimation for OFDM Systems" adlı bildiriyle sunulmuştur.

Bu çalışmaların sonuçları, "European Transactions on Telecommunications" dergisinin yayınladığı "Special Issue on Multi Carrier Spread Spectrum & Related Topics" adlı özel baskısında "Maximum A Posteriori Multipath Fading Channel Estimation for OFDM Systems" başlığıyla Eylül/Ekim 2002 sayısında yayınmıştır.

2.7.3 Çokyollu Sönümlemenin Varlığında, Uzay-Zaman Blok Kodlanmış OFDM Sistemler için Kanal Kestirimi

Bu güne kadar kanal parametresi kestirimi için literatürde türlü teknikler önerilmiş bulunmaktadır. Özellikle OFDM sistemler için, tekil değer ayrıştırması veya frekans bölgesi süzgeçlemesine dayalı kanal kestirim teknikleri ve ayrıca kanal kestirimcisinin başarımını daha iyiye götürmek için, zamanla değişen ayırgan kanalın, zaman-frekans ilintisini en iyi kullanan en küçük ortalama karesel hata (MMSE) kanal kestirimcileri, günümüzde başarı ile uygulanmaktadır. Bu teknik sonradan, verici çeşitlemeli ve uzay zaman kodlaması kullanan OFDM sistemleri için genişletilmiştir. Verici çeşitlemesi, mobil, çokyollu, telsiz kanallardaki sönümlemeyle başetmek için etkili bir yöntemdir. Son dönemlerde, yüksek veri hızında telsiz iletişim için, uzay-zaman kodlaması geliştirilmiş, ve OFDM sistemlerde incelenmiştir. Bununla birlikte, uzay-zaman kodlarının çözümü, elde edilmesi güç olan kanal bilgisini gerektirir. Yakın zamanda Alamouti, iletim için, iki verici antenin kullanıldığı, dikkate değer bir iletim çeşitleme yöntemi önermiştir. Bu yöntem daha sonradan keyfi sayıda verici anteni için genelleştirilmiş, ve verici-alıcı anten çifti ile elde edilebilen en yüksek çeşitlemeyi başardığı görülmüştür. Uzay-zaman kodlarının dikey yapısı, en büyük olabilirlik kod çözümünün, sinyalin birleşik algılanmasından daha basit olarak, farklı antenlerden iletilen sinyalin ayrıştırılması yoluyla uygulanmasını mümkün kılmaktadır. Bildirinin tam metninde de görüleceği gibi, uzay-zaman blok kodlama, kanal kestirimini oldukça kolay hale getirmektedir. Bu çalışmada, uzay-zaman blok kodlaması kullanarak, verici çeşitlemeli ortogonal frekans bölmeli çoğullama (OFDM) sistemleri için, hesaplamasal olarak verimli, veri desteksiz bir MAP (maximum a posteriori) kanal kestirim algoritması önerilmektedir. Bu amaçla Alamouti'nin iki verici antenli iletim çeşitleme yöntemi kullanılmış ve OFDM sistemler için genelleştirilmiştir. Algoritma, ayrık çokyollu sönümlemeli kanalın, uygun Karhunen-Loeve dikey açılımı ile uygun modellenmesini gerektirir, ve bilinmeyen kanalın gerçek MAP kestirimine yakınsayarak beklenti enbüyükleme (EM) yöntemiyle her alt taşıyıcı için, özyineli şekilde, karmaşık kanal katsayılarını kestirir. Bilinmeyen kanal parametrelerine ilişkin güvenilir başlangıç değerleri seçmek için, pilot simgeler kullanılmıştır. Önerilen MAP kanal kestirimcisinin, değiştirilmiş Cramer-Rao alt sınırı için analitik bir ifadesi elde edilmiştir. Ayrıca kestirimcinin, kanal ilintisi ve işatet/gürültü oranındaki değişimlere dayanıklılığı da analiz edilmiştir. QPSK sinyalleşme kullanan bir sistem için başarım, ortalama karesel hata ve simge/hata oranı aracılığıyla gösterilmiştir. Kapsamlı bilgisayar benzetimleri, önerilen kanal kestirimcisiyle birlikte verici çeşitlemeli OFM (orthogonal frekans çoğullaması) kullanan uzay-zaman blok kodlaması, telsiz mobil kanallarda yüksek verimlilikte veri iletimi için umut vadedici bir teknik olduğunu göstermiştir. Bu çalışma, Channel Estimation for Space-Time Block Coded OFDM Systems in the Presence of Multipath Fading adlı bildiriyle Kasım 2002'de Taiwan'da düzenlenen GLOBECOM 2002'de sunulmuştur.

2.7.4 OFDM Sistemi için Ortak Kanal İzleme ve Sembol Sezim Yöntemi

Bu çalışmada çok yollu sönümlemeli kanalda pilot sembol yardımlı OFDM sistemi için ortak kanal izleme ve sembol sezim yöntemi önerilmiştir. Önerilen yöntem hem kanal kestirimi hem de sembol sezimi için Kalman filtresi kullanarak iki işlemi bir alıcı yapısı altında birleştirmektedir. Çok yollu sönümlemeli kanalın olasılıksal süreç (AR) olarak modellenmesi durum-uzay modelinin oluşturulmasını ve dolayısıyla kanal parametrelerinin Kalman filtresi ile izlenmesine izin vermektedir. Ancak, önerilen yöntem ile kanal değişimlerinin izlenmesi için iletilen sembollerin bilinmesini gerekmektedir. Bundan dolayı kanal değişimlerinin izlenmesi ile birlikte iletilen sembollerin sezimi için iteratif bir algoritma oluşturulmuştur. Son olarak oluşturulan yöntemin performansı simulasyon sonuçları ile desteklenmiştir.

Bu çalışma, 10-11 Eylül 2002 tarihlerinde Hamburg, Almanya'da düzenlenmiş olan 7th International OFDM-Workshop(InOWo'02) Kurultayında "Joint Channel Tracking and Symbol Detection for OFDM Systems with Kalman Filtering" adlı bildiriyle sunulmuştur. Yukarıdaki paragrafta özetlenen çalışmalar daha genişletilerek sürdürülmüş ve bu "International Journal of Electronics and Communications" dergiside yayınlanmak üzere kabul edilmiştir.

2.8 Kaynakça

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Bölüm III

UZAY-ZAMAN KODLAMALI, MSK-OFDM TÜMLEŞİK SİSTEM TASARIMI

3.1 Giriş

Bu bölümde, uzay-zaman kodlamalı, MSK Modülasyonlu OFDM tümleşik sistemlerin tasarımları için projede gerçekleştirilen çalışmalar ve üretilen sonuçlar, bunu izleyen alt bölümlerde, ayrıntılı bir biçimde sunulmaktadır. Bu çalışmalar kısaca şöyle özetlenebilir.

Ortogonal frekans bölmeli çoğullama (Orthogonal Frequency Division Multiplexing, OFDM), frekans-seçici kanallar üzerinden iletimde sağladığı yüksek başarımdan dolayı, çok taşıyıcılı sistemlerin günümüzde tercih edilen bir türüdür. Bölüm 3.2 de, MSK modülasyonlu sinyallerin iletilmesi durumunda frekans-seçici sönümleme etkilerini enaza indirmak amacıyla OFDM sistemi kullanımı önerilmiş, değişik kodlar tasarlanmış, bilgisayar benzetimleri yardımıyla bu sistemlerin hata başarımları incelenmiştir.

Uzay-zaman kodlaması, kullandığı iletim çeşitlemesi tekniği sayesinde gezgin sönümlemeli kanallarda önemli kodlama kazançları sağlamaktadır. Şimdiye dek bu teknik PSK ve QAM modülasyon tekniklerine uygulanmışken, Bölüm 3.3 de ilk kez Hızlı Frekans Kaydırmalı Anahtarlama (Minimum Shift Keying, MSK) modülasyonuna uygulanmaktadır. Duruğumsu ve hızlı sönümlemeli kanallarda tasarım ölçütleri göz önüne alınarak bir bilgisayar arama programı geliştirilerek optimum kodlar elde edilmekte ve elde edilen kodların hata başarımları bilgisayar benzetimi yardımıyla değerlendirilmektedir.

Uzay-zaman kodları birden çok verici ve/veya alıcı anten kullanımının beraberinde getirdiği kapasite artışından yararlanarak sönümlemeli kanallarda yüksek hata başarımına ulaşırlar. Bu teknik gerekli iletim band genişliğini arttırmaksızın uzay çeşitlemesi sağlar. Uzay-zaman kodlarının bir türü olan dik (ortogonal) uzay-zaman kodları iki verici anten kullanımının sağlayabileceği en yüksek çeşitleme kazancına sahiptir. Ancak, kanal durum bilgisinin alıcıda ideal olarak kestirilemediği durumda bu yapıya ilişkin diklik özelliği ortadan kalktığından hata başarımı oldukça düşmektedir. Bölüm 3.4 te vericide iki, alıcıda ise birden fazla anten kullanan sistemler için geliştirilmiş bir güç kontrol tekniği sunulmaktadır.

OFDM sinyallerinin frekans-seçici kanallar üzerinden iletimde sağladığı yüksek başarımdan dolayı çok-taşıyıcılı sistemlerin günümüzde önem kazanan bir türüdür. Sönümlemeli kanallarda hata başarımını yükseltmenin en iyi yollarından biri çeşitlemeden yararlanmaktır. Bilginin kopyalarının bağımsız alt-kanallar üzerinden iletilmesi ilkesine dayanan bu teknik, Bölüm 3.5 te OFDM yapısına uygulanmaktadir. İletilmek istenen veri önce kafes kodlanmakta, daha sonra ise dik uzay-zaman kodlamasından geçirilerek kanala OFDM bloğu üzerinden iletilmektedir. Önerilen bu sistem uzay, zaman ve frekans çeşitlemesi türlerini birlikte içermektedir.

3.2 MSK Modülasyonlu OFDM Sistemleri

3.2.1 Giriş

Günümüzde önemi hızla artan gezgin ve telsiz iletişim sistemleri, iletişim kanalının neden olduğu toplamsal Gauss gürültüsüne ek olarak sönümlemeden (fading) ve faz seyirmesinden de büyük ölçüde etkilenmektedir. Bu tür sistemler için, gerek kullanıcı sayısının artması ve gerekse gezgin anten boyutları veya uydunun ışınım sınırlamaları nedeniyle band ve güç verimliliği yüksek iletişim tekniklerinin geliştirilmesi gerekmektedir. Bu teknikler genellikle kafes kodlamalı modülasyon (trellis coded modulation, TCM) tekniğine dayalıdır. Sabit zarf, sürekli faz ve içerdiği doğal kodlama gibi özellikleriyle gezgin iletişim kanallarında yeğlenen minimum kaydırmalı anahtarlama (minimum shift keying, MSK), bir önkodlayıcı yardımıyla kodlanarak band ve güç verimliliği yüksek çoklu kafes kodlamalı (multiple trellis coded modulation, MTCM) MSK sistemlerin oluşturulması için de oldukça uygun bir modülasyon tekniğidir. Diğer taraftan geniş bandlı iletişim sistemlerinde, özellikle frekans seçici kanallar üzerinden iletimde yüksek başarımlarından dolayı, "çok taşıyıcılı (multicarrier)" sistemler günümüzde yaygın olarak kullanılmaya başlamıştır. Bu sistemlerin OFDM (orthogonal frequency division multiplexing) olarak adlandırılan biçimi [1],[2], tüm iletişim kanalını belli sayıda alt-kanallara bölerek bilgiyi birbirine dik (orthogonal) seçilmiş alt-taşıyıcı frekanslarda iletme ilkesine dayanmaktadır. OFDM sistemlerde alt-taşıyıcıların spektrumlarının örtüşmelerine izin verildiğinden band verimliliği açısından da bir üstünlük sağlanmış olur. Öte yandan, OFDM tek taşıyıcılı sistemlerle karşılaştırıldığında oldukça uzun bir işaretleşme peryoduna sahip olduğundan hızlı sönümlemelere karşı daha iyi başarıma sahiptir.

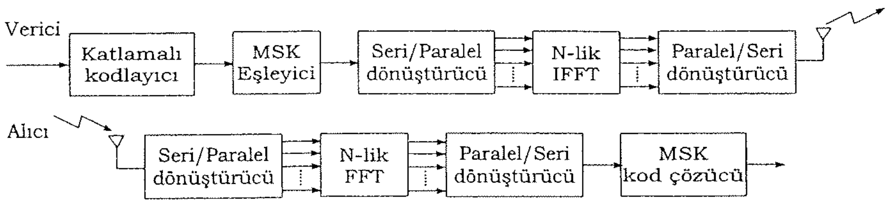
Literatürde, son yıllarda çok taşıyıcılı sistemler arasında oldukça ön plana çıkan OFDM ile çeşitli kodlama türlerinin birleştirilmesi [3], [4] ve giriş işaret kümesi sınırlandırma [5] konularında araştırmalara rastlamak mümkündür. Bu bildiride çoklu kafes kodlu MSK modülasyonunu OFDM iletim tekniği ile birleştirerek özellikle sönümlemeli kanallar için yüksek başarıma sahip bir iletişim sistemi önerilmektedir.

3.2.2 Sistem Modeli

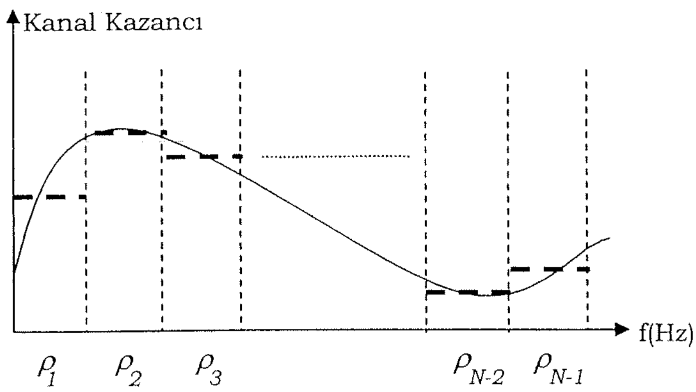
Ele alınan iletişim sisteminin blok diyagramı Şekil 3.2.1 ' de görülmektedir. Verici kısımda bilgi dizisi, katlamalı kodlayıcı yardımıyla kodlanarak çoklu kafes kodlu MSK işaretini oluşturmak üzere önceden belirlenmiş kanal simgelerine eşleştirilir. OFDM sisteminin en karakteristik özelliği olan dik taşıyıcıları gerçekleştirmek için uygulaması kolay ve maliyeti düşük olan hızlı Fourier (Fast Fourier Transform, FFT) ve ters hızlı Fourier dönüştürücülerden (Inverse Fast Fourier Transform) yararlanılmaktadır. Kodlanmış ve eşlenmiş simge dizisini alt-taşıyıcılara ötelemek amacı ile N 'lik IFFT bloğu kullanılır. Burada, N sayısı alt-kanal sayısını gösterir ve pratikte 512, 1024 gibi değerler alabilir. Dikkat edilmesi gereken nokta ise IFFT ve FFT işlemlerinin hızlı ve etkin yapılabilmesi için N değerinin 2'nin bir üssü olarak seçilmesidir. IFFT işlemine uygun biçime getirilmek amacıyla kodlanmış işaret dizisi 1-giriş, N-çıkışlık bir seriden paralele dönüştürücü devresine uygulanarak N li bir paralel dizi oluşturulur. IFFT bloğu çıkışı yeniden seri biçime dönüştürülerek simgeler arka arkaya kanala gönderilir. Alıcı tarafta ise peşpeşe alınan her N kanal simgesi paralele dönüştürülüp FFT si alınır. Tekrar seriye dönüştürülünce çoklu kafes kodlanmış MSK işaret dizisinin kanalın sönümleme ve gürültü etkileriyle bozulmuş biçimi elde edilir. Alıcının son adımında ise MTCM kod çözücü kafes kodların en büyük olabilirlikli çözümünü gerçekleştiren Viterbi algoritmasını kullanarak iletilen bilgiyi yeniden elde etmeye çalışır. Geniş bandlı iletişimde en kötü durumlardan birisi de iletişim kanalının frekans seçici olmasıdır. OFDM tekniği böyle bir kanalı çok sayıda (N adet) birbiriyle örtüşebilen alt-bandlara bölerek kullanmayı sağlarken, eldeki frekans-seçici kanalı frekans-seçici olmayan yani düzgün sönümlemeye (flat fading) sahip N adet alt-banda böler,(Şekil 3.2.2). Böylece, her bir banddan iletilen işaretler alıcıda daha kolay çözülebilir. Viterbi algoritmasını kullanmayı kolaylaştıran başka bir etken ise,(3.2.1) ilişkisinden de görüleceği gibi, kanal girişindeki IFFT ve kanal çıkışındaki FFT işlemleri nedeniyle sönümlemesinin alt-kanallar üzerinde etkisinin, $\rho_k, 0 < k < N-1$ sönümleme katsayılarının FFT çıkışında kodlanmış simgeler ile çarpımsal biçiminde ortaya çıkmasına neden olmasıdır.

$$r_k = \rho_k C_k + n_k, \quad 0 < k < N-1 \tag{3.2.1}$$

Burada, r_k, N 'lik bir iletim çerçevesi içersinde k. zaman diliminde kod çözücü girişine gelen örnektir. C_k bu çerçevede k. alt-kanaldan iletilmiş olan çoklu kafes kodlu MSK simgesini gösterirken, n_k, sıfır ortalamalı, boyut başına $N_0/2$ varyanslı istatistiksel bağımsız Gauss dağılımlı gürültü örneğidir. ρ_k ise kanalın k. alt-kanalının bu çerçeve süresince geçerli sönümleme katsayısıdır ve kompleks Gauss dağılımı ile modellenebilir. Böylece, iletimde problemlere neden olan frekans-seçici kanal , alıcı tarafından bakıldığında zaman-seçici kanala dönüştürülmüş olup sisteme oldukça büyük bir çeşitleme (diversity) eklemiştir.



Şekil 3.2.1: Çoklu kafes kodlamalı MSK modülasyonlu OFDM sistemi



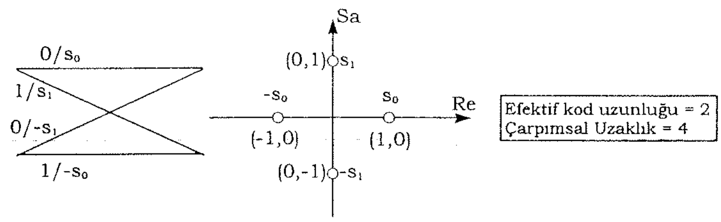
Şekil 3.2.2: Frekans-seçici kanal

3.2.3 Çoklu MSK Kodları

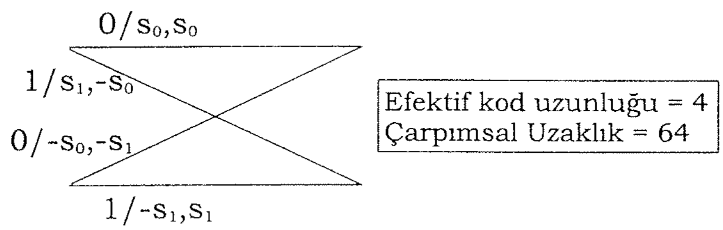
Yukarıda verilen (3.2.1) ilişkisinden de kolaylıkla anlaşılacağı gibi frekans-seçici kanalın giriş ve çıkışına eklenen IFFT ve FFT işlemlerinin bir sonucu olarak iletişim kanalının bilgi üzerine etkisi zaman-seçici sönümlemeli kanalda olduğu gibi gözlenmektedir. Buna göre, bu iletişim sisteminde kullanılmak üzere tasarlanacak kodların da zaman-seçici kanallara özgü ölçütlere göre tasarlanması gerekmektedir. Özellikle yüksek işaret/gürültü oranlarında zaman-seçici sönümlemeli kanallar için iki önemli ölçüt bulunmaktadır. Bunlardan ilki olan etkin kod uzunluğu (effective code length, ECL), aynı durumdan başlayıp aynı durumda sona eren yol çiftleri üzerindeki farklı kanal simgesi sayısıdır. İkinci derecede önemli hata başarım ölçütü ise çarpımsal uzaklıktır. (product distance, d_p^2). Çarpımsal uzaklık, ilk ölçütün sağlandığı zaman aralıkları için hesaplanan kanal simgesi karesel uzaklıklarının çarpılması ile elde edilir. Yüksek hata başarımına sahip bir kod tasarlarken ilk hedef efektif kod uzunluğunun büyütülmesi, ikinci olarak da karesel uzaklığın artırılmasıdır. Bu ölçütlere göre kodlanmamış ve çeşitli oranlarda kodlanmış türlü sistemler tasarlanmış ve bilgisayar benzetimleri yardımıyla hata başarımları incelenmiştir. Karşılaştırma amacıyla referans olarak kodlanmamış MSK alınmıştır, (Şekil 3.2.3). Şekil 3.2.4 ve 3.2.5' te 1/2 oranlı kodlanmış 2 ve 4-durumlu MSK kodları verilmiştir. Şekil 3.2.6 ve 3.2.7' de ise kodlama oranı yükseltilerek 2/3 oranlı 2 ve 4-durumlu MSK kodları verilmiştir.

3.2.4 Hata Başarımı

Bilgisayar benzetimleriyle Bölüm 3.2.3' te verilen kodların hata başarımları incelenmiştir.

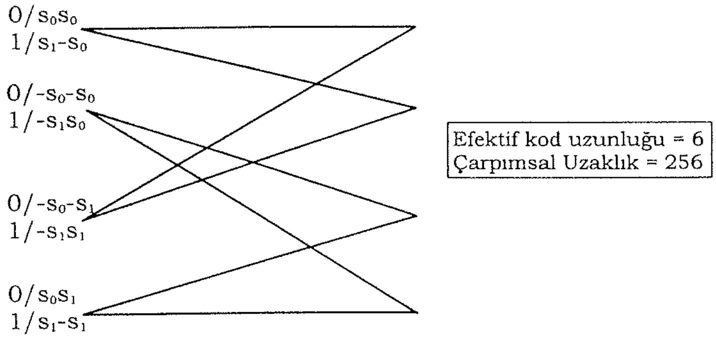


Şekil 3.2.3: Kodlanmamış MSK

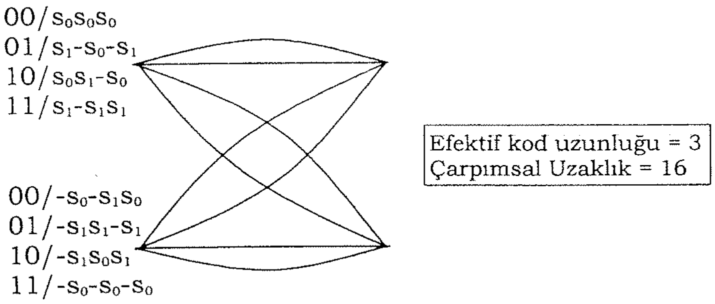


Şekil 3.2.4: 1/2 oranlı 2-durumlu kodlanmış MSK

Rayleigh ve Rician dağılımlı sönümlemeye sahip kanallar üzerinde yapılan benzetimlerde alıcı tarafta kanal durum bilgisinin ideal olarak kestirildiği ve eşzamanlamanın ideal olduğu varsayımları yapılmıştır. FFT ve IFFT tabanı 128 olarak seçilmiştir. Benzetim ile sözkonusu kodların değişik işaret-gürültü oranlarında Rayleigh ve Rician sönümlemeli kanallarda bit hata olasılıkları elde edilmiştir. Kodlanmamış , 1/2 oranlı kodlanmış ve 2/3 oranlı kodlanmış MSK kodlarının Rayleigh sönümlemeli kanaldaki hata başarım eğrileri Şekil 3.2.8'de, Rician sönümlemeli kanaldaki hata başarımları ise Şekil 3.2.9' da verilmiştir. Benzetim

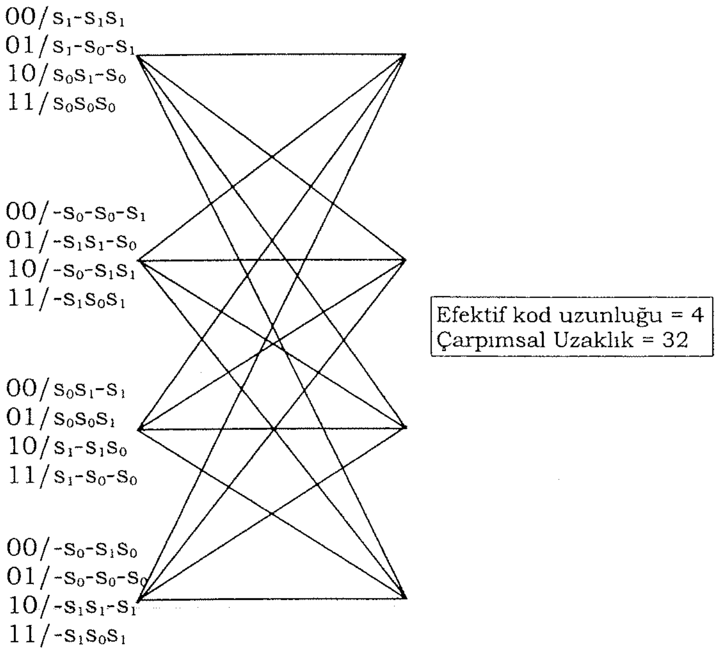


Şekil 3.2.5: 1/2 oranlı 4-durumlu kodlanmış MSK



Şekil 3.2.6: 2/3 oranlı 2-durumlu kodlanmış MSK

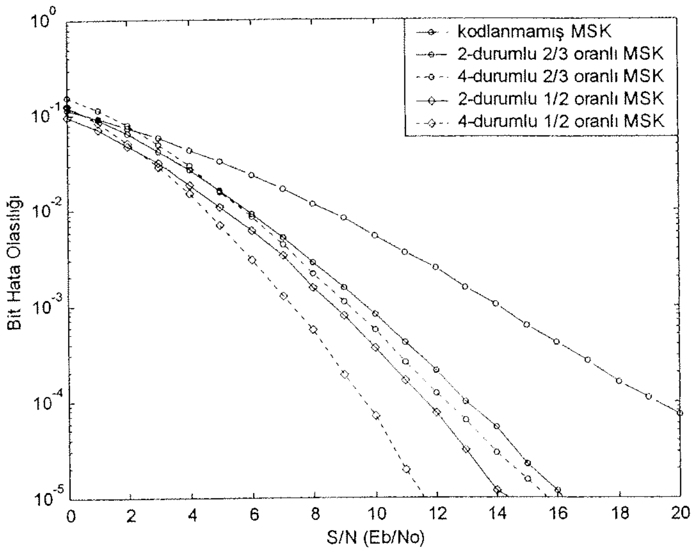
sonuçları incelendiğinde Rayleigh ve Rician kanallarda kodlanmamış MSK göre Bölüm 3.2.3' te önerilen kodların oldukça yüksek bir hata başarımına sahip oldukları görülebilir. Bunun başlıca nedeni, zaman-seçici sönümlemeli kanallar için verilen ölçütlerin kodlama yaparak iyileştirilmesidir. Rayleigh sönümlemeli kanalda, 10^{-5} bit hata olasılığına ulaşmak için kodlanmamış MSK kullanılması durumunda yaklaşık 24 dB işaret-gürültü oranına gereksinim duyulurken, kodlayıcı oranından özveride bulunularak elde edilen çoklu kodlanmış MSK için bu değer 12-16 dB aralığında değişmektedir. Rician sönümlemeli kanalda ise, yine 10^{-5} bit hata olasılığına ulaşmak için kodlanmamış MSK için 12 dB işaret-gürültü oranı gerekirken, kodlama sonucunda bu değer de 7-9 dB değerlerine inmiştir. Benzetim sonuçlarından da görüldüğü gibi kodlayıcı oranı arttıkça kodun ulaşabileceği hata başarımı da düşmektedir. Böyle bir sistem tasarlanırken istenen veri hızı ile hata başarımı birlikte gözönünde tutularak bir denge noktasına ulaşılmalıdır.



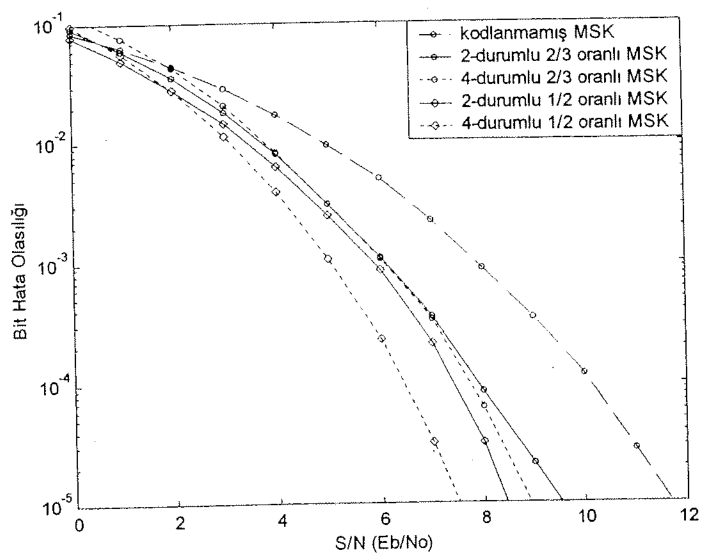
Şekil 3.2.7: 2/3 oranlı 4-durumlu kodlanmış MSK

3.2.5 Sonuç

Bu çalışmada, ortogonal frekans bölmeli çoğullama ile çoklu kafes kodlu MSK birleştirilmiş ve frekans-seçici sönümlemeli kanallar üzerinden iletişime uygun bir sistem önerilmiştir. Kullanılan FFT ve IFFT nedeniyle kanalın frekans-seçicilikten zaman-seçiciliğe dönüşmesi ile oluşan yeni hata başarım ölçütleri gözönüne alınarak çeşitli MSK kodları tasarlanmış, bilgisayar benzetimleri yardımıyla hata başarımları incelenmiştir.



Şekil 3.2.8: Tasarlanan MSK kodlarının Rayleigh sönümlemeli kanalda hata başarımları



Şekil 3.2.9: Tasarlanan MSK kodlarının Rician sönümlemeli (K=10) kanalda hata başarımları

3.3 Uzay-Zaman Kodlamalı Çoklu MSK Modülasyonu

3.3.1 Giriş

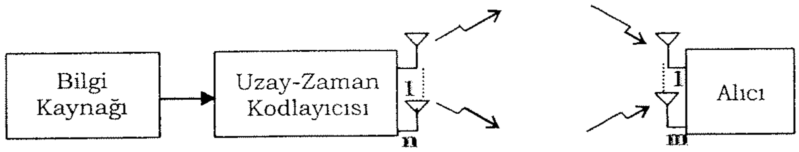
Minimum kaydırmalı anahtarlamalı (Minimum Shift Keying) MSK modülasyonu, sürekli faz modülasyonunun özel bir biçimi olup sabit zarf ve band verimliliği gibi özellikleri nedeniyle band ve/veya güç sınırlı iletişim ortamları için oldukça uygun bir iletişim tekniğidir. Yapısında içerdiği kodlamaya [6] ek olarak band verimliliğinden bir miktar özveride bulunularak güç verimliliğinin kodlama işlemi yardımıyla daha da artırılabiliyor olması, bu modülasyon tekniğini söz konusu iletişim ortamları için daha da çekici duruma getirmektedir. Son yıllarda yaygın olarak incelenen ve sistemin kodlama kazancını artıran bu tür yöntemler genellikle kafes kodlamalı modülasyon (trellis coded modulation, TCM) tekniğine dayanır. MSK modülasyonu, toplamsal beyaz Gauss gürültülü kanalların yanısıra özellikle gezgin iletişim sistemlerinde karşılaşılan sönümlemeli (fading) kanallar için de çok uygun bir modülasyon tekniğidir.

Kafes kodlamalı sistemler için Gauss gürültülü kanallarda, özellikle yüksek işaret/gürültü oranlarında, hata başarım ölçütü serbest Öklid uzaklığı olmasına karşın, sönümlemeli kanallarda, yüksek işaret/gürültü oranlarında en önemli hata başarım ölçütü etkin kod uzunluğu (effective code length, ECL), ikincil olarak da çarpımsal uzaklıktır (product distance, d_p^2). İyi bir kod tasarımında, etkin kod uzunluğunun ve çarpımsal uzaklığının olabildiğince büyük yapılmasına çalışılır. MSK modülasyonunda etkin kod uzunluğu ve çarpımsal uzaklığı artırmanın bir yolu da çoklu kafes kodlaması kullanılmasıdır. Bu yaklaşımda birden çok kafes adımının birleştirilmesi sonucunda oluşturulan yeni kafesde her dala birden fazla simge eşleştirilerek kodun etkin kod uzunluğu ve çarpımsal uzaklığı oldukça artırılır.

Sönümlemeli kanallarda hata başarımını artırmanın bir yolu da çeşitleme (diversity) tekniğinden yararlanmaktır. Çeşitleme, tüm kanallarda birden derin sönümleme olasılığının küçük olacağı varsayımı altında, aynı bilgiyi birden fazla bağımsız kanaldan iletmeye dayanır. Bu bağımsız kanallar frekansta, zamanda ve/veya uzayda (farklı anten) çeşitleme yoluyla yaratılır. Çeşitleme türleri birlikte veya ayrı ayrı bir iletişim sisteminde kullanılabilir. Zaman çeşitlemesinde, gönderilen işaretin en az zayıflamış bir kopyası farklı iletim anlarından birinde elde edilirken frekans çeşitlemesinde bu kopya aynı zaman aralığında farklı frekans bölgelerinden birinden elde edilir. Uzay ya da anten çeşitlemesi ise birden çok verici ve/veya alıcı anten kullanılarak hata başarımının artırılmasına dayanır. Alıcı anten sayısını artırarak çeşitleme sağlama literatürde yeterince incelenmiş bir konu olmasına karşın çeşitleme için verici anten sayısını artırmak ve bu antenleri kod tasarımı sırasında birlikte göz önüne alarak her biri için farklı kodlayıcılar geliştirmek yeni bir konudur. Bu yeni tekniğe uzay-zaman kodlaması denmektedir [7],[8]. Bu çalışmada, uzay-zaman kodlama tekniği MSK modülasyonuna uygulanmakta, iki verici ve bir alıcı anten için iki, dört ve sekiz durumlu uzay-zaman kodlamalı çoklu MSK sistemler önerilmektedir. Bu sistemlerin tasarımlarında, düzgün ve yavaş sönümlemeli kanallarda uzay-zaman kodlarının tasarım ölçütlerini oluşturan rank ve determinant ölçütlerinin eniyileştirilmesi yoluna gidilmiş ve bu amaçla geliştirilen bir kod arama algoritmasından yararlanılmıştır. Önerilen kodların hata başarımları geliştirilen bir bilgisayar benzetim programı yardımıyla incelenmiş, tek verici anten kullanılması ve her iki verici antende aynı MSK kafes kodunun kullanılması durumlarına olan üstünlükleri Rayleigh sönümlemeli kanallar için ortaya konulmuştur.

3.3.2 Sistem Modeli

Uzay-zaman kodlaması genel olarak n verici ve m alıcı anten kullanılması ilkesine dayanır (Şekil 3.3.1) ve sönümlemeli kanallara yönelik klasik kafes kodlamalı modülasyon tekniği uzay-zaman kodlama tekniğinin bir verici/bir alıcı anten özel durumu olarak düşünülebilir.



Şekil 3.3.1: Uzay-zaman kodlamalı sistem

Uzay-zaman kodlamasında aynı veri bloğu her verici anten için ayrı bir kafes kodlayıcıdan geçirilerek n verici antenden iletilmekte, her alıcı antende verici antenlerden gelen, aynı bilgiye ilişkin farklı işaretler toplanmakta ve alınan işaret dizileri Viterbi algoritması yardımıyla çözülmektedir. Kanal çıkışında, her bir alıcı antende, t. zaman aralığında elde edilen işaret (3.3.1)'deki biçimde verilebilir.

$$r_t^j = \sum_{i=1}^{n} \alpha_{i,j} c_t^i \sqrt{E_s} + n_t^j \tag{3.3.1}$$

Burada, $1 \leq j \leq m$ alıcı anten sayısını, c_t^i, t. zaman aralığında i. verici anten tarafından iletilen simgeyi, E_s, ortalama simge enerjisini ve n_t^j ise t. zaman aralığında j. alıcı antende ortaya çıkan Gauss dağılımlı, sıfır ortalamalı, boyut başına $N_0/2$ varyanslı istatistiksel bağımsız gürültü örneğini göstermektedir. $\alpha_{i,j}$, i. verici antenden j. alıcı antene kanal kazancı olup sıfır ortalamalı, boyut başına 0.5 varyanslı bağımsız kompleks Gauss dağılımı ile modellenmektedir. Kanal iki değişik biçimde modellenmektedir. İlkinde, kanal kazancı her işaretleşme aralığında istatistiksel bağımsız olarak değişmektedir. Bu model hızlı sönümleme olarak adlandırılmaktadır. İkinci modelde ise kanal kazancının değeri bir çerçeve boyunca sabit kalmakta, bir çerçeveden diğerine istatistiksel bağımsız değişmektedir. Bu modele de duruğumsu (quasistatic) sönümleme denmektedir.

Şimdi, uzay-zaman kodlamalı sistem kullanarak n antenden l süre boyunca (3.3.2)'de verilen c. kod sözcüğünün iletildiği varsayılsın,

$$c = c_1^1 c_1^2 \cdots c_1^n c_2^1 c_2^2 \cdots c_2^n \cdots c_l^1 c_l^2 \cdots c_l^n. \tag{3.3.2}$$

Burada, c kod sözcüğü dizisini oluşturan simgelerin alt indisleri gönderildikleri zaman aralıklarını, üst simgeleri ise o aralık boyunca gönderildikleri verici anteni göstermektedir. Kanaldaki gürültü ve sönümleme etkileri nedeniyle bozulan işaret alınıp Viterbi algoritması yardımıyla çözülmeye çalışıldığında hatayla e kod sözcüğü dizisine karar verilebilir,

$$e = e_1^1 e_1^2 \cdots e_1^n e_2^1 e_2^2 \cdots e_2^n \cdots e_l^1 e_l^2 \cdots e_l^n. \tag{3.3.3}$$

Bu hataya yol çiftleri hatası denir ve Tarokh ve diğerleri tarafından her iki kanal modeli için de bu olasılığın üst sınırı analitik yollarla elde edilmiştir, [7]. Buna göre, duruğumsu(quasistationary) sönümlemeli kanallar için iki adet başarım ölçütü vardır. Bu ölçütler oluşturulan yol matrisi yardımıyla hesaplanmaktadır.

$$B(c,e) = \begin{bmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & e_l^1 - c_l^1 \\ e_1^2 - c_1^2 & e_2^2 - c_2^2 & \cdots & e_l^2 - c_l^2 \\ e_1^3 - c_1^3 & e_2^3 - c_2^3 & \cdots & e_l^3 - c_l^3 \\ \cdots & \cdots & \cdots & \cdots \\ e_1^n - c_1^n & e_2^n - c_2^n & \cdots & e_l^n - c_l^n \end{bmatrix} \tag{3.3.4}$$

$$A(c,e) = B(e,c)B^*(c,e) \tag{3.3.5}$$

Duruğumsu sönümlemeli kanallar için en önemli hata başarım ölçütü (3.3.4) ve (3.3.5) yardımıyla hesaplanan $A(c,e)$ yol matrisinin kertesinin(rankının) maksimum yapılmasıdır. Bu matrisin kertesinin en yüksek değeri n verici anten sayısıdır. Bu ölçüt yardımıyla tasarlanacak kodların değişik verici antenlerden gönderdikleri simge dizilerinin bağımsızlığı sağlanır. İkincil ölçüt ise yine bu matrisin özdeğerlerinin çarpımıdır. Bu ölçüt ise iletilen kod sözcüğü dizileri arasındaki mesafenin uzaklığını etkiler. Herhangi bir uzay-zaman kodlamalı sistemin hata başarımı incelenmek isteniyorsa hataya neden olabilecek, yani aynı durumdan başlayıp aynı durumda son bulan tüm yol çiftleri için ölçütler gözlenmelidir. Bu ölçütlerin bir kod içersindeki en düşük değerleri hata başarımında etkin terimlerdir.

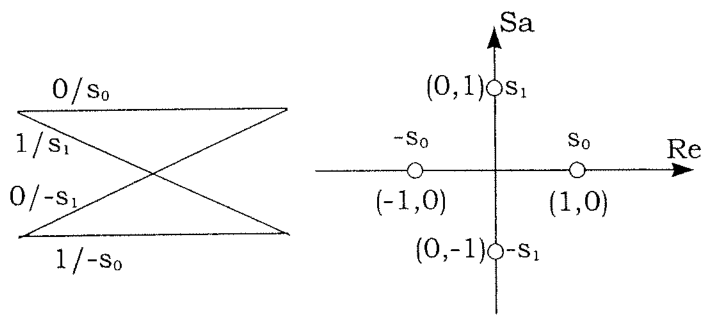
Hızlı sönümlemeli kanallarda ise ölçütler kafes kodlamalı modülasyonda kullanılan temel ölçütleri andırmaktadır. Kod sözcüğü dizileri göz önüne alınarak uzaklık ve çarpım kriterleri yine her yol çifti için hesaplanır. $1 \leq t \leq l$ zaman aralığında $c_l^1 c_l^2 \cdots c_l^n \neq e_l^1 e_l^2 \cdots e_l^n$ eşitsizliği en az ν zaman aralığı için sağlanmalıdır. Bu ölçüte uzaklık ölçütü denir. İkinci derecede önemli kriter ise uzaklık ölçütünün içerdiği eşitsizliği sağlayan zaman aralıkları için çarpımsal uzaklıktır ve (3.3.6) ve (3.3.7) yardımıyla hesaplanabilir.

$$\text{Çarpımsal Uzaklık} = \prod_{t \in \nu(c,e)} |c_t - e_t|^2 \tag{3.3.6}$$

$$|c_t - e_t|^2 = \sum_{i=1}^{n} |c_t^i - e_t^i|^2 \tag{3.3.7}$$

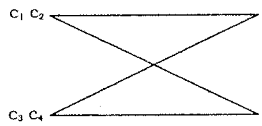
3.3.3 MSK için Uzay-Zaman Kodları

Kodlanmamış MSK modülasyonunun işaret vektörleri ve kafes diyagramı Şekil 3.3.2'de görülmektedir.



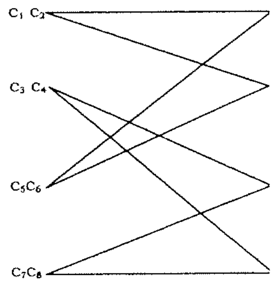
Şekil 3.3.2: MSK Kafesi ve İşaret-uzayı diyagramı

Bu şekildeki MSK kafesinin birden fazlasını art arda getirip birleştirerek çoklu MSK modülasyonuna ulaşılır. Bu modülasyonda kafes dalı başına birden fazla simge eşlenir. Buna göre, bilgisayar arama algoritması yardımıyla iki verici bir alıcı anten için her iki tür kanal için ayrı ayrı optimum ve optimuma yakın kodlar tasarlanmıştır. Arama sonucunda duruğumsu ve hızlı sönümlemeli kanallar için 2 ve 4 durumlu kafeslerde optimum kodlar (Şekil 3.3.3 ve 3.4.4) , 8 durumlu kafes içinse optimuma yakın kod bulunmuştur, (Şekil 3.3.5). Duruğumsu sönümlemeli kanallar için elde edilen 2, 4 ve 8 durumlu kodların hepsinin kertesi 2 iken, özdeğer çarpımları, sırasıyla, 64,128 ve 96'dır. Hızlı sönümlemeli kanallar için bulunan 2, 4 ve 8 durumlu kodların ise, uzaklıkları sırasıyla, 4, 6 ve 7 iken, çarpımsal uzaklıkları 1024, 36864 ve 6912'dir. Kod araması sırasında karşılaşılan temel problem bilgisayar arama programının 8 durumlu optimum kodu bulması için gereken işlem süresinin oldukça büyük olmasıdır. Bu nedenle, her iki kanal tipi için de 8 durumlu optimum kodlar bulunamamış, bunlar yerine kısıtlı seçenek kümesi içersinden optimuma yakın kodlar elde edilmiştir.



	Duruğumsu s.		Hızlı s.	
	$verici_1$	$verici_2$	$verici_1$	$verici_2$
c_1	s_0 s_0	s_1 $-s_1$	s_1 $-s_1$	s_1 $-s_1$
c_2	s_1 $-s_0$	s_0 s_1	s_0 s_1	s_0 s_1
c_3	$-s_0$ $-s_1$	$-s_1$ s_0	$-s_1$ s_0	$-s_1$ s_0
c_4	$-s_1$ s_0	$-s_0$ $-s_0$	$-s_0$ $-s_0$	$-s_0$ $-s_0$

Şekil 3.3.3: Duruğumsu ve hızlı sönümlemeli kanallar için 2 durumlu 1/2 oranlı optimum MSK kodları



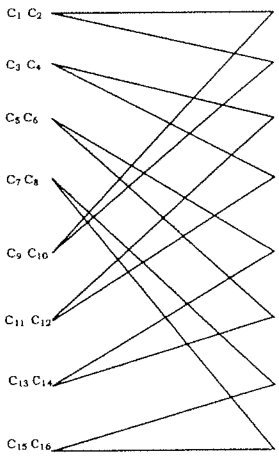
	Duruğumsu s.		Hızlı s.	
	$verici_1$	$verici_2$	$verici_1$	$verici_2$
c_1	s_0 s_0	s_1 $-s_1$	s_0 s_0	s_1 $-s_1$
c_2	s_1 $-s_0$	s_0 s_1	s_1 $-s_0$	s_0 s_1
c_3	$-s_0$ $-s_0$	$-s_1$ s_1	$-s_1$ s_1	$-s_1$ s_1
c_4	$-s_1$ s_0	$-s_0$ $-s_1$	$-s_0$ $-s_1$	$-s_0$ $-s_1$
c_5	$-s_0$ $-s_1$	$-s_1$ s_0	$-s_0$ $-s_1$	$-s_1$ s_0
c_6	$-s_1$ s_1	$-s_0$ $-s_0$	$-s_1$ s_1	$-s_0$ $-s_0$
c_7	s_0 s_1	s_1 $-s_0$	s_1 $-s_0$	s_1 $-s_0$
c_8	s_1 $-s_1$	s_0 s_0	s_0 s_0	s_0 s_0

Şekil 3.3.4: Duruğumsu ve hızlı sönümlemeli kanallar için 4 durumlu 1/2 oranlı optimum MSK kodları

3.3.4 Hata Başarımı

Bölüm 3.3.3' te tasarlanan uzay-zaman kodların hata başarımını incelemek amacıyla bilgisayar benzetimleri yapılmıştır. Bu benzetimler sırasında ele alınan sistemde bir çerçeve 100 kafes adımından oluşmaktadır. Alıcı tarafta sönümlemeli kanal durum bilgisinin ideal olarak kestirildiği varsayılmıştır. Duruğumsu kanallarda uzay-zaman kodlaması yararını vurgulamak amacıyla ilk olarak Şekil 3.3.3'teki kafese sahip

$$(c_1, c_2, c_3, c_4) = (s_0 s_0, s_0 s_1, -s_1 s_0, -s_0 - s_0)$$



	Duruğumsu s.		Hızlı s.	
	verici$_1$	verici$_2$	verici$_1$	verici$_2$
C_1	S_0 S_0	S_0 S_0	S_1 $-S_1$	S_0 S_0
C_2	S_0 S_1	S_1 $-S_0$	S_0 S_1	S_1 $-S_0$
C_3	$-S_0$ $-S_1$	$-S_1$ S_0	$-S_1$ S_0	$-S_1$ S_0
C_4	$-S_0$ $-S_0$	$-S_0$ $-S_0$	$-S_0$ $-S_0$	$-S_0$ $-S_0$
C_5	S_1 $-S_1$	S_0 S_0	S_0 S_0	S_0 S_0
C_6	S_1 $-S_0$	S_1 $-S_0$	S_1 $-S_0$	S_1 $-S_0$
C_7	$-S_1$ S_0	$-S_1$ S_0	$-S_0$ S_1	$-S_1$ S_0
C_8	$-S_1$ S_1	$-S_0$ $-S_0$	$-S_1$ S_1	$-S_0$ $-S_0$
C_9	S_1 $-S_1$	S_0 S_0	S_0 S_0	S_1 $-S_1$
C_{10}	S_1 $-S_0$	S_1 $-S_0$	S_1 $-S_0$	S_0 S_1
C_{11}	$-S_1$ S_0	$-S_1$ S_0	$-S_0$ $-S_1$	$-S_1$ S_0
C_{12}	$-S_1$ S_1	$-S_0$ $-S_0$	$-S_1$ S_1	$-S_1$ S_1
C_{13}	S_0 S_0	S_0 S_0	S_1 $-S_1$	S_0 S_0
C_{14}	S_0 S_1	S_1 $-S_0$	S_0 S_1	S_1 $-S_0$
C_{15}	$-S_0$ $-S_1$	$-S_1$ S_0	$-S_1$ S_0	$-S_0$ $-S_1$
C_{16}	$-S_0$ $-S_0$	$-S_0$ $-S_0$	$-S_0$ $-S_0$	$-S_0$ $-S_0$

Şekil 3.3.5: Duruğumsu ve hızlı sönümlemeli kanallar için 8 durumlu 1/2 oranlı optimuma yakın MSK kodları

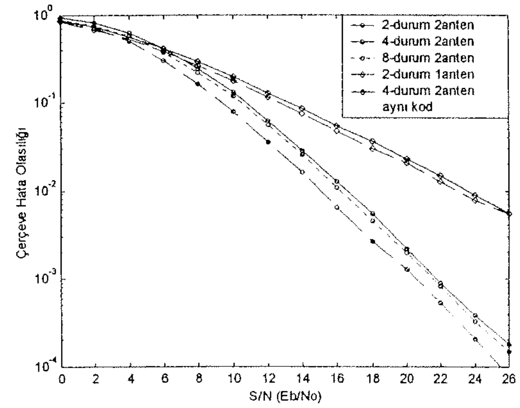
MSK kodunu kullanan tek verici-tek alıcılı sistem ele alınmıştır. Bu kodun kertesi 1, özdeğer çarpımı ise 12'dir. İkinci olarak da 4 durumlu kod için iki antenden aynı

$$(c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8) = (s_1 - s_1, s_0 s_1, -s_0 - s_0, -s_1 s_0, -s_1 s_0, -s_0 - s_0, s_0 s_1, s_1 - s_1)$$

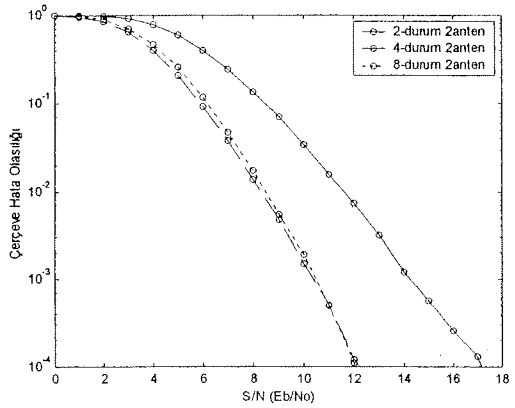
MSK kodunu kullanan sistem ele alınmıştır. Bu kodun da kertesi 1, özdeğer çarpımı 32' dir. Benzetim ile sözkonusu kodların değişik işaret-gürültü oranlarında Rayleigh dağılımlı sönümleli kanalda çerçeve hata olasılığı elde edilmiştir. Benzetim sonuçları karşılaştırmalı olarak Şekil 3.3.6 ve Şekil 3.3.7' de sırasıyla, duruğumsu ve hızlı sönümlemeli kanallar için benzetim sonuçları verilmiştir.

Şekil 3.3.6' daki benzetim sonuçları incelendiğinde, tek antenlik kod ile iki antenden aynı işareti gönderen kodun diğerlerine karşı oldukça kötü bir hata başarımına sahip olduğu kolayca görülebilir. Bunun nedeni, bu iki koda ilişkin kertenin 1 olmasıdır. Kertesi 2 olan diğer kodlar incelendiğinde ise aynı kerteye sahip bu kodların hata başarım eğrileri özdeğer çarpımları sırasındadır. En yüksek özdeğer çarpımına sahip 4-durumlu kodun hata başarımı beklendiği üzere en yüksektir.

Hızlı sönümlemeli kanallar için tasarlanan kodların bilgisayar benzetim sonuçları incelendiğinde ise, en önemli hata başarım ölçütü olan uzaklık ölçütünün hata başarımı üzerine etkisi açıkça görülmektedir. 10-4 hata olasılığına ulaşmak için 2-durumlu kod 17dB işaret-gürültü oranı isterken 4 ve 8 durumlu kodlar için 12dB' dir. Burada, 4 ve 8 durumlu



Şekil 3.3.6: Duruğumsu sönümlemeli kanal için benzetim sonuçları



Şekil 3.3.7: Hızlı sönümlemeli kanal için benzetim sonuçları

kodlar için yakın değerler alan uzaklık ölçütünün düşük işaret-gürültü oranlarında etkisini gösteremediği, yüksek işaret-gürültü oranlarına yaklaşıldıkça 8 durumlu kodun daha yüksek hata başarımına sahip olacağı söylenebilmektedir.

3.3.5 Sonuç

Bu çalışmada, yakın zamanda ortaya atılmış olan iletim çeşitlemesi temelli uzay-zaman kodlaması tekniği ilk kez MSK modülasyonuna uygulanmış ve gezgin kanallarda uzay-zaman kodlamasının getirdiği güç verimliliği ile MSK modülasyonunun getirdiği bandverimliliğinden birlikte yararlanılmıştır. Duruğumsu ve hızlı sönümlemeli kanallar için tasarım ölçütleri göz önüne alınarak optimum kodlar elde edilmiş ve bu kodların hata başarımları bilgisayar benzetimi yardımıyla incelenmiş, referans sistemlere üstünlükleri açıkça ortaya konmuştur.

3.4 Çok Alıcılı Antenli Dik Uzay-Zaman Kodları İçin Güç Kontrolu

3.4.1 Giriş

Sönümlemenin gezgin iletişim sistemleri üzerindeki etkisini azaltmanın en iyi yolu çeşitleme tekniklerinden yararlanmaktır. Uzay, zaman ve frekans çeşitlemesini de içeren çeşitleme tekniklerinin amacı, iletim ortamında bağımsız kanallar ortaya çıkararak aynı bilgiye ilişkin çeşitli işaretlerin alıcıya ulaşmasını sağlamaktır. Böylece, kullanılan bağımsız kanallardan biri üzerinden iletilen bilgi derin sönümleme etkisi sonucunda alıcıya çok zayıflamış olarak ulaşsa bile bir diğer kanaldan alıcıya ulaşabilecek daha az zayıflamış kopya alıcının hata başarımını arttıracaktır.

Verici anten çeşitlemesi son yıllarda giderek artan bir öneme sahip olmuş ve dikkatleri üzerine çekmiştir. Çok verici/çok alıcı antenli yapılarla kanal sığasının arttığının gösterilmesi ile başlayan bu süreç, Tarokh *et al.* [7] [8] in uzay–zaman kodlaması tekniğini ortaya atmaları ile literatürde önemli bir yere oturmuştur. Tarokh *et al.*, çalışmalarında birden çok verici/alıcı anten kullanılması durumunda elde edilecek iletişim sisteminin hata olasılığı üst sınırı ifadelerini elde ederek kod tasarım ölçütlerini vermişlerdir. Bu ölçütlere dayanılarak tasarlanan iletişim sistemleri tam çeşitleme kazancı ve yüksek kodlama kazancına sahip olabilmektedirler. Öte yandan, Alamouti [9], karmaşıklığı az olan dik verici çeşitlemesini (OTD) ortaya atmıştır. İki verici, M alıcı anten kullanan bu yapı tam çeşitleme kazancı sağlamaktadır.

3.4.2 Sistem Modeli

OTD tekniğinde kanal kazançlarının (sönümleme katsayılarının) arka arkaya iki simge aralığı boyunca değişmediği, herhangi iki iki simgelik aralıkta istatistiksel bağımsız olarak değiştiği varsayılmıştır. s_0, s_1 kanal simgesi çifti iki işaretleşme aralığı boyunca kanaldan iletilmektedir. İlk zaman diliminde s_0 simgesi ilk verici anten yardımıyla kanala iletilirken, s_1 simgesi ikinci antenden iletilir. İkinci zaman diliminde ise birinci ve ikinci antenlerden, sırasıyla, $-s_1^*$ ve s_0^* simgeleri iletilir. Bu durumda 0. ve 1. zaman dilimlerinde alıcı antene ulaşan işaretler, sırasıyla, r_0 ve r_1 ile gösterilirse

$$\begin{aligned} r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= -h_0 s_1^* + h_1 s_0^* + n_1 \end{aligned} \qquad (3.4.1)$$

yazılır. Burada, $h_0 = |h_0|e^{j\theta_0}$ ve $h_1 = |h_1|e^{j\theta_1}$, sırasıyla, birinci ve ikinci verici antenler ile alıcı anten arasındaki kanal kazançlarını gösterir. Rayleigh dağılımlı zarfa sahip olan bu sönümleme katsayıları sıfır–ortalamalı, boyut başına 0.5 varyanslı karmaşık Gauss dağılımı ile modellenebilir. Kanaldaki toplamsal beyaz Gauss gürültüsü ise her bir kanal ve ardışıl simge aralıkları için istatistiksel bağımsız, sıfır ortalamalı, boyut başına $N_0/2$ varyanslı karmaşık Gauss dağılımlı n_0 ve n_1 rastlantı değişkenleridir.

Kanal kazançlarının alıcıda ideal olarak kestirilebildiği varsayımı altında, r_0 ve r_1 işaretleri

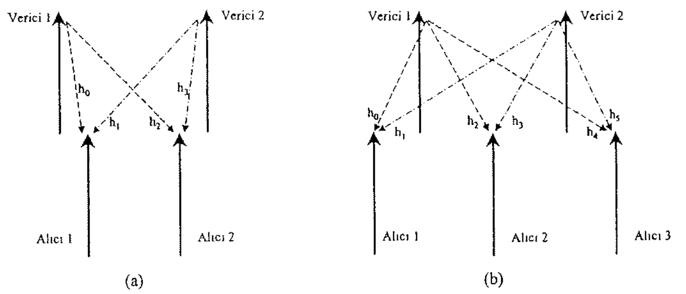
$$\begin{aligned}\tilde{s_0} &= h_0^* r_0 + h_1 r_1^* \\ &= (|h_0|^2 + |h_1|^2)s_0 + h_0^* n_0 + h_1 n_1^* \\ \tilde{s_1} &= h_1^* r_0 - h_0 r_1^* \\ &= (|h_0|^2 + |h_1|^2)s_1 - h_0 n_1^* + h_1^* n_0 \end{aligned} \quad (3.4.2)$$

işlemleri yardımıyla birleştirilerek iletilmiş olan s_0 ve s_1 işaretlerine ilişkin kestirim değerleri elde edilir. (3.4.2) denkleminden görülebileceği gibi iki kanaldan biri derin sönümleme etkisi altında kalsa bile diğer kanaldan alınan işaret yardımıyla iletilen işaret kestirilebilir. Ancak, gerçekte durum tam olarak böyle olmayabilir. Kanal kazançlarının alıcı tarafta kestirimi hatalı yapılırsa yukarıda anlatılan çeşitleme tekniği ideal durumdaki sonuçları vermemektedir. Kestirim hataları da göz önüne alındığında alıcıda kestirilen kazançlar $\hat{h_0} = h_0 + \epsilon_0$, $\hat{h_1} = h_1 + \epsilon_1$ ifadeleri ile verilmektedir. Burada, kestirim hatasını gösteren ϵ_0 ve ϵ_1 rastlantı değişkenleri sıfır–ortalamalı, boyut başına σ_h^2 varyanslı karmaşık Gauss dağılımlıdır. Kestirim işleminin başarımı, işaret/kanal sönümleme katsayısı kestirim hatası oranı SECR (σ_s^2/σ_h^2) ile belirlenebilir. Burada, σ_s^2 değeri ortalama işaret gücünü göstermektedir. Bu durumda, alıcı hatalı kestirilmiş kanal kazançlarını kullanarak

$$\begin{aligned}\tilde{s_0} &= (|h_0|^2 + |h_1|^2 + h_0\epsilon_0^* + h_1^*\epsilon_1)s_0 + (h_1\epsilon_0^* - h_0^*\epsilon_1)s_1 + (h_0^* + \epsilon_0^*)n_0 + (h_1 + \epsilon_1)n_1^* \\ \tilde{s_1} &= (|h_0|^2 + |h_1|^2 + h_1\epsilon_1^* + h_0^*\epsilon_0)s_1 + (h_0\epsilon_1^* - h_1^*\epsilon_0)s_0 + (h_1^* + \epsilon_1^*)n_0 - (h_0 + \epsilon_0)n_1^* \end{aligned} \quad (3.4.3)$$

çıkışlarını elde edecektir. (3.4.3) denkleminde görüldüğü gibi artık çıkışta yalnız gürültüye bağlı terimler değil, kestirim hatası nedeniyle ortaya çıkan simgelerarası girişim terimleri de bulunmaktadır. Buna göre, sistemin hata başarımı artık yalnız işaret–gürültü oranına (SNR) değil, işaret–girişim oranına (SIR) da bağlıdır.

Bu çalışmada, Fan *et al.* [10] tarafından iki verici, tek alıcı antenli dik verici çeşitlemeli sistemler için ortaya atılan güç kontrol tekniği geliştirilerek birden fazla alıcı anten kullanan iletişim sistemlerine uygunlaştırılmıştır. Bu sistemlere ilişkin kestirilen simge değerleri $\tilde{s_0}$ ve $\tilde{s_1}$'in ifadeleri elde edilerek çeşitli kestirim hatası değerleri için hata başarımını belirlemek amacıyla bilgisayar benzetimleri yapılmıştır. Tasarlanan iki ve üç alıcı antenli yapılar Şekil.3.4.1'de verilmiştir.



Şekil 3.4.1: (a)İki alıcı antenli dik verici çeşitlemesi (b) Üç alıcı antenli dik verici çeşitlemesi

İki verici anten ile iki alıcı anten arasında ortaya çıkan dört bağımsız kanalın kazançları $h_0 = |h_0|e^{j\theta_0}$, $h_1 = |h_1|e^{j\theta_1}$, $h_2 = |h_2|e^{j\theta_2}$ ve $h_3 = |h_3|e^{j\theta_3}$ ile gösterilmiştir (Şekil 3.4.1a). İlk verici antenden iletilen işaretler a ile, ikinci antenden iletilenler ise b ile ağırlaştırıldığında, ilk alıcı antenin birinci ve ikinci zaman dilimlerinde aldığı işaretler, sırasıyla,

$$\begin{aligned} r_0 &= ah_0 s_0 + bh_1 s_1 + n_0 \\ r_1 &= -ah_0 s_1^* + bh_1 s_0^* + n_1 \end{aligned} \tag{3.4.4}$$

ile verilebiliyorken, ikinci alıcı antene ulaşan işaretler, sırasıyla,

$$\begin{aligned} r_2 &= ah_2 s_0 + bh_3 s_1 + n_2 \\ r_3 &= -ah_2 s_1^* + bh_3 s_0^* + n_3 \end{aligned} \tag{3.4.5}$$

olarak ifade edilebilir. Burada a ve b güç kontrolu için ağırlaştırma katsayıları olup $a^2 + b^2 =$ 1'dir. Alıcı tarafta kullanılan birleştirme işlemi ve ağırlaştırma katsayılarının seçimi tek alıcı antenli yapıya göre oldukça farklıdır. Alıcıda birleştirme işlemi her alıcı anten için değişik katsayılar kullanabilmektedir. Buna göre, alıcıda $\tilde{s}_0$ ve $\tilde{s}_1$ kestirim değerlerinin elde edilmesi için

$$\begin{aligned} \tilde{s}_0 &= c\hat{h}_0^* r_0 + d\hat{h}_1 r_1^* + e\hat{h}_2^* r_2 + f\hat{h}_3 r_3^* \\ \tilde{s}_1 &= d\hat{h}_1^* r_0 - c\hat{h}_0 r_1^* + f\hat{h}_3^* r_2 - e\hat{h}_2 r_3^* \end{aligned} \tag{3.4.6}$$

denklemlerinden yararlanılmaktadır. Birleştirme işlemi sonucunda elde edilen $\tilde{s}_0$ ve $\tilde{s}_1$ kestirim değerleri

$$\begin{aligned} \tilde{s}_0 = {} & (ac|h_0|^2 + bd|h_1|^2 + ae|h_2|^2 + bf|h_3|^2 + ac\epsilon_0^* h_0 + bd\epsilon_1 h_1^* + ae\epsilon_2^* h_2 + bf\epsilon_3 h_3^*) s_0 \\ & + (bch_0^* h_1 - adh_0^* h_1 + beh_2^* h_3 - afh_2^* h_3 + bc\epsilon_0^* h_1 - ad\epsilon_1 h_0^* + be\epsilon_2^* h_3 - af\epsilon_3 h_2^*) s_1 \end{aligned}$$

$$
\begin{aligned}
& +c(h_0^* + \epsilon_0^*)n_0 + d(h_1 + \epsilon_1)n_1^* + e(h_2^* + \epsilon_2^*)n_2 + f(h_3 + \epsilon_3)n_3^* \\
\tilde{s}_1 &= (ac|h_0|^2 + bd|h_1|^2 + ae|h_2|^2 + bf|h_3|^2 + ac\epsilon_0 h_0^* + bd\epsilon_1^* h_1 + ae\epsilon_2 h_2^* + bf\epsilon_3^* h_3)s_1 \\
& +(adh_0 h_1^* - bch_0 h_1^* + afh_2 h_3^* - beh_2 h_3^* + ad\epsilon_1^* h_0 - bc\epsilon_0 h_1^* + af\epsilon_3^* h_2 - be\epsilon_2 h_3^*)s_0 \\
& +d(h_1^* + \epsilon_1^*)n_0 - c(h_0 + \epsilon_0)n_1^* + f(h_3^* + \epsilon_3^*)n_2 - e(h_2 + \epsilon_2)n_3^* \qquad (3.4.7)
\end{aligned}
$$

biçimindedir. Alıcıda kullanılan anten sayısındaki artışla birlikte, kestirilen $\tilde{s}_0$ ve $\tilde{s}_1$ işaretlerinin içerisindeki rastgele girişim teriminin öneminin de arttığı görülmektedir. Güç kontrol yapısı tasarımındaki ana amaç, başarımı gürültüden daha fazla bozan, girişim terimlerini olabildiğince zayıflatmaktır. İki alıcı antenli yapıda kullanılan kanal sayısı dört olduğundan, geribesleme ve güç kontrolu işlemleri tek alıcılı yapıda olduğu kadar kolay gerçekleştirilemez. Tek alıcı antenli yapıda alıcının vereceği karar iki durumdan biri ($|h_0| > |h_1|$), ($|h_0| < |h_1|$) iken iki alıcı anten durumunda verilebilecek karar dört farklı biçimdedir: ($|h_0| > |h_1|$ VE $|h_2| > |h_3|$), ($|h_0| > |h_1|$ VE $|h_2| < |h_3|$), ($|h_0| < |h_1|$ VE $|h_2| > |h_3|$), ($|h_0| < |h_1|$ VE $|h_2| < |h_3|$). Bu durumda her bir simge çifti iletimi için geri besleme kanalından iki kontrol bitinin gönderilmesi ve o duruma ilişkin kontrol katsayılarının (a, b, c, d, e, f) uygun seçilmesi gereklidir. Bu seçim sırasında göz önünde bulundurulması gereken iki ölçüt bulunmaktadır. Bunlardan birincisi, çıkıştaki işaret–gürültü oranını mümkün olduğu kadar arttırmak (kuvvetli kanaldan yüksek güçlü işaret iletimi) iken, ikinci ölçüt, işaret–girişim oranını arttırmak olmalıdır. Buna göre, ilk verici anten ile alıcı antenler arasındaki kanalların iyi olduğu ($|h_0| > |h_1|$ VE $|h_2| > |h_3|$) durumda $a = c = e = 1.0$, $b = d = f = 0.0$ seçilerek tüm iletim gücü ilk verici antene yoğunlaştırılmıştır. Benzer bir durum olan ikinci verici anten ile alıcı antenler arasındaki kanalların iyi olması durumunda ($|h_0| < |h_1|$ VE $|h_2| < |h_3|$) ise $a = c = e = 0.0$ ve $b = d = f = 1.0$ seçilerek tüm iletim gücü ikinci verici antene yoğunlaştırılmıştır. Bu katsayıların kullanılması sonucunda kestirim değerleri $\tilde{s}_0$ ve $\tilde{s}_1$ içerisindeki tüm girişim terimlerinin bastırıldığı ve sistemin seçmeli çeşitlemeye denk olduğu görülebilir. Alıcı antenlerin, hangi verici antene ilişkin kanalın daha kuvvetli olduğunu ayırt edemedikleri diğer durumlarda ise hatalı güç kontrolu uygulamasına neden olup işaret–gürültü oranını düşürmemek için $a = b = c = d = e = f = \sqrt{0.5}$ seçilmiş ve güç kontrolu yapılmamıştır.

Üç alıcı antenli yapı için kanal kazançları h_0,h_1,h_2,h_3,h_4 ve h_5 ile gösterilmektedir (Şekil 3.4.1b). Verici tarafta iletim gücünü ağırlaştırmak için a ve b katsayıları kullanıldığında birinci, ikinci ve üçüncü alıcı antene, ilk ve ikinci zaman aralıklarında ulaşan işaretler, sırasıyla,

$$
\begin{aligned}
r_0 &= ah_0 s_0 + bh_1 s_1 + n_0 \\
r_1 &= -ah_0 s_1^* + bh_1 s_0^* + n_1 \\
r_2 &= ah_2 s_0 + bh_3 s_1 + n_2 \qquad (3.4.8)
\end{aligned}
$$

$$\begin{aligned} r_3 &= -ah_2s_1^* + bh_3s_0^* + n_3 \\ r_4 &= ah_4s_0 + bh_5s_1 + n_4 \qquad (3.4.9) \\ r_5 &= -ah_4s_1^* + bh_5s_0^* + n_5 \qquad (3.4.10) \end{aligned}$$

ile verilebilir.

Alıcıda güç kontrol katsayısı çiftleri (c,d), (e,f) ve (g,h), sırasıyla, birinci, ikinci ve üçüncü antene ulaşan işaretleri ağırlaştırmak için kullanılır. s_0 ve s_1 işaretlerini kestirmek amacıyla üç alıcı antende iki zaman aralığında alınan işaretler

$$\begin{aligned} \tilde{s}_0 &= c\tilde{h}_0^*r_0 + d\tilde{h}_1r_1^* + e\tilde{h}_2^*r_2 + f\tilde{h}_3r_3^* + g\tilde{h}_4^*r_4 + h\tilde{h}_5r_5^* \\ \tilde{s}_1 &= d\tilde{h}_1^*r_0 - c\tilde{h}_0r_1^* + f\tilde{h}_3^*r_2 - e\tilde{h}_2r_3^* + h\tilde{h}_5^*r_4 - g\tilde{h}_4r_5^* \qquad (3.4.11) \end{aligned}$$

biçiminde birleştirilir. (3.4.8) ile verilen ifadeler (3.4.11) denkleminde yerlerine konarak kestirim değerleri

$$\begin{aligned} \tilde{s}_0 = &\ (ac|h_0|^2 + bd|h_1|^2 + ae|h_2|^2 + bf|h_3|^2 + ag|h_4|^2 + bh|h_5|^2 + ac\epsilon_0^*h_0 + bd\epsilon_1h_1^* + ae\epsilon_2^*h_2 \\ &+bf\epsilon_3h_3^* + ag\epsilon_4^*h_4 + bh\epsilon_5h_5^*)s_0 + (bch_0^*h_1 - adh_0^*h_1 + beh_2^*h_3 - afh_2^*h_3 + bgh_4^*h_5 - ahh_4^*h_5 \\ &+bc\epsilon_0^*h_1 - ad\epsilon_1h_0^* + be\epsilon_2^*h_3 - af\epsilon_3h_2^* + bg\epsilon_4^*h_5 - ah\epsilon_5h_4^*)s_1 + c(h_0^* + \epsilon_0^*)n_0 + d(h_1 + \epsilon_1)n_1^* \\ &+f(h_3 + \epsilon_3)n_3^* + g(h_4^* + \epsilon_4^*)n_4 + h(h_5 + \epsilon_5)n_5^* \qquad (3.4.12) \end{aligned}$$

$$\begin{aligned} \tilde{s}_1 = &\ (ac|h_0|^2 + bd|h_1|^2 + ae|h_2|^2 + bf|h_3|^2 + ag|h_4|^2 + bh|h_5|^2 + ac\epsilon_0h_0^* + bd\epsilon_1^*h_1 + ae\epsilon_2h_2^* \\ &+(adh_0h_1^* - bch_0h_1^* + afh_2h_3^* - beh_2h_3^* + bf\epsilon_3^*h_3 + ag\epsilon_5h_4^* + bh\epsilon_5^*h_5)s_1 + ahh_4h_5^*bgh_4h_5^* \\ &+ad\epsilon_1^*h_0 - bc\epsilon_0h_1^* + af\epsilon_3^*h_2 - be\epsilon_2h_3^* + ah\epsilon_5^*h_4 - bg\epsilon_4h_5^*)s_0 + d(h_1^* + \epsilon_1^*)n_0 - c(h_0 + \epsilon_0)n_1^* \\ &+f(h_3^* + \epsilon_3^*)n_2 - e(h_2 + \epsilon_2)n_3^* + h(h_5^* + \epsilon_5^*)n_4 - g(h_4 + \epsilon_4)n_5^* \qquad (3.4.13) \end{aligned}$$

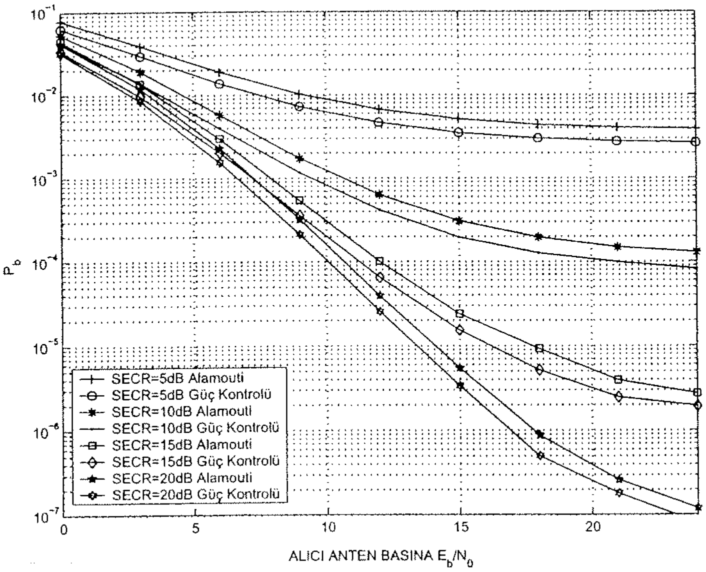
şeklinde belirlenir. İki alıcı antenli yapıda olduğu gibi bu durumda da dört farklı güç kontrol durumu bulunmaktadır. İlk verici anten ile alıcı antenler arasındaki kanallara ilişkin kanal kazançları her alıcı anten için ikinci verici antene göre daha yüksek ise ($|h_0| > |h_1|$ VE $|h_2| > |h_3|$ VE $|h_4| > |h_5|$) güç kontrol katsayıları $a = c = e = g = 1.0$ ve $b = d = f = h = 0.0$ seçilerek tüm iletim gücü ilk verici antene yoğunlaştırılır. Eşdeğer olarak, ikinci verici antene ilişkin kazançların daha yüksek olduğu durumda ($|h_0| < |h_1|$ VE $|h_2| < |h_3|$ VE $|h_4| < |h_5|$) tüm iletim gücü ikinci verici antene yoğunlaştırılacaktır ($a = c = e = g = 0.0$, $b = d = f = h = 1.0$). Tüm diğer olası durumlarda verici antenlerden biri üzerine hatalı güç yoğunlaştırmasını engellemek amacıyla $a = b = c = d = e = f = g = h = \sqrt{0.5}$ seçilerek verici antenlerin eş güçte iletim yapması sağlanacaktır.

3.4.3 Benzetim Sonuçları

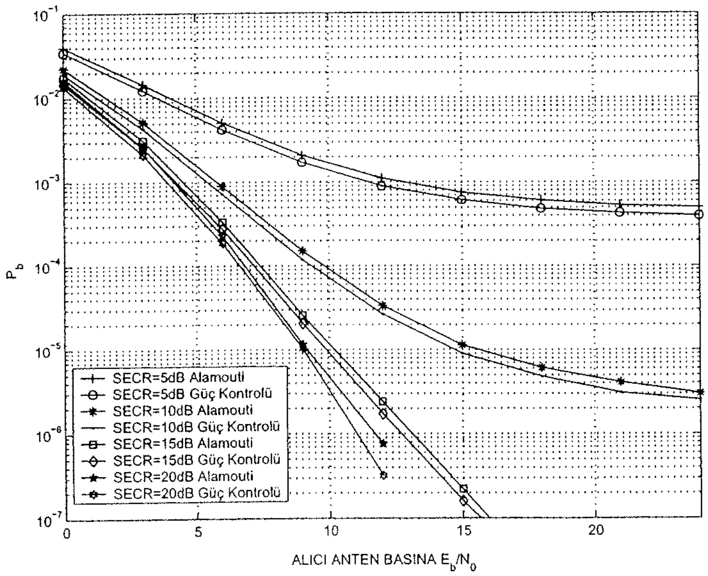
İki ve üç alıcı antenli dik uzay–zaman kodları için önerilen güç kontrol yapılarına ilişkin hata başarımları bilgisayar benzetimleri yardımıyla elde edilmiştir. Değişik SECR değerleri için elde edilen sonuçlar güç kontrolu uygulanmamış durum ile karşılaştırmalı olarak Şekil 3.4.2 ve Şekil 3.4.3'te sunulmuştur. İki alıcı anten kullanan yapıya ilişkin hata başarım eğrilerine göre, $10dB$ SECR değeri için, güç kontrolu uygulanmış yapı 10^{-3} bit hata olasılığına ulaşmak için kontrol uygulanmamış yapıya göre $2dB$ daha az işaret–gürültü oranına gereksinim duymakta iken, 10^{-4} bit hata olasılığı için bu fark $9dB$'i geçmektedir. Üç alıcı anten kullanan yapının hata başarımı incelendiğinde güç kontrol yapısının getirdiği kazancın iki alıcı antenli duruma göre azaldığı görülmektedir. Bunun nedeni, her üç alıcı antenin de aynı verici antene ilişkin kanal kazançlarını daha yüksek görmesi olasılığının artan alıcı anten sayısı ile birlikte azalmasıdır. Sadece bu durumlarda güç yoğunlaştırması yapılarak başarım iyileştirildiği için bu durumun ortaya çıkış olasılığının azalmasıyla birlikte kazancın da düşeceği kestirilebilmektedir.

3.4.4 Sonuçlar

Bu çalışmada, birden fazla alıcı anten kullanan dik uzay–zaman kodlamalı sistemlerin sönümlemeli kanallardaki hata başarımını yükseltmek için bir güç kontrol yapısı önerilmiştir. Kanal kazançlarının alıcıda ideal olarak kestirilememesi durumunda, önerilen yapıların hata başarımları bilgisayar benzetimleri yardımıyla elde edilmiş, önerilen güç kontrol yapısının dik uzay–kodlamalı sistemlerin hata başarımlarını yükselttiği görülmüştür.



Şekil 3.4.2: İki alıcı antenli yapının bit hata olasılığı



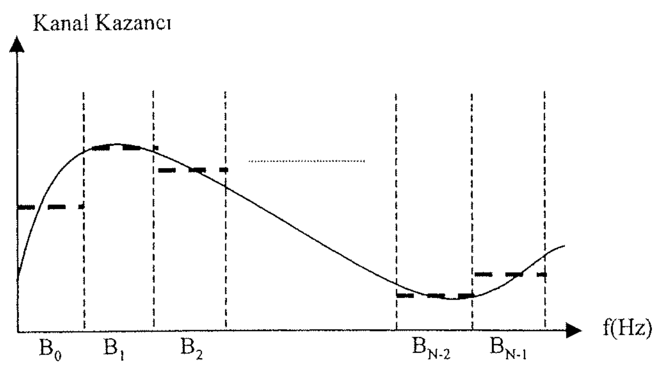
Şekil 3.4.3: Üç alıcı antenli yapının bit hata olasılığı

3.5 İlintili Sönümlemeli Kanallarda Dik Uzay-Zaman Kodlamalı OFDM

3.5.1 Giriş

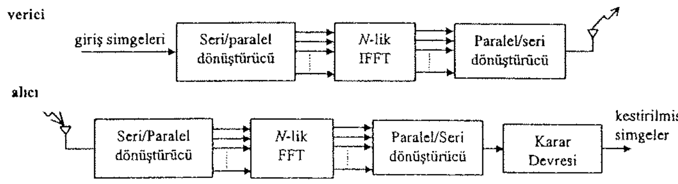
Genişbandlı iletişim sistemlerinde, özellikle frekans seçici kanallar üzerinden iletimde yüksek başarımlarından dolayı, çok–taşıyıcılı sistemler günümüzde yaygın olarak kullanılmaktadır. Bu sistemlerin OFDM olarak adlandırılan biçimi, genişbandlı iletişim kanalını belli sayıda alt kanallara bölerek bilgiyi birbirine dik seçilmiş alt taşıyıcı frekanslarda iletme ilkesine dayanmaktadır. Geleneksel frekans bölmeli çoğullamalı sistemler ile karşılaştırıldığında, dik alt kanalların örtüşmesine izin verildiğinden band verimliliği açısından bir üstünlük sağlanmaktadır. Öte yandan, OFDM tek taşıyıcılı iletişim sistemlerine göre oldukça uzun bir işaretleşme peryoduna sahip olduğundan hızlı sönümlemelere karşı daha iyi başarıma sahiptir.

Genişbandlı iletişimde karşılaşılabilecek en kötü durumlardan biri ise kanalda sönümlemenin iletilen işaretin her frekans bileşenini aynı miktarda etkilememesidir (frekans–seçicilik). Bu özelliğe sahip bir kanalda OFDM tekniği kanalı N dik alt kanala $\{B_0, B_1, ..., B_{N-1}\}$ bölerek frekans seçici kanalı N adet frekans seçici olmayan alt kanala dönüştürür (Şekil 3.5.1).



Şekil 3.5.1: Frekans seçici kanal

Böylece, iletişim tek bir frekans seçici kanal yerine N adet frekans seçici olmayan kanal üzerinden eşzamanlı olarak sağlanır. Dik frekans bölmeli çoğullama tekniğinin en belirgin özelliği olan dik taşıyıcıları gerçekleştirmek amacıyla, uygulaması kolay ve maliyeti düşük olan hızlı Fourier (FFT) ve ters hızlı Fourier dönüştürücülerinden (IFFT) yararlanılmaktadır (Şekil.3.5.2).



Şekil 3.5.2: Dik Frekans Bölmeli Çoğullama

Bilgiye ilişkin simge dizisini alt taşıyıcılara ötelemek amacı ile N noktalık IFFT bloğu kullanılır. IFFT ve FFT bloklarının hızlı ve etkin kullanılması için alt kanal sayısı N'nin ikinin bir kuvveti olarak seçilmesi gerekmektedir. IFFT işlemine uygun biçime getirilmek amacıyla girişteki simge dizisi, 1-giriş N-çıkışlık bir seri/paralel dönüştürücüden geçirilerek veri simgeleri N uzunluklu çerçeveler haline getirilir. IFFT bloğu çıkışında elde edilen simgeler tekrar seri biçime sokularak verici anten üzerinden kanala verilirler. Alıcı tarafta ise peşpeşe alınan her N kanal simgesi paralele dönüştürülüp FFT işlemi gerçekleştirilir. FFT işlemi sonrasında elde edilen, kanaldaki sönümleme ve gürültü etkileri tarafından bozulmuş simgeler bir karar devresi girişine uygulanarak iletilmiş olan bilgi simgelerinin kestirilmesine çalışılır.

Bu çalışmada, OFDM kullanan iletişim sistemlerinde uzay, zaman ve frekans çeşitleme teknikleri birlikte uygulanarak kanaldaki ilintili sönümleme ve toplamsal beyaz Gauss gürültüsüne karşın yüksek hata başarımına sahip bir tümleşik iletişim sistemi önerilmektedir.

3.5.2 Kanal Modeli

OFDM tekniği kullanılarak frekans–seçici sönümleme etkisinden kurtarılan iletişim sistemi ortaya çıkan frekans seçici olmayan ancak büyük olasılıkla ilintili sönümlemeye sahip alt kanallar üzerinden bilgiyi iletecektir. Kullanılan iletişim kanalının üstel azalan güç gecikme profiline sahip olduğu varsayılmıştır, [11]. τ_l, çok–yollu iletişim kanalındaki l. yolun gecikmesi, τ_{max}, çok–yollu yapının sahip olduğu en yüksek gecikme miktarı ve C bir sabit olmak üzere böyle bir kanalın güç gecikme profili

$$\theta(\tau_l) = C\exp(-\tau_l/\tau_{max}) \tag{3.5.1}$$

ifadesi ile verilir. Bu güç gecikme profiline sahip kanala ait N alt kanalın etkilendikleri sönümleme katsayılarının ilintisini gösteren normalize ilinti matrisi

$$r(k,k') = \frac{1 - \exp[-L(1/\tau_{rms} + j2\pi(k-k')/N)]}{\tau_{rms}(1 - \exp(-L/\tau_{rms}))(1/\tau_{rms} + j2\pi(k-k')/N)} \quad (3.5.2)$$

ifadesi yardımıyla hesaplanabilir. $r(k,k')$ terimi normalize ilinti matrisi R'nin k. satır ($0 \leq k \leq N-1$), k'. sütun ($0 \leq k' \leq N-1$) elemanını gösterir ve k. ve k'. alt kanalların sahip olduğu sönümleme katsayıları arasındaki ilinti katsayısını verir. (3.5.2) ifadesinde kullanılan τ_{rms}, yol gecikme sürelerinin standart sapması iken, L, OFDM için kullanılacak çevrimsel önek uzunluğunu göstermektedir.

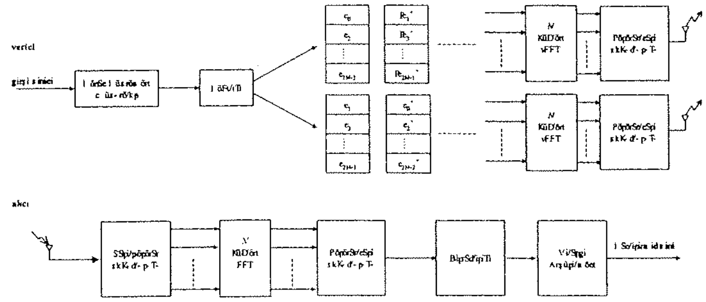
Bu çalışmada kanalın sönümleme etkisinin iki çerçeve süresi boyunca değişmediği, her iki iki çerçevelik zaman aralığı arasında ise istatistiksel bağımsız değiştiği varsayılmıştır.

3.5.3 Sistem Modeli

OFDM kullanımıyla oluşan alt kanallarda başarımını iyileştirmek için çeşitleme tekniklerinden yararlanılır. Uzay, zaman ve frekans çeşitlemesini de içeren çeşitleme tekniklerinin amacı, iletim ortamında bağımsız kanallar ortaya çıkararak aynı bilgiye ilişkin çeşitli işaretlerin alıcıya ulaşmasını sağlamaktır. Böylece, kullanılan bağımsız kanallardan biri üzerinden iletilen işaret derin sönümleme etkisi sonucunda alıcıya çok zayıflamış olarak ulaşsa bile bir diğer kanaldan alıcıya ulaşabilecek daha az zayıflamış bir kopya alıcının hata başarımını arttıracaktır.

Bu çalışmada uzay çeşitlemesi sağlamak amacıyla son zamanlarda uygulamaları sıklıkla karşımıza çıkan dik uzay–zaman kodları kullanılmıştır. Alamouti[9] tarafından ortaya atılan bu yapı iki verici, M alıcı anten kullanarak tam çeşitleme kazancı sağlamaktadır. İlintili sönümlemeye sahip alt kanallar üzerinde çeşitleme sağlamak amacıyla ise sistemin girişine kafes kodlamalı modülatör eklenmiştir. Böylece, kafes kodlamalı modülasyonun içinde barındırdığı zaman çeşitlemesinden de yararlanılmaktadır. Ancak, kullanılan IFFT işlemi nedeniyle modülatör çıkışındaki simgeler frekans bölgesindeki alt–kanallar üzerinden iletildiği için kafes kodlamalı modülasyonun sahip olduğu bu özellik tasarlanan iletişim sisteminde frekans boyutunda çeşitleme sağlayarak ilintili sönümlemeye karşı hata başarımını iyileştirir.

Önerilen tümleşik yapıda (şekil 3.5.3), her $2N$ adet ikili simgeden oluşan bilgi dizisi kafes kodlamalı modülatör ile kodlanarak $\{s_0, s_1, ..., s_{2N-2}, s_{2N-1}\}$ kodlanmış simge dizisini oluşturmaktadır. Üretilen $2N$ adet kodlanmış kanal simgesi iki OFDM çerçevesi süresince kanaldan iletilecektir. Bunun için, dağıtıcı, girişine gelen kanal simgelerini iki verici antene ait OFDM çerçevelerine dağıtmaktadır. $S_0 = diag\{s_0, s_2, \cdots, s_{2N-2}\}$ ve $S_1 = diag\{s_1, s_3, \cdots, s_{2N-1}\}$



Şekil 3.5.3: Tümleşik iletişim sistemi

köşegen matrislerinin köşegen elemanları birinci çerçeve süresinde, sırasıyla, birinci ve ikinci antenden iletilecek OFDM çerçevelerine yerleştirilir. İkinci çerçeve süresince ise $-S_1^\dagger$ ve $S_0^\dagger$ matrislerinin köşegen elemanları, sırasıyla, ilk ve ikinci antenlerden iletilecek OFDM çerçevesini oluştururlar. Burada † matrisin devrik eşleniğini göstermektedir. Her iki antene ait OFDM bloğunda üretilen çerçevelerin N noktalı IFFT'si alınıp kanala ilişkin verici anten üzerinden seri biçimde iletilmektedir. Kanal sönümleme katsayılarının iki çerçeve süresi boyunca sabit kaldığı varsayımı kullanılırsa, alıcıda, N noktalı FFT'si alınmış işaret

$$\mathbf{r} = S\mathbf{h} + \mathbf{n} \tag{3.5.3}$$

olarak elde edilir. Burada iletim matrisi S

$$S = \begin{bmatrix} S_0 & S_1 \\ -S_1^\dagger & S_0^\dagger \end{bmatrix}$$

biçimindedir. $\mathbf{r_0} = [r_0 r_2 \cdots r_{2N-2}]^T$ ilk çerçeve süresinde alınan işaretin, $\mathbf{r_1} = [r_1 r_3 \cdots r_{2N-1}]^T$ ise ikinci çerçeve süresince alınan işaretin FFT çıkışını göstermek üzere $\mathbf{r}$ vektörü $\mathbf{r} = [\mathbf{r_0}^T \mathbf{r_1}^T]^T$ ile gösterilebilir. Burada ^{T} matris devriğini göstermektedir. Sönümleme vektörü $\mathbf{h}$ ise birinci ve ikinci kanallara ilişkin sönümleme vektörleri cinsinden $\mathbf{h} = [\mathbf{h_0}^T \mathbf{h_1}^T]^T$ olarak ifade edilebilir. $\mathbf{h_0} = [h_{0,0} h_{0,1} \cdots h_{0,N-1}]^T$ ve $\mathbf{h_1} = [h_{1,0} h_{1,1} \cdots h_{1,N-1}]^T$ kanal sönümleme vektörlerini göstermektedir. Bu vektörlere ilişkin $h_{i,j}$ elemanı i. verici anten ile alıcı anten arasındaki j. alt-kanalın iki çerçeve boyunca sabit kalan sönümleme katsayısını göstermektedir. $h_{i,j}$ değerleri değişik i değerleri için istatistiksel bağımsız iken j değerleri için (3.5.2) ile verilen ilintiye sahiptir. İstatistiksel özellikleri ise sıfır ortalamalı, boyut başına 0.5 varyanslı karmaşık Gauss dağılımı ile modellenebilmektedir. Alıcıya etkiyen gürültü bileşenleri $\mathbf{n} = [\mathbf{n_0}^T \mathbf{n_1}^T]^T$ vektörü ile gösterilmiştir. $\mathbf{n_0} = [n_0 n_2 \cdots n_{2N-2}]^T$ ilk çerçeve süresince FFT çıkışında görülen gürültü örneklerini gösterirken $\mathbf{n}_1 = [n_1 n_3 \cdots n_{2N-1}]^T$ ikinci çerçeve süresince çıkışta görülecek gürültü örneklerini göstermektedir. Gürültü bileşeni n_i'ler her i değeri için istatistiksel bağımsız, sıfır ortalamalı, boyut başına $N_0/2$ varyanslı karmaşık Gauss dağılımı ile modellenmiştir. Birleştirici bloğu, dik uzay–zaman kodlarının çözülmesi için gerekli birleştirme işlemini $k = 0, 1, \cdots, N-1$ için

$$\begin{aligned} \tilde{s}_{2k} &= h_{0,k}^* r_{2k} + h_{1,k} r_{2k+1}^* && (3.5.4) \\ \tilde{s}_{2k+1} &= h_{1,k}^* r_{2k} - h_{0,k} r_{2k+1}^* && (3.5.5) \end{aligned}$$

ifadeleri yardımıyla yapıp $\{\tilde{s}_0, \tilde{s}_1, ..., \tilde{s}_{2N-1}\}$ kestirim değerlerini oluşturarak kod çözme işleminin gerçekleştirilmesi için Viterbi algoritması bloğuna iletir. Viterbi algoritması kafes üzerinde $M(\mathrm{s}, \bar{\mathrm{s}})$ optimum metriğini minimize eden $\bar{\mathrm{s}}$ dizisini $\hat{\mathrm{s}}$ olarak seçer:

$$\begin{aligned} \hat{\mathrm{s}} &= \arg \min_{\bar{\mathrm{s}}} M(\mathrm{s}, \bar{\mathrm{s}}) \\ &= \arg \min_{\bar{\mathrm{s}}} \sum_{k=0}^{N-1} m(s_{2k}, \bar{s}_{2k}) + m(s_{2k+1}, \bar{s}_{2k+1}). && (3.5.6) \end{aligned}$$

Viterbi algoritmasının kullandığı dal metrikleri $k = 0, 1, \cdots, N-1$ için

$$\begin{aligned} m(s_{2k}, \bar{s}_{2k}) &= (|h_{0,k}|^2 + |h_{1,k}|^2 - 1) |\bar{s}_{2k}|^2 + d^2(\tilde{s}_{2k}, \bar{s}_{2k}) \\ m(s_{2k+1}, \bar{s}_{2k+1}) &= (|h_{0,k}|^2 + |h_{1,k}|^2 - 1) |\bar{s}_{2k+1}|^2 + d^2(\tilde{s}_{2k+1}, \bar{s}_{2k+1}) && (3.5.7) \end{aligned}$$

olarak verilebilir. Burada, $d^2(.,.)$ operatörü $d^2(x, y) = (x - y)(x - y)^*$ ile tanımlıdır. İşaret enerjilerinin eşit olduğu kümelerin (M-PSK gibi) kullanılması durumunda kullanılan metrikler

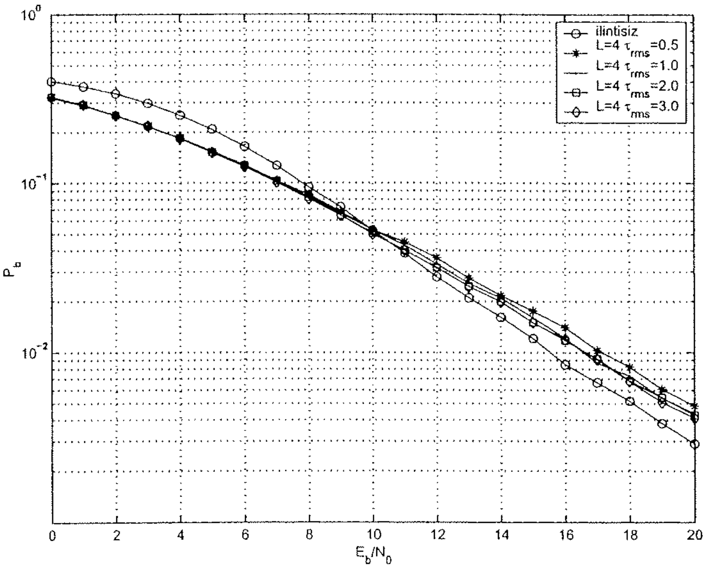
$$\begin{aligned} m(s_{2k}, \bar{s}_{2k}) &= d^2(\tilde{s}_{2k}, \bar{s}_{2k}) \\ m(s_{2k+1}, \bar{s}_{2k+1}) &= d^2(\tilde{s}_{2k+1}, \bar{s}_{2k+1}) && (3.5.8) \end{aligned}$$

biçimine indirgenir.

3.5.4 Benzetim Sonuçları

Kafes kodlamalı OFDM ile kafes kodlanmış dik uzay–zaman kodlamalı OFDM iletişim sistemlerinin ilintisiz ve ilintili sönümlemeli frekans–seçici kanallar üzerinde bit hata olasılıkları çeşitli işaret–gürültü oranlarında bilgisayar benzetimleri yardımıyla elde edilmiştir. Elde edilen hata başarımları şekil 3.5.4 ve şekil 3.5.5'te sunulmuştur.

Bilgisayar benzetimleri sırasında $N = 64$ alt–kanal kullanan OFDM yapıları ele alınmış ve çevrimsel öneki uzunluğu $L = 4$ alınarak çeşitli güç gecikme dağılımı parametresi τ_{rms}

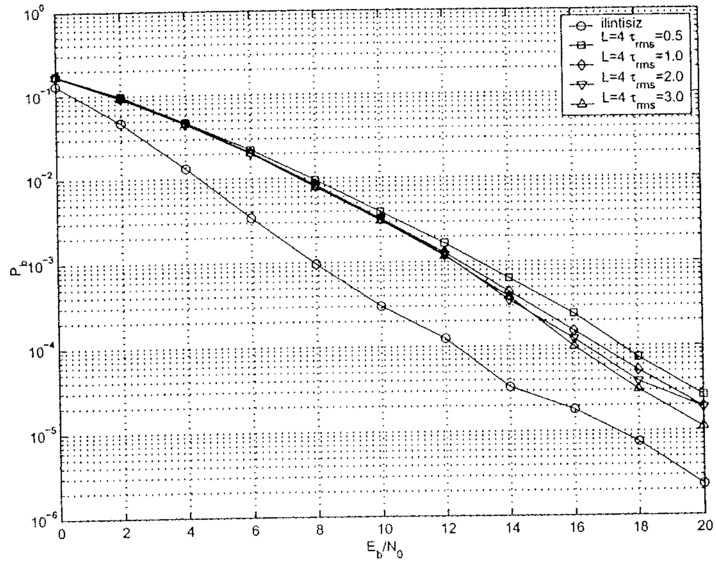


Şekil 3.5.4: Kafes kodlamalı OFDM sistemin bit hata olasılığı

değeri için hata başarımları elde edilmiştir. şekil 3.5.4'te hata başarımı verilen referans kafes kodlamalı OFDM yapısında yalnızca zaman ve frekans çeşitlemeleri bulunmakta iken şekil 3.5.5'te hata başarımı verilmiş olan yapıda dik uzay–zaman kodlarının kullanımının getirdiği uzay çeşitlemesi de bulunmaktadır. Benzetim sonuçları incelendiğinde, 10^{-3}'lük bit hata olasılığına ulaşmak için önerilen yapının ilintisiz kanalda kafes kodlamalı OFDM'den yaklaşık $15dB$ daha az işaret–gürültü oranı gerektirdiği; ilintili kanalda ise bu farkın yaklaşık $13dB$ olduğu görülmektedir.

3.5.5 Sonuçlar

Bu çalışmada, frekans–seçici kanallar üzerinde yüksek hata başarımına sahip kafes kodlanmış dik uzay–zaman kodlamalı bir OFDM sistemi önerilmiştir. OFDM alt kanallarına ilişkin sönümleme etkilerinin ilintisiz ve ilintili olduğu durumlarda önerilen tümleşik iletişim sistemine ait bilgisayar benzetimleri yapılmış ve çeşitli işaret–gürültü oranları için hata başarımları elde edilmiştir. Benzetim sonuçları önerilen yapının içerisinde barındırdığı uzay çeşitlemesi sayesinde kafes kodlamalı OFDM'e göre büyük miktarda kazanç sağladığını göstermektedir.



Şekil 3.5.5: Kafes ve uzay–zaman kodlamalı OFDM sistemin bit hata olasılığı

3.6 Kaynakça

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Bölüm IV
OFDM SİSTEMLERİNDE YENİ EŞZAMANLAMA TEKNİK VE ALGORİTMALARI

4.1 Giriş

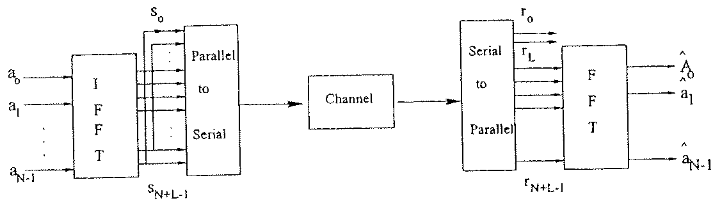
OFDM sinyalleşmesi, haberleşme kanalındaki sönümleme ve çokyolluluğun etkileriyle başa çıkmak için, kanalı alt-taşıyıcılar kullanarak alt-kanallara ayıran etkili bir yöntemdir. Tek taşıyıcılı yöntemlerle kıyaslandığında, OFDM sistemlerinde kanal denkleştirme daha kolay gerçekleştirilebilir olup, ayrıca, kanal kestirimi ile sistemin senkronizasyon hatalarına karşı duyarlılığı azaltılabilmektedir [1], [2]. Buna karşılık OFDM sistemler verici ile alıcı osilatörler arasındaki uyuşmazlıktan kaynaklanan frekans ve faz kaymalarına karşı tek taşıyıcılı sistemlerden daha duyarlıdır. Değişken frekans hatası, alt taşıyıcılar arasındaki dikliği zedelediği gibi, alt-taşıyıcı senkronizasyonunun sağlanmasını ve korunmasını da zorlaştırmaktadır [3], [4]. Örnek olarak, gürültüsüz ortamda, 30 dB ve daha yüksek sinyal gürültü oranı (SGO) elde edebilmek için, frekans kayması $|\epsilon| < 1.3.10^{-2}$ eşitsizliğini sağlamalıdır. Bu nedenle, taşıyıcı ve faz senkronizasyonunun sağlanması OFDM sistemlerdeki en önemli problemlerden biridir. Frekans kayması, alıcıda, bilinen pilot semboller kullanılarak [5], [6] veya ortalama *log-likelihood* fonksiyonu enbüyüklenerek [7] kestirilebilir. [8]'de frekans kayması kestirimi için, kod çözücü başarımında ihmal edilebilir kötüleşme ile beraber yüksek hızlı senkronizasyon sağlayan, veri destekli (VDi) bir algoritma sunulmuştur. Senkronizasyon sağlamak için, iletilen OFDM sinaylindeki artık bilgiden faydalanılabilir. Frekans kayması için böyle bir yaklaşım [9]-[12]'de gösterilmiştir. Bu yaklaşımla beraber, Van de Beek [9] frekans kaymasını ve OFDM çerçeve zamanlamasını hesaplamak için, çevrimsel önekle ilintiyi kullanan bir yöntem vermiştır. Ancak burada enbüyük olabilirlik(maximun likelihood)oranı türetilirken, iletilen veri sembolleri için Gauss yaklaşıklığı kullanılmıştır ve dolayısıyla modülasyonun etkisi gözardı edilmiştir. OFDM taşıyıcı fazı kestirimi, literatürde çokça incelenmiş OFDM kanal kestiriminin bir parçasıdır [13]-[15].

Bu çalışmanın temel amacı, M-PSK modülasyonlu OFDM sistemlerinde taşıyıcı frekansı ve faz kaymasının ayrık kestirimi için bir takım yeni veri desteksiz (VDz) enbüyük olabilirlik (EO) kestirim algoritmaları sunmaktır. Bölüm 4.4'de frekans kayması kestirilmiş ve [9]'da önerilen ve çevrimsel önekteki artıklığa dayanan teknikle dengelenmiştir. Bildiride Van de Beek'in çalışması, şu yönlerden geliştirilmiştir: i) OFDM sistemlerindeki frekans kayması kestirimi için, veri sembollerine yönelik bir Gauss varsayımı olmaksızın, enbüyük olabilirlik kestirim algoritması türetilmiştir. ii) Yöntemin ortalama karesel başarımı analitik olarak hesaplanmış ve benzetim sonuçlarıyla karşılaştırılmıştır. iii) Kestirimcinin hem ön kestirim, hem de izleme kiplerinde çalışabileceği gösterilmiştir. iv) Frekans kaymasını izlemek için bir izleme algoritması önerilmiş ve kestirilen frekans kayması ile senkronizasyondan önce ve sonra simge hata oranındaki (SHO) azalma, bir kapalı çevrim sistemle hesaplanmıştır. Bölüm 4.4'de M-PSK işaret gösterilimi üzerinden ortalama alındığında alt SGO sınırını enbüyükleyen VDz EO taşıyıcı faz senkronizasyonu incelenmiştir. Kestirimcinin ortalama karesel hata düzeyi analitik olarak türetilmiş ve başarımı benzetim sonuçlarıyla karşılaştırılmıştır. Sonuçta, sistem başarımının, artan SGO ile Cramer-Rao sınırına yaklaştığı görülmektetir. Son olarak, Bölüm 4.5'te sonuçlar özetlenmiştir.

4.2 OFDM Sistemleri

OFDM'deki temel fikir, bir veri dizisini, daha düşük veri hızında N paralel diziye dönüştürerek her paralel diziyi farklı bir alt-taşıyıcıyla iletmektir. Bu taşıyıcılar, aralarındaki frekans aralığı uygun seçilerek, dik (ortogonal) hale getirilirler. Böylece alt-taşıyıcılar arasında spektral örtüşmeye izin verilebilir ve dolayısı ile basit frekans bölmeli çoğullamaya (FDM) kıyasla daha iyi bir spektral verimlilik elde edilir. Tipik bir OFDM sistemi blok diagramı Şekil 4.2.1'de gösterilmiştir. Her $m$inci OFDM sembol süresince, iletilen veri simgeleri seri paralel dönüştürücü ile, her biri bir alt-taşıyıcıyı modüle eden daha düşük hızlı N diziye dönüştürülür, $\{a_m^k\}_{k=0}^{N-1}$. Bu alt-taşıyıcılar, ters FFT işlemi ile $\{s_m^k\}_{k=0}^{N-1}$ olarak ifade edilen N zaman işaretine dönüştürülür ve buna L uzunlukta çevrimsel önek eklenerek bir OFDM çerçevesi oluşturulur. Çevrimsel önek, N uzunluklu OFDM sembolünün sonundaki L tane simgeyi çerçevenin en önüne kopyalayarak oluşturulur. Bu nedenle, iletilen bir OFDM çerçevesinin etkin uzunluğu $N + L$ sembol olacaktır. Çevrimsel önekin eklenmesi, semboller arası girişimin (ISI) önlenmesi ve alt-taşıyıcılar arasındaki dikliğin korunması açısından önem taşır. Bu bölümde, çevrimsel önekte taşınan artık bilgi, frekans kayması kestiriminin, ek pilotlara gerek duymaksızın, etkin biçimde gerçekleştirilebilmesine olanak sağlayacağı gösterilecektir. Bölüm 4.4 de görülecegi gibi, kalan OFDM simgeleri (N tane) faz senkronizasyonunu sağlamak için etkin olarak kullanılabilir.

İletilen N karmaşık verinin, M-PSK modülasyonu ile iletildiği ve kanal gürültüsünün toplamsal beyaz Gauss gürültüsü (AWGN) olduğu varsayılmıştır. Kanal süzgeçlemesinin, verici ve alıcı arasında eşit bölündüğü ve frekans ve faz kayması olmadığı durumda kanal yanıtı Nyquist biçiminde olduğu varsayılmaktadır. Literatürden, mükemmel zamanlama bilgisinin, veri hızının $10 - 20\%$'si kadar frekans hataları seviyesinde bile elde edilebildiği



Şekil 4.2.1: N-blok veri ileten temelband OFDM sistemi

bilinmektedir, [16]. Çevrimsel önek ile sağlanan koruma süresi kanal dürtü yanıtından uzun olduğu sürece, semboller arası girişimin önüne geçileceği [17]'de gösterilmiştir. Alıcıda veri ayrık Fourier dönüşümü (DFT) ile yeniden elde edilir. OFDM çerçevesi ve sembol zamanlaması senksonizayonunun sağlandığı varsayılırsa, $m$inci OFDM sembolü için uyumlu süzgeç çıkışında elde edilen karmaşık sinyal zarfı şöyle ifade edilebilir;

$$r_m(k) = s_m(k) + n_m(k), \qquad m = 1, 2, \cdots, L_0. \tag{4.2.1}$$

Burada,

$$s_m(k) = \begin{cases} s_m^{k+N-L} \exp\{j(2\pi\epsilon k/N + \phi)\}, & k = 0, 1, \ldots, L-1 \\ \\ s_m^{k-L} \exp\{j(2\pi\epsilon k/N + \phi)\}, & k = L, L+1, \cdots, N+L-1 \end{cases} \tag{4.2.2}$$

ve s_m^k'lar ters FFT işlemi ile, veri dizisinden (4.2.3) teki gibi elde edilir

$$s_m^k = (1/N) \sum_{n=0}^{N-1} a_m^n \exp(j2\pi kn/N). \tag{4.2.3}$$

Burada a_m^n'ler, $n$inci alt-taşıyıcıdan, $m$inci OFDM sembol süresince iletilen ve $\{e^{j\frac{2\pi r}{M}}, r = 0, 1, \ldots M-1\}$ kümesinden değerler alan M-PSK sembolünü belirtmektedir. ϵ kanalın bağıl frekenas kaymasını (gerçek frekans kaymasının taşıyıcılar arası uzaklığa oranı), ϕ kanal faz kaymasını ve $n(k)$ ise varyansı $\sigma_n^2 = E\{|n(k)|^2\}$ olan eklenir beyaz Gauss gürültüsünün karmaşık zarfını ifade eder.

4.3 EO Frekans kayması kestirimcisi

Önceden (4.2.1)'de tanımlandığı gibi $m$inci OFDM sembolüne ilişkin $N + L$ simge uzunluklu gözlem vektörü,

$$\mathbf{r_m} = [r_m(0), r_m(1), \ldots, r_m(N + L - 1)]^T.$$

biçiminde yazılabilir. Gösterim sadeliği için m indisini belirtmeden, *likelihood* fonksiyonu şöyle yazılabilir,

$$L(\epsilon,\phi,\{a^n\}) \equiv p(\mathbf{r}|\epsilon,\phi,\{a^n) = \prod_{k=0}^{L-1} f[r(k)-s(k)] \prod_{k=L}^{N-1} f[r(k)-s(k)] \prod_{k=N}^{N+L-1} f[r(k)-s(k)] \quad (4.3.4)$$

$$= \prod_{k=0}^{L-1} f[r(k)-s(k)]f[r(k+N)-s(k+N)] \prod_{k=L}^{N-1} f[r(k)-s(k)]. \quad (4.3.5)$$

Burada $f(.)$, varyansı σ_n^2 olan karmaşık Gauss yoğunluk fonksiyonunu belirtir. M-PSK modülasyonu için benzerlik fonksiyonu [14]-[15]'de

$$L(\epsilon,\phi,\{a^n\}) \doteq \exp\left\{\frac{2}{\sigma_n^2}\sum_{k=0}^{L-1} Re\left[r(k)s^*(k)+r(k+N)s^*(k+N)\right]\right\}, \quad (4.3.6)$$

olarak verilmiştir, burada (*) karmaşık eşlenik ifadesidir. (4.3.6)'daki veri bağımlılığı, benzerlik oranının, $\{a\}$'nın tüm olurlu değerleri üzerinden ortalaması alınarak yokedilebilir. Bu ortalama işleminin, küçük *SGO* varsayımı yapılmadıkça, matematiksel olarak analizi pek olanaklı gözükmemektedir. Bu nedenle, (4.3.6)'yı kendisinin, karesel terimlere kısaltılmış Taylor serisi ile şu şekilde değiştirebiliriz,

$$L(\epsilon,\phi,\{a^n\}) \doteq 1+u+v+uv+\frac{1}{2}u^2+\frac{1}{2}v^2. \quad (4.3.7)$$

(4.7) de, u ve v şöyle tanımlanmıştır.

$$u = \frac{2}{\sigma_n^2}\sum_{k=0}^{L-1} Re[r(k)s^*(k)+r(k+N)s^*(k+N)] \quad (4.3.8)$$

$$v = \frac{2}{\sigma_n^2}\sum_{k=L}^{N-1} Re[r(k)s^*(k)]. \quad (4.3.9)$$

Bu durumda, *likelihood* fonksiyonunun veri sembolleri üzerinden (4.3.7)'deki şekliyle ortalaması alınabilir. (4.3.8),(4.3.9) eşitliklerinden ve $s(k)$'nın (4.2.2) ve (4.2.3)'de verilen tanımları kullanılarak

$$E(u) = E(v) = E(uv) = 0$$

olduğu gösterilebilir. Bu durumda, (4.3.7)'deki ifadenin son iki terimine ilişkin beklenti ifadeleri aşağıdaki gibi elde edilir;

$$E(u^2) = \frac{1}{N\sigma_n^4}\sum_{k=0}^{L-1}\{|r(k)|^2+|r(k+N)|^2+2Re[r(k)r^*(k+N)e^{j2\pi\epsilon}]\} \quad (4.3.10)$$

$$E(u^2) = \frac{1}{N\sigma_n^4}\sum_{k=L}^{N-1}|r(k)|^2. \quad (4.3.11)$$

Bu türetimlerin ayrıntıları [13] da sunulmuştur. Sonuç olarak ortalamasi alınmış *likelihood* fonksiyonu ϕ fazından ve ϵ'nin deneme değeri $\hat{\epsilon}$'den bağımsız olur ve şu son hali alır:

$$L(\hat{\epsilon}) = 1 + \frac{1}{N\sigma_n^4}\left\{\sum_{k=0}^{N+L-1}|r(k)|^2 + 2\sum_{k=0}^{L-1}Re[r(k)r^*(k+N)e^{j2\pi\hat{\epsilon}}]\right\}. \qquad (4.3.12)$$

Eğer,

$$\nu = \sum_{k=0}^{L-1} r(k)r^*(k+N)$$
$$\rho = \sum_{k=0}^{N+L-1}|r(k)|^2,$$

ise, o zaman

$$L(\hat{\epsilon}) = C_1|\nu|\cos(2\pi\hat{\epsilon} + \angle\nu) + C_2\rho \qquad (4.3.13)$$

olur. Burada C_1 ve C_2, $\hat{\epsilon}$'den bağımsız sabitleri göstermektedir. $\hat{\epsilon}$'nin VDz EO kestirimi (4.3.13)'ün $\hat{\epsilon}$'ye göre türevinin alınıp sıfıra eşitlenmesiyle bulunur;

$$\hat{\epsilon}_{ML} = -\frac{1}{2\pi}\arg\left[\sum_{k=0}^{L-1} r(k)r^*(k+N)\right]. \qquad (4.3.14)$$

Türetilen algoritma, M-PSK yıldız kümesi üzerinden ortalaması alınmış *likelihood* fonksiyonunun alt SGO sınırını enbüyüklerken, [9]'da önerilen ve gözlemlenen sinyalin Gauss olduğu varsayımı altinda elde edilen SGO sınırının düştüğü görülmektedir.

4.3.1 *Taşıyıcı Frakans Kaymasının izleyici kipte kestirimi*

(4.3.12)'nin $\hat{\epsilon}$'ye göre türevi,

$$\frac{dL(\hat{\epsilon})}{d\hat{\epsilon}} = (-4\pi/N\sigma_n^4)\sum_{k=0}^{L-1} Im\{q_k\} \qquad (4.3.15)$$

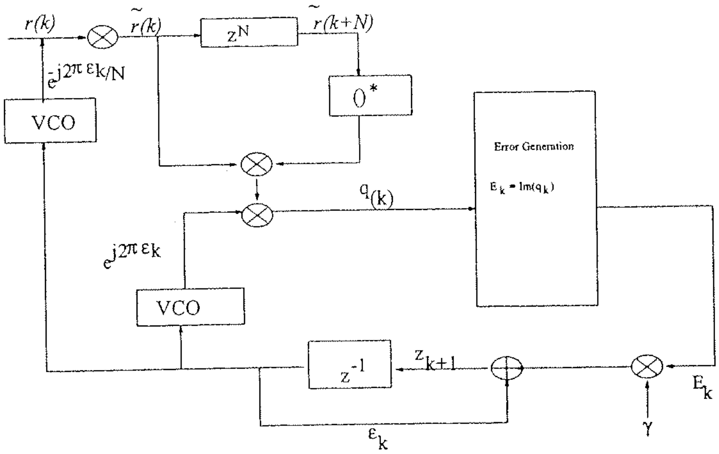
eşliğini verir, burada $q_k = r(k)r^*(k+N)\exp(j2\pi\hat{\epsilon})$'dır. Eğer $L=1$ alınırsa (4.3.15)den, $\hat{\epsilon}$'nin eniyilenmesi için [8] da verilen özyineli bir algoritma türetilebilir,

$$\hat{\epsilon}_{k+1} = \hat{\epsilon}_k + \gamma E_k. \qquad (4.3.16)$$

Burada γ pozitif bir sabittir ve algoritmanın yakınsama hızını ve artık ortalama karesel hatasını (MSE) kontrol eder. Pratikte γ, çevrim kazancı 1'den çok küçük olacak şekilde seçilir, böylece durağan MSE'nin de küçük olması sağlanır. $\hat{\epsilon}_k$ ve $\hat{\epsilon}_{k+1}$, $\hat{\epsilon}$'nin k ve $(k+1)$inci adımdaki türetimleridir. E_k ise hata kontrol sinyalidir ve şöyle verilir,

$$E_k = -Im\{q_k\}.$$

Yukarıda açıklanan algoritma, Şekil 4.2.2'de gösterildiği gibi, otomatik frekans kontrol çevrimi kullanılarak gerçekleştirilebilir. İzleme kipi için de $N+L$ uzunluklu OFDM sembolü gerekmektedir. Kayma, $\hat{\epsilon}_k$'yı elde etmek için her kanal boyunca sadece $r(k)$ ve $r(k+N)$ çiftini kullanarak ilk ve son L simge ile izlenir. Diğer simgeler en son $k = L$ durumunda bulunan $\hat{\epsilon}_k$ kayması ile düzeltilirler.



Şekil 4.3.2: Frekans kayması izlenmesi

4.3.2 *Frekans Kayması Kestirimcisinin Ortalama-Karesel Başarımı*

Kestirimcinin başarımını belirlemek için *log-likelihood* oranının türevinin ortalaması kullanılabilir. Bu bağlamda aşağıdaki ifadeler yararlı olacaktır.

$$W(\hat{\epsilon}) = \frac{dL}{d\hat{\epsilon}}, \quad F(\hat{\epsilon}) = E[W(\hat{\epsilon})], \quad A \cong \frac{dF}{d\hat{\epsilon}}|_{\hat{\epsilon}=\epsilon}. \tag{4.3.17}$$

(4.3.17) deki ifadedeki $F(.)$ fonksiyonu, kestirimcinin "S-eğrisi" ya da "ayrımsama karakteristiği" olarak tanımlanır. Eğer EO kestirimcisi $\hat{\epsilon}_{EO}$ olursa, bu durumda küçük değişimler için yaklaşık varyansı [18]

$$Var[\hat{\epsilon}_{EO} - \epsilon] = A^{-2}Var[W(\epsilon)] = \frac{E[W^2(\hat{\epsilon})]}{A^2}|_{\hat{\epsilon}=\epsilon}. \tag{4.3.18}$$

biçiminde tanımlanmaktadır. (4.3.13) ilişkisinden

$$W(\hat{\epsilon}) = \frac{dL}{d\hat{\epsilon}} = -2\pi|\nu|\sin(2\pi\hat{\epsilon} + \angle\nu), \tag{4.3.19}$$

elde edilir ve şu sonuç ortaya çıkar,

$$E[W(\hat{\epsilon})] = -2\pi Im\left\{E(\nu)e^{j2\pi\hat{\epsilon}}\right\}, \tag{4.3.20}$$

$$E[W^2(\hat{\epsilon})] = 2\pi^2\left[E(|\nu|^2) - Re\left\{E(\nu^2)e^{j4\pi\hat{\epsilon}}\right\}\right]. \tag{4.3.21}$$

A için [13] da aşağıdaki ifade türetilmiştir;

$$A = -4\pi^2\sigma_s^2 L \tag{4.3.22}$$

burada, $\sigma_s^2 = E(|s(k)|^2)$'dir. $E(|\nu|^2)$ ve $E(\nu^2)$ için [13] da türetilen ifadeler (4.3.21)'de yerlerine konulursa, $E[W^2(\hat{\epsilon})]$ için

$$E[W^2(\hat{\epsilon})] = 2\pi^2 L[\sigma_n^4 + 2\sigma_s^2\sigma_n^2]. \tag{4.3.23}$$

eşitliği elde edilir. $E[W^2(\hat{\epsilon})]$ ve A ifadeleri (4.3.18)'de yerlerine konulursa yaklaşıklama varyansı,

$$Var[\hat{\epsilon}_{ML} - \epsilon] = \frac{1 + 2SGO}{8\pi^2 L(SGO)^2} \tag{4.3.24}$$

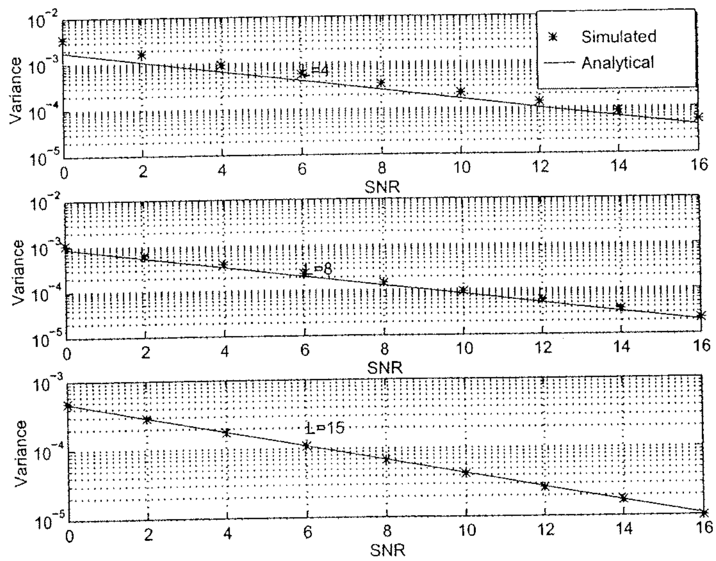
olarak bulunur, burada $SGO = \sigma_s^2/\sigma_n^2$'dir.

4.3.3 *Bilgisayar Benzetimleri*

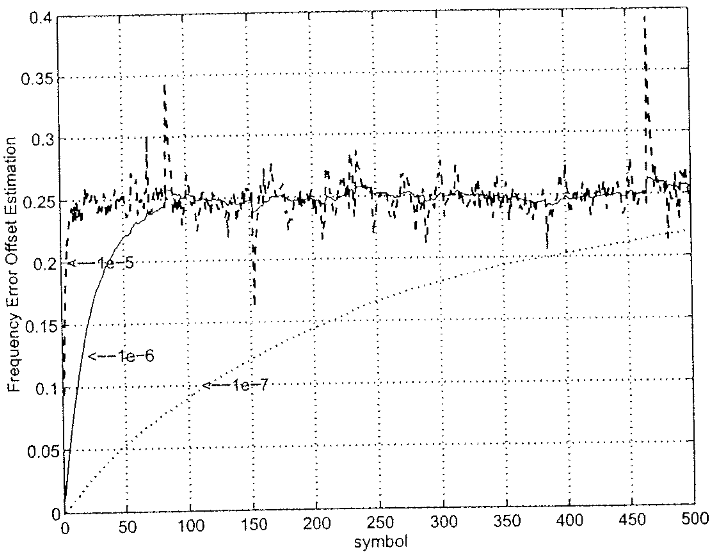
Temelband OFDM sisteminin bilgisayar benzetimi için şu parametre değerleri seçilmiştir. Toplam altkanalların sayısı, $N = 256$, M-MPK modülasyonunda düzey sayısı $M = 16$, normalize taşıyıcı frekans kayması, $\epsilon = 0.25$, önek uzunluğu, $L = 4, L = 7$ ve $L = 15$. Benzetim çalışmalarında sadece toplamsal Gauss gürültüsü gözönüne alınmıştır. 4.3.3,frekans kestirim varyansı için (4.3.24) ilişkisinden analitik olarak hesaplanan değerlerle benzetim çalışmaları sonucu elde edilen değerlerin oluşturduğu eğrileri göstermektedir. Bu eğrilerden, analitik ve benzetim sonuçlarının birbirleriyle mükemmel biçimde uyuştuğu gözlemlenmektedir.

Kestirim algoritmasının performansı, 4.3.4'de gösterilen blok şemadaki izleyici kipinde çalışması durumunda da bilgisayar benzetimi yoluyla incelenmiş ve yakınsama parametresi $\gamma > 10^{-5}$ seçilmesi durumunda ϵ için kestirilen değerlerin çok fazla dalgalandığı gözlenmiştir. Bu nedenle benzetim çalışmalarında γ için $10^{-5}, 10^{-6}$ ve 10^{-7} değerleri seçilmiştir. 4.3.4 den de görüleceği gibi, $\gamma = 10^{-7}$ seçildiğinde ilk 500 sembol için algoritmanın durağan bir değere yakınsayamadığı anlaşılmaktadır. Yapılan yoğun bilgisayar benzetim çalışmaları sonucu γ için en uygun değerin 10^{-6} olduğuna karar verilmiştir. 4.3.5 algoritmanın izleyici kipinde kestirilen varyansın sinyal-gürültü oranı(SNR) ile değişimini göstermektedir. Benzetim, 10000 sembol kullanılarak gerçekleştirilmiş ve her durumda, karşılaştırmak amacı ile, $L = 4$ seçilmiştir. Bu eğrilerden, kestirilen varyansın izleme modunda, normal moddan daha iyi sonuçlar verdiği anlaşılmaktadır.

OFDM sisteminin, belli bir frekans kayması altında, sembol hata başarımı (Symbol error rate, SER) da bilgisayar benzetimi ile incelenmiştir. Elde edilen sonuçlar 4.3.6 ve 4.4.10 özetlenmektedir. Başarım eğrileri SER in SNR ile değişimi biçiminde oluşturulmuş ve farklı önek uzunlukları için farklı başarım eğrileri elde edilmiştir. Frekans kayması, izleyici kipinde bir kapalı çevrim algoritması ile kompanse edilmiştir. 4.3.6 dan kestirilmiş varyans değerlerinin SNR ve önek uzunluğu arttıkça küçüldüğü gözlenmektedir. 4.4.10 de ise OFDM sisteminin SER başarımının normal ve izleyici kipindeki değişimlerini göstermektedir. Bu durumda önek uzunluğu $L = 4$ seçilmiştir.



Şekil 4.3.3: Varyans kestirimin analitik ve benzetim sonuçlarının karşılaştırılması



Şekil 4.3.4: $\gamma = 10^{-5}, 10^{-6}, 10^{-7}$, SNR= 10 dB için frakans kaymasının izleme başarımı

4.4 Faz Kayması için EO Kestirimi

Bölüm 3'de taşıyıcı frekans kayması kestirimi için her OFDM çerçevesinin ilk L sembolünün yeterli olduğu gösterilmiştir. Bu vesileyle OFDM sembol zamanlaması ve frekans kayması senkronizasyonunun sağlanıp, dengelendiği varsayılırsa, her sembolde kalan N simge taşıyıcı faz senkronizasyonu için kullanılabilir. Kusursuz faz kestirimi varsayımı altında $m$inci OFDM sembolü için gözlemlenen işaretin karmaşık zarfı aşağıdaki gibi ifade edilebilir.

$$r_m(k) = s_m(k) + n(k), \quad m = 1, 2, \ldots, L_0, \tag{4.4.25}$$

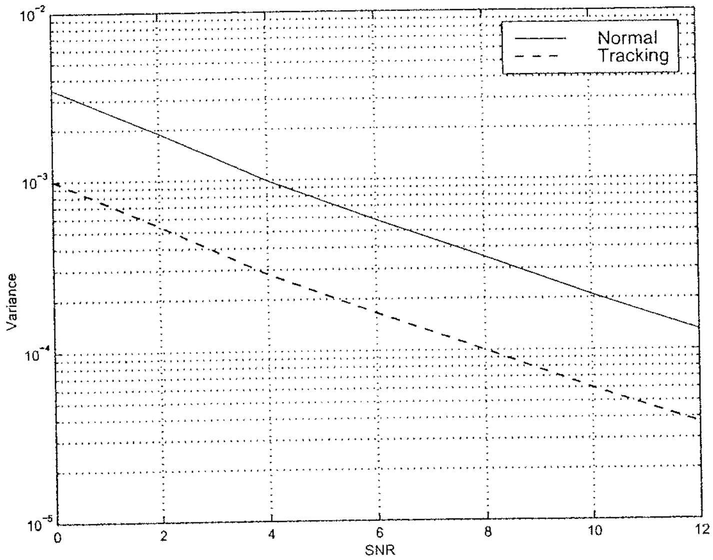
burada

$$s_m(k) = s_m^k e^{j\phi}, \quad k = 0, 2 \ldots N-1 \tag{4.4.26}$$

ve s_m^k'lar (4.2.2)'de gösterildiği gibi veri dizisinin ters FFT'sidir. Ayrıca, (4.4.26)'daki ϕ kanal faz kaymasını, (4.4.25)'deki $n(k)$ ise varyansı $\sigma_n^2 = E\{|n(k)|^2\}$ olan eklenir beyaz Gauss gürültüsünün zarfını temsil eder.

Herbiri N simge içeren L_0 tane OFDM sembolünden oluşan $\mathbf{r}$ gözlem vektörü dikkate alınsın,

$$\begin{aligned}\mathbf{r} &= [\mathbf{r}_1 \mathbf{r}_2 \ldots \mathbf{r}_{L_0}]^T \\ \mathbf{r}_m &= [r_m(0), r_m(1) \ldots, r_m(N-1)]^T, \qquad m = 1, 2, \ldots L_0.\end{aligned}$$



Şekil 4.3.5: $L = 4$ için normal ve izleyici varyans kestirimi başarımlarının karşılaştırılması

Veri dizisi $\{a_m^n\}$ ve ϕ verildiği taktirde (4.4.25),(4.4.26) ve (4.2.3)'den, **r**'nin gözlemlenen örneklerinin benzerlik fonksiyonu orjinal veri dizisi cinsinden ifade edilebilir.

$$L(\phi, \{a_m^n\}) = \exp\left\{\frac{2}{N\sigma_n^2}\sum_{m=1}^{L_0}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1} Re\left[r_m(k)e^{-j2\pi nk/N}a_m^{*n}e^{-j\phi}\right]\right\}. \tag{4.4.27}$$

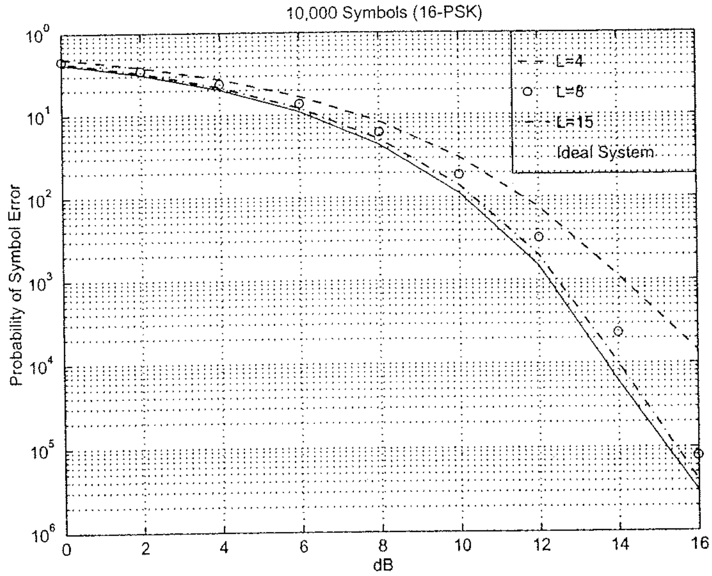
(4.4.27)'deki $R_m(n) \equiv \sum_{k=0}^{N-1} r_m(k)\exp(-j2\pi nk/N)$ teriminin $r_m(k)$ gözlem dizisinin ayrık Fourier dönüşümü olduğu kolayca görülebilir. Bu durum gözönüne alınırsa, (4.4.27) şu biçime dönüşür.

$$L(\phi, \{a_m^n\}) = \exp\left\{\frac{2}{N\sigma_n^2}\sum_{m=1}^{L_0}\sum_{n=0}^{N-1} Re\left[R_m(n)a_m^{*n}e^{-j\phi}\right]\right\}. \tag{4.4.28}$$

Buradan, sadece ϕ'ye bağlı bir **log-likelihood** fonksiyonu elde edebilmek için, öncelikle (4.4.28)'in a_m^n, M-PSK veri dizisi üzerinden beklenen değeri bulunur. Daha sonra bulunan değerin logaritması alınarak aşağıdaki ifade elde edilir.

$$\Lambda(\phi) = \sum_{m=1}^{L_0}\sum_{n=0}^{N-1}\ln\left(\frac{1}{M}\sum_{r=0}^{M-1}\exp\left\{\frac{2}{N\sigma_n^2}|R_m(n)|\cos(\phi + 2\pi r/M - \arg R_m(n))\right\}\right). \tag{4.4.29}$$

Düşük SGO için geçerli olan aynı matematiksel yaklaşıklığı kullanılarak aşağıdaki fonksiyonuna ulaşılır.



Şekil 4.3.6: Farklı SNR ve önek değerleri için OFDM sisteminin SER başarımı

$$\Lambda(\phi) = \sum_{m=1}^{L_0} \sum_{n=0}^{N-1} \ln \left\{ I_0(\frac{2}{N\sigma_n^2}|R_m(n)|) + 2I_M(\frac{2}{N\sigma_n^2}|R_m(n)|)\cos[M(\phi - \arg R_m(n))] \right\}. \tag{4.4.30}$$

Burada, I_0 ve I_M, sırasıyla 0ıncı ve Minci dereceden değiştirilmiş birinci türden Bessel fonksiyonlarıdır. ϕ'nin EO kestirimi, (4.4.30)'un ϕ'ye göre türevi alınarak ve paydada sadece düşük dereceli terim bırakılarak türetilebilir.

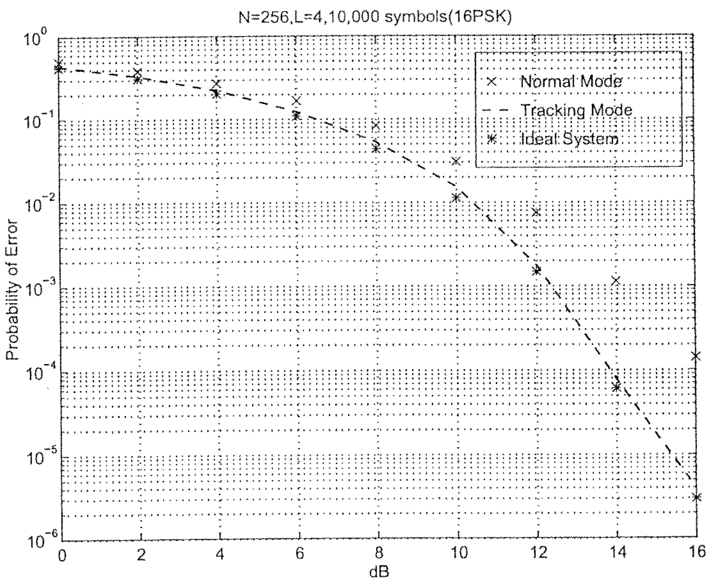
$$W(\phi) \triangleq \frac{d\Lambda(\phi)}{d\phi} = \gamma_c \sin M\phi - \gamma_s \cos M\phi \tag{4.4.31}$$

burada,

$$\gamma_c = \sum_{m=1}^{L_0} \sum_{n=0}^{N-1} A_{n,m} \cos[M \arg(R_m(n))], \tag{4.4.32}$$

$$\gamma_s = \sum_{m=1}^{L_0} \sum_{n=0}^{N-1} A_{n,m} \sin[M \arg(R_m(n))], \tag{4.4.33}$$

$$A_{n,m} = \frac{-2MI_M(\frac{2}{N\sigma_n^2}|R_m(n)|)}{I_0(\frac{2}{N\sigma_n^2}|R_m(n)|)}. \tag{4.4.34}$$



Şekil 4.3.7: $L = 4$ için normal ve izleyici modunda OFDM systeminin SER başarımının karşılaştırılması

Daha sonra (4.4.31) bağıntısı sıfıra eşitlenerek faz kayması için VDz EO kestirim bulunur.

$$\hat{\phi}_{ML} = \frac{1}{M}\tan^{-1}(\frac{\gamma_s}{\gamma_c}). \tag{4.4.35}$$

Yeterince küçük SGO rı için, elde edilen EO kestirimcisi, [20]'de belirtilen VDz ileri beslemeli taşıyıcı eşzamanlayıcı sınıfına ait Minci kuvvet eşzamanlayıcısına [14-15] dönüştüğü gösterilebilir. $SGO = \sigma^2{}_s/\sigma_n^2 = 1/(N\sigma_n^2) << 1$, için (M-PSK için $\sigma_s^2 = 1/N$ olduğu (4.2.3)'den görülebilir), (4.4.34)'deki $I_0(z)$ ve $I_M(z)$ şöyle yaklaşıklanabilir,

$$I_0(z) \approx 1, \quad I_M(z) \approx \frac{(z/2)^M}{M!}.$$

Bu değerler (4.4.34),(4.4.32) ve (4.4.33)'de kullanılırsa, (4.4.35)'deki faz kestirimi ifadesi aşağıdaki gibi olur.

$$\hat{\phi}_{ML} = \frac{1}{M}\arg\sum_{m=1}^{L_0}\sum_{n=0}^{N-1} R_m^M(n). \tag{4.4.36}$$

Böylece, OFDM sistemler için EO yaklaşımıyla türetilen faz eşzamanlayıcıların da düşük SGO bölgesinde Minci kuvvet eşzamanlayıcısıyla yaklaşıklanabileceği gösterilmiştir.

4.4.1 *Faz Kayması Kestirimcisinin Ortalama-Karesel Başarımı*

Küçük değişiklikler olduğu durumda faz kestiriminin yaklaşıklık varyansı,

$$Var[\hat{\phi}_{ML} - \phi] = A^{-2}Var[W(\phi)] = \frac{E[W^2(\hat{\phi})]}{A^2}|_{\hat{\phi}=\phi}. \tag{4.4.37}$$

olur, buradaki A ve $W(.)$ ifadeleri (13)'dekine benzer biçimde tanımlanmıştır. Ayrıntıları (9) da görüleceği gibi, sonuçta olan ara adımlar atlanırsa A ve $E[W^2(\hat{\phi})]$ için şu bağıntılar türetilmiştir;

$$A = \frac{-2L_0M}{\sigma_n^{2M}N^{M-1}(M-1)!}, \tag{4.4.38}$$

$$E[W^2(\hat{\phi})] = \frac{1}{2}Q^2L_0^2N^2\{\frac{E[P]-1}{L_0N}\}, \tag{4.4.39}$$

elde edilir. Burada,

$$Q = -2M/(N\sigma_n^2)^M M!,$$

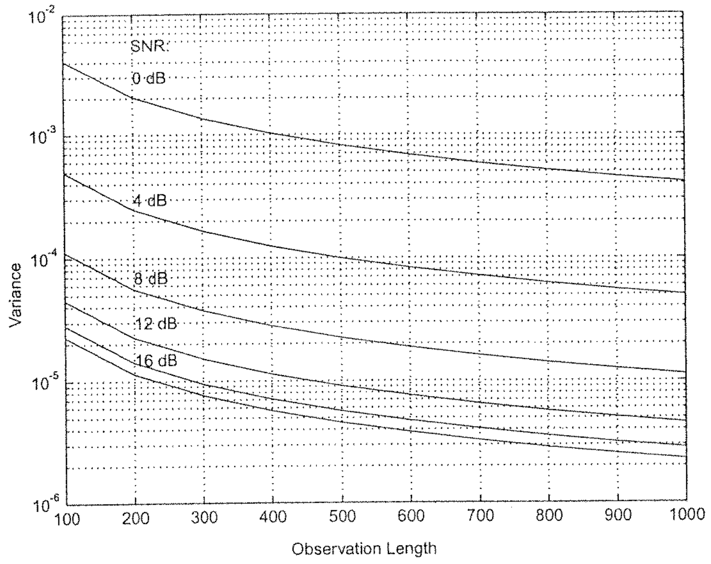
$$E[P] = M!(2\sigma_n^2)^M \sum_{m=0}^{M} \binom{M}{m}\frac{1}{m!(2\sigma_n^2)^m}.$$

Bu sonuçlar (4.4.37)'de yerine konulursa faz kestirimi varyansı için şu son ifade elde edilir;

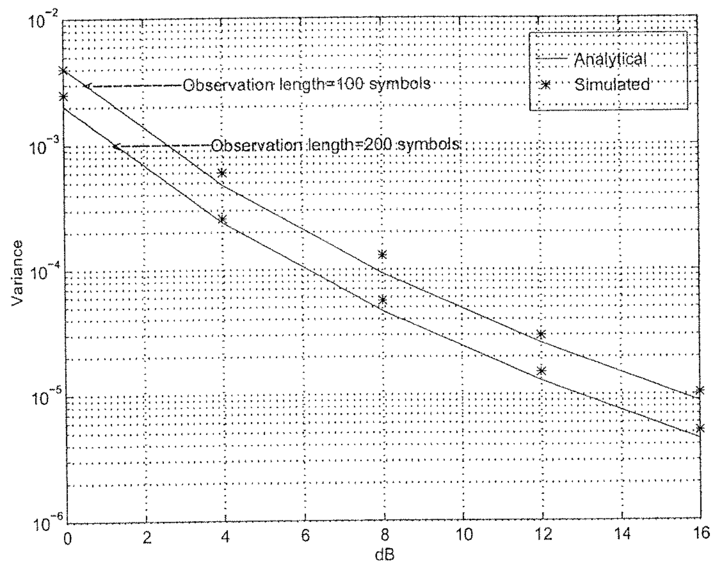
$$Var[\hat{\phi}_{ML} - \phi] = \frac{1}{L_0N}\frac{M!}{2M^2}\sum_{m=0}^{M-1}\binom{M}{m}\frac{2^{M-m}}{m!(SNR)^{M-m}} \quad . \tag{4.4.40}$$

4.4.2 *Bilgisayar Benzetimleri*

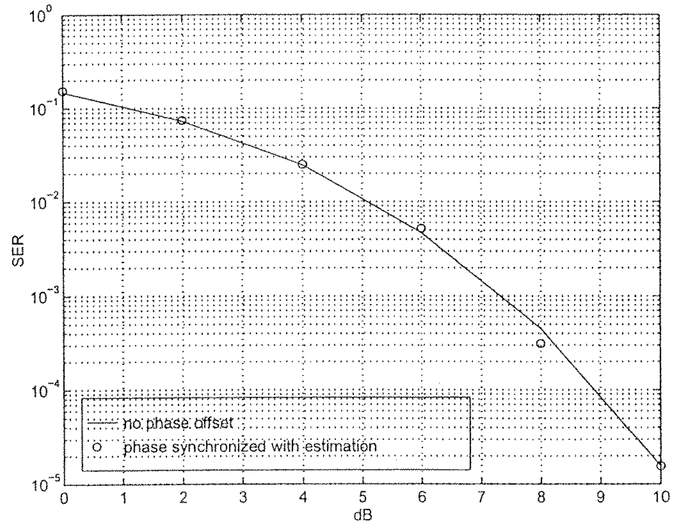
Bu çalışmada elde edilen faz kestirim algoritmasının başarımını değerlendirmek için yapılan bilgisayar benzetimlerinde şu parametreler seçilmişitir. Toplam altkanalların sayısı, $N = 256$, M-MPK modülasyonda düzey sayısı $M = 4$, normalize taşıyıcı faz kayması, $\phi = \pi/32$, gözlem uzunluğu, $L_0 = 100, 200$. Benzetim çalışmalarında sadece toplamsal Gauss gürültüsü gözönüne alınmıştır. 4.4.8 faz kestirim varyansının SNR ile değişim sonuçlarını göstermektedir. Bu grafiğe ayrıca faz kestirimi için (4.4.40) ta elde edilen analitik varyans eğrisi de eklenmiştir. Bu grafiklerden, benzetim ve analitik sonuçların birbirleri ile mükemmel biçimde uyum içinde olduğu görülmektedir. OFDM sisteminin, belli bir faz kayması altında, sembol hata başarımı (Symbol error rate, SER) da bilgisayar benzetimi ile incelenmiştir. Elde edilen sonuçlar 4.4.9 özetlenmektedir. Başarım eğrileri, SER in SNR ile değişimi biçiminde verilmiş olup, faz kayması izleyici kipinde bir kapalı çevrim algoritması ile kompanse edilmiştir. ??dan kestirilmiş varyans değerlerinin SNR ve önek uzunluğu arttıkça küçüldüğü gözlenmektedir.



Şekil 4.4.8: EO faz kestirimcisinin gözlem uzunluğuna bağlı olarak analitik başarımı



Şekil 4.4.9: Analitik ve bilgisayar benzetim faz kestirimlerinin SNR in bir fonksiyonu olarak değişimleri



Şekil 4.4.10: OFDM sistemlerinin faz eşzamanlaması altında SER başarımı

4.5 Sonuç

Bu çalışmada, VDz EO taşıyıcı frekans, faz senkronizasyonu için, M-PSK işaret kümesi üzerinden ortalama olabilirlik (averaged-likelihood) fonksiyonunun alt SGO sınırını enbüyükleyen ayrı algoritmalar türetilmiş ve ortalama karesel başarımları hem analitik olarak hem de bilgisayar benzetimleriyle incelenmiş ve analitik sonuçların benzetim sonuçlarıyla tam bir uyum sergilediği gözlemlenmiştir. Frekans kayması kestirimi için, başarımın SGO ile arttığı ancak çevrimsel önekin başarım kontrolünde önemli rol oynadığı sonucuna varılmıştır. Faz kayması kestirimi için ise EO kestirimcisinin yeterince küçük SGO değerleri için, literatürde daha önceden önerilmiş ve VDz ileri beslemeli taşıyıcı eşzamanlayıcılarından Minci üstten eşzamanlayıcısına indirgendiği gözlemlenmiştir. Kestirimcinin ortalama karesel başarımı analitik olarak elde edilmiş ve benzetim sonuçlarıyla karşılaştırılmıştır. Sonuç olarak önerilen algoritma, büyük faz kayması durumunda bile çok doğru kestirimler elde ettiği, öz-gürültüsü (self noise) üretmediği ve orta ila yüksek SGO seviyelerinde Cramer-Rao sınırını yakaladığı görümektedir. Son olarak, (4.4.40)'da çıkarılan varyans ifadesinin,[[20], Denklem (14)]de verilen yaklaşık varyans formülünün genelleştirilmiş bir durumu olduğu görümektedir. Bunun dışında, öz-gürültünün olmadığı ve VDz algoritmanın başarımının orta ila yüksek SGO değerleri için Cramer-Rao sınırına eşit olduğu gözlemlenmiştir. Benzer gözlemler [9] ve [20]'de de yapılmıştır.

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Bölüm V

UZAY-ZAMAN KODLAMALI OFDM TÜMLEŞİK SİSTEMİ İÇİN YENİ KANAL KESTİRİM ALGORİTMALARI

5.1 Giriş

Özellikle, uzay-zaman kodlanmış ve OFDM sinyaller tarafından uyarılmış bayılmalı (fading) kanalların kestirimi, telsiz iletişim sistemlerinin alıcılarının tasarımlarında büyük önem taşımaktadır. Bu tür sistemlerin alıcılarında demodülasyon ve sezim (detection) işlemleri ancak kanal parametrelerinin bilindiği varsayılarak gerçekleşebilmektedir. Bu nedenle alıcıda demodülasyon ve bunu izleyen sayısal sinyalin sezimi işlemine başlamadan önce kanal katsayılarının bir şekilde kestirilmesi gerekmektedir. Bu bölümde kanal kestirimi için projede yürütülen araştırmalarda varılan ilginç ve yeni bir takım sonuçlar sunulmaktadır. Bu çalışmalar aşağıdaki paragraflarda kısaca özetlenmiştir.

Bunu izleyen Bölüm 5.2 de, M-PSK sinyal ile iletişim yapan OFDM sistemleri için EM (Expectation-Maximization) yöntemine dayanan, hesaplama yönünden çok hızlı, bir MAP(Maximum a-posteriori) kanal kestirim algoritması geliştirilmektedir. Temel yaklaşım, İletilen M-PSK verileri üzerinden istatistiksel bir ortalama alınarak, kestirim algoritmasının eğitim verilerine gereksinim duymayacak biçimde tasarlanması gerçekleştirilmektedir.

Bölüm 5.3 te ise uzay-zaman kodlanmış işaretlerin iletildiği çokyollu telsiz iletişim senaryosu gözönüne alınarak, sönümlemeli kanalın kanal katsayıları matrisi ile iletilen işaretlerin ortak kestirimi için yeni bir gözü-kapalı(blind) kestirim yöntemi önerilmektedir.

5.2 OFDM Sistemlerle Uyarılmış Zamanla-Değişen Bayılmalı Kanallar için Yeni bir Kanal Kestirim Algoritması

Bu çalışmada M-PSK sinyal ile iletişim yapan OFDM sistemleri için EM (Expectation-Maximization) yöntemine dayanan, hesaplama yönünden çok hızlı, yeni bir bir MAP(Maximum a-posteriori) kanal kestirim algoritması geliştirilmektedir. İletilen M-PSK verileri üzerinden istatistiksel bir ortalama alınarak, kestirim algoritmasının eğitim verilerine gereksinim duymayacak biçimde tasarlanması gerçekleştirilmektedir (Non-data-aided). Ayrık, çok-yollu bayılmalı kanalı belirleyen, ilintili(correlated) ve çok sayıda kanal parametreleri, Karhunen-Loeve dik açılımından yararlanılarak ilintisiz(uncorrelated) ve az sayıdaki kanal parametrelerine dönüştürülmekte ve bu parametreler de yukarıda belirtilen hızlı algoritma ile kestirilmektedir. Geliştirilen algoritma daha sonra QPSK sinyalleri ile module edilmiş OFDM sistemlerine uygulanmış ve kanal parametre kestirimi için kesin analitik sonuçlar elde edilmiştir.

OFDM sinyalleşme sönümlemeli kanal ve çokyolluluğun etkilerinin üstesinden gelmek için, kanalı uygun seçilmiş alt-taşıyıcı frekanslara karşılık gelen kanallara bölerek, iletişimi verimli bir biçimde gerçekleştiren bir yoldur. OFDM, şu anda, telsiz yerel alan ağ (WLAN) standartları (IEEE 802.11), ikinci tip ETSI yüksek performanslı yerel alan ağı (HIPERLAN/2) ve Japonya'nın mobil çokluortam erişim haberleşme sistemleri için kabul edilmiştir [1]. OFDM sistemlerde evre uyumlu kod çözümü için verici ve alıcı anten çiftlerinin arasındaki kanal durum bilgisi gereklidir. Bu amaçla kanal parametresi kestirimi için farklı teknikler önerilmiş bulunmaktadır; tekil değer ayrıştırması veya frekans bölgesi süzgeçlemesine dayalı kanal kestirim teknikleri [2][3] ve zaman bölgesi süzgeçleme [4] teknikleri bunlara örnek olarak verilebilir. Son yıllarda, kanal kestirimcisinin performansını daha iyiye götürmek için, zamanla değişen ayırgan(dispersive) kanalın, zaman-frekans ilintisini en iyi kullanan en küçük ortalama karesel hata (MMSE) kanal kestirimcisi önerilmiştir [5]. Bu teknik, sonradan, verici çeşitlemeli ve uzay zaman kodlaması kullanan OFDM sistemleri için genişletilmiştir [6].

Verici çeşitlemesi, mobil ve çokyollu, telsiz kanallardaki sönümlemeyle başa çıkmak için etkili bir yöntemdir. Son dönemlerde, yüksek veri hızında telsiz iletişim için, uzay-zaman kodlaması geliştirilmiş [7], ve OFDM sistemlere uygulanmıştır [8]. Bununla birlikte, uzay-zaman kodlarının çözümü, elde edilmesi güç olan, kanal bilgisini gerektirir. [8]'deki çalışmada ideal kanal durum bilgisine sahip olunduğu varsayılmıştır. Yakın zamanda, Alamouti [9], iletim için, iki verici antenin kullanıldığı, dikkate değer bir verici çeşitleme yöntemi önermiştir. Bu yöntem daha sonradan her hangi sayıda verici anteni için genelleştirilmiş [10][11], ve verici-alıcı anten çifti ile elde edilebilen en başarılı çeşitlemeyi gerçekleştirdiği görülmüştür. Uzay-zaman kodlarının dik yapısı, en büyük olabilirlik kod çözümünün, sinyalin birleşik algılanmasından daha basit olarak, farklı antenlerden iletilen sinyalin ayrıştırılması yoluyla uygulanmasını mümkün kılmaktadır. İlerleyen bölümlerde de gösterileceği gibi, uzay-zaman blok kodlama, kanal kestirimini oldukça kolaylaştırmaktadır.

Bu çalışmada, ilk defa Alamouti tarafından önerilmiş olan iki-verici çeşitlemeli OFDM sistemler için çokyollu sönümlemeli kanal kestirimine, Siala'nın yöntemi [12] uygulanmıştır. Algoritma, veri-yardımsız, beklenti enbüyükleme yöntemini kulanarak ve MAP ölçütlerini dikkate alarak, özyineli kanal kestirimi yapılmasına dayanmakta ve kanal kestiriminin eniyilenmesi sırasında pilot simgelerin yanında bilgi taşıyan simgelerden de faydalanılmaktadır.

Algoritma, OFDM alıcısı tarafından gözlemlenen ayrık, çokyollu, sönümlemeli kanalın Karhunen-Loeve (KL) dikey açılımı ile ifade edilmesini gerektirmektedir. Kanal kestirimcisi farklı frekanslardaki kanal frekans yanıtlarının birbirleriyle ilintisinden en yüksek düzeyde yararlanmaktadır. Bilgisayar benzetimleri, önerilen kanal kestirimine dayalı, evre uyumlu modülasyon çözümü gerçekleştiren OFDM sistemlerin performanslarının önemli şekilde iyileştiğini göstermektedir.

5.2.1 Alamouti'nin OFDM Sistemler için İletim çeşitleme Yöntemi

Bu çalışmada, Alamouti tarafından önerilen [9] uzay-zaman blok kodlanmış verici çeşitleme yöntemi, ikinci dereceden bir çeşitleme sağlamak amacıyla, 2 verici ve 1 alıcı anteni ile tanımlanan OFDM sistemlere uygulanmıştır.

Her n'inci zaman dilimi için, k'ıncı alt-taşıyıcı (ton) tarafından modüle edilmiş veri simgeleri $A_l(2n,k)$ ve $A_l(2n+1,k); k=0,1,\cdots,N-1$, iki antenden $l=1,2$, eş anlı olarak iletilirler. İletilen simgeler birim varyansa sahip ve farklı k ve n değerleri için bağımsız varsayılmaktadır. Sistemin performansını arttırmak için, evre uyumlu, faz kaydırmalı bir modülasyon (PSK) tekniği kullanılmıştır. Kanal sönümlemesinin ardışıl iki OFDM sembol süresince ($2T$) sabit, ancak farklı $2T$ aralıklarında değişken olduğu kabul edilmiştir. l'inci verici ile alıcı anten arasındaki kanal kazançlarının ayrık frekans cevabına dair vektörler, $\mathbf{H}_l(2n) = [H_l(2n,0), H_l(2n,1),\cdots,H_l(2n,N-1)]^T; l=1,2; n=0,1,\cdots,L-1$, Gauss sürecinin, frekans bölgesinde, ilintili örnek değerleridir ve şöyle ifade edilebilir;

$$\mathbf{H}_l(2n) = \Psi\ \mathbf{G}_l(2n) \tag{5.2.1}$$

burada $\mathbf{G}_l(2n)$, elemanları $\mathbf{G}_l(2n)[k] = G_l(2n,k)$ olan, $N\times 1$ boyutlu, sıfır ortalamalı i.i.d. Gauss vektörüdür ve kovaryans matrisi $\mathbf{\Lambda} = diag(\lambda_0,\lambda_1,\cdots,\lambda_{N-1})$ olarak tanımlanır. $\mathbf{G}_l(2n)$'i oluşturan elemanların varyansları, $\mathbf{r} = E\{\mathbf{H}_l(2n)\mathbf{H}_l^\dagger(2n)\}$ olarak tanımlanan ve $\mathbf{r}\psi_j = \lambda_j\psi_j$ eşitliğini sağlayan ayrık kanal özilinti matrisi $\mathbf{r}$'nin dikeyleştirilmiş özişlevlerini $\Psi = [\psi_0,\psi_1\cdots,\psi_{N-1}]$ kullanan Karhunen-Loeve (KL) dönüşümüne ait özdeğerlere $\{\lambda_j\}$ eşittirler.

Alamouti'nin kodlama yöntemi, k. ton için, tüm ardışıl $A(2n,k)$ ve $A(2n+1,k)$ sembollerini aşağıdaki 2×2 matrise eşlemler:

$$\begin{array}{cc} & uzay \longrightarrow \\ zaman\downarrow & \begin{bmatrix} A(2n,k) & A(2n+1,k) \\ -A^*(2n+1,k) & A^*(2n,k) \end{bmatrix} \end{array}$$

burada, belirli bir satırdaki birinci ve ikinci semboller sırasıyla birinci ve ikinci antenden aynı anda ve satırlar, ardışıl zaman aralıklarında iletilmektedirler. Alamouti'nin iki vericili ve bir alıcılı verici çeşitleme yönteminin kullanılması sonucunda, eğer alıcıda gözlemlenen sinyal dizisi, N ton'dan oluşan bloklara ayrılırsa, $\mathbf{R}(2n) = [R(2n,0),\cdots,R(2n,N-1)]^T$ and $\mathbf{R}(2n+1) = [R(2n+1,0),\cdots,R(2n+1,N-1)]^T$, ardarda gözlemlenen her sinyal vektörü çifti şu şekilde ifade edilebilir,

$$\begin{aligned}\mathbf{R}(2n)&=\mathbf{A}(2n)\mathbf{H}_1(2n)+\mathbf{A}(2n+1)\mathbf{H}_2(2n)+\mathbf{W}(2n)\\ \mathbf{R}(2n+1)&=-\mathbf{A}^{\dagger}(2n+1)\mathbf{H}_1(2n)+\mathbf{A}^{\dagger}(2n)\mathbf{H}_2(2n)+\mathbf{W}(2n+1)\end{aligned} \tag{5.2.2}$$

burada $\mathbf{A}(2n)$ ve $\mathbf{A}(2n+1)$, elemanları, sırasıyla, $\mathbf{A}(2n)[k,k] = A(2n,k)$ olan $\mathbf{A}(2n+1)[k,k] = A(2n+1,k)$, $N\times N$ köşegen matrislerdir. $\mathbf{W}(2n)$ ve $\mathbf{W}(2n+1)$, N tondaki toplanır gürültüyü modelleyen, $N\times 1$, sıfır-ortalamalı ve σ^2 varyanslı i.i.d. Gauss vektörleridir.

Her n için, $\mathbf{R} = [\mathbf{R}^T(2n)\ \ \mathbf{R}^T(2n+1)]^T$ tanımlanır ve (2) matris formda yazılır

$$\mathbf{R} = \mathbf{A}\ \mathbf{H} + \mathbf{W} \tag{5.2.3}$$

burada $\mathbf{H} = [\mathbf{H}_1^T(2n)\ \ \mathbf{H}_2^T(2n)]^T$, $\mathbf{W} = [\mathbf{W}^T(2n)\ \ \mathbf{W}^T(2n+1)]^T$ ve

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}(2n) & \mathbf{A}(2n+1) \\ -\mathbf{A}^*(2n+1) & \mathbf{A}^*(2n) \end{bmatrix}. \tag{5.2.4}$$

olarak tanımlanmıştır.

5.2.2 EM-Tabanlı MAP Kanal Kestirimi

Rastgele değişkenlerin ortak olasılık yoğunluk işlevi alıcı tarafından bilindiği ve,

$$p(\mathbf{G}) \sim \exp(-\mathbf{G}^{\dagger}\widetilde{\Lambda}^{-1}\mathbf{G})\ , \tag{5.2.5}$$

şeklinde ifade edilebildiği için, -burada $\mathbf{G} = [\mathbf{G}_1^T,\mathbf{G}_2^T]^T$ ve $\widetilde{\Lambda} = diag(\Lambda\ \Lambda)$'dır-, OFDM alıcının FFT çıkışında görüleceği üzere, sönümlemeli kanalda MAP kriterleri kullanılmıştır. İletilen $\mathbf{A}$ sinyallerinin Alamouti'nin yöntemine göre kodlandığı ve $\mathbf{G}$ ayrık kanal gösterimleri gözönünde tutulursa, gürültü bileşenlerinin bağımsız olmalarından faydalanılarak, alıcıda gözlemlenen $\mathbf{R}$ sinyalinin koşullu olasılık yoğunluk işlevi şöyle tanımlanabilir;

$$p(\mathbf{R}|\mathbf{A},\mathbf{G}) \sim \exp\left[-(\mathbf{R}-\mathbf{A}\widetilde{\Psi}\mathbf{G})^{\dagger}\widetilde{\Sigma}^{-1}(\mathbf{R}-\mathbf{A}\widetilde{\Psi}\mathbf{G})\right] \tag{5.2.6}$$

burada $\widetilde{\Sigma} = diag(\Sigma\ \Sigma)$ ve Σ elemanları $k = 0,1,\cdots,N-1$ için $\Sigma[k,k] = \sigma^2$ olarak tanımlanmış $N\times N$ köşegen matris, ve $\widetilde{\Psi} = diag(\Psi\ \Psi)$'dir.

G'nin MAP kestirimi $\widehat{\mathbf{G}}_{map} = \arg\max_{\mathbf{G}} p(\mathbf{G}|\mathbf{R})$ olarak belirlenmiştir. Bu denklemin doğrudan çözümü oldukça karmaşıktır. Buna karşılık çözüm, özyineli EM algoritması ile kolayca elde edilebilir. Bu algoritma tümevarımsal olarak $\mathbf{G}$ için yeni bir kestirim yapar, böylece

(5.2.7)deki sonsal koşullu olasılık yoğunluk işlevinin tekdüze(monotone) artışı gerçekleştitilir. Tekdüze artışın gerçeklenmesi,

$$Q(\mathbf{G}|\mathbf{G}^{(m)}) = \sum_{\mathbf{A}} p(\mathbf{R},\mathbf{A},\mathbf{G}) \log p(\mathbf{R},\mathbf{A},\mathbf{G}^{(m)}) \tag{5.2.7}$$

yardımcı fonksiyonunun enbüyüklenmesi ile sağlanır; burada $\mathbf{G}^{(m)}$, $\mathbf{G}$'nin m'inci iterasyondaki kestirimidir.

$\mathbf{A} = \{A_l(n,k)\}$ veri sembollerinin birbirlerinden bağımsız ve özdeşçe dağıldığı varsayıldığı ve $\mathbf{A}$'nın $\mathbf{G}$'den bağımsız olduğu için $p(\mathbf{R},\mathbf{A},\mathbf{G}) \sim p(\mathbf{R}|\mathbf{A},\mathbf{G})p(\mathbf{G})$ olur. Böylece (5.2.7), (5.2.5) ve (5.2.6) kullanılarak hesaplanabilir.

Gözlemlenen $\mathbf{R}$ sinyali için EM algoritması bilinmeyen $\mathbf{G}$ kanal parametrelerine ilişkin $\mathbf{G}^0$ başlangıç değerini kullanarak, $\mathbf{G}$'nin $(q+1)$'inci kestirimini $\mathbf{G}^{(q+1)} = \arg\max_{\mathbf{G}} Q(\mathbf{G}|\mathbf{G}^{(q)})$ enbüyükleme adımıyla hesaplar.

İlkdeğerleme: Bilinmeyen kanal parametrelerine dair uygun başlangıç değerleri seçmek için her OFDM çerçevesindeki N_{PS} veri simgeleri $\{A(2n,k)\}$ $A(2n+1,k)$; $k \in S_{PS}$, alıcıda bilinen pilot simgeler olarak kullanılır. Kanal kestirimlerini aradeğerlemek için pilotlar arasında, ilk başta, $l_{SC} < 1/\tau_{max}$ ile verilen ve kanalın frekans bölgesindeki en büyük gecikme dağılımını gösteren τ_{max} ile ters orantılı, minimum alt-taşıyıcı aralığı l_{SC} bulunmaktadır. Bu nedenle PSK modülasyonlu bir işaret kümesi için, kanal parametrelerinin başlangıç değerleri $\mathbf{G}_l^{(0)}(2n)$ $l = 1,2$ şu veri destekli yöntemle seçilebilir:

$\mathbf{H}_l^p(2n)$, $l = 1,2$, için elemanları $\mathbf{H}_l^p(2n)[k] = H_l(2n,k)$ olan $N_{PS} \times 1$ boyutlu bir vektör ve bunun sonucu olarak $k \in S_{PS}$ frekanslarında kanal kazancı olduğu düşünülürse, ardışıl iki OFDM simgesindeki $2N_{PS}$ pilot veri simgeleri kullanılarak $\widehat{\mathbf{H}}_l^p(2n)$ için [2]'da belirtildiği gibi,

$$\widehat{\mathbf{H}}_l^p(2n) = \mathbf{\Psi}_p \mathbf{\Delta}_p \mathbf{\Psi}_p^\dagger \widehat{\mathbf{H}}_{l,ls}^p(2n) \tag{5.2.8}$$

olur, burada $\widehat{\mathbf{H}}_{l,ls}^p(2n)$, $\mathbf{H}_l^p(2n)$'nin en küçük karesel kestirimi ([2], sayfa 932), $\mathbf{\Psi}_p$ elemanları $r_p[k,k'] = r(k,k'), k,k' \in S_{PS}$ olan, $N_{PS} \times N_{PS}$ boyutlu $\mathbf{r_p}$, kanal kovaryans matrisinin özdeğerlerinden oluşan birimcil matris, $\mathbf{\Delta}_p$ ise elemanları $\delta_k = 1/(1+\sigma^2/\mu_k)$ olan köşegen matrisdir; burada μ_k'lar $\mathbf{r_p}$'nün özdeğerleridir. $2N_{PS}$ kanal kestirimli örnekler $\widehat{\mathbf{H}}_l^p[k], k \in S_{PS}$ sayesinde tüm başlangıç kanal kazançları $H_l^0(k), k = 0,1,\cdots,N-1$ Lagrange aradeğerleme algoritması gibi bir aradeğerleme tekniğiyle, kolayca bulunabilir. Sonuç olarak $\mathbf{G}_l^{(0)}(2n)$'ın başlangıç değerleri $\mathbf{G}_l^{(0)}(2n) = \mathbf{\Psi}^\dagger \mathbf{H}_l^{(0)}(2n)$ şeklinde bulunur.

Pilot simgeler gözönünde tutularak yapılan hesaplamalardan sonra, $\mathbf{G}_l^{(i+1)}(2n)$ $(l = 1,2)$ yeni kestiriminin ifadesi,

$$\mathbf{G}_1^{(i+1)} = (\mathbf{I} + \mathbf{\Sigma}\mathbf{\Lambda}^{-1})^{-1}\mathbf{\Psi}^\dagger \left[\mathbf{V}_1^{(i)}\mathbf{R}(2n) - \mathbf{V}_2^{\dagger(i)}\mathbf{R}(2n+1)\right]$$

$$\mathbf{G}_2^{(i+1)} = (\mathbf{I} + \Sigma\Lambda^{-1})^{-1}\Psi^\dagger \left[\mathbf{V}_2^{(i)}\mathbf{R}(2n) - \mathbf{V}_1^{\dagger(i)}\mathbf{R}(2n+1)\right] \qquad (5.2.9)$$

şeklinde bulunur. Bu ifadede $(\mathbf{I} + \Sigma\Lambda^{-1})^{-1} = diag([(1+\sigma^2/\lambda_0)^{-1}, \cdots, (1+\sigma^2/\lambda_{N-1})^{-1}$ ve $\mathbf{V}_l^{(i)} = diag[v_l^{(i)}(0), v_l^{(i)}(1), \cdots, v_l^{(i)}(N-1)]$ buadaki $v_l^{(i)}(k)$ şöyle verilmiştir:

$$v_1^{(i)}(k) = \begin{cases} A(2n,k); & \text{if } k \in S_{PS} \\ \Gamma_1^{(i)}(k); & \text{if } k \in S_{PS}^c \end{cases}, \qquad v_2^{(i)}(k) = \begin{cases} A(2n+1,k); & \text{if } k \in S_{PS} \\ \Gamma_2^{(i)}(k); & \text{if } k \in S_{PS}^c \end{cases}.$$

$k = 0, 1, \cdots, N-1$ için, veri simgelerinin i. iterasyon adımındaki *sonsal* olasılıklarını ifade eden $\Gamma_l^i(k)$,

$$\Gamma_l^{(i)}(k) = \sum_{a_1,a_2 \in S_k} a_l^* P(A(2n,k) = a_1, A(2n+1,k) = a_2 | \mathbf{R}, \mathbf{G}^{(i)}) \qquad (5.2.10)$$

olarak tanımlanır, S_k ise k. OFDM simgesinin aldığı işaret kümesidir.

5.2.2.1 QPSK Sinyalleşme için $\Gamma_l^{(q)}(k)$'nın hesaplanması:

QPSK taşıyıcısını modüle eden veri dizisi $S_l(k)$, bağımsız ve eşit olasılıklı olarak üretilen $a = (\pm 1 \pm j)$ sinyallerinden oluşsun. Veri dizisi $s_l(k)$, $l = 1, 2$ ve $k = 0, 1, \cdots, N-1$ için bağımsız olduğundan (10)'daki $\Gamma_m(k)$ aşağıdaki gibi hesaplanabilir:

$$\Gamma_l^{(i)}(k) = \tanh\left[\frac{2}{\sigma^2} Re(Z_l^{(i)}(k))\right] - j\tanh\left[\frac{2}{\sigma^2} Im(Z_l^{(i)}(k))\right] \qquad (5.2.11)$$

burada

$$Z_1^{(i)}(k) = R(1,k)\sum_m G_1^{(i)*}(m)\psi_m^*(k) + R^*(2,k)\sum_m G_2^{(i)}(m)\psi_m(k)$$
$$Z_2^{(i)}(k) = R(1,k)\sum_m G_2^{(i)*}(m)\psi_m^*(k) - R^*(2,k)\sum_m G_1^{(i)}(m)\psi_m(k)\ .$$

5.2.2.2 Rastlantısal $\{\mathrm{G}_l(m)\}$ Parametrelerinin Kestirimi için değiştirilmiş Crámer-Rao Sınırı (MCRB)

$l = 1, 2$ ve $m = 0, 1 \cdots, N-1$ için $\{G_l(m)\}$'ler kestirilecek rastlantısal parametreler olsun. Buna göre, bazı hesaplamalar sonrasında değiştirilmiş Crámer-Rao sınırı

$$MCRB(G_l(m)) = 2(\frac{1}{\lambda_m} + \frac{1}{\sigma^2})$$

biçiminde elde edilir. Burada σ^2 gürültü varyansı, λ_m ise çokyollu sönümlemeli kanala ait ayrık özilişki fonksiyonu $r(k, k')$nın özdeğerleridir.

5.2.3 Benzetim Sonuçları

Bu bölümde OFDM sistemler için EM algoritması ile kanal parametrelerinin kestirimine dair benzetim sonuçları sunulmaktadır. Benzetim için güç gecikme profili üstel olarak azalan $\{\theta(\tau_l) = C\exp(-\tau_l/\tau_{max})\}$ çokyollu sönümlemeli kanal kullanılmıştır. τ_l gecikmeleri çevrimsel önek (cyclic prefix) boyunca düzgün ve bağımsız dağılmışlardır. C normalizasyon sabitidir. Bu kanal modelinin farklı alt taşıyıcıları için normalize edilmiş ayrık kanal ilintisi [2] de sunulmuştur;

$$r_2(k,k') = \frac{1-\exp\left[-L(\frac{1}{\tau_{rms}} + 2\pi j(k-k')/N)\right]}{\tau_{rms}\left(1-\exp(-L/\tau_{rms})\right)\left(\frac{1}{\tau_{rms}} - j2\pi(k-k')/N\right)}$$

ayrıca farklı blok için ayrı kanal ilintileri (9)da şöyle verilmiştir;

$$r_1(n,n') = J_0(2\pi(n-n')f_dT_s$$

burada J_0, sıfırıncı dereceden, birinci tip Bessel fonksiyonu, ve f_d Doppler frekansıdır. Benzetimde kullanılan senaryo, 500 kHz band genişliğinde çalışan, 64 tona bölünmüş ve $8\mu s$'si çevrimsel önek (L=4) olan toplam $136\mu s$ sembol süresine (T_s) sahip, telsiz QPSK OFDM sistemi içermektedir. Sistemin kodlanmamış veri hızı 0.95 Mbit/s olarak belirlenmiş, güç gecikme profili için rms süresi $\tau_{rms} = 1$ örnek $(2\mu s)$ varsayılmış, ve doppler frekansı $f_d = 100$ Hz kabul edilmiştir.

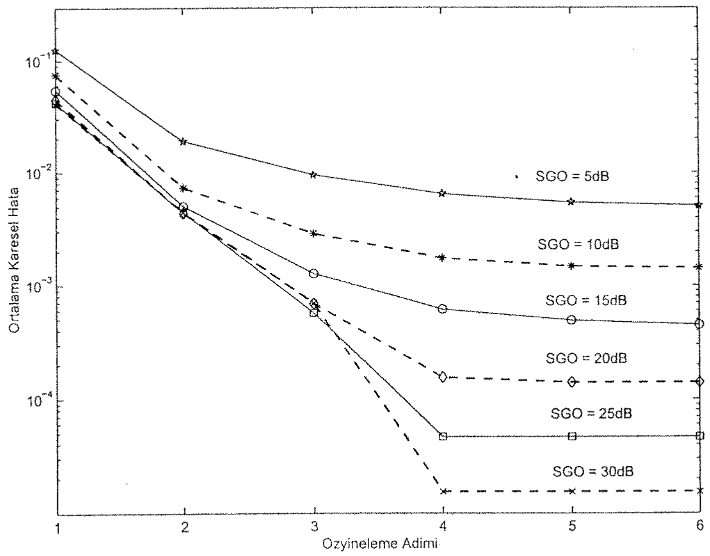
Şekil5.2.1'de EM tabanlı algoritmanın MSE performansı, ortalama SNR'ın fonksiyonu olarak görülmektedir. Ortalama SNR $E[|H(n,k)|^2\}E[|A(n,k)|^2]/\sigma^2$ olarak tanımlanmıştır. QPSK sinyalleşme için $E[|A(n,k)|^2] = 1$ ve sönümlemeli kanalın normalize edilmiş frekans yanıtı için $E[|H(n,k)|^2] = 1$ olduğundan dolayı, düzgelenmiş SNR, $1/\sigma^2$ olur, buradaki σ^2 sisteme giren karmaşık beyaz Gauss gürültüsünün varyansıdır. $G^{(0)}(n,k)$'nın başlangıç değeri (16)daki gibi seçilmiştir. Ortalama karesel hata (MSE), gerçek $\{\mathbf{G} = [G(n,k)]\}$ ve kestirilmiş $\{\widehat{\mathbf{G}} = [\widehat{G}(n,k)]\}$ kanal parametrelerini gösteren matrisler arasındaki farkın normu olarak tanımlanmıştır;

$$MSE = ||\mathbf{G} - \widehat{\mathbf{G}}|| = \sqrt{\sum_{n=0}^{N-1}\sum_{k=0}^{K-1}(G(n,k) - \widehat{G}(n,k))^2}$$

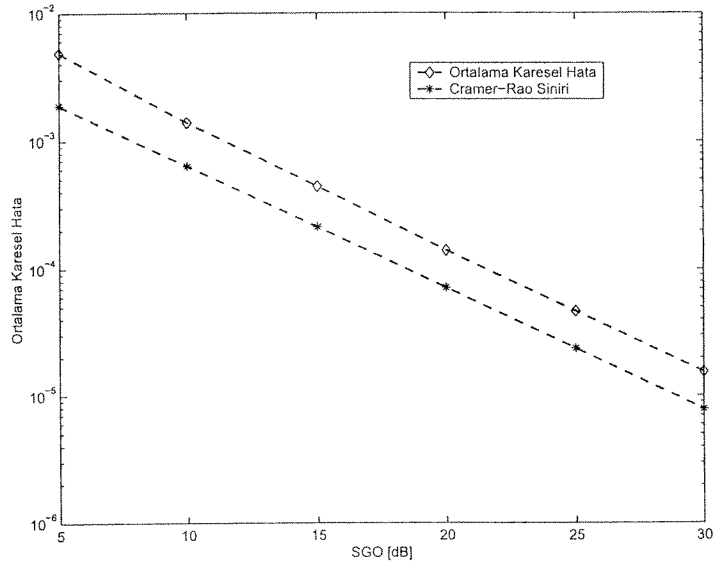
Şekil 5.2.2'de EM tabanlı algoritmanın MSE performansı, özyine sayısının fonksiyonu olarak görülmektedir. şekil 5.2.1'den EM tabanlı algoritmanın MSE performansının, SNR'a bağlı olarak 3-10 iterasyon arasında yakınsadığı sonucuna varılmaktadır.

5.2.4 Sonuç

Bu çalışmada, OFDM sistemler için bir optimum kanal kestirim algoritması önerilmiştir. Bu algoritma, MAP kriterlerine göre kanalın özyineli kestirimini yapmakta ve bunu yaparken toplamsal Gauss gürültülü M-PSK modülasyonu kullanan EM algoritmasından faydalanmaktadır. Ayrık çokyollu iletişim kanalı, Karhunen-Loeve açınımı ile ifade edilmiş ve böylece zamanla değişen ayırgan sönümlemeli kanalın frekans yanıtına ait zaman ve frekans bölgesi ilişkilerinden yararlanılmıştır. Zamanla değişen kanalın kestirimi için, M-PSK işaret gösterilimi üzerinden ortalama alınarak, veri-desteksiz bir kestirim yöntemi ele alınmıştır. Bu amaçla her alt taşıyıcı için frkans bölgesinde karmaşık kanal parametrelerini kestiren ve bilinmeyen kanalın gerçek MAP kestirimine yakınsayan bir EM algoritması türetilmiştir. Algoritma QPSK ile modüle edilmiş OFDM sistemine uygulanmıştır. Önerilen algoritmanın verimliliği bilgisayar benzetimleriyle gösterilmiştir.



Şekil 5.2.1: Yakın-Alan Senaryosu



Şekil 5.2.2: Kestirilen Açıklık, Yükseklik Açısı ve Uzaklık Parametrelerine ait ortalama karesel hataların Sinyal Gürültü Oranı ile değişimi.

5.3 OFDM Sistemlerinde Kanal Kestirimi İçin Koşulsuz En Büyük Olabilirlik Yaklaşımı

5.3.1 Giriş

Önümüzdeki yıllarda telsiz haberleşme sistemlerinde yüksek haberleşme hızı ihtiyacının en üst seviyeye çıkacağı oldukça kabul gören bir düşüncedir. Elde edilen büyük gelişmelere karşın arzu edilen en üst seviye data hızları, telsiz haberleşme sistemlerinin doğası gereği ortaya çıkan çok yollu yayılım ve istenmeyen hücre içi ve hücreler arası karışma nedeniyle sınırlandırılmaktadır. Çok-taşıyıcılı ya da ayrık çok-tonlu modülasyon olarak da adlandırılan OFDM, bir kullanıcıdan başka bir kullanıcıya bilgi iletmek için çok sayıda alttaşıyıcıdan yararlanır. OFDM tabanlı bir sistem yüksek hızlı seri bilgiyi çok sayıda daha az hızlı alt işaretlere böler öyle ki sistem bilgiyi paralel olarak farklı frekanslarda eşzamanlı olarak iletir. OFDM' in üstünlüğü, RF ve daha düşük çok yol bozunumuna karşı esnekliğidir.OFDM' in dik doğası, izgesel etkinlik üzerinde pozitif katkıya sahip olarak altkanalların örtüşmesine izin verir. Bilgi taşıyan alt taşıyıcılardan her biri teorik olarak karışmadan etkilenmeyecek biçimde birbirinden uzaktadır.

OFDM sistemlerinde iletilen işaretler ile kanal parametrelerinin gözü kapalı kestirimi oldukça önemli bir problem olarak karşımıza çıkmaktadır. Bu problemin çözümü için koşullu işaret modeline dayalı kestirim algoritmaları (örneğin sayısal işaretleri deterministik diziler gibi ele almak) [16], [17], [18] ileri sürülmüştür. Koşullu işaret modeline dayalı kestirim yöntemine karşılık, iletilen işaretler, olasılıksal IID dizileri olarak ele alınan koşulsuz en büyük olabilirlik yaklaşımı bu bildiride sunulmuştur. Ortaya çıkan koşulsuz en büyük olabilirlik maliyet işlevinin etkin çözümü ise özyineli sabit nokta algoritması ile elde edilmiştir. Ayrıca, bu yöntem kanal parametrelerini en büyük olabilirlik kestirimi ile, modülasyonlu işaretlerin en büyük sonsal ortak kestirimini vermektedir.

5.3.2 OFDM İşaret Modeli

Bir OFDM sistemi elde edilebilen band genişliğini N tane örtüşen dar frekans bandına böler. Etkin sembol uzunluğu, T_s sistemin örnekleme uzunluğu olmak üzere, $T = NT_s$' dir. $g(\tau; t)$ kanalının yavaş sönümlü olduğu varsayılmakta ve bir OFDM sembolü boyunca sabit olduğu düşünülmektedir. Sistemde, T_{cp} uzunluklu çevrimsel öntakı kullanmak altkanalların dikliğini korur ve ardışık OFDM işaretleri arasındaki sembollerarası girişimi ortadan kaldırır. Bu durumda sistemi paralel Gauss kanalların bir kümesi olarak tanımlayabiliriz. Altkanal k

üzerindeki alınan işaret

$$y_k = h_k x_k + v_k, \quad k = 0 \ldots K-1, \tag{5.3.1}$$

şeklinde tanımlanır. Burada x_k iletilen sembol, v_k toplanır kanal gürültüsü,

$$h_k = G\left(\frac{k}{KT_s}; t\right), k = 0 \ldots K-1, \tag{5.3.2}$$

k'ıncı alt taşıyıcıdaki zayıflama ve $G(f;t)$ t anında OFDM sembolü süresince $g(\tau;t)$ kanalının frekans yanıtıdır. (5.3.1)'deki alınan işaret modeli matris formunda yazıldığında

$$\mathbf{y}(n) = \mathbf{H} \cdot \mathbf{x}(n) + \mathbf{v}(n) \qquad 0 < n < N-1 \tag{5.3.3}$$

Burada $\mathbf{y}(n) = [y_0(n), \cdots, y_{K-1}(n)]^T$, $\mathbf{x}(n) = [x_0(n), \cdots, x_{K-1}(n)]^T$ ve

$$\mathbf{H} = \begin{bmatrix} h_0 & 0 & \cdots & 0 \\ 0 & h_1 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & h_{K-1} \end{bmatrix} \tag{5.3.4}$$

olarak verilmiştir.

Bu bildiride ele alınan temel problem, toplanır Gauss gürültüsü tarafından bozulmuş işaretten $\boldsymbol{y} = [\mathrm{y}(0) \quad \mathrm{y}(1) \ldots \mathrm{y}(N-1)]^T$, kanal parametrelerinin $\boldsymbol{\theta} = [h(0) \quad h(1) \ldots h(N-1)]^T$ kestirimidir. Problemin çözümünde asimptotik olarak etkin, koşulsuz en büyük olabilirlik kestirim yöntemi kullanılacaktır. Bu amaçla öncelikle problemin koşulsuz işaret modeli geliştirilecektir.

5.3.2.1 Koşulsuz İşaret Modeli

Koşullu ve koşulsuz işaret modelleri arasındaki tek farklılık $\mathbf{x}(n)$ işaret vektörleri ile ilgili varsayımlardır. Koşullu işaret modelinde işaret vektörleri bilinmeyen fakat deterministik büyüklükler olarak ele alınırlar ve bilinmeyen parametreler kümesinin bir parçasını oluştururlar. Böylece, bilinmeyen parametrelerin sayısı data vektörlerinin sayısının artmasıyla birlikte artar. Bu da tutarsız kestirimlere yol açar. Koşullu işaret modeline karşılık, koşulsuz işaret modelinde işaret vektörleri rastgele büyüklükler olarak ele alınırlar ve parametre kümesine dahil edilmezler. Bunun bir sonucu olarak, bilinmeyen parametrelerin sayısı sabittir ve tutarlı kestirimler elde etmek olasıdır. Elimizdeki problem için işaret vektör elemanları ± 1 (BPSK) gibi düşünülecektir. $S = \{s_m\}, \quad m = 1, \ldots, 2^K$, ± 1 in olası bütün K elemanlı vektörlerinin kümesi olsun öyle ki S, K boyutlu sinyal uzayını temsil etsin.

İşaretin sonlu yapısı ve gürültü vektörlerinin beyazlığı kullanılarak, data matrisi **A**'nın olasılık yoğunluk işlevi

$$f_{\mathbf{H}}(\mathbf{A}) = \frac{1}{2^{KN}(\pi\sigma^2)^{KN}} \prod_{n=1}^{N} \sum_{m=1}^{2^K} exp\left\{-\frac{\| \mathbf{y}(n) - \mathbf{H}s_m \|^2}{\sigma^2}\right\} \tag{5.3.5}$$

şeklinde yazılır.

A'nın olasılık yoğunluğunun karmaşık normal yoğunlukların sonlu bir karışımı olduğu görülmektedir. Burada $f_H(\cdot)$ dağılımları bilinmeyen $\mathbf{H} \in \mathcal{C}^{K\times 1}$ matrisi tarafından parametrelendirilmiştir.

(5.3.5)'deki koşulsuz yoğunluk işlevinden negatif logaritmik olabilirlik işlevi aşağıdaki gibi elde edilir:

$$\mathcal{L}(\mathbf{H}) = -\prod_{n=1}^{N} \log \sum_{m=1}^{2^K} exp\left\{-\frac{\| \mathbf{y}(n) - \mathbf{H}s_m \|^2}{\sigma^2}\right\} + sabit\ . \tag{5.3.6}$$

Burada **H**'nın koşulsuz en büyük olabilirlik kestirimi L(**H**)'ın global en küçültenidir. Oldukça doğrusal olmayan böyle bir maliyet işlevi (5.3.6) için global yakınsak algoritmaların bulunması oldukça güçtür. Buna rağmen birinci derece olabilirlik denklemlerine

$$\frac{\partial \mathcal{L}(\mathbf{H})}{\partial \mathbf{H}} = 0 \tag{5.3.7}$$

dayanan yöresel yakınsak bir algoritma formülüze edilebilir. Burada $\partial\mathcal{L}(\mathbf{H})/\partial\mathbf{H}$'ın i,j'inci elemanı $\partial\mathcal{L}(\mathbf{H})/\partial\mathbf{H}_{i,j}$'dir.

Sabit nokta özyineleme algoritması yardımıyla **H**'ın koşulsuz en büyük olabilirlik kestirim ifadesi

$$\mathbf{H}_u \left(\sum_{n=1}^{N} \sum_{m=1}^{2^K} p_m(n)\mathbf{s}_m\mathbf{s}_m^T \right) = \sum_{n=1}^{N} \sum_{m=1}^{2^K} p_m(n)\mathbf{y}(n)\mathbf{s}_m^T \tag{5.3.8}$$

şeklinde yazılabilir [17]. Burada

$$p_m(n) = \frac{\exp\{-\frac{1}{\sigma^2} \| \mathbf{y}(n) - \mathbf{H}\mathbf{s}_m \|^2\}}{\sum_{l=1}^{2^K} \exp\{-\frac{1}{\sigma^2} \| \mathbf{y}(n) - \mathbf{H}\mathbf{s}_l \|^2\}} \tag{5.3.9}$$

$\mathbf{x}(n)$ verildiğinde $\mathbf{x}(n) = \mathbf{s}_k$ sonsal olasılığıdır.

(5.3.7)'deki doğrusal olmayan denklem kümesinin sabit nokta yaklaşımı ile özyineli çözümünün adımları aşağıda sıralanmıştır.

5.3.2.2 Önerilen Algoritma

Sabit nokta tekniği (SNT)

1. $\mathbf{H}^{(0)}$'ın başlangıç kestirimi verilsin,

2. $i = 1, 2, \ldots,$ için

$$p_m^{(i)}(n) = \frac{\exp\{-\frac{1}{\sigma^2} \parallel \mathbf{y}(n) - \mathbf{H}^i \mathbf{s}_m \parallel^2\}}{\sum_{l=1}^{2^K} \exp\{-\frac{1}{\sigma^2} \parallel \mathbf{y}(n) - \mathbf{H}^i \mathbf{s}_l \parallel^2\}} \tag{5.3.10}$$

olmak üzere

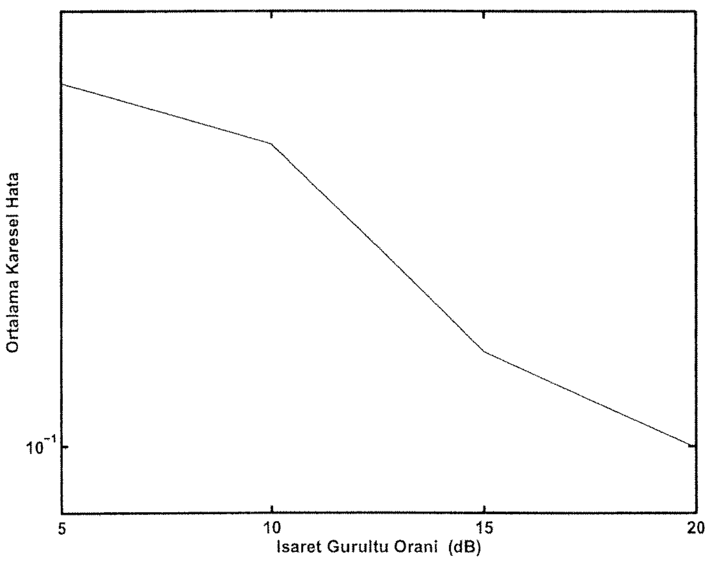
$$\mathbf{H}_u^{(i+1)} \left(\sum_{n=1}^{N} \sum_{m=1}^{2^K} p_k^{(i)}(n) \mathbf{s}_m \mathbf{s}_m^T \right) = \sum_{n=1}^{N} \sum_{m=1}^{2^K} p_m^{(i)}(n) \mathbf{y}(n) \mathbf{s}_m^T \tag{5.3.11}$$

(5.3.11)'deki denklem setinden $\mathbf{H}_u^{(i+1)}$'yı hesaplayın.

3. $\mid \mathcal{L}(\mathbf{H}^{(i+1)}) - \mathcal{L}(\mathbf{H}^{(i)}) \mid < \epsilon$ oluncaya kadar ikinci basamağı tekrar edin. Burada ϵ önceden tanımlanmış bir tolerans değişkenidir.

4. $n = 1, \ldots, N$, için $m_n = argmax_m p_m^f(n)$'yi bulun. Burada $p_m^f(n)$ nihai sonsal olasılığı gösterir. İşaret vektörlerinin en büyük sonsal kestirimleri $\mathbf{x}(n) = \mathbf{s}_m$'dir.

Önerilen algoritmanın detayları ve başarım analizi tam bildiride sululacaktır.



Şekil 5.3.1: Önerilen Algoritmanın Başarımı

5.3.3 Benzetim Örneği

Önerilen yöntemin uygulanabilirliğini göstermek için bu bölümde bir benzetim örneği verilmiştir. 12 alt kanal ve 2 çevrimsel öntakıdan oluşan BPSK OFDM sistemi gözönüne alınmıştır. Kanala ait güç gecikme profilinin r.m.s. genişliği $\tau_{rms} = 0.2\mu s$ olarak seçilmiş ve h_m ve h_n zayıflamaları arasındaki kanal ilintisi aşağıdaki şekilde ifade edilmiştir:

$$r_{m,n} = \frac{1 - e^{-L((1/\tau_{rms})+2\pi j(m-n)/N)}}{\tau_{rms}(1 - e^{-(L/\tau_{rms})})(\frac{1}{\tau_{rms}} + j2\pi\frac{m-n}{N})} \qquad (5.3.12)$$

Önerilen sabit nokta özyineleme kanal kestirim algoritması 0dB-20dB aralığındaki işaret gürültü oranları için sınanmıştır ve elde edilen sonuçlar Şekil 1'de sunulmuştur. Bu şekilden ortalama karesel hatanın yüksek işaret gürültü oranları için azaldığı gözlemlenmiştir.

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Bölüm VI

SONUÇLAR

Bu projede, günümüzde çok önem kazanan gezgin-telsiz (mobil/wireless) iletişim alanında son yıllarda yaygın olarak kullanılmaya başlanan uzay-zaman kodlama tekniği ile OFDM teknikleri birleştirilerek yeni bir "uzay-zaman kodlamalı OFDM tümleşik geniş bandlı gezgin iletişim sistemi"nin verici ve alıcı kısımlarınının tasarlanması gerçekleştirilmiş ve böyle bir sistem için gerekli bir takım yeni ve hızlı eşzamanlama (senkronizasyon) ve kanal kestirim algoritmaları geliştirilmiştir.

Gezgin ve telsiz iletişim sistemleri (uydu iletişimi, gezgin radyo, "indoor" iletişim), iletişim kanalının neden olduğu toplamsal Gauss gürültüsüne ek olarak sönümlemeden(fading) ve faz seyirmesinden de büyük ölçüde etkilenmektedir. Bu sönümleme etkisini azaltmanın etkin bir yolu da çeşitleme (diversity) yöntemlerinden yararlanmaktır. Bu amaçla son yıllarda, "uzay-zaman kodlama (space-time coding)" adıyla yeni bir teknik ortaya atılmış ve kodlamanın, modülasyonun ve çeşitlemenin optimum bir şekilde birleştirildiği bu yöntemle tasarlanmış gezgin iletişim sistemlerinin başarımlarında büyük iyileşmeler sağlandığı görülmüştür. Diğer taraftan geniş bandlı gezgin iletişim sistemlerinde, özellikle frekans seçici kanallar üzerinden iletimde yüksek başarımlarından dolayı, "çok taşıyıcılı (multicarrier)" sistemler günümüzde yaygın olarak kullanılmaya başlamıştır. Bu sistemlerin OFDM diye adlandırılan versiyonu, tüm iletişim kanalını belli sayıda alt-kanallara bölerek bilgiyi birbirine dik(orthogonal) seçilmiş alt-taşıyıcı frekanslarla iletme ilkesine dayanmaktadır. Ancak, OFDM sistemlerinin gerek taşıyıcı frekans ve gerekse faz kaymalarına çok duyarlı olduğu bilinmektedir. Bu nedenle OFDM sistemlerinde frekans ve faz eşzamanlama probleminin bu duyarlığı da aşacak şekilde çözülmesi gerekir.

Yukarıda verilen bilgilerin ışığı altında bu projede

- uzay-zaman kodlama tekniği ile OFDM tekniğini birleştirerek ve sürekli faz modülasyonunu (Minimum Shif Keying) da kullanarak, yeni bir tümleşik geniş bandlı "Uzay-zaman Kodlamalı OFDM Gezgin İletişim Sisteminin verici ve alıcı kısımları tasarlanmış,
- böyle bir sistemin, özellikle frekans seçici kanallar üzerinden iletişim yapması durumunda başarım analizi, analitik yöntemlerle ve bilgisayar benzetimleri ile gerçekleştirilmiş,

- frekans seçici ve sönümlemeli kanallarda, uzay- zaman kodlama tekniğinin sağladığı çeşitlemenin olumlu etkileri ile OFDM tekniğinin getirdiği sistemin frekans seçici etkilere bağışıklığının incelenerek klasik sistemlerle karşılaştırılmış ve
- OFDM sistemleri için, yeni frekans ve faz eşzamanlama algoritmaları ve alıcılarının tasarımı için gerekli bir takım yeni kanal kestirim algoritmaları geliştirilmiştir.

Böylece, bu projede, hem çeşitleme sağlayan ve hem de kanalın frekans seçiciliğine duyarlı olmayan yeni bir tümleşik iletişim sisteminin mimarisi ortaya çıkartılmıştır. Bu amaçlara şu aşamalardan geçerek varılmıştır.

1. Önce, çoklu kafes kodlu MSK modülasyonu OFDM tekniği ile tümleştirilerek özellikle sönümlemeli kanallar için yüksek başarıma sahip yeni bir iletişim sistemi önerilmiş ve çeşitli çoklu MSK kafes kodların OFDM sistemlerde hata başarımları bilgisayar benzetimleri yardımıyla incelenmiştir.
2. Daha sonra bu çalışma genelleştirilerek, MSK modülasyonunun band verimliliği ile uzay-zaman kodlarının güç verimliliğini biraraya getiren çoklu MSK modülasyonlu bir uzay-zaman kodlamalı sistemin tasarımı gerçekleştirilmiştir. Çalışmada, uzay-zaman kodlama tekniği MSK modülasyonuna uygulanmakta, iki verici ve bir alıcı anten için iki, dört ve sekiz durumlu uzay-zaman kodlamalı türlü çoklu MSK sistemler önerilmektedir. Bu sistemlerin tasarımlarında, düzgün ve yavaş sönümlemeli kanallarda uzay-zaman kodlarının tasarım ölçütlerini oluşturan rank ve determinant ölçütlerinin eniyileştirilmesi yoluna gidilmiş ve bu amaçla geliştirilen bir kod arama algoritmasından yararlanılmıştır. Önerilen kodların hata başarımları geliştirilen bir bilgisayar benzetim programı yardımıyla incelenmiş, tek verici anten kullanılması ve her iki verici antende aynı MSK kafes kodunun kullanılması durumlarına olan üstünlükleri Rayleigh sönümlemeli kanallar için ortaya konmuştur.
3. Projede ele alınan diğer bir konu da, dik uzay-zaman kodlarında güç kontrolunun eniyi bir biçimde nasıl gerçekleştirileceğidir. Bu çalışmada, literatürde bir alıcı antenli dik uzay-zaman kodları için ortaya atılan güç kontrol yapısı birden fazla alıcı anten kullanan dik uzay-zaman kodlamalı iletişim sistemleri için genişletilerek sönümlemeli kanallarda yüksek hata başarımına sahip bir iletişim sistemi önerilmiştir. Ayrıca, kanal kazançlarının alıcıda hatalı kestirilmesi durumunda yüksek başarım sağlayan iki ve üç alıcı antenli iletişim yapıları tasarlanarak güç kontrolü uygulanmadığı duruma göre olan kazançları bilgisayar benzetimleri yardımıyla belirlenmitir.

4. Bu bağlamda, ilintili frekans-seçici sönümlemeli kanallar için dik uzay-zaman kodlamalı OFDM tümleşik bir iletişim sisteminin tasarımı ele alınmıştır. Çalışmada OFDM kullanan iletişim sistemlerinde uzay, zaman ve frekans çeşitleme teknikleri birlikte uygulanarak kanaldaki ilintili sönümleme ve toplamsal beyaz Gauss gürültüsüne karşın yüksek hata başarımına sahip bir tümleşik iletişim sistemi önerilmektedir. Bu yapıda uzay ve zaman çeşitlemesini sağlamak amacıyla son zamanlarda uygulamaları sıklıkla karşımıza çıkan dik uzay-zaman kodları kullanılmıştır. OFDM alt kanallarına ilişkin sönümleme etkilerinin ilintili olduğu durumda önerilen tümleşik iletişim sistemine ait bilgisayar benzetimleri yapılmış ve çeşitli sinyal-gürültü oranları için hata başarımları elde edilerek ilintisiz sönümleme durumuyla karşılaştırılmıştır.

5. Son olarak uzay-zaman kodlamalı OFDM tümleşik sistemi için bir takım yeni kanal kestirim algoritmaları geliştirilmiştir. Özellikle, uzay-zaman kodlanmış ve OFDM sinyaller tarafından uyarılmış bayılmalı (fading) kanalların kestirimi, telsiz iletişim sistemlerinin alıcılarının tasarımlarında büyük önem taşımaktadır. Bu tür sistemlerin alıcılarında demodülasyon ve sezim (detection) işlemleri ancak kanal parametrelerinin bilindiği varsayılarak gerçekleşebilmektedir. Çalışmalarda kanal kestirimi için şu iki temel yaklaşım izlenmiştir.

   a) Uzay-zaman kodlanmış sinyaller tarafından uyarılmış kanalların gözü-kapalı(Blind) kestiriminde koşulsuz en büyük olabilirlik işlevinin enküçültülmesine dayalı Baum-Welch algoritmasının kullanılmasının uygulamada çok yararlı olabileceği sonucuna ulaşılmıştır.

   b) OFDM Sistemler tarafından uyarılmış zamanla-değişen bayılmalı kanallar için EM-Tabanlı eğitim verilerine (Non-Data-Aided) gereksinim duymayan kanal kestirim yaklaşımı ile son derece hızlı, basit olarak gerçeklenebilir kanal kestirim algoritmaları tasarlanmıştır.

Projede Üretilen Önemli Sonuçlar

1. Projede teklif edilen uzay-zaman kodlama tekniği ile OFDM tekniğinin tümleştirildiği eniyi tümleşik telsiz iletişim sisteminin özellikle frekans bayılmalı seçici kanallar üzerinde yüksek bir başarımla çalıştığı sonucuna varılmıştır

2. Ortaya çıkan tümleşik sistemin frekans ve faz eşzamanlaması için eniyi olabilirlik yöntemi kullanılarak basit, iteratif ve işaret-gürültü oranından bağımsız algoritmalar geliştirilmiştir.

3. Yeni bir takım kanal kestirim algoritmaları geliştirilmiştir.

4. Proje sonuçlarının uygulamaya aktarılabilmesi için, Avrupa 6. Çerçeve Programı kapsamında Avrupa'da başlatılan çok geniş bir *Network of Excellence* Projesine (NEWCOM Projesi) IŞIK Üniversitesi de, bu konularda katkılar vermesi için, kabul edilmiştir. Ayrıca Türkiye içinde de yine bu konularla ilgili diğer bir *Nework of Excellence* projesi(SWIM Projesi) içinde de IŞIK Üniversitesi yer almıştır.

5. Bu projede elde edilen sonuçlardan ve bilgi birikiminden yararlanarak ileriye dönük çalışmalarda, özellikle 4. kuşak mobil iletişim sistemlerinin spektral verimi çok yüksek, çok taşıyıcılı tabanlı fiziksel katmanlarının verici ve alıcı ünitelerinin tasarımlarında, büyük önem taşıyacaktır. Yukarıdaki paragrafta sözü edilen gerek NEWCOM ve gerekse SWIM Avrupa projelerinin temeli de bu problemin çözümüne yöneliktir.

6. Projede önerilen mobil iletişim sisteminin tasarımının uygulamaya dönüştürülebilmesi için ülkemizde bu konularda uygulayıcı olarak çalışan TELETAŞ, NETAŞ ve ASELSAN gibi firmaların bu proje raporuna ulaşması gerekir. Bu nedenle, TÜBİTAK ın bu rapordan bu firmalara birer kopya göndermesini öneririz.

7. Proje kapsamında yapılan yayınlar proje Eklerinde verilmiştir.

Bölüm VII

PROJE SONUCUNDA GERÇEKLEŞTİRİLEN YAYINLAR

7.1 Uluslararası Dergi Makaleleri

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ULUSLARASI DERGİ MAKALELERİ

Maximum A Posteriori Multipath Fading Channel Estimation for OFDM Systems*

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Abstract. In this paper, a non-data-aided maximum a posteriori (MAP) channel estimation technique for OFDM systems employing M-PSK modulation scheme is proposed. The technique requires a convenient representation of the discrete multipath fading channel based on the Karhunen-Loeve orthogonal expansion and estimates the complex channel parameters of each subcarriers iteratively in frequency domain using the Expectation-Maximization (EM) algorithm. Pilot symbols are employed to choose reliable initial values of the unknown channel parameters. An analytical expression is derived for the exact Cramer-Rao lower bound of the proposed MAP channel estimator. Moreover, robustness of estimator to changes in channel correlation and signal-to-noise ratio is also analyzed. The performance is presented in terms of the mean-square error and the uncoded symbol error rate for a system employing QPSK signaling. Computer simulations demonstrate that the performance of OFDM systems using coherent demodulation based on our channel estimation can be significantly improved.

1 INTRODUCTION

OFDM signaling is proven to be an efficient way to overcome the effects of fading channel and multi-path by dividing the frequency selective channel into a number of sub-channels corresponding to the OFDM sub-carrier frequencies. OFDM has already been accepted for the new wireless local area network (WLAN) standards (IEEE 802.11), the ETSI High Performance Local Area Network type 2 (HIPERLAN/2) and Japan's Mobil Multimedia Access Communications (MMAC) systems [1]. In OFDM, channel state information between transmit and receive antenna pairs is required for coherent decoding. Several channel parameter estimation techniques were proposed in literature. In [2-3] a channel estimator for OFDM systems has been proposed based on the singular-value decomposition or frequency-domain filtering. Time domain filtering has been proposed in [4]. To further improve the channel estimator performance, a MMSE channel estimator, which makes full use of the time-frequency correlation of the time-varying dispersive channel was proposed in [5]. This technique has been extended later in [6] to develop a channel estimation in OFDM systems with transmitter diversity using space time coding. However, all these approaches assume that the data transmitted is known through a training sequence. In this paper we apply the method of Siala [7] to the estimation of fading channels in a non-data-aided fashion for OFDM systems. This algorithm performs an iterative channel estimation according to the maximum a posteriori (MAP) criterion, using the Expectation-Maximization (EM) algorithm. It uses profitably not only pilot symbols but also information-carrying symbols on the optimization of the channel estimation. It requires a conventional representation of the multipath channel, based on a discrete Karhunen-Loeve (KL) orthogonal expansion of the discrete multipath channel seen by the OFDM receiver. The channel estimator makes full use of only the frequency correlations of the channel response at different frequencies. Whether their level of performance, this may be improved with the addition of the time correlations in the algorithm [8]. In particular, for mobile wireless channels, the correlation of the channel frequency response at different times and frequencies can be separated into the multiplication of the time-and frequency-domain correlation functions and this would decrease the computational complexity of the channel estimation substantially [5].

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The rest of the paper is organized as follows. In Section 2, the OFDM system and the channel model are introduced. In Section 3, multipath channel and its orthogonal representation by means of the discrete Karhunen-Loeve transformation are presented. Section 4 introduces an EM based MAP channel estimation algorithm on a single received OFDM symbol block as well as the exact analytical expressions for the Cramer-Rao bounds. Section 5 provides simulation results on the convergence of the EM algorithms and overall analysis of the symbol-error rate(SER) performance under both the channel and SNR mismatch. Finally, in Section 6, a summary and final remarks are presented.

2 OFDM SYSTEMS WITH CHANNEL MODEL

The OFDM system with channel estimation considered in this paper is shown in Figure 1. The independent data symbols $A(k)$ are modulated by N subcarriers and inverse discrete Fourier transform (IDFT) and the last L samples are copied and put as cyclic prefix (CP) to form the OFDM symbol. This data vector is transmitted over the channel, whose impulse response is shorter than L samples. The cyclic prefix is removed at the receiver and the signal is demodulated with a discrete Fourier Transform (DFT). We assume that the use of CP both preserves the orthogonality of the subcarrier frequencies (tones) and eliminates intersymbol interference (ISI) between consecutive OFDM symbols. Further, the channel is assumed to be constant during one OFDM symbol. Under these assumptions we can describe the system as a set of parallel Gaussian channels with correlated channel attenuation $H(k)$. The attenuations on each tone are given by [3]

$$H(k) = H(\frac{k}{NT_s}),\ k = 0, 1, \cdots, N-1.$$

where $H(.)$ is the frequency response of the channel $h(\tau)$ during the OFDM symbol and T_s is the sampling period of the system. The received signal after demodulation (performing a DFT), can be expressed in vector form as

$$\mathbf{R} = \mathbf{A}\mathbf{H} + \mathbf{W}. \qquad (1)$$

Here, $\mathbf{A}$ is an $N \times N$ diagonal matrix with $\mathbf{A}[k,k] = A(k)$ representing the symbol transmitted over the kth tone. Since the phase of each subchannel can be obtained by the channel estimator, coherent phase-shift keying (PSK) modulation is used here to enhance the system performance. Therefore $A(k) \in \exp(j2\pi r/M), r = 0, 1, \cdots, M-1$. $\mathbf{H}$ is an $N \times 1$ vector with $\mathbf{H}[k] = H(k)$. Finally, $\mathbf{W}$ is an $N \times 1$ zero-mean, i.i.d Gaussian vector that models additive noise in the N sub-channels (tones). We have

$$E[\mathbf{W}^H\mathbf{W}] = \sigma^2\mathbf{I}_N \qquad (2)$$

where $\mathbf{I}_N$ represents an $N \times N$ identity matrix and σ^2 is the variance of the additive noise entering the system. The frequency response of the fading channel at the kth subcarrier, $H(k)$ are correlated samples, in frequency, of a complex Gaussian process. At the receiver, a Viterbi algorithm which needs the channel parameters $H(k)$ is used to compute the appropriate metrics to implement the decoding process. In the absence of channel state information, the decoder must estimate the channel states and thus, there has been extensive affords in the direction of channel parameter estimation. However, most of the works done tries to achieve this goal with employing least-square channel estimation technique assuming the transmitted data is known either by means of the training symbols or through a decision-directed fashion. But a drawback of this approach is that the calculation of the inverse of a square matrix is needed whose size is proportional to the length of the discrete-channel impulse response [3, 6]. This requires intensive computation for large matrix sizes. Moreover, for OFDM systems, channel estimation is challenging if we assume that this should be implemented in a non-data aided fashion [9,10]. In this paper a novel channel estimation algorithm is presented by representing the discrete multipath channel based on the Karhunen-Loeve orthogonal representation and make use of the EM technique. EM provides an iterative and more easily implementable solution.

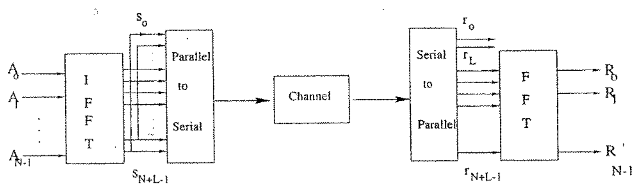

Figure 1: Baseband OFDM system, transmitting N-blocks of data.

3 REPRESENTATION OF MULTIPATH FADING CHANNELS

The complex baseband representation of a fading multipath channel impulse response can be described as [11]

$$h(\tau) = \sum_l \alpha_l \delta(\tau - \tau_l T_s) \qquad (3)$$

where τ_l is the delay of the lth path and α_l is the corresponding complex amplitude with a power-delay profile $\theta(\tau_l)$. Note that α_l's are zero-mean, complex Gaussian random variables, which are assumed to be independent for different paths. We now briefly describe the channel statistics. The correlation function of the frequency response of the multipath fading channel for different frequencies is

$$r(f, f') = E[H(f)H^*(f')] \qquad (4)$$

where

$$H(f) = \int_{-\infty}^{+\infty} h(\tau)e^{-j2\pi f\tau} = \sum_l \alpha_l e^{-j2\pi f\tau_l} . \quad (5)$$

It can be shown that (4) has the form [11]

$$r(f, f') = \sigma_H^2 r_f(f - f') \quad (6)$$

$$r_f(\Delta f) = (1/\sigma_H^2) \sum_l \sigma_l^2 e^{-j2\pi\Delta f\tau_l} \quad (7)$$

where σ_l^2 is the average power of the lth path and σ_H^2 is the total average power of the channel impulse response defined as

$$\sigma_H^2 = \sum_l \sigma_l^2.$$

For an OFDM system with subchannel spacing Δf, the discrete correlation function for subcarriers defined by $r(k, k') = E[H(k)H^*(k')]$ can be written as

$$r(k, k') = \sigma_H^2 r_1(k, k'), \quad k, k' = 0, 1, \cdots, N-1 \quad (8)$$

where

$$r_1(k, k') = r_f((k - k')\Delta f) .$$

By means of the discrete Karhunen-Loeve (KL) transformation, the frequency response vector $\mathbf{H}$ of the multipath channel can be expressed as

$$\mathbf{H} = \mathbf{\Psi G} \quad (9)$$

where $\mathbf{G}$ is an $N \times 1$ zero mean i.i.d Gaussian vector with $\mathbf{G}[k] = G(k)$ whose covariance matrix is $\mathbf{\Lambda} = diag(\lambda_0, \lambda_1, \cdots, \lambda_{N-1})$. The variances of the components of $\mathbf{G}$, arranged in decreasing order, are equal to the eigenvalues $\{\lambda_j\}$ of the KL transformation with the orthogonalized eigenvectors $\mathbf{\Psi} = [\psi_0, \psi_1 \cdots, \psi_{N-1}]$ of the discrete channel autocorrelation matrix $\mathbf{r}$ defined as

$$\mathbf{r} = E\{\mathbf{HH}^H\} \quad (10)$$

where $\mathbf{r}[k, k'] = r(k, k')$, which satisfies $\mathbf{r}\psi_j = \lambda_j \psi_j$ for $j = 0, 1 \cdots, N-1$.

4 EM-BASED MAP CHANNEL ESTIMATION

4.1 CHANNEL ESTIMATION

The MAP criterion is used in the fading channel as seen at the FFT output of the OFDM receiver since the joint probability density function of the random variables are known by the receiver and can be expressed as

$$p(\mathbf{G}) \sim \exp(-\mathbf{G}^H \mathbf{\Lambda}^{-1}\mathbf{G}). \quad (11)$$

Given the transmitted signal $\mathbf{A}$ and the discrete channel representation $\mathbf{G}$, and taking into account the independence of the noise components, we can express the conditional probability density function of the received signal $\mathbf{R}$ as

$$p(\mathbf{R}|\mathbf{A}, \mathbf{G}) \sim \exp\left[-(\mathbf{R} - \mathbf{A\Psi G})^H \mathbf{\Sigma}^{-1}(\mathbf{R} - \mathbf{A\Psi G})\right], \quad (12)$$

where $\mathbf{\Sigma}$ is an $N \times N$ diagonal matrix with $\mathbf{\Sigma}[k, k] = \sigma^2$, for $k = 0, 1, \cdots, N-1$.

The MAP estimate $\widehat{\mathbf{G}}$ is given by

$$\widehat{\mathbf{G}}_{map} = \arg\max_{\mathbf{G}} p(\mathbf{G}|\mathbf{R}) . \quad (13)$$

Directly solving this equation is mathematically intractable. However, the solution can be obtained easily by means of the iterative EM algorithm. Since the EM algorithm has been studied and applied to a number of problems in communications over the years, the details of the algorithm will not be presented in this paper. The reader is suggested to read [12] for a general exposition to EM algorithm and [13] for its application to the estimation problem related to the work herein. Basically, this algorithm inductively reestimate $\mathbf{G}$ so that a monotonic increase in the *a posteriori* conditional pdf in (13) is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q(\mathbf{G}|\mathbf{G}^{(m)}) = \sum_{\mathbf{A}} p(\mathbf{R}, \mathbf{A}, \mathbf{G}) \log p(\mathbf{R}, \mathbf{A}, \mathbf{G}^{(m)}) \quad (14)$$

where the sum is taken over all possible transmitted data symbols and $\mathbf{G}^{(m)}$ is the estimation of $\mathbf{G}$ at the mth iteration. Note that $p(\mathbf{R}, \mathbf{A}, \mathbf{G}) \sim p(\mathbf{R}|\mathbf{A}, \mathbf{G})p(\mathbf{G})$ since the data symbols $A(k)$ are assumed to be transmitted independent of each other and identically distributed and the fact that $\mathbf{A}$ is independent of $\mathbf{G}$. By similar argument, we have $p(\mathbf{R}, \mathbf{A}, \mathbf{G}^{(m)}) \sim p(\mathbf{R}|\mathbf{A}, \mathbf{G}^{(m)})p(\mathbf{G}^{(m)})$. Therefore, (14) can be evaluated by means of the Expressions (11) and (12).

Given the received signal $\mathbf{R}$, the EM algorithm starts with an initial value $\mathbf{G}^0$ of the unknown channel parameters $\mathbf{G}$. The $(m+1)$th estimate of $\mathbf{G}$ is obtained through the maximization step described by

$$\mathbf{G}^{(m+1)} = \arg\max_{\mathbf{G}} Q(\mathbf{G}|\mathbf{G}^{(m)}).$$

After long algebraic manipulations the expression of the reestimate $\mathbf{G}^{(m+1)}$ can be obtained as follows:

$$\mathbf{G}^{(m+1)} = (\mathbf{I} + \mathbf{\Sigma\Lambda}^{-1})^{-1}\mathbf{\Psi}^H \mathbf{\Gamma}^{(m)}\mathbf{R} \quad (15)$$

where, it can be easily seen that

$$(\mathbf{I}+\mathbf{\Sigma\Lambda}^{-1})^{-1} = diag[(1+\sigma^2/\lambda_0)^{-1}, \cdots, (1+\sigma^2/\lambda_{N-1})^{-1}]$$

and $\mathbf{\Gamma}^m$ in (15) is an $N \times N$ dimensional diagonal matrix representing the *a posteriori* probabilities of the data symbols at the mth iteration step whose kth diagonal component is defined as

$$\Gamma^{(m)}(k) = \sum_{a \in S_k} a^* P(A(k) = a|\mathbf{R}, \mathbf{G}^{(m)}). \quad (16)$$

S_k denotes alphabet set taken by the kth OFDM symbol. It is proved that $\mathbf{G}^{(m)}$ will converge to the true MAP channel estimator $\widehat{\mathbf{G}}_{map}$ as m, the number of iterations, gets larger [12].

In order to be able to choose good initial values for the unknown channel parameters and to ensure a fast start up in the equalization/detection operation following the channel estimation process, the N_{PS} data symbols $A(k), k \in S_{PS}$ in each OFDM frame are generally used as pilot symbols known by the receiver. Here, S_{PS} denotes the set of pilot symbols indices. Note that, $N_{PS} \geq L$ in order to identify the channel. When N is large, however, this does not create a significant degradation in spectrum efficiency since L, the number of prefix symbols, takes small values with respect to the total number of subcarriers carrying the data. To interpolate the channel estimates, initially, there exists a minimum subcarrier spacing, l_{SC}, between pilots given by $l_{SC} < 1/\tau_{max}$, where τ_{max} is the maximum delay spread of the channel in the frequency domain. Therefore for PSK modulated alphabet set, the initial value of the channel parameters $\mathbf{G}^{(0)}$ can be selected according to the following data-aided scheme.

Let $\mathbf{H}_p$ denote an $N_{PS} \times 1$ vector with $H_p[k] = H(k)$, resulting the channel attenuations at frequencies k/NT_s for $k \in S_{PS}$. Using N_{PS} pilot data symbols $A(k), k \in S_{PS}$, the linear minimum mean-square error (LMMSE) estimate of $\widehat{\mathbf{H}}_p$ is given by [3]

$$\widehat{\mathbf{H}}_p = \mathbf{\Psi}_p \mathbf{\Delta}_p \mathbf{\Psi}_p^H \widehat{\mathbf{H}}_{ls} \quad (17)$$

where $\mathbf{\Psi}_p$ is an unitary matrix containing the eigenvectors of the $N_{PS} \times N_{PS}$ dimensional channel covariance matrix $\mathbf{r_p}$ with $r_p[k,k'] = r(k,k'), k, k' \in S_{PS}$. $\mathbf{\Delta}_p$ is an diagonal matrix with entries

$$\delta_k = \frac{1}{1 + \beta\sigma^2/\mu_k}$$

where, μ_k's are the eigenvalues of $\mathbf{r_p}$ and,

$$\beta = E\{|A(k)|^2\}E\{|1/A(k)|^2\}$$

is a constant depending on the signal constellation [3]. In the case of MPSK signaling, $\beta = 1$. Then, given N_{PS} channel attenuation samples $H_p[k], k \in S_{PS}$, the complete initial channel attenuation sample values $H^0(k), k = 0, 1, \cdots, N-1$ can easily be determined using an interpolation technique, i.e., Lagrange interpolation algorithm. Finally the initial values of $\mathbf{G}^{(0)}$ can be determined from (9) as follows

$$\mathbf{G}^{(0)} = \mathbf{\Psi}^H \mathbf{H}^{(0)}. \quad (18)$$

Taking the pilot symbols into account, the final expression of $\mathbf{G}^{(m+1)}$ can be expressed as follows.

$$\mathbf{G}^{(m+1)} = (\mathbf{I} + \Sigma\mathbf{\Lambda}^{-1})^{-1}\mathbf{\Psi}^H \mathbf{V}^{(m)}\mathbf{R} \quad (19)$$

where $\mathbf{V}^{(m)} = diag[v^m(0), v^m(1), \cdots, v^m(N-1)]$ and $v^m(k)$ is given as

$$v^m(k) = \begin{cases} A^*(k) & \text{if } k \in S_{PS} \\ \Gamma^m(k) & \text{if } k \in S_{PS}^c. \end{cases}$$

Note that implementation complexity of the EM algorithm, presented above, can be reduced substantially due to the fact that the magnitude of the eigenvalues $\lambda_k, k = 0, 1, \cdots, N-1$ of the channel correlation matrix in (10) becomes negligible for $k > 2BT + 1$ where B is the one-sided bandwidth and T is the length of the channel impulse response. As pointed out in [3], for an OFDM system $2BT = L$, where L is number of symbols in the cyclic prefix since $T = LT_s$ and $2B = 1/T_s$. Since L is much smaller than the total number of subcarriers, N, the complexity of the MAP estimation algorithm based on the Karhunen-Loeve expansion proposed in this paper will be low while it is being optimal.

Computation of $\Gamma^{(m)}(k)$ for QPSK Signaling:

If $a = (\pm 1 \pm j)/\sqrt{2}$ represents unit power, independent and identically distributed data sequence modulating the QPSK carrier, $\Gamma^{(m)}(k)$ in (16) can be expressed as follows.

$$\Gamma^{(m)}(k) = \frac{\sum_{a \in S_k} a^* P(R(k)|A(k)=a, \mathbf{G}^{(m)})P(A(k)=a)}{\sum_{a \in S_k} P(R(k)|A(k)=a, \mathbf{G}^{(m)})P(A(k)=a)}. \quad (20)$$

From (12) it follows that

$$\Gamma^{(m)}(k) = \frac{\sum_{a \in S_k} a^* \exp\left(\frac{2}{\sigma^2} Re[a^* Z^{(m)}(k)]\right)}{\sum_{a \in S_k} \exp\left(\frac{2}{\sigma^2} Re[a^* Z^{(m)}(k)]\right)}$$

where

$$Z^{(m)}(k) = R(k) \sum_j G^{(m)*}(j)\psi_j^*(k).$$

Then, taking summations in the numerator and the denominator of (20) over the values of QPSK symbols a, we have the final result as follows:

$$\begin{aligned} \Gamma^{(m)}(k) &= \frac{1}{\sqrt{2}} \left\{ \tanh\left[\frac{\sqrt{2}}{\sigma^2} Re(Z^{(m)}(k))\right] \right. \\ &\quad - \left. j \tanh\left[\frac{\sqrt{2}}{\sigma^2} Im(Z^{(m)}(k))\right] \right\}. \end{aligned} \quad (21)$$

4.2 Cramer-Rao Bound (CRB) for Estimating the Random Parameters $\{G(j)\}$

Let $\{G(j)\}$ s be the random parameters to be estimated. The (q,s)th element of the Fisher information matrix is defined as

$$J_{q,s} = -E\left[\frac{\partial^2 \ln p(\mathbf{R}|\mathbf{G})}{\partial G(q)\partial G(s)}\right] - E\left[\frac{\partial^2 \ln p(\mathbf{G})}{\partial G(q)\partial G(s)}\right]. \quad (22)$$

Since $\mathbf{G}$ and $\mathbf{A}$ are independent of each other, $\ln p(\mathbf{R}|\mathbf{G})$ in (22) can be computed as follows

$$\ln p(\mathbf{R}|\mathbf{G}) \equiv \sum_{\mathbf{A}} \ln p(\mathbf{R}|\mathbf{A}, \mathbf{G}) + \ln p(\mathbf{G})), \quad (23)$$

where the joint probability density functions $p(\mathbf{G})$ and $p(\mathbf{R}|\mathbf{A},\mathbf{G})$ are given by (11) and (12), respectively. Taking into account that the data symbols are independent of each other, after some algebra, (23) can be expressed as

$$\begin{aligned}\ln p(\mathbf{R}|\mathbf{G}) &\equiv \sum_{k=0}^{N-1}\Bigg[-\frac{1}{\sigma^2}\left(|R(k)|^2+|H(k)|^2\right)\\ &+ \ln\cosh\left(\frac{\sqrt{2}}{\sigma^2}Re\{R^*(k)H(k)\}\right)\\ &+ \ln\cosh\left(\frac{\sqrt{2}}{\sigma^2}Im\{R^*(k)H(k)\}\right)\\ &- \frac{|G(k)|^2}{\lambda(k)}\Bigg]. \qquad (24)\end{aligned}$$

Performing now the derivatives in (22) and after taking expectations over both $\mathbf{R}$ and $\mathbf{G}$ and taking into fact that the eigenfunctions $\psi_j(k)$ are orthogonal, it follows that

$$J_{q,s} = \begin{cases} 4(1/\lambda_q+1/\sigma^2)-(4/\sigma^4)\Phi(\sigma^2) & \text{if } q=s\\ 0 & \text{otherwise}\end{cases}$$

where

$$\Phi(\sigma^2) \equiv E\left(\frac{|R(k)|^2}{\cosh(\frac{\sqrt{2}}{\sigma^2}Re\{R^*(k)H(k)\})}\right).$$

It seems that evaluation of the above expectation analytically is mathematically intractable. Therefore, instead, we try to evaluate $\Phi(\sigma^2)$ as a function of σ^2 by computer simulations and then to fit a curve on it. By doing so we have

$$\begin{aligned}\Phi(x) &= (1.94\times10^{-7})x^6-(1.177\times10^{-5})x^5\\ &+ (2.04\times10^{-4})x^4-(7.77\times10^{-4})x^3\\ &+ (8.41\times10^{-3})x^2-(2.570\times10^{-1})+1.07\,.\end{aligned}$$

Finally the Cramer-Rao bound for the unknown channel parameters $G(q), q=0,1,\cdots,N-1$ are given by

$$CRB(G(q)) = J_{q,q}^{-1}\,.$$

5 Simulation Results

The simulation results for estimating the channel parameters in OFDM systems with EM algorithm are now presented. We consider the fading multipath channel given by (3) with an exponentially decaying power delay profile $\theta(\tau_l)=C\exp(-\tau_l/\tau_{rms})$ and delays τ_l that are uniformly and independently distributed over the length of the cyclic prefix. C is a normalizing constant. Note that the normalized discrete channel-correlations for different subcarriers of this channel model was presented in [3] as follows,

$$r_1(k,k') = \frac{1-\exp\left(-LB(k,k')\right)}{\tau_{rms}(1-\exp(-L/\tau_{rms}))B(k,k')}$$

where

$$B(k,k') = ((1/\tau_{rms})+2\pi j(k-k')/N)\,.$$

The scenario for our simulation study consists of a wireless QPSK OFDM system employing the transmitted pulse having a unit-energy Nyquist-root raised-cosine shape with rolloff $\alpha=0.2$. The symbol period(T_s) is chosen to be 0.167 μs, corresponding to an uncoded symbol rate of 6 Mbit/s. Transmission bandwidth (3.6 MHz) is divided into $N=256$ tones. We assume that the multipath channel model consists of 5 impulses with uniformly spaced intervals of durations T_s. Therefore, the maximum channel delay $\tau_{max}=4$ sample (0.668 μs) long. On the other hand, the duration of the transmitter impulse response after matched filter at the receiver is chosen to be $L_g=7$ symbols interval. It is truncated at ±3 sample interval around its center. Note that, in order to prevent ISI and ICI, the length of cyclic prefix (L) should be longer than the overall channel response length ($\tau_{max}+L_g-1$), i.e., $L\geq10$ samples. As explained previously, this puts a constraint on the number of pilot symbols to be chosen as $N_{PS}\geq L$. For this simulation study we chose $N_{PS}=10$. To get insight into the average behavior of the channel estimator, we have averaged the performance over 100 Monte-Carlo runs.

Figure 2 demonstrates the average MSE performance of the EM-based channel estimation algorithm as a function of the average SNR under different τ_{rms} values ($\tau_{rms}=0.2$, 4 and ∞) together with the Cramer-Rao bound. The average SNR was defined as $E[|H(k)|^2]E[|A(k)|^2/\sigma^2$. Since $E[|A(k)|^2=1$ for QPSK signaling and $E[|H(k)|^2]=1$ for normalized frequency response of the fading channel, the normalized SNR simply becomes $1/\sigma^2$, where σ^2 is the variance of the complex white Gaussian noise entering the system. Average Mean-square-error(MSE) is defined as the norm of the difference between the vectors $\mathbf{G}=[G(k)]$ and $\widehat{\mathbf{G}}_{map}=[\widehat{G}_{map}(k)]$, representing the true and the estimated values of channel parameters, respectively. Namely,

$$MSE = \frac{1}{N}||\mathbf{G}-\widehat{\mathbf{G}}_{map}||^2.$$

The initial values, $G^{(0)}(k)$, were chosen according to (18). Note that, since the estimator has more leakage for the channel with $\tau_{rms}=\infty$, the channel estimator has a slightly more performance degradation than channels with $\tau_{rms}=0.2$ and $\tau_{rms}=4$. This can also be observed from Figure 2. However, performance degradation vanishes at high SNRs.

In Figures 3, 4 and 5, the average MSE performance of the EM-based algorithm are presented as a function of the number of iterations for $\tau_{rms}=0.2$, 4 and ∞ respectively. It is concluded from these curves that the MSE performance of the EM-based algorithm converges within 2-5 iterations, depending on the average SNR and τ_{rms}.

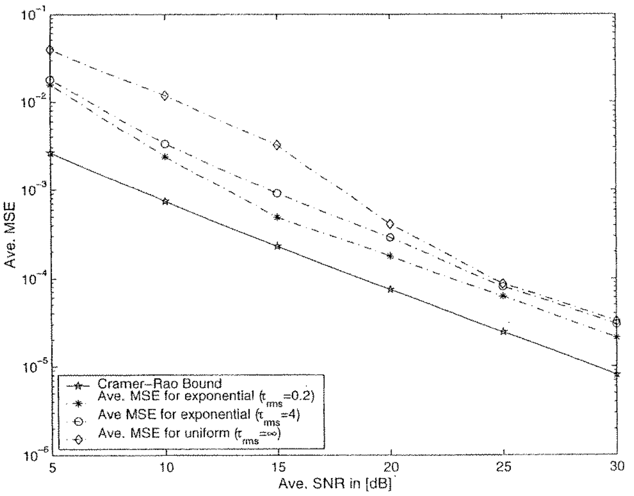

Figure 2: MSE performance of the EM algorithm as a function of average SNR.

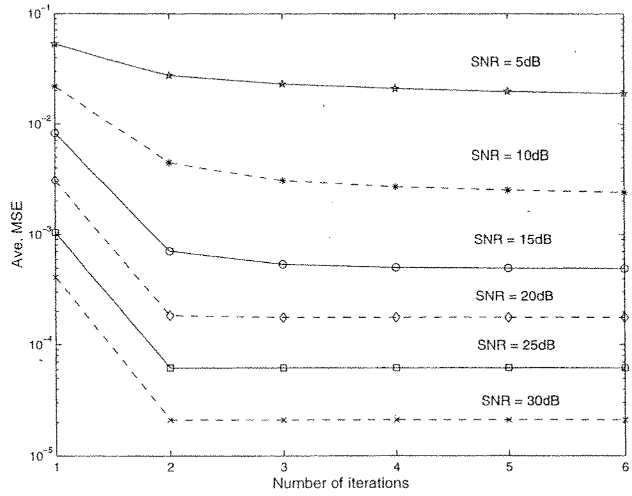

Figure 3: MSE performance of the EM algorithm as a function of number of iterations ($\tau_{rms} = 0.2$ sample for the exponentially decaying power delay profile).

Mismatch Analysis:

Once the true frequency-domain correlation, characterizing the channel statistics and the SNR, are known the optimal channel estimator can be designed as indicated in Section 4. However, in mobile wireless communications, the channel statistics depend on the particular environment, for example, indoor or outdoor, urban or suburban, and change with time. Hence, it is important to analyze the performance degradation due to a mismatch of the estimator to the channel statistics as well as the SNR, and to study the choice of the channel correlation, and SNR for this estimator so that it is robust to variations in the channel statistics. As a performance measure, we use uncoded Symbol Error Rate (SER) for QPSK signaling. The SER expression for this case is given in [14] as a function of the SNR and the average MSE as follows:

$$SER_{QPSK} = \frac{3}{4} - \frac{\mu}{2} - \frac{\mu}{\pi}\arctan(\mu) \qquad (25)$$

where

$$\mu = \sigma_H^2 / \sqrt{(\sigma_H^2 + MSE)(1 + 1/SNR)}\,,$$

and σ_H^2 represents the normalized variance of the channel gains ($\sigma_H^2 = 1$) and $SNR = 1/\sigma^2$. In practice, the true channel correlations and SNR are not known. If the MAP channel estimator is designed to match a channel with frequency domain correlation $\mathbf{r}$ and SNR, but the real channel $\widetilde{\mathbf{H}}$ has the correlation $\widetilde{\mathbf{r}}$ and the real $\widetilde{SNR}$, the average MSE for the designed channel estimator is

$$MSE = \frac{1}{N}E\|\widehat{\mathbf{H}}_{map} - \widetilde{\mathbf{H}}_{map}\|^2 \qquad (26)$$

where

$$\widehat{\mathbf{H}}_{map} = \mathbf{\Psi}\widehat{\mathbf{G}}_{map}.$$

Evaluation of (26) analytically does not seem to be possible. Therefore, we determine it by computer simulations. To analyze MAP estimator's performance sensitivity to design errors, we designed the estimator for a uniform channel correlation which gives the worst MSE performance among all channels [3, 5] and evaluated for an exponentially decaying power-delay profile. Since design for high SNR is preferred for SNR mismatch, we chose $SNR = 20$ dB. Figure 6 demonstrates the estimator's sensitivity to the channel statistics and SNR design mismatch. As it can be seen from Figure 6, only small performance loss is observed for low SNRs when the estimator is designed for mismatched channel statistics. However, the system performance degrades significantly for low SNR design and high SNR values.

6 Conclusion

In this paper, we have presented the design of a channel estimator for OFDM systems that make full use of frequency correlations of the multipath channel. This algorithm performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing M-PSK modulation scheme with additive Gaussian noise. It exploits the representation of the multipath channel, based on the discrete Karhunen-Loeve expansion of the multipath channel seen by the OFDM receiver. A non-data aided estimation scheme is developed by averaging over the M-PSK signal constellation. To be able to obtain good initial estimates, pilot symbols are used to estimate the initial value of the corresponding channel parameters according to a data-aided scheme, then the initial values of the complete channel parameters are determined using an interpolation technique. Moreover, we derive Cramer-Rao

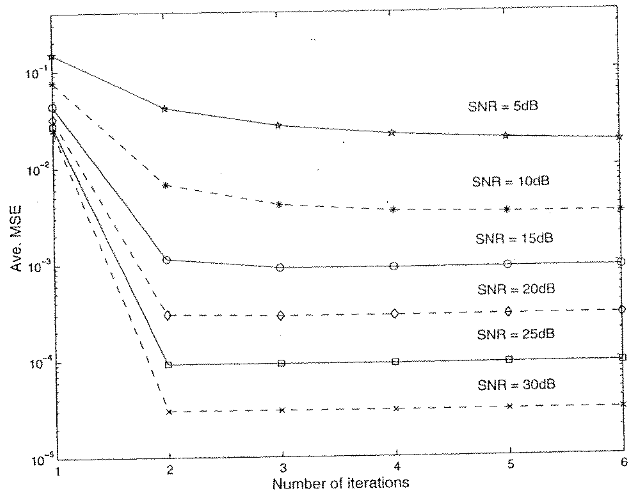

Figure 4: MSE performance of the EM algorithm as a function of number of iterations (τ_{rms} = 4 sample for the exponentially decaying power delay profile).

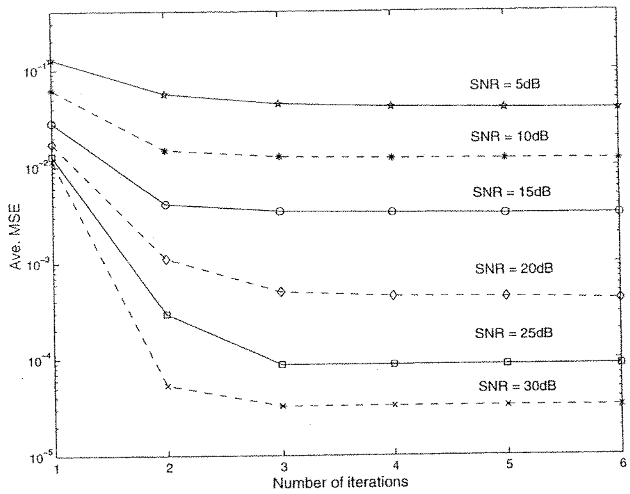

Figure 5: MSE performance of the EM algorithm as a function of number of iterations (uniform power delay profile obtained by letting $\tau_{rms} \to \infty$).

bounds for the MAP estimation technique and analyze the estimator's sensitivity to design errors. Computer simulations demonstrate that the proposed EM-based algorithm converges within 2-5 iterations, depending on the average SNR and channel rms width. Multipath channel with exponentially decaying power delay profile for different rms width values is also studied in simulations. One can observe from these results that channel estimator has slightly better performance for small rms width values. Finally it is concluded that the EM-based estimator is computationally efficient and it is robust to various channel profiles.

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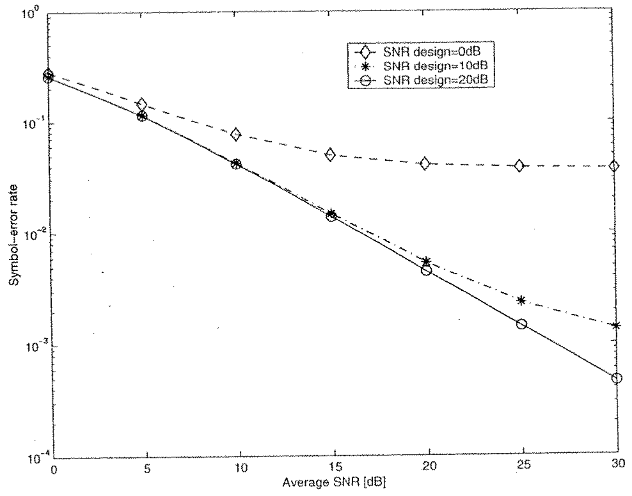

Figure 6: *Symbol-error rate under channel correlation and SNR mismatch.*

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Maximum Likelihood Blind Channel Estimation for Space-Time Coding Systems

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Sophisticated signal processing techniques have to be developed for capacity enhancement of future wireless communication systems. In recent years, space-time coding is proposed to provide significant capacity gains over the traditional communication systems in fading wireless channels. Space-time codes are obtained by combining channel coding, modulation, transmit diversity, and optional receive diversity in order to provide diversity at the receiver and coding gain without sacrificing the bandwidth. In this paper, we consider the problem of blind estimation of space-time coded signals along with the channel parameters. Both conditional and unconditional maximum likelihood approaches are developed and iterative solutions are proposed. The conditional maximum likelihood algorithm is based on iterative least squares with projection whereas the unconditional maximum likelihood approach is developed by means of finite state Markov process modelling. The performance analysis issues of the proposed methods are studied. Finally, some simulation results are presented.

Keywords and phrases: blind channel estimation, conditional and unconditional maximum likelihood.

1. INTRODUCTION

The rapid growth in demand for a wide range of wireless services is a major driving force to provide high-data rate and high quality wireless access over fading channels [1]. However, wireless transmission is limited by available radio spectrum and impaired by path loss, interference from other users and fading caused by destructive addition of multipath. Therefore, several physical layer related techniques have to be developed for future wireless systems to use the frequency resources as efficiently as possible. One approach that shows real promise for substantial capacity enhancement is the use of diversity techniques [2]. Diversity techniques basically reduce the impact of fading due to multipath transmission and improve interference tolerance which in turn can be traded for increase capacity of the system. In recent years, the use of antenna array at the base station for transmit diversity has become increasingly popular, since it is difficult to deploy more than one or two antennas at the portable unit. Transmit diversity techniques make several replicas of the signal available to the receiver with the hope that at least some of them are not severally attenuated. Moreover, the methods of transmitter diversity combined with channel coding have been employed at the transmitter, which is referred to as space-time coding, to introduce temporal and spatial correlation into signals transmitted from different antennas [2, 3]. The basic idea is to reuse the same frequency band simultaneously for parallel transmission channels to increase channel capacity [2, 3].

Unfortunately, employing antenna diversity at the transmitter is particularly challenging, since the signals are combined in space prior to reception. Moreover, estimation of fading channels in space-time systems is further complicated, since the receiver estimates the path gain from each transmit antenna to each receive antenna. It is also important to note that space-time decoding requires multi-channel state information. Thus the achievable diversity gain comes at the price of proportional increase in the amount of training which results in efficiency loss, especially in a rapidly varying environment. Clearly, the practical advantages of eliminating

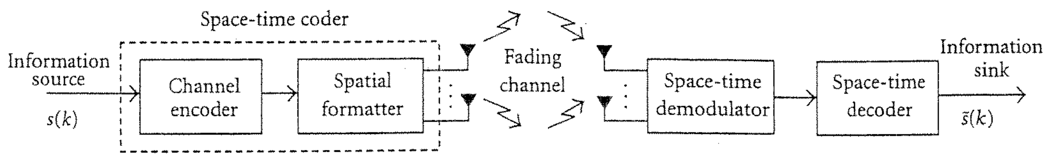

FIGURE 1: Space-time coding and decoding system.

the need for a training sequence numerous. This motivates the development of receiver structures with blind channel estimation capabilities. There has been considerable work reported in the literature on the estimation of channel information to improve performance of space-time coded systems operating on fading channels [4, 5, 6, 7]. In this paper, we consider the problem of blind estimation of space-time coded signals along with the matrix of path gains. We propose two different approaches based on the assumptions on the input sequences. Our proposed approaches also exploit the finite alphabet property of the space-time coded signals. We treat both conditional and unconditional maximum likelihood (ML) approaches. The first approach (conditional ML) results in joint estimation of the channel matrix and the input sequences, and is based on the iterative least squares and projection [8]. The second approach, which is known as unconditional ML, treats the input sequence as stochastic independent identically distributed (i.i.d.) sequences. In contrast, the unconditional ML approach formulates the blind estimation problem in discrete-time finite state Markov process framework [9, 10, 11]. Since the proposed algorithms obtain ML estimates of channel matrix and the space-time coded signals, they enjoy many attractive properties of the ML estimator including consistency and asymptotic normality. Moreover, it is asymptotically unbiased and its error covariance approaches Cramér-Rao lower bound (CRB).

The performance of the proposed ML approaches are explored based on the evaluation of CRB. The CRB is a well-known statistical tool that provides benchmarks for evaluating the performance of actual estimators. For the conditional estimator, the CRB derived in [12], is adapted to the present scenario. In unconditional case, since, the computation of the exact CRB is analytically intractable, some alternative methods must therefore be considered for simplifying CRB calculation [13]. The derivation technique used for unconditional ML have the advantage of eliminating the need to evaluate computationally intractable averaging over all possible input sequences. However, it provides a looser bound which is not as tight as the exact CRB, but it is computationally easier to evaluate.

The outline of the paper is as follows. In Section 2, we describe a basic model for a communication system that employs space-time coding with n transmit and m receive antennas. In Section 3, we derive both conditional and unconditional ML estimators for the blind estimation of space-time coded signals along with the channel matrix. In Section 4, we develop CRB for the covariance of the estimation errors for the achievable variance of any unbiased estimator for these parameter set. Finally, we present some numerical examples that illustrate the performance of the ML estimators in Section 5.

Notations used in this paper are standard. Symbols for matrices (in capital letter) and vector (lower case) are in boldface. $(\cdot)^T$, $(\cdot)^H$, $(\cdot)^*$, and $\otimes$ denote transpose, Hermitian, conjugate, and Kronecker product, respectively. The symbol $\mathbf{I}$ stands for identity matrix with proper dimension; $\hat{\boldsymbol{\theta}}$ denotes the estimate of parameter vector $\boldsymbol{\theta}$; and $\|\cdot\|$ denotes the 2-norm.

2. SYSTEM MODEL

In the sequel, we consider a mobile communication system equipped with n transmit antennas and optional m receive antennas. A general block diagram for the systems of interest is depicted in Figure 1. In this system, the source generates bit sequence $s(k)$, which are encoded by an error control code to produce codewords. The encoded data are parsed among n transmit antennas and then mapped by the modulator into discrete complex-valued constellation points for transmission across channel. The modulated streams for all antennas are transmitted simultaneously. At the receiver, there are m receive antennas to collect the transmissions. Spatial channel link between each transmit and receive antenna is assumed to experience statistically independent fading.

The signals at each receive antenna is a noisy superposition of the faded versions of the n transmitted signals. The constellation points are scaled by a factor of E_s, so that the average energy of transmitted symbols is 1. Then we have the following complex base-band equivalent received signal at receive antenna j:

$$r_j(k) = \sum_{i=1}^{n} \alpha_{i,j}(k)c_i(k) + n_j(k), \tag{1}$$

where $\alpha_{i,j}(k)$ is the complex path gain from transmit antenna i to receive antenna j, $c_i(k)$ is the coded symbol transmitted from antenna i at time k, $n_j(k)$ is the additive white Gaussian noise sample for receive antenna j at time k.

Equation (1) can be written in a matrix form as

$$\mathbf{r}(k) = \boldsymbol{\Omega}(k)\mathbf{c}(k) + \mathbf{n}(k), \tag{2}$$

where $\mathbf{r}(k) = [r_1(k), \ldots, r_m(k)]^T \in \mathbb{C}^{m\times 1}$ is the received signal vector, $\mathbf{c}(k) = [c_1(k), \ldots, c_n(k)]^T \in \mathbb{C}^{n\times 1}$ is the code vector transmitted from the n transmit antennas at time k, $\mathbf{n}(k) = [n_1(k), \ldots, n_m(k)]^T \in \mathbb{C}^{m\times 1}$ is the noise vector at the receive antennas, and $\boldsymbol{\Omega}(k) \in \mathbb{C}^{m\times n}$ is the fading channel gain matrix given as

$$\Omega(k) = \begin{bmatrix} \alpha_{1,1}(k) & \cdots & \alpha_{1,n}(k) \\ \vdots & \cdots & \vdots \\ \alpha_{m,1}(k) & \cdots & \alpha_{m,n}(k) \end{bmatrix}. \quad (3)$$

We impose the following assumptions on model (2) for the rest of the paper:

(AS1) the coded symbol $c_i(k)$ is adopting finite complex values;

(AS2) the noise vector $\mathbf{n}(k) = [n_1(k), \dots, n_m(k)]^T$ is Gaussian distributed with zero-mean and

$$\mathbb{E}[\mathbf{n}(k)\mathbf{n}^H(l)] = \sigma^2 \mathbf{I}\delta_{k,l}, \qquad \mathbb{E}[\mathbf{n}(k)\mathbf{n}^T(l)] = 0, \quad (4)$$

where $\mathbb{E}$ denotes expectation operator and $\delta_{k,l}$ is the Kronecker delta ($\delta_{k,l} = 1$ if $k = l$ and 0 otherwise). Thus $\mathbf{n}(k)$ is assumed to be uncorrelated both temporally and spatially;

(AS3) the fading channel is assumed to be quasi-static flat fading, so that during the transmission of L codeword symbols across anyone of the links, the complex path gains do not change with time k, but are independent from one codeword transmission to the next, that is,

$$\alpha_{i,j}(k) = \alpha_{i,j}, \quad k = 1, 2, \dots, L. \quad (5)$$

The problem of estimating matrix of path gains along with the space-time coded signals from noisy observations $r(L) = [\mathbf{r}^T(1), \dots, \mathbf{r}^T(L)]^T$ is the main concern of the paper. The traditional solution to this problem is to first estimate $\boldsymbol{\theta} = [\boldsymbol{\Omega}, \sigma^2]$ from training sequence embedded in the input signal, and then use these estimates as if they were the true parameters to obtain estimates of input sequence. As an alternative, we propose ML blind approaches based on finite alphabet property of the space-time coded signals. Then we derive ML cost functions for our proposed approaches in the next section.

3. ML ESTIMATION

Regarding the input sequence, two different assumptions can be considered: (i) conditional model which assumes the input sequences to be deterministic unknown parameters and (ii) unconditional model which assumes the input sequences to be stochastic processes. These two signal models lead to corresponding ML solutions. In the first approach, the input sequences are treated as unknown but deterministic quantities, therefore they are part of the set of unknown parameters. The number of unknown parameters in deterministic case grows with the increase in the number of observations which usually results in inconsistent estimates. In contrast, under the unconditional signal model, the input sequences are treated as random quantities, and are not included in the parameter set. As a result, the number of unknown parameters is fixed and it is therefore possible to obtain consistent estimates. Now we develop corresponding ML estimation algorithms.

3.1. *Conditional ML approach*

In this section, an ML approach is developed under (AS1), (AS2), (AS3), and the conditional signal model assumption. The log-likelihood function is then given by

$$\mathcal{L} = -\text{const} - mL\log\sigma^2 - \frac{1}{\sigma^2}\sum_{k=1}^{L} \|\mathbf{r}(k) - \boldsymbol{\Omega}\mathbf{c}(k)\|^2. \quad (6)$$

The conditional ML estimation can be obtained by jointly maximizing $\mathcal{L}$ over the unknown parameters $\boldsymbol{\Omega}$ and $c(L) = [\mathbf{c}^T(1), \dots, \mathbf{c}^T(L)]^T$. After neglecting unnecessary terms, conditional ML yields the following minimization problem:

$$\min_{\boldsymbol{\Omega}, c(L)} \|r(L) - \boldsymbol{\Omega}c(L)\|^2. \quad (7)$$

Since the elements of $c(L)$ are restricted to be finite alphabet, (7) results in a nonlinear separable optimization problem with mixed integer and continuous variables. Typically, the minimization problem in (7) is solved in two steps by alternatively minimizing with respect to $\boldsymbol{\Omega}$ and $c(L)$ while keeping other parameters fixed. First, we minimize (7) with respect to $\boldsymbol{\Omega}$ by the least squares solution. Then substitute $\hat{\boldsymbol{\Omega}}$ back into (7) and solve it for $c(L)$. The ML estimate of $c(L)$ in the second step can be obtained by enumeration. However, this search is computationally very demanding since the number of possible $c(L)$ matrices that need to be checked grows exponentially both with L and n. Therefore, the iterative approaches attempt to solve this problem with lower computational complexity.

We now adopt a block conditional ML algorithm that has a lower computational complexity [8]. The proposed algorithm is based on iterative least squares and projection (ILSP). It takes advantage of the ML estimator being separable in its continuous and integer variables. Note that the dimension of the channel gain matrix $\boldsymbol{\Omega}$ is chosen to satisfy $n \le m$ for this particular approach.

Given an initial estimate $\hat{\boldsymbol{\Omega}}$ of $\boldsymbol{\Omega}$, the minimization of (7) with respect to $c(L)$ is a least squares problem that can be solved in closed form. Each element of the solution is rounded-off to its closest discrete values (coded MPSK signals). Then a better estimate of $\boldsymbol{\Omega}$ is obtained by minimizing (7) with respect to $\boldsymbol{\Omega}$, keeping $\hat{c}(L)$ fixed. This minimization also results in least squares. This process continues until $\boldsymbol{\Omega}$ converges. In practice, we can stop when the difference $\|\boldsymbol{\Omega}_i - \boldsymbol{\Omega}_{i-1}\|$ is within a threshold ϵ.

The following steps summarize the conditional ML algorithm:

Start with initial estimate $\boldsymbol{\Omega}_{(0)}$, $i = 0$

(1) $i = i + 1$
- $c_i(L) = (\boldsymbol{\Omega}_{i-1}^* \boldsymbol{\Omega}_{i-1})^{-1} \boldsymbol{\Omega}_{i-1}^* r(L)$.
- Project each element of $c_i(L)$ to closest discrete values.
- $\boldsymbol{\Omega}_i = r c_i^*(L)(c_i(L) c_i^*(L))^{-1}$.

(2) Continue until $\|\boldsymbol{\Omega}_i - \boldsymbol{\Omega}_{i-1}\| \le \epsilon$.

Clearly, due to nonlinear operation in projecting $c_i(L)$ to its closest discrete values, the convergence is not guaranteed.

However, sufficiently good initialization provided from suboptimal techniques improve the possibility of global convergence and also reduce the number of iterations required.

3.2. Unconditional ML approach

Under (AS2), (AS3), and the signal model (2), we can formulate the probability density function of the received vector r (given $\mathbf{u}$) as

$$f_{\theta}(r \mid \mathbf{u}) = \frac{1}{(\pi\sigma^2)^{mL}} \prod_{k=1}^{L} \exp\left\{-\frac{\|\mathbf{r}(k) - \Omega\mathbf{g}(\mathbf{u}(k))\|^2}{\sigma^2}\right\}, \quad (8)$$

where $\mathbf{g}(\cdot)$ is the same nonlinear mapping that describes channel coder, spatial formatter, and modulator, $\mathbf{u}(k)$ is the input sequence influencing the space-time coded symbols.

In general, trying to estimate θ and $\mathbf{u}$ jointly from (8) is computationally demanding except for small data alphabet size and small data record. Therefore, the goal is to obtain a cost function that is dependent only on θ, in this way it is possible to avoid least squares based on two step procedures for blind ML estimation. To this end, we therefore consider an unconditional signal model and compute the corresponding ML cost function via the expectation of the conditional ML function with respect to the statistics of the input sequences

$$f_{\theta}(\mathbf{r}) = \mathbb{E}_{\mathbf{u}}\left[f_{\theta}(\mathbf{r} \mid \mathbf{u})\right]. \quad (9)$$

However, the expectation $\mathbb{E}_{\mathbf{u}}$ in (9) leads to complicated cost function. The maximization of this cost function is therefore computationally demanding. At this point, we modified (AS1) for the unconditional case in the following form:

($AS1_u$) information sequence $s(k)$ is an i.i.d. sequence adopting equiprobable finite values.

If we exploit the assumption ($AS1_u$) on the input sequence and use the conditional ML function (8), we can obtain the unconditional ML function specifically for the problem at hand as

$$f_{\theta}(\mathbf{r}) = \frac{1}{2^{(l+t-1)}(\pi\sigma^2)^{mL}} \prod_{k=1}^{L} \sum_{p=1}^{2^{(l+t-1)}} \exp\left\{-\frac{\|\mathbf{r}(k) - \Omega\mathbf{g}(\zeta_p)\|^2}{\sigma^2}\right\}, \quad (10)$$

where $\zeta_p = [s(lk+l-1), \ldots, s(lk-t)]^T$ is the input vector influencing the coded symbols at time k, t is the number of memory elements in the encoder, $l = \log_2 M$ is the block length of information bits that are transmitted (if we restrict ourselves to $MPSK$). Since each element of the ζ_p takes on 2 possible values, $2^{(l+t-1)}$ is the set of all possible $(l+t-1)$ vectors of 2.

The log-likelihood function for the unconditional signal model is then given by

$$\mathcal{L}(\theta) = \sum_{k=1}^{L} \log\left(\sum_{p=1}^{2^{(l+t-1)}} \exp\left\{-\frac{\|\mathbf{r}(k) - \Omega\mathbf{g}(\zeta_p)\|^2}{\sigma^2}\right\}\right) + \text{constant}, \quad (11)$$

and the unconditional ML estimation of θ is the global maximizer of $\mathcal{L}(\theta)$. Unfortunately, existence of the globally convergent algorithm for this nonlinear cost function is unlikely. Moreover, the direct maximization of (11) still results in computationally demanding nonlinear optimization problem. In finding the ML estimator, it is quite common to resort numerical techniques of maximization such as the Newton-Raphson and scoring methods. However, the Newton-Raphson and scoring methods may suffer from convergence problems. As an alternative, the problem can be cast in a finite-state Markov chain framework by employing the Baum-Welch algorithm which reduces computational burden significantly. The Baum-Welch algorithm although iterative in nature, is guaranteed under certain mild conditions to converge and at convergence to produce a local maximum.

In the sequel, we exploit finite-state Markov process modelling property of the space-time coded signals and employed associated estimation algorithm to provide computationally efficient solution to resulting optimization problem. Let us then introduce unconditional ML framework based on finite-state Markov process modelling first.

3.2.1 Function of a Markov chain

Many important problems in digital communications such as inter-symbol interference, partial response signalling can be modelled by means of finite-state Markov process with unknown parameters observed in independent noise [10, 11]. Based on ($AS1_u$), codeword produced by the channel encoder in space-time coder can be characterized as a finite-state Markov process. Moreover, the received signal vector at an antenna array in the presence of spatial formatting, fading channel and noise can also be viewed as a stochastic process (function of Markov chain) that has an underlying Markovian finite-state structure.

The space-time coder is characterized by a memory of length t and $2^{(l+t-1)}$ state trellis, where the state $\zeta(k)$ at time k labels the coder memory $(s(lk+l-2), \ldots, s(lk-t))$,

$$\zeta(k) \in \Pi = \left\{\tau_p,\ p = 1, \ldots, 2^{(l+t-2)}\right\}. \quad (12)$$

The transition from state $\zeta(k)$ to $\zeta(k+1)$ is represented on the trellis by a branch denoted by the vector

$$\phi(k) = \left[s(lk+l-1), \ldots, s(lk-t)\right]^T \quad (13)$$

and $\phi(k) \in \Phi = \{\xi_n, n = 1, \ldots, 2^{(l+t-1)}\}$. Then both the $\{\zeta(k)\}$ sequence and the $\{\phi(k)\}$ sequence form a first-order finite Markov chains, that is,

$$\Pr\left[\phi(k) = \xi_n\right] = \Pr\left[\zeta(k) = \tau_q,\ \zeta(k-1) = \tau_s\right] \quad (14)$$

for some q, s depending on k.

The observation vector $r(k)$ can therefore be modelled as a probabilistic function of the Markov chain. In the received signal model, the unknown channel matrix Ω enter in a linear way, while the nonlinear part of the function $\mathbf{g}(\cdot)$ is due to the space-time coder and is known. Let $\mathbf{g}(\xi_n)$ denote the space-time coder output corresponding to the event $\phi(k) = \xi_n$. The sample $\phi(k) = \xi_n$ is a realization of the complex random sample $\mathbf{g}(\phi(k))$ which takes $2^{(l+t-1)}$ possible values depending on the $\phi(k) = \xi_n$. Moreover, every realization of a sequence of symbols corresponds to the sequence of branches $\{\mathbf{x}_k\}$ of length L, given as

$$\mathcal{X} = (\mathbf{x}_1, \dots, \mathbf{x}_L), \quad \mathcal{X} \in \Xi \ |\Xi| \in 2^{L(l+t-1)}. \tag{15}$$

The underlying Markovian structure of our signal model can then be characterized by the following model parameters:

(i) $\Pr[\zeta(k) = \tau_q \mid \zeta(k-1) = \tau_s]$ is a predetermined transition probability. If no information about the transmitted sequence is available, all permissible state transitions have the same probability, that is, $\Pr[\zeta(k) = \tau_q \mid \zeta(k-1) = \tau_s] = 1/2^{(l+t-1)}$, if state τ_s leads to state τ_q;

(ii) $\hat{\pi}(0) = [\hat{\pi}_1(0), \dots, \hat{\pi}_{2^{(l+t-1)}}(0)]$ initial state probability vector. If no assumption on the starting bits is made, the initial probability is same for all states;

(iii) the conditional density $f(\mathbf{r}(k) \mid \zeta(k) = \tau_q, \zeta(k-1) = \tau_s) = f(\mathbf{r}(k) \mid \phi(k) = \xi_n)$ is that of a Gaussian complex random vector with mean $\mathbf{\Omega}\,\mathbf{g}(\xi_n)$ and variance σ^2.

Since the state transition probability and the initial state probability vector are predetermined, the only model parameter of the Markov chain left to be estimated is $f(\mathbf{r}(k) \mid \phi(k) = \xi_n)$ for the current model. We therefore devise the Baum-Welch algorithm to estimate the Markov chain model parameter (iii) or equivalently to estimate $\boldsymbol{\theta}$.

3.2.2 Baum-Welch algorithm

The Baum-Welch algorithm is a commonly used iterative technique for estimating the parameters of a probabilistic functions of a Markov chain. It maximizes an auxiliary function related to the Kullback-Leibler information measure instead of the likelihood function [9]. The auxiliary function is defined as a function of two sets of parameters $\boldsymbol{\theta}, \boldsymbol{\theta}'$

$$Q(\boldsymbol{\theta}, \boldsymbol{\theta}') = \sum_{\mathcal{X} \in \Xi} f_{\boldsymbol{\theta}}(r, \mathcal{X}) \log \left(f_{\boldsymbol{\theta}'}(r, \mathcal{X})\right), \tag{16}$$

where $f_{\boldsymbol{\theta}}(r, \mathcal{X})$ represents the conditional likelihood, given a particular branch sequences $\mathcal{X}$, weighted by $\Pr[\mathcal{X}]$, the a priori probability of $\mathcal{X}$ (e.g., [10]).

The theorem that forms the basis for the Baum-Welch algorithm explains the reason why Kullback-Leibler information measure can be used instead of the average likelihood.

Theorem 1. The maximization of $Q(\boldsymbol{\theta}, \boldsymbol{\theta}')$ leads to increased likelihood, that is, $Q(\boldsymbol{\theta}, \boldsymbol{\theta}') \geq Q(\boldsymbol{\theta}, \boldsymbol{\theta}) \Rightarrow f_{\boldsymbol{\theta}'}(r) \geq f_{\boldsymbol{\theta}}(r)$.

For the proof of the theorem, see [9].

To obtain the explicit form of the auxiliary function for the current problem, we start with

$$\log f_{\boldsymbol{\theta}'}(r, \mathcal{X}) = \log \Pr[\mathcal{X}] + \log f_{\boldsymbol{\theta}'}(r \mid \mathcal{X}). \tag{17}$$

Since sequences $\mathcal{X}$ have equal probability, the first term $\log \Pr[\mathcal{X}]$ is constant. For the second term, we use the fact that the noise samples are independent and obtain

$$\begin{aligned} &\sum_{k=1}^{L} \log f_{\boldsymbol{\theta}'}(\mathbf{r}(k), \mathbf{x}_k) \\ &\quad = \sum_{k=1}^{L} \sum_{p=1}^{2^{(l+t-1)}} \log f_{\boldsymbol{\theta}'}(\mathbf{r}(k), \mathbf{x}_k = \xi_p)\delta(\mathbf{x}_k, \xi_p), \end{aligned} \tag{18}$$

where $\delta(\mathbf{x}_k, \xi_p) = 1$ when $\mathbf{x}_k = \xi_p$ and 0 otherwise, and

$$\begin{aligned} &\log f_{\boldsymbol{\theta}'}(\mathbf{r}(k), \mathbf{x}_k = \xi_p) \\ &\quad = -\frac{1}{\sigma'^2}\left\|\mathbf{r}(k) - \mathbf{\Omega}'\mathbf{g}(\xi_p)\right\|^2 - \log\left(\sigma'^2\right). \end{aligned} \tag{19}$$

Substitution of (18) in (16) yields

$$\begin{aligned} &Q(\boldsymbol{\theta}^{(i)}, \boldsymbol{\theta}') \\ &\quad = C + \sum_{k=1}^{L} \sum_{p=1}^{2^{(l+t-1)}} \left\{\left[-\frac{1}{\sigma'^2}\left\|\mathbf{r}(k) - \mathbf{\Omega}'\mathbf{g}(\xi_p)\right\|^2 - \log\left(\sigma'^2\right)\right]\right. \\ &\qquad \left. \times \sum_{\mathcal{X} \in \Xi} f_{\boldsymbol{\theta}^{(i)}}(r, \mathcal{X})\delta(\mathbf{x}_k, \xi_p)\right\}. \end{aligned} \tag{20}$$

It was shown in [10], that the sum over Ξ is equal to $f_{\boldsymbol{\theta}^{(i)}}(\mathbf{r}, \phi(k) = \xi_p)$. We thus have

$$\begin{aligned} Q(\boldsymbol{\theta}^{(i)}, \boldsymbol{\theta}') &= C + \sum_{k=1}^{L} \sum_{p=1}^{2^{(l+t-1)}} f_{\boldsymbol{\theta}^{(i)}}(\mathbf{r}, \phi(k) = \xi_p) \\ &\quad \times \left\{-\frac{1}{\sigma'^2}\left\|\mathbf{r}(k) - \mathbf{\Omega}'\mathbf{g}(\xi_p)\right\|^2 - \log\left(\sigma'^2\right)\right\}, \end{aligned} \tag{21}$$

where $\boldsymbol{\theta}^{(i)}$ is the old parameter estimates obtained at the ith iteration while $\boldsymbol{\theta}' = [\mathbf{\Omega}', \sigma'^2]$ is the new parameter set to be estimated at the $(i+1)$th iteration and $f_{\boldsymbol{\theta}^{(i)}}(\mathbf{r}, \phi(k) = \xi_p)$ is the weighted conditional likelihood. The direct computation of weighted conditional likelihood is computationally intensive. Fortunately, there exist recursive procedures (called forward and backward procedures), for computing $f_{\boldsymbol{\theta}^{(i)}}(\mathbf{r}, \phi(k) = \xi_p)$ whose complexity increases only linearly with data length L [9].

The following explicit expression for the array response matrix is obtained from $\partial Q/\partial \mathbf{\Omega}' = 0$:

$$\begin{aligned} \mathbf{\Omega}^{(i+1)} &= \left(\sum_{k=1}^{L} \sum_{p=1}^{2^{(l+t-1)}} f_{\boldsymbol{\theta}^{(i)}}(\mathbf{r}, \phi(k) = \xi_p)\mathbf{r}(k)\mathbf{g}(\xi_p)^H\right) \\ &\quad \times \left(\sum_{k=1}^{L} \sum_{p=1}^{2^{(l+t-1)}} f_{\boldsymbol{\theta}^{(i)}}(\mathbf{r}, \phi(k) = \xi_p)\mathbf{g}(\xi_p)\mathbf{g}(\xi_p)^H\right)^{-1}. \end{aligned} \tag{22}$$

The last equality follows from the definition of the partial derivative with respect to a complex quantity (see, e.g., [14])

$$\frac{\partial Q}{\partial \Omega'_{ij}} = \frac{1}{2}\left[\frac{\partial Q}{\partial \mathrm{Re}\{\Omega'_{ij}\}} + j\frac{\partial Q}{\partial \mathrm{Im}\{\Omega'_{ij}\}}\right], \tag{23}$$

where $\mathbf{\Omega}_{ij}$ is the ijth element of $\mathbf{\Omega}$.

From $\partial Q/\partial \sigma'^2 = 0$, the iterative estimation formula can also be derived for the noise variance

$$\sigma'^2 = \frac{\sum_{k=1}^{L}\sum_{p=1}^{2^{(l+t-1)}} f_{\theta^{(i)}}(\mathbf{r}, \phi(k) = \xi_p)\left\|\mathbf{r}(k) - \mathbf{\Omega}'\mathbf{g}(\xi_p)\right\|^2}{\sum_{k=1}^{L}\sum_{p=1}^{2^{(l+t-1)}} f_{\theta^{(i)}}(\mathbf{r}, \phi(k) = \xi_p)}. \tag{24}$$

Based on this results, the steps of the proposed unconditional ML algorithm are summarized as follows:

Set the parameters to some initial value $\theta^{(0)} = (\mathbf{\Omega}^{(0)}, \sigma^{2(0)})$.

(1) Compute the forward and backward variables to obtain $f_{\theta^{(i)}}(\mathbf{r}, \zeta(k) = \zeta_p)$.
(2) Compute $\mathbf{\Omega}'^{(i+1)}$ from (22).
(3) Compute $\sigma'^{2(i+1)}$ from (24).
(4) Repeat steps (1)–(3) until $\|\theta^{(i+1)} - \theta^{(i)}\| < \epsilon$, where ϵ is a predefined tolerance parameter.
(5) Use $f_{\theta^{(i)}}(\mathbf{r}, \phi(k) = \xi_p)$'s to recover the transmitted symbols.

Since the proposed method exploits the finite alphabet structure of the space-time coded signals and implements a stochastic ML solution, it is expected to exhibit better performance than suboptimal estimation techniques, especially when short data records are available. For a sufficiently good initialization, the proposed algorithm converges rapidly to the ML estimate of $\hat{\theta}$. In practice, however, we did not observe convergence problem when we initialized parameters according to suggestions of [11] (while initial guess on σ^2 is large enough to avoid overflow, $\mathbf{\Omega}$ is initialized arbitrarily (e.g., $\mathbf{\Omega}^{(0)} \approx 0$)).

4. PERFORMANCE ANALYSIS

The performance of the conditional and unconditional ML methods are assessed here by deriving their CRBs for the unbiased estimates of the nonrandom parameters. The CRB depends on the information on vector parameter θ quantified by the Fisher information matrix (FIM) and provides a lower bound on the variance of the unbiased estimate (i.e., $\mathbb{E}\{\hat{\theta}\} = \theta$). Then the CRB for an unbiased estimator $\hat{\theta}$ is bounded by the inverse of the FIM $\mathbf{J}(\theta)$:

$$\mathbb{E}\left\{(\theta - \hat{\theta})(\theta - \hat{\theta})^T\right\} \geq \mathbf{J}^{-1}(\theta). \tag{25}$$

4.1. Conditional CRB

The derivation of $\mathbf{J}(\theta)$ in (25) follows along the lines of [12]. We start constructing FIM by calculating the derivative of (6) with respect to

$$\tau = \left[\mathbf{c}_r^T(1)\ \mathbf{c}_c^T(1)\ \cdots\ \mathbf{c}_r^T(L)\ \mathbf{c}_c^T(L)\ \boldsymbol{\alpha}_r^T\ \boldsymbol{\alpha}_c^T\right]^T, \tag{26}$$

where

$$\begin{aligned}
\mathbf{c}_r(k) &= \mathrm{Re}\left\{\left[c_1(k), \ldots, c_n(k)\right]^T\right\},\\
\mathbf{c}_c(k) &= \mathrm{Im}\left\{\left[c_1(k), \ldots, c_n(k)\right]^T\right\},\\
\boldsymbol{\alpha}_r^i &= \mathrm{Re}\left\{\left[\alpha_{1,i}, \ldots, \alpha_{m,i}\right]^T\right\},\\
\boldsymbol{\alpha}_r &= \mathrm{Re}\left\{\left[\boldsymbol{\alpha}_1^T, \ldots, \boldsymbol{\alpha}_n^T\right]^T\right\},\\
\boldsymbol{\alpha}_c^i &= \mathrm{Im}\left\{\left[\alpha_{1,i}, \ldots, \alpha_{m,i}\right]^T\right\},\\
\boldsymbol{\alpha}_c &= \mathrm{Im}\left\{\left[\boldsymbol{\alpha}_1^T, \ldots, \boldsymbol{\alpha}_n^T\right]^T\right\}.
\end{aligned} \tag{27}$$

Taking the partial derivatives of (6), we then have

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \mathbf{c}_r(k)} &= \frac{\partial}{\partial \mathbf{c}_r(k)}\left(\text{const.} - \frac{1}{\sigma^2}\sum_{k=1}^{L}\mathbf{n}^H(k)\mathbf{n}(k)\right) \quad k = 1, \ldots, L\\
&= \frac{1}{\sigma^2}\left(\mathbf{\Omega}^H\mathbf{n}(k) + \mathbf{\Omega}^T\mathbf{n}^*(k)\right)\\
&= \frac{2}{\sigma^2}\mathrm{Re}\left\{\mathbf{\Omega}^H\mathbf{n}(k)\right\},\\
\frac{\partial \mathcal{L}}{\partial \mathbf{c}_c(k)} &= \frac{\partial}{\partial \mathbf{c}_c(k)}\left(\text{const.} - \frac{1}{\sigma^2}\sum_{k=1}^{L}\mathbf{n}^H(k)\mathbf{n}(k)\right) \quad k = 1, \ldots, L\\
&= \frac{1}{\sigma^2}\left(-j\mathbf{\Omega}^H\mathbf{n}(k) + j\mathbf{\Omega}^T\mathbf{n}^*(k)\right)\\
&= \frac{2}{\sigma^2}\mathrm{Im}\left\{\mathbf{\Omega}^H\mathbf{n}(k)\right\},\\
\frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_r^i} &= \frac{2}{\sigma^2}\sum_{k=1}^{L}\mathrm{Re}\left\{c_i^*(k)\mathbf{n}(k)\right\} \quad i = 1, \ldots, n,\\
\frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_r} &= \frac{2}{\sigma^2}\sum_{k=1}^{L}\mathrm{Re}\left\{\mathbf{c}^*(k) \otimes \mathbf{n}(k)\right\},\\
\frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_c^i} &= \frac{1}{\sigma^2}\left\{-jc_i^*(k)\mathbf{n}(k) + jc_i(k)\mathbf{n}^*(k)\right\} \quad i = 1, \ldots, n\\
&= \frac{2}{\sigma^2}\sum_{k=1}^{L}\mathrm{Im}\left\{\mathbf{c}_i(k)\mathbf{n}(k)\right\},\\
\frac{\partial \mathcal{L}}{\partial \boldsymbol{\alpha}_c} &= \frac{2}{\sigma^2}\sum_{k=1}^{L}\mathrm{Im}\left\{\mathbf{c}^*(k) \otimes \mathbf{n}(k)\right\}.
\end{aligned} \tag{28}$$

We need the following assumption and results to obtain FIM, (see [12]):

$$\begin{aligned}
E\left[\mathbf{n}(n)\mathbf{n}^H(m)\right] &= \sigma^2\mathbf{I},\\
E\left[\mathbf{n}(n)\mathbf{n}^T(m)\right] &= 0,\\
E\left[\mathbf{n}^H(n)\mathbf{n}(n)\mathbf{n}^T(m)\right] &= 0.
\end{aligned} \tag{29}$$

Using (28), (29), and taking expectations , we then obtain the entries of the FIM for the conditional case, which are given by

$$E\left\{\left(\frac{\partial\mathcal{L}}{\partial c_r(n)}\right)\left(\frac{\partial\mathcal{L}}{\partial c_r(m)}\right)^T\right\}=\frac{2}{\sigma^2}\operatorname{Re}\{\Omega^H\Omega\}\delta_{n,m}=\mathrm{A},$$
$$E\left\{\left(\frac{\partial\mathcal{L}}{\partial c_r(n)}\right)\left(\frac{\partial\mathcal{L}}{\partial c_c(m)}\right)^T\right\}=-\frac{2}{\sigma^2}\operatorname{Im}\{\Omega^H\Omega\}\delta_{n,m}=\mathrm{B},$$
$$E\left\{\left(\frac{\partial\mathcal{L}}{\partial c_c(n)}\right)\left(\frac{\partial\mathcal{L}}{\partial c_c(m)}\right)^T\right\}=\frac{2}{\sigma^2}\operatorname{Re}\{\Omega^H\Omega\}\delta_{n,m},$$
$$E\left\{\left(\frac{\partial\mathcal{L}}{\partial c_r(k)}\right)\left(\frac{\partial\mathcal{L}}{\partial \alpha_r}\right)^T\right\}=\frac{2}{\sigma^2}\operatorname{Re}\{\Omega^H\otimes \mathbf{c}^H(k)\}=\mathrm{C}_k,$$
$$E\left\{\left(\frac{\partial\mathcal{L}}{\partial c_c(k)}\right)\left(\frac{\partial\mathcal{L}}{\partial \alpha_r}\right)^T\right\}=\frac{2}{\sigma^2}\operatorname{Im}\{\Omega^H\otimes \mathbf{c}^H(k)\}=\mathrm{D}_k,$$
$$E\left\{\left(\frac{\partial\mathcal{L}}{\partial c_r(k)}\right)\left(\frac{\partial\mathcal{L}}{\partial \alpha_c}\right)^T\right\}=-\frac{2}{\sigma^2}\operatorname{Im}\{\Omega^H\otimes \mathbf{c}^H(k)\},$$
$$E\left\{\left(\frac{\partial\mathcal{L}}{\partial c_c(k)}\right)\left(\frac{\partial\mathcal{L}}{\partial \alpha_c}\right)^T\right\}=\frac{2}{\sigma^2}\operatorname{Re}\{\Omega^H\otimes \mathbf{c}^H(k)\},$$
$$\begin{aligned}E\left\{\left(\frac{\partial\mathcal{L}}{\partial \alpha_r}\right)\left(\frac{\partial\mathcal{L}}{\partial \alpha_r}\right)^T\right\}&=\frac{2}{\sigma^2}\sum_{n=1}^{L}\sum_{m=1}^{L}\operatorname{Re}\left[\mathbf{c}^*(k)\otimes \mathbf{n}(k)\times\mathbf{n}^H(m)\otimes\mathbf{c}^H(m)\right],\\&=\frac{2}{\sigma^2}\sum_{k=1}^{L}\operatorname{Re}\left[\mathbf{c}^*(k)\otimes\mathbf{I}_m\otimes\mathbf{c}^H(k)\right]=\mathrm{E},\end{aligned}$$
$$\begin{aligned}E\left\{\left(\frac{\partial\mathcal{L}}{\partial \alpha_c}\right)\left(\frac{\partial\mathcal{L}}{\partial \alpha_c}\right)^T\right\}&=\frac{2}{\sigma^2}\sum_{n=1}^{L}\sum_{m=1}^{L}\operatorname{Re}\left[\mathbf{c}^*(k)\otimes \mathbf{n}(k)\times\mathbf{n}^H(m)\otimes\mathbf{c}^H(m)\right],\\&=\frac{2}{\sigma^2}\sum_{k=1}^{L}\operatorname{Re}\left[\mathbf{c}^*(k)\otimes\mathbf{I}_m\otimes\mathbf{c}^H(k)\right],\end{aligned}$$
$$\begin{aligned}E\left\{\left(\frac{\partial\mathcal{L}}{\partial \alpha_r}\right)\left(\frac{\partial\mathcal{L}}{\partial \alpha_c}\right)^T\right\}&=-\frac{2}{\sigma^2}\sum_{n=1}^{L}\sum_{m=1}^{L}\operatorname{Im}\left[\mathbf{c}^*(k)\otimes \mathbf{n}(k)\times\mathbf{n}^H(m)\otimes\mathbf{c}^H(m)\right],\\&=-\frac{2}{\sigma^2}\sum_{k=1}^{L}\operatorname{Im}\left[\mathbf{c}^*(k)\otimes\mathbf{I}_m\otimes\mathbf{c}^H(k)\right]=-\mathrm{F}.\end{aligned}\tag{30}$$

Then the FIM can be written in partitioned form as

$$\mathrm{J}=\left[\begin{array}{ccc|c}\mathcal{H}&&0&\mathcal{C}_1\\&\ddots&&\vdots\\0&&\mathcal{H}&\mathcal{C}_L\\\hline\mathcal{C}_1^T&\cdots&\mathcal{C}_L^T&\mathcal{E}\end{array}\right],\tag{31}$$

where

$$\mathcal{H}=\begin{bmatrix}\mathrm{A}&-\mathrm{B}\\\mathrm{B}&\mathrm{A}\end{bmatrix},\quad\mathcal{C}_k=\begin{bmatrix}\mathrm{C}_k&-\mathrm{D}_k\\\mathrm{D}_k&\mathrm{C}_k\end{bmatrix},\quad\mathcal{E}=\begin{bmatrix}\mathrm{E}&-\mathrm{F}\\\mathrm{F}&\mathrm{E}\end{bmatrix}.\tag{32}$$

The FIM can now be directly constructed. We can numerically compute the variance of individual parameter estimate by inverting the FIM $\mathrm{CRB}(\boldsymbol{\tau})=\operatorname{diag}\{\mathrm{J}^{-1}(\boldsymbol{\tau})\}$.

4.2. Unconditional CRB

We now turn to the evaluation of the unconditional CRB. Under ($\mathrm{AS1}_u$), the computation of the exact CRB is analytically intractable, we therefore consider an alternative approach for simplifying CRB calculation [13].

The evaluation of the exact form of the unconditional CRB requires the Hessian matrix for the unconditional log-likelihood function. The corresponding log-likelihood function explicitly for the current problem is given by

$$\log\left[f_\theta(\mathbf{r})\right]=-nL\log(2)-mL\log\left(\pi\sigma^2\right)+\sum_{k=1}^{L}\log\left(\sum_{p=1}^{2^{(l+t-1)}}\exp\left\{-\frac{\|\mathbf{r}(k)-\Omega\mathbf{g}(\zeta_p)\|^2}{\sigma^2}\right\}\right).\tag{33}$$

Unfortunately, due to the nature of (33) the evaluation of the Hessian matrix is analytically intractable. However, it is common to adopt (see, e.g., [13]) an approximate log-likelihood function to obtain valid CRB. Due to concavity of the log-likelihood function and Jensen's inequality, we obtain from (33) the following approximate log-likelihood function:

$$\log\left[f_\theta(\mathbf{r})\right]\le\sum_{k=1}^{L}\sum_{p=1}^{2^{(l+t-1)}}\log\left[\exp\left\{-\frac{\|\mathbf{r}(k)-\Omega\mathbf{g}(\zeta_p)\|^2}{\sigma^2}\right\}\right].\tag{34}$$

If we further simplify (35), we obtain

$$\log\left[f_\theta(\mathbf{r})\right]\le-\frac{1}{\sigma^2}\sum_{k=1}^{L}\sum_{p=1}^{2^{(l+t-1)}}\|\mathbf{r}(k)-\Omega\mathbf{g}(\zeta_p)\|^2.\tag{35}$$

At this point, we should point out that the Hessian matrix from the approximate log-likelihood function can be easily obtained. However, (35) leads to a CRB called modified CRB(MCRB) which is not as tight as exact CRB, but it is computationally easier to evaluate.

It turns out from the approximate log-likelihood function of (34) that the entries of the FIM are as

$$\mathrm{J}_{\sigma^2,\sigma^2}=\frac{nL}{\sigma^4},\qquad\mathrm{J}_{\sigma^2,\Omega}=\mathbf{0},\qquad\mathrm{J}_{\Omega,\sigma^2}=\mathbf{0}.\tag{36}$$

Moreover, the submatrix $\mathrm{J}_{\Omega,\Omega}$ can also be obtained as

$$\mathrm{J}_{\Omega,\Omega}=\frac{2}{\sigma^2}\sum_{p=1}^{2^{(l+t-1)}}\mathbf{g}(\zeta_p)\mathbf{g}^H(\zeta_p).\tag{37}$$

The i.i.d. input sequence coded with orthogonal space-time codes results in uncorrelated coded sequence. It is therefore possible to further simplify the valid MCRB's. In this case, the valid MCRB can be easily obtained as follows:

$$\mathrm{J}^{-1}=\sigma^2\begin{bmatrix}\dfrac{\sigma^2}{nL}&0\\0&\dfrac{2\,\mathrm{I}}{2^{2(l+t-1)}}\end{bmatrix}.\tag{38}$$

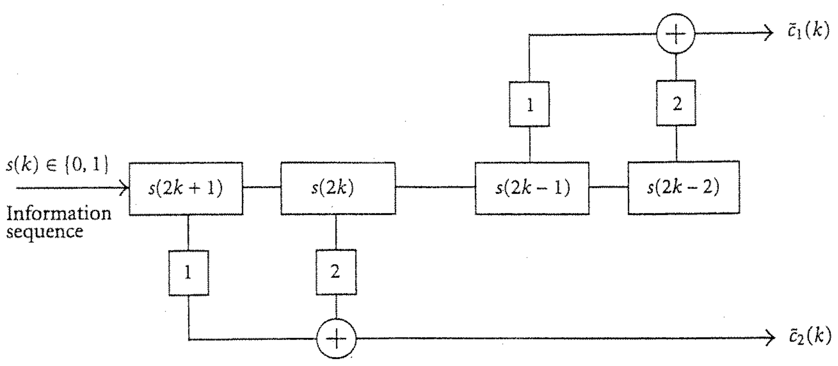

FIGURE 2: 4-state space-time coding system model.

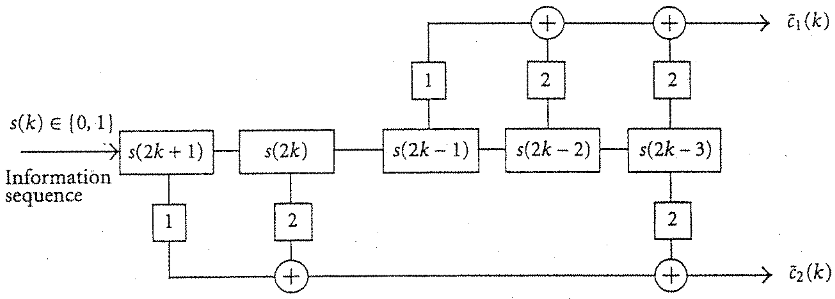

FIGURE 3: 8-state space-time coding system model.

5. SIMULATIONS

In this section, we illustrate some simulation results to evaluate the effectiveness and applicability of the proposed ML approaches. We consider the generator matrix form representation of the space-time coding system [15]. In this representation the stream of coded complex $MPSK$ symbols are obtained by applying the mapping function $\mathcal{M}$ to the following matrix multiplication:

$$\mathbf{c}(k) = \mathcal{M}(\mathbf{u}(k) \cdot \mathbf{G}(\text{mod}M)), \qquad (39)$$

where $\mathbf{u}(k) = [s(lk+t-1), \ldots, s(lk-t)]^T$ and $\mathbf{G}$ is the generator matrix with n columns and $l+s$ rows and $\mathcal{M}$ is a mapping function that maps integer values $\bar{c}_i$ to the coded $MPSK$ symbols, $\mathcal{M}(\bar{c}_i) = \exp(2\pi j\bar{c}_i/M)$.

The performance of the proposed methods was evaluated as a function of SNR (signal-to-noise ratio) based on the Monte Carlo simulations. Both conditional and unconditional ML methods were tested for 200 Monte Carlo trials per SNR point across range of SNRs. In each trial, the estimation error of each parameter estimate from conditional and unconditional ML for the channel parameters were recorded. We consider the following two different cases.

Case 1. 4PSK space-time code example shown in Figure 2 is considered with $n = 2$, $t = 2$ and the generator matrix

$$\mathbf{G} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}. \qquad (40)$$

In this case, the coded 4PSK symbols obtained from two current information bits are transmitted over the first antenna, whereas the coded 4PSK symbols obtained from two preceding bits are transmitted over the second antenna simultaneously. The coded symbols are then transmitted through quasi-static fading channel matrix.

In Figure 4, we have plotted the estimation error obtained from conditional and unconditional ML for the channel parameters as well as the corresponding CRBs. The estimation error experienced by the proposed estimation procedures at each iteration (SNR = 10 dB) is shown in Figure 6.

Case 2. A slightly more complicated space-time encoder with $n = 2$, $t = 3$ and the generator matrix

$$\mathbf{G} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \\ 2 & 2 \end{bmatrix} \qquad (41)$$

is considered in this case. This example would be an 8-state code as shown in Figure 3.

In Case 2, the coded 4PSK symbols generated from $[s(2k+1), s(2k), s(2k-3)]$ are transmitted over the first antenna, whereas the coded 4PSK symbols obtained from $[s(2k-1), s(2k-2), s(2k-3)]$ are transmitted over the second antenna simultaneously. The coded symbols are then transmitted through the quasi-static fading channel matrix.

Figure 5 shows the experimental estimation error for

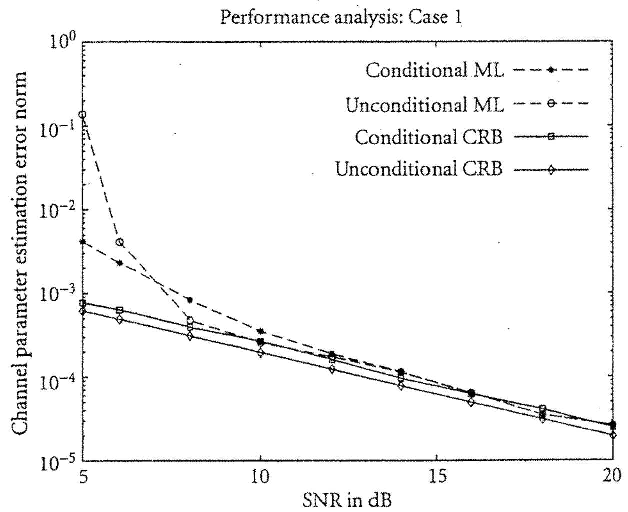

FIGURE 4: Case 1: Channel matrix estimation error norm.

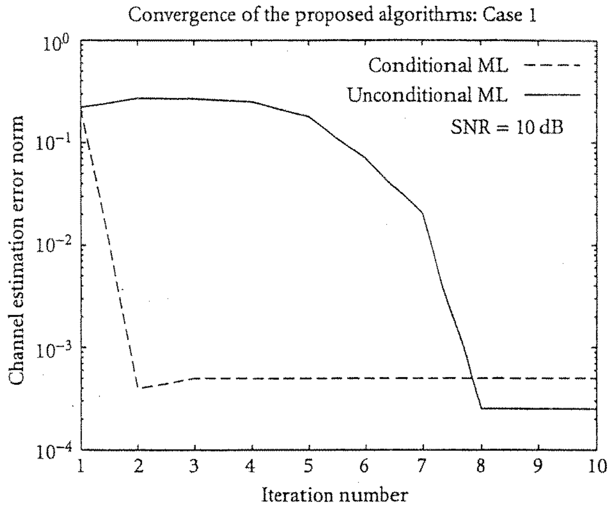

FIGURE 6: Case 1: Convergence of the channel matrix.

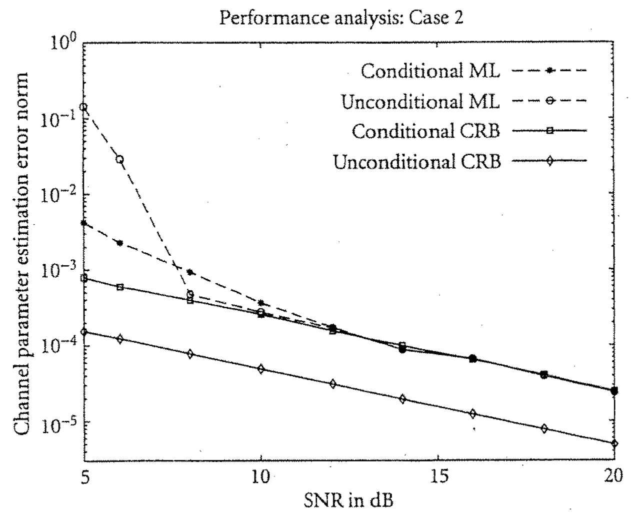

FIGURE 5: Case 2: Channel matrix estimation error norm.

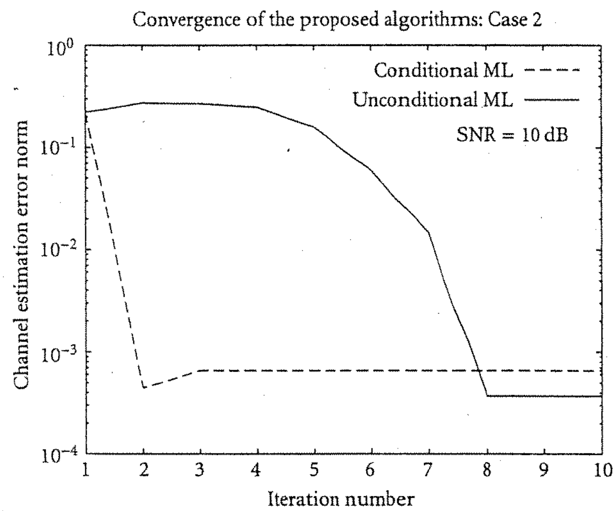

FIGURE 7: Case 2: Convergence of the channel matrix.

both the conditional and unconditional ML together with their corresponding CRB's for a range of SNR's. Figure 7 shows the estimation error experienced by the proposed estimation procedures at each iteration (SNR = 10 dB).

Based on the simulations we made the following observations:

(i) the proposed conditional and unconditional ML approaches perform almost identically for high SNR values. Moreover, conditional ML achieve conditional CRB for high SNRs;

(ii) since the unconditional cost function is dominated by only one term for high SNR, it results in exactly the same cost function as one would obtain for conditional ML estimation of $\boldsymbol{\theta}$. It is therefore expected that both conditional and unconditional cost functions yield similar estimates of $\boldsymbol{\theta}$ at high SNR. Thus the unconditional ML approach also achieves conditional CRB for high SNR;

(iii) the unconditional approach requires more iterations than the conditional approach to converge, however, unconditional approach is more successful in reducing channel estimation error norm at convergence for moderate SNR values.

6. CONCLUSIONS

In this paper, we presented the conditional and unconditional approaches to the problem of blind estimation of the channel parameters along with the space-time coded sequence. We derived iterative ML algorithms based on the conditional and unconditional signal models. Furthermore, the performance of the proposed algorithms are explored based on the derivation of their associated CRBs. We also presented Monte Carlo simulations to verify the theoretically predicted estimator's performance. The examples demonstrated that proposed ML approaches achieve the conditional

CRB for high SNR values. Since the unconditional CRB provides a looser bound, it is not as tight as exact CRB.

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Joint Channel Tracking and Symbol Detection for OFDM Systems with Kalman Filtering

Adnan Şen, Hakan A. Çırpan, and Erdal Panayırcı

Abstract This paper proposes a new joint channel tracking and symbol detection scheme for pilot symbol assisted OFDM systems in multipath fading. The proposed scheme uses Kalman filters for both channel tracking and subsequent equalization which are combined in the coupled estimator structure. Modelling the multipath fading channel as random processes to describe channel's variations in a general AR framework lends itself to a state-space representation that enables application of Kalman filtering for tracking of channel variations. However, the proposed tracking algorithm requires knowledge of the transmitted symbols. This implies that an iterative method should be sought to obtain alternatively either channel or transmitted symbols. To compose the coupled estimator structure, a linear Kalman filter equalizer with the corresponding state-space model is therefore proposed for the detection of transmitted symbols. With the proposed Kalman filters, iterative structure is utilized to decode transmitted symbols and subsequently to track channel parameters. Finally, the performance of the proposed method is investigated through the experimental results.

Keywords OFDM Systems, Joint Channel Estimation and Symbol Detection.

1. Introduction

The rapid growth in demand for a wide range of wireless services is a major driving force to provide high-data rate and high quality wireless access over multipath fading channels. However, wireless transmission is limited by available radio spectrum and impaired by path loss, interference from other users and fading caused by destructive addition of multipath. Therefore several physical layer related techniques have to be developed for future wireless systems to use the frequency resources as efficiently as possible. One approach that shows real promise for substantial capacity enhancement is to employ Multicarrier transmission and, in particular, orthogonal frequency division multiplexing (OFDM) [1]. OFDM has emerged as an attractive and powerful alternative to conventional modulation schemes recently, mainly due to its various advantageous in lessening the severe effect of frequency selective fading. Therefore, OFDM is currently being adopted and tested for many standards, including terrestrial digital broadcasting (DAB and DVB) in Europe, and high speed modems over Digital Subscriber Lines in the US. It has also been implemented for broadband indoor wireless systems including IEEE802.11a, MMAC and HIPERLAN/2.

An OFDM system operating over a wireless communication channel effectively forms a number of parallel frequency nonselective fading channels thereby reducing intersymbol interference (ISI) and obviating the need for complex equalization thus greatly simplifying channel estimation/equalization task. Moreover, OFDM is bandwidth efficient since the spectra of the neighboring subchannels overlap, yet channels can still be separated through the use of orthogonality of the carriers. Furthermore, its structure also allows efficient hardware implementations using FFTs and polyphase filtering [1].

Although the structure of OFDM signalling avoids ISI arising due to channel memory, fading multipath channel still introduces random attenuations on each tone. Hence, accurate channel estimation technique have to be used to improve the performance of the OFDM systems. Recently, several channel parameter estimation techniques were proposed in the literature [2-8]. In [2], a channel estimator for OFDM systems has been proposed based on the singular value decomposition. Time domain filtering has been proposed in [3]. To further improve the channel estimator performance, a MMSE channel estimator, which makes full use of the time and frequency correlation of the time-varying dispersive channel was proposed in [4]. In [5], a non-data-aided maximum a posteriori channel estimation technique was proposed which requires a convenient representation of the discrete multipath fading channel based on the Karhunen-Loeve orthogonal expansion and estimates the complex channel parameters of each subcarriers iteratively in frequency domain using the Expectation-Maximization(EM) algorithm. However, these methods do not explicitly take into account the channel's variations. One possi-

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57 (2003) No. 1

ble approach assumes channel's taps as random processes. In this approach, the channel taps are usually assumed to be low-pass circular complex Gaussian processes in order to model progressive variations. The information on the channel dynamics, provided by the stochastic framework, can therefore be exploited with accurate tracking algorithms. In [6], random phase introduced by Rayleigh fading in OFDM systems modelled as a multichannel autoregressive (AR) process. Based on the proposed multichannel AR model, Kalman filtering technique was applied for tracking the channel taps and maximum a posteriori (MAP) optimum detection technique was utilized for joint channel estimation and detection. However, the proposed optimum detector complexity was too high to be of practical use [6].

In this paper, we address the estimation and equalization problem for OFDM systems with pilot symbols. Among various channel models, the stochastic approach has been used to describe channel's variations in a general AR framework. The information theoretic results in [7] has shown that the lower order AR models provide a sufficiently accurate model for multipath fading channels. Fortunately, the AR modelling lends itself to a state-space representation that enables application of Kalman filtering for tracking of channel variations. We therefore propose Kalman filtering to derive minimum variance estimators for the fading coefficients yielding an adaptive channel tracking algorithm. However, it requires the knowledge of the transmitted symbols. This implies that an iterative method should be sought to obtain alternatively either channel or transmitted symbols. To complete detection-estimation algorithm for OFDM systems with the distributed training, a linear Kalman filter equalization technique [8] is therefore proposed for the detection of transmitted symbols.

The rest of the paper is organized as follows: A discrete-time received signal model for a OFDM sytem in multipath fading is presented in section 2. Section 3 first considers the Kalman filter in a general context, associates modelling of the fading channel taps with an AR process and then gives Kalman based channel tracking and symbol detection algorithms. In Section 4, the proposed coupled estimator structure is briefly discussed . Finally, we present some numerical examples that illustrate the performance of the proposed Kalman based receiver in Section 5 and conclusions are made in Section 6.

Notations used in this paper are standard. Symbols for matrices (in capital letter) and vector (lower case) are in boldface. $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$ denote transpose, conjugate and conjugate-transpose respectively. $\hat{\theta}$ denotes the estimate of parameter vector θ.

2. Problem Formulation

Before developing Kalman filter based channel estimation and data detection algorithm, we briefly describe the OFDM system model and the channel statistics in this section.

2.1 A Model for the Received Signal

The OFDM system baseband model under consideration is shown in Fig. 1. In OFDM, the entire information stream is split in many parallel low-rate channels, which are then regularly multiplexed and transmitted through narrow-band subcarriers. Consider an OFDM system with N subcarriers, and let $a(k)$ be the independent data symbol to be placed on subcarrier k, $0 \leq k \leq N-1$. Thus, the data symbols $a(k)$ are modulated by N subcarriers using Inverse Fast Fourier Transform (IFFT) and the last L samples are copied and put as cyclic prefix (CP) to form the complete OFDM symbols of $N+L$ samples long. This data is transmitted serially over the channel, whose impulse response is shorter that L samples. The cyclic prefix is removed at the receiver and the received signal is demodulated with a Fast Fourier Transform (FFT). We assume that the use of CP both preserves the orthogonality of the subcarrier frequencies (tones) and eliminates ISI between consecutive OFDM symbols, resulting the simple input/output relation. In spite of the loss of transmission power and bandwidth associated with the CP, the preservation of orthogonality in the system and the simple channel equalization generally motivate the use of the CP. Further, the channel is assumed to be constant during one OFDM symbol. Under these assumptions we can describe the system as a set of parallel Gaussian channels with correlated channel attenuation $h(k)$, shown in Fig. 2. The attenuation at the kth sub-carrier is given by [4]

$$h(k) = G(\frac{k}{NT_s}), \qquad k = 0, 1, \cdots, N-1 \qquad (1)$$

where $G(.)$ is the frequency response of the channel $g(\tau)$ during the OFDM symbol and T_s is the sampling period of the system.

The received signal after demodulation (performing a FFT) at the kth tone, can be expressed as

$$y(k) = a(k)\ h(k) + v(k), \qquad k = 0, \cdots, N-1. \quad (2)$$

In the above expression $v(k)$ is additive independent complex white Gaussian noise at the kth tone with zero mean and variance σ^2.

Given noisy observations $\{y(0), y(1), \cdots y(N-1)\}$, our main objective is to estimate channel taps $\{h(0), h(1), \cdots h(N-1)\}$ along with the signal $\{a(0), a(1), \cdots, a(N-1)\}$ modulating the tones.

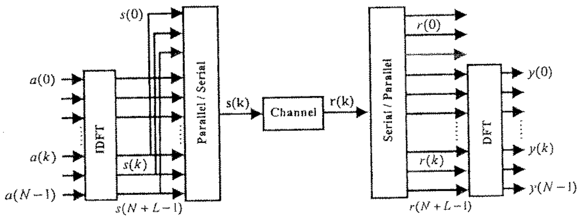

Fig. 1. Baseband OFDM system, transmitting N-blocks of data

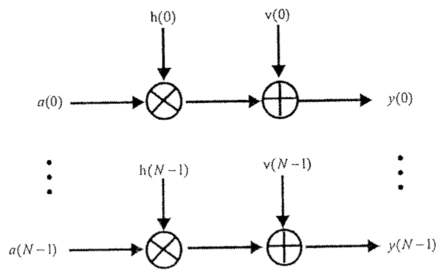

Fig. 2. Baseband OFDM system described as a set of parallel Gaussian Channels

Without imposing structures on $h(k)$, this goal is ill-posed problem because for every incoming received data one more unknown appears in addition to the unknown $a(k)$. In wireless mobile communications, channel frequency variations arise mainly due to multipath effect. Consequently, channel variations evolve in a progressive fashion and hence fit in some frequency-evolution model. Among different models, the AR model is adopted herein in order to model channel dynamics. We therefore exploit channel taps as an AR process with a *priori* known structure in the development of estimation technique. Before we start the channel estimation analysis, we introduce multipath fading channel and its corresponding statistical properties in the next section.

2.2 Channel Statistics

The complex baseband representation of a fading multipath channel impulse response can be described as [11]

$$g(\tau) = \sum_l \alpha_l \delta(\tau - \tau_l T_s) \tag{3}$$

where τ_l is the delay of the lth path and α_l is the corresponding complex amplitude with a power-delay profile $\theta(\tau_l)$. Note that α_l's are zero-mean, complex Gaussian random variables, which are assumed to be independent for different paths.

We now briefly describe the channel statistics. The correlation function of the frequency response of the multipath fading channel for different frequencies is

$$r(f, f') = E[G(f)G^*(f')] \tag{4}$$

where

$$G(f) = \int_{-\infty}^{+\infty} g(\tau) e^{-j2\pi f\tau} = \sum_l \alpha_l e^{-j2\pi f\tau_l} \,. \tag{5}$$

It can be shown that (4) has the form [11]

$$r(f, f') = \sigma_g^2 r_f(f - f') \tag{6}$$

$$r_f(\Delta f) = (1/\sigma_g^2) \sum_l \sigma_l^2 e^{-j2\pi\Delta f\tau_l} \tag{7}$$

where σ_l^2 is the average power of the lth path and σ_g^2 is the total average power of the channel impulse response defined as

$$\sigma_g^2 = \sum_l \sigma_l^2 .$$

For an OFDM system with tone spacing Δf, the discrete correlation function for different tones can be written more compactly as

$$r_f(m) = E\left\{h(k)h^*(k+m)\right\} \,. \tag{8}$$

More frequently used channel model could be explicitly derived by using an exponentially decaying power delay profile $\theta(\tau_l) = Ce^{-\tau_l/\tau_{rms}}$ and special delays τ_l that are uniformly and independently distributed over length of CP. In [2], it is shown that the normalized exponential discrete channel correlation for different subcarriers is

$$r_f(m) = \frac{1 - \exp\left(-L\left(1/\tau_{rms} + 2\pi jm/N\right)\right)}{\tau_{rms}(1 - \exp(-L/\tau_{rms}))\left(1/\tau_{rms} + 2\pi jm/N\right)} \,. \tag{9}$$

Furthermore, the uniform channel correlation between the attenuations $h(k)$ and $h(k+m)$ can be obtained by letting $\tau_{rms} \to \infty$ in (9), resulting in

$$r_f(m) = \frac{1 - \exp\left(2\pi jLm/N\right)}{2\pi jk/N} \,. \tag{10}$$

Note that, the correlation function of the channel taps for different frequencies depends, in general only on the multipath delay spread and is separated from the effect of Doppler frequency. By only exploiting the frequency correlation in the channel estimation task, we are able to reduce complexity of the channel estimator.

3. Channel Estimation and Equalization

Progressive variations of the multipath fading channel requires the tap coefficients of the equalizer to be adjusted continuously. The conventional strategy to treat the variations of the multipath channel is to design an adaptive equalizer to directly update equalizer coefficients by using recursive least-squares (RLS) algorithm. In this case, explicit estimation of the channel parameters are avoided, but these approaches are unable to adequately compensate for channel impairments [9]. An alternative approach is that the adaptation of equalizer is performed indirectly via a channel estimator. The later approach, which is more robust against the channel variations, is used in this study. We employ Kalman recursive state estimation algorithms to acquire the channel, then use estimates to design adaptive equalizer.

The algorithm to be discussed in this paper is based on the Kalman recursive state estimation algorithm, which is well known in statistical estimation and control theory but perhaps not so in communications [10]. Therefore, we first consider its application in general context then explore applications in communications.

Kalman filter is a useful channel estimator if the channel model embedded in the Kalman filter closely matches the underlying communications channel. To build a channel model for the multipath fading channel, we match spectral characteristics of the multipath fading with an AR process. We will now consider the application of Kalman filter in a general context.

3.1 Kalman Filter Formulation

Consider a linear, discrete-time dynamic system, the state vector is any set of quantities that would be sufficient to uniquely describe the unforced dynamic behavior of the system. The two equations for Kalman filter are given by

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{\Phi}_k \mathbf{x}(k-1) + \mathbf{w}(k) \qquad (11) \\ \mathbf{y}(k) &= \mathbf{H}_k \mathbf{x}(k) + \mathbf{v}(k) \end{aligned}$$

where (11) and (12) are known as the signal (state vector) model and the observation (measurement) model equations, respectively, $\mathbf{x}(k)$ is a signal state vector containing system variables which may not all directly measurable, $\mathbf{y}(k)$ is the measurement vector, representing quantities which are observed and thus known, $\mathbf{\Phi}_k$ is the state transition matrix, which determines variation of $\mathbf{x}(k)$ together with $\mathbf{w}(k)$, a random vector known as the plant noise, $\mathbf{H}_k$ is the observation matrix and $\mathbf{v}(k)$ is the measurement noise vector, which is independent of $\mathbf{w}(k)$.

Since the state-space representation includes both time-varying and time-invariant systems and also encompasses stochastic and deterministic inputs, it is more flexible and powerful than the transfer function form. Moreover, it also allows us to define observability concept which is useful in determining whether the desired unknown parameters of a system can be estimated from the given observations. Assuming that the system state-space model is observable, the state estimation problem may be stated as follows: Given the observations $\mathbf{y}(0), \cdots, \mathbf{y}(N-1)$ and the state-space model (11) and (12), find the optimal estimate of $\mathbf{x}(k)$ denoted as $\hat{\mathbf{x}}(k \mid k)$. Based on the assumptions that the noise vectors $\mathbf{w}(k)$ and $\mathbf{v}(k)$ are individually and mutually uncorrelated with correlation matrices, i.e,

$$\begin{aligned} E[\mathbf{w}(i)\mathbf{w}^T(j)] = \mathbf{Q}_i\delta_{ij}, \quad E[\mathbf{v}(i)\mathbf{v}^T(j)] = \mathbf{R}_i\delta_{ij}, \\ E[\mathbf{w}(i)\mathbf{v}^T(j)] = 0 \qquad (12) \end{aligned}$$

where δ_{ij} is the Kronecker delta function, then the Kalman filter provides the way to recursively find the optimal linear estimation of the state vector $\mathbf{x}(k)$ based on a set of observed data $\mathbf{y}(k)$, through recursively minimizing the mean-squared error

$$\hat{\mathbf{x}}_{MMSE}(k \mid k) = \arg \min_{\hat{\mathbf{x}}(k|k)} E\left\{\|\mathbf{x}(k) - \hat{\mathbf{x}}(k \mid k)\|^2\right\}. \qquad (13)$$

The MMSE state estimate $\hat{\mathbf{x}}(k)$ can be obtained by solving (11) and (12) with Kalman filter recursions based on the standard Riccati equations listed in Table I.

Estimation of the state vector $\hat{\mathbf{x}}(k)$ are obtained recursively as

$$\hat{\mathbf{x}}(k \mid k) = \mathbf{\Phi}_k\hat{\mathbf{x}}(k \mid k-1) + \mathbf{K}(k)\left(\mathbf{y}(k) - \mathbf{H}_k\hat{\mathbf{x}}(k \mid k-1)\right) \qquad (14)$$

where $\hat{\mathbf{x}}(k \mid k)$ is the filtered state estimate, and $\mathbf{\Phi}_k\hat{\mathbf{x}}(k \mid k-1)$ is the corresponding predicted state estimate. If we define $\mathbf{e}(k) = \hat{\mathbf{x}}(k \mid k) - \mathbf{\Phi}_k\hat{\mathbf{x}}(k \mid k-1)$ and $\varepsilon(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k \mid k)$ as the predicted and filtered state error vector, respectively then $\mathbf{S}(k \mid k-1) = E[\mathbf{e}(k)\mathbf{e}^H(k)]$ and $\mathbf{S}(k \mid k) = E[\varepsilon(k)\varepsilon^H(k)]$ are the predicted and estimated error covariance matrices, respectively, and $\mathbf{K}(k)$ is the Kalman gain vector.

3.2 Channel Estimation

Since only the first few correlation terms are important to finitely parametrize structured variations of wireless channel in the design of channel estimator, low-order AR models, or even a simple Markov model can capture most of the channel tap dynamics and lead to effective tracking algorithms. In this paper, we follow the common practice and model the channel's variations as a random process. Thus, this paper associates the multiplicative multipath fading effect of the channel with an AR process.

TABLE I: GENERAL KALMAN RECURSIONS

Model:	$\mathbf{x}(k) = \Phi_k \mathbf{x}(k-1) + \mathbf{w}(k)$ $\mathbf{y}(k) = \mathbf{H}_k \mathbf{x}(k) + \mathbf{v}(k)$
Riccati Equations:	$\mathbf{S}(k \mid k-1) = \Phi_k \mathbf{S}(k-1 \mid k-1)\Phi_k^T + \mathbf{Q}_k$ $\mathbf{K}(k) = \mathbf{S}(k \mid k-1)\mathbf{H}_k^T \left(\mathbf{R}_k + \mathbf{H}_k \mathbf{S}(k \mid k-1)\mathbf{H}_k^T\right)^{-1}$ $\mathbf{S}(k \mid k) = (\mathbf{I} - \mathbf{K}(k)\mathbf{H}_k)\,\mathbf{S}(k \mid k-1)$
State Estimation Update:	$\hat{\mathbf{x}}(k \mid k-1) = \Phi_k \hat{\mathbf{x}}(k-1 \mid k-1)$ $\hat{\mathbf{x}}(k \mid k) = \hat{\mathbf{x}}(k \mid k-1) + \mathbf{K}(k)\left(\mathbf{y}(k) - \mathbf{H}_k \hat{\mathbf{x}}(k \mid k-1)\right)$

3.2.1 AR Model Considerations

We will approximate the multiplicative multipath fading effect in OFDM system with a general AR model order p,

$$h(k) = -\sum_{i=1}^{p} c_i h(k-i) + w(k), \quad k = 0, 1, \cdots, N-1 \tag{15}$$

where $w(k)$ is a white gaussian random process with variance σ_w^2. The choice of p is a trade-off between the accuracy of the model and the difficulty in estimating its parameters. For low-pass processes, however, even a low-order AR model may be sufficient [13]. The AR parameters c_i, are complex in general. However, if it is assumed that the real and imaginary parts of $h(k)$ are independent, these AR parameters will be real. The parameters c_i are closely related to the physical parameters of the underlying fading process.

The estimation of c_i is still not trivial since the channel $h(k)$ is not observed directly. One needs to somehow acquire the channel correlations $r_f(m)$ in order to solve the Yule-Walker equations [10]

$$r_f(m) = \begin{cases} -\sum_{i=1}^{p} c_i r_f(m-i) & m \geq 1 \\ -\sum_{i=1}^{p} c_i r_f(-i) + \sigma_w^2 & m = 0 \end{cases} \tag{16}$$

and obtain c_i. For example, a channel correlation model given by (9) can be used here to determine c_i coefficients. However, the problem of estimating the statistics of a random channel taps has been previously studied. A useful method to obtain the channel correlation during a training mode is provided in [13], via higher than second order statistics. Their method is effective and requires only reasonable assumptions about the transmitted sequence and noise. Optimal maximum likelihood solutions has been reported in [11]. Moreover, a combination of two Kalman filters was utilized for tracking the channel and estimating channel's statistics [12]. However, we assume in the sequel that channel statistics are either known a priori or estimated from received data. Thus, given channel statistics $r_f(m)$, AR parameters can be directly obtained by solving normal equations [10]. Once the AR model parameters are identified, Kalman filtering ideas may be employed to track the variations of channel coefficients. Since the Kalman filter would require state-space representation of the multipath fading channel, we will now formulate the state-space description of the fading channel model based on AR model parameters.

3.2.2 State-space Representation

When a Kalman filter is used for estimating a process, the model which describes the dynamics of the process and the observation is formed using the state-space representation. If we define $\mathbf{h}(k) = [h(k), h(k-1), \cdots, h(k-p)]^T$, then (15) can be written in state-space form as

$$\mathbf{h}(k) = \begin{bmatrix} -c_1 & -c_2 & \dots & -c_p \\ 1 & 0 & \dots & 0 \\ & \ddots & \ddots & \vdots \\ 0 & \dots & 1 & 0 \end{bmatrix} \mathbf{h}(k-1) + \begin{bmatrix} w(k) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{17}$$

Let $\mathbf{A}$ be the $(p \times p)$ square matrix in the right hand side of (17), then the state equations becomes

$$\mathbf{h}(k) = \mathbf{A}\mathbf{h}(k-1) + \mathbf{b}w(k) \tag{18}$$

where $\mathbf{b} = [1, 0, \cdots, 0]^T$. In order to obtain the measurement equation, define $\mathbf{a}(k) = [a(k), 0, \cdots, 0]^T$. Then (2) can be written as

$$y(k) = \mathbf{a}^T(k)\mathbf{h}(k) + v(k) \ . \tag{19}$$

Equations (18) and (19) offer a state-space representation of the multiplicative multipath fading model with transition matrix $\mathbf{A}$ (which is assumed to be known in this section). Based on this representation, the minimum variance estimator for the state vector, i.e., the conditional expectation of $\mathbf{h}(k)$ given $\{a(k), y(k)\}_{k=0}^{N-1}$ can be computed from Kalman filtering. The recursions are summarized in Table II. Matrices $\mathbf{K}(k)$ and $\mathbf{S}(k \mid k)$ denote the Kalman filter gain and the covariance of the state vector $\mathbf{x}(k)$.

The proposed method yields an adaptive channel tracking algorithm. However, it requires knowledge of $a(k)$, which is not available for all tones. We therefore apply alternately channel tracking and equalization algorithm to acquire the channel and the decode the information symbols.

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TABLE II: KALMAN BASED CHANNEL TRACKING ALGORITHM

Initialization:	$\mathbf{h}(0 \mid -1) = 0\ , \quad \mathbf{S}(0 \mid -1) = 0$
Riccati Equations:	$\mathbf{S}(k \mid k-1) = \mathbf{A}\mathbf{S}(k-1 \mid k-1)\mathbf{A}^T + \mathbf{b}^T\mathbf{Q}\mathbf{b}$ $\mathbf{K}(k) = \mathbf{S}(k \mid k-1)\mathbf{a}(k)\left(\sigma_v^2 + \mathbf{a}^T(k)\mathbf{S}(k \mid k-1)\mathbf{a}(k)\right)^{-1}$ $\mathbf{S}(k \mid k) = \left(\mathbf{I} + \mathbf{K}(k)\mathbf{a}^T(k)\right)\mathbf{S}(k \mid k-1)$
State Estimation Update:	$\hat{\mathbf{h}}(k \mid k-1) = \mathbf{A}\hat{\mathbf{h}}(k-1 \mid k-1)$ $\hat{\mathbf{h}}(k \mid k) = \hat{\mathbf{h}}(k \mid k-1) + \mathbf{K}(k)\left(y(k) - \mathbf{a}^T(k)\hat{\mathbf{h}}(k \mid k-1)\right)$

3.3 Adaptive Kalman Equalization

A Kalman filtering procedure was developed in Section 3.2 to track the variations of the channel taps based on some modelling parameters. However, Kalman based adaptive channel estimation technique have to be coupled with a equalization technique in order to eventually compose a practical system. A number of different approaches on equalization may be used. Traditionally, tap estimates are used to decode the transmitted symbols, either in a maximum likelihood sequence estimation, or using simpler decision feedback schemes. In this paper, we adopt adaptive Kalman equalizer to couple proposed Kalman channel estimator. Adaptive Kalman equalizer approach [8] uses non-Gaussian linear state-space model due to discrete nature of the input which results in the plant and observation noise terms being correlated. Thus, this approach does not yield conditional mean estimates.

Adaptive Kalman equalizer was originally developed for FIR channel model [8]. In this section, we first summarize the adaptive Kalman equalizer, then we modify it to apply for OFDM systems.

If we assume FIR channel model, the elements of the state vector would be the inputs to the channel, i.e.,

$$\mathbf{a}_e(k) = [a(k), a(k-1), \cdots, a(k-d)]^T \tag{20}$$

where $(d{+}1)$ is the number of taps of the channel. This choice of the state vector is in contrast with Kalman based channel estimator, where the channel taps are used to construct the state vector.

For the adaptive Kalman equalizer, the state transition equation has the following form:

$$\mathbf{a}_e(k) = \mathbf{F}\mathbf{a}_e(k-1) + \mathbf{g}a(k) \tag{21}$$

where $\mathbf{F}$ is the $(d+1) \times (d+1)$ shift matrix, i.e,

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & \cdots\cdots & 0 \\ 1 & 0 & \cdots\cdots & 0 \\ 0 & 1 & \ddots\ \ddots & \vdots \\ \vdots & \ddots & \ddots\ \ddots & \vdots \\ 0 & \cdots\cdots & 1 & 0 \end{bmatrix}$$

and $\mathbf{g}$ is a vector with $(d+1)$ elements, $\mathbf{g} = [1, 0, \cdots, 0]^T$ or more concisely,

$$\mathbf{a}_e(k) = \begin{bmatrix} 0 & 0 & \cdots\cdots & 0 \\ 1 & 0 & \cdots\cdots & 0 \\ 0 & 1 & \ddots\ \ddots & \vdots \\ \vdots & \ddots & \ddots\ \ddots & \vdots \\ 0 & \cdots\cdots & 1 & 0 \end{bmatrix} \mathbf{a}_e(k-1) + \begin{bmatrix} a(k) \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix} .$$

Then, the observation equation for the Kalman equalizer is

$$y(k) = \mathbf{a}_e^T(k)\mathbf{h}_e(k) + v(k)\ . \tag{22}$$

where $\mathbf{h}_e(k) = [h(k), h(k-1), \cdots, h(k-d)]^T$ is a vector with channel taps. Based on the state-space representation for FIR channel, adaptive Kalman equalizer recursions are summarized in [8]. However, the state-space representation (21) and (22) for FIR equalizer could be adopted for OFDM systems in a very simple form since OFDM overcomes ISI arising from channel memory and only introduces random attenuations on each tone. Thus, the simplified form of state-space representation for OFDM systems becomes

$$\begin{aligned} a(k) &= f\ a(k-1) + a(k) \\ y(k) &= a(k)h(k) + v(k)\ . \end{aligned} \tag{23}$$

where f ($f = 0$) in (23) superficially introduced parameter in order to put (23) in a form given by (21). (23) is therefore simply a scalar form of the state-space representation. With the initialization for $a(1)$ (from pilot symbol) and pre-selected power σ_v^2 for the measurement noise, Kalman filter for equalization could be obtained with scalar Kalman filter recursions summarized in Table III. In the steps of Table III, $K(k)$ is the Kalman gain, $S(k \mid k-1)$ and $S(k)$ are the *a priori* and the *posteriori* variance of the estimated state vector respectively.

4. Receiver Structure

The proposed receiver uses Kalman filters for both channel tracking and subsequent equalization. Therefore, the Kalman filters are combined in the proposed coupled estimator structure of Figure 3.

TABLE III: ADAPTIVE KALMAN EQUALIZER

Initialization:	$a(1) = \text{known},\ S(1) = 1$
Riccati Equations:	$S(k \mid k-1) = 1$ $K(k) = S(k \mid k-1)h(k)\left(\sigma_v^2 + \mid h(k) \mid^2 S(k \mid k-1)\right)^{-1}$ $S(k) = (1 - K(k)h(k))\,S(k \mid k-1)$
State Estimation Update:	$\hat{a}(k) = K(k)y(k)$

Note that, in the joint detection-estimation problem, both $\mathbf{h}(k)$ and $a(k)$ are unknown. With the knowledge of pilot symbol $a(1)$ and the observation $y(1)$, $\hat{\mathbf{h}}(1)$ can be obtained using a Kalman recursion following the steps of Table II. However, the detection of $a(k)$, $k = 2, \cdots, N-1$ relies on the estimates of $\mathbf{h}(k)$, $k = 2, \cdots, N-1$ that in turn require the knowledge of $a(k)$, $k = 2, \cdots, N-1$. Therefore, iterative structure is employed in the proposed receiver to obtain alternatively either $a(k)$ or $\mathbf{h}(k)$. According to (18) and (19), a coarse prediction of $\mathbf{h}(k \mid k-1)$ can be obtained directly from prediction step. It can be observed from Kalman recursions in Table II that the coarse channel prediction obey the recursion

$$\hat{\mathbf{h}}(k \mid k-1) = \mathbf{A}\hat{\mathbf{h}}(k-1 \mid k-1) \qquad (24)$$

that are initialized by $\mathbf{h}(0 \mid -1) = \mathbf{0}$. Next, we use the coarse channel estimates in adaptive Kalman equalizer to obtain coarse symbol estimates for $\hat{a}(k)$ that are denoted by $\hat{a}^c(k)$. These estimates are subsequently transformed into $\hat{a}^r(k)$ using the nearest neighbor criterion with a slicer.

Replacing $a(k)$ by $\hat{a}^r(k)$, we rely on Kalman filter to obtain refined channel estimates $\hat{\mathbf{h}}(k \mid k)$. Thus, we summarize our algorithm for channel tracking and symbol detection, in the following steps:

Initialization: Obtain $\mathbf{h}(1 \mid 1)$ from pilot symbol;
1. Obtain $\hat{\mathbf{h}}(k \mid k-1)$ using (24);
2. Use Kalman equalizer in Table III. to decode $\hat{a}^c(k)$;
3. Use slicer to obtain $\hat{a}^r(k)$ from $\hat{a}^c(k)$;
4. Perform Kalman channel estimator in Table II. to retrieve $\hat{\mathbf{h}}(k \mid k)$ using $\hat{a}^r(k)$;
5. Repeat Steps 1-4 for $k+1 \leftarrow k$.

In general, the issue of convergence for coupled estimator's remains open. It is clear, however, for the coupled estimator that if the Kalman filter provides correct channel taps then the Kalman equalizer converge to correct estimates of symbols, and vice versa.

Next, we test the performance of our joint channel tracker and equalizer through simulations.

5. Simulations

We now present the simulation results for tracking the channel taps and decoding transmitted symbols in OFDM systems with Kalman filtering. We consider the fading multipath channel given by (9) with an exponentially decaying power delay profile and delays τ_l that are uniformly and independently distributed over the length of the cyclic prefix.

The scenario for our simulation study consists of a wireless QPSK OFDM system employing the pulse shape as a unit-energy Nyquist-root raised-cosine shape with rolloff $\alpha = 0.2$ with a symbol period(T_s) of 0.277 μs. Transmission bandwidth(3.6 MHz) is divided into 128 tones. We assume that the fading multipath channel has exponentially decaying power delay profile (9) with an $\tau_{rms} = 4$ sample (1.08 μs) long.

Since the first-order AR model provides a sufficiently accurate model for multipath fading channels, AR(1) process ($p = 2$) is adopted in the development of the state-space description. Channel model AR(1) parameters are obtained by solving Yule-Walker equations in terms of the correlations. QPSK-OFDM sequence passes through channel taps and corrupted by AWGN (30dB, 20dB and 10dB respectively. We use a pilot symbol every twelve (P=12) symbols.

In the following, the coupled estimator is used to obtain alternatively either transmitted symbols or channel taps in a iterative receiver scheme shown in Figure 3. The results of the Kalman tracking algorithm for both real part and imaginary part of $h(k)$ are obtained and shown in Figure 4. In order to better evaluate the performance of the proposed Kalman based tracking algorithm, we compare it with other previously developed recursive least squares (RLS) adaptive algorithm. The RLS algorithm minimizes the average weighted squared error over an interval. Therefore, the channel tracker coefficients are chosen to minimize the cost function

$$\zeta(i) = \sum_{k=0}^{i} \lambda^{i-k} \mid y(k) - \mathbf{a}^T(k)\mathbf{h}(k) \mid^2 \qquad (25)$$

where λ is the forgetting factor and the cost function (25) is minimized by the following recursions:

$$\begin{aligned} \hat{\mathbf{h}}(k+1) &= \hat{\mathbf{h}}(k) + \mathbf{K}(k)(y(k) - \mathbf{a}^T(k)\hat{\mathbf{h}}(k)) \\ \mathbf{K}(k) &= \mathbf{S}(k)\mathbf{a}(k)\left(\lambda + \mathbf{a}^T(k)\mathbf{S}(k)\mathbf{a}(k)\right)^{-1} \\ \mathbf{S}(k+1) &= \lambda^{-1}\left(\mathbf{S}(k) - \mathbf{K}(k)\mathbf{a}^T(k)\mathbf{S}(k)\right) \qquad (26) \end{aligned}$$

By these equations, the estimator uses the information of the received signal to update its state estimates. We can therefore observe that the RLS algorithm is basically same as the measurement update equations of the Kalman filter. Thus, the RLS is a more appropriate algorithm if we do not have enough information about the system parameters. For instance, it is assumed in this paper that the channel parameters **A** is not available in the RLS.

Thick solid lines in these figures represent the true channel taps, thick solid lines with x-mark represent channel tracking with Kalman filtering where as thin solid lines with plus represents the channel tracking with RLS.

It can be seen from simulations that, both the proposed pilot symbol assisted coupled receiver (Kalman based) and the RLS based channel tracking algorithm together with Kalman equalizer perform almost identical performances especially at high SNR values. However, the proposed Kalman based receiver yield better performance for low SNR values.

Since we match spectral characteristics of the multipath fading channel with an AR process, a closer match can be obtained with a higher order AR model. However we focus on the order 2 AR model here due to its simplicity. Moreover, we investigate the sensitivity of performance to such model order selection. We compare tracking performance of the AR order 2 with a order 1 model for SNR=30dB, as shown by Figure 5. It is thus advisable to select AR model order $p \geq 2$ in the proposed method to obtain appropriate tracking. Of course more computational complexity is introduced along with the tracking process if you increase the model order.

Next we wish to illustrate and compare the tracking performance of the proposed Kalman based receiver. The tracking ability of pilot symbol assisted method can be observed from Figure 6A. In a non-data aided set-up (No pilot case), the proposed techniques performance degrades significantly after 20 tones. Thus, periodic retraining scheme is therefore suggested to avoid run-away effect caused by Kalman equalizer.

Finally, we wish to evaluate the performance of the algorithms by plotting the respective error probabilities as a function of SNR. Figure 6B shows the symbol error rate for the proposed method (solid line with x-mark), and the RLS method (solid line with plus) and the adaptive Kalman equalizer with perfect channel knowledge (solid line with circle). The performance of the proposed method surpassed that RLS method. This figure also shows that the multipath fading is the major impeding factor than the effects of the additive noise.

6. Conclusions

We have developed a novel Kalman filter based scheme for joint iterative channel tracking and symbol recovery of pilot symbol assisted OFDM systems in multipath fading channels. Modelling multipath fading channel as AR processes, Kalman filter was employed to track the variations of the channel. Moreover, to compose the joint iterative estimator structure, a linear Kalman filter equalizer with the corresponding state-space model was proposed for the recovery of transmitted symbols. Although, the adaptive Kalman equalizer does not yield minimum variance estimates, its structure is very simple and can be implemented with scalar Kalman filter recursions. In this paper, we also address parameter adjustment for the Kalman filter and show that the Kalman filter that utilizes a low-complexity AR model order 2 is useful for estimating the multipath fading channel taps in OFDM systems. The simulation results show that the resulting algorithms is efficient and can be effectively employed in such applications. Future topics include extensions to Space-Time coded OFDM systems and blind estimation of the AR parameters.

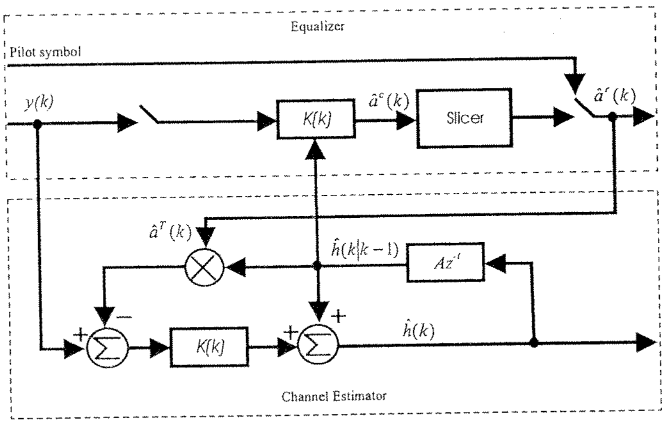

Fig. 3. Proposed Kalman filter based channel tracking/equalization structure

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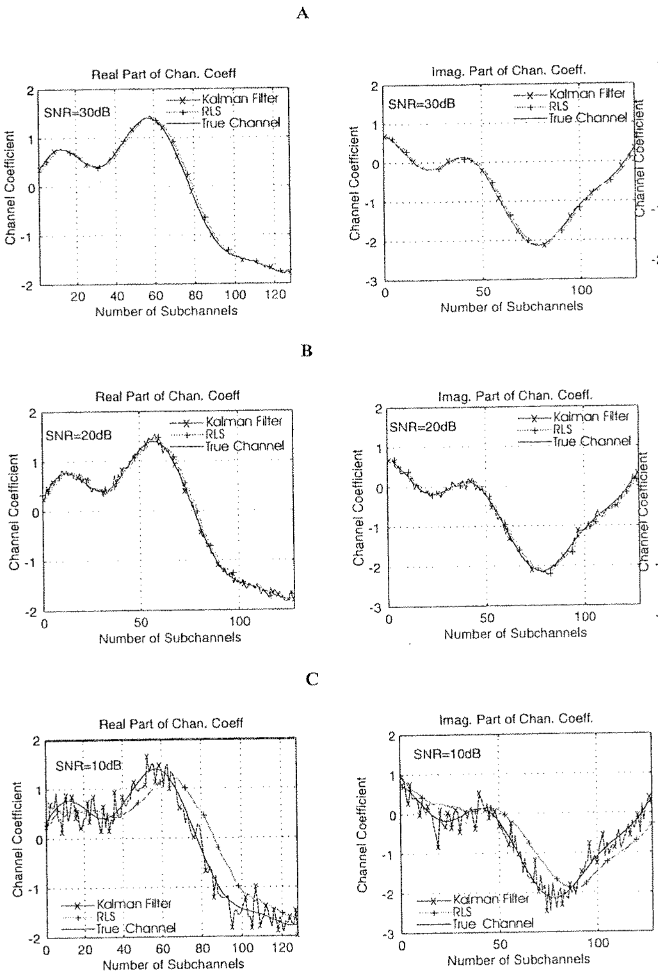

Fig. 4. Comparison of channel tracking performance between the proposed Kalman-based method and the method with RLS: A. SNR=30dB, B. SNR=20dB, C. SNR=10dB.

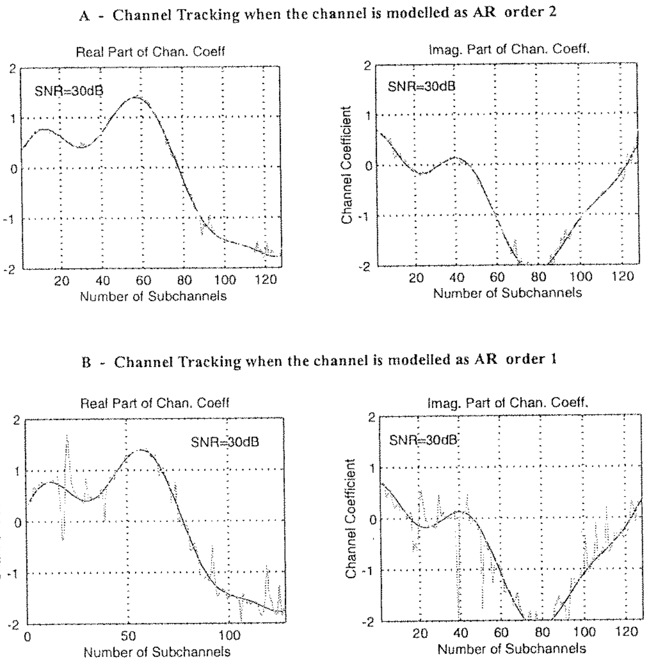

Fig. 5. Performance comparison of the tracking algorithm based on the channel modelling: A- Channel is modelled with AR order 2. B- Channel is modelled with AR 1.

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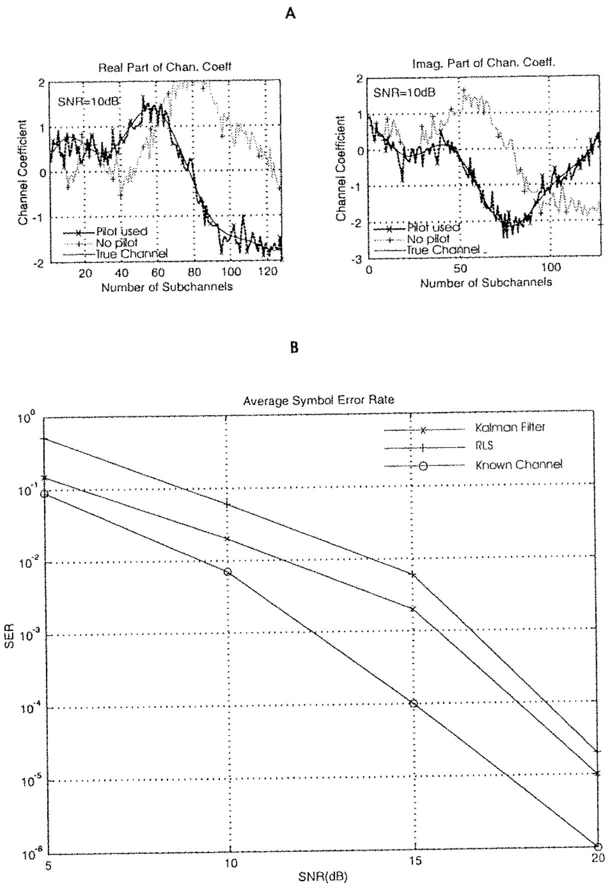

Fig. 6. Performance comparison of the tracking algorithm: A- the Tracking performance of the proposed Kalman based receiver. B- SER performance of the proposed receiver .

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Sequence Estimation with Transmit Diversity for Wireless Communications

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Abstract In this paper, an optimum sequence estimation algorithm for wireless systems with Alamouti's two transmitter diversity in the presence of multipath fading is proposed. The algorithm is based on a jointly iterative channel and sequence estimation according to the maximum likelihood (ML) criterion, using the Expectation-Maximization (EM) algorithm employing an M-level phase-shift keying(M-PSK) modulation scheme with additive Gaussian noise. The discrete multipath channel is represented in terms of the channel gains from each transmit antenna to the receive antenna. EM algorithm estimates jointly the complex channel parameters of each channel and the data sequence transmitted, iteratively, which converges to the true ML solution. The channel estimation is achieved in a simple way through the iterative equations by decoupling of the signals transmitted from different antennas. The algorithm is applied to the trellis coded modulation systems and the efficiency of the algorithm proposed has been shown with computer simulations. The simulation results show that the EM algorithm converges quickly for fast fading channels. The performance of the EM-based decoder approaches that of the ML receiver which has perfect knowledge of the channel.

Keywords Sequence estimation, transmit diversity, EM algorithm, multipath fading channels.

1. Introduction

Transmitter diversity is an effective technique for combating fading in multipath wireless channels. It has been observed recently that transmitter (spatial) diversity may be the only option when the frequency and time diversity techniques are not always available. For instance, frequency diversity cannot be achieved in a frequency non-selective channel, and in a slow fading channel, either time diversity is not effective, or significant delays must be introduced to achieve it because of the required large interleaver size. Transmit diversity has been studied only recently to reduce the detrimental effects in wireless fading channels because of its relative simplicity of implementation and feasibility of having multiple antennas at the base stations.

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Several transmit diversity techniques were studied extensively in the past. Wittneben [1] proposed the first bandwidth efficient transmit scheme and subsequently, a delay diversity scheme was introduced by Seshadri and Winters, [2]. More recently, space-time trellis coding has been proposed by Tarokh, Seshadri and Calderbank [3] which combines signal processing at the receiver with coding techniques appropriate to multiple transmit antennas. These so-called space-time codes perform well in slowly-fading channels, assuming perfect channel information(CSI) at the receiver. With the presence of channel mismatch, however, system performance suffers a significant degradation.

Recently, Alamouti proposed a remarkable transmit diversity scheme for transmission using two transmit antennas, [4]. This scheme has been generalized later in [5, 6] to an arbitrary number of transmit antennas and is thus able to achieve the full diversity promised by the transmit and receive antennas. Assuming that the channel state information is somehow available, the orthogonal structure of these space-time block codes enables the ML decoding to be implemented in a simple way through decoupling of the signal transmitted from different antennas rather than by joint detection. However, channel state information is usually difficult to obtain. In the absence of perfect channel state information, evaluation of the ML function requires the expectation over the joint statistics of the channel fading coefficients, which is usually mathematically intractable. To cope with this technical difficulty, in this paper, we apply the method of Georghiades and Han [7] to the sequence estimation in the presence of multipath fading channels for wireless systems with two-transmitter diversity. The algorithm is based on a jointly iterative channel and sequence estimation according to the ML criterion, using the EM algorithm, [8, 9, 10]. The last part of the paper provides simulation results on the convergence of the EM algorithm. The performance is presented in terms of the bit error rate for a system employing trellis coded 8-PSK signaling. The extensive computer simulations show that a formulation of the sequence estimation based on the EM algorithm is a promising technique for highly efficient data transmission over mobile wireless channels, and it performs close to the performance of a maximum likelihood decoder that assumes perfect CSI.

The paper is organized in four sections following this introduction. In Section 2, the system model is introduced, Section 3, includes the EM-based algorithm, Section 4 presents the simulation results and finally the conclusions are presented in Section 5.

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2. System Model

We consider the wireless communication system as shown in Figure 1 with transmitter diversity using a space-time block coded transmit diversity scheme as first proposed by Alamouti, [4]. The scheme is described with 2 transmit and 1 receive antennas to provide a diversity of order 2. Note that, the method can be easily extended to the more general orthogonal space-time block coded systems introduced by Tarokh *et al.*[5] involving more than two transmit and one receive antennas.

The information data can be either uncoded or encoded by a trellis coded modulation (TCM) encoder, then fed into the space-time block encoder. At each time slot, the output symbols are modulated and transmitted simultaneously, each from a different transmit antenna. At the receiver end, the space-time block decoder followed by symbol-by-symbol decoder, or by a Viterbi decoder, for uncoded and coded cases, respectively, can be used to decode the received sequence. The generated complex constellation symbols characterizing the input bits are fed into the space-time block encoder proposed by Alamouti whose transmission matrix is given as

$$\begin{array}{c} \quad space \longrightarrow \\ time \downarrow \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \end{array} \qquad (1)$$

whose rows are transmitted in successive time intervals with the first and second symbol in a given row sent simultaneously through the first and second antenna, respectively. Based on this scheme, at each time slot k $(k = 0, 1, \cdots, L-1)$, the signal transmitted from the first antenna is s_{2k} and the signal transmitted from the second antenna is s_{2k+1}. In the next time slot $(k+1)$, the signal $-s_{2k+1}^*$ is transmitted from the first antenna, and the signal s_{2k}^* is transmitted from the second antenna. Coherent M-PSK modulation is used here to enhance the system performance.

The wireless channel is assumed to be a fast fading channel where the maximum Doppler spread normalized by the symbol rate is of the order of 10^{-2}. Since we use Alamouti's scheme, it means that channel fading is required to be constant over two consecutive symbol periods($2T$), but it independently varies from one time interval $2T$ to another. Therefore, the system also offers time diversity through the TCM encoding and/or the interleaving of the data.

Define now $\boldsymbol{h}_0 = [h_{0,0}, h_{0,2}, \cdots, h_{0,(2L-2)}]^T$ and $\boldsymbol{h}_1 = [h_{1,0}, h_{1,2}, \cdots, h_{1,(2L-2)}]^T$, where $h_{i,j}$ denotes the channel gains from the first and second transmit antennas to the receive antenna, respectively, at the jth symbol period, $j = 0, 2, \cdots, 2L-2$. They are modeled as complex zero-mean Gaussian random variables with autocorrelation $r_l = E[h_{i,2k}h_{i,\ 2k+2l}^*]$, $i = 0, 1$; $l = 0, 1, \cdots, L-1$ and that $\boldsymbol{h}_0$ and $\boldsymbol{h}_1$ are independent of each other. For mobile fading channels, the autocorrelations are given by $r_l = v^2 J_0(2\pi f_d T l)$ where v^2 is the unnormalized variance of the fading gains, $J_0(.)$ is the zero-order Bessel function of the first kind, f_d is the maximum Doppler frequency in Hz, and T represents the signaling interval. Thus, for $i = 0, 1$, vector $\boldsymbol{h}_i$ has a normalized Toeplitz covariance matrix $\boldsymbol{R} = (1/v^2)[r_l]$. For $k = 0, 1, \cdots, L-1$, each pair of the two consecutive received signals can then be expressed as

$$\begin{aligned} r_{2k} &= s_{2k}h_{0,2k} + s_{2k+1}h_{1,2k} + n_{2k} \\ r_{2k+1} &= -s_{2k+1}^* h_{0,2k} + s_{2k}^* h_{1,2k} + n_{2k+1} \end{aligned} \qquad (2)$$

where n_{2k} and n_{2k+1} are independent samples of an additive Gaussian random variable with variance σ^2, representing the additive white Gaussian noise entering the system.

Letting $\boldsymbol{r} = [\boldsymbol{r}_0^T\ \boldsymbol{r}_1^T]^T$ where $\boldsymbol{r}_0 = [r_0, r_2, \cdots, r_{2L-2}]^T$ and $\boldsymbol{r}_1 = [r_1, r_3, \cdots, r_{2L-1}]^T$, (2) can be expressed in a matrix form

$$\boldsymbol{r} = \boldsymbol{S}\boldsymbol{h} + \boldsymbol{n} \qquad (3)$$

where, $\boldsymbol{h} = [\boldsymbol{h}_0^T\ \boldsymbol{h}_1^T]^T$, $\boldsymbol{n} = [\boldsymbol{n}_0^T\ \boldsymbol{n}_1^T]^T$,

$$\boldsymbol{S} = \begin{bmatrix} \boldsymbol{S}_0 & \boldsymbol{S}_1 \\ -\boldsymbol{S}_1^\dagger & \boldsymbol{S}_0^\dagger \end{bmatrix} \qquad (4)$$

and, $\boldsymbol{S}_0 = \mathrm{diag}\{s_0, s_2, \cdots, s_{2L-2}\}$, $\boldsymbol{S}_1 = \mathrm{diag}\{s_1, s_3, \cdots, s_{2L-1}\}$. $\dagger$ denotes conjugated transpose.

3. Sequence Estimation with EM Algorithm

Now consider the classical problem of estimating data sequence $\boldsymbol{s} = (s_0, s_1, \cdots, s_{2L-1})$ from observations of received data $\boldsymbol{r} = (r_0, r_1, \cdots, r_{2L-1})$. A ML receiver then performs

$$\max_{\boldsymbol{s}} p(\boldsymbol{r}|\boldsymbol{s}) = \max_{\boldsymbol{s}} E_{\boldsymbol{h}}\left[p(\boldsymbol{r}|\boldsymbol{s}, \boldsymbol{h})\right]. \qquad (5)$$

Note that evaluation of the likelihood function above requires the expectation over the joint statistics of the random channel parameters $\boldsymbol{h}$, a task that more often is mathematically intractable, even if the likelihood function can be obtained analytically off line. However, it is invariably a nonlinear function of $\boldsymbol{s}$, which makes the maximization step computationally infeasible in real time. For long and/or coded sequences transmitted over fading channels, the problem of optimum sequence estimation is known to be especially difficult or intractable. In such cases, an iterative formulation of the sequence estimation problem based on the EM algorithm can provide an implementable solution. We now briefly present the EM algorithm following the notation employed in [7]. However, the reader is urged to review the original paper of Dempster, Laird and Rubin, [8]. For an application of the EM algorithm to fading channels and a tutorial on the EM algorithm to read [9] and [11], respectively.

Let in general $\boldsymbol{s}$ be a set of parameters to be estimated from the some observed data $\boldsymbol{r}$. Then, The ML estimate $\widehat{\boldsymbol{s}}_{ML}$ of $\boldsymbol{s}$ is a solution to

$$\widehat{\boldsymbol{s}}_{ML} = \arg\max_{\boldsymbol{s}} p(\boldsymbol{r}|\boldsymbol{s}). \qquad (6)$$

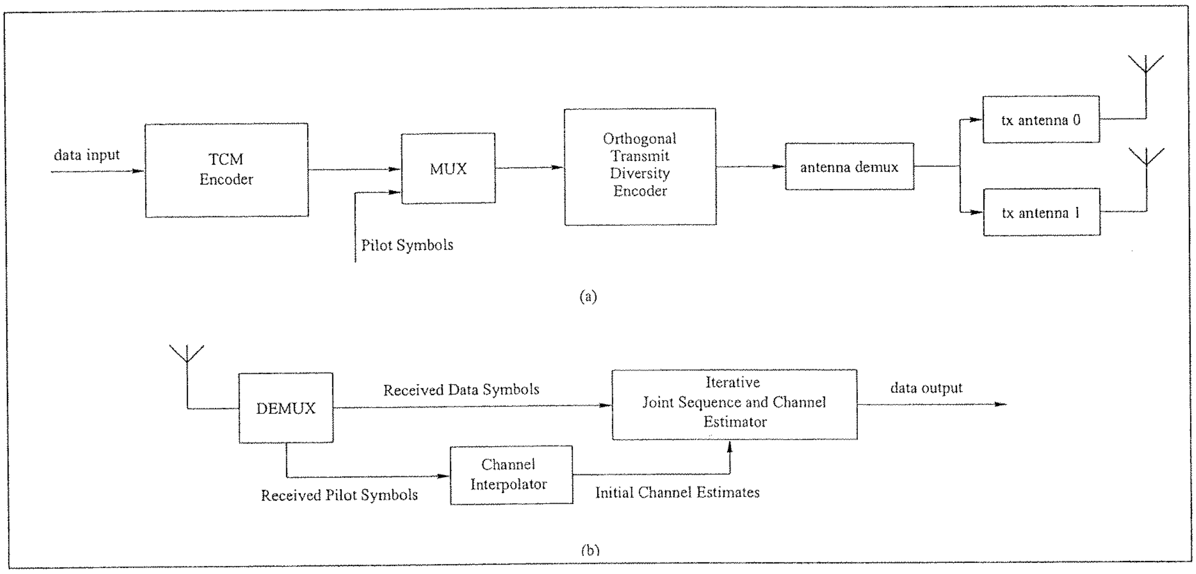

Fig. 1. (a) Transmitter and (b) Receiver block diagrams of the transmit diversity system

where $p(r|s)$ is the conditional density of the data given the parameter vector to be estimated. Note that, in many cases an explicit expression for this conditional density does not exist, or hard to obtain. In other cases, such an expression may exist, but it can not be easy to maximize efficiently over the set of parameters. In such situations, and under some conditions, the EM algorithm may provide an iterative solution to the ML estimation problem.

The EM algorithm based solution proceeds as follows. Suppose that instead of the data r that is actually available, one had access to data x. The data x is such that r could be obtained through a many-to-one mapping $x \rightarrow r(x)$, and their knowledge makes the estimation problem easy (for example, the conditional density $p(x|r)$ is easily obtained). The two sets of data r and x are referred to as the *incomplete* and *complete* data, respectively. The EM algorithm makes use of the log-likelihood function for the complete data in a two-step iterative procedure which under some conditions converges to the ML estimates given in (6) [12, 8]. At each step of the EM iteration, the likelihood function can be shown to be nondecreasing [12, 8]; if it is also bounded (which is mostly the case in practice) the algorithm converges. The two-step procedure at the $i$the iteration is as follows:

1. Expectation Step(E-step): Compute $Q(s|s^{(i)}) \equiv E[\log(p(x|s)|r, s^{(i)}]$;
2. Maximization Step(M-Step): Solve $s^{(i+1)} = \arg\max_s Q(s|s^{(i)})$

where $s^{(i)}$ is the parameter vector estimate at the ith iteration. Note that since the complete data x is not actually available, and the quantity $\log p(x|s)$ is a random variable, the algorithm maximizes its conditional expectation instead, given the incomplete data and the most recent estimate of the parameter vector to be estimated. In the beginning, the EM algorithm starts with an initial value $s^{(0)}$ of the unknown parameters s. The initial estimate is chosen based on the data and/or side information available. Otherwise, an *a priori* estimate is used.

Let us now apply the algorithm specifically to our problem. An obvious choice for the complete data is the observed data sequence r, and the unwanted random channel parameter vector h, namely, $x = (r, h)$. In this case,

$$p(x|s) = p(r|h, s)p(h)$$

where $p(h)$ is a *a priori* density of h, under our assumption that s and h are independent. Then

$$Q'(s|s^{(i)}) = E[\log p(r|h, s)|r, s^{(i)}] + E[\log p(h)|r, s^{(i)}]$$

and since the second term above is not a function of s (and thus it is a constant as far as the M-step is concern) it can be dropped to yield

$$Q(s|s^{(i)}) = E[\log p(r|h, s)|r, s^{(i)}]. \qquad (7)$$

We now briefly show that the EM algorithm converges to the ML algorithm [7]. The conditional expectation above is with respect to the conditional density of the random parameter vector h given the incomplete data r and assuming that $s = s^{(i)}$. The conditional density $p(h|r, s^{(i)})$ can be expressed as

$$p(h|r, s^{(i)}) = C.p(r|h, s^{(i)})p(h)$$

where C is some constant not a function of s that can be evaluated, but need not be. Thus

$$Q(s|s^{(i)}) = C.\int_h \log[p(r|h, s)]p(r|h, s^{(i)})p(h)dh.$$

Then a necessary condition for a maximum of $Q(s|s^{(i)})$ (assuming differentiability) is

$$\int_h \frac{p(r|h,s^{(i)})}{p(r|h,s)} \frac{\partial p(r|h,s)}{\partial s} p(h)dh|_{s=s^{(i+1)}} = 0.$$

It is easy to see that at convergence, in which case $s = s^{(i)} = s^{(i+1)}$, the condition reduces to

$$\frac{\partial p(r|s)}{\partial s}|_{s=\widehat{s}_{ML}} = 0$$

which is the necessary condition satisfied by the ML estimate of s.

We now return to our original problem. The log-likelihood function of r given s and h needed in (7) to compute the expectation step is easily obtained from (3) as follows

$$\ell(r|s,h) \equiv \log p(r|s,h) \sim p(||r - Sh||^2).$$

Dropping unnecessary terms and rearranging slightly it follows that

$$\ell(r|s,h) = \mathcal{R}e[r^\dagger Sh] - \frac{1}{2}||S||^2. \quad (8)$$

Assuming the PSK signaling is used we can drop the second term in the right hand side of (8). Then the expectation step of the EM algorithm at the ith iteration yields,

$$\begin{aligned} Q(s|s^{(i)}) &= \mathcal{R}e[r^\dagger S\widehat{h}^{(i)}] \\ &= \sum_{k=0}^{L-1} \Big[\mathcal{R}e\left\{ \left(r_{2k}^* s_{2k} - r_{2k+1}^* s_{2k+1}^*\right) \widehat{h}_{0,2k}^{(i)} \right\} \\ &\quad + \mathcal{R}e\left\{ \left(r_{2k}^* s_{2k+1} + r_{2k+1}^* s_{2k}^*\right) \widehat{h}_{1,2k}^{(i)} \right\} \Big] \quad (9) \end{aligned}$$

where $s^{(i)}$ is the most recent sequence estimate at the ith iteration and

$$\widehat{h}^{(i)} = E[h|r, s^{(i)}]. \quad (10)$$

The above conditional mean can be evaluated as follows: After some algebra, it is shown in Appendix-A that

$$p(h|r,s^{(i)}) \sim \exp\left[-(h - \widehat{h}^{(i)})^\dagger \Psi^{-1} (h - \widehat{h}^{(i)}) \right] \quad (11)$$

where, for a zero-mean, independent and identically distributed complex Gaussian noise vector, replacing the autocorrelation matrix $\Sigma_{\mathrm{n}} = \sigma^2 I$ in (A.4),(A.5), and for PSK signaling, replacing $S^\dagger S = I$ in (A.5) it follows that

$$\widehat{h}^{(i)} = (v^2/\sigma^2)\Psi S^{\dagger(i)} r, \quad (12)$$

$$\Psi = \left[R_{\mathrm{h}}^{-1} + (v^2/\sigma^2) I \right]^{-1}. \quad (13)$$

Here, R_{h} is a $2L \times 2L$ block diagonal matrix defined by $R_{\mathrm{h}} = \mathrm{diag}\{R\ R\}$, where R is the normalized autocorrelation matrix of the random fading vector, as defined earlier, whose main diagonal elements are unity. v^2 is the unnormalized variance of the random fading gains. σ^2 is the variance of the noise.

The EM algorithm starts with the initial channel estimates $\{\widehat{h}_{0,2k}^{(0)}, \widehat{h}_{1,2k}^{(0)}\}$ and uses them in (9) to produce, by maximization over s, a sequence estimate. This is then used in (12) to obtain the next channel estimate which, in turn, is used again in (9) to produce the next sequence estimate, and so on, until convergence occurs within two to three iterations. At convergence, $s^{(i+1)} = s^{(i)}$, the algorithm produces both a sequence estimate and a fading channel estimate.

We now turn to the maximization step of the EM algorithm, where we distinguish between the coded and the uncoded transmission. First we observe from (9) that in the absence of coding, maximizing $Q(s|s^{(i)})$ with respect to sequence s is equivalent to maximizing each individual term in the sum, i.e., making symbol-by-symbol decisions. Then, if $s^{(i+1)}$ is the maximizing sequence, for $k = 0, 1, \cdots, L-1$, its components are given by

$$\begin{aligned} s_{2k}^{(i+1)} &= \arg\max_{s_{2k}} \mathcal{R}e\left\{ r_{2k}^* s_{2k} \widehat{h}_{0,2k}^{(i)} + r_{2k+1}^* s_{2k}^* \widehat{h}_{1,2k}^{(i)} \right\} \\ s_{2k+1}^{(i+1)} &= \arg\max_{s_{2k+1}} \mathcal{R}e\left\{ -r_{2k+1}^* s_{2k+1}^* \widehat{h}_{0,2k}^{(i)} \right. \\ &\quad \left. + r_{2k}^* s_{2k+1} \widehat{h}_{1,2k}^{(i)} \right\} \quad (14) \end{aligned}$$

where we have used the expression for $Q(s|s^{(i)})$ in (9).

When trellis coding is used, the maximization over all trellis sequences can be done efficiently using the Viterbi algorithm. It is seen that in contrast to directly evaluating the likelihood function in (5), the EM algorithm yields at each step of iteration a likelihood function that allows the use of the Viterbi algorithm for efficient computations.

Initialization

In order to be able to choose good initial values for $s^{(0)}$, the N_{PS} data symbols $\{s_{2k}, s_{2k+1}\}$ for $k \in S_{PS}$, in each observation block are generally used as pilot symbols known by the receiver. They are inserted periodically in the sequence. Here, S_{PS} denotes the set of pilot symbols indices. To interpolate the channel estimates, initially, there exist a minimum spacing, l_{SC}, between pilots given by $l_{SC} < 1/\tau_{max}$, where τ_{max} is the maximum delay spread of the channel ($B_{coh} = 1/\tau_{max}$, channel coherence bandwidth).

To initialize the receiver, we determine $\widehat{h}_{0,2k}^{(0)} = \widehat{h}_0^{(0)}[2k]$ and $\widehat{h}_{1,2k}^{(0)} = \widehat{h}_1^{(0)}[2k], k \in S_{PS}$ in terms of the pilot symbols and the received signals corresponding to the pilot symbols from the following equations.

$$\widehat{h}_0^{(0)} = \Psi_{11}^{(0)}(s_0^{\dagger(0)} r_0 - s_1^{(0)} r_1) + \Psi_{12}^{(0)}(s_1^{\dagger(0)} r_0 + s_0^{(0)} r_1)$$

$$\widehat{h}_1^{(0)} = \Psi_{21}^{(0)}(s_0^{\dagger(0)}r_0 - s_1^{(0)}r_1) + \Psi_{22}^{(0)}(s^{\dagger(1)}r_0 + s^{(0)}r_1), (15)$$

and

$$\Psi^{(0)} = \begin{bmatrix} \Psi_{11}^{(0)} & \Psi_{12}^{(0)} \\ -\Psi_{21}^{(0)} & \Psi_{22}^{(0)} \end{bmatrix}.$$

The complete initial channel gains $\left\{\widehat{h}_{0,2k}^{(0)}, \widehat{h}_{1,2k}^{(0)}\right\}$ for $k = 0, 1, \cdots, L-1$ can be easily determined using an interpolation technique, i.e., Lagrange interpolation algorithm.

The EM algorithm is now summarized briefly as follows:

Step 1. Set $i = 0$ and choose the initial values $s^{(0)}$, and determine $\widehat{h}_0^{(0)}, \widehat{h}_1^{(0)}$, as explained above

Step 2. Compute $s^{(i+1)}$ by maximizing $Q(s|\widehat{s}^{(i)})$ in (9) over all sequences by Viterbi algorithm if trellis coding is present. Use (14) to perform the maximization if coding is not present.

Step 3. Compute $\widehat{h}_0^{(i+1)}, \widehat{h}_1^{(i+1)}$ from (12) and go to Step 2, repeat until the algorithm converges, in which case the last sequence estimate is produced as the ML estimate.

Note that a computation of the number of iterations needed to implement the EM algorithm indicates that it increases linearly in the sequence length compared to the more than exponential increase for direct implementation. Also, the maximization step can be implemented easily due to the fact that $Q(s|s^{(i)})$ can be expressed in recursive form as in (9), and thus, the Viterbi algorithm can be employed.

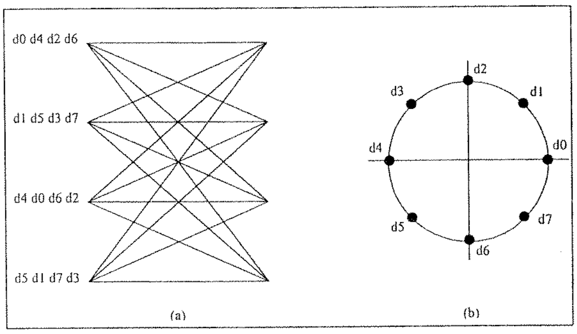

Fig. 2. (a) Trellis diagram of the 4-state 2/3 rate 8-PSK trellis code (b) 8-PSK signal set

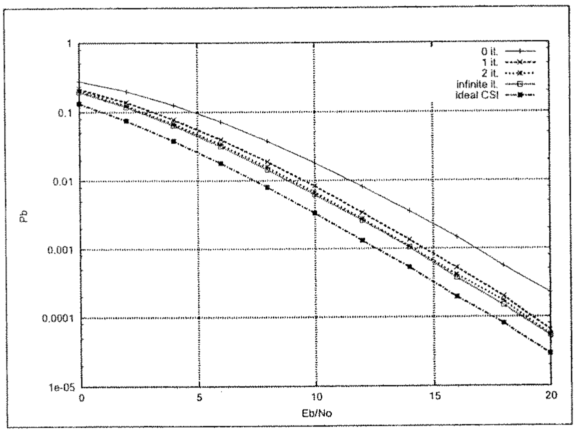

Fig. 3. Bit error performance of trellis coded 8-PSK code for $f_D T = 0.01$

4. Simulation Results

Error performance of the proposed iterative decoder has been investigated via computer simulations. The fading channel is modeled as the Jakes fading with normalized autocorrelation ($v^2 = 1$) $r_l = J_0(2\pi f_d T l)$ where $J_0(.)$, f_d and T were defined previously. Data bits are first encoded by a rate 2/3, 4-state 8-PSK TCM encoder to produce the coded data symbol sequence of length 100. The encoder whose trellis diagram is given in Figure 2 was recently proposed in [13] and has optimum performance when used in combination with Alamouti's transmit diversity scheme.

In order to initialize the EM algorithm, the receiver has to have good estimates of the channel. These estimates have been provided using pilot symbol assisted modulation (PSAM), [14]. Six pairs of pilot symbols, which are already known at the receiver, are added periodically to the data symbol sequence with a period of 20. At the receiver, channel fading coefficients are first estimated at the pilot symbol positions. The unknown data fading coefficients are then estimated by applying Lagrange interpolation technique on the pilot fading coefficients, according to the initialization procedure as explained in Section 3. The EM algorithm uses these channel estimates to initialize and converge to the maximum likelihood decoding within two or three iterations. The maximization step of the EM algorithm is efficiently performed using the Viterbi algorithm. Bit error probability curves have been presented for a channel with a normalized maximum Doppler frequency of 0.01 in Fig. 3.

The proposed scheme seems to converge to the ML decoding in two iterations. This provides an SNR gain of 3 dB in the high SNR region. The performance improvement is caused by the reduction in the channel estimation error which can be seen in Fig. 4, where the mean square estimation error (MSEE) values versus iteration numbers are presented for different SNR values.

The channel estimation errors converge to the maximum likely estimates in two iterations. For a channel with higher Doppler frequency ($f_D T = 0.03$), the bit error probability curves again converge in two iterations (Fig. 5), but this time resulting in an error floor.

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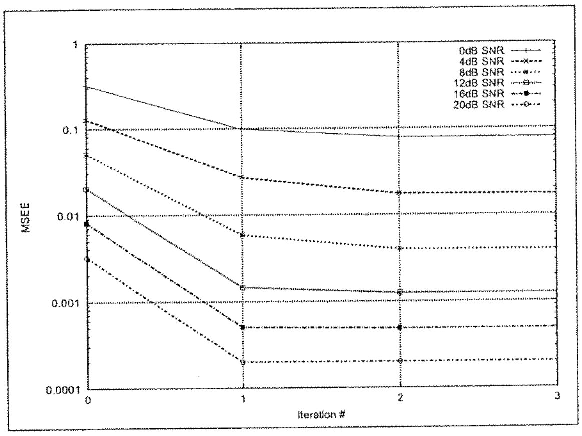

Fig. 4. Mean square estimation error for $f_D T = 0.01$

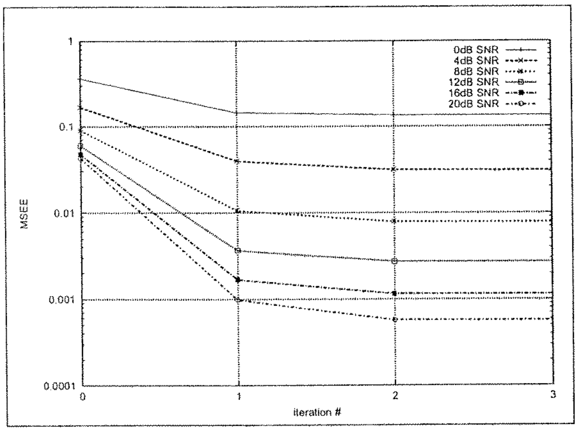

Fig. 6. Mean square estimation error for $f_D T = 0.03$

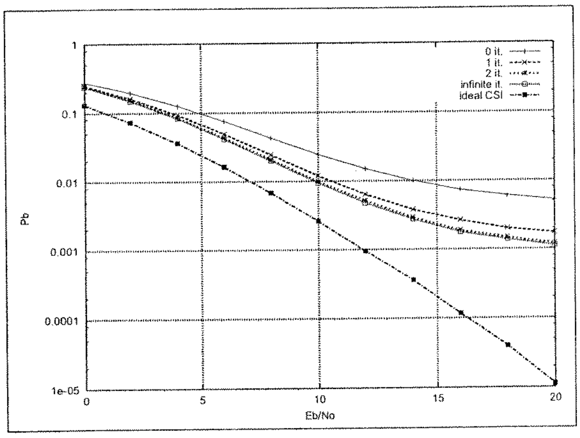

Fig. 5. Bit error performance of trellis coded 8-PSK code for $f_D T = 0.03$

Alamouti's transmit diversity scheme loses its orthogonality property in the presence of channel estimation error and an error floor is observed.

Since, in the fast fading channel, PSAM with a pilot separation of 20 symbols, loses its effectiveness in estimating the channel fading coefficients, the algorithm converges to a local maximum which results in a high estimation error (Fig. 6). In both cases, the proposed decoder is shown to converge to the ML decoding in just two iterations.

5. Conclusion

In this paper, we proposed an optimum sequence estimation algorithm for wireless communications systems employing a transmit diversity. This algorithm performs an iterative estimation of the transmitted sequence of data symbols according to the ML criterion, using the EM algorithm employing M-PSK modulation scheme with additive Gaussian noise. The discrete multipath channel was represented in terms of the channel gains from each transmit antenna to the receive antenna. EM algorithm estimates jointly the complex channel parameters of each channel and the data sequence transmitted, iteratively, which converges to the true ML solution. The algorithm is applied to the trellis coded 8-PSK modulated wireless systems and efficiency of the algorithm proposed has been shown by the computer simulations. Simulation results show that the EM algorithm converges quickly for fast fading channels. The performance of the EM-based decoder approaches that of the ML receiver which has perfect knowledge of the channel. In addition, the EM-based detector is rather simple to implement since the maximization step of the algorithm can be done using the Viterbi algorithm.

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Appendix

Derivation of Eq. (11)

Let,

$$r = Sh + n \tag{A1}$$

where n and h are $N \times 1$ dimensional zero-mean identically distributed Gaussian random vectors with covariance matrices Σ_{n} and R_{h}, respectively. S represents the $N \times N$ dimensional data matrix, assumed to be independent of h. That is, if $s = (s_1, s_2, \cdots, s_N)$ is the data vector then S is a diagonal matrix with element of s as diagonal elements. Note that for the special case of phase shift keying, it is easy to see that $S^{\dagger} S = I$.

Using the Bayes formula $p(h|r,s)$ can be expressed as

$$p(h|r,s) = \frac{p(r|h,s)p(h)}{\int_h p(r|h,s)p(h)dh} \quad \text{(A2)}$$

where

$$p(h|r,s) = \frac{1}{(2\pi)^N |\det\Sigma_\mathrm{n}|} \exp\left\{-\frac{1}{2}(r-Sh)^\dagger \Sigma_\mathrm{n}^{-1}(r-Sh)\right\}$$

$$p(h) = \frac{1}{(2\pi)^N |\det R_\mathrm{h}|} \exp\left\{-\frac{1}{2}h^\dagger R_\mathrm{h}^{-1} h\right\} \quad \text{(A3)}$$

Defining

$$\widehat{h} = \Psi S^\dagger \Sigma_\mathrm{n}^{-1} r, \quad \text{(A4)}$$

and

$$\Psi = \left(R_\mathrm{h}^{-1} + S^\dagger \Sigma_\mathrm{n}^{-1} S\right)^{-1}, \quad \text{(A5)}$$

It can be easily shown that the denominator of (A.2) can be expressed as

$$p(r|h,s)p(h) \sim \exp\left\{-\frac{1}{2}\left[(h-\widehat{h})^\dagger \Psi^{-1}(h-\widehat{h}) + r^\dagger \Sigma_\mathrm{n}^{-1} r - \widehat{h}^\dagger \Psi^{-1}\widehat{h}\right]\right\} \text{(A6)}$$

From (A.6) it follows that

$$\int_h p(r|h,s)p(h)dh = \frac{1}{(2\pi)^N |\det\Sigma_\mathrm{n}|} \exp\left\{-\frac{1}{2}\left(r^\dagger \Sigma_\mathrm{n}^{-1} r - \widehat{h}^\dagger \Psi_\mathrm{h}^{-1}\widehat{h}\right)\right\} \text{(A7)}$$

Substituting (A.6) and (A.7) into (A.2), we have the final result.

$$p(h|r,s) = \frac{1}{(2\pi)^N |\det R_\mathrm{h}|} \exp\left\{-\frac{1}{2}\left[(h-\widehat{h})^\dagger \Psi^{-1}(h-\widehat{h})\right]\right\}. \quad \text{(A8)}$$

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ULUSLARARASI KİTAP BÖLÜMLERİ

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NON-DATA AIDED EM-BASED CHANNEL ESTIMATION FOR OFDM SYSTEMS WITH TIME-VARYING FADING CHANNELS(*)

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Abstract- In this paper, a computationally efficient algorithm is presented for maximum *a posteriori* (MAP) channel estimation for OFDM systems employing M-PSK modulation scheme with additive Gaussian noise, based on the Expectation Maximization (EM) method. A non-data-aided scheme is considered for the estimation of a multipath time-varying channel by averaging over the M-PSK signal constellation. For this, an EM algorithm is derived which estimates the complex channel parameters of each subcarriers iteratively in frequency domain and which converges to the true MAP estimation of the unknown channel. The algorithm requires a convenient representation of the discrete multipath fading channel based on the Karhunen-Loeve orthogonal expansion. The algorithm is applied to the QPSK modulated OFDM systems and efficiency of the method proposed is shown by the computer simulations.

1. INTRODUCTION

OFDM signaling is proven to be an efficient way to overcome the effects of fading channel and multi-path by dividing the frequency selective channel into a number of sub-channels corresponding to the OFDM sub-carrier frequencies. OFDM has already been accepted for the new wireless local area network (WLAN) standards (IEEE 802.11), the ETSI High Performance Local Area Network type 2 (HIPERLAN/2) and Japan's Mobil Multimedia Access Communications (MMAC) systems [1].

In OFDM, channel state information between transmit and receive antenna pairs is required for coherent decoding. Therefore, several channel parameter estimation techniques were proposed in literature. In [2-3] a channel estimator for OFDM systems has been proposed based on the singular-value decomposition or frequency-domain filtering. Time domain filtering has been proposed in [4].

To further improve the channel estimator performance, a MMSE channel estimator, which makes full use of the time-frequency correlation of the time-varying dispersive channel were proposed in [5]. This technique has been extended later in [6] to develop a channel estimation in OFDM systems with transmitter diversity using space time coding. In this paper we apply the method of Siala [7] to the estimation of time-varying fading channels for OFDM systems. This algorithm performs an iterative channel estimation according to the maximum *a posteriori* (MAP) criterion, using the Expectation-Maximization (EM) algorithm. It uses profitably not only pilot symbols but also information-carrying symbols on the optimization of the channel estimation. It requires a conventional representation of the multipath Doppler channel, based on a discrete Karhunen-Loeve(KL) orthogonal expansion of the discrete multipath Doppler channel seen by the OFDM receiver. The channel estimator makes full use of the correlation of the channel frequency response at different times and frequencies. In particular, for mobile wireless channels, the correlation of the channel frequency response at different times and frequencies can be separated into the multiplication of the time-and frequency-domain correlation functions and this would decrease the computational complexity of the channel estimation substantially [5]. Computer simulations demonstrate that the computational complexity of our channel estimation algorithm is significantly improved.

2. OFDM SYSTEMS WITH TIME-VARYING CHANNEL ESTIMATOR

The received signal after demodulation (performing a DFT), can be expressed as

$$R(n,k) = H(n,k)A(n,k) + W(n,k), \quad k = 0,1,....K-1; \quad n = 0,1,....N-1 \qquad (1)$$

where $A(n,k)$ is the signal modulating the $k$the subcarrier during time $nT_s \leq t \leq (n+1)T_s$, T_s being the OFDM symbol duration. They are assumed to have unit variance and be independent for different k's and n's. Since the phase of each subchannel can be obtained by the channel estimator, coherent phase-shift keying (PSK) modulation is used here to enhance the system performance. $W(n,k)$ is represents the additive complex Gaussian noise with variance σ^2, entering the system. $H(n,k)$ is the frequency response of the fading channel at the kth subcarrier at time n. They are correlated samples, both in time and frequency, of a complex Gaussian process.

In the absence of absence of channel state information, decoder must estimate the channel states and there has been extensive affords in the direction of channel parameter estimation. However, most of the works done tries to achieve this goal with employing the training symbols. For OFDM systems channel estimation is challenging if we assume that this should be implemented in a non-data aided fashion [8, 9]. In this paper a novel time-varying channel estimation algorithm is presented by representing the discrete multipath channel based on the Karhunen-Loeve orthogonal representation and make use of the Expectation Maximization technique.

3. REPRESENTATION OF DISCRETE MOBILE RADIO

For an OFDM system with block length T_s and subchannel spacing Δf, the discrete correlation function for different blocks and subcarriers of the frequency response of the time-varying multi-path radio channel for different discrete times and frequencies defined by $r(n,k;n',k')=E[H(n,k)H^*(n',k')]$ can be written as

$$r(n,k;n',k')=\sigma_H^2 r_1(n,n')r_2(k,k'),\quad n,n'=0,1,\dots,N-1;\quad k,k'=0,1,\dots K-1 \qquad (2)$$

where $\sigma_H^2(t)$ is the total average power of the channel impulse response defined as $\sigma_H^2=\sum_\ell \sigma_\ell^2$. $\sigma_l^2(t)$ is the average power of the lth path and

$$r_f(k,k')=\left(1/\sigma_H^2 \sum_l \sigma_l^2 e^{-j2\pi(k-k')\Delta f \tau_l} \right)$$

From Jakes' model [11]

$$r_t(n,n')=J_0(2\pi(n-n')f_d T_s) \qquad (3)$$

where J_0 is the zeroth-order Bessel function of the first kind and f_d is the Doppler frequency which is related to the vehicle speed v and the carrier frequency f_c by $f_d=vf_c/c$, where c is the speed of light.

Discrete frequency response of the wireless channel, $H(n,k)$ can be expressed as

$$H(n,k)=\sum_{i=0}^{N-1}\sum_{j=0}^{K-1} G(i,j)\psi_{i,j}(n,k).\quad n=0,1,\dots,N-1;\quad k=0,1,\dots,K-1 \qquad (4)$$

where the random variables $\{G(i,j)\}$ are independent complex zero-mean Gaussian coefficients. The variance of these coefficients, arranged in decreasing order, are equal to the eigenvalues $\{\lambda_{i,j}\}$ of the Karhunen Loeve (KL) transformation with the orthogonalized eigenfunctions $\psi_{i,j}(n,k)$'s of the discrete autocorrelation function $r(n,k;n',k')$ defined by

$$\sum_{n'=0}^{N-1}\sum_{k'=0}^{K-1} r(n,k;n',k')\psi_{i,j}(n',k')=\lambda_{i,j}\psi_{i,j}(n,k),\quad n=0,1,\dots,N-1;\quad k=0,1,\dots,K-1. \qquad (5)$$

Note that when the autocorrelation function is separable as in (2), then it can be shown that $\{\psi_{i,j}(n,k)\}$'s become also separable. That is,

$$\psi_{i,j}(n,k)=\phi_l(n,i)\phi_2(k,j) \tag{6}$$

where $\phi_l(n,i)$ and $\phi_2(k,j)$ are the components of the normalized eigenvectors of the autocorrelations $r_l(n,n')$ and $r_2(k,k')$, respectively. The corresponding eigenvalues are β_i, and γ_j, $i=0,1,...,N-1$; $j=0,1,...,K-1$. From (1) and (5), they satisfy the following relationship.

$$\lambda_{i,j}=\beta_i\gamma_j/\sigma_H^2. \tag{7}$$

The advantage in having the autocorrelation function by a separable function is that instead of solving NK x NK matrix eigenvalue problem of (5), only two N x N and K x K matrix eigenvalue problems need to solved. Since the required computations to solve these problems are $O(NK^3)$ and $O(N^3)+O(K^3)$, respectively, the reduction in dimensionality achieved by the separable model is quite significant.

4. EM-BASED MAP CHANNEL ESTIMATION

The MAP criterion is used in the fading channel as seen at the FFT output of the OFDM receiver since the joint probability density function of the random variables are known by the receiver and can be expressed as

$$p(\mathrm{G})\approx\prod_i\prod_j\exp\left(-\frac{|G(i,j)|^2}{\lambda_{i,j}}\right) \tag{8}$$

where $G=\{G(i,j)\}$. Given the transmitted signal $A=\{A(n,k)\}$, and the discrete channel representation G, and taking into account the independence of the noise components, we can express the conditional probability density function of the received signal $R=\{R(n,k)\}$ as

$$p(\mathrm{R}|\mathrm{A},\mathrm{G})\approx\prod_n\prod_k\exp\left\{-\frac{1}{\sigma^2}\left|R(n,k)-A(n,k)\sum_i\sum_j G(i,j)\psi_{i,j}(n,k)\right|^2\right\} \tag{9}$$

The MAP estimate $\{\mathrm{G}\}$ is given by

$$\hat{\mathrm{G}}=\arg\max_{\mathrm{G}} p(\mathrm{G}|\mathrm{R}). \tag{10}$$

Directly solving this equation is mathematically intractable. However, the solution can be obtained easily by means of the iterative EM algorithm. This algorithm inductively reestimate G so that a monotonic increase in the *a posteriori* conditional pdf in (9) is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q\left(\mathrm{G}|\mathrm{G}^{(m)}\right)=\sum_A p(\mathrm{R},\mathrm{A},\mathrm{G})\log p\left(\mathrm{R},\mathrm{A},\mathrm{G}^{(m)}\right) \tag{11}$$

where sum is taken over all possible transmitted data coded signals and $G^{(m)}$ is the estimation of G at the mth iteration. Given the received signal R, the EM algorithm starts with an initial value $G^{(0)}$ of the unknown channel parameters G. The $(m+1)$th estimate of G is obtained by the maximization step described by

$$\mathbf{G}^{(m+1)} = \arg\max_{\mathbf{G}} Q\left(\mathbf{G}|\mathbf{G}^{(m)}\right).$$

After long algebraic manipulations the expression of the (p,q)th component $G^{(m)}(p, q)$, $(p=0,1,\ldots,N-1;\ q=0,1,\ldots,K-1)$ of the re-estimate $G^{(m+1)}$ can be obtained as follows:

$$G^{(m+1)}(p,q) = \frac{1}{\left(1+\sigma^2/\lambda_{p,q}\right)}\sum_n \sum_k \Gamma^{(m)}(n,k)R(n,k)\psi^*_{p,q}(n,k) \tag{12}$$

where,

$$\Gamma^{(m)}(n,k) = \sum_{a\in S_{n,k}} a * P\left(A(n,k)=a|\mathbf{R},\mathbf{G}^{(m)}\right) \tag{13}$$

and $S_{n,k}$ denotes alphabet set taken by the (n,k)th OFDM symbol. In order to be able to choose good initial values for the unknown channel parameters and to ensure a fast start up in the equalization/detection operation following the channel estimation process, the leading L data symbols $D(n,k)$, $k=0,1,\ldots,L-1$ in each OFDM frame are generally used as pilot symbols known by the receiver. When K is large, however, this does not create a significant degradation in spectrum efficiency since L takes small values with respect to the total number of subcarriers carrying the data. Therefore for PSK modulated alphabet set, the initial value of the channel parameters can be selected according to the following data-aided estimates.

$$G^{(0)}(p,q) = \frac{1}{\left(1+\sigma^2/\lambda_{p,q}\right)}\sum_n \sum_{k=0}^{L-1} D^*(n,k)R(n,k)\psi^*_{p,q}(n,k) \tag{14}$$

4.1 Computation of $\Gamma^{(m)}(n,k)$ for QPSK Signaling

If a=(±1 ± j) represents independent identically distributed data sequence modulating the QPSK carrier, $\Gamma^{(m)}(n,k)$ in (13) can be expressed as follows.

$$\Gamma^{(m)}(n,k) = \frac{\sum_{a\in S_{n,k}} a * P\left(R(n,k)|A(n,k)=a,\mathbf{G}^{(m)}\right)P\left(A(n,k)=a\right)}{\sum_{a\in S_{n,k}} P\left(R(n,k)|A(n,k)=a,\mathbf{G}^{(m)}\right)P\left(A(n,k)=a\right)} \tag{15}$$

From (11) it follows that

$$\Gamma^{(m)}(n,k) = \frac{\sum_{a\in S_{n,k}} a * exp\left(\frac{2}{\sigma^2} Re\left[a * Z^m(n,k)\right]\right)}{\sum_{a\in S_{n,k}} exp\left(\frac{2}{\sigma^2} Re\left[a * Z^m(n,k)\right]\right)} \tag{16}$$

where

$$Z^m(n,k) = R(n,k)\sum_i \sum_j G^{(m)*}(i,j)\psi^*_{i,j}(n,k).$$

Then, taking summations in the numerator and the denumerator of (16) over the values of QPSK symbols a, we have the final result as follows.

$$r^{(m)}(n,k) = \tanh\left[\frac{2}{\sigma^2}\mathrm{Re}\left(Z^m(n,k)\right)\right] - j\tanh\left[\frac{2}{\sigma^2}\mathrm{Im}\left(Z^m(n,k)\right)\right] \tag{17}$$

Note that the Modified-Cramer-Rao-Bound (MCRM) can be derived for the estimated random parameters $\{G(i,j)\}$ as follows. Performing the derivatives in (8) and (9) with respect to $\{G(i,j)\}$, taking expectations over $\boldsymbol{R}$, $\boldsymbol{A}$ and $\boldsymbol{G}$ and then taking into fact that the eigenfunctions $\psi\_{i,j}(n,k)$ are orthogonal, it follows that

$$\mathrm{MCRB}(G(p,q)) = 2\,(1/\sigma^2 + 1/\lambda_{p,q})^{-1} \tag{18}$$

where σ^2 is the noise variance and $\lambda_{p,q}$ are the eigenvalues of the discrete autocorrelation function $r(n,k;n',k')$

5. SIMULATION RESULTS

The performance of the proposed EM based ML channel estimation technique was evaluated as a function of signal-to-noise ratio (SNR) based on the Monte Carlo simulations. We considered the fading multipath channel with an exponentially decaying power delay profile $\Theta(\tau_l) = C\exp(-\tau_l/\tau_{max})$ per path delays τ_l that are uniformly and independently distributed over the length of the cyclic prefix. C is a normalizing constant. Note that the normalized discrete channel-correlations for different subcarriers of this channel model was presented in [3] as follows:

$$r_2(k,k') = \frac{1 - \exp\left[-\frac{1}{\tau_{rms}} + \frac{2\pi j(k-k')}{N}\right]}{\tau_{rms}\left(1 - \exp(-L\cdot\tau_{rms})\right)\left(\frac{1}{\tau_{rms}} + \frac{j2\pi(k-k')}{N}\right)}.$$

The discrete channel correlations for different block is given by (3) The scenario for our simulation study consists of a wireless QPSK OFDM system operating with a 500 kHz bandwidth and is divided into 16 tones with a total symbol period (T_s) of 136 μs, of which 27 μs constitute the cyclic prefix ($L=4$). The uncoded data rate of the system is 0.24 Mbit/s. We assume that the rms width is $\tau_{rms}=1$ sample (6.8 μs) for the power-delay profile and the dopler frequency is $f_d=100$ Hz.

The proposed algorithm was tested for 100 Monte Carlo trials per SNR point accross a range of SNRs (5-15 dB). The average SNR was defined as $E[|H(n,k)|^2]E[|A(n,k)|^2]/\sigma^2$. Since $E[|A(n,k)|^2]=1$ for QPS signaling and $E[|H(n,k)|^2]=1$ for the normalized frequency response of the fading channel, the normalized SNR simply becomes $1/\sigma^2$, where σ^2 is the variance of the complex white Gaussian noise entering the system. The initial values of $G^{(0)}(n,k)$'s were according to (14). Root Mean-square-error (RMSE) is defined as the difference between the matrices

$G=[G(n,k)]$ and $\hat{G}=[\ \hat{G}(n,k)\ ]$, representing the true and the estimated values of channel parameters, respectively . Namely,

$$RMSE=\left\|G-\hat{G}\right\|=\left(\sum_{n=0}^{N-1}\sum_{k=0}^{K-1}\left(G(n,k)-G(\hat{n},k)\right)^2\right)^{\frac{1}{2}}.$$

In each trial, the RMS of the estimation error for the channel parameters were recorded. In Fig. 1, we have plotted the experimental estimation RMS error as well as the corresponding modified CRBs. Fig. 2 shows the estimation RMS error experienced by the proposed technique at each iteration (SNR=10dB and SNR=20 dB respectively). Based on the experimental results, we made the following observations:

-Since MCRB given by (18) provides an approximate bound, it is not tight however it is much easier to compute.

-For low SNR, the proposed approach requires more iterations to converge. It is concluded from Fig.2 that the MSE performance of the EM-based algorithm converges within 3-10 iterations, depending on the SNR.

6. CONCLUSIONS

In this paper, we proposed an optimum channel estimation algorithm for OFDM systems. This algorithm performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing M-PSK modulation scheme with additive Gaussian noise. The discrete multipath channel was represented in terms of a Karhunen-Loeve expansion which makes full use of time and frequency-domain correlations of the frequency response of the time-varying dispersive fading channel. A non-data aided estimation scheme was considered for time-varying channel estimation by taking averaging over the M-PSK signal constellation. For this, an EM algorithm is derived which estimates the complex channel parameters of each sub carriers iteratively in frequency domain and which converges to the true MAP estimation of the unknown channel. The algorithm is applied to the QPSK modulated OFDM systems and efficiency of the algorithm proposed is shown by the computer simulations.

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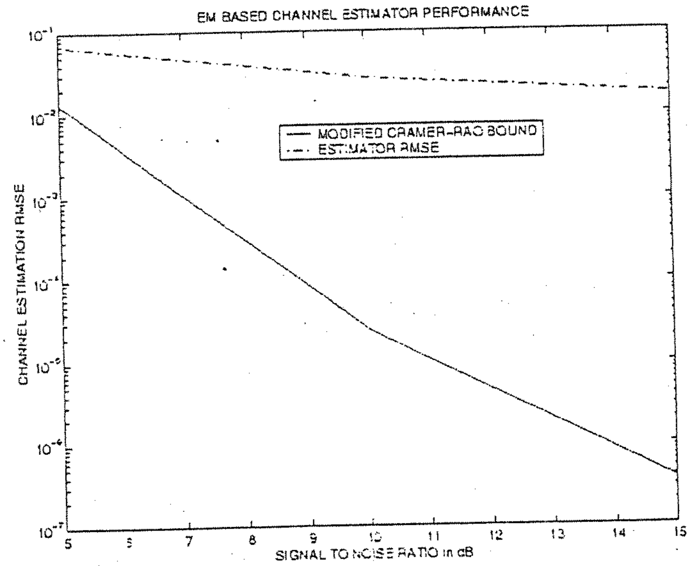

Figure 1. Performance of the proposed method

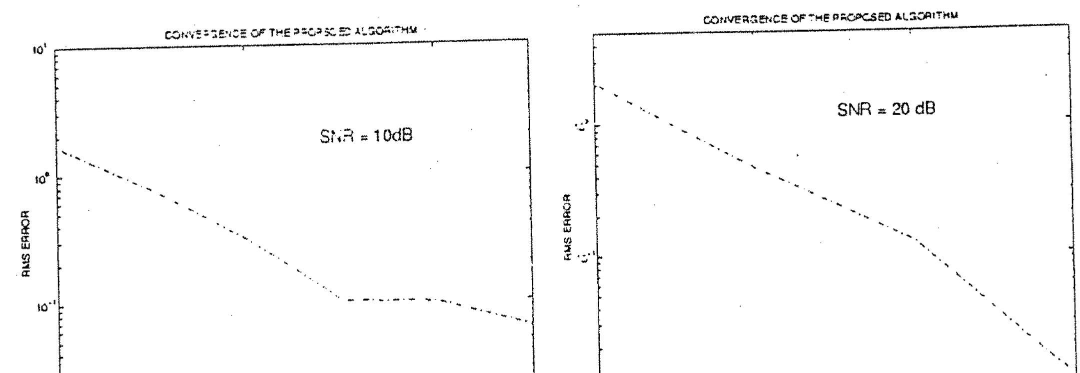

Figure 2. Convergence of the proposed method (SNR = 10 & 20 dB)

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Chapter 1

FEEDFORWARD NON-DATA-AIDED PHASE SYNCHRONIZATION TECHNIQUES FOR OFDM SYSTEMS

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Abstract In this paper, first, a non-data-aided(NDA), maximum likelihood(ML) algorithm is derived for the carrier phase offset in OFDM systems, employing M-PSK modulation scheme, in the presence of nondispresive channels. It is shown that for sufficiently small SNR the ML phase estimator obtained reduces to the familiar Mth order power synchronizer which belongs to the class of NDA feedforward carrier synchronizers introduced earlier in the literature. Its mean-squared performance is obtained analytically and compared with simulation results. Then, a computationally efficient algorithm is presented for NDA maximum likelihood (ML) carrier phase estimation of OFDM systems for transmission over frequency selective channels based on the Expectation-Maximization (EM) algorithm. For this, an EM algorithm is derived which estimates the phase rotations of each subcarriers iteratively and which converges to the true ML estimation of the unknown phases. The algorithm is applied to the QPSK modulated OFDM systems and it is demonstrated by simulation that the phase error variances of estimated subcarrier phase rotations do not depend on the number of subcarriers.

Keywords: phase synchronization, OFDM systems,ML algorithm, EM algorithm

O. Sunay (ed.), High Speed Wireless Communications Systems, 1-30.

1. Introduction

OFDM signaling is proven to be an efficient way to overcome the effects of fading channel and multi-path by dividing the frequency selective channel into a number of sub-channels corresponding to the OFDM subcarrier frequencies. OFDM has already been accepted for the new wireless local area network (WLAN) standards (IEEE 802.11), ETSI High Performance Local Area Network type 2 (HIPERLAN/2) and Japan's Mobile Multimedia Access Communication (MMAC) systems. The technique is recommended in Europe for Digital Audio Broadcasting(DAB) over satellite, it is also being adopted by EBU for terrestial digital video broadcasting (DVB-T)transmission. OFDM is now being considered for the fourth generation mobile communication systems.

When compared with single carrier systems, channel equalization is less complex, and sensitivity to channel estimation and frame synchronization error can be reduced. See [1] and [2] and the references therein. However, OFDM systems are more sensitive to carrier frequency and phase offset, caused by the mismatch of the oscillators in the transmitter and the receiver, than single carrier systems. A time varying frequency error not only disturbs the frequency orthogonality, but also makes subcarrier synchronization much more difficult to achieve and maintain [3], [20]. For example, in the absence of additive noise, the frequency offset must satisfy $|\epsilon| < 1.3.10^{-2}$ in order to obtain an effective signal-to-noise ratio of 30 dB or higher. Therefore, assuming that the symbol timing has already been known, carrier and phase synchronization in OFDM systems is one of the major tasks to be implemented.

A frequency offset estimate may be generated at the receiver with the aid of pilot symbols known at the receiver [4], [5] or as in [6], by maximizing the average log-likelihood function. In [7] a data aided(DA)frequency offset estimation algorithm is presented ensuring high speed synchronization with negligible decoder performance degradation at a low implementation cost. Redundancy in the transmitted OFDM signal also offers the opportunity for synchronization. Such an approach is found in [8],[9] for a frequency offset. Along with this approach, [8] describes a method of using a correlation with the cyclic prefix to find the frequency offset and the OFDM frame timing. However they make a Gaussian approximation for data in deriving the maximum likelihood ratio and therefore the effect of modulation of data symbols is ignored. In [10, 12] the frequency offset is estimated and compensated by the technique based on the redundant information contained in the cyclic prefix preceeding the OFDM symbols. The paper extends the Van de Beek's[8] method in several ways as follows: i) A maximum likelihood(ML) estimation algorithm is derived for frequency offset estimation in OFDM systems without any Gaussian assumption for data symbols. ii) Its mean-squared performance is obtained analytically and compared with simulation results. iii) It is shown that the derived estimator can work also in tracking mode as well as the forward estimation mode. iv) A tracking algorithm is provided to track the frequency offset and the symbol error rate(SER) degradation before and after synchronization with estimated offset is also calculated for a closed loop system

The OFDM carrier phase estimation is a part of OFDM channel estimation, which is widely known and treated in the literature. See [13], [14, 15] and the references therein. As with frequency recovery, ML estimation method plays a central role in carrier phase estimation. Various approximation to the ML formulation lead to different phase estimation techniques. This is also a consequence of many scenarios depending on the specific modulation format and avaibility of data information. Phase Shift Keying(PSK) is a popular modulation technique which is widely employed in digital radio transmission. Optimum demodulation of PSK signals requires that a phase coherent carrier be reconstructed at the receiver. Assuming that the data is not available to the synchronizer, non-data-aided(NDA) carrier recovery methods have been proposed in the literature including classical M-th power loop [13, 16]

The main objective of this paper is to derive NDA ML estimation algorithms for carrier phase in OFDM systems employing M-PSK modulation scheme, suitable for frequency-nonselective and frequency-selective channels [12, 11, 17, 18]. After a brief description of an OFDM system in Section 2, Section 3 then considers the NDA ML carrier phase synchronization maximizing the low SNR limit averaged over M-PSK signal constellation. The mean-squared error of the estimator is also derived analytically and its performance is compared with the simulation results and is shown that the performance converges to the Cramer-Rao bound with increasing SNR. In Section 4, a computationally efficient algorithm is derived for the phase synchronization of ODFM systems in the presence of frequency-selective channels via the Expectation Maximization(EM) technique. Finally, Section 5 summarizes the main conclusions of the paper.

2. OFDM Systems

The main idea behind OFDM is to split the data stream to be transmitted into N parallel streams of reduced data rate and to transmit each of them on a separate sub-carrier. These carriers are made orthogonal by appropriately choosing the frequency spacing between them.

Therefore, spectral overlapping among the sub-carriers is allowed, since the orthogonality will ensure that the receiver can separate the OFDM sub-carriers, and a better spectral efficiency can be achieved than by using simple frequency division multiplex. A typical block diagram of an OFDM system is shown in Fig.1.1. During each mth OFDM symbol

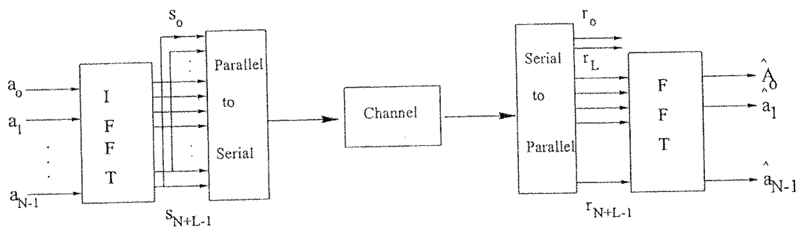

Figure 1.1. Baseband OFDM system, transmitting N-blocks of data

interval, the transmitted data symbols split by a serial-to-parallel conversion block into N lower bit rate streams $\{a_m^k\}_{k=0}^{N-1}$ each modulating a sub-carrier. These sub-carriers are converted via an inverse FFT processor into N time signals $\{s_m^k\}_{k=0}^{N-1}$, and then a cyclic prefix of length L is added to form a complete OFDM symbol. The cyclic prefix is generated by copying the last L samples of the OFDM symbol (N samples long) and appending them to the beginning of the frame. Therefore, the effective length of the OFDM symbol as transmitted is $N + L$ samples long. The insertion of a cyclic prefix is an accepted means of avoiding inter symbol interference and preserving orthogonality between sub-carriers. The redundant information contained within the cyclic prefix enables also the frequency offset estimation very effectively and without additional pilots. The rest of the samples of the OFDM symbol(N samples long) can be used effectively to implement the phase synchronization.

We assume that the N complex data being transmitted is derived from M-PSK (M-ary phase shift keying) scheme and the channel noise is additive, white and Gaussian. The channel filtering is equally split between transmitter and receiver and the over all channel response is Nyquist in the absence of frequency offset as well as that the clock recovery is ideal. It is known that excellent timing information can normally be driven even with frequency errors on the order of 10-20% of the symbol rate [19]. It is also shown in [20] that the guard time provided by the cyclic prefix will eliminate the Intersymbol and interchannel interferences as long as the duration of the channel impulse response(the Nyquist pulse) is smaller than the guard time. At the receiver the data are recovered by means of a DFT (Discrete Fourier Transform). Assuming that the OFDM frame and symbol timing synchronization have already been achieved, the complex signal envelope of the received signal obtained from the matched filter output for the mth OFDM symbol can be expressed as

$$r_m(k) = s_m(k) + n_m(k), \qquad m = 1, 2, \cdots, L_0. \tag{1.1}$$

where

$$s_m(k) = \begin{cases} s_m^{k+N-L} \exp\{j(2\pi\epsilon k/N + \phi)\}, & k = 0, 1, \ldots, L-1 \\ s_m^{k-L} \exp\{j(2\pi\epsilon k/N + \phi)\}, & k = L, L+1, \cdots, N+L-1 \end{cases} \tag{1.2}$$

and s_m^k's are the IFFT of the data sequence given by

$$s_m^k = (1/N) \sum_{n=0}^{N-1} a_m^n \exp(j2\pi kn/N), \tag{1.3}$$

and a_m^n denotes M-PSK symbol transmitted on the nth subcarrier during the mth OFDM symbol, taking values in the set $\{e^{j\frac{2\pi r}{M}}, r = 0, 1, \ldots M-1\}$. ϵ is the relative frequency offset of the channel (the ratio of the actual frequency offset to the intercarrier spacing) and ϕ represents the channel phase offset. $n_m(k)$ is the complex envelope of the additive white Gaussian noise with variance $\sigma_n^2 = E\{|n_m(k)|^2\}$.

3. ML Estimation for Phase Estimation in OFDM systems

It is shown in [8] that, first L samples of each OFDM symbol are sufficient to determine the carrier frequency offset estimate. Therefore, assuming that the OFDM frequency offset synchronization have already been achieved and compensated, the the rest of the N samples of each symbol can be employed for the carrier phase synchronization. Under the assumption that the frequency offset estimation is achieved perfectly, the complex signal envelope of the received signal for the mth OFDM symbol can then be expressed as

$$r_m(k) = s_m(k) + n_m(k), \qquad m = 1, 2, \ldots, L_0, \tag{1.4}$$

where

$$s_m(k) = s_m^k e^{j\phi}, \quad k = 0, 2 \ldots N-1 \tag{1.5}$$

and s_m^k's are the IFFT of the data sequence given by (1.3). In (1.5), ϕ represents the channel phase offset and $n_m(k)$ in (1.4) is the complex envelope of the additive white Gaussian noise with variance $\sigma_{n_m}^2 = E\{|n(k)|^2\}$.

Consider now an observation vector $\mathbf{r}$ containing L_0 number of OFDM symbols each containing N samples,

$$\begin{aligned}\mathbf{r} &= [\mathbf{r}_1\mathbf{r}_2\ldots\mathbf{r}_{L_0}]^T\\ \mathbf{r}_m &= [r_m(0), r_m(1)\ldots, r_m(N-1)]^T, \qquad m = 1, 2, \ldots L_0.\end{aligned}$$

Thus, from (1.4),(1.5) and (1.3), given ϕ and the data sequence $\{a_m^n\}$, the likelihood function of the observed samples $\mathbf{r}$ can be expressed in terms of the original data sequence and takes the form,

$$L(\phi, \{a_m^n\}) = \exp\left\{\frac{2}{N\sigma_n^2}\sum_{m=1}^{L_0}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1} Re\left[r_m(k)e^{-j2\pi nk/N}a_m^{*n}e^{-j\phi}\right]\right\}. \tag{1.6}$$

It is clear that, the term $R_m(n) \equiv \sum_{k=0}^{N-1} r_m(k)\exp(-j2\pi nk/N)$ in (1.6) is the DFT of the observation sequence $r_m(k)$. Taking this into account, (1.6) becomes

$$L(\phi, \{a_m^n\}) = \exp\left\{\frac{2}{N\sigma_n^2}\sum_{m=1}^{L_0}\sum_{n=0}^{N-1} Re\left[R_m(n)a_m^{*n}e^{-j\phi}\right]\right\}. \tag{1.7}$$

Now, in order to get a log-likelihood function only depending on ϕ, expectation of (1.7) is to be taken first over the M-PSK data sequence a_m^n. Taking then the logarithm of the averaged quantity yields,

$$\begin{aligned}\Lambda(\phi) &= \sum_{m=1}^{L_0}\sum_{n=0}^{N-1}\ln\Big(\frac{1}{M}\sum_{r=0}^{M-1}\exp\{\frac{2}{N\sigma_n^2}|R_m(n)|\\ &\quad\times\cos(\phi + 2\pi r/M - \arg R_m(n))\}\Big).\end{aligned} \tag{1.8}$$

By making same mathematical approximations, that are valid at low SNR, the following log-likelihood function is obtained.

$$\begin{aligned}\Lambda(\phi) &= \sum_{m=1}^{L_0}\sum_{n=0}^{N-1}\ln\{I_0((2/N\sigma_n^2)|R_m(n)|) + 2I_M(\frac{2}{N\sigma_n^2}|R_m(n)|)\\ &\quad\times\cos[M(\phi - \arg R_m(n))]\}.\end{aligned} \tag{1.9}$$

Here, I_0 and I_M are 0th and Mth order modified Bessel functions of the first kind, respectively. The ML estimation of ϕ can be derived by taking derivative of (1.9) with respect to ϕ and retaining the only lower order term in the denominator as follows

$$W(\phi) \triangleq \frac{d\Lambda(\phi)}{d\phi} = \gamma_c \sin M\phi - \gamma_s \cos M\phi \tag{1.10}$$

where,

$$\gamma_c = \sum_{m=1}^{L_0} \sum_{n=0}^{N-1} A_{n,m} \cos[M \arg(R_m(n))], \tag{1.11}$$

$$\gamma_s = \sum_{m=1}^{L_0} \sum_{n=0}^{N-1} A_{n,m} \sin[M \arg(R_m(n))], \tag{1.12}$$

$$A_{n,m} = \frac{-2MI_M(\frac{2}{N\sigma_n^2}|R_m(n)|)}{I_0(\frac{2}{N\sigma_n^2}|R_m(n)|)}. \tag{1.13}$$

Then, setting Equation (1.10) to zero gives NDA ML estimate for the phase offset

$$\hat{\phi}_{ML} = \frac{1}{M} \tan^{-1}(\frac{\gamma_s}{\gamma_c}). \tag{1.14}$$

We now show that for sufficiently low SNR, the ML phase estimator obtained above reduces to the familiar Mth power synchronizer [14, 15] which belongs to the class of NDA feedforward carrier synchronizers introduced in [16]. For $SNR = \sigma^2{}_s/\sigma_n^2 = 1/(N\sigma_n^2) << 1$,((1.3) it follows that $\sigma_s^2 = 1/N$ for M-PSK data), $I_0(z)$ and $I_M(z)$ in (1.13) can be approximated as

$$I_0(z) \approx 1, \qquad I_M(z) \approx \frac{(z/2)^M}{M!}.$$

Using them in (1.13),(1.11) and (1.12), the phase estimate in (1.14) can be expressed as

$$\hat{\phi}_{ML} = \frac{1}{M} \arg \sum_{m=1}^{L_0} \sum_{n=0}^{N-1} R_m^M(n). \tag{1.15}$$

Hence, we have shown that also for OFDM systems the phase synchronizers resulting from ML approach can be approximated for Mth power synchronizer in the low SNR region.

3.1 Mean-Squared Performance of the Phase Estimator

For the case of relatively small jitter the approximation of variance is,

$$Var[\hat{\phi}_{ML} - \phi] = A^{-2} Var[W(\phi)] = \frac{E[W^2(\hat{\phi})]}{A^2}|_{\hat{\phi}=\phi}. \tag{1.16}$$

where the quantities A and $W(.)$ are defined as follows [23].

$$W(\hat{\epsilon}) = \frac{d\Lambda}{d\hat{\epsilon}}, \quad F(\hat{\epsilon}) = E[W(\hat{\epsilon})], \quad A \cong \frac{dF}{d\hat{\epsilon}}|_{\hat{\epsilon}=\epsilon}.$$

After some algebra, we drive the following expressions for A and $E[W^2(\hat{\phi})]$ whose details are given in the Appendix.

$$A = \frac{-2L_0M}{\sigma_n^{2M}N^{M-1}(M-1)!}, \tag{1.17}$$

$$E[W^2(\hat{\phi})] = \frac{1}{2}Q^2L_0^2N^2\{\frac{E[P]-1}{L_0N}\}, \tag{1.18}$$

where,

$$Q = -2M/(N\sigma_n^2)^M M!,$$

$$E[P] = M!(2\sigma_n^2)^M \sum_{m=0}^{M} \binom{M}{m} \frac{1}{m!(2\sigma_n^2)^m}.$$

So, substituting these results in (1.16), the final analytical expression for variance of phase estimation is obtained as follows

$$Var[\hat{\phi}_{ML} - \phi] = \frac{1}{L_0N}\left\{\frac{M!}{2M^2}\sum_{m=0}^{M-1}\binom{M}{m}\frac{2^{M-m}}{m!(SNR)^{M-m}}\right\}. \tag{1.19}$$

The variance is a function of symbol length N, number of observed OFDM symbols L_0, number of levels of M-ary signaling and finally the $SNR(= 1/(N\sigma_n^2))$. The performance of the estimator can be obtained by varying any of these parameters.

Fig.1.2 shows the analytical variance of the phase estimator for, $N = 256, L_0 = 100, M = 4$ as a function of observation length for different values of SNR. Fig.1.2 shows that the performance improves in higher SNR and higher observation length.

We note that the variance expression given by (1.19) is an extension of the approximate variance formula appeared in ([21], Equation(14)) for M-PSK constellations. We also observe that the self noise is absent and the performance of the NDA algorithm is basically the same as the Cramer-Rao bound for moderate to high SNR. Similar observations have been made in [8, 16].

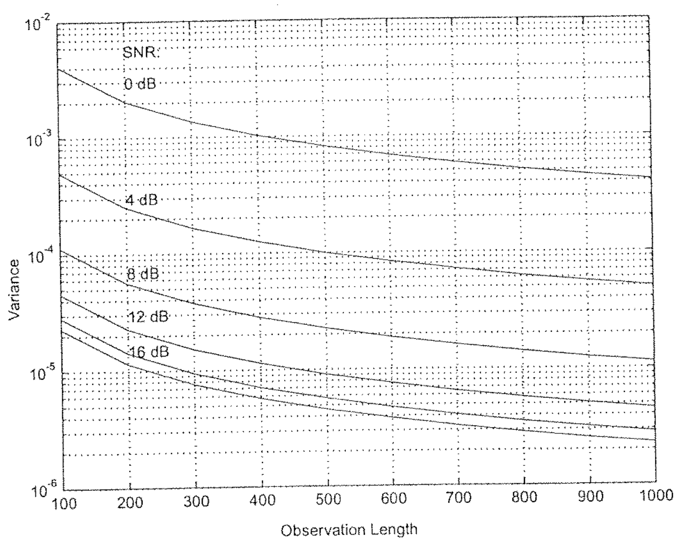

Figure 1.2. Analytical performance of ML phase estimator as a function of observation length

3.2 Simulation Results for Phase Offset Estimation

In order to evaluate the performance of the presented phase estimation algorithm as well as to make some performance comparison, computer simulations were carried out. The following parameters were selected for this purpose: Total number of sub-channels $N = 256$; Number of levels in M-ary signaling $M = 4$; Carrier Phase offset $\phi = \frac{\pi}{32}$; Length of observation symbol $L_0 = 100, 200$. Only additive white Gaussian noise is present.

The simulation was carried out over 1500 OFDM symbols and the phase estimation was obtained according to Equation (1.15). Because of the low SNR assumption, simulation particularly covered the range for 0 to 16 dB.

Fig. 1.3 shows the variance of estimator as a function of SNR, for 4-PSK signaling and for the observation length of 100 and 200. Analytical variance curves were also included in the same plot. From the comparison of these curves, it is seen that there is an excellent agreement between the analytical and simulation results. It is also observed that the performance is better at higher SNR and longer observation

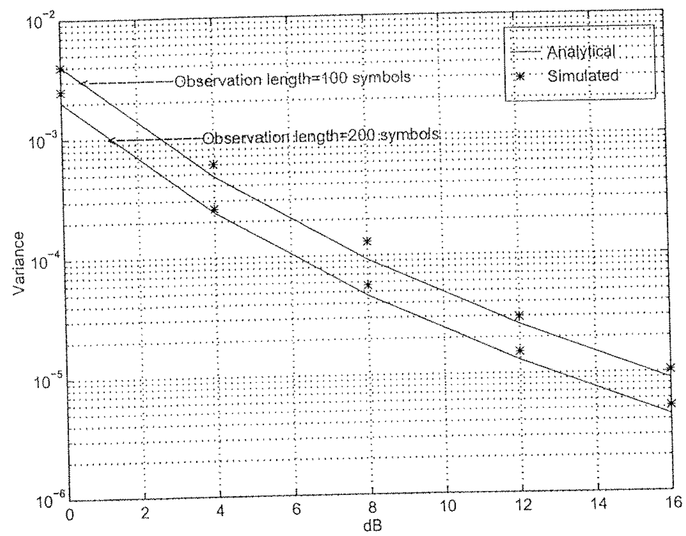

Figure 1.3. Simulated and analytical variance of ML phase estimator as a function of SNR

intervals, as expected. Performance of the OFDM system as depicted in Fig.1.4 was also determined in terms of symbol error rate (SER) plot for different signal-to-noise ratio. To correct phase offset, a closed-loop realization of the algorithms was carried out for a certain observation interval. It is shown in Fig.1.4 that as variance of estimation becomes better with higher SNR, the SER is also low under these conditions. The curves almost coincides because of low variance of the estimator.

4. Phase Synchronization of OFDM Systems over Frequency-Selective Channels via the EM Algorithm

As mentioned in Sec. 1, OFDM signaling is proven to be an efficient way to overcome the effects of fading channel and multi-path by dividing the frequency selective channel into a number of sub-channels corresponding to the OFDM sub-carrier frequencies. In the presence of the nondispersive channels, the channel shift is identical for all carriers. For this case, a nondata-aided(NDA) Maximum Likelihood(ML) estimation algorithm was derived in Sec. 3 for carrier phase synchronization in OFDM systems which maximizes the low SNR limit of the likelihood function averaged over M-PSK signal constellation.

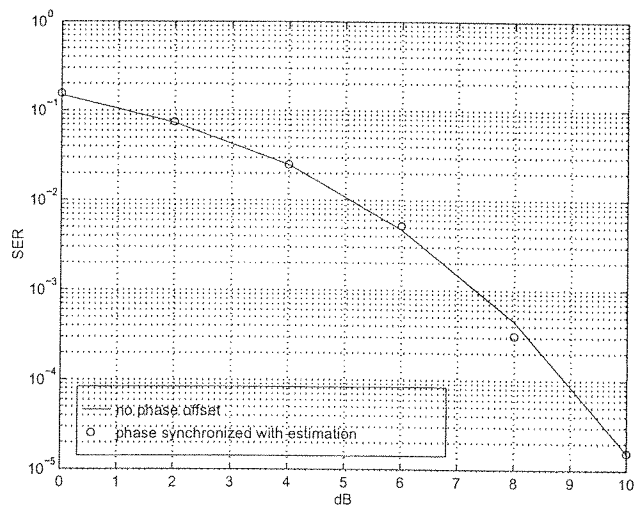

Figure 1.4. SER performance of OFDM system with phase synchronization

On the contrary, the frequency selective channels or dispersive channels (e.g. the terrestial mobile channel) give rise to a phase shift which is different for each of the modulated carriers in the OFDM signal. In this case the phase synchronizer will estimate the phase rotation of the useful signal on each carrier. Estimating these phase shifts by maximum likelihood technique does not seem to be analytically intractable. Even if the likelihood function can be evaluated offline, however, it is invariably a nonlinear function of the parameters to be estimated, which makes the maximization step (which must be performed in real-time) computationally infeasible. In such cases, an iterative formulation of the parameter estimation problem based on the expectation-Maximization(EM) algorithm [22] can provide an implementable solution.

The main objective of this section is to investigate the use of the EM algorithm to the problem of carrier phase estimation of OFDM systems employing M-PSK modulation scheme for transmission over frequency selective channels with additive Gaussian noise. The algorithm iterates back and forth, using the current phase estimates to decompose the observe data better and thus improves the next phase estimates. It was shown that under some regularity conditions [22] the algorithm converges to a stationary point of the likelihood function where each iteration cycle increases the liklihood of the estimated phases. We consider here also the nondata-aided(NDA) carrier phase synchronization and show that performance of the synchronization algorithm converges to the Cramer-Rao bound with increasing SNR.

4.1 Phase Estimation in OFDM systems with EM algorithm

Assuming that the OFDM symbol timing and the frequency offset synchronization have already been achieved and compensated, and that the OFDM signal is transmitted over an AWGN channel, the complex signal envelope of the received signal for the mth OFDM frame can be expressed as

$$r_m(k) = s_m(k) + n_m(k), \tag{1.20}$$

$m = 1, 2, \ldots, L_0$ and $k = 0, 1, \cdots, N-1$. where

$$s_m(k) = (1/N) \sum_{n=0}^{N-1} a_m^n |H_n| \exp(j\theta_n) \exp(j2\pi kn/N). \tag{1.21}$$

Here, $\{a_m^n\}$ denotes M-PSK symbols transmitted on the nth subcarrier during the mth OFDM frame, taking values in the set $\{e^{j\frac{2\pi r}{M}}, r = 0, 1, \ldots M-1\}$. $|H_n|$ and θ_n represent the samples taken from the amplitude and the phase spectra of the frequency selective channel, including the influence of a pulse-shaping filter in the system(e.g. Nyquist filter), and $n_m(k)$ is the complex envelope of the additive white Gaussian noise with variance $\sigma^2 = E\{|n_m(k)|^2\}$.

Consider now an observation vector $\mathbf{r}$ containing L_0 number of OFDM symbols(frames) each containing N samples,

$$\mathbf{r} = [\mathbf{r}_1 \mathbf{r}_2 \ldots \mathbf{r}_{L_0}],$$

$$\mathbf{r}_m = [r_m(0), r_m(1) \ldots, r_m(N-1)]^T, \qquad m = 1, 2, \ldots L_0$$

where $r_m(k)$ for samples $k = 1, 2 \ldots N$ in mth frame is given by (1.20).

Thus, from (1.20) and (1.21), given $\boldsymbol{\theta} = [\theta_0, \theta_1, \cdots, \theta_{N-1}]$ and the data sequence $\mathbf{a} = \{a_m^n\}$, the likelihood function of the observed samples can be written as [20],

$$p(\mathbf{r}|\mathbf{a}, \boldsymbol{\theta}) \approx \exp\left[c \sum_{m=1}^{L_0} \sum_{n=0}^{N-1} |H_n| Re\{R_m(n) a_m^{n*} e^{-j\theta_n}\}\right]. \tag{1.22}$$

where $c \triangleq 2/N\sigma^2$ and (*) denotes complex conjugate. The term $R_m(n) = \sum_{k=0}^{N-1} r_m(k) \exp(-j2\pi nk/N)$ in (1.22) is the DFT of the observation sequence $r_m(k)$. We assume the channel gain parameters, $|H_n|$, to be known at the receiver.

For the comparison of the results obtained later we now give the Cramer-Rao lower bound [20] on the variance of the estimation error, $E\left[|\theta_n - \widehat{\theta}_n|^2\right]$ as follows

$$E\left[|\theta_n - \widehat{\theta}_n|^2\right] \geq \frac{1}{|H_n|^2}\frac{\sigma^2}{L_0}, \quad n = 0, 1, \cdots, N-1, \tag{1.23}$$

where L_0 denotes the number of symbols per carrier within the observation interval.

The EM Algorithm

The key element of EM algorithm is to replace the maximization process of the likehood ratio over the parameters to be estimated with an iterative maximization of the new objective function

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^i) = E_{\mathrm{a}}\left\{\log p(\mathbf{r}|\mathbf{a}, \boldsymbol{\theta}) \times p(\mathbf{r}|\mathbf{a}, \boldsymbol{\theta}^i)\right\}$$

where $\boldsymbol{\theta}^i$ is the estimation of $\boldsymbol{\theta}$ at the ith iteration. Referring to $\mathbf{x} = (\mathbf{r}, \mathbf{a})$ as the *complete data*, $Q(\boldsymbol{\theta}|\boldsymbol{\theta}^i)$ may be viewed as a smootehed version of the complete-data loglikelihood. Given initial estimates $\boldsymbol{\theta}^0$, the $i$the iteration of the EM algorithm is described by

1. **Expectation step (E-step):** Compute

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^i) = E_{\mathrm{a}}\left\{\log p(\mathbf{r}|\mathbf{a}, \boldsymbol{\theta}) \times p(\mathbf{r}|\mathbf{a}, \boldsymbol{\theta}^i)\right\},$$

It can be shown that

$$Q(\boldsymbol{\theta}|\boldsymbol{\theta}^i) = \sum_{n=0}^{N-1} |H_n| \left[\gamma_n^c(\mathbf{r}, \boldsymbol{\theta}^i)\cos\theta_n + \gamma_n^s(\mathbf{r}, \boldsymbol{\theta}^i)\sin\theta_n\right]. \tag{1.24}$$

where

$$\gamma_n^c(\mathbf{r}, \boldsymbol{\theta}^i) = E_{\mathrm{a}}\left\{\sum_{m=1}^{L_0} Re[R_m(n)a_m^{n*}] \times p(\mathbf{r}|\mathbf{a}, \boldsymbol{\theta}^i)\right\} \tag{1.25}$$

$$\gamma_n^s(\mathbf{r}, \boldsymbol{\theta}^i) = E_{\mathrm{a}}\left\{\sum_{m=1}^{L_0} Im[R_m(n)a_m^{n*}] \times p(\mathbf{r}|\mathbf{a}, \boldsymbol{\theta}^i)\right\}. \tag{1.26}$$

2. **Maximization Step(M-Step):** Solve

$$\boldsymbol{\theta}^{i+1} = \arg\max_{\boldsymbol{\theta}\in\Theta} Q(\boldsymbol{\theta}|\boldsymbol{\theta}^i).$$

Performing the maximization step, we obtain the recursion

$$\boldsymbol{\theta}^{i+1} = \arg \gamma(\mathbf{r}, \boldsymbol{\theta}^i), \tag{1.27}$$

where

$$\gamma(\mathbf{r}, \boldsymbol{\theta}^i) = \gamma^c(\mathbf{r}, \boldsymbol{\theta}^i) + j\gamma^s(\mathbf{r}, \boldsymbol{\theta}^i) \tag{1.28}$$

and

$$\begin{aligned}
\gamma^c(\mathbf{r}, \boldsymbol{\theta}^i) &= \left[\gamma_0^c(\mathbf{r}, \boldsymbol{\theta}^i), \gamma_1^c(\mathbf{r}, \boldsymbol{\theta}^i), \cdots, \gamma_{N-1}^c(\mathbf{r}, \boldsymbol{\theta}^i)\right]^T, \\
\gamma^s(\mathbf{r}, \boldsymbol{\theta}^i) &= \left[\gamma_0^s(\mathbf{r}, \boldsymbol{\theta}^i), \gamma_1^s(\mathbf{r}, \boldsymbol{\theta}^i), \cdots, \gamma_{N-1}^s(\mathbf{r}, \boldsymbol{\theta}^i)\right]^T.
\end{aligned}$$

Computation of $\gamma_n^c(\mathbf{r}, \boldsymbol{\theta}^i)$ *and* $\gamma_n^s(\mathbf{r}, \boldsymbol{\theta}^i)$

From (1.22) and (1.25), $\gamma_n^c(\mathbf{r}\boldsymbol{\theta}^i)$ can be expressed as

$$\begin{aligned}
\gamma_n^c(\mathbf{r}, \boldsymbol{\theta}^i) &= E_{\mathrm{a}}\Bigg[\sum_m Re\{R_m a_m^{n*}\} \\
&\quad \times \exp\left(c\sum_s \sum_l |H_l| Re\{R_s(l) a_m^{n*} e^{-j\theta_l^i}\}\right)\Bigg] \\
&= \sum_m E_{\mathrm{a}}\left[Re\{R_m a_m^{n*}\}\right. \\
&\quad \times \exp\left(c|H_n| Re\{R_m(n) a_m^{n*} e^{-j\theta_n^i}\}\right)\Big] \\
&\quad \times \prod_{\substack{s \\ s\neq m}} \prod_{\substack{l \\ l\neq n}} E\left[\exp\left(c|H_l| Re\{R_s(l) a_m^{n*} e^{-j\theta_l^i}\}\right)\right]
\end{aligned} \tag{1.29}$$

from which we can easily obtain,

$$\gamma_n^c(\mathbf{r}, \boldsymbol{\theta}^i) = K\sum_m \frac{E_{\mathrm{a}}\left[Re\{R_m a_m^{n*}\} \exp\left(c|H_n| Re\{R_m(n) a_m^{n*} e^{-j\theta_n^i}\}\right)\right]}{E_{\mathrm{a}}\left[\exp\left(c|H_n| Re\{R_m(n) a_m^{n*} e^{-j\theta_n^i}\}\right)\right]}. \tag{1.30}$$

Using the same steps as above, we also obtain a similar expression for $\gamma_n^s(\mathbf{r}, \boldsymbol{\theta}^i)$ as follows:

$$\gamma_n^s(\mathrm{r},\boldsymbol{\theta}^i) =$$
$$K\sum_m \frac{E_{\mathrm{a}}\left[Im\{R_m a_m^{n*}\}\exp\left(c|H_n|Re\{R_m(n)a_m^{n*}e^{-j\theta_n^i}\}\right)\right]}{E_{\mathrm{a}}\left[\exp\left(c|H_n|Re\{R_m(n)a_m^{n*}e^{-j\theta_n^i}\}\right)\right]}, \qquad (1.31)$$

where the constant K is defined as

$$K = \prod_s\prod_l E_{\mathrm{a}}\left[\exp\left(c|H_l|Re\{R_s(l)a_m^{n*}e^{-j\theta_l^i}\}\right)\right]$$

and it is immaterial since it is canceled when taking arguments in (1.30) and (1.31).

4.2 An Example: Phase estimation for QPSK Signaling

We first obtain an exact analytical expression for $\gamma_n^c(\mathrm{r},\boldsymbol{\theta}^i)$ and $\gamma_n^s(\mathrm{r},\boldsymbol{\theta}^i)$ as follows. If $a_m^n = (\pm 1 \pm j)$ represents independent identically distributed data sequence modulating the QPSK carrier, the expectation in the numerator of (1.30) can be evaluated as follows:

$$E_{\mathrm{a}} e^{c|H_n|Re\{R_m(n)a_m^{n*}e^{-j\theta_n^i}\}} =$$
$$E_{\mathrm{a}}\left\{e^{c|H_n|\left(X_m^i(n)Re\{a_m^n\}+Y_m^i(n)Im\{a_m^n\}\right)}\right\} \qquad (1.32)$$

where

$$X_m^i(n) \triangleq Re\{R_m(n)e^{-j\theta_n^i}\}$$
$$Y_m^i(n) \triangleq Im\{R_m(n)e^{-j\theta_n^i}\}$$

Then, taking expectations in (1.32) with respect to the data sequence after taking into account the fact that the $Re\{a_m^n\}$ and $Im\{a_m^n\}$ are independent random variables taking values ± 1 with equal probability, it follows that

$$E_{\mathrm{a}} e^{c|H_n|Re\{R_m(n)a_m^{n*}e^{-j\theta_n^i}\}} =$$
$$\cosh\left(c|H_n|X_m^i(n)\right)\cosh\left(c|H_n|Y_m^i(n)\right).$$

In a similar way, the expectations of the denumerators of (1.30) and (1.31) can be evaluated yielding,

$$E_{\rm a}\left[Re\{R_m(n)a_m^{n*}\}e^{c|H_n|Re\{R_m(n)a_m^{n*}e^{-j\theta_n^i}\}}\right] =$$

$$Re\{R_m(n)\}\sinh\left(c|H_n|X_m^i(n)\right)\cosh\left(c|H_n|Y_m^i(n)\right)$$
$$+Im\{R_m(n)\}\sinh\left(c|H_n|Y_m^i(n)\right)\cosh\left(c|H_n|X_m^i(n)\right)$$

and,

$$E_{\rm a}\left[Im\{R_m(n)a_m^{n*}\}e^{c|H_n|Re\{R_m(n)a_m^{n*}e^{-j\theta_n^i}\}}\right] =$$

$$Im\{R_m(n)\}\sinh\left(c|H_n|X_m^i(n)\right)\cosh\left(c|H_n|Y_m^i(n)\right)$$
$$-Re\{R_m(n)\}\sinh\left(c|H_n|Y_m^i(n)\right)\cosh\left(c|H_n|X_m^i(n)\right).$$

Finally, substituting these results in (1.30) and (1.31), we have

$$\gamma_n^c(\mathbf{r},\boldsymbol{\theta}^i) = \sum_{m=1}^{L_0} Re\{R_m(n)\}\tanh\left(c|H_n|X_m^i(n)\right) + Im\{R_m(n)\}\tanh\left(c|H_n|Y_m^i(n)\right) \quad (1.33)$$

and

$$\gamma_n^s(\mathbf{r},\boldsymbol{\theta}^i) = \sum_{m=1}^{L_0} Im\{R_m(n)\}\tanh\left(c|H_n|X_m^i(n)\right) - Re\{R_m(n)\}\tanh\left(c|H_n|X_m^i(n)\right). \quad (1.34)$$

It can be shown from (1.33) and (1.34) that (1.28) can be expressed as

$$\gamma_n(\mathbf{r},\boldsymbol{\theta}^i) = \gamma_n^c(\mathbf{r},\boldsymbol{\theta}^i) + j\gamma_n^s(\mathbf{r},\boldsymbol{\theta}^i) = \sum_{m=1}^{L_0} R_m(n)\Gamma_m^{i*}(n) \quad (1.35)$$

where,

$$\Gamma_m^i(n) = \tanh\left(c|H_n|X_m^i(n)\}\right) + j\tanh\left(c|H_n|Y_m^i(n)\right), \quad (1.36)$$

and $R_m(n)$ was defined earlier as the DFT of the observation sequence $r_m(k), k = 0, 1, \cdots, N-1$ as follows:

$$R_m(n) = \sum_{k=0}^{N-1} r_m(k)\exp(-j2\pi nk/N).$$

We now give a brief summary of the final EM algorithm which estimates the phase vector $\boldsymbol{\theta}$. Given an observation sequence $\mathbf{r}$, the channel gain parameters $|H_n|$ and the signal-to-noise ratio,

1 Set $i = 0$ and choose the components of the initial phase vector, $\boldsymbol{\theta}^0 = [\theta_0^0, \theta_1^0, \cdots, \theta_{N-1}^0]^T$, independently from a uniform distribution on $(-\pi, \pi)$.

2 Compute $\gamma(\mathbf{r}, \boldsymbol{\theta}^0) = [\gamma_0(\mathbf{r}, \boldsymbol{\theta}^0), \gamma_1(\mathbf{r}, \boldsymbol{\theta}^0), \cdots, \gamma_{N-1}(\mathbf{r}), \boldsymbol{\theta}^0)]^T$, where
$$\gamma_n(\mathbf{r}, \boldsymbol{\theta}^0) = \gamma_n^c(\mathbf{r}, \boldsymbol{\theta}^0) + j\gamma_n^s(\mathbf{r}, \boldsymbol{\theta}^0)$$
and the quantities $\gamma_n^c(\mathbf{r}, \boldsymbol{\theta}^0)$ and $\gamma_n^s(\mathbf{r}, \boldsymbol{\theta}^0)$ are computed from (1.33) and (1.34), respectively.

3 Compute $\boldsymbol{\theta}^{i+1} = [\theta_0^{i+1}, \theta_1^{i+1}, \cdots, \theta_{N-1}^{i+1}]^T$, from (1.27).

4 Compute $\gamma(\mathbf{r}, \boldsymbol{\theta}^{i+1})$ and go to Step 3. Repeat until the algorithm converges, in which case the last phase estimate is produced as the ML estimate.

It is now instructive to consider the two extreme cases of high and low SNR. For high SNR ($\sigma^2 \to 0$) we may approximate
$$\tanh(x) \approx \text{sign}(x)$$
to obtain $\Gamma_m^i(n)$
$$\Gamma_m^i(n) = c|H_n|\text{sign}\left(X_m^i(n)\right) + jc|H_n|\text{sign}\left(Y_m^i(n)\right). \tag{1.37}$$

It is very interesting to observe from (1.35) and (1.27) that for high SNR case, the EM algorithm becomes independent of both the signal-to-noise ratio and the channel gain parameters. This reduces the computational complexity of the algorithm and removes the need to estimate also these parameters separately.

For small SNR($\sigma^2 >> 1$), we may approximate
$$\tanh(x) \approx x - x^3/3$$
to obtain
$$\Gamma_m^i(n) = c|H_n|R_m(n)e^{-j\theta_n^i}(1 - \frac{1}{3}c^2|H_n|^2 R_m(n)e^{-j2\theta_n^i})$$

from which the recursion in (1.27) can be computed as

$$\theta_n^{i+1} = \theta_n^i + \arg \sum_{m=1}^{L_0} |R_m|^2 \left(1 - \frac{1}{3} c^2 |H_n|^2 R_m(n) e^{-j2\theta_n^i}\right).$$

From the above, it follows that for low SNR case, the EM algorithm depends both on the signal-to-noise ratio and the channel gain parameters.

4.3 Simulation Results

We now present simulation results with EM algorithm for carrier phase estimation OFDM systems operating over dispersive channels. The dispersive chanel is chosen as a multipath Rayleigh model which is suitable for wireless systems operating at the outdoor dispersive environment. We assume that the channel response is only slowly time varying with respect to the OFDM symbol period. That is, it is assumed that the channel is quasi-stationary and its impulse response stay constant throughout all of the L_0 OFDM symbols. Note that this assumption is realistic for high bit rate OFDM systems, such as those being planned for terrestial digital video broadcasting services in which the time selectivity can be neglected for several consecutive OFDM blocks [24]. In general, however, the validity of this assumption should be checked against the value of the Doppler bandwidth B_D of the multipath fading channel normalized to the L_0 OFDM block blocks rate $1/L_0NT$. As a rule of thumb, the multipath channel can be considered stationary within L_0 blocks if $B_D L_0 NT < 0.01$.

The following multipath model was employed for the channel impulse response for the duration of L_0 OFDM frames,

$$h(t) = \sum_{i=1}^{K_p} A_i \delta(t - \tau_i), \qquad A_i = \rho_i e^{j\theta_i} \tag{1.38}$$

where ρ_i and θ_i are the amplitude and phase of the path associated with the delay τ_i and K_p is the number of paths. The random variables $\{A_i\}$ are zero-mean complex-valued Gaussian and are mutually independent. The random independent delays $\{\tau_i\}$ are generated so as to provide an exponential power delay profile with an average delay τ_{av} and a maximum delay τ_{max}. As an example, we adopted the same parameters values for the quasi-stationary channel model as employed in [24]: $K_p = 30$, $\tau_{av} = 5\mu s$, and $\tau_{max} = 20\mu s$. The values of $\{\tau_i\}, \{\theta_i\}$ and $\{\rho_i\}$ for the channel are listed in Table 1.1. We assume in our simulation that the information bits are mapped onto QPSK symbols. The symbol interval is chosen to be $T_s = 0.167\mu s$, corresponding to a symbol rate of 6

Mbaud. Finally, in the simulations, the number of subcarriers is chosen as $N = 1024$ or $N = 2048$. The actual phases $\boldsymbol{\theta} = [\theta_0, \theta_1, \cdots, \theta_{N-1}]^T$ and the amplitudes $\mathbf{H} = [|H_0|, |H_1|, \cdots, |H_{N-1}|]^T$ were determined from the samples taken from the amplitude and the phase spectra of the dispersive channel whose parameters are given in Table 1.1. That is

$$|H_k| = H(k/NT), \quad \theta_k = \arg(H(k/NT)).$$

First, we have run extensive computer simulations to check the sensivity of the EM algorithm to the initial starting point scanning wide range of SNR values and for different values of L_0. We have selected all N initial phase values randomly from an uniform distribution in $(-\pi, \pi)$. Our main conclusion is that the algorithm converges within the Cramer-Rao lower bound to ML estimates of all the unknown parameters, regardless of the initial guess.

Secondly, our simulations have revealed the fact that the estimation error variances of each of the phases are equal to each other and are independent of the channel gains, $|H_k|$ and the number of sub-channels, N for wide range of SNR values. This implies that the EM method iteratively decompose the observe signal and estimate the parameters of each signal components separately leading to a numerically as well as statistically stable algorithm.

Fig.1.5 presents the simulation results of the phase error variance versus SNR, for different values of L_0, the number of symbols per carrier within the observation interval. In simulations, we applied the EM algorithm summarized in Sec.4 and set the starting point for the initial phases to zero. As seen in Fig.1.5, the variance of the estimation error of each phase value converges to the Cramer-Rao bound for SNR values greater that 10 dB. Figs. Fig.1.6 and Fig.1.7 show, for $L_0 = 50$, several phase-error variance curves each corresponding to a different number of iteration varying from 1 to 5. We see that for SNR values greater that 10 dB, the algorithm converges to the Cramer-Rao bound after first iteration. For SNR values less than 10 dB the convergency is reached in at most three iterations.

5. Conclusion

In this paper, we have presented several feedforward carrier phase synchronization algorithms for OFDM systems. First, maximizing the low SNR limit of the likelihood function averaged over M-PSK signal constellation, an NDA ML phase estimation algorithm has been derived for OFDM systems operating over nondispersive channels and their mean-squared performances were determined analytically as well as by computer simulation. It was shown that there is an excellent agreement

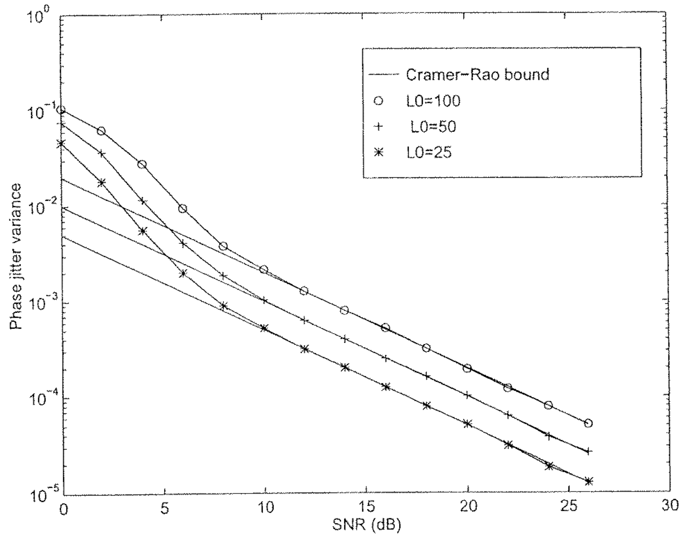

Figure 1.5. Jitter performance of ML phase estimator

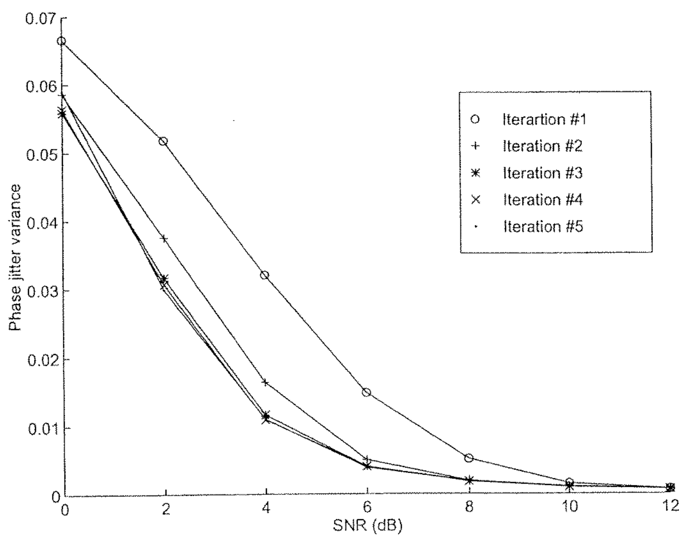

Figure 1.6. Phase jitter with EM iterations

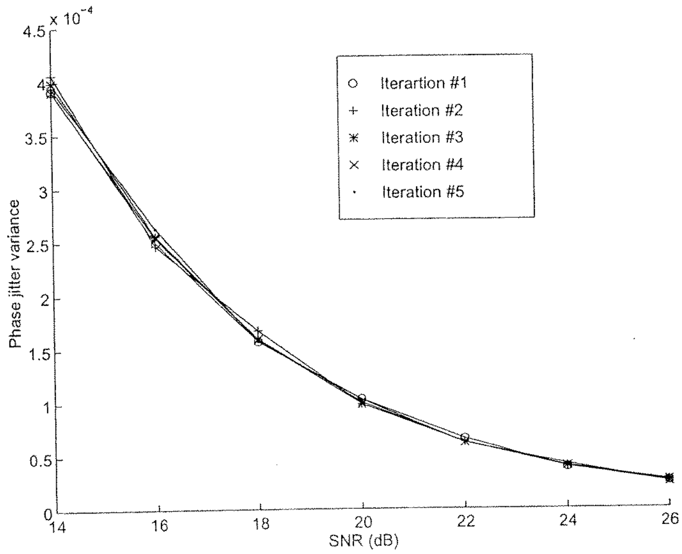

Figure 1.7. Phase jitter with EM iterations

between the analytical and simulation results. It was concluded that for sufficiently small SNR the ML phase estimator obtained reduces to the familiar Mth order power synchronizer which belongs to the class of NDA feedforward carrier synchronizers introduced earlier in the literature. We observed that the resulting algorithm generates very accurate estimation even when the phase offset is high, self noise is absent and that the performance of the NDA algorithm is basically the same as the Cramer-Rao bound for moderate to high SNR.

We have then obtained a computationally efficient algorithm for ML carrier phase estimation of OFDM systems employing M-PSK modulation scheme for transmission over frequency selective channels with additive Gaussian noise, based on the EM algorithm. We considered the non-data-aided(NDA) carrier phase synchronizer and derived an EM algorithm which estimates the phase rotations of each subcarriers iteratively and which converges to the true ML estimation of the unknown phases. The algorithm was applied to the QPSK modulated OFDM system and demonstrated by simulation that the algorithm converges iteratively to the exact ML estimate of all unknown phase simultaneously where each iteration increases the likelihood. It was concluded that most of the time the convergence is achieved in maximum two iterations and the convergence is independent of the initial stating points and of the number of the parameters to be estimated.

Table 1.1. Parameters of the Channel Model

i	Delay τ_i (μ s)	Amplitude ρ_i	Phase θ_i (deg)
1	0.0120	0.4213	5.9010
2	0.2892	0.1543	0.2147
3	0.5593	0.4401	3.9968
4	0.6919	0.4380	4.6862
5	1.0266	0.1864	4.4331
6	1.2347	0.0669	1.1484
7	1.3056	0.0809	4.0282
8	1.9643	0.1647	3.3214
9	2.0906	0.1503	4.0649
10	2.3076	0.1714	3.8432
11	2.3907	0.1289	2.8815
12	2.8962	0.2123	2.8152
13	3.7334	0.3531	5.0859
14	3.7415	0.0982	6.2326
15	3.7630	0.0808	0.7662
16	4.0452	0.1157	5.6671
17	5.4348	0.2199	2.3719
18	5.5246	0.2016	6.0266
19	5.9653	0.1228	5.1854
20	6.6460	0.2004	1.1537
21	6.8295	0.2102	1.3142
22	7.5086	0.2630	4.4436
23	7.9602	0.1199	6.0964
24	8.2400	0.3210	5.0876
25	8.8824	0.1907	1.4835
26	9.7827	0.2379	4.7438
27	10.1142	0.1800	0.1396
28	11.1587	0.2539	1.8221
29	17.6513	0.2767	1.7052
30	18.3765	0.1208	5.3582

Appendix: Derivations of A and $E[W^2(\hat{\phi})]$

In Sec. 3.1, the slope A was defined as $A \cong dE[W(\hat{\phi})/d\hat{\phi}|_{\hat{\phi}=\phi}$. From (1.10), it follows that

$$E[W(\hat{\phi})] = E[\gamma_c]\sin M\hat{\phi} - E[\gamma_s]\cos M\hat{\phi}. \tag{1.A.1}$$

From (1.11)and (1.13), it follows that

$$E[\gamma_c] = \sum_{m=1}^{L_0}\sum_{n=0}^{N-1}\frac{-2M}{(N\sigma_n^2)^M M!}Re\{E[R_m^M(n)]\}.$$

Now, recalling,

$$R_m(n) = \sum_{k=0}^{N-1} r_m(k)e^{-j2\pi nk/N} = \sum_{k=0}^{N-1} e^{j\phi}[s_m(k) + n_m(k)]e^{-j2\pi n\frac{k}{N}} = e^{j\phi}(a_m^n + N_m^n) \tag{1.A.2}$$

we have,

$$E[R_m^M(n)] = e^{jM\phi}E(a_m^n + N_m^n)^M = e^{jM\phi}\{E[(N_m^n)^M] + \sum_{l=0}^{M}\binom{M}{l}E(a_m^n)^l E(N_m^n)^{M-l}\} = 1.$$

From the expression of MPSK symbols, it can be easily shown that $E[(a_m^n)^l] = 1$ if $l = M, 2M, \ldots$ and since the noise is zero-mean complex Gaussian, it follows that $E[(N_m^n)^l] = 0$ for all $l \neq 0$. As a result,

$$E[\gamma_c] = \sum_{m=1}^{L_0}\sum_{n=0}^{N-1}\frac{-2M}{(N\sigma_n^2)^M M!}\cos M\phi = \frac{-2L_0\cos M\phi}{N^{M-1}\sigma_n^{2M}(M-1)!}.$$

Similarly,

$$E[\gamma_s] = \frac{-2L_0\sin M\phi}{N^{M-1}\sigma_n^{2M}(M-1)!}.$$

Putting these expressions in (1.A.1) and taking derivative with respect to $\hat{\phi}$ and setting $\hat{\phi} = \phi$, the final result for A is obtained, as given by (1.17).

Derivation of $E[W^2(\hat{\phi})]$

From (1.10) it follows that

$$E[W^2(\hat{\phi})] = \frac{1}{2}E[\gamma_c^2 + \gamma_s^2] - \frac{1}{2}E[\gamma_c^2 - \gamma_s^2]\cos 2M\hat{\phi} - E[\gamma_c\gamma_s]\sin 2M\hat{\phi}.$$

We now compute $E[\gamma_c^2 + \gamma_s^2]$, $E[\gamma_c^2 - \gamma_s^2]$ and $E[\gamma_c\gamma_s]$ as follows.

From expressions of γ_c, γ_s and Q given by (1.11), (1.12) and (1.18),respectively, we have

$$\gamma_c = \sum_{m=1}^{L_0}\sum_{n=0}^{N-1} QRe[R_m^M(n)], \quad \gamma_s = \sum_{m=1}^{L_0}\sum_{n=0}^{N-1} QIm[R_m^M(n)]$$

from which it follows that

$$\gamma_c^2 + \gamma_s^2 = Q^2 \sum_{m=1}^{L_0} \sum_{n=0}^{N-1} \sum_{i=1}^{L_0} \sum_{j=0}^{N-1} \{Re[R_m^M(n) R_j^{*M}(i)]\}. \tag{1.A.3}$$

Similarly,

$$\gamma_c^2 - \gamma_s^2 = Q^2 \sum_{m=1}^{L_0} \sum_{n=0}^{N-1} \sum_{i=1}^{L_0} \sum_{j=0}^{N-1} \{Re[R_m^M(n) R_j^M(i)]\}, \tag{1.A.4}$$

and,

$$\gamma_c \gamma_s = Q^2 \sum_{m=1}^{L_0} \sum_{n=0}^{N-1} \sum_{i=1}^{L_0} \sum_{j=0}^{N-1} Re[R_m^M(n)] Im[R_j^M(i)]. \tag{1.A.5}$$

From (1.A.2) it follows that

$$E\{R_m^M(n) R_j^{*M}(i)\} = E\{[a_m^n + N_m^n]^M [a_j^{*i} + N_j^{*i}]^M\}.$$

After some algebra, it can be shown that

$$E\{[a_m^n + N_m^n]^M [a_j^{*i} + N_j^{*i}]^M\} = \begin{cases} E\{|a_m^n + N_m^n|^{2M}\} \equiv E[P] & \text{if } n = i \text{ and } m = j \\ 1 & \text{otherwise.} \end{cases} \tag{1.A.6}$$

Using these results in (1.A.3) yields,

$$E[\gamma_c^2 + \gamma_s^2] = Q^2 L_0 N \left(E[P] - 1 + L_0 N\right).$$

Similarly, to compute $E[\gamma_c^2 - \gamma_s^2]$, we note from (1.A.2) that

$$E\{R_m^M(n) R_j^M(i)\} = e^{j2M\phi} E\{[a_m^n + N_m^n]^M [a_j^i + N_j^i]^M\}$$

and it can be shown that for all i, j, n and m,

$$E\{[a_m^n + N_m^n]^M [a_j^i + N_j^i]^M\} = 1.$$

Using these results in (1.A.4) yields

$$E[\gamma_c^2 - \gamma_s^2] = Q^2 L_0^2 N^2 Re\{e^{j2M\phi}\} = Q^2 L_0^2 N^2 \cos 2M\phi.$$

Finally, to compute $E[\gamma_c \gamma_s]$ it can be shown in (1.A.2) that for all i, j, n and m,

$$E\{Re[R_m^M(n)] Im[R_j^M(i)]\} = \frac{1}{2} E\{Im[R_m^{2M}(n)]\} = \frac{1}{2} \sin 2M\phi$$

Substituting this in (1.A.5) yields

$$E[\gamma_c \gamma_s] = \frac{1}{2} Q^2 L_0^2 N^2 \sin 2M\phi$$

Computation of $E[P]$

The quantity P is defined in (1.A.6) by

$$P = |a_m^n + N_m^n|^{2M}.$$

Here, $a_m^n \epsilon \{e^{\frac{j2\pi r}{M}}, r = 0, 1 \ldots M-1\}$ with i.i.d distribution and

$$N_m^n = \sum_{k=0}^{N-1} n_m^k \exp(-j2\pi kn/N).$$

Since n_m^k has distribution of $N(0, \sigma_n^2)$, N_m^n has distribution of $N(0, N\sigma_n^2)$. Let, $V = |a_m^n + N_m^n|$. Probability density function of V is given by [25], as follows.

$$p(V) = \frac{V}{\sigma^2} e^{-\frac{V^2+1}{2\sigma^2}} I_0\left(\frac{V}{\sigma^2}\right),$$

where $\sigma^2 = N\sigma_n^2/2$. Therefore,

$$E[P] = E[V^{2M}] = \int_0^\infty \frac{V^{2M+1}}{\sigma^2} e^{-\frac{V^2+1}{2\sigma^2}} I_0\left(\frac{V}{\sigma^2}\right) dV.$$

Using the integral formula,

$$\int_0^\infty e^{-a^2} x^{m-1} I_0(bx) dx = \frac{\Gamma(\frac{m}{2})}{2a^m} \mathbf{M}(\frac{m}{2}, 1, \frac{b^2}{4a^2}),$$

we have

$$E[P] = e^{-\frac{1}{2\sigma^2}} M!(2\sigma)^M \mathbf{M}(M+1, 1, \frac{1}{2\sigma^2}), \tag{1.A.7}$$

where $\mathbf{M}$(a,b,c) is the M-function and is defined as,

$$\mathbf{M}(M+1, 1, \frac{1}{2\sigma^2}) = e^{\frac{1}{2\sigma^2}} \mathbf{M}(-M, 1, -\frac{1}{2\sigma^2}) = e^{\frac{1}{2\sigma^2}} L_M^{(0)}(-\frac{1}{2\sigma^2}).$$

Here, $L_M^{(0)}$ is the Laguerre function defined as,

$$L_M^{(0)}(-\frac{1}{2\sigma^2}) = \sum_{m=0}^{M} (-1)^m \binom{M}{M-m} \frac{1}{m!} \frac{-1}{(2\sigma^2)^m} = \sum_{m=0}^{M} \binom{M}{m} \frac{1}{m!(2\sigma^2)^m}$$

Using these expressions in (1.A.7), the final result for $E[P]$ is obtained

$$E[P] = M!(2\sigma^2)^M \sum_{m=0}^{M} \binom{M}{m} \frac{1}{m!(2\sigma^2)^m}$$

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Channel Estimation for Space-Time Block Coded OFDM Systems in the Presence of Multipath Fading

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Abstract— **In this paper, a computationally efficient, non-data-aided maximum a posteriori(MAP) channel estimation algorithm is proposed for orthogonal frequency division multiplexing (OFDM) systems with transmitter diversity using space-time block coding. The Alamouti's transmit diversity scheme with two transmit antennas is employed here and generalized for OFDM systems. The algorithm requires a convenient representation of the discrete multipath fading channel based on the Karhunen-Loeve orthogonal expansion and estimates the complex channel parameters of each subcarriers iteratively using the Expectation Maximization(EM) method, which converges to the true MAP estimation of the unknown channel. An analytical expression is derived for the Modified Cramer-Rao lower bound of the proposed MAP channel estimator. The performance is presented in terms of the mean-square error for a system employing QPS signaling.**

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has recently emerged as an attractive and powerful alternative to conventional modulation and multiple access schemes for achieving the high bit rates required for a wireless multimedia service [1]. In OFDM, the entire channel is divided into many narrow sub-channels, thereby enabling high data rate transmission in time-dispersive channels with relatively low-complexity. On the other hand, the dispersive property of the wireless channel causes deep fades for those subchannels. Hence, diversity techniques have to be used to compensate for the frequency selectivity. Transmitter diversity is an effective technique for combatting fading in mobile in multipath wireless channels. More recently, space-time coding has been developed for high data-rate wireless communications [2]. In [3], space-time coding with OFDM has been studied. However, decoding of space time codes requires channel state information, which is usually difficult to obtain. The work in [3] assumes ideal channel state information. Recently, Alamouti proposed a remarkable transmit diversity scheme for transmission using two transmit antennas [4]. This scheme has been generalized later in [5] to an arbitrary number of transmit antennas and is able to achieve the full diversity promised by the transmit and receive antenna. The orthogonal structure of these space-time block codes enable the Maximum likelihood decoding to be implemented in a simple way through decoupling of the signal transmitted from different antennas rather than joint detection. As will be shown in the following sections, the space-time block coding make the channel estimation quite easy.

In this paper we apply the method of Siala [6] to the estimation of multipath fading channels for OFDM systems with two-transmitter diversity proposed first by Alamouti. The algorithm is based on a non-data-aided iterative channel estimation according to the maximum a posteriori(MAP) criterion, using the Expectation-Maximization (EM) algorithm [8]. It uses profitably not only pilot symbols but also information-carrying symbols on the optimization of the channel estimation. It requires a conventional representation of the fading channel, based on a discrete Karhunen-Loeve(KL) orthogonal expansion of the discrete multipath channel seen by the OFDM receiver.

II. Alamouti's Transmit Diversity Scheme for OFDM Systems

In this paper, we consider OFDM systems with transmitter diversity using a space-time block coded transmit diversity scheme first proposed by Alamouti [4]. We describe the scheme with 2 transmit and 1 receive antennas to provide a diversity of order 2. Note that, the method can be easily extended to the more general orthogonal space-time block coded systems introduced by Tarokh *et.al.* involving more than two transmit and one receive antennas. The Alamouti's scheme can be generalized for OFDM systems with two antennas transmitter diversity as follows.

At each time slot n, the data symbols $A_l(n,k), l = 1,2; k = 0,1,\cdots,N-1$, modulated by the kth subcarrier(tone)during the OFDM symbol time T_s, are simultaneously transmitted from the two antennas $l = 1,2$. They are assumed to have unit variance and be independent for different k's and n's. Since the phase of each subchannel can be obtained by the channel estimator, coherent phase-shift keying(PSK) modulation is used here to enhance the system performance. The wireless channel is assumed to be a quasi-static so that path gains are constant over a frame of L_{frame} and vary from one frame to another. The channel frequency response for the kth tone corresponding to the ith transmitter antenna and the receiver antenna is defined to be channel attenuations $H_l(k), l = 1,2;\ k = 0,1,\cdots,N-1$. The attenuations on each tone are given by [7]

$$H_l(k) = H(k/NT_s), k = 0,1,\cdots,N-1,$$

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where $H_l(.)$ is the frequency response of the channel $h_l(t)$ between the lth transmitter and the receiver. They are correlated samples, in frequency, of a complex Gaussian process.

For the kth tone, Alamouti's encoding scheme maps every consecutive symbol $A(2n, k)$ and $A(2n+1, k)$ to the following 2×2 matrix:

$$\begin{array}{c} \\ time \downarrow \end{array} \overset{space \longrightarrow}{\begin{bmatrix} A(2n,k) & A(2n+1,k) \\ -A^*(2n+1,k) & A^*(2n,k) \end{bmatrix}} \quad (1)$$

whose rows are transmitted in successive time intervals with the first and second symbol in a given row sent simultaneously through the first and second antenna respectively.

Since the Alamouti's two branch transmit diversity scheme with one receiver is employed here, $for \quad n = 0, 1, 2, \cdots, \frac{L_{frame}}{2} - 1$ (L_{frame} is even integer.) each pair of the two consecutive received signal can be expressed as

$$\begin{aligned} R(2n,k) &= A(2n,k)H_1(k) + A(2n+1,k)H_2(k) \\ &+ W(2n,k) \\ R(2n+1,k) &= -A^*(2n+1,k)H_1(k) + A^*(2n,k)H_2(k) \\ &+ W(2n+1,k) \end{aligned} \quad (2)$$

where $W(2n, k)$ and $W(2n+1, k)$ are independent samples of an additive Gaussian random variable with variance σ^2, representing the additive white Gaussian noise entering the system.

If the received signal sequence is parsed in blocks of N tones, $\boldsymbol{R}(2n) = [R(2n, 0), \cdots, R(2n, N-1)]^T$ and $\boldsymbol{R}(2n+1) = [R(2n+1, 0), \cdots, R(2n+1, N-1)]^T$, (2) can be expressed in vector form as

$$\begin{aligned} \boldsymbol{R}(2n) &= \boldsymbol{A}(2n)\boldsymbol{H}_1 + \boldsymbol{A}(2n+1)\boldsymbol{H}_2 + \boldsymbol{W}(2n) \\ \boldsymbol{R}(2n+1) &= -\boldsymbol{A}^*(2n+1)\boldsymbol{H}_1 + \boldsymbol{A}^*(2n)\boldsymbol{H}_2 + \boldsymbol{W}(2n+1) \end{aligned} \quad (3)$$

where $\boldsymbol{A}(2n)$ and $\boldsymbol{A}(2n+1)$ are an $N \times N$ diagonal matrices with $\boldsymbol{A}(2n)[k, k] = A(2n, k)$ and $\boldsymbol{A}(2n+1)[k, k] = A(2n+1, k)$ respectively. $\boldsymbol{H}_l = [H_l(0), \cdots, H_l(N-1)]^T$ denotes the channel attenuations for the N tones between the lth transmitter and the receiver. Finally, $\boldsymbol{W}(2n)$ and $\boldsymbol{W}(2n+1)$ are an $N \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive noise in the N tones.

Equation (3) shows that the information symbols $\boldsymbol{A}(2n)$ and $\boldsymbol{A}(2n+1)$ are transmitted twice in two consecutive time intervals through two different channels. In order to estimate the channels and decode $\boldsymbol{A}$ with the embedded diversity gain through the repeated transmission, for each n, we define, $\boldsymbol{R} = [\boldsymbol{R}^T(2n) \;\; \boldsymbol{R}^T(2n+1)]^T$ and write (3) into a matrix form

$$\boldsymbol{R} = \boldsymbol{A}\,\boldsymbol{H} + \boldsymbol{W} \quad (4)$$

where $\boldsymbol{H} = [\boldsymbol{H}_1^T \;\; \boldsymbol{H}_2^T]^T$, $\boldsymbol{W} = [\boldsymbol{W}^T(2n) \;\; \boldsymbol{W}^T(2n+1)]^T$ and

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}(2n) & \boldsymbol{A}(2n+1) \\ -\boldsymbol{A}^*(2n+1) & \boldsymbol{A}^*(2n) \end{bmatrix}. \quad (5)$$

Obviously, channel estimation is very essential for decoding space-time codes. In the absence of channel state information, decoder must estimate the channel states and there has been extensive affords in the direction of channel parameter estimation. In this paper a novel channel estimation algorithm is presented by representing the discrete multipath channel based on the Karhunen-Loeve orthogonal representation and make use of the Expectation Maximization technique.

III. Representation of Discrete Mobile Radio Channels

The complex baseband representation of a mobile wireless channel impulse response can be described as [7]

$$h(t, \tau) = \sum_i \alpha_i(t)\delta(\tau - \tau_i) \quad (6)$$

where τ_i is the delay of the $i$the path and $\alpha_i(t)$ is the corresponding complex amplitude. Due to the motion of the vehicle, $\alpha_i(t)$'s are wide-sense stationary narrowband complex Gaussian processes, which are assumed to be independent for different paths. The correlation function of the frequency response of the multipath radio channel for different frequencies is

$$r(f, f') = E[H(f)H^*(f')] \quad (7)$$

where $H(f) = \int_{-\infty}^{+\infty} h(\tau)e^{-j2\pi f\tau} = \sum_i \alpha_i e^{-j2\pi f\tau_i}$.

It can be shown that

$$\begin{aligned} r(f, f') &= \sigma_H^2 r_f(f - f') \\ r_f(\Delta f) &= (1/\sigma_H^2)\sum_i \sigma_i^2 e^{-j2\pi\Delta f\tau_i} \end{aligned} \quad (8)$$

where σ_i^2 is the average power of the ith path and σ_H^2 is the total average power of the channel impulse response.

For an OFDM system with subchannel spacing Δf, the discrete correlation function for different blocks and subcarriers defined by $r(k, k') = E[H(k)H^*(k')]$ can be written as

$$r(k, k') = \sigma_H^2 r_n(k, k'), \quad k, k' = 0, 1, \cdots, N-1 \quad (9)$$

where $r_n(k, k') = r_f((k - k')\Delta f)$.

Discrete frequency response vector of the wireless channel, $\boldsymbol{H}_l$, $l = 1, 2$ between the antenna l and the receive can be expressed as

$$\boldsymbol{H}_l = \Psi\,\boldsymbol{G}_l \quad (10)$$

where the random variables $\boldsymbol{G}_l$, $l = 1, 2$ is an $N \times 1$ zero-mean i.i.d. Gaussian vector with $\boldsymbol{G}_l(k) = G_l(k)$ whose covariancce matrix is $\Lambda = diag(\lambda_0, \lambda_1, \cdots, \lambda_{N-1})$. The variances of the components of $\boldsymbol{G}_l$, arranged in decreasing order, are equal to the eigenvalues λ_j of the Karhunen Loeve(KL) transformation with the orthogonalized eigenfunctions $\boldsymbol{\Psi} = [\boldsymbol{\psi}_0, \boldsymbol{\psi}_1 \cdots, \boldsymbol{\psi}_{N-1}]$ of the discrete channel autocorrelation matrix $\boldsymbol{r}$ defined as

$$\boldsymbol{r} = E\{\boldsymbol{H}_l \boldsymbol{H}_l^H\} \quad (11)$$

which satisfies $\boldsymbol{r}\boldsymbol{\psi}_j = \lambda_j \boldsymbol{\psi}_j$.

IV. EM-Based MAP Channel Estimation

A. Channel Estimation

The MAP criterion is used in the fading channel as seen at the FFT output of the OFDM receiver since the joint probability density function of the random variables are known by the receiver and can be expressed as

$$p(G_l) \sim \exp(-G_l^H \Lambda^{-1} G_l), \ \ l = 1, 2. \qquad (12)$$

Given the transmitted signals $\mathbf{A}$ as coded according to Alamouti's scheme and the discrete channel representations $\mathbf{G} = [\mathbf{G}_1^T, \mathbf{G}_2^T]^T$ and taking into account the independence of the noise components, the conditional probability density function of the received signal $\mathbf{R}$ can be expressed as,

$$p(R|A, G) \sim \exp\left[-(R - A\widetilde{\Psi}G)^H \Sigma^{-1}(R - A\widetilde{\Psi}G)\right] \qquad (13)$$

where Σ is an $2N \times 2N$ diagonal matrix with $\Sigma[k,k] = \sigma^2$, for $k = 0, 1, \cdots, 2N-1$ and

$$\widetilde{\Psi} = \begin{bmatrix} \Psi & 0 \\ 0 & \Psi \end{bmatrix}. \qquad (14)$$

The MAP estimate $\widehat{\mathbf{G}}$ is given by

$$\widehat{\mathbf{G}} = \arg\max p(\mathbf{G}|\mathbf{R}). \qquad (15)$$

Directly solving this equation is mathematically intractable. However, the solution can be obtained easily by means of the iterative EM algorithm. This algorithm inductively reestimate $\mathbf{G}$ so that a monotonic increase in the *a posteriori* conditional pdf in (15) is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q(\mathbf{G}|\mathbf{G}^{(m)}) = \sum_{\mathbf{A}} p(\mathbf{R}, \mathbf{A}, \mathbf{G}) \log p(\mathbf{R}, \mathbf{A}, \mathbf{G}^{(m)}) \qquad (16)$$

where sum is taken over all possible transmitted data coded signals and $\mathbf{G}^{(m)}$ is the estimation of $\mathbf{G}$ at the mth iteration.

Note that $p(\boldsymbol{R}, \boldsymbol{A}, \boldsymbol{G}) \sim p(\boldsymbol{R}|\boldsymbol{A}, \boldsymbol{G})p(\boldsymbol{G})$ since the data symbols $\boldsymbol{A} = \{A_l(n,k)\}$ are assumed to be transmitted independent of each other and identically distributed and the fact that $\boldsymbol{A}$ is independent of $\boldsymbol{G}$. By similar argument, we have $p(\boldsymbol{R}, \boldsymbol{A}, \boldsymbol{G}^{(m)}) \sim p(\boldsymbol{R}|\boldsymbol{A}, \boldsymbol{G}^{(m)})p(\boldsymbol{G}^{(m)})$.

Therefore, (16) can be evaluated by means of the expressions (12) and (13).

Given the received signal $\mathbf{R}$, the EM algorithm starts with an initial value $\mathbf{G}^0$ of the unknown channel parameters $\mathbf{G}$. The $(q+1)$th estimate of $\mathbf{G}$ is obtained by the maximization step described by

$$\mathbf{G}^{(q+1)} = \arg\max_{\mathbf{G}} Q(\mathbf{G}|\mathbf{G}^{(q)}).$$

After long algebraic manipulations the expression of the reestimate $G_l^{(q+1)}$ $(l = 1, 2)$ can be obtained as follows:

$$\begin{aligned} G_1^{(q+1)} &= (I + \Sigma\Lambda^{-1})^{-1}\Psi^H \left[\Gamma_1^{(q)} R(2n) \right. \\ &- \left. \Gamma_2^{H(q)} R(2n+1)\right] \\ G_2^{(q+1)} &= (I + \Sigma\Lambda^{-1})^{-1}\Psi^H \left[\Gamma_2^{(q)} R(2n) \right. \\ &- \left. \Gamma_1^{H(q)} R(2n+1)\right] \end{aligned} \qquad (17)$$

where it can be easily seen that

$$(I + \Sigma\Lambda^{-1})^{-1} = diag[(1+\sigma^2/\lambda_0)^{-1}, \cdots, (1+\sigma^2/\lambda_{N-1})^{-1}]$$

and Γ_l^q in (17) is an $N \times N$ dimensional diagonal matrix representing the *a posteriori* probabilities of the data symbols at the qth iteration step whose kth diagonal component is defined as

$$\Gamma_l^{(q)}(k) = \sum_{a \in S_k} a^* P(A(2n,k) = a_1, A(2n+1,k) = a_2|\boldsymbol{R}, \boldsymbol{G}^{(q)}) \qquad (18)$$

and S_k denotes alphabet set taken by the kth OFDM symbol.

B. Initialization

In order to be able to choose good initial values for the unknown channel parameters and to ensure a fast start up in the equalization/detection operation following the channel estimation process, the N_{PS} data symbols $\{A(2n,k)\}$ $A(2n+1,k)$ $for\ k \in S_{PS}$, in each OFDM block are generally used as pilot symbols known by the receiver.

Here, S_{PS} denotes the set of pilot symbols indices. Note that, $N_{PS} \geq L$ in order to identify the channel. When N is large, however, this does not create a significant degradation in spectrum efficiency since L, the number of prefix symbols, takes small values with respect to the total number of subcarriers carrying the data. To interpolate the channel estimates, initially, there exist a minimum subcarrier spacing, l_{SC}, between pilots given by $l_{SC} < 1/\tau_{max}$, where τ_{max} is the maximum delay spread of the channel in the frequency domain. Therefore for PSK modulated alphabet set, the initial value of the channel parameters $G_l^{(0)}$ $l = 1, 2$, can be selected according to the following data-aided scheme.

Let D_l^p denote an $N_{PS} \times 1$ vector with $D_l[k] = H_l(k)$, resulting the channel attenuations at frequencies k/NT_s for $k \in S_{PS}$. Using $2N_{PS}$ pilot data symbols in the successive two OFDM blocks, $k \in S_{PS}$, the linear minimum mean-square error (LMMSE) estimate of $\widehat{D}_l$ is given by [7]

$$\widehat{D}_l = \Psi_p \Delta_p \Psi_p^H \widehat{H}_{l,ls} \qquad (19)$$

where Ψ_p is an unitary matrix containing the eigenvectors of the $N_{PS} \times N_{PS}$ dimensional channel covariance matrix $\mathbf{r_p}$ with $r_p[k,k'] = r(k,k'), k, k' \in S_{PS}$. Δ_p is an diagonal matrix with entries

$$\delta_k = \frac{1}{1 + \beta\sigma^2/\mu_k}$$

where, μ_k's are the eigenvalues of $\mathbf{r_p}$ and,

$$\beta = E\{|A(k)|^2\}E\{|1/A(k)|^2\}$$

is a constant depending on the signal constellation [7]. In the case MPSK signaling, $\beta = 1$. Then, given N_{PS} channel attenuation samples $H_l^p[k], k \in S_{PS}$, the complete initial channel attenuation sample values $H_l^0(k), k = 0, 1, \cdots, N-1$ can easily be determined using an interpolation technique, i.e., Lagrange interpolation algorithm. Finally the initial values of $G_l^{(0)}$ can be determined as follows

$$G_l^{(0)} = \Psi^H H_l^{(0)} \quad l = 0, 1.$$

Taking the pilot symbols into account, the final expression of $G_l^{(q+1)}$ can be expressed as follows:

$$\begin{aligned} G_1^{(q+1)} &= (I + \Sigma\Lambda^{-1})^{-1}\Psi^H \left[V_1^{(q)} R(2n) \right. \\ &- \left. V_2^{H(q)} R(2n+1)\right] \\ G_2^{(q+1)} &= (I + \Sigma\Lambda^{-1})^{-1}\Psi^H \left[V_2^{(q)} R(2n) \right. \\ &- \left. V_1^{H(q)} R(2n+1)\right] \end{aligned} \quad (20)$$

where $V_l^{(q)} = diag[v_l^{(q)}(0), v_l^{(q)}(1), \cdots, v_l^{(q)}(N-1)]$ and $v_l^q(k), l = 1, 2$, is given as

$$v_1^{(q)}(k) = \begin{cases} A(2n, k) & \text{if } k \in S_{PS} \\ \Gamma_1^{(q)}(k) & \text{if } k \in S_{PS}^c \end{cases},$$

$$v_2^{(q)}(k) = \begin{cases} A(2n+1, k) & \text{if } k \in S_{PS} \\ \Gamma_2^q(k) & \text{if } k \in S_{PS}^c \end{cases}.$$

Note that implementation complexity of the EM algorithm, presented above, can be reduced substantially due to the fact that the magnitude of the eigenvalues $\lambda_k, k = 0, 1, \cdots, N-1$ of the channel correlation matrix in (11) becomes negligible for $k > 2BT + 1$ where B is the one-sided bandwidth and T is the length of the channel impulse response. As pointed out in [7], for an OFDM system $2BT = L$, where L is number of symbols in the cyclic prefix since $T = LT_s$ and $2B = 1/T_s$. Since L is much smaller than N, the total number of subcarriers, the complexity of the MAP estimation algorithm based on the Karhunen-Loeve expansion proposed in this paper will be low while it is being optimal.

C. Computation of $\Gamma_l^{(q)}(k)$ for QPSK Signaling

Let $a = (\pm 1 \pm j)$ represents independent identically distributed data sequence modulating the QPSK carrier. Since for $l = 1, 2$ and $k = 0, 1, \cdots, N-1$, the data sequence $s_l(k)$ is independent, $\Gamma_m(k)$ in (17) can be computed as follows:

$$\Gamma_l^{(q)}(k) = \frac{\sum_{a_1, a_2 \in S_k} a_l^* \exp\left(\frac{2}{\sigma^2} Re[a_l^* Z_l^m(k)]\right)}{\sum_{a_1, a_2 \in S_k} \exp\left(\frac{2}{\sigma^2} Re[a_l^* Z_l^m(n, k)]\right)} \quad (21)$$

where

$$\begin{aligned} Z_1^{(q)}(k) &= R(1,k) \sum_m G_1^{(q)*}(m) \psi_m^*(k) \\ &+ R^*(2,k) \sum_m G_2^{(q)}(m) \psi_m(k) \\ Z_2^{(q)}(k) &= R(1,k) \sum_m G_2^{(q)*}(m) \psi_m^*(k) \\ &- R^*(2,k) \sum_m G_1^{(q)}(m) \psi_m(k) \end{aligned} \quad (22)$$

Then, taking summations in the numerator and the denominator of (21) over the values of QPSK symbols a_1, a_2, for $l = 1, 2$ and $k = 0, 1, \cdots, N-1$, we have the final result as follows:

$$\Gamma_l^{(q)}(k) = \tanh\left[\frac{2}{\sigma^2} Re(Z_l^{(q)}(k))\right] - j \tanh\left[\frac{2}{\sigma^2} Im(Z_l^{(q)}(k))\right]. \quad (23)$$

V. Modified-Cramer-Rao Bound(MCRB)

Let for $l = 1, 2$ and $m = 0, 1, \cdots, N-1$, $\{G_l(m)\}$'s be the random parameters to be estimated. The (m, n)th element of the Fisher information matrix is defined as

$$J_l(m,n) = -E\left[\frac{\partial^2 \ln p(\mathbf{R}|\mathbf{A}, \mathbf{G}_l)}{\partial G_l(m) \partial G_l(n)}\right] + E\left[\frac{\partial^2 \ln p(\mathbf{G}_l)}{\partial G_l(m) \partial G_l(n)}\right]$$

where the joint probability density functions $p(\mathbf{G})$ and $p(\mathbf{R}|\mathbf{A}, \mathbf{G})$ are given by (14) and (15), respectively and, expectations should be taken over $\mathbf{R}$, $\mathbf{A}$ and $\mathbf{G}$. Performing the the above derivatives and taking into fact that the eigenfunctions $\psi_m(k)$ are orthogonal, it follows that

$$J_l(m,n) = \begin{cases} 2(\frac{1}{\lambda_m} + \frac{1}{\sigma^2}) & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$$

Therefore

$$MCRB(G_l(m)) = J^{-1}(m,m)$$

where σ^2 is the noise variance and λ_m are the eigenvalues of the discrete autocorrelation function $r(k, k')$ of the multipath fading channel.

VI. Simulations

The simulation results for estimating the channel parameters of OFDM systems with transmitter diversity via EM algorithm are now presented. We consider the scheme with 2 transmit and 1 receive antennas with the fading multipath channels between transmitters and the receiver. $H_l(k)$'s are with an exponentially decaying power delay profile $\theta(\tau_l) = C\exp(-\tau_l/\tau_{rms})$ and delays τ_l that are uniformly and independently distributed over the length of the cyclic prefix. C is a normalizing constant. Note that the normalized discrete channel-correlations for different subcarriers of this channel model was presented in [3] as follows,

$$r_1(k,k') = \frac{1 - \exp\left[-L\left[1/\tau_{rms} + 2\pi j(k-k')/N\right)\right]}{\tau_{rms}(1 - \exp(-L/\tau_{rms}))\left(1/\tau_{rms} + j2\pi(k-k')/N\right)}.$$

The scenario for our simulation study consists of a wireless QPSK OFDM system employing the transmitted pulse having a unit-energy Nyquist-root raised-cosine shape with rolloff $\alpha = 0.2$. The symbol period(T_s) is chosen to be 0.167 μs, corresponding to an uncoded symbol rate of 6 Mbit/s. Transmission bandwidth (3.6 MHz) is divided into $N = 256$ tones. We assume that the multipath channel models consist of 5 impulses with uniformly spaced intervals of durations T_s. Therefore, the maximum channel delay $\tau_{max} = 4$ sample (0.668 μs) long. On the other hand, the duration of the transmitter impulse responses after matched filter at the receiver are chosen to be $L_g = 7$ symbols interval. They are truncated at ± 3 sample interval around their center. Note that, in order prevent ISI and ICI, the length of cyclic prefix (L) should be longer than the overall channel response length ($\tau_{max} + L_g - 1$), i.e., $L \geq 10$ samples. As explained previously, this puts a constraint on the number of pilot symbols to be chosen as $N_{PS} \geq L$. For this simulation study we chose $N_{PS} = 10$. To get insight into the average behavior of the channel estimator, we have averaged the performance over 100 Monte-Carlo runs.

Fig. 1 demonstrates the average MSE performance of the EM-based channel estimation algorithm as a function of the average SNR for $\tau_{rms} = 04$ together with the modified CRB. The average SNR was defined as $E[|H_l(k)|^2]E[|A_l(k)|^2/\sigma^2$. Since $E[|A_l(k)|^2 = 1$ for QPSK signaling and $E[|H_l(k)|^2] = 1$ for normalized frequency response of the fading channel, the normalized SNR simply becomes $1/\sigma^2$, where σ^2 is the variance of the complex white Gaussian noise entering the system. Average Mean-square-error(MSE) is defined as the norm of the difference between the vectors $G = [G_0^T, G_1^T]$ and $\widehat{G}_{map}$, representing the true and the estimated values of channel parameters, respectively. Namely,

$$MSE = \frac{1}{2N} \|G - \widehat{G}_{map}\|^2.$$

Notice that the modified CRB provides a looser bound which gets closer to MSE as SNR increases. In Figs. 3, the average MSE performance of the EM-based algorithm are presented as a function of the number of iterations for $\tau_{rms} = 4$. It is concluded from these curves that the MSE performance of the EM-based algorithm converges within 2-5 iterations, depending on the average SNR.

VII. Conclusions

In this paper, we proposed an optimum channel estimation algorithm for OFDM systems. This algorithm performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing M-PSK modulation scheme with additive Gaussian noise. The discrete multipath channel was represented in terms of a Karhunen-Loeve expansion which makes full use of frequency-domain correlation of the frequency response of the time-varying dispersive fading channel.

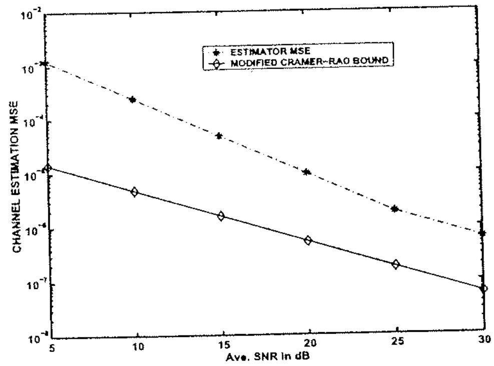

Fig. 1. MSE performance of the EM algorithm as a function of average SNR

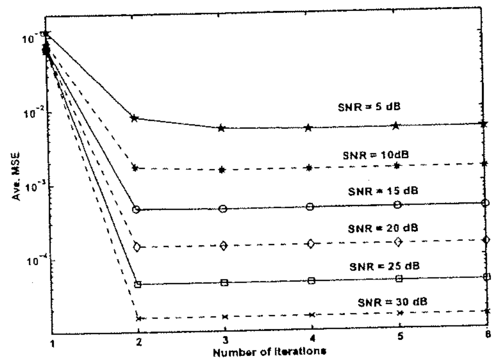

Fig. 2. MSE performance of the EM algorithm as a function of number of iterations ($\tau_{rms} = 4$ sample for the exponentially decaying power delay profile

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Joint Channel Tracking and Symbol Detection for OFDM Systems with Kalman Filtering

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Abstract— **This paper proposes a new joint channel tracking and symbol detection scheme for pilot symbol assisted OFDM systems in multipath fading. The proposed scheme uses Kalman filters for both channel tracking and subsequent equalization which are combined in the coupled estimator structure. Modelling the multipath fading channel as random processes to describe channel's variations in a general AR framework lends itself to a state-space representation that enables application of Kalman filtering for tracking of channel variations. However, the proposed tracking algorithm requires knowledge of the transmitted symbols. This implies that an iterative method should be sought to obtain alternatively either channel or transmitted symbols. To compose the coupled estimator structure, a linear Kalman filter equalizer with the corresponding state-space model is therefore proposed for the detection of transmitted symbols. With the proposed Kalman filters, iterative structure is utilized to decode transmitted symbols and subsequently to track channel parameters. Finally, the performance of the proposed method is studied through the experimental results.**

Index Terms—**OFDM receiver, Kalman Filtering.**

I. Introduction

OFDM has emerged as an attractive and powerful alternative to conventional modulation schemes in the recent past due to its various advantageous in lessening the severe effect of frequency selective fading. Therefore, OFDM is currently being adopted and tested for many standards, including terrestrial digital broadcasting (DAB and DVB) in Europe, and high speed modems over Digital Subscriber Lines in the US. It has also been proposed for broadband indoor wireless systems including IEEE802.11a, MMAC and HIPERLAN/2.

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An OFDM system operating over a wireless communication channel effectively forms a number of parallel frequency nonselective fading channels thereby reducing intersymbol interference (ISI) and obviating the need for complex equalization thus greatly simplifying channel estimation/equalization task. Moreover, OFDM is bandwidth efficient since the spectra of the neighboring subchannels overlap, yet channels can still be separated through the use of orthogonality. Furthermore, its structure also allows efficient hardware implementations using FFTs and polyphase filtering [1].

Although the structure of OFDM signalling avoids ISI arising due to channel memory, fading multipath channel still introduces random attenuations on each tone. Hence, accurate channel estimation technique have to be used to improve the performance of the OFDM systems. Recently, several channel parameter estimation techniques were proposed in the literature [2-5]. In this paper, we address the estimation and equalization problem for OFDM systems with pilot symbols. Among various channel models, the stochastic approach has been used to describe channel's variations in a general AR framework. The information theoretic results in [6] has shown that the lower order AR models provide a sufficiently accurate model for multipath fading channels. Fortunately, the AR modelling lends itself to a state-space representation that enables application of Kalman filtering for tracking of channel variations. We therefore propose Kalman filtering to derive minimum variance estimators for the fading coefficients yielding an adaptive channel tracking algorithm. However, it requires the knowledge of the transmitted symbols. This implies that an iterative method should be sought to obtain alternatively either channel or transmitted symbols. To complete detection-estimation algorithm for OFDM systems with the pilot symbols, a linear Kalman filter equalization technique [9] is therefore proposed for the detection of transmitted symbols.

II. Problem Formulation

Before developing Kalman filter based channel estimation and data detection algorithm, we briefly describe the OFDM system model and the channel statistics in this section.

A. A Model for the Received Signal

In OFDM, the entire information stream is split in many parallel low-rate channels, which are then regularly multiplexed and transmitted through narrow-band subcarriers. Consider an OFDM system with N subcarriers, and let $a(k)$ be the independent data symbol to be placed on subcarrier k, $0 \leq k \leq N-1$. Thus, the data symbols $a(k)$ are modulated by N subcarriers using inverse fast Fourier transform (IFFT) and the last L samples are copied and put as cyclic prefix (CP) to form the complete OFDM symbols of $N+L$ samples long. This data is transmitted over the channel, whose impulse response is shorter that L samples. The cyclic prefix is removed at the receiver and the signal is demodulated with a Fast Fourier Transform (FFT). We assume that the use of CP both preserves the orthogonality of the subcarrier frequencies (tones) and eliminates ISI between consecutive OFDM symbols. Further, the channel is assumed to be constant during one OFDM symbol. Under these assumptions we can describe the system as a set of parallel Gaussian channels with correlated channel attenuation $h(k)$. The attenuations on each tone are given by [3]

$$h(k) = G(\frac{k}{NT_s}), \qquad k = 0, 1, \cdots, N-1 \qquad (1)$$

where $G(.)$ is the frequency response of the channel $g(\tau)$ during the OFDM symbol and T_s is the sampling period of the system.

The received signal after demodulation (performing a DFT) at the kth tone, can be expressed as

$$y(k) = a(k)\, h(k) + v(k), \qquad k = 0, \cdots, N-1. \quad (2)$$

In the above expression $v(k)$ is additive complex white Gaussian noise at the kth tone with zero mean and variance σ^2.

The problem of estimating channel taps $\{h(0), h(1), \cdots h(N-1)\}$ along with the signals modulating the tones $\{a(0), a(1), \cdots, a(N-1)\}$ from noisy observations $\{y(0), y(1), \cdots y(N-1)\}$ is the main concern of the paper. In this paper, we model channel taps as AR process with *a priori* known structure in the development of estimation technique. Let us now briefly describe the channel statistics in the multipath fading environment.

B. Channel Statistics

The complex baseband representation of a fading multipath channel can be described as

$$g(\tau) = \sum_l \alpha_l \delta(\tau - \tau_l T_s) \qquad (3)$$

where τ_l is the delay of the lth path and α_l is the corresponding amplitude with a power-delay profile $\theta(\tau_l)$ which are zero-mean, narrowband complex Gaussian processes independent for different paths.

Using the channel model in (1) and (3), the attenuations on each tone k becomes

$$h(k) = \sum_l \alpha_l e^{-j2\pi(k/N)\tau_l} . \qquad (4)$$

The correlation between the attenuations $h(k)$ and $h(k+m)$

$$r_h(m) = E\{h(k)h^*(k+m)\} . \qquad (5)$$

More frequently used channel model could be explicitly derived by using an exponentially decaying power delay profile $\theta(\tau_l) = Ce^{-\tau_l/\tau_{rms}}$ and special delays τ_l that are uniformly and independently distributed over length of CP. In [2], it is shown that the normalized exponential discrete channel correlation for different subcarriers is

$$r_h(m) = \frac{1 - \exp\left(-L\left(1/\tau_{rms} + 2\pi jm/N\right)\right)}{\tau_{rms}(1 - \exp(-L/\tau_{rms}))\left(1/\tau_{rms} + 2\pi jm/N\right)} . \qquad (6)$$

Note that, the correlation function of the channel taps for different frequencies depends, in general only on the multipath delay spread and is separated from the effect of Doppler frequency. By only exploiting the frequency correlation in the channel estimation task, we are able to reduce complexity of the channel estimator.

III. Channel Estimation and Equalization

In this paper, the algorithms employed to acquire the channel and equalize information symbols are based on the Kalman recursive state estimation algorithm. Kalman filter is a useful channel estimator if the channel model embedded in the Kalman filter closely matches the underlying communications channel. To build a channel model for the multipath fading channel, we match spectral characteristics of the multipath fading with an AR process.

A. Channel Estimation

Since only the first few correlation terms are important to finitely parametrize structured variations of wireless channel in the design of channel estimator, low-order AR models, or even a simple Markov model can capture most of the channel tap dynamics and lead to effective tracking algorithms. In this paper, we follow the common practice and model the channel's variations as a random process. Thus, this paper associates the multiplicative multipath fading effect of the channel with an AR process.

1) AR Model Considerations: We will approximate the multiplicative multipath fading effect in OFDM system with a general AR model order p,

$$h(k) = -\sum_{i=1}^{p} c_i h(k-i) + w(k), \quad k = 0, 1, \cdots, N-1 \tag{7}$$

where $w(k)$ is a white gaussian random process with variance σ_w^2. The estimation of c_i is still not trivial since the channel $h(k)$ is not observed directly. One needs to somehow acquire the channel correlations $r_h(m)$ in order to solve the Yule-Walker equations

$$r_h(m) = \begin{cases} -\sum_{i=1}^{p} c_i r_h(m-i) & m \geq 1 \\ -\sum_{i=1}^{p} c_i r_h(-i) + \sigma_w^2 & m = 0 \end{cases} \tag{8}$$

and obtain c_i. For example, a channel correlation model given by (8) can be used here to determine c_i coefficients. However, the problem of estimating the statistics of a random channel taps has been previously studied in [7], [8]. However, we assume in the sequel that channel statistics are either known a priori or estimated from received data. Thus, given channel statistics $r_h(m)$, AR parameters can be directly obtained by solving normal equations. Once the AR model parameters are identified, Kalman filtering ideas may be employed to track the variations of channel coefficients. Since the Kalman filter would require state-space representation of the multipath fading channel, we will now formulate the state-space description of the fading channel model based on AR model parameters.

2) State-space Representation: If we define $\mathbf{h}(k) = [h(k), h(k-1), \cdots, h(k-p)]^T$, then (7) can be written in state-space form as

$$\mathbf{h}(k) = \begin{bmatrix} -c_1 & -c_2 & \cdots & -c_p \\ 1 & 0 & \cdots & 0 \\ & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix} \mathbf{h}(k-1) + \begin{bmatrix} w(k) \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{9}$$

Let $\mathbf{A}$ be the $(p \times p)$ square matrix in the right hand side of (9), then the state equations becomes

$$\mathbf{h}(k) = \mathbf{A}\mathbf{h}(k-1) + \mathbf{b}w(k) \tag{10}$$

where $\mathbf{b} = [1, 0, \cdots, 0]^T$. In order to obtain the measurement equation, define $\mathbf{a}(k) = [a(k), 0, \cdots, 0]^T$. Then (2) can be written as

$$y(k) = \mathbf{a}^T(k)\mathbf{h}(k) + v(k)\,. \tag{11}$$

Equations (10) and (11) offer a state-space representation of the multiplicative multipath fading model with transition matrix $\mathbf{A}$ (which is assumed to be known in this section). Based on this representation, the minimum variance estimator for the state vector, i.e., the conditional expectation of $\mathbf{h}(k)$ given $\{a(k), y(k)\}_{k=0}^{N-1}$ can be computed from Kalman filtering recursions.

B. Adaptive Kalman Equalization

In this paper, we adopt adaptive Kalman equalizer to couple proposed Kalman channel estimator. Adaptive Kalman equalizer was originally developed for FIR channel model [9]. In this section, we first summarize the adaptive Kalman equalizer, then we modify it to apply for OFDM systems.

If we assume FIR channel model, the elements of the state vector would be the inputs to the channel, i.e.,

$$\mathbf{a}_e(k) = [a(k), a(k-1), \cdots, a(k-d)]^T \tag{12}$$

where $(d+1)$ is the number of taps of the channel. This choice of the state vector is in contrast with Kalman based channel estimator, where the channel taps are used to construct the state vector.

For the adaptive Kalman equalizer, the state transition equation has the following form:

$$\mathbf{a}_e(k) = \mathbf{F}\mathbf{a}_e(k-1) + \mathbf{g}a(k) \tag{13}$$

where $\mathbf{F}$ is the $(d+1) \times (d+1)$ shift matrix, i.e,

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 & 0 \end{bmatrix}$$

and $\mathbf{g}$ is a vector with $(d+1)$ elements, $\mathbf{g} = [1, 0, \cdots, 0]^T$ or more concisely,

$$\mathbf{a}_e(k) = \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 & 0 \end{bmatrix} \mathbf{a}_e(k-1) + \begin{bmatrix} a(k) \\ 0 \\ \vdots \\ \vdots \\ 0 \end{bmatrix}.$$

Then, the observation equation for the Kalman equalizer is

$$y(k) = \mathbf{a}_e^T(k)\mathbf{h}_e(k) + v(k)\,. \tag{14}$$

where $\mathbf{h}_e(k) = [h(k), h(k-1), \cdots, h(k-d)]^T$ is a vector with channel taps. Based on the state-space representation for FIR channel, adaptive Kalman equalizer recursions are summarized in [9]. However, the state-space representation (13) and (14) for FIR equalizer could be adopted for OFDM systems in a very simple form since OFDM overcomes ISI arising from channel memory and only introduces random attenuations on each tone. Thus, the simplified form of state-space representation for OFDM systems becomes

$$\begin{aligned} a(k) &= f\, a(k-1) + a(k) \\ y(k) &= a(k)h(k) + v(k)\,. \end{aligned} \tag{15}$$

where f ($f = 0$) in (15) superficially introduced parameter in order to put (15) in a form given by (13). (15) is therefore simply a scalar form of the state-space representation. With the initialization for $a(1)$ (from pilot symbol) and pre-selected power σ_v^2 for the measurement noise Kalman filter for equalization could be obtained with scalar Kalman filter recursions.

IV. Receiver Structure

The proposed receiver uses Kalman filters for both channel tracking and subsequent equalization. Therefore, the Kalman filters are combined in the proposed coupled estimator structure of Figure 1. Note that, in the joint detection-estimation problem, both $\mathbf{h}(k)$ and $a(k)$ are unknown. With the knowledge of pilot symbol $a(1)$ and the observation $y(1)$, $\hat{\mathbf{h}}(1)$ can be obtained using a Kalman recursions. However, the detection of $a(k),\ k = 2, \cdots, N-1$ relies on the estimates of $\mathbf{h}(k),\ k = 2, \cdots, N-1$ that in turn require the knowledge of $a(k),\ k = 2, \cdots, N-1$. Therefore, iterative structure is employed in the proposed receiver to obtain alternatively either $a(k)$ or $\mathbf{h}(k)$. According to (10) and (11), a coarse prediction of $\mathbf{h}(k \mid k-1)$ can be obtained directly from prediction step. It can be observed from Kalman recursions that the coarse channel prediction obey the recursion

$$\hat{\mathbf{h}}(k \mid k-1) = \mathbf{A}\hat{\mathbf{h}}(k-1 \mid k-1) \tag{16}$$

that are initialized by $\mathbf{h}(0 \mid -1) = \mathbf{0}$. Next, we use the coarse channel estimates in adaptive Kalman equalizer to obtain coarse symbol estimates for $\hat{a}(k)$ that are denoted by $\hat{a}^c(k)$. These estimates are subsequently transformed into $\hat{a}^r(k)$ using the nearest neighbor criterion with a slicer.

Replacing $a(k)$ by $\hat{a}^r(k)$, we rely on Kalman filter to obtain refined channel estimates $\hat{\mathbf{h}}(k \mid k)$. Thus, we summarize our algorithm for channel tracking and symbol detection, in the following steps:

Initialization: Obtain $\mathbf{h}(1 \mid 1)$ from pilot symbol;

1. Obtain $\hat{\mathbf{h}}(k \mid k-1)$ using (16);
2. Use Kalman equalizer to decode $\hat{a}^c(k)$;
3. Use slicer to obtain $\hat{a}^r(k)$ from $\hat{a}^c(k)$;
4. Perform Kalman channel estimator to retrieve $\hat{\mathbf{h}}(k \mid k)$ using $\hat{a}^r(k)$;
5. Repeat Steps 1-4 for $k+1 \leftarrow k$.

Next, we test the performance of our joint channel tracker and equalizer through simulations.

V. Simulations

We now present the simulation results for tracking the channel taps and decoding transmitted symbols in OFDM systems with Kalman filtering. We consider the fading multipath channel given by (6) with an exponentially decaying power delay profile and delays τ_l that are uniformly and independently distributed over the length of the cyclic prefix.

The scenario for our simulation study consists of a wireless QPSK OFDM system employing the pulse shape as a unit-energy Nyquist-root raised-cosine shape with rolloff $\alpha = 0.2$ with a symbol period(T_s) of 0.277 μs. Transmission bandwidth(3.6 MHz) is divided into 128 tones. We assume that the fading multipath channel has exponentially decaying power delay profile (6) with an $\tau_{rms} = 4$ sample (1.08 μs) long.

Since the first-order AR model provides a sufficiently accurate model for multipath fading channels, AR(1) process ($p = 2$) is adopted in the development of the state-space description. Channel model AR(1) parameters are obtained by solving Yule-Walker equations in terms of the correlations. QPSK-OFDM sequence passes through channel taps and corrupted by AWGN (30dB, 20dB and 10dB respectively). We use a pilot symbol every twelve (P=12) symbols.

In the following, the coupled estimator is used to obtain alternatively either transmitted symbols or channel taps in a iterative receiver scheme shown in Figure 1. The results of the Kalman tracking algorithm for both real part and imaginary part of $h(k)$ are obtained and shown in Figure 2 at each SNR. In order to better evaluate the performance of the proposed tracking algorithm, we compare it with other previously developed recursive least squares (RLS) adaptive algorithm. Thick solid lines in these figures represent the true channel taps, thick solid lines with x-mark represent channel tracking with Kalman filtering where as thin solid lines with plus represents the channel tracking with RLS.

It can be seen from simulations that, both the proposed pilot symbol assisted coupled receiver (Kalman based) and the RLS based channel tracking algorithm together with Kalman equalizer perform almost identical performances especially at high SNR values. However, the proposed Kalman based receiver yield better performance for low SNR values.

VI. Conclusions

We have developed a novel Kalman filter based scheme for joint iterative channel tracking and symbol recovery of pilot symbol assisted OFDM systems in multipath fading channels. Modelling multipath fading channel as AR processes, Kalman filter was employed to track the variations of the channel. Moreover, to compose the joint iterative estimator structure, a linear Kalman filter equalizer with the corresponding state-space model was proposed for the recovery of transmitted symbols. Although, the adaptive Kalman equalizer does not yield minimum variance estimates, it is structure is very simple and can be implemented with scalar Kalman filter recursions. The simulation results show that the resulting algorithms is efficient and can be effectively employed in such applications.

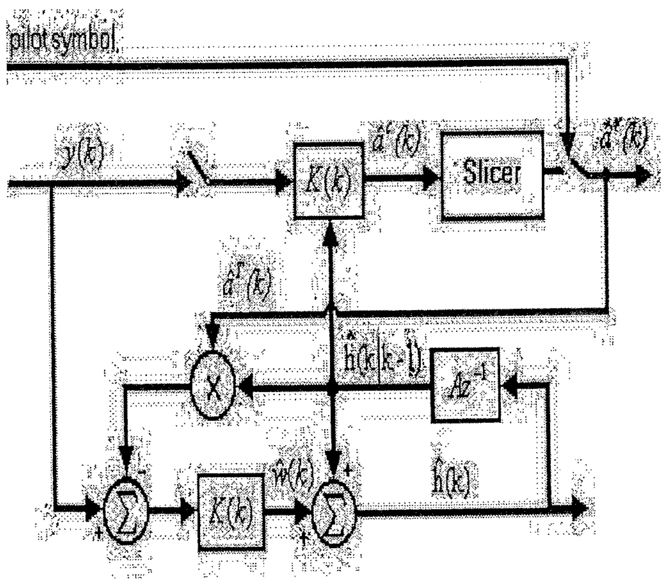

Fig. 1. Proposed Kalman filter based channel tracking/equalization structure

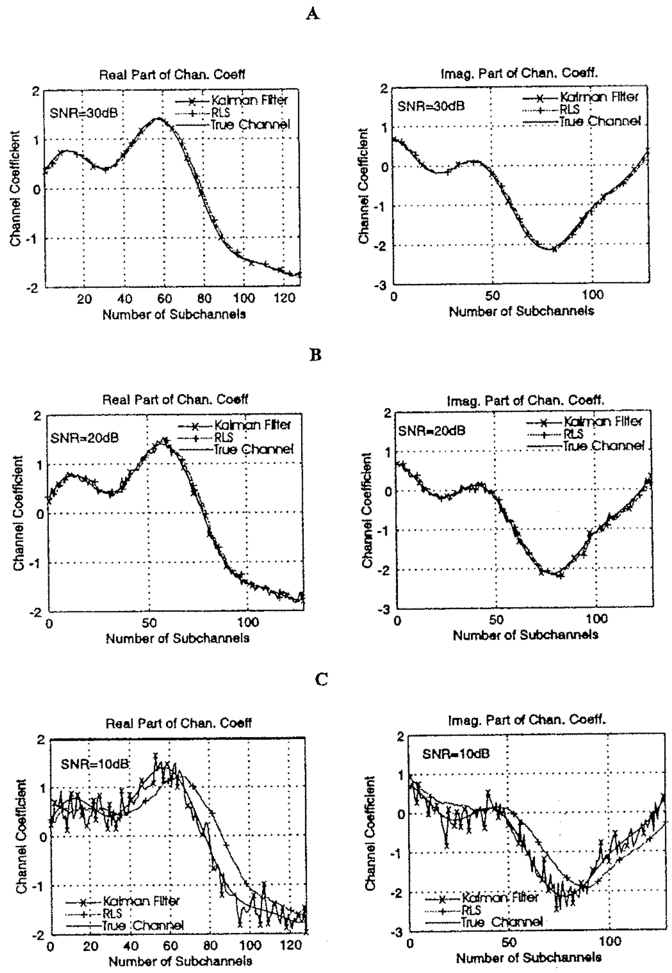

Fig. 2. Comparison of channel tracking performance between the proposed Kalman-based method and the method with RLS: A) SNR=30dB, B) SNR=20dB, C) SNR=10dB.

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Pilot-/Data-Aided Estimation of Rayleigh Flat-Fading Channels

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***Abstract*—We consider pilot- and data-aided channel estimation for orthogonal frequency division multiplexing (OFDM) in frequency-flat Rayleigh fading. Our goal is to derive the optimum channel estimator which processes noisy observations of both the pilot and the data symbols, aiming at a minimum mean square error (MMSE). For simplicity, we restrict the analysis to a two-carrier OFDM model, in which one carrier is assigned to carry a reference pilot whereas the other is used for data transmission. The MSE performance of the optimum estimator can be bounded by the modified Cramér-Rao bound (MCRB) and we show that this bound is asymptotically attained.**

I. INTRODUTION

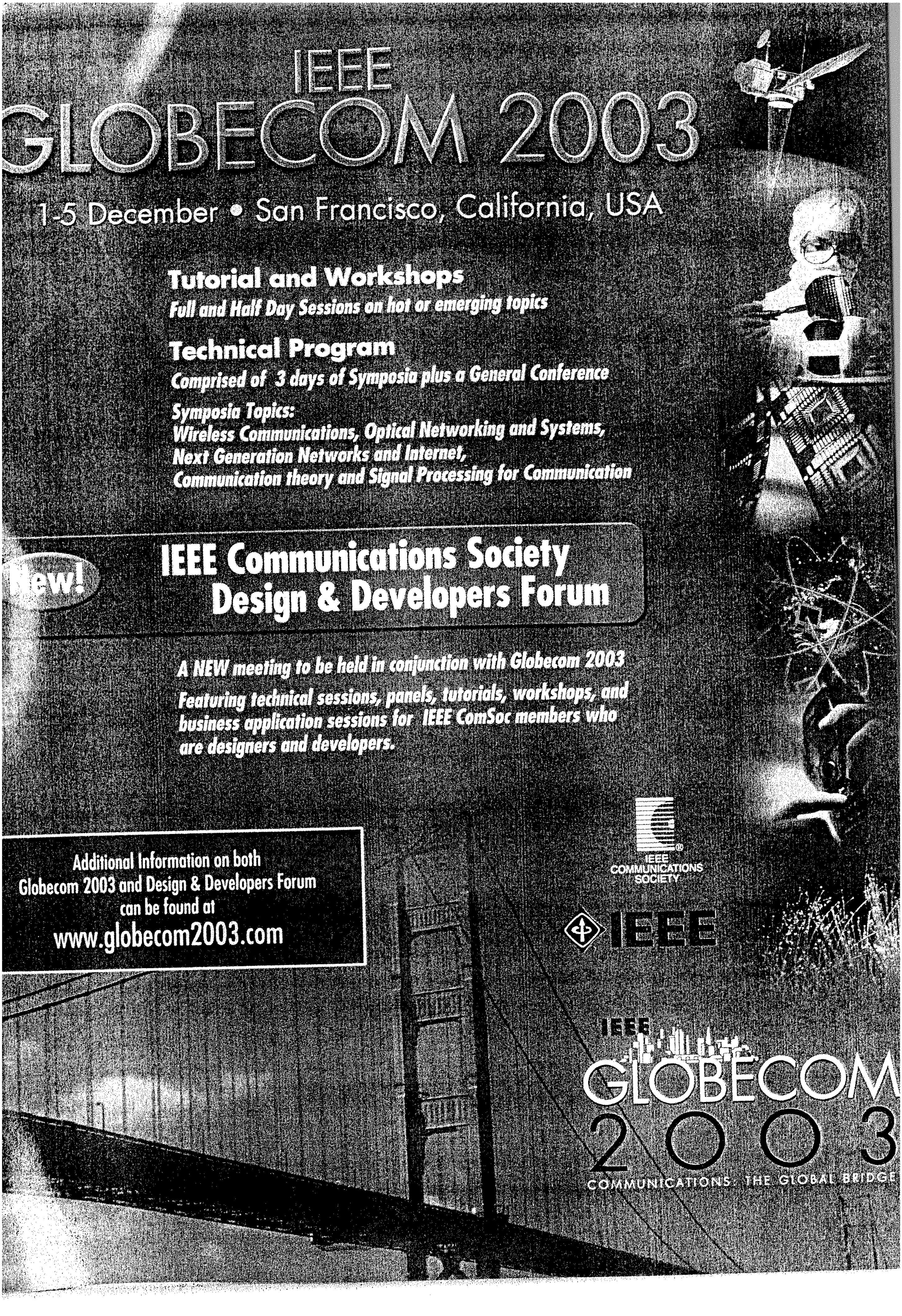
IN order to provide accurate estimates, pilot-aided channel estimation schemes are often deployed in orthogonal frequency division multiplexing (OFDM). Basically, there are two ways of processing the pilot receive symbols. First, the channel can be considered as deterministic but unknown, which is the underlying assumption behind maximum-likelihood (ML) estimation. Second, we can regard the channel as to be a collection of random parameters and may utilize the statistical properties of the channel in the design of an estimator.

A broad overview of various pilot-aided channel estimation schemes can be found in [1], which also provides references for further reading. Model-based channel estimation by Wiener filtering belongs to the category where statistical channel knowledge is available and utilized in the estimation process, for example, see [2]. Wiener filters minimize the mean square error (MSE) between the estimate and the parameter of interest, applying linear filtering of the observed data. Since the Wiener filter is a linear MMSE filter, only the first-order and second-order moments of the channel statistics define the filter coefficients. Consequently, the Wiener filter achieves the smallest possible MSE only if the channel model is linear and the random parameters which are involved in the estimation process are Gaussian [3]. For OFDM, a Gaussian linear model is the common wide-sense stationary uncorrelated scattering (WSSUS) channel model with slowly-fading Rayleigh taps.

If the assumption of normality does not hold, the linear MMSE estimator is no more optimum. In the general case, the MMSE estimate is the conditional mean (for example, see [4]), which in general is a nonlinear estimate. These considerations gain importance if one is interested in improving the channel estimate by using both pilot and data symbols in the estimation process. Unlike the deterministic pilot symbols, the data symbols are random variables which are not Gaussian. Consequently, the optimum pilot-/data-aided MMSE estimator is a nonlinear estimator.

In this paper, our goal is to derive and analyze the optimum pilot- and data-aided (PDA) MMSE channel estimator for a simplified two-carrier OFDM system. Though is it possible to extend the theory to an entire OFDM system, we confine the study to a two-carrier model to keep the mathematics tractable and comprehensive. The channel is assumed to cause frequency-flat Rayleigh fading. In our analysis, we compare the PDA estimator to the conventional pilot-aided (PA) estimator. The MSE of the latter is easily obtained, but the derivation of the MSE of the PDA estimator is cumbersome. However, the PDA-MSE can be lower bounded by the modified Cramér-Rao bound. It is shown that this bound is asymptotically attained which allows to calculate the asymptotic performance gain of PDA over conventional PA estimation.

The remainder of this paper is organized as follows. We introduce the system model in Sec. II and derive the conventional PA and the enhanced PDA estimator in Sec. III. In Sec. IV, the MSE performance of the PDA estimator is studied and an expression for the high-SNR performance gain is given. Finally, Sec. V shows simulation results for 4-ASK and 4-PSK.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Maximum A Posteriori Channel Estimation for Space-Frequency Block Coded OFDM Systems

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Abstract— **Incorporating subchannel grouping, space-frequency coding for transmit diversity orthogonal frequency division multiplexing (OFDM) systems has been proposed recently to achieve maximum diversity gain. Focusing on space-frequency transmit diversity OFDM transmission through frequency selective channels, this paper proposes a computationally efficient, non-data-aided maximum a posteriori(MAP) channel estimation algorithm. The algorithm requires a convenient representation of the discrete multipath fading channel based on the Karhunen-Loeve orthogonal expansion and estimates the complex channel parameters of each subcarriers iteratively using the Expectation Maximization(EM) method, which converges to the true MAP estimation of the unknown channel. An analytical expression is derived for the Modified Cramer-Rao lower bound of the proposed MAP channel estimator.**

I. Introduction

The overwhelming growth of broadband wireless services usage together with the scarcity of bandwidth resources, motivate intense focus of research toward developing efficient coding and modulation schemes that improve the quality and bandwidth efficiency of wireless systems. One approach that shows real promise to overcome the limitations imposed by fading channel, and provide reliable transmission and high spectrum efficiency is to combine two powerful technologies in the physical layer: space-time coding (STC) and OFDM modulation [1],[2].

STC has been proved effective in combating fading and increasing channel capacity without necessarily sacrificing bandwidth efficiency [3],[4],[5]. There is in fact a diversity gain that results from multiple paths between base station and user terminal, and a coding gain that results from how symbols are correlated across transmit antennas. Unfortunately, most existing space-time coding schemes have been developed for flat fading channels initially. Therefore, their successful implementation over broad-band frequency selective channel requires the development of sophisticated signal processing algorithms for channel estimation, and joint equalization/decoding. This task is quite challenging with multiple transmit antennas due to the long delay spread of broad-band channels, which increases the number of channel parameters to be estimated and the number of states in joint equalization/decoding. This, in turn, places significant additional computational load which motivates a more practical reduced-complexity space-time coded OFDM (ST-OFDM) structure [1]. OFDM is chosen over a single-carrier solution due to lower complexity of equalizers for long delay spread channels [6]. In OFDM, a broadband signal is broken down into multiple narrowband carriers, where each carrier is more robust to multipath. OFDM can be implemented efficiently by using fast Fourier transforms (FFTs) at the transmitter and receiver. At the receiver, FFT reduces the channel response into a multiplication constant on each tone. The combined application of OFDM modulation and space-time coding allows us to avoid the complexity of space-time equalizers and therefore yields a unique reduced-complexity physical layer capabilities [1].

The use of OFDM in transmitter diversity systems also offers the possibility of coding in a form of space-frequency OFDM (SF-OFDM) [3]. In [2], it was shown that the SF-OFDM system has the same performance as a previously reported ST-OFDM scheme in slow fading environments but shows better performance in the more difficult fast fading environments. This paper therefore focuses on channel estimation approach for SF-OFDM systems. In this paper, a computationally efficient, non-data-aided maximum a posteriori(MAP) channel estimation algorithm is proposed for orthogonal frequency division multiplexing (OFDM) systems with transmitter diversity using space-frequency block coding. In the development of the MAP channel estimation algorithm, the channel taps are assumed to be random processes. Moreover, orthogonal series expansion based on the Karhunen-Loeve expansion of a random process is applied which makes the expansion coefficient r.v.'s uncorrelated. Thus, the algorithm estimates the uncorrelated complex expansion coefficients iteratively using the Expectation Maximization(EM) method.

II. Alamouti's Transmit Diversity Scheme for OFDM Systems

In this paper, we consider a transmitter diversity scheme in conjunction with OFDM modulation. Many transmit diversity schemes have been proposed in the literature offering different complexity vs. performance trade-offs. We choose Alamouti's space-time block code (STBC) scheme due to its simple implementation and good performance. The Alamouti's scheme

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imposes an orthogonal spatio-temporal structure on the transmitted symbols that guarantees full (i.e., order 2) spatial diversity. In addition to spatial, to realize multipath diversity gains over frequency selective channels, the Alamouti STBC scheme is implemented at a block level in frequency domain.

The Alamouti STBC system with 2 transmit antennas and 1 receive antenna, where utilizing N_c subcarriers per antenna transmissions is employed in this paper. The fading channel between the μth transmit antenna and the receive antenna is assumed to be frequency selective but time-flat and is described by the discrete-time baseband equivalent impulse response $\boldsymbol{h}_\mu(n) = [h_{\mu,0}(n), \cdots, h_{\mu,L}(n)]$, with L standing for the channel order.

Let $A_{k,\mu}(n)$ be the data symbol transmitted on the kth subcarrier frequency (frequency bin) from the μth transmit antenna during the nth OFDM symbol interval. As defined, the symbols $\{A_{k,\mu}(n), \mu = 1, 2, \ k = 0, 1, \cdots, N_c - 1\}$ are transmitted in parallel on N_c subcarriers by 2 transmit antennas.

At the receiver, the antenna receives a noisy superposition of the multiantenna transmissions through the fading channels. We assume ideal carrier synchronization, timing and perfect symbol-rate sampling. We also assume that a cyclic prefix (CP) of length L has been inserted per OFDM symbol and is removed at the receiver end. After FFT processing, the received data sample $R_k(n)$ at the receive antenna can be expressed as

$$R_k(n) = \sum_{\mu=1}^{2} H_{k,\mu}(n) A_{k,\mu}(n) + W_k(n) \tag{1}$$

where $H_{k,\mu}(n)$ is the subchannel gain from the μth transmit antenna to the receive antenna evaluated at the kth subcarier

$$H_{k,\mu}(n) = \sum_{l=0}^{L} h_{\mu,l}(n) e^{-j(2\pi k/N_c)l} \tag{2}$$

and the additive noise $W_k(n)$ is circularly symmetric, zero-mean, complex Gaussian with variance σ^2 that is also assumed to be statistically independent with respect to n and k.

Equation (1) represents a general model for transmit diversity OFDM systems. However, the generation of $A_{k,\mu}(n)$ from the information symbols lead to corresponding transmit diversity OFDM scheme. In our system, the generation of $A_{k,\mu}(n)$ is performed via space-frequency coding, which was first suggested in [3].

A. Space-Frequency Coding

We consider a strategy which basically consists of coding across OFDM tones and is therefore called space-frequency coding. Since an OFDM communication system can be considered as a block transmission system, the serial input data symbols is converted into a data vector $\boldsymbol{A}(n) = [A_0(n), A_1(n) \cdots, A_{N_c-1}(n)]^T$. The space-frequency encoder then codes data symbol vector into two vectors $\boldsymbol{A}_1(n)$ and $\boldsymbol{A}_2(n)$ as

$$\begin{aligned} \boldsymbol{A}_1(n) &= [A_0(n), -A_1^*(n), \cdots, A_{N_c-2}(n), -A_{N_c-1}^*(n)]^T \\ \boldsymbol{A}_2(n) &= [A_1(n), -A_0^*(n), \cdots A_{N_c-1}(n), -A_{N_c-2}^*(n)]^T \end{aligned} \tag{3}$$

In space-frequency Alamouti scheme, $\boldsymbol{A}_1(n)$ and $\boldsymbol{A}_2(n)$ are transmitted through the first and second antenna element respectively during the block instant n.

The operations of the space-frequency encoder can best be described in terms of even and odd polyphase component vectors. If we denote even and odd component vectors of $\boldsymbol{A}(n)$ as

$$\begin{aligned} \boldsymbol{A}_e(n) &= [A_0(n), A_2(n), \cdots, A_{N_c-4}(n), A_{N_c-2}(n)]^T \\ \boldsymbol{A}_o(n) &= [A_1(n), A_3(n), \cdots, A_{N_c-3}(n), A_{N_c-1}(n)]^T \end{aligned} \tag{4}$$

then the space-frequency encoder maps every two consecutive frequency blocks to the following matrix:

$$\begin{array}{cc} & \textit{space} \rightarrow \\ \textit{frequency} \downarrow & \left[\begin{array}{cc} \boldsymbol{A}_e(n) & \boldsymbol{A}_o(n) \\ -\boldsymbol{A}_o^*(n) & \boldsymbol{A}_e^*(n) \end{array} \right] \end{array}. \tag{5}$$

B. Vector Signal Model

If the received signal sequence is parsed in even and odd blocks of N_c tones, $\boldsymbol{R}_e(n) = [R_0(n), R_2(n), \cdots, R_{N_c-2}(n)]^T$ and $\boldsymbol{R}_o(n) = [R_1(n), R_3(n), \cdots, R_{N_c-1}(n)]^T$, the received signal can be expressed in vector form as

$$\begin{aligned} \boldsymbol{R}_e(n) &= \mathcal{A}_e(n)\boldsymbol{H}_{1,e} + \mathcal{A}_o(n)\boldsymbol{H}_{2,e} + \boldsymbol{W}_e(n) \\ \boldsymbol{R}_o(n) &= -\mathcal{A}_o^*(n)\boldsymbol{H}_{1,o} + \mathcal{A}_e^*(n)\boldsymbol{H}_{2,o} + \boldsymbol{W}_o(n) \end{aligned} \tag{6}$$

where $\mathcal{A}_e(n)$ and $\mathcal{A}_o(n)$ are an $N_c/2 \times N_c/2$ diagonal matrices with diag$\mathcal{A}_e(n) = \boldsymbol{A}_e$ and diag$\mathcal{A}_o(n) = \boldsymbol{A}_o$ respectively. $\boldsymbol{H}_{\mu,e}(n) = [H_{0,\mu}(n), H_{2,\mu}(n), \cdots, H_{N_c-2,\mu}(n)]^T$ and $\boldsymbol{H}_{\mu,o}(n) = [H_{0,\mu}(n), H_{1,\mu}(n) \cdots, H_{N_c-1,\mu}(n)]^T$ be $N_c/2$ length vectors denoting the even and odd component vectors of the channel attenuations between the lth transmitter and the receiver. Finally, $\boldsymbol{W}_e(n)$ and $\boldsymbol{W}_o(n)$ are an $N_c/2 \times 1$ zero-mean, i.i.d. Gaussian vectors that model additive noise in the N_c tones.

Equation (6) shows that the information symbols $\mathcal{A}_e(n)$ and $\mathcal{A}_o(n)$ are transmitted twice in two consecutive adjacent subchannel groups through two different channels. In order to estimate the channels and decode $\boldsymbol{A}$ with the embedded diversity gain through the repeated transmission, for each n, we define, $\boldsymbol{R} = [\boldsymbol{R}_e^T(n) \ \ \boldsymbol{R}_o^T(n)]^T$ and write (6) into a matrix form[1]

$$\boldsymbol{R} = \mathcal{A}\,\boldsymbol{H} + \boldsymbol{W} \tag{7}$$

where $\boldsymbol{H} = [\boldsymbol{H}_{1,e}^T \ \ \boldsymbol{H}_{2,e}^T]^T$, $\boldsymbol{W} = [\boldsymbol{W}_e^T(n) \ \ \boldsymbol{W}_o^T(n)]^T$ and

$$\mathcal{A} = \left[\begin{array}{cc} \mathcal{A}_e(n) & \mathcal{A}_o(n) \\ -\mathcal{A}_o^*(n) & \mathcal{A}_e^*(n) \end{array} \right]. \tag{8}$$

[1] We assume that the complex channel gains between adjacent subcarriers are approximately constant, i.e., $\boldsymbol{H}_{1,e} \approx \boldsymbol{H}_{1,o}$ and $\boldsymbol{H}_{2,e} \approx \boldsymbol{H}_{2,o}$.

Obviously, channel estimation is very essential for decoding space-frequency codes. In the absence of channel state information, decoder must estimate the channel states and there has been extensive affords in the direction of channel parameter estimation. Based on the received signal model (7), we will propose a novel channel estimation algorithm in this paper by representing the discrete multipath channel based on the Karhunen-Loeve orthogonal representation and make use of the Expectation Maximization technique.

C. *Karhunen-Loeve Representation of the Multipath Channel*

The Karhunen-Loeve expansion methodology has been used for efficient simulation of multipath fading environments [10]. An exception to this approach, we model discrete frequency response vector of the wireless channel gain vector, $\boldsymbol{H}_{\mu,e}(n)$ based on the Karhunen-Loeve expansion [7], [9] since it makes the expansion coefficient random variable's uncorrelated. Thus, correlated channel gains, in frequency, of a Gaussian process can be expressed as

$$\boldsymbol{H}_{\mu,e}(n) = \Psi\, \boldsymbol{G}_{\mu,e}(n) \tag{9}$$

where $\boldsymbol{G}_{\mu,e}(n)$ is an $N_c/2 \times 1$ zero-mean i.i.d. Gaussian vector whose covariance matrix is $\Lambda = diag(\lambda_0, \lambda_1, \cdots, \lambda_{N-1})$. The variances of the components of $\boldsymbol{G}_{\mu,e}(n)$, arranged in decreasing order, are equal to the eigenvalues λ_j of the Karhunen Loeve(KL) transformation with the orthogonalized eigenfunctions $\Psi = [\psi_0, \psi_1 \cdots, \psi_{N_c-1}]$ of the discrete channel autocorrelation matrix r_μ defined by $\boldsymbol{r}_\mu = E\{\boldsymbol{H}_\mu(n)\boldsymbol{H}_\mu^\dagger(n)\}$ which satisfies $r_\mu \psi_j = \lambda_j \psi_j$ where † denotes conjugate transpose.

III. EM-Based MAP Channel Estimation

In the MAP estimation approach we choose $\widehat{G}$ to maximize the posterior PDF or

$$\widehat{G} = \arg\max_{G} p(G|R)\,. \tag{10}$$

To find MAP estimator, we must equivalently maximize $p(R|G)p(G)$. The prior PDF of the Karhunen-Loeve expansion coefficient r.v.'s of the fading channel can be expressed as

$$p(G) \sim \exp(-G^\dagger \widetilde{\Lambda}^{-1} G)\,, \tag{11}$$

where $G = [G_{1,e}^T, G_{2,e}^T]^T$ and $\widetilde{\Lambda} = diag(\Lambda\ \ \Lambda)$.

Given the transmitted signals $\mathbf{A}$ as coded according to space-frequency transmit diversity scheme and the discrete channel orthonormal series expansion representation coefficients G and taking into account the independence of the noise components, the conditional probability density function of the received signal $\mathbf{R}$ can be expressed as,

$$p(R|A,G) \sim \exp\left[-(R - A\widetilde{\Psi}G)^\dagger \widetilde{\Sigma}^{-1}(R - A\widetilde{\Psi}G)\right] \tag{12}$$

where $\widetilde{\Sigma} = diag(\Sigma\ \ \Sigma)$ and Σ is an $N \times N$ diagonal matrix with $\Sigma[k,k] = \sigma^2$, for $k = 0, 1, \cdots, N-1$ and $\widetilde{\Psi} = diag(\Psi\ \ \Psi)$.

Direct maximization of (10) is mathematically intractable. However, the solution can be obtained easily by means of the iterative EM algorithm. This algorithm inductively reestimate G so that a monotonic increase in the *a posteriori* conditional pdf in (7) is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q(\mathbf{G}|\mathbf{G}^{(i)}) = \sum_{\mathbf{A}} p(\mathbf{R}, \mathbf{A}, \mathbf{G}) \log p(\mathbf{R}, \mathbf{A}, \mathbf{G}^{(i)}) \tag{13}$$

where $\mathbf{G}^{(i)}$ is the estimation of $\mathbf{G}$ at the ith iteration.

Note that $p(R, A, G) \sim p(R|A, G)p(G)$ since the data symbols $A = \{A_{k,\mu}(n)\}$ are assumed to be independent of each other and identically distributed and the fact that A is independent of G. Therefore, (13) can be evaluated by means of the expressions (10) and (12).

Given the received signal R, the EM algorithm starts with an initial value G^0 of the unknown channel parameters $\mathbf{G}$. The $(i+1)$th estimate of G is obtained by the maximization step described by $G^{(i+1)} = \arg\max_{\mathbf{G}} Q(G|G^{(i)})$.

A. *Initialization*

To choose good initial values for the unknown channel parameters, the N_{PS} data symbols $\{A_{k,\mu}(n)\}$ for $k \in S_{PS}$, in each OFDM frame are generally used as pilot symbols known by the receiver. To interpolate the channel estimates, initially, there exist a minimum subcarrier spacing, l_{SC}, between pilots given by $l_{SC} < 1/\tau_{max}$, where τ_{max} is the maximum delay spread of the channel in the frequency domain. Therefore for PSK modulated alphabet set, the initial value of the channel parameters $G_{\mu,e}^{(0)}(n)$ $\mu = 1, 2$, can be selected according to the following data-aided scheme.

Let for $\mu = 1, 2$, $\boldsymbol{H}_{\mu,e}^p(n)$ denote an $N_{PS} \times 1$ vector with $\boldsymbol{H}_{\mu,e}^p(n)[k] = H_{\mu,k}(n)$, resulting the channel gains at frequencies $k \in S_{PS}$. Using $2N_{PS}$ pilot data symbols in the subcarrier groups, the linear minimum mean-square error (LMMSE) estimate of $\widehat{\boldsymbol{H}}_{\mu,e}^p(n)$ is given by [8]

$$\widehat{\boldsymbol{H}}_{\mu,e}^p(n) = \Psi^p \Delta^p \Psi^{p\dagger} \widehat{\boldsymbol{H}}_{\mu,e,ls}^p(n) \tag{14}$$

where $\widehat{\boldsymbol{H}}_{\mu,e,ls}^p(n)$ is the least-square estimate of $\boldsymbol{H}_{\mu,e}^p(n)$ as defined in ([8], page 932), Ψ^p is an unitary matrix containing the eigenvectors of the $N_{PS} \times N_{PS}$ dimensional channel covariance matrix r_μ^p with $r_\mu^p[k,k'] = r_\mu(k,k'), k, k' \in S_{PS}$. Δ_μ^p is an diagonal matrix with entries $\delta_{k,\mu} = 1/(1 + \sigma^2/\lambda_{k,\mu})$ where, $\lambda_{k,\mu}$'s are the eigenvalues of $\mathbf{r}_\mu^{\mathrm{p}}$. Then, given $2N_{PS}$ channel estimated samples $\widehat{\boldsymbol{H}}_{\mu,e}^p(n)[k]$, $k \in S_{PS}$ and $\mu = 1, 2$, the complete initial channel gains $\boldsymbol{H}_{\mu,e}^0[k], k = 0, 1, \cdots, N_c - 1$ can easily be determined using an interpolation technique, i.e., Lagrange interpolation algorithm. Finally the initial values of $G_{\mu,e}^{(0)}(n)$ can be determined as $\boldsymbol{G}_{\mu,e}^{(0)}(n) = \Psi^\dagger \boldsymbol{H}_{\mu,e}^{(0)}(n)$.

Taking the pilot symbols into account, after long algebraic manipulations, the expression of the reestimate $G_{\mu,e}^{(i+1)}(n)$ ($\mu = 1, 2$) can be obtained as follows:

$$G_{1,e}^{(i+1)} = (I + \Sigma\Lambda^{-1})^{-1}\Psi^{\dagger}\left[V_1^{(i)}R_e(n) - V_2^{\dagger(i)}R_o(n)\right]$$
$$G_{2,e}^{(i+1)} = (I + \Sigma\Lambda^{-1})^{-1}\Psi^{\dagger}\left[V_2^{(i)}R_e(n) - V_1^{\dagger(i)}R_o(n)\right] \quad (15)$$

where $(I + \Sigma\Lambda_{\mu}^{-1})^{-1} = \mathrm{diag}([(1 + \sigma^2/\lambda_{\mu,0})^{-1}, \cdots, (1 + \sigma^2/\lambda_{\mu,N_c-2})^{-1}])$ and $V_l^{(i)} = \mathrm{diag}[v_{\mu}^{(i)}(0), v_{\mu}^{(i)}(2), \cdots, v_{\mu}^{(i)}(N_c - 2)]$ and $v_{\mu}^{(i)}(k)$, is given as

$$v_1^{(i)}(k) = \begin{cases} A_{k,1}(n); & \text{if } k \in S_{PS} \\ \Gamma_1^{(i)}(k); & \text{if } k \in S_{PS}^c \end{cases},$$

$$v_2^{(i)}(k) = \begin{cases} A_{k,2}(n); & \text{if } k \in S_{PS} \\ \Gamma_2^{(i)}(k); & \text{if } k \in S_{PS}^c \end{cases}.$$

Here, for $k \in S_{PS}^c$, $\Gamma_{\mu}^{i}(k)$ represents the *a posteriori* probabilities of the data symbols at the ith iteration step and is defined by

$$\Gamma_{\mu}^{(i)}(k) = \sum_{a_1, a_2 \in S_k} a_{\mu}^{*} P(A_{k,1}(n) = a_1, A_{k,2}(n) = a_2 | R, G^{(i)}) \quad (16)$$

and S_k denotes alphabet set taken by the kth OFDM symbol.

B. Computation of $\Gamma_{\mu}^{(i)}(k)$ for QPSK Signaling

Let $a = (\pm 1 \pm j)$ represents independent identically distributed data sequence modulating the QPSK carrier. Since for $\mu = 1, 2$ and $k = 0, 1, \cdots, N_c - 1$, the data sequence $s_{\mu}(k)$ is independent, Γ_{μ} in (15) can be computed as follows:

$$\Gamma_{\mu}^{(i)} = \tanh\left[\frac{2}{\sigma^2}Re(Z_{\mu}^{(i)})\right] - j\tanh\left[\frac{2}{\sigma^2}Im(Z_{\mu}^{(i)})\right] \quad (17)$$

where

$$Z_1^{(i)} = R_{e_d}\Psi^{*}G_{1,e}^{*(i)} + R_{o_d}\Psi^{*}G_{2,e}^{(i)}$$
$$Z_2^{(i)} = R_{e_d}\Psi^{*}G_{2,e}^{*(i)} - R_{o_d}\Psi^{*}G_{1,e}^{(i)}$$

and $R_{e_d} = \mathrm{diag}R_e$ and $R_{o_d} = \mathrm{diag}R_o$.

IV. Modified-Cramer-Rao Bound(MCRB)

Let for $\mu = 1, 2$ and $m = 0, 1, \cdots, N_c - 1$, $\{G_{\mu}(m)\}$'s be the random parameters to be estimated. The (m, n)th element of the Fisher information matrix is defined as

$$J_{\mu}(m,n) = -E\left[\frac{\partial^2 \ln p(\mathbf{R}|\mathbf{A},\mathbf{G}_{\mu})}{\partial G_{\mu}(m)\partial G_{\mu}(n)}\right] + E\left[\frac{\partial^2 \ln p(\mathbf{G}_{\mu})}{\partial G_{\mu}(m)\partial G_{\mu}(n)}\right]$$

where the joint probability density functions $p(\mathbf{G})$ and $p(\mathbf{R}|\mathbf{A},\mathbf{G})$ are given by (11) and (12), respectively and, expectations should be taken over $\mathbf{R}$, $\mathbf{A}$ and $\mathbf{G}$. Performing the the above derivatives and taking into fact that the eigenfunctions $\psi_m(k)$ are orthogonal, it follows that

$$J_{\mu}(m,n) = \begin{cases} 2(\frac{1}{\lambda_m} + \frac{1}{\sigma^2}) & \text{if } m = n \\ 0 & \text{otherwise} \end{cases}$$

Therefore

$$MCRB(G_{\mu}(m)) = J^{-1}(m,m)$$

where σ^2 is the noise variance and λ_m are the eigenvalues of the discrete autocorrelation function $r(k,k')$ of the multipath fading channel.

V. Simulations

The simulation results for estimating the channel parameters of OFDM systems with transmitter diversity via EM algorithm are now presented. We consider the scheme with 2 transmit and 1 receive antennas with the fading multipath channels between transmitters and the receiver. $H_{\mu}(k)$'s are with an exponentially decaying power delay profile $\theta(\tau_{\mu}) = C\exp(-\tau_{\mu}/\tau_{rms})$ and delays τ_{μ} that are uniformly and independently distributed over the length of the cyclic prefix. C is a normalizing constant. Note that the normalized discrete channel-correlations for different subcarriers and blocks were presented in [3] as follows,

$$r_1(k,k') = \frac{1 - \exp\left[-L\left[\frac{1}{\tau_{rms}} + \frac{2\pi j(k-k')}{N}\right)\right]}{\tau_{rms}(1 - \exp(\frac{-L}{\tau_{rms}}))\left(\frac{1}{\tau_{rms}} + \frac{j2\pi(k-k')}{N}\right)}$$

$$r(n,n') = J_o(2\pi(n - n'f_d)T_s)$$

where $J_o$is the zeroth-order Bessel function of the first kind and f_d is the Doppler frequency .

The scenario for our ST-OFDM simulation study consists of a wireless QPSK OFDM system operating with a 2MHz bandwidth and is divided into 256 tones with a total period (Ts) of 136 μs,of which 8 μs constitute the cyclix prefix (L=4). The uncoded data rate 3.76 Mbit/s. we assume that the rms width is $\tau_{rms} = 1$ sample (2 μs) for the power-delay profile and the dopler frequencies are $f_d = 50, 100, 200Hz$.

Fig. 1 demonstrates the average MSE performance of the EM-based channel estimation algorithm as a function of the average SNR and different doppler frequencies. The average SNR was defined as $E[|H_l(k)|^2]E[|A_l(k)|^2/\sigma^2$. Since $E[|A_l(k)|^2 = 1$ for QPSK signaling and $E[|H_l(k)|^2] = 1$ for normalized frequency response of the fading channel, the normalized SNR simply becomes $1/\sigma^2$, where σ^2 is the variance of the complex white Gaussian noise entering the system. Average Mean-square-error(MSE) is defined as the norm of the difference between the vectors $G = [G_{1,e}^T, G_{2,e}^T]$ and $\widehat{G}_{map}$, representing the true and the estimated values of channel parameters, respectively. Namely,

$$MSE = \frac{1}{2N}\|G - \widehat{G}_{map}\|^2.$$

Notice that EM based channel estimation algorithm for SF-OFDM system outperforms in fast fading environments and gets closer to modified CRB for high SNR values. However, the performance of proposed channel estimation algorithm for ST-OFDM degrades significantly for fast fading environments since ST-OFDM is more sensitive to channel gain variation over time.

VI. Conclusions

In this paper, we proposed an optimum channel estimation algorithm for SF-OFDM systems. This algorithm performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing M-PSK modulation scheme with additive Gaussian noise. The discrete multipath channel was represented in terms of a Karhunen-Loeve expansion which makes full use of frequency-domain correlation of the frequency response of the time-varying dispersive fading channel. Simulation results verify that the proposed MAP based iterative estimation technique is well suited in OFDM transmitter diversity systems. Moreover, it also exploits the advantages of SF-OFDM systems in fast fading environments.

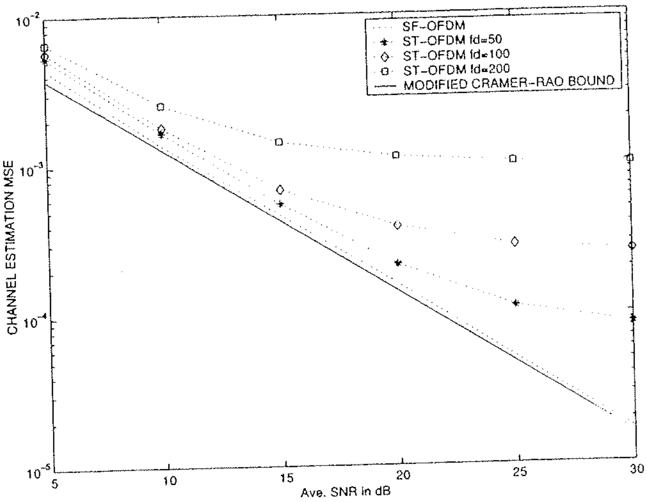

Fig. 1. MSE performance of the the proposed EM based channel estimation algorithm as a function of average SNR

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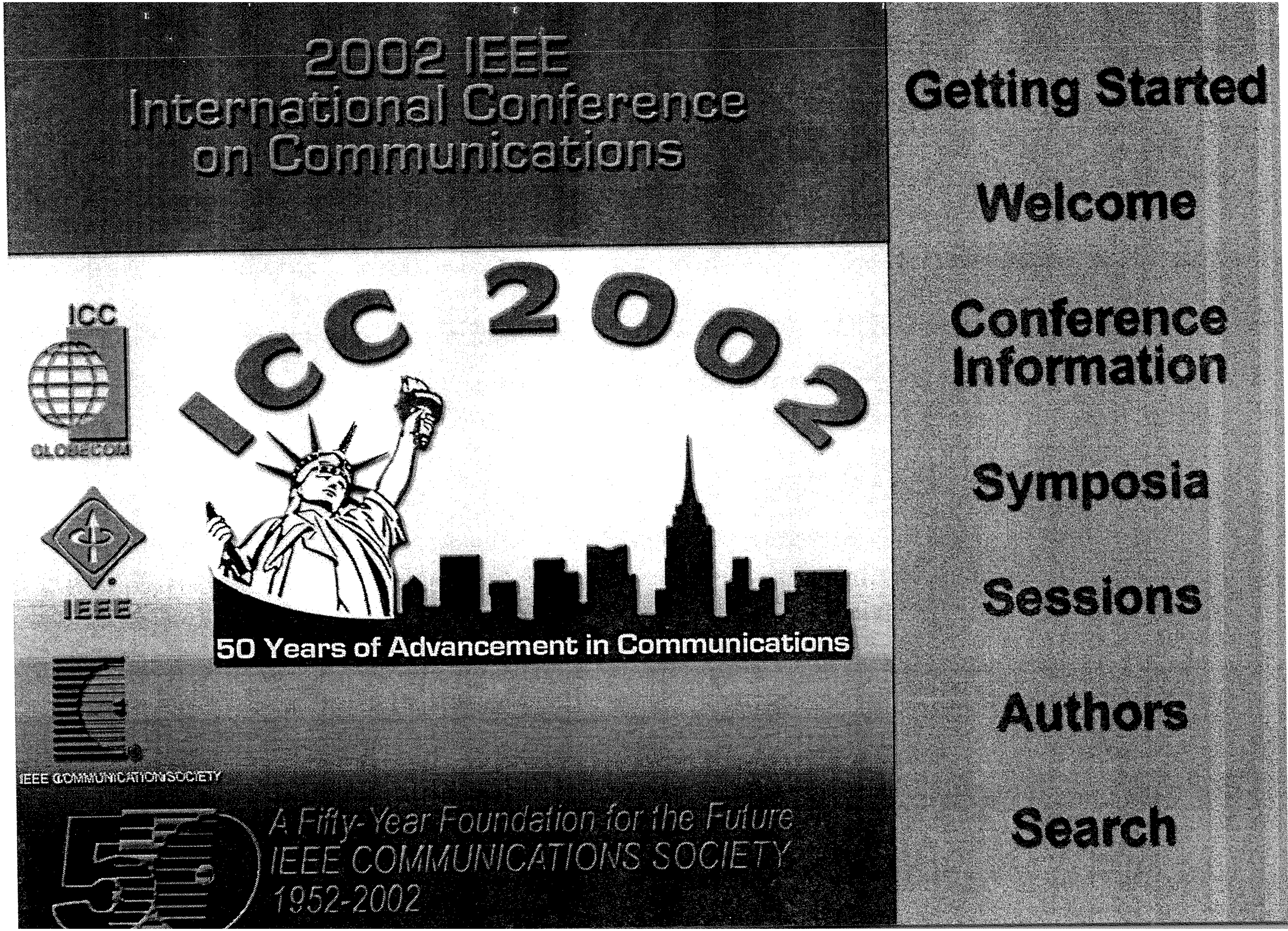
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Blind Channel Estimation for Space-Time Coding Systems with Baum-Welch Algorithm

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Abstract—**In recent years, space-time coding is proposed to provide significant capacity gains over the traditional communication systems in fading wireless channels. In this paper, we consider the problem of blind estimation of the channel parameters along with space-time coded signals. Our proposed approach exploits the finite alphabet property of the space-time coded signals and is based on the unconditional signal model by treating the information sequence as stochastic I.I.D. sequences. The iterative Baum-Welch algorithm is then adapted to solve resulting unconditional ML optimization cost function. Finally, some simulation results are presented.**

I. INTRODUCTION

The rapid growth in demand for a wide range of wireless services is a major driving force to provide high-data rate and high quality wireless access over fading channels [1]. However, wireless transmission is limited by available radio spectrum and impaired by path loss, interference from other users and fading caused by destructive addition of multipath. Therefore several physical layer related techniques have to be developed for future wireless systems to use the frequency resources as efficiently as possible. One approach that shows real promise for substantial capacity enhancement is the use of diversity techniques [2]. Diversity techniques basically reduce the impact of fading due to multipath transmission and improve interference tolerance which in turn can be traded for increase capacity of the system. In recent years, the use of antenna array at the base station for transmit diversity has become increasingly popular since it is difficult to deploy more than one or two antennas at the portable unit. Moreover, the methods of transmitter diversity and channel coding have been integrated into a single system, which is referred to as space-time coding, to provide significant capacity gains over the traditional communication systems in fading wireless channels [2], [3].

Achieving diversity gain for transmit diversity is particularly challenging since the transmitter does not typically know the channel. One problem in making space-time coding system feasible is then the derivation of a fading channel estimation technique. There has been considerable work reported in the literature on the estimation of channel information to improve performance of coded, coded modulated and space-time coded systems operating on fading channels [4], [5], [6]. In this paper we consider the problem of blind estimation of the matrix of path gains along with the space-time coded signals. Our proposed approach exploits the finite alphabet property of the space-time coded signals and is based on the unconditional signal model by treating the coded signals as stochastic I.I.D. sequences. We formulate the blind estimation problem in the unconditional maximum likelihood(ML) framework based on the discrete-time finite state Markov process modeling. Since the proposed algorithm jointly obtains the unconditional ML estimates of channel matrix and the space-time coded signals, it enjoys many attractive properties of the ML estimator.

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II. SYSTEM MODEL

In the sequel, we consider a mobile communication system equipped with n transmit antennas and optional m receive antennas $m \leq n$. A general block diagram for the systems of interest is depicted in Figure 1. In this system, the source generates bit sequence $s(k)$, which are encoded by an error control code to produce codewords. The encoded data are parsed among n transmit antennas and then mapped by the modulator into discrete complex valued constellation points for transmission across channel. The modulated streams for all antennas are transmitted simultaneously. At the receiver, there are m receive antennas to collect the transmissions. Spatial channel link between each transmit and receive antenna is assumed to experience statistically independent fading.

The signals at each receive antenna is a noisy superposition of the faded versions of the n transmitted signals. The constellation points are scaled by a factor of E_s so that the average energy of constellation points is 1. Then we have the following complex base-band equivalent received signal at receive antenna j:

$$r_j(k) = \sum_{i=1}^{n} \alpha_{i,j}(k) c_i(k) + n_j(k) \tag{1}$$

where $\alpha_{i,j}(k)$ is the complex path gain from transmit antenna i to receive antenna j, $c_i(k)$ is the coded symbol transmitted from antenna i at time k, $n_j(k)$ is the additive white Gaussian noise sample for receive antenna j at time k. (1) can be written in a matrix form as

$$\mathbf{r}(k) = \mathbf{\Omega}(k)\ \mathbf{c}(k) + \mathbf{n}(k) \tag{2}$$

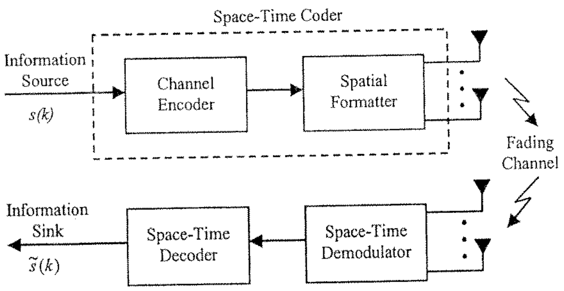

Fig. 1. Space-time coding and decoding system

where $\mathbf{r}(k) = [r_1(k), ..., r_m(k)]^T \in \mathbb{C}^{m\times 1}$ is the received signal vector, $\mathbf{c}(k) = [c_1(k), ..., c_n(k)]^T \in \mathbb{C}^{n\times 1}$ is the code vector transmitted from the n transmit antennas at time k, $\mathbf{n}(k) = [n_1(k), ..., n_m(k)]^T \in \mathbb{C}^{m\times 1}$ is the noise vector at the receive antennas and $\mathbf{\Omega}(k) \in \mathbb{C}^{m\times n}$ is the fading channel gain matrix given as

$$\mathbf{\Omega}(k) = \begin{bmatrix} \alpha_{1,1}(k) & \cdots & \alpha_{n,1}(k) \\ \vdots & \cdots & \vdots \\ \alpha_{1,m}(k) & \cdots & \alpha_{n,m}(k) \end{bmatrix} .$$

We impose following assumptions on model (2) for the rest of the paper:

AS1: Information sequence $s(k)$ is adopting finite complex values.

AS2: The noise vector $\mathbf{n}(k) = [n_1(k), ..., n_m(k)]^T$ is Gaussian distributed with zero-mean and

$$\begin{aligned} \mathbb{E}\left[\mathbf{n}(k)\mathbf{n}^H(l)\right] &= \sigma^2 \mathbf{I}\delta_{k,l} \\ \mathbb{E}\left[\mathbf{n}(k)\mathbf{n}^T(l)\right] &= \mathbf{0} \end{aligned} \tag{3}$$

where $\mathbb{E}$ denotes expectation operator and $\delta_{k,l}$ is the Kronecker delta ($\delta_{k,l} = 1$ if $k = l$ and 0 otherwise).

Thus $\mathbf{n}(k)$ is assumed to be uncorrelated both temporally and spatially.

AS3: The fading channel is assumed to be quasi-static flat fading, so that during the transmission of L codeword symbols across any one of the links, the complex path gains do not change with time k, but are independent from one codeword transmission to the next, i.e.,

$$\alpha_{i,j}(k) = \alpha_{i,j}, \quad k = 1, 2, \cdots, L . \tag{4}$$

The problem of estimating matrix of path gains along with the space-time coded signals from noisy observations $\boldsymbol{r}(L) = [\mathbf{r}^T(1), \cdots, \mathbf{r}^T(L)]^T$ is the main concern of the paper. The traditional solution to this problem is to first estimate $\boldsymbol{\theta} = [\mathbf{\Omega}, \sigma^2]$ from training sequence embedded in the input signal and then use these estimates as if they were the true parameters to obtain estimates of input sequence. As an alternative, we propose ML blind approach based on finite alphabet property of the space-time coded signals in this paper. Let us then derive ML cost function for our proposed approach in the next section.

III. ML Estimation

Under **AS2**, **AS3** and signal model (2), we can formulate the joint ML function of $\mathbf{u}$ as

$$\begin{aligned} f_{\boldsymbol{\theta}}(\mathbf{r} \mid \mathbf{u}) &= \frac{1}{2^{nL}(\pi\sigma^2)^{mL}} \\ &\times \prod_{k=1}^{L} \exp\left\{-\frac{\|\mathbf{r}(k) - \Omega\, \mathbf{g}(\mathbf{u}(k))\|^2}{\sigma^2}\right\} \end{aligned} \tag{5}$$

where $\mathbf{g}(\cdot)$ is the some nonlinear mapping that describes channel coder, spatial formatter and modulator, $\mathbf{u}(k)$ is the input sequence influencing the space-time coded symbols.

In general, trying to estimate $\boldsymbol{\theta}$ and $\mathbf{u}$ jointly from (5) is computationally demanding except for small data alphabet size and small data record. Therefore the goal is to obtain a cost function that is dependent only on $\boldsymbol{\theta}$, in this way it is possible to avoid least squares based two step procedures for blind ML estimation. To this end, we therefore consider an unconditional signal model and compute the corresponding ML cost function via the expectation of the joint ML function with respect to the statistics of the input sequences

$$f_{\boldsymbol{\theta}}(\mathbf{r}) = \mathbb{E}_{\mathbf{u}}\left[f_{\boldsymbol{\theta}}(\mathbf{r} \mid \mathbf{u})\right] . \tag{6}$$

However, the expectation $\mathbb{E}_{\mathbf{u}}$ in (6) leads to complicated cost function. The maximization of this cost function is therefore computationally demanding. At this point, if we exploit **AS1** and use the joint ML function (5), we can obtain the unconditional ML function specifically for the problem at hand as

$$\begin{aligned} f_{\boldsymbol{\theta}}(\mathbf{r}) &= \frac{1}{2^{nL}(\pi\sigma^2)^{mL}} \prod_{k=1}^{L} \\ &\times \sum_{p=1}^{M^{(l+t-1)}} \exp\left\{-\frac{\|\mathbf{r}(k) - \Omega\, \mathbf{g}(\boldsymbol{\zeta}_p)\|^2}{\sigma^2}\right\} \end{aligned} \tag{7}$$

where $\boldsymbol{\zeta}_p = [s(lk+l-1), ..., s(lk-t)]^T$ is the input vector influencing the coded symbols at time k, t is the number of memory elements in the encoder, $l = \log_2 M$ is the information bits that are transmitted (If we restrict ourselves to M-PSK). Since each element of the $\boldsymbol{\zeta}_p$ takes on M possible values, $M^{(l+t-1)}$ be the set of all possible $(l+t-1)$ vectors of M.

The log-likelihood function for the unconditional signal model is then given by

$$L(\boldsymbol{\theta}) = \mathrm{C} + \sum_{k=1}^{L} \log \sum_{p=1}^{M^{(l+t-1)}} \exp\left\{-\frac{\|\mathbf{r}(k) - \Omega\, \mathbf{g}(\boldsymbol{\zeta}_p)\|^2}{\sigma^2}\right\} \tag{8}$$

and the unconditional ML estimation of $\boldsymbol{\theta}$ is the global maximizer of $L(\boldsymbol{\theta})$. Unfortunately, existence of the globally convergent algorithm for this nonlinear cost function is unlikely. Moreover, the direct maximization of (8) still results in computationally demanding nonlinear optimization problem. In finding the ML estimator, it is quite common to resort numerical techniques of maximization such as the Newton-Raphson and scoring methods. However, the Newton-Raphson and scoring methods may suffer from convergence problems. As an alternative, the problem can be cast in a finite-state Markov chain framework by employing the Baum-Welch algorithm which reduces computational burden significantly. The Baum-Welch algorithm although iterative in nature, is guaranteed under certain mild conditions to converge and at convergence to produce a local maximum.

In the sequel, we exploit finite-state Markov process modeling property of the space-time coded signals and employed associated estimation algorithm to provide computationally efficient solution to resulting optimization problem.

A. Function of a Markov Chain

Many important problems in digital communications such as inter-symbol interference, partial response signaling can be modeled by means of finite-state Markov process with unknown parameters observed in independent noise [8]. Based on the **AS1**, codeword produced by the channel encoder in space-time coder can be characterized as a finite-state Markov process. Moreover, the received signal vector at an antenna array in the presence of spatial formatting, fading channel and noise can also be viewed as a stochastic process (function of Markov chain) that has an underlying Markovian finite-state structure.

The space-time coder is characterized by a memory of length t and $M^{(l+t-1)}$ state trellis, where the state $\zeta(k)$ at time k labels the coder memory $(s(lk+l-2), ..., s(lk-t))$,

$$\zeta(k) \in \Pi = \left\{\tau_p, \quad p = 1, ..., M^{(l+t-2)}\right\} . \tag{9}$$

The transition from state $\zeta(k)$ to $\zeta(k+1)$ is represented on the trellis by a branch denoted by the vector

$$\phi(k) = [s(lk+l-1), ..., s(lk-t)]^T \tag{10}$$

and $\phi(k) \in \Phi = \{\xi_n, n = 1, \cdots, M^{(l+t-1)}\}$. Then both the $\{\zeta(k)\}$ sequence and the $\{\phi(k)\}$ sequence form a first order finite Markov chains, i.e.,

$$Pr(\phi(k) = \xi_n) = Pr(\zeta(k) = \tau_q, \zeta(k-1) = \tau_s) \tag{11}$$

for some q, s depending on k.

The observation vector $\mathbf{r}(k)$ can therefore be modeled as a probabilistic function of the Markov chain. In the received signal model, the unknown channel matrix Ω enter in a linear way, while the nonlinear part of the function $\mathbf{g}(\cdot)$ is due to the space-time coder and is known. Let $\mathbf{g}(\xi_n)$ denote the space-time coder output corresponding to the event $\phi(k) = \xi_n$. The sample $\phi(k) = \xi_n$ is a realization of the complex random sample $\mathbf{g}(\phi(k))$ which takes $M^{(l+t-1)}$ possible values depending on the $\phi(k) = \xi_n$. Moreover, every realization of a sequence of symbols corresponds to a branch sequence of length L, given as

$$\mathcal{X} = (x_1, \cdots, x_L), \quad \mathcal{X} \in \Xi \quad | \Xi | \in | M |^L \tag{12}$$

The underlying Markovian structure of our signal model can be characterized by the following model parameters: **i)** $Pr[\zeta(k) = \tau_q \mid \zeta(k-1) = \tau_s]$ is a predetermined transition probability. If no information about the transmitted sequence is available, all permissible state transitions have the same probability, i.e., $Pr[\zeta(k) = \tau_q \mid \zeta(k-1) = \tau_s] = \frac{1}{M^{(l+t-1)}}$, if state τ_s leads to state τ_q. **ii)** $\hat{\pi}(0) = [\hat{\pi}_1(0), \cdots, \hat{\pi}_{M^{(l+t-1)}}(0)]$ initial state probability vector. If no assumption on the starting bits is made, the initial probability is same for all states. **iii)** The conditional density $f(\mathbf{r}(k) \mid \zeta(k) = \tau_q, \zeta(k-1) = \tau_s) = f(\mathbf{r}(k) \mid \phi(k) = \xi_n)$ is that of a Gaussian complex random vector with mean $\Omega\ \mathbf{g}(\xi_n)$ and variance σ^2. Since the state transition probability and the initial state probability vector are predetermined, the only model parameter of the Markov chain left to be estimated is $f(\mathbf{r}(k) \mid \phi(k) = \xi_n)$ for the current model. We therefore devise the Baum-Welch algorithm to estimate the Markov chain model parameter (**iii**) or equivalently to estimate $\boldsymbol{\theta}$.

B. Baum-Welch Algorithm

The Baum-Welch algorithm is a commonly used iterative technique for estimating the parameters of a probabilistic functions of a Markov chain. It maximizes an auxiliary function related to the Kullback-Leibler information measure instead of the likelihood function [7]. The auxiliary function is defined as a function of two set of parameters $\boldsymbol{\theta}_1, \boldsymbol{\theta}_2$

$$Q(\boldsymbol{\theta}_1, \boldsymbol{\theta}_2) = \sum_{\mathcal{X} \in \Xi} f_{\boldsymbol{\theta}_1}(\mathbf{r}, \mathcal{X})\ \log(f_{\boldsymbol{\theta}_2}(\mathbf{r}, \mathcal{X})) \tag{13}$$

where $\mathcal{X}$ takes all possible branch sequences, (e.g., [8]).

The theorem that that forms the basis for the Baum-Welch algorithm explains the reason why Kullback-Leibler information measure can be used instead of the average likelihood.

Theorem: $Q(\boldsymbol{\theta}, \boldsymbol{\theta}') \geq Q(\boldsymbol{\theta}, \boldsymbol{\theta}) \Rightarrow f_{\boldsymbol{\theta}'}(\mathbf{r}) \geq f_{\boldsymbol{\theta}}(\mathbf{r})$.

For the proof of the theorem, see [7].

The explicit form of the auxiliary function for the current problem is [7],

$$\begin{aligned} Q(\theta^{(i)}, \theta') &= C + \sum_{k=1}^{L} \sum_{p=1}^{M^{(l+t-1)}} \gamma_p^{(i)}(k) \\ &\times \left\{ -\frac{1}{\sigma'^2} \|\mathbf{r}(k) - \Omega'\ \mathbf{g}(\xi_p)\|^2 - \log(\sigma'^2) \right\} \end{aligned} \tag{14}$$

where $\theta^{(i)}$ is the old parameter estimates obtained at the ith iteration while $\boldsymbol{\theta}' = [\Omega', \sigma'^2]$ is the new parameter set to be estimated at the $(i+1)$th iteration and $\gamma_p^{(i)}(k) = f_{\boldsymbol{\theta}^{(i)}}(\mathbf{r}, \phi(k) = \xi_p)$ is the weighted conditional likelihood. The direct computation of weighted conditional likelihood is computationally intensive. Fortunately, there exists recursive procedures (called forward and backward procedures), for computing $\gamma_p^{(i)}(k)$ whose complexity increases only linearly with data length N [7].

The following explicit expression for the array response matrix is obtained from $\partial Q/\partial \Omega' = 0$:

$$\Omega^{(i+1)} = \left(\sum_{k=1}^{L} \sum_{p=1}^{M^{(l+t-1)}} \gamma_p^{(i)}(k)\, \mathbf{r}(k) \mathbf{g}(\xi_p)^H \right) \times \left(\sum_{k=1}^{L} \sum_{p=1}^{M^{(l+t-1)}} \gamma_p^{(i)}(k)\, \mathbf{g}(\xi_p) \mathbf{g}(\xi_p)^H \right)^{-1} \quad (15)$$

where $(\circ)^H$ denotes the complex conjugate transpose.

The last equality follows from the definition of the partial derivative with respect to a complex quantity (see e.g. [11])

$$\frac{\partial Q}{\partial \Omega'_{ij}} = \frac{1}{2} \left[\frac{\partial Q}{\partial \mathrm{Re}\{\Omega'_{ij}\}} + j \frac{\partial Q}{\partial \mathrm{Im}\{\Omega'_{ij}\}} \right] \quad (16)$$

where Ω_{ij} is the ijth element of Ω.

From $\partial Q/\partial \sigma'^2 = 0$, the iterative estimation formula can also be derived for the noise variance:

$$\sigma'^2 = \frac{\sum_{k=1}^{L} \sum_{p=1}^{M^{(l+t-1)}} \gamma_p^{(i)}(k) \|\mathbf{r}(k) - \Omega' \mathbf{g}(\xi_p)\|^2}{\sum_{k=1}^{L} \sum_{p=1}^{M^{(l+t-1)}} \gamma_p^{(i)}(k)} . \quad (17)$$

Based on this results, the steps of the proposed unconditional ML algorithm are summarized as follows:

Set the parameters to some initial value $\theta^{(0)} = (\Omega^{(0)}, \sigma^{2(0)})$.

1. Compute the forward and backward variables to obtain $\gamma_p^{(i)}(k)$.
2. Compute $\Omega'^{(i+1)}$ from (15).
3. Compute $\sigma'^{2(i+1)}$ from (17).
4. Repeat Steps 1-3 until $\|\theta^{(i+1)} - \theta^{(i)}\| < \epsilon$, where ϵ is a predefined tolerance parameter.
5. Use $f_{\theta^{(i)}}(\mathbf{r}, \phi(k) = \xi_p)$'s to recover the transmitted symbols.

Since the proposed method exploits the finite alphabet structure of the space-time coded signals and implements a stochastic ML solution, it is expected to exhibit better performance than suboptimal estimation techniques, especially when short data records are available. For a sufficiently good initialization, the proposed algorithm converges rapidly to the ML estimate of $\hat{\theta}$.

IV. Performance Analysis: CRB

The evaluation of the exact form of the CRB requires the Hessian matrix for the log-likelihood function. Under **AS1**, the computation of the exact CRB is analytically intractable, we therefore consider an alternative approach for simplifying CRB calculation [9].

The corresponding log-likelihood function explicitly for the current problem is given by

$$\log\left[f_{\theta}(\mathbf{r})\right] = -\log(2^{nL}(\pi\sigma^2)^{mL}) + \sum_{k=1}^{L} \log \sum_{p=1}^{M^{(l+t-1)}} \exp\left\{-\frac{1}{\sigma^2}\|\mathbf{r}(k) - \mathbf{\Omega}\, \mathbf{g}(\zeta_p)\|^2\right\} . \quad (18)$$

Unfortunately, due to nature of (18) the evaluation of the Hessian matrix is analytically intractable. However it is common to adopt (see e.g. [9]) an approximate log-likelihood function to obtain valid CRB. Due to concavity of the log-likelihood function and Jensen's inequality, we obtain from (18) the following approximate log-likelihood function:

$$\log\left[f_{\theta}(\mathbf{r})\right] \leq \sum_{k=1}^{L} \sum_{p=1}^{M^{(l+t-1)}} \log\left[\exp\left\{-\frac{1}{\sigma^2}\|\mathbf{r}(k) - \mathbf{\Omega}\, \mathbf{g}(\zeta_p)\|^2\right\}\right] . \quad (19)$$

If we further simplify (20), we obtain

$$\log\left[f_{\theta}(\mathbf{r})\right] \leq -\frac{1}{\sigma^2} \sum_{k=1}^{L} \sum_{p=1}^{M^{(l+t-1)}} \|\mathbf{r}(k) - \mathbf{\Omega}\, \mathbf{g}(\zeta_p)\|^2 . \quad (20)$$

At this point, we should point out that the Hessian matrix from the approximate log-likelihood function can be easily obtained. However, (20) leads to a CRB which is not as tight as exact CRB, but it is computationally easier to evaluate.

It turns out from the approximate log-likelihood function of (19) that the entries of the FIM are as

$$\mathbf{J}_{\sigma^2,\sigma^2} = \frac{nL}{\sigma^4}, \quad \mathbf{J}_{\sigma^2,\mathbf{\Omega}} = 0, \quad \mathbf{J}_{\mathbf{\Omega},\sigma^2} = \mathbf{0} . \quad (21)$$

Moreover, the submatrix $\mathbf{J}_{\mathbf{\Omega},\mathbf{\Omega}}$ can also be obtained as

$$\mathbf{J}_{\mathbf{\Omega},\mathbf{\Omega}} = \frac{2}{\sigma^2} \sum_{p=1}^{M^{(l+t-1)}} \mathbf{g}(\zeta_p)\, \mathbf{g}^H(\zeta_p) . \quad (22)$$

The I.I.D. input sequence coded with orthogonal space-time codes results in uncorrelated coded sequence. It is therefore possible to further simplify the valid CRB's. In this case, the valid CRB can be easily obtained as follows:

$$\mathbf{J}^{-1} = \sigma^2 \begin{bmatrix} \frac{\sigma^2}{nL} & \mathbf{0} \\ \mathbf{0} & \frac{2\,\mathbf{I}}{M^{2(l+t-1)}} \end{bmatrix} . \quad (23)$$

V. Simulations

In this section, we illustrate some simulation results to evaluate the effectiveness and applicability of the proposed unconditional ML approach. We consider generator matrix form representation of the space-time coding system in this paper [10]. In

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this representation the stream of coded complex M-PSK symbols are obtained by applying mapping function $\mathcal{M}$ to the following matrix multiplication

$$\mathbf{c}(k) = \mathcal{M}(\mathbf{u}(k) \cdot \mathbf{G} \quad \mathrm{mod} M)) \tag{24}$$

where $\mathbf{u}(k) = [s(lk+t-1), ..., s(lk-t)]^T$ and $\mathbf{G}$ is the generator matrix with n columns and $l+s$ rows and $\mathcal{M}$ is a mapping function that maps integer values to the M-PSK symbols, $\mathcal{M}(x) = \exp(2\pi jx/M)$.

A space-time encoder example shown in Fig. 2 is considered with $n=2, t=3$ and generator matrix,

$$\mathbf{G} = \begin{bmatrix} 2 & 0 \\ 1 & 0 \\ 0 & 2 \\ 0 & 1 \\ 2 & 2 \end{bmatrix}$$

This example would be the 8 state code with the systematic depicted in Fig. 2. In this particular example, the coded 8-PSK symbols are generated from $[s(2k+1), s(2k), s(2k-3)]$ are transmitted over the first antenna, whereas the coded 8-PSK symbols obtained from $[s(2k-1), s(2k-2), s(2k-3)]$ are transmitted over the second antenna simultaneously. The coded symbols are then transmitted through quasi-static fading channel matrix.

The performance of the proposed method was evaluated as a function of SNR (signal to noise ratio) based on the Monte Carlo simulations (200 trials per SNR point). In each trial, the norm of the estimation error from unconditional ML for the channel parameters were recorded. In Fig. 3, we have plotted the estimation error norm obtained from conditional and unconditional ML for the channel parameters as well as the corresponding approximate CRB.

Based on the simulations we made the following observations:

i) The unconditional and conditional ML approaches perform almost identically for high SNR values.
ii) Since the unconditional cost function is dominated by only one term for high SNR, it results in exactly the same cost function as one would obtain for conditional ML estimation of $\boldsymbol{\theta}$. It is therefore expected that both conditional and unconditional cost functions yield similar estimates of $\boldsymbol{\theta}$ at high SNR.

VI. Conclusions

In this paper, we presented unconditional ML approach to the problem of blind estimation of channel parameters along with the space-time coded sequence. We derived iterative ML algorithm based on the unconditional signal model. Furthermore, the performance of the proposed algorithms is explored based on the derivation of approximate CRB. We also presented Monte Carlo simulations to verify the theoretically predicted estimator's performance.

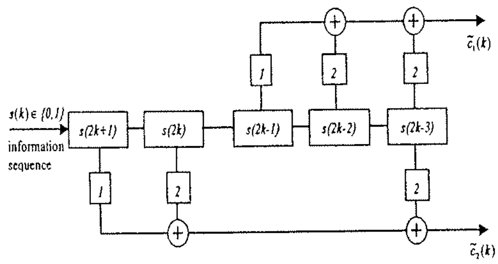

Fig. 2. 8-state space-time coding system model

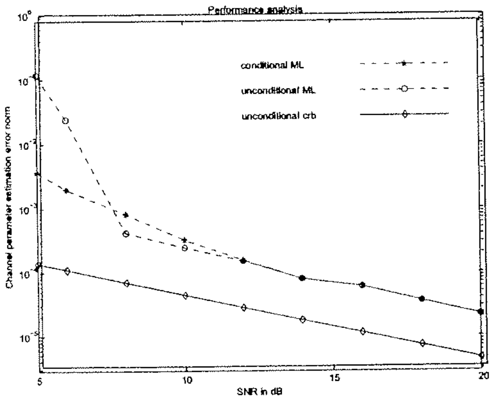

Fig. 3. Channel Matrix Estimation Error Norm

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PROCEEDINGS

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Space–Time Coded Multiple MSK

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Abstract

Minimum Shift Keying (MSK) is a bandwidth– and power–efficient modulation scheme with its continuous phase and constant envelope properties. MSK is widely used in satellite communication channels where bandwidth–efficiency takes an important role on the cost of the communication system. A new technique that is used for reliable transmission of data over fading channels is known as space–time coding. Space–Time codes employ multiple transmit and receive antennas to ensure diversity. In this paper[1], the combination of these two techniques is presented. This new scheme, namely Space–Time Coded Multiple MSK, has both high bandwidth– and power–efficiency. A computer–based code–search algorithm has been developed to find codes with high error performance. The exact bit error probabilities for the proposed codes have been investigated over rapid fading channel. Simulation results show that the new codes outperform the given reference MSK codes. The exact error probability and simulation results are also shown to converge for high signal–to–noise ratios.

1 Introduction

Wireless channel has severe problems that do not allow high–speed reliable data communications. Some of these problems are the additive white Gaussian noise (AWGN), multipath fading and interchannel interference (ICI). Data rate at which reliable communications can take place is strictly limited by these destructive effects. For years, the main approach to the solution of this problem have been on the design of better codes that would ensure reliable communications at rates closer to the channel capacity. Recently, Tarokh *et. al.* [1, 2, 3] came up with a new coding approach exploiting the benefits of using multiple transmit and/or receive antennas, such as the increase in channel capacity. Channel capacity calculations for this case have been independently presented by Telatar [4] and by Foschini and Gans [5]. Another technique that is used in order to increase the error performance in fading channels is diversity. Diversity systems transmit replicas of data over independent channels to ensure that at least one less–attenuated copy arrives at the receiver. These independent channels may be in temporal, frequency and spatial domain. Diversity techniques can be implemented separately or together. In temporal diversity technique, a less–attenuated replica may be obtained in the temporal

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domain, where, in frequency diversity technique, diversity can be achieved using different frequency regions. Spatial diversity technique, however, uses multiple transmit and/or receive antennas to ensure independent channels in space. While the diversity schemes employing multiple receive antennas are widely implemented in the literature, using multiple transmit antennas and designing different codes for each transmit antenna by jointly considering the overall system error performance is a new approach.

Minimum Shift Keying (MSK) is a spectrally efficient modulation scheme with its constant envelope and continuous phase properties. Using synchronous demodulation and exploiting phase continuity, MSK has the same error probability as Binary Phase Shift Keying (BPSK). MSK trellis and signal constellation are presented in Figure 1. Since it involves an inherent coding due to the phase continuity,

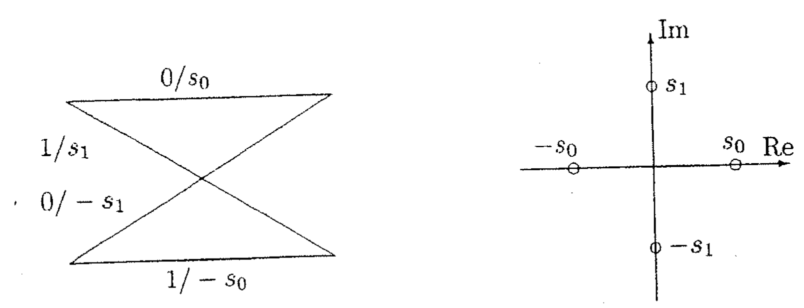


Figure 1: MSK trellis and signal constellation

MSK modulation can be represented in terms of a convolutional encoder followed by a memoryless mapper [8](Figure 2).

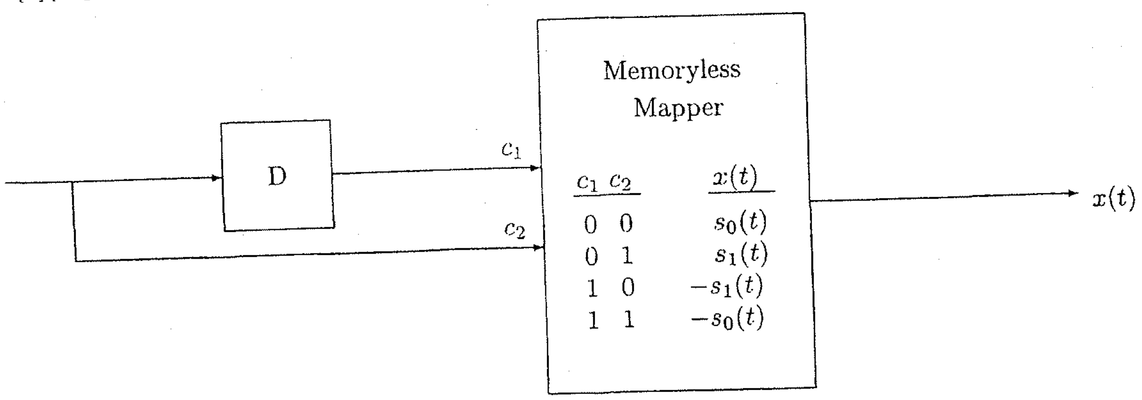


Figure 2: Coded-modulation representation of MSK

Until now, Space–Time codes have been applied to PSK and QAM modulation schemes, where, in this paper, we apply to MSK modulation to exploit the additional bandwidth–efficiency as a result of the continuous phase. The code search is performed on the set of the multiple trellis coded MSK channel symbol pairs, but can be easily extended to sets with more than two consecutive MSK symbols. Optimum and near–optimum codes for 2–, 4– and 8– state trellises are obtained. The exact pairwise error probability curves for the proposed codes are obtained using the technique given for trellis–coded modulation by Cavers and Ho [7] and extended to space–time codes by Uysal and Georghiades [6]. Based on this approach, the bit error probabilities of the proposed codes are also estimated for rapid fading channels.

The organization of the paper is as follows. We describe the system model used in this study in Section II. In Section III, the development of the code–search algorithm is presented and a general block diagram is given. In Section IV, exact bit error probability calculations for our codes are derived and in Section V, simulation results are presented. Finally, the conclusions are given in Section VI.

2 System Model

The block diagram of the considered system is given in Figure 3. The input to the space–time encoder

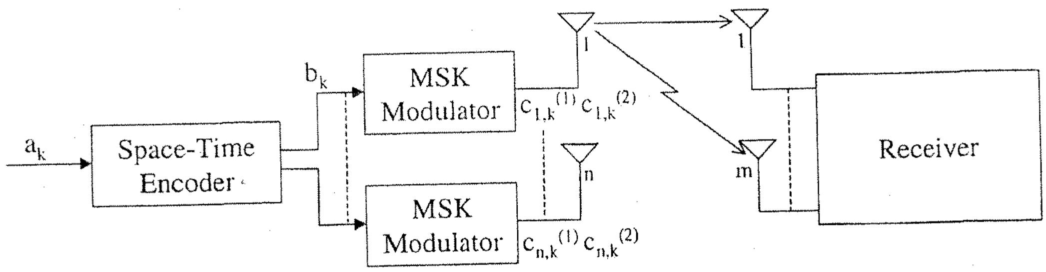


Figure 3: Space–Time coded MSK System Block Diagram

is a sequence of binary digits denoted by $\mathbf{a} = (a_1, a_2, ..., a_k, ...)$ and the outputs are sequences of coded binary digits. Each output sequence $\mathbf{b}$ is MSK modulated to arrive at one channel symbol pair per data bit resulting in a rate 1/2 multiple trellis coded MSK denoted by $\mathbf{c}_{i,k} = (c_{1,1}^{(1)}, c_{1,1}^{(2)}, ..., c_{i,k}^{(1)}, c_{i,k}^{(2)}, ...)$ where $1 \leq i \leq n$ and $1 \leq k \leq L$. n and L stand for the number of transmit antennas and frame length, respectively. Each channel symbol pair $c_{i,k}^{(1)} c_{i,k}^{(2)}$ is transmitted simultaneously via transmit antenna i at the time interval $kT \leq t \leq (k+1)T$.

The channel gains $\alpha_{i,j}$, corresponding to the channel between transmit antenna i and receive antenna j, are assumed as independent samples of complex Gaussian random variables with zero-mean and variance 0.5 per dimension. This is equivalent to the assumption that signals transmitted from different antennas undergo independent fades. The temporal characteristics of the channel gains can be modelled either by rapid flat fading, where the channel gains vary from one symbol interval to another, or by quasi–static flat fading, where the channel gains stay constant for a frame and vary independently from one frame to another.

The additive white Gaussian noise samples $n_{j,k}$ are, as usual, modelled as zero–mean complex Gaussian distributed random variables with variance $N_o/2$ per dimension.

2.1 Quasi–static Flat Fading Case

The received signal at receive antenna j in time interval k for the quasi-static flat fading case can be given by

$$d_{j,k}^{(p)} = \sum_{i=1}^{n} \alpha_{i,j} c_{i,k}^{(p)} \sqrt{E_S} + n_{j,k}^{(p)} \quad , p = 1, 2, j = 1, 2, ..., m \tag{1}$$

where E_S is defined as the average energy per symbol. Assuming ideal channel state information (CSI), the probability of transmitting $\mathbf{c}$ and deciding in favor of the erroneous sequence $\mathbf{e}$ at the decoder is upper bounded by

$$P(\mathbf{c} \rightarrow \mathbf{e} | \alpha_{i,j}, i = 1, 2, ..., n, \ j = 1, 2, ..., m) \leq \exp(-d^2(c, e) E_S / 4N_o) \tag{2}$$

where $d^2(\mathbf{c},\mathbf{e})$ is defined as the Euclidean distance between the correct and erroneous sequences, $\mathbf{c}$ and $\mathbf{e}$. $d^2(\mathbf{c},\mathbf{e})$ can be calculated as

$$d^2(\mathbf{c},\mathbf{e}) = \sum_{j=1}^{m}\sum_{k=1}^{L}\sum_{p=1}^{2}\left|\sum_{i=1}^{n}\alpha_{i,j}(c_{i,k}^{(p)} - e_{i,k}^{(p)})\right|^2 . \quad (3)$$

After simple manipulations, we can arrive at the pairwise error probability upper bound for space–time codes over quasi–static flat Rayleigh fading channels, [1]:

$$P(\mathbf{c}\rightarrow\mathbf{e}) \leq \left(\prod_{i=1}^{r}\lambda_i\right)^{-m}(E_S/4N_o)^{-rm} \quad (4)$$

where r and λ_i denote the rank and i^{th} non–zero eigenvalue of the path difference matrix $A(\mathbf{c},\mathbf{e})$. $A(\mathbf{c},\mathbf{e})$ can be easily computed by its square–root $B(\mathbf{c},\mathbf{e})$:

$$A(\mathbf{c},\mathbf{e}) = B(\mathbf{c},\mathbf{e}).B^*(\mathbf{c},\mathbf{e}) \quad (5)$$

$$B(\mathbf{c},\mathbf{e}) = \begin{pmatrix} e_{1,1}^{(1)} - c_{1,1}^{(1)} & e_{1,1}^{(2)} - c_{1,1}^{(2)} & \cdots & \cdots & e_{1,L}^{(1)} - c_{1,L}^{(1)} & e_{1,L}^{(2)} - c_{1,L}^{(2)} \\ e_{2,1}^{(1)} - c_{2,1}^{(1)} & e_{2,1}^{(2)} - c_{2,1}^{(2)} & \cdots & \cdots & e_{2,L}^{(1)} - c_{2,L}^{(1)} & e_{2,L}^{(2)} - c_{2,L}^{(2)} \\ e_{3,1}^{(1)} - c_{3,1}^{(1)} & e_{3,1}^{(2)} - c_{3,1}^{(2)} & \cdots & \cdots & e_{3,L}^{(1)} - c_{3,L}^{(1)} & e_{3,L}^{(2)} - c_{3,L}^{(2)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & (\ldots) \\ e_{n,1}^{(1)} - c_{n,1}^{(1)} & e_{n,1}^{(2)} - c_{n,1}^{(2)} & \cdots & \cdots & e_{n,L}^{(1)} - c_{n,L}^{(1)} & e_{n,L}^{(2)} - c_{n,L}^{(2)} \end{pmatrix} \quad (6)$$

with $(.)^*$ representing the complex conjugate operation. From (4), to minimize the pairwise error probability, one should first maximize the minimum rank of the path difference matrix $A(\mathbf{c},\mathbf{e})$ (**Rank Criterion**), then maximize the minimum sum of determinants of all the principal $r * r$ cofactors of A (**Determinant Criterion**) for all possible error events.

2.2 Rapid Flat Fading Case

Similarly, the received signal at receive antenna j at time interval k for the rapid flat fading can be given by

$$d_{j,k}^{(p)} = \sum_{i=1}^{n}\alpha_{i,j,k}^{(p)}c_{i,k}^{(p)}\sqrt{E_S} + n_{j,k}^{(p)} \quad ,, p = 1,2, j = 1,2,...,m \quad . \quad (7)$$

This time, the path gains $\alpha_{i,j,k}^{(p)}$ for $k = 1,2,...,L$, $i = 1,2,...,n$, $j = 1,2,...,m$ can be modelled as independent samples of a complex Gaussian random variable with zero–mean and variance 0.5 per dimension. This assumption corresponds to rapid Rayleigh flat fading. Also, $n_{j,k}^{(p)}$ are samples of independent zero–mean complex Gaussian random variables with variance $N_o/2$ per dimension. Pairwise error probability upper bound can be obtained as

$$P(\mathbf{c}\rightarrow\mathbf{e}|\alpha_{i,j,k}^{(p)}, i = 1,2,...,n,\ j = 1,2,...,m,\ k = 1,2,...,L,\ p = 1,2) \leq \exp(-d^2(\mathbf{c},\mathbf{e})E_S/4N_o) \quad (8)$$

where $d^2(\mathbf{c},\mathbf{e})$ can be defined as

$$d^2(\mathbf{c},\mathbf{e}) = \sum_{j=1}^{m}\sum_{k=1}^{L}\sum_{p=1}^{2}\left|\sum_{i=1}^{n}\alpha_{i,j,k}^{(p)}(c_{i,k}^{(p)} - e_{i,k}^{(p)})\right|^2 . \quad (9)$$

Averaging with respect to the Rayleigh distribution, we arrive at

$$P(\mathbf{c}\rightarrow\mathbf{e}) \leq \prod_{k=1}^{L}\prod_{p=1}^{2}\left(1 + |c_k^{(p)} - e_k^{(p)}|^2\frac{E_S}{4N_o}\right)^{-m} . \quad (10)$$

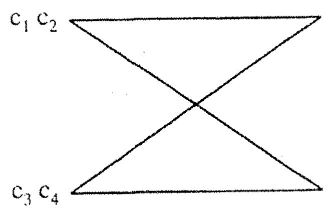
Let $\nu(\mathbf{c},\mathbf{e})$ denote the set of time instances such that $|c_k^{(p)} - e_k^{(p)}| \neq 0$ and let $|\nu(\mathbf{c},\mathbf{e})|$ denote the number of elements of $\nu(\mathbf{c},\mathbf{e})$. Then (10) can be rewritten as

$$P(c \to e) \leq \prod_{k,p\in\nu(c,e)} \left(|c_k^{(p)} - e_k^{(p)}|^2 \frac{E_S}{4N_o} \right)^{-m}. \qquad (11)$$

To minimize the pairwise error probability, we should first maximize $|\nu(\mathbf{c},\mathbf{e})|$ (**Distance Criterion**), then maximize the minimum of the products $\prod_{k,p\in\nu(c,e)} |c_k^{(p)} - e_k^{(p)}|^2$ taken over distinct codewords $\mathbf{c}$ and e where $|c_k^{(p)} - e_k^{(p)}|^2 = \sum_{i=1}^{n} |c_{i,k}^{(p)} - e_{i,k}^{(p)}|^2$ (**Product Criterion**).

3 Code Search Algorithm

A computer–based code search algorithm has been developed to search for space–time coded MSK schemes that have good error performance over quasi–static and rapid fading channels. Our search set is limited to trellis coded MSK symbol pairs, but can be easily extended to sets with more than two consecutive MSK symbols. 2–, 4–, and 8– state space–time coded MSK schemes that ensure full diversity for both quasi–static and rapid flat fading channels are found. Search results are presented in Figures 4–6.



	Quasi-static F.		Rapid F.	
	Tx1	Tx2	Tx1	Tx2
c_1	$s_0\ s_0$	$s_1\ -s_1$	$s_1\ -s_1$	$s_1\ -s_1$
c_2	$s_1\ -s_0$	$s_0\ s_1$	$s_0\ s_1$	$s_0\ s_1$
c_3	$-s_0\ -s_1$	$-s_1\ s_0$	$-s_1\ s_0$	$-s_1\ s_0$
c_4	$-s_1\ s_0$	$-s_0\ -s_0$	$-s_0\ -s_0$	$-s_0\ -s_0$

Figure 4: 2–state space–time coded multiple MSK schemes

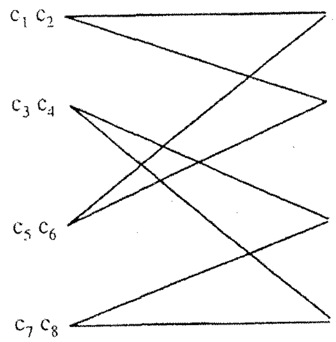
While the 2– and 4–state codes are fully optimized for the two criteria for both type of channels, 8–state codes are only optimized for the first criteria.This is because of the increase in processing time for the second criteria over quasi–static and rapid fading channels with the growing number of states.

The proposed codes have the following parameters for the quasi–static (QF) and rapid (RF) fading channels.

	QF Rank C.	QF Determinant C.	RF Distance C.	RF Product C.
2 State	2	64	4	1024
4 State	2	128	6	36864
8 State	2	96	7	6912

4 Exact Error Probability Evaluation

The exact expressions for the bit error probabilities of the proposed codes under rapid fading has been evaluated by the technique given for trellis–coded modulation by Cavers and Ho [7] and extended to space–time codes by Uysal and Georghiades [6]. In this technique, it is assumed that the received signal



	Quasi-static F.		Rapid F.	
	Tx1	Tx2	Tx1	Tx2
c_1	$s_0\ s_0$	$s_1\ -s_1$	$s_0\ s_0$	$s_1\ -s_1$
c_2	$s_1\ -s_0$	$s_0\ s_1$	$s_1\ -s_0$	$s_0\ s_1$
c_3	$-s_0\ -s_0$	$-s_1\ s_1$	$-s_1\ s_1$	$-s_1\ s_1$
c_4	$-s_1\ s_0$	$-s_0\ -s_1$	$-s_0\ -s_1$	$-s_0\ -s_1$
c_5	$-s_0\ -s_1$	$-s_1\ s_0$	$-s_0\ -s_1$	$-s_1\ s_0$
c_6	$-s_1\ s_1$	$-s_0\ -s_0$	$-s_1\ s_1$	$-s_0\ -s_0$
c_7	$s_0\ s_1$	$s_1\ -s_0$	$s_1\ -s_0$	$s_1\ -s_0$
c_8	$s_1\ -s_1$	$s_0\ s_0$	$s_0\ s_0$	$s_0\ s_0$

Figure 5: 4–state space–time coded multiple MSK schemes

is passed through a matched filter having an impulse response with a scaling factor of $1/\sqrt{N_o}$. The received signal is

$$d_{j,k}^{(p)} = \sum_{i=1}^{n} \alpha_{i,j,k}^{(p)} x_{i,k}^{(p)} + n_{j,k}^{(p)} \tag{12}$$

where $x_{i,k}^{(p)}$ denotes the p^{th} transmitted symbol from the i^{th} transmit antenna in the k^{th} time interval. For rapid fading channels, the maximum likelihood receiver chooses the sequence $\mathbf{x}$ which minimizes

$$m(\mathbf{d},\mathbf{x}) = \sum_{j=1}^{m}\sum_{k=1}^{L}\sum_{p=1}^{2} \left| d_{j,k}^{(p)} - \sum_{i=1}^{n} \alpha_{i,j,k}^{(p)} x_{i,k}^{(p)} \right|^2 . \tag{13}$$

Then, the probability of the receiver to decode $\hat{\mathbf{x}}$ when, in fact, $\mathbf{x}$ was transmitted can be given by

$$P(\mathbf{x} \rightarrow \hat{\mathbf{x}}) = Pr[m(\mathbf{d},\hat{\mathbf{x}}) \leq m(\mathbf{d},\mathbf{x})] = Pr[D \leq 0] \tag{14}$$

where D is defined as

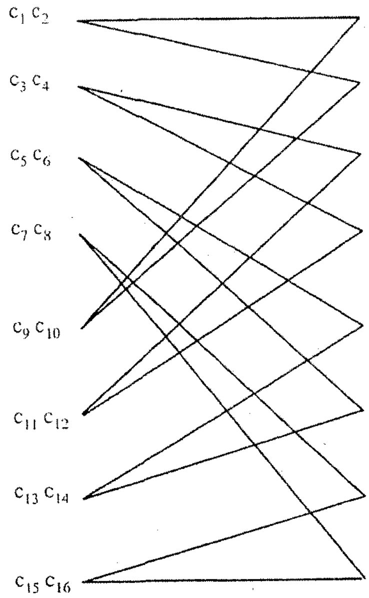
$$D = \sum_{j=1}^{m}\sum_{k=1}^{L}\sum_{p=1}^{2} D_{j,k}^{(p)} = \sum_{j=1}^{m}\sum_{k=1}^{L}\sum_{p=1}^{2} \left(m(d_{j,k}^{(p)}, \hat{x}_{i,k}^{(p)}) - m(d_{j,k}^{(p)}, x_{i,k}^{(p)}) \right) . \tag{15}$$

After some simplification, $D_{j,k}^{(p)}$ can be obtained by

$$D_{j,k}^{(p)} = \left| \sum_{i=1}^{n} \alpha_{i,j,k}^{(p)} (\hat{x}_{i,k}^{(p)} - x_{i,k}^{(p)}) \right|^2 - \sum_{i=1}^{n} \alpha_{i,j,k}^{(p)} (\hat{x}_{i,k}^{(p)} - x_{i,k}^{(p)}) n_{j,k}^{(p)*} - \sum_{i=1}^{n} \alpha_{i,j,k}^{(p)*} (\hat{x}_{i,k}^{(p)*} - x_{i,k}^{(p)*}) n_{j,k}^{(p)} \tag{16}$$

which is similar to the well–known quadratic equation $D_{j,k}^{(p)} = A_{j,k}^{(p)} |r_{j,k}^{(p)}|^2 + B_{j,k}^{(p)} |v_{j,k}^{(p)}|^2 + C_{j,k}^{(p)} r_{j,k}^{(p)} v_{j,k}^{(p)*} + C_{j,k}^{(p)*} r_{j,k}^{(p)*} v_{j,k}^{(p)}$, where $r_{j,k}^{(p)} = n_{j,k}^{(p)}$, $v_{j,k}^{(p)} = \sum_{i=1}^{n} \alpha_{i,j,k}^{(p)} (\hat{x}_{i,k}^{(p)} - x_{j,k}^{(p)})$, and $A_{j,k}^{(p)} = 0, B_{j,k}^{(p)} = 1, C_{j,k}^{(p)} = -1$. Because of the spatial and temporal independency, the characteristic function of D can be given as

$$\Phi_D(s) = \prod_{j=1}^{m}\prod_{k=1}^{L}\prod_{p=1}^{2} \varphi_{j,k}^{(p)}(s) \tag{17}$$



	Quasi-static F.		Rapid F.	
	Tx1	Tx2	Tx1	Tx2
c_1	$s_0\ s_0$	$s_0\ s_0$	$s_1\ -s_1$	$s_0\ s_0$
c_2	$s_0\ s_1$	$s_1\ -s_0$	$s_0\ s_1$	$s_1\ -s_0$
c_3	$-s_0\ -s_1$	$-s_1\ s_0$	$-s_1\ s_0$	$-s_1\ s_0$
c_4	$-s_0\ -s_0$	$-s_0\ -s_0$	$-s_0\ -s_0$	$-s_0\ -s_0$
c_5	$s_1\ -s_1$	$s_0\ s_0$	$s_0\ s_0$	$s_0\ s_0$
c_6	$s_1\ -s_0$	$s_1\ -s_0$	$s_1\ -s_0$	$s_1\ -s_0$
c_7	$-s_1\ s_0$	$-s_1\ s_0$	$-s_0\ s_1$	$-s_1\ s_0$
c_8	$-s_1\ s_1$	$-s_0\ -s_0$	$-s_1\ s_1$	$-s_0\ -s_0$
c_9	$s_1\ -s_1$	$s_0\ s_0$	$s_0\ s_0$	$s_1\ -s_1$
c_{10}	$s_1\ -s_0$	$s_1\ -s_0$	$s_1\ -s_0$	$s_0\ s_1$
c_{11}	$-s_1\ s_0$	$-s_1\ s_0$	$-s_0\ -s_1$	$-s_1\ s_0$
c_{12}	$-s_1\ s_1$	$-s_0\ -s_0$	$-s_1\ s_1$	$-s_1\ s_1$
c_{13}	$s_0\ s_0$	$s_0\ s_0$	$s_1\ -s_1$	$s_0\ s_0$
c_{14}	$s_0\ s_1$	$s_1\ -s_0$	$s_0\ s_1$	$s_1\ -s_0$
c_{15}	$-s_0\ -s_1$	$-s_1\ s_0$	$-s_1\ s_0$	$-s_0\ -s_1$
c_{16}	$-s_0\ -s_0$	$-s_0\ -s_0$	$-s_0\ -s_0$	$-s_0\ -s_0$

Figure 6: 8–state space–time coded multiple MSK schemes

where $\varphi_{j,k}^{(p)}(s)$ is defined as the two-sided Laplace transform of the pdf of the Gaussian distributed random variable $D_{j,k}^{(p)}$. $\varphi_{j,k}^{(p)}(s)$ is given by

$$\varphi_{j,k}^{(p)}(s) = \frac{p_{1,j,k}^{(p)} p_{2,j,k}^{(p)}}{(s - p_{1,j,k}^{(p)})(s - p_{2,j,k}^{(p)})} \tag{18}$$

with

$$\begin{bmatrix} p_{1,j,k}^{(p)} \\ p_{2,j,k}^{(p)} \end{bmatrix} = w_{j,k}^{(p)} \mp \sqrt{(w_{j,k}^{(p)})^2 + \frac{1}{4(\mu_{rr}\mu_{vv} - |\mu_{rv}|^2)(|C_{j,k}^{(p)}|^2 - A_{j,k}^{(p)} B_{j,k}^{(p)})}}$$

and

$$w_{j,k}^{(p)} = \frac{A_{j,k}^{(p)}\mu_{rr} + B_{j,k}^{(p)}\mu_{vv} + C_{j,k}^{(p)}\mu_{rv} + C_{j,k}^{(p)*}\mu_{rv}^{*}}{4(\mu_{rr}\mu_{vv} - |\mu_{rv}|^2)(|C_{j,k}^{(p)}|^2 - A_{j,k}^{(p)} B_{j,k}^{(p)})} .$$

μ_{rr},μ_{vv} and μ_{rv} denote the variances of random variables $r_{j,k}^{(p)}$ and $v_{j,k}^{(p)}$, and covariance between them, respectively. Assuming ideal CSI, $\Phi_D(s)$ can be expressed as

$$\Phi_D(s) = \left[\prod_{k,p\in\nu(x,\hat{x})} \left(\frac{E_S}{4N_o} \sum_{i=1}^{n} \left| \hat{x}_{i,k}^{(p)} - x_{i,k}^{(p)} \right|^2 \right) \right]^{-m} \left[\prod_{k,p\in\nu(x,\hat{x})} \frac{-1}{16(s - p_{1,j,k}^{(p)})(s - p_{2,j,k}^{(p)})} \right]^{m} \tag{19}$$

where

$$\begin{bmatrix} p_{1,j,k}^{(p)} \\ p_{2,j,k}^{(p)} \end{bmatrix} = \frac{1}{4} \mp \sqrt{\frac{1}{16} + \frac{1}{4\frac{E_S}{N_o} \sum_{i=1}^{n} \left| \hat{x}_{i,k}^{(p)} - x_{i,k}^{(p)} \right|}} .$$

The probability of error can be expressed in terms of the Laplace transform of D as

$$P(\mathbf{x} \rightarrow \hat{\mathbf{x}}) = \Pr[D \leq 0] = -\sum Residue\,[\Phi_D(s)/s]_{Right\,Plane\,Poles} \quad (20)$$

Up to now, we have presented the exact expression for the pairwise error probability which is usually not enough to estimate the error performance of the code. Therefore, we use the transfer function approach. We estimate the bit error rate of the code by considering only some error events of lengths up to a pre-determined number. This estimation can be given by the equation:

$$P_b = \frac{1}{M} \sum_{\mathbf{x} \neq \hat{\mathbf{x}}} q(\mathbf{x} \rightarrow \hat{\mathbf{x}}) P(\mathbf{x} \rightarrow \hat{\mathbf{x}}) \quad (21)$$

where M is the number of input bits per trellis transition and $q(\mathbf{x} \rightarrow \hat{\mathbf{x}})$ is the number of bit errors associated with the considered error event.

5 Simulation Results

Computer simulations have been obtained for the proposed codes over quasi–static and rapid flat fading channels. Error performances of the proposed rate 1/2 coded multiple MSK codes have been investigated over a communication system employing two transmit and one receive antennas. Channel state information is assumed to be known perfectly at the receiver (ideal CSI). Frame size is taken as 100 trellis transitions which corresponds to 200 channel symbols. Simulation results are presented in Figures 7–9.

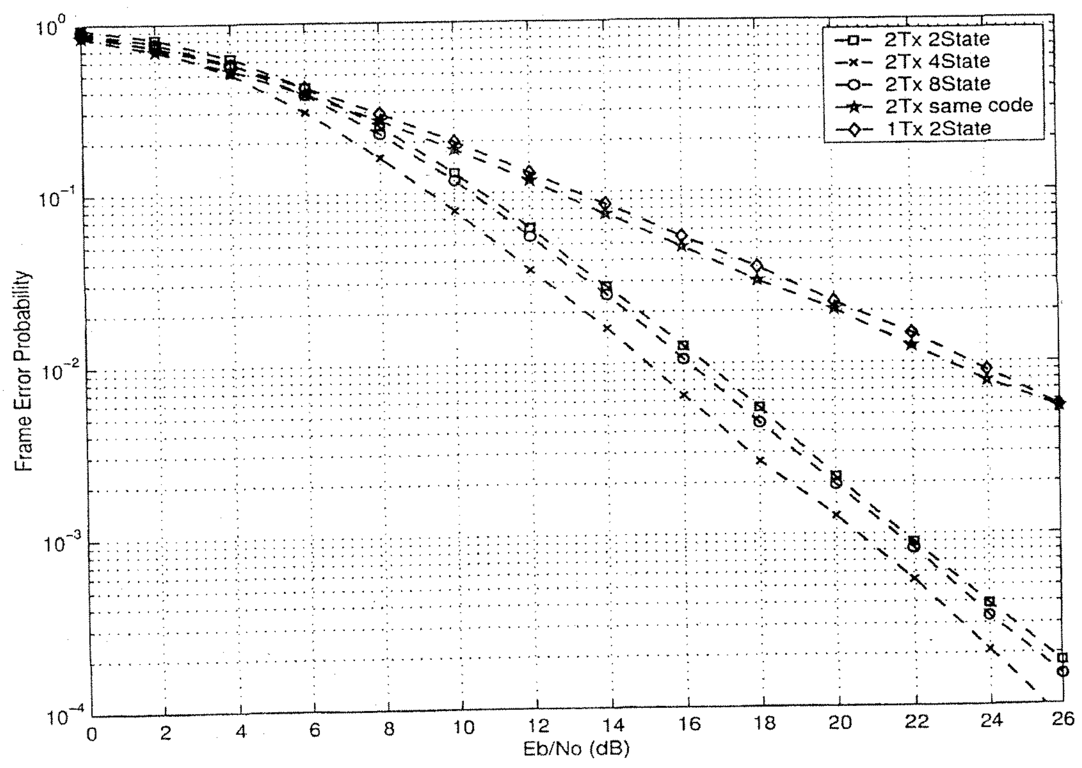


Figure 7: Frame error probabilities of the proposed multiple space–time coded MSK schemes for quasi–static flat Rayleigh fading channel

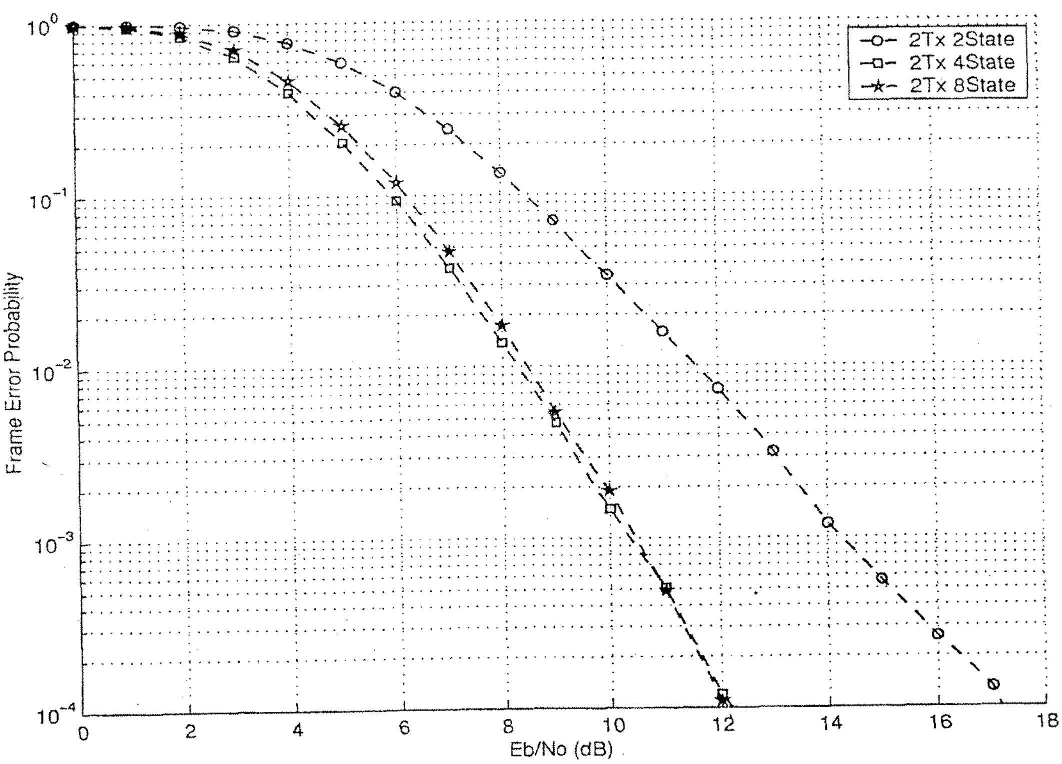


Figure 8: Frame error probabilities of the proposed multiple space–time MSK codes for the rapid flat Rayleigh fading channel

Reference space–time and trellis coded MSK schemes have been used to emphasize the benefits of using multiple transmit antennas and employing proper coding for them. First reference scheme employs one transmit and one receive antennas. It is designed to have optimal performance over quasi-static flat fading channels. The second one employs two transmit and one receive antennas but two antennas transmit the same channel symbols simultaneously.

It is easily seen from Figure 7 that, codes that guarantee rank 1 have poor error performance compared to the others with rank 2. Full rank codes have close error performances, while their order is determined by the value of the determinant criterion.

According to the simulation results over rapid fading channels, codes with higher distance parameter have better error performance for high signal–to–noise ratios. To achieve a frame error probability of 10^{-4}, the 2–state code requires a signal–to–noise ratio of $17dB$, where 4– and 8–state codes need only $12dB$.

For both type of channels, while the primary criterion effects the slope of the error curves, secondary criterion effects their values.

According to Figure 9, it is proven that the bit error probabilities of space–time codes are well–estimated by using the exact error probability technique. Simulation and analytical bit error probability curves converge at high SNR values as seen from Figure 9.

6 Conclusions

In this paper, the space–time coding technique is applied to MSK modulation technique to achieve a power– and bandwidth–efficient communication system. Optimum and near–optimum codes for both quasi-static and rapid fading channels have been designed. Error performances of the proposed codes

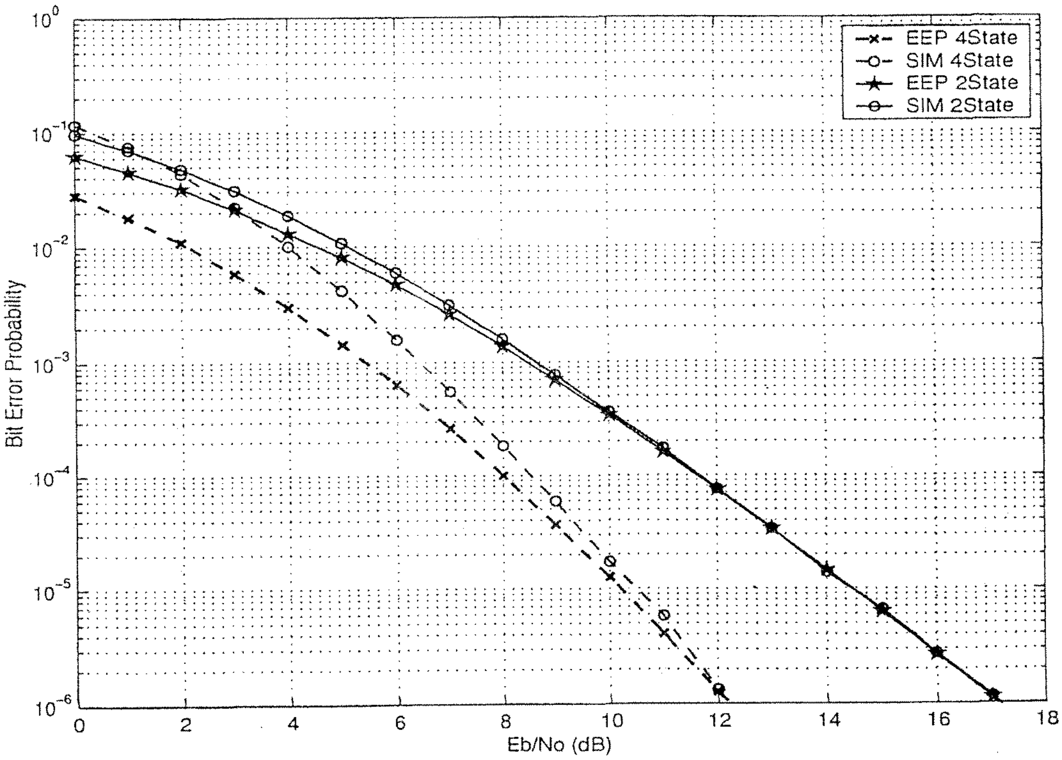


Figure 9: Bit error probabilities of the 2– and 4–state multiple space–time coded MSK schemes for the rapid flat Rayleigh fading channel

have been investigated analytically and by means of computer simulation. Exact error probability calculation technique is applied to estimate the error probabilities of the new codes over rapid flat Rayleigh fading channel. It is shown that our codes outperform the considered reference MSK systems.

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POWER CONTROL FOR ORTHOGONAL SPACE–TIME CODING WITH MULTIPLE RECEIVE ANTENNAS

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Abstract - Performance of data communication systems over wireless channels is severely degraded by the multipath fading effects. Space–time codes exploit the channel capacity increase by using multiple transmit and/or receive antennas. This ensures space diversity without expanding the required transmission bandwidth. A type of space–time codes, namely orthogonal transmit diversity (OTD), has recently attracted much attention. This scheme has low complexity and can achieve maximum diversity. However, error performance of OTD systems decreases in the absence of perfect channel state information at the receiver. In this paper, we present an enhanced power control scheme for OTD systems with two transmit and multiple receive antennas. Simulation results have been presented to show the performance improvement.

Keywords - Orthogonal Transmit Diversity, Fading Channels

I INTRODUCTION

Wireless communication systems suffer from several destructive effects, such as the additive white Gaussian noise and the fading effect caused by the multipath structure of the communication channel. Data rate at which reliable communications can take place is strictly limited by these disturbances. At high mobility conditions, the performance of wireless communication system is severely degraded by the multipath fading effect.

A well–known technique used in order to increase the error performance of communication systems in fading channels is diversity. Diversity systems transmit replicas of data over independent channels to ensure that at least one less–attenuated copy arrives at the receiver. These independent channels may be formed in temporal, frequency and spatial domains resulting in time, frequency or space diversity which can be implemented separately or together. In temporal diversity technique, diversity can be achieved in time, while, in frequency diversity technique, independent channels can be formed over different frequency regions. Space diversity technique generally employs multiple transmit and/or receive antennas. These antennas are located at enough distance of each other to ensure that the formed channels are independent. Schemes employing multiple antennas and using signal processing based combination at the receiver are widely investigated in the literature. However, using multiple antennas at each mobile unit would result in an increase of the system cost. This is the first motivation to design schemes employing multiple antennas at the base station.

The first studies on this issue were done when Telatar [1] and Foschini and Gans [2] independently calculated the capacity of multi–antenna channels. Results of these works have shown that multi–antenna systems have higher capacities compared to that of the single antenna schemes. Then, Tarokh *et al.* [3, 4] came up with a new coding approach exploiting the benefits of using multiple transmit and/or receive antennas. This new scheme is known as space–time coding. Main idea of this technique is using multiple transmit antennas and designing different codes for each transmit antenna by jointly considering the overall system error performance. On the other hand, Alamouti [5] proposed orthogonal transmit diversity (OTD) scheme employing two transmit and M receive antennas. This scheme ensures full diversity over fading channels with low complexity.

II SYSTEM MODEL

A. Alamouti's Scheme

According to this scheme, for a communication system employing two transmit and one receive antennas, channel symbols s_0 and s_1 are transmitted over a period of two signalling intervals. It is assumed that channel gains (fading coefficients) remain constant over a period of two signalling intervals and vary independently from one period to the other. In the first interval, s_0 is transmitted from the first transmit antenna, while, s_1 is transmitted from the second. In the second signalling interval, the symbol $-s_1^*$ is transmitted from the first transmit antenna and s_0^* is transmitted from the second one. Thus, each channel symbol has been transmitted over the two independent channels. The received signals at the first and second intervals can be given as

$$\begin{aligned} r_0 &= h_0 s_0 + h_1 s_1 + n_0 \\ r_1 &= -h_0 s_1^* + h_1 s_0^* + n_1 \end{aligned} \tag{1}$$

with $(.)^*$ representing the complex conjugate operation. $h_0 = |h_0|e^{j\theta_0}$ and $h_1 = |h_1|e^{j\theta_1}$ denote the channel gains defined from the first and second transmit antennas to the receive antenna, respectively. These fading coefficients can be modelled by zero–mean, complex Gaussian distribution with variance 0.5 per dimension. h_0 and h_1 are assumed to be uncorrelated random variables. Noise samples n_0 and n_1 are also modelled as zero–mean, complex Gaussian random variables with variance $N_0/2$ per dimension and are assumed to be statistically independent for different channels and subsequent signalling intervals.

Assuming ideal channel state information (CSI), the received signals r_0 and r_1 are combined as follows,

$$\begin{aligned}\tilde{s_0} &= h_0^* r_0 + h_1 r_1^* \\ &= (|h_0|^2 + |h_1|^2)s_0 + h_0^* n_0 + h_1 n_1^* \\ \tilde{s_1} &= h_1^* r_0 - h_0 r_1^* \\ &= (|h_0|^2 + |h_1|^2)s_1 - h_0 n_1^* + h_1^* n_0\end{aligned} \tag{2}$$

where $\tilde{s_0}$ and $\tilde{s_1}$ are the estimated values for s_0 and s_1. These estimates are used for the decoding of the received signals due to the maximum likelihood decision rule. The estimated value for a transmitted channel symbol (s_0 or s_1) depends only on itself and some noise components.

It can be seen from (2) that if one of the channels is effected by severe fading ($h_0 \approx 0$), the replica of the symbol transmitted over another channel could improve the error performance. The performance of orthogonal transmit diversity systems decreases in the absence of perfect CSI. Erroneous estimate of the channel gains can be given as

$$\begin{aligned}\tilde{h}_0 &= h_0 + \epsilon_0 \\ \tilde{h}_1 &= h_1 + \epsilon_1.\end{aligned} \tag{3}$$

In (3), channel estimation errors ϵ_0 and ϵ_1 are assumed to have zero–mean complex Gaussian distribution with variance σ_h^2. The quality of the estimation process can be given by the signal–to–estimation error of channel fading parameters ratio (SECR) which is given by σ_s^2/σ_h^2. σ_s^2 stands for the average signal energy of the constellation used. Receiver uses the erroneous channel estimates $\tilde{h}_0$ and $\tilde{h}_1$ for the combination process. Therefore, transmitted symbols are then estimated as

$$\begin{aligned}\tilde{s}_0 =& \tilde{h}_0^* r_0 + \tilde{h}_1 r_1^* \\ =& (h_0^* + \epsilon_0^*)r_0 + (h_1 + \epsilon_1)r_1^* \\ =& (|h_0|^2 + |h_1|^2 + h_0\epsilon_0^* + h_1^*\epsilon_1)s_0 \\ &+ (h_1\epsilon_0^* - h_0^*\epsilon_1)s_1 \\ &+ (h_0^* + \epsilon_0^*)n_0 + (h_1 + \epsilon_1)n_1^* \\ \tilde{s}_1 =& \tilde{h}_1^* r_0 - \tilde{h}_0 r_1^* \\ =& (h_1^* + \epsilon_1^*)r_0 - (h_0 + \epsilon_0)r_1^* \\ =& (|h_0|^2 + |h_1|^2 + h_1\epsilon_1^* + h_0^*\epsilon_0)s_1 \\ &+ (h_0\epsilon_1^* - h_1^*\epsilon_0)s_0 \\ &+ (h_1^* + \epsilon_1^*)n_0 - (h_0 + \epsilon_0)n_1^*.\end{aligned} \tag{4}$$

According to (4), the estimates include a dominant signal term, an intersymbol interference term and noise terms. At high signal–to–noise ratios (SNR), intersymbol interference becomes the primary effect degrading the error performance. This term causes an error floor which is independent of the SNR value.

B. Power Control Scheme for Single Receive Antenna

To increase the error performance of orthogonal transmit diversity systems employing two transmit and one receive antennas in the absence of ideal CSI, Fan *et al.* [6] have presented a power control scheme. According to this scheme, the signals to be transmitted from the first and second transmit antennas are multiplied by constants a and b, respectively. In order to keep the average transmit power constant, power control coefficients a and b are chosen to satisfy $a^2 + b^2 = 1$. At the receiver, each received signal is multiplied by the corresponding power control constant. Power controlled estimation gives

$$\begin{aligned}\tilde{s}_0 =& a\tilde{h}_0^* r_0 + b\tilde{h}_1 r_1^* \\ =& a(h_0^* + \epsilon_0^*)r_0 + b(h_1 + \epsilon_1)r_1^* \\ =& (a^2|h_0|^2 + b^2|h_1|^2 + a^2 h_0\epsilon_0^* + b^2 h_1^*\epsilon_1)s_0 \\ &+ ab(h_1\epsilon_0^* - h_0^*\epsilon_1)s_1 \\ &+ a(h_0^* + \epsilon_0^*)n_0 + b(h_1 + \epsilon_1)n_1^* \\ \tilde{s}_1 =& b\tilde{h}_1^* r_0 - a\tilde{h}_0 r_1^* \\ =& b(h_1^* + \epsilon_1^*)r_0 - a(h_0 + \epsilon_0)r_1^* \\ =& (a^2|h_0|^2 + b^2|h_1|^2 + b^2 h_1\epsilon_1^* + a^2 h_0^*\epsilon_0)s_1 \\ &+ ab(h_0\epsilon_1^* - h_1^*\epsilon_0)s_0 \\ &+ b(h_1^* + \epsilon_1^*)n_0 - a(h_0 + \epsilon_0)n_1^*.\end{aligned} \tag{5}$$

The selection of the power control parameters depends on the channel gains of the corresponding signalling intervals. If the gain of the first channel is higher than the second ($|h_0| > |h_1|$), power control parameter a is chosen greater than b to concentrate the transmit power to the channel with higher gain (better channel). The transmitter side has to know which channel has better characteristics over this interval. This information is sent from the receiver to the transmitter by a feedback channel. This feedback channel is assumed to be error free.

This scheme improves the signal–to–noise ratio of the received signal. Moreover, it can increase the signal–to–interference ratio (SIR). As an example, if the power control parameters in (5) are chosen as $a = 1$ and $b = 0$, the intersymbol interference terms will disappear and the communication system will be equivalent to a selective diversity system.

C. Power Control Scheme for Multiple Receive Antennas

In this paper, the power control scheme proposed by Fan *et al.* for two transmit antennas and one receive antenna is extended to the cases where the receiver is equipped with two and three antennas. The complexity of the power control scheme increases for the multiple receive antennas case. The increasing number of independent channels used makes the power control parameter selection much more difficult. Channel symbol estimates $\tilde{s}_0$ and $\tilde{s}_1$ have been expressed for these schemes and computer simulations have been performed for the proposed power control schemes for different values of SNR and SECR. The considered schemes are given in Figure 1.

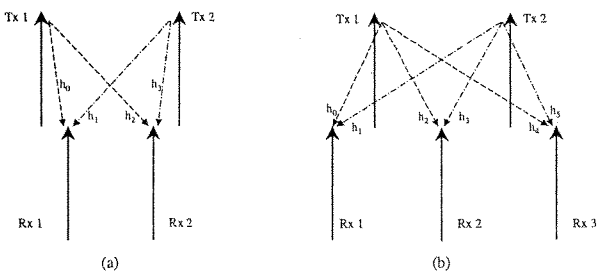

Figure 1: OTD scheme employing (a) two receive antennas (b) three receive antennas

For the OTD system employing two receive antennas, the channel gains between the two transmit and two receive antennas are shown as h_0, h_1, h_2 and h_3, as depicted in Figure 1. If the symbols transmitted from the first and second transmit antennas are weighted by the parameters a and b, respectively, signals received by the first and second receive antennas in the subsequent signalling intervals can be given by

$$\begin{aligned} r_0 &= a h_0 s_0 + b h_1 s_1 + n_0 \\ r_1 &= - a h_0 s_1^* + b h_1 s_0^* + n_1 \end{aligned} \tag{6}$$

and

$$\begin{aligned} r_2 &= a h_2 s_0 + b h_3 s_1 + n_2 \\ r_3 &= - a h_2 s_1^* + b h_3 s_0^* + n_3 \end{aligned} \tag{7}$$

respectively. Although the power control scheme at the transmitter side is the same, receive antenna power controls and selection of the parameters are quite different than the one receive antenna case. Combination process may use different selection of power control parameters at each receive antenna. (c,d) parameter pair is used for the signals received at the first receive antenna, while, (e,f) is used for the second antenna.

Thus, the estimation values can be calculated as

$$\begin{aligned} \tilde{s}_0 &= c\tilde{h}_0^* r_0 + d\tilde{h}_1 r_1^* + e\tilde{h}_2^* r_2 + f\tilde{h}_3 r_3^* \\ \tilde{s}_1 &= d\tilde{h}_1^* r_0 - c\tilde{h}_0 r_1^* + f\tilde{h}_3^* r_2 - e\tilde{h}_2 r_3^* \end{aligned} \tag{8}$$

which gives

$$\begin{aligned} \tilde{s}_0 = {} & (ac|h_0|^2 + bd|h_1|^2 + ae|h_2|^2 + bf|h_3|^2 \\ & + ac\epsilon_0^* h_0 + bd\epsilon_1 h_1^* + ae\epsilon_2^* h_2 + bf\epsilon_3 h_3^*) s_0 \\ & + (bch_0^* h_1 - adh_0^* h_1 + beh_2^* h_3 - afh_2^* h_3 \\ & + bc\epsilon_0^* h_1 - ad\epsilon_1 h_0^* + be\epsilon_2^* h_3 - af\epsilon_3 h_2^*) s_1 \\ & + c(h_0^* + \epsilon_0^*) n_0 + d(h_1 + \epsilon_1) n_1^* \\ & + e(h_2^* + \epsilon_2^*) n_2 + f(h_3 + \epsilon_3) n_3^* \end{aligned} \tag{9}$$

$$\begin{aligned} \tilde{s}_1 = {} & (ac|h_0|^2 + bd|h_1|^2 + ae|h_2|^2 + bf|h_3|^2 \\ & + ac\epsilon_0 h_0^* + bd\epsilon_1^* h_1 + ae\epsilon_2 h_2^* + bf\epsilon_3^* h_3) s_1 \\ & + (adh_0 h_1^* - bch_0 h_1^* + afh_2 h_3^* - beh_2 h_3^* \\ & + ad\epsilon_1^* h_0 - bc\epsilon_0 h_1^* + af\epsilon_3^* h_2 - be\epsilon_2 h_3^*) s_0 \\ & + d(h_1^* + \epsilon_1^*) n_0 - c(h_0 + \epsilon_0) n_1^* \\ & + f(h_3^* + \epsilon_3^*) n_2 - e(h_2 + \epsilon_2) n_3^*. \end{aligned} \tag{10}$$

With the increasing number of receive antennas, random interference terms in $\tilde{s}_0$ and $\tilde{s}_1$ become more effective on the error performance. The most important aim of power control is to minimize the disturbing effects of these terms on the overall system performance. The number of independent channels is four for the communication system employing two receive antennas. Consequently, feedback and power control schemes will not be as simple as the single receive antenna case. For single receive antenna, the number of states to be decided at the receiver is only two representing the cases $(|h_0| > |h_1|)$ or $(|h_0| < |h_1|)$. On the other hand, for two receive antennas case, four states represent all possible cases which are

$$\begin{aligned} &(|h_0| > |h_1| \text{ AND } |h_2| > |h_3|), \\ &(|h_0| > |h_1| \text{ AND } |h_2| < |h_3|), \\ &(|h_0| < |h_1| \text{ AND } |h_2| > |h_3|), \\ &(|h_0| < |h_1| \text{ AND } |h_2| < |h_3|). \end{aligned}$$

At the beginning of each signalling interval, transmitter has to know the channel state in order to implement power control. This information is supplied by the transmission of two control bits over the feedback channel. Transmitter determines the values for the power control parameters upon this feedback information. SNR and SIR values at the receiver should be maximized in order to increase the error performance of the communication system. If the channels between the first transmit antenna and the receive antennas are in better condition compared to the second one, for which

$$(|h_0| > |h_1| \text{ AND } |h_2| > |h_3|),$$

the parameters are chosen as $a = c = e = 1.0$, $b = d = f = 0.0$ to concentrate the total transmit power on the first transmit antenna. Similarly, when the channels between the second transmit antenna and the receive antennas are in better condition, for which

$$(|h_0| < |h_1| \text{ AND } |h_2| < |h_3|),$$

the parameters are chosen as $a = c = e = 0.0$, $b = d = f = 1.0$ to concentrate the total transmit power to the second transmit antenna. These values ensure that the intersymbol interference terms will disappear in the resulting estimation values and the communication system will behave like a selection diversity system. The worst case appears when the receive antennas can not distinguish the transmit antenna with higher channel gains which occurs for

$$(|h_0| > |h_1| \text{ AND } |h_2| < |h_3|),$$
$$(|h_0| < |h_1| \text{ AND } |h_2| > |h_3|).$$

In these cases, parameters are chosen as $a = b = c = d = e = f = \sqrt{0.5}$ and no power control is applied to avoid erroneous power concentration on one of the transmit antennas.

For the case of three receive antennas, the channel gains are denoted by h_0, h_1, h_2, h_3, h_4 and h_5 (Fig.1). Transmit power weighting parameters a and b are used at the transmitter which gives

$$\begin{aligned} r_0 &= ah_0s_0 + bh_1s_1 + n_0 \\ r_1 &= -ah_0s_1^* + bh_1s_0^* + n_1 \end{aligned} \tag{11}$$

at the first receive antenna in the subsequent signalling intervals. The received signals at the second receive antenna can be given by

$$\begin{aligned} r_2 &= ah_2s_0 + bh_3s_1 + n_2 \\ r_3 &= -ah_2s_1^* + bh_3s_0^* + n_3 \end{aligned} \tag{12}$$

while

$$\begin{aligned} r_4 &= ah_4s_0 + bh_5s_1 + n_4 \\ r_5 &= -ah_4s_1^* + bh_5s_0^* + n_5 \end{aligned} \tag{13}$$

are received at the third receive antenna over the same intervals.

At the receiver, the combinator uses the power control parameter pairs (c, d), (e, f) and (g, h) for the signals received at the first, second and third receive antennas, respectively. The received signals are combined to determine the values

$$\begin{aligned} \tilde{s}_0 &= c\tilde{h}_0^*r_0 + d\tilde{h}_1r_1^* + e\tilde{h}_2^*r_2 + f\tilde{h}_3r_3^* + g\tilde{h}_4^*r_4 + h\tilde{h}_5r_5^* \\ \tilde{s}_1 &= d\tilde{h}_1^*r_0 - c\tilde{h}_0r_1^* + f\tilde{h}_3^*r_2 - e\tilde{h}_2r_3^* + h\tilde{h}_5^*r_4 - g\tilde{h}_4r_5^* \end{aligned} \tag{14}$$

in order to estimate s_0 and s_1. Using (11), (12) and (13) in (14) yields

$$\begin{aligned} \tilde{s}_0 = {} & (ac|h_0|^2 + bd|h_1|^2 + ae|h_2|^2 + bf|h_3|^2 + ag|h_4|^2 \\ & + bh|h_5|^2 + ac\epsilon_0^*h_0 + bd\epsilon_1h_1^* + ae\epsilon_2^*h_2 + bf\epsilon_3h_3^* \\ & + ag\epsilon_4^*h_4 + bh\epsilon_5h_5^*)s_0 \\ & + (bch_0^*h_1 - adh_0^*h_1 + beh_2^*h_3 - afh_2^*h_3 + bgh_4^*h_5 \\ & - ahh_4^*h_5 + bc\epsilon_0^*h_1 - ad\epsilon_1h_0^* + be\epsilon_2^*h_3 - af\epsilon_3h_2^* \\ & + bg\epsilon_4^*h_5 - ah\epsilon_5h_4^*)s_1 \\ & + c(h_0^* + \epsilon_0^*)n_0 + d(h_1 + \epsilon_1)n_1^* + e(h_2^* + \epsilon_2^*)n_2 \\ & + f(h_3 + \epsilon_3)n_3^* + g(h_4^* + \epsilon_4^*)n_4 + h(h_5 + \epsilon_5)n_5^* \end{aligned} \tag{15}$$

$$\begin{aligned} \tilde{s}_1 = {} & (ac|h_0|^2 + bd|h_1|^2 + ae|h_2|^2 + bf|h_3|^2 + ag|h_4|^2 \\ & + bh|h_5|^2 + ac\epsilon_0h_0^* + bd\epsilon_1^*h_1 + ae\epsilon_2h_2^* + bf\epsilon_3^*h_3 \\ & + ag\epsilon_5h_4^* + bh\epsilon_5^*h_5)s_1 \\ & + (adh_0h_1^* - bch_0h_1^* + afh_2h_3^* - beh_2h_3^* + ahh_4h_5^* \\ & - bgh_4h_5^* + ad\epsilon_1^*h_0 - bc\epsilon_0h_1^* + af\epsilon_3^*h_2 - be\epsilon_2h_3^* \\ & + ah\epsilon_5^*h_4 - bg\epsilon_4h_5^*)s_0 \\ & + d(h_1^* + \epsilon_1^*)n_0 - c(h_0 + \epsilon_0)n_1^* + f(h_3^* + \epsilon_3^*)n_2 \\ & - e(h_2 + \epsilon_2)n_3^* + h(h_5^* + \epsilon_5^*)n_4 - g(h_4 + \epsilon_4)n_5^*. \end{aligned} \tag{16}$$

There are again four different cases for power control. If all the channel gains from the first transmit antenna to the receive antennas are higher than those of the second transmit antenna, mainly if

$$(|h_0| > |h_1| \text{ AND } |h_2| > |h_3| \text{ AND } |h_4| > |h_5|),$$

the power control parameters are chosen as $a = c = e = g = 1.0$ and $b = d = f = h = 0.0$ and all transmit power is concentrated on the first transmit antenna. Equivalently, if the channel gains assigned to the second transmit antenna are higher, mainly if

$$(|h_0| < |h_1| \text{ AND } |h_2| < |h_3| \text{ AND } |h_4| < |h_5|),$$

the total transmit power is used for the second transmit antenna by choosing $a = c = e = g = 0.0$ and $b = d = f = h = 1.0$. On any other cases where

$$(|h_0| < |h_1| \text{ AND } |h_2| < |h_3| \text{ AND } |h_4| > |h_5|),$$
$$(|h_0| < |h_1| \text{ AND } |h_2| > |h_3| \text{ AND } |h_4| < |h_5|),$$
$$(|h_0| > |h_1| \text{ AND } |h_2| < |h_3| \text{ AND } |h_4| < |h_5|),$$
$$(|h_0| < |h_1| \text{ AND } |h_2| > |h_3| \text{ AND } |h_4| > |h_5|),$$
$$(|h_0| > |h_1| \text{ AND } |h_2| < |h_3| \text{ AND } |h_4| > |h_5|),$$
$$(|h_0| > |h_1| \text{ AND } |h_2| > |h_3| \text{ AND } |h_4| < |h_5|),$$

the power control parameters are chosen as $a = b = c = d = e = f = g = h = \sqrt{0.5}$ to avoid erroneous power concentration.

III SIMULATION RESULTS

Bit error probabilities of the proposed power control schemes employing one transmit and multiple receive antennas have been investigated for different SECR values by means of computer simulations. Simulation results are presented in Fig.2 and Fig.3, where simulation results for the corresponding Alamouti's schemes are also included for the comparison purposes.

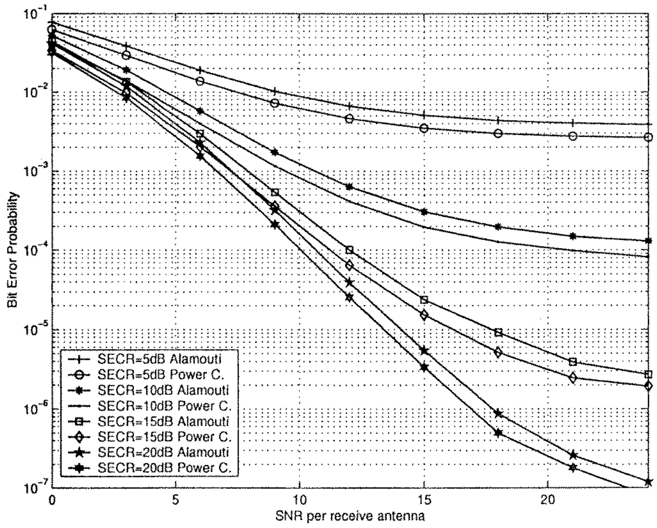

Figure 2: Simulation results for two receive antennas

According to these results, for $10\,dB$ of SECR, the scheme with two receive antennas needs $2\,dB$ lower SNR than the conventional orthogonal transmit diversity scheme with two receive antennas to achieve a BER of 10^{-3}, while this gain becomes more than $5\,dB$ for 10^{-4}.

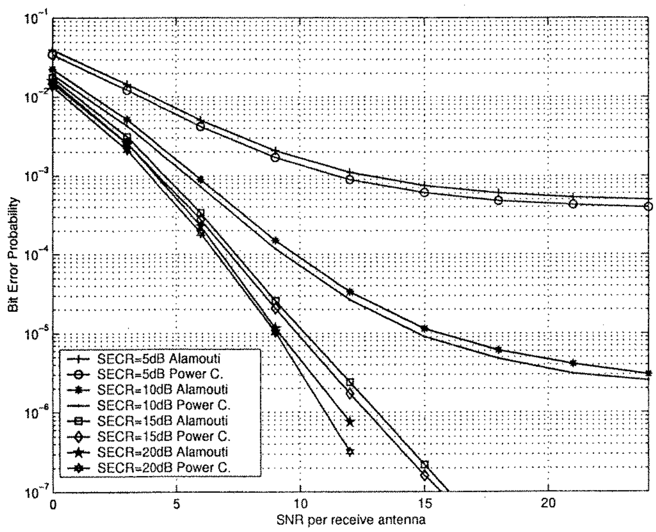

Figure 3: Simulation results for three receive antennas

For the case of three receive antennas, the gain achieved by the power control scheme seems to decrease compared to the two receive antennas case. With the increasing number of receive antennas, the probability of them to agree on the same transmit antenna as having higher gains decreases. This forces to the equal power concentration on two transmit antennas more often which consequently forces the power control scheme to approach the Alamouti's scheme.

IV CONCLUSIONS

In this paper, power control schemes for orthogonal transmit diversity systems employing multiple receive antennas have been proposed in order to increase the error performances of these systems in multipath fading channels. Bit error probabilities of the proposed schemes have been investigated by means of computer simulation. Performance curves have been compared to the corresponding reference orthogonal transmit diversity systems in order to emphasize the error performance improvement of the proposed schemes with power control.

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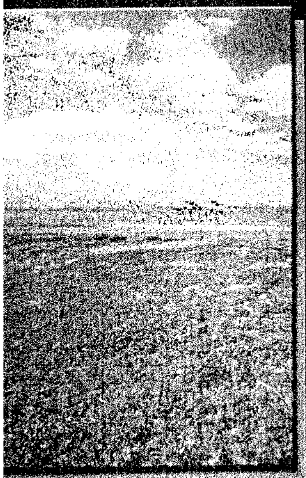

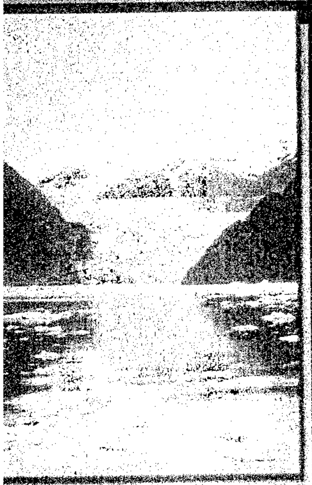

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EM-Based Sequence Estimation for Wireless Systems with Orthogonal Transmit Diversity

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Abstract—**In this paper, an optimum sequence estimation algorithm for wireless systems with Alamouti's two transmitter diversity in the presence of multipath fading is proposed. The algorithm is based on a jointly iterative channel and sequence estimation according to the maximum likelihood (ML) criterion, using the Expectation-Maximization (EM) algorithm employing M-PSK modulation scheme with additive Gaussian noise. The discrete multipath channel is represented in terms of the channel gains from each transmit antenna to the receive antenna. EM algorithm derived estimates jointly the complex channel parameters of each channel and the data sequence transmitted, iteratively, which converges to the true ML solution. The channel estimation is achieved in a simple way through the iterative equations by decoupling of the signals transmitted from different antennas. The algorithm is applied to the trellis coded modulation systems and efficiency of the algorithm proposed has been shown by the computer simulations. Simulation results show that the EM algorithm converges quickly for fast fading channels. The performance of the EM-based decoder approaches that of the ML receiver which has perfect knowledge of the channel.**

I. Introduction

Transmitter diversity is an effective technique for combating fading in multipath wireless channels. It has been observed recently that transmitter(spatial) diversity may be the only option when the frequency and time diversity techniques are not always available. Transmit diversity has been studied only recently to reduce the detrimental effects in wireless fading channels because of its relative simplicity of implementation and feasibility of having multiple antennas at the base stations. Several transmit diversity techniques were studied extensively in the past. Wittneben [1] proposed the first bandwidth efficient transmit scheme and subsequently, a delay diversity scheme was introduced by Seshadri and Winters, [2]. More recently, space-time trellis coding has been proposed by Tarokh, Seshadri and Calderbank [3] which combines signal processing at the receiver with coding techniques appropriate to multiple transmit antennas. These so-called space-time codes perform well in slowly-fading channels, assuming perfect channel information(CSI) at the receiver. With the presence of channel mismatch, however, system performance suffers a significant degradation.

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Recently, Alamouti proposed a remarkable transmit diversity scheme for transmission using two transmit antennas, [4]. This scheme has been generalized later in [5], [6] to an arbitrary number of transmit antennas and is able to achieve the full diversity promised by the transmit and receive antennas. Assuming that the channel state information is available somehow, the orthogonal structure of these space-time block codes enables the ML decoding to be implemented in a simple way through decoupling of the signal transmitted from different antennas rather than joint detection. However, channel state information is usually difficult to obtain. In the absence of perfect channel state information, evaluation of the ML function requires the expectation over the joint statistics of the channel fading coefficients, which is usually mathematically intractable. To cope with this technical difficulty, in this paper, we apply the method of Georghiades and Han [7] and use the results of Li,Georghiades and Huang [8] to the sequence and channel estimation for specifically Alamouti's orthogonal space-time coded systems in the presence of multipath fading channels with two-transmitter diversity. The algorithm is based on a jointly iterative channel and sequence estimation according to the ML criterion, using the EM algorithm, [9], [10], [11]. The last part of the paper provides simulation results on the convergence of the EM algorithm. The performance is presented in terms of the bit error rate for a system employing trellis coded 8-PSK signaling. The extensive computer simulations show that a formulation of the sequence estimation based on the EM algorithm is a promising technique for highly efficient data transmission over mobile wireless channels and it performs close to the performance of a maximum likelihood decoder that assumes perfect CSI.

The paper is organized in four sections following this introduction. In Section 2, the system model is introduced, Section 3, includes the EM-based algorithm, Section 4 presents the simulation results and finally conclusions are presented in Section 5.

II. System Model

We consider the wireless communication system as shown in Figure 1 with transmitter diversity using a space-time block coded transmit diversity scheme first proposed by Alamouti, [4]. The scheme is described with 2 transmit and 1 receive

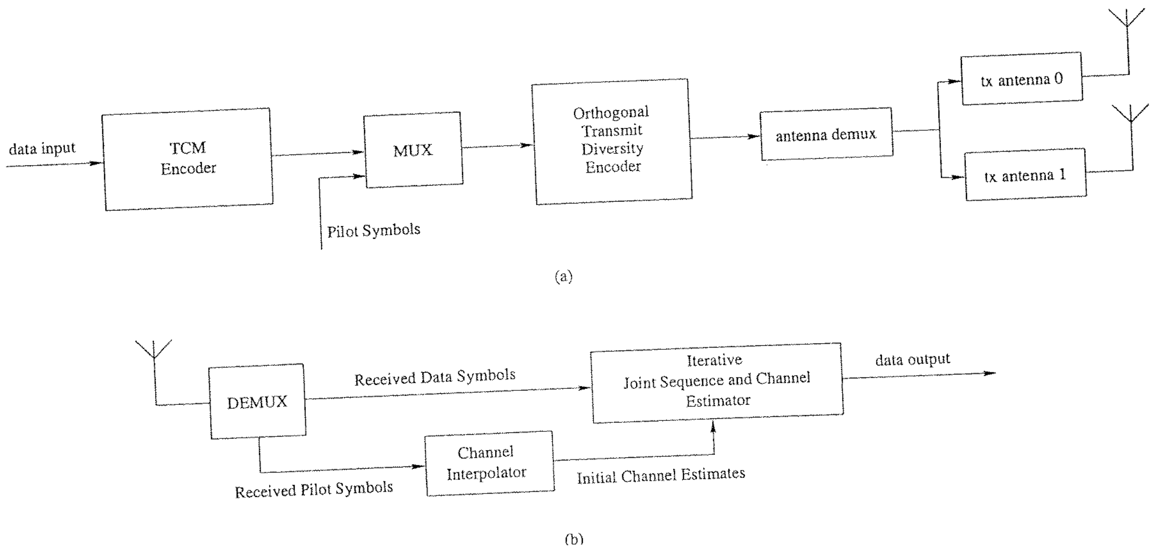

Fig. 1. (a) Transmitter and (b) Receiver block diagrams of the transmit diversity system

antennas to provide a diversity of order 2. Note that, the method can be easily extended to the more general orthogonal space-time block coded systems introduced by Tarokh *et al.*[5] involving more than two transmit and one receive antennas.

The information data can be either uncoded or encoded by a TCM encoder, then fed into the space-time block encoder. At each time slot, the output symbols are modulated and transmitted simultaneously each from a different transmit antenna. At the receiver end, the space-time block decoder followed by symbol-by-symbol decoder or by Viterbi decoder, for uncoded and coded cases, respectively, can be used to decode the received sequence. The generated complex constellation symbols characterizing the input bits are fed to the space-time block encoder proposed by Alamouti whose transmission matrix is given as

$$\begin{array}{c} \quad space \rightarrow \\ time \downarrow \left[\begin{array}{cc} x_1 & x_2 \\ -x_2^* & x_1^* \end{array} \right] \end{array} \tag{1}$$

whose rows are transmitted in successive time intervals with the first and second symbol in a given row sent simultaneously through the first and second antenna, respectively. Based on this scheme, at each time slot k ($k = 0, 1, \cdots, L-1$), the signal transmitted from the first antenna is s_{2k} and the signal transmitted from the second antenna is s_{2k+1}. In the next time slot $(k+1)$, the signal $-s_{2k+1}^*$ is transmitted from the first antenna, and the signal s_{2k}^* is transmitted from the second antenna. Coherent phase-shift keying(PSK) modulation is used here to enhance the system performance.

The wireless channel is assumed to be a fast fading channel where the maximum Doppler spread normalized by the symbol rate is of the order of 10^{-2}. Since we use Alamouti's scheme, it means that channel fading is required to be constant over two consecutive symbol periods($2T$), but varies from one time interval $2T$ to another. Define $\boldsymbol{h}_0 = [h_{0,0}, h_{0,2}, \cdots, h_{0,(2L-2)}]^T$ and $\boldsymbol{h}_1 = [h_{1,0}, h_{1,2}, \cdots, h_{1,(2L-2)}]^T$, where $h_{i,j}$ denotes the channel gains from the first and second transmit antennas to receive antenna, respectively, at the jth symbol period, $j = 0, 2, \cdots, 2L-2$. They are modeled as complex zero-mean Gaussian random variables with autocorrelation $r_l = E[h_{i,2k} h_{i,\ 2k+2l}^*]$, $i = 0, 1$; $l = 0, 1, \cdots, L-1$ and that $\boldsymbol{h}_0$ and $\boldsymbol{h}_1$ are independent of each other. For mobile fading channels, the autocorrelations are given by $r_l = v^2 J_0(2\pi f_d T l)$ where v^2 us the unnormalized variance of the fading gains, $J_0(.)$ is the zero-order Bessel function of the first kind, f_d is the maximum Doppler frequency in Hz and T represents the signaling interval. Thus, for $i = 0, 1$, vector $\boldsymbol{h}_i$ has a normalized Toeplitz covariance matrix $\boldsymbol{R} = (1/v^2)[r_l]$. For $k = 0, 1, \cdots, L-1$, each pair of the two consecutive received signals can then be expressed as

$$\begin{aligned} r_{2k} &= s_{2k} h_{0,2k} + s_{2k+1} h_{1,2k} + n_{2k} \\ r_{2k+1} &= -s_{2k+1}^* h_{0,2k} + s_{2k}^* h_{1,2k} + n_{2k+1} \end{aligned} \tag{2}$$

where n_{2k} and n_{2k+1} are independent samples of an additive Gaussian random variable with variance σ^2, representing the additive white Gaussian noise entering the system.

Letting $\boldsymbol{r} = [\boldsymbol{r}_0^T\ \boldsymbol{r}_1^T]^T$ where $\boldsymbol{r}_0 = [r_0, r_2, \cdots, r_{2L-2}]^T$ and $\boldsymbol{r}_1 = [r_1, r_3, \cdots, r_{2L-1}]^T$, (2) can be expressed into a matrix form

$$\boldsymbol{r} = \boldsymbol{S}\boldsymbol{h} + \boldsymbol{n} \tag{3}$$

where, $\boldsymbol{h} = [\boldsymbol{h}_0^T\ \boldsymbol{h}_1^T]^T$, $\boldsymbol{n} = [\boldsymbol{n}_0^T\ \boldsymbol{n}_1^T]^T$,

$$\boldsymbol{S} = \left[\begin{array}{cc} \boldsymbol{S}_0 & \boldsymbol{S}_1 \\ -\boldsymbol{S}_1^\dagger & \boldsymbol{S}_0^\dagger \end{array} \right] \tag{4}$$

and, $\boldsymbol{S}_0 = \mathrm{diag}\{s_0, s_2, \cdots, s_{2L-2}\}$, $\boldsymbol{S}_1 = \mathrm{diag}\{s_1, s_3, \cdots, s_{2L-1}\}$. $\dagger$ denotes conjugated transpose.

III. Sequence Estimation with EM Algorithm

Now consider the classical problem of estimating data sequence $\boldsymbol{s} = (s_0, s_1, \cdots, s_{2L-1})$ from the observations of received data $r = (r_0, r_1, \cdots, r_{2L-1})$. A ML receiver then performs

$$\max_s p(r|s) = \max_s E_h [p(r|s,h)]. \quad (5)$$

Note that evaluation of the likelihood function above requires the expectation over the joint statistics of the random channel parameters h, a task that more often is mathematically intractable. Even if the likelihood function can be obtained analytically off line, however, it is invariably a nonlinear function of s, which makes the maximization step computationally infeasible in real time. Especially for long and/or coded sequences transmitted over fading channels, the problem of optimum sequence estimation is known to be difficult or intractable. In such cases, an iterative formulation of the sequence estimation problem based on the EM algorithm can provide an implementable solution. The ML estimate $\widehat{s}_{ML}$ is given by

$$\widehat{s}_{ML} = \arg\max_s p(r|s). \quad (6)$$

The EM algorithm inductively reestimates $\widehat{s}_{ML}$ so that a monotonic increase in the *a posteriori* conditional pdf above is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q(s|s^{(i)}) = E[\log p(r|s,h)|r, s^{(i)}]. \quad (7)$$

Given the received signal r, the EM algorithm starts with an initial value $s^{(0)}$ of the unknown channel parameters s. The $(i+1)th$ estimate of s is obtained by the maximization step described by

$$s^{(i+1)} = \arg\max_s Q(s|s^{(i)}). \quad (8)$$

The log-likelihood function of r given s and h needed in (7) to compute the expectation step is easily obtained from (3) as follows

$$\ell(r|s,h) \equiv \log p(r|s,h) \sim p(||r - Sh||^2)$$

Dropping unnecessary terms and rearranging slightly it follows that

$$\ell(r|s,h) = \mathcal{R}e[r^\dagger Sh] - \frac{1}{2}||S||^2. \quad (9)$$

Assuming the PSK signaling is used we can drop the second term in the right hand side of (8).

Then, the expectation step of the EM algorithm at the ith iteration yields,

$$\begin{aligned} Q(s|s^{(i)}) =& \mathcal{R}e[r^\dagger S\widehat{h}^{(i)}] \\ =& \sum_{k=0}^{L-1} \Big[\mathcal{R}e\left\{(r_{2k}^* s_{2k} - r_{2k+1}^* s_{2k+1}^*)\widehat{h}_{0,2k}^{(i)}\right\} + \\ & \mathcal{R}e\left\{(r_{2k}^* s_{2k+1} + r_{2k+1}^* s_{2k}^*)\widehat{h}_{1,2k}^{(i)}\right\}\Big] \end{aligned} \quad (10)$$

where

$$\widehat{h}^{(i)} = E[h|r, s^{(i)}]. \quad (11)$$

After some algebra, the above conditional mean can be obtained as follows: It can be shown that

$$p(h|r,s^{(i)}) \sim \exp\left[-(h - \widehat{h}^{(i)})^\dagger \Psi^{-1}(h - \widehat{h}^{(i)})\right], \quad (12)$$

where,

$$\widehat{h}^{(i)} = (v^2/\sigma^2)\Psi S^{\dagger(i)} r,$$

and

$$\Psi = \left(R_h^{-1} + (v^2/\sigma^2)I\right)^{-1}.$$

Here, R_h is a $2L \times 2L$ block diagonal matrix defined by $R_h = \text{diag}\{R \ R\}$, where R is the normalized autocorrelation matrix of the random fading vector, as defined earlier, whose main diagonal elements are unity. v^2 is the unnormalized variance of the random fading gains. σ^2 is the variances of the noise.

The EM algorithm starts with an initial estimate of the channel estimates $\{\widehat{h}_{0,2k}^{(0)}, \widehat{h}_{1,2k}^{(0)}\}$ and uses them in (6) to produce, by maximization, a sequence estimate. This sequence estimate is then used in (8) to produce the next sequence estimate, and so on, until convergence within two to three iterations. At convergence, $s^{(i+1)} = s^{(i)}$, the algorithm produces both a sequence estimate and a fading channel estimate.

We now turn to the maximization step of the EM algorithm, where we distinguish between the coded and the uncoded transmission. First we observe from (10) that in the absence of coding, maximizing $Q(s|s^{(i)})$ with respect to sequence s is equivalent to maximizing each individual term in the sum, i.e., making symbol-by-symbol decisions. Then, if $s^{(i+1)}$ is the maximizing sequence, for $k = 0, 1, \cdots, L-1$, its components are given by

$$\begin{aligned} s_{2k}^{(i+1)} &= \arg\max_{s_{2k}} \mathcal{R}e\left\{r_{2k}^* s_{2k}\widehat{h}_{0,2k}^{(i)} + r_{2k+1}^* s_{2k}^* \widehat{h}_{1,2k}^{(i)}\right\} \\ s_{2k+1}^{(i+1)} &= \arg\max_{s_{2k+1}} \mathcal{R}e\left\{-r_{2k+1}^* s_{2k+1}^* \widehat{h}_{0,2k}^{(i)} + r_{2k}^* s_{2k+1}\widehat{h}_{1,2k}^{(i)}\right\} \end{aligned} \quad (13)$$

where we have used the expression for $Q(s|s^{(i)})$ in (10).

When trellis coding is used, the maximization over all trellis sequences can be done efficiently using the Viterbi algorithm. It is seen that in contrast to directly evaluating the likelihood function in (9), the EM algorithm yields at each step of iteration a likelihood function that allows the use of the Viterbi algorithm for efficient computations.

Initialization

In order to be able to choose good initial values for $s^{(0)}$, the N_{PS} data symbols $\{s_{2k}, s_{2k+1}\}$ for $k \in S_{PS}$, in each observation block are generally used as pilot symbols known by the receiver. They are inserted periodically in the sequence. Here, S_{PS} denotes the set of pilot symbols indices. To interpolate the channel estimates, initially, there exist a minimum spacing,

l_{SC}, between pilots given by $l_{SC} < 1/\tau_{max}$, where τ_{max} is the maximum delay spread of the channel ($B_{coh} = 1/\tau_{\max}$, channel coherent bandwidth).

To initialize the receiver we determine $\widehat{h}_{0,2k}^{(0)} = \widehat{h}_0^{(0)}[2k]$ in terms of the pilot symbols and the received signals corresponding to the pilot symbols from the following equations. $\widehat{h}_{1,2k}^{(0)} = \widehat{h}_1^{(0)}[2k], k \in S_{PS}$, where

$$\begin{aligned} \widehat{h}_0^{(0)} &= \Psi_{11}^{(0)}(s_0^{\dagger(0)} r_0 - s_1^{(0)} r_1) + \Psi_{12}^{(0)}(s_1^{\dagger(0)} r_0 + s_0^{(0)} r_1) \\ \widehat{h}_1^{(0)} &= \Psi_{21}^{(0)}(s_0^{\dagger(0)} r_0 - s_1^{(0)} r_1) + \Psi_{22}^{(0)}(s_{\dagger 0}^{(1)} r_0 + s^{(0)} r_1), \end{aligned} \quad (14)$$

and

$$\Psi^{(0)} = \begin{bmatrix} \Psi_{11}^{(0)} & \Psi_{12}^{(0)} \\ -\Psi_{21}^{(0)} & \Psi_{22}^{(0)} \end{bmatrix}.$$

The complete initial channel gains $h_{0,2k}^{(0)}, h_{1,2k}^{(0)}$ for $k = 0, 1, \cdots, L-1$ can be easily determined using an interpolation technique, i.e., Lagrange interpolation algorithm.

The EM algorithm can be summarized briefly as follows:

Step 1. Set $i = 0$ and choose the initial values $s^{(0)}$, and determine $\widehat{h}_0^{(0)}, \widehat{h}_1^{(0)}$, as explained above

Step 2. Compute $s^{(i+1)}$ by maximizing $Q(s|\widehat{s}^{(i)})$ in (8)and (10) over all sequences by Viterbi algorithm if trellis coding is present. Use (13) to perform the maximization if coding is not present.

Step 3. Compute $\widehat{h}_0^{(i+1)}, \widehat{h}_1^{(i+1)}$ from (11) and goto Step 2, repeat until the algorithm converges, in which case the last sequence estimate is produced as the ML estimate.

Note that a computation of the number of iterations needed to implement the EM algorithm indicates that it increases linearly in the sequence length compared to the more than exponential increase for direct implementation. Also, the maximization step in (8) can be implemented easily due to the fact that $Q(s|s^{(i)})$ can be expressed as in recursive form as in (10), and thus, the Viterbi algorithm can be employed.

IV. SIMULATION RESULTS

Error performance of the proposed iterative decoder has been investigated via computer simulations. The fading channel is modeled as the Jakes fading with autocorrelation $v^2 = 1$, $r_l = J_0(2\pi f_d T l)$ where $J_0(.)$ is the zero-order Bessel function of the first kind, f_d is the maximum Doppler frequency in Hz and T represents the signaling interval. Data bits are first encoded by a rate 2/3, 4-state 8-PSK TCM encoder to produce the coded data symbol sequence of length 100. The encoder used was recently proposed in [12] and has optimum performance when used in combination with Alamouti's transmit diversity scheme.

In order to initialize the EM algorithm, the receiver has to have good estimates of the channel. These estimates have been provided using pilot symbol assisted modulation (PSAM), [13]. Six pairs of pilot symbols, which are already known at the receiver, are added periodically to the data symbol sequence with a period of 20. At the receiver, channel fading coefficients are first estimated at the pilot symbol positions. The unknown data fading coefficients are then estimated by applying Lagrange interpolation technique on the pilot fading coefficients, according to the initialization procedure as explained in Section 3. The EM algorithm uses these channel estimates to initialize and converge to the maximum likelihood decoding within two or three iterations. The maximization step of the EM algorithm is efficienty performed using the Viterbi algorithm. Bit error probability curves have been presented for a channel with normalized maximum Doppler frequency of 0.01 in Fig. 2.

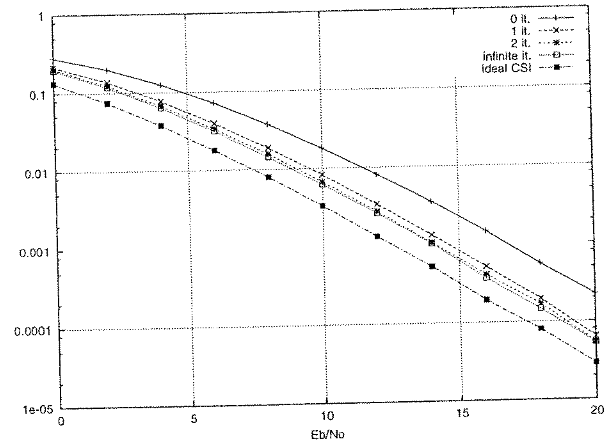

Fig. 2. Bit error performance of trellis coded 8-PSK code for $f_D T = 0.01$

The proposed scheme seems to converge to the ML decoding in two iterations. This provides an SNR gain of $3\,dB$ in the high SNR region. The performance improvement is caused by the reduction in the channel estimation error which can be seen in Fig. 3, where the minimum square estimation error (MSEE) values versus iteration numbers are presented for different SNR values.

The channel estimation errors converge to the maximum likely estimates in two iterations. For a channel with higher Doppler frequency ($f_D T = 0.03$), the bit error probability curves again converge in two iterations (Fig. 4), but this time resulting in an error floor. Alamouti's transmit diversity scheme loses its orthogonality property in the presence of channel estimation error and an error floor is observed.

Since, in the fast fading channel, PSAM with a pilot separation of 20 loses its effectiveness in estimating the channel fading coefficients, the algorithm converges to a local maximum which results in a high estimation error (Fig. 5). In both cases, the proposed decoder is shown to converge to the ML decoding in just two iterations.

V. CONCLUSION

In this paper, we proposed an optimum sequence estimation algorithm for wireless communications systems employing a

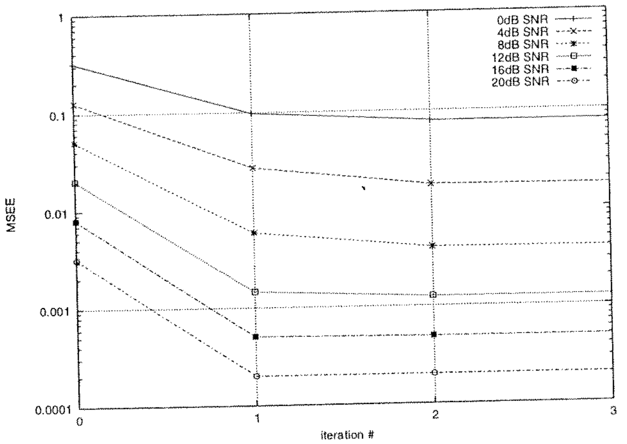

Fig. 3. Mean square estimation error for $f_D T = 0.01$

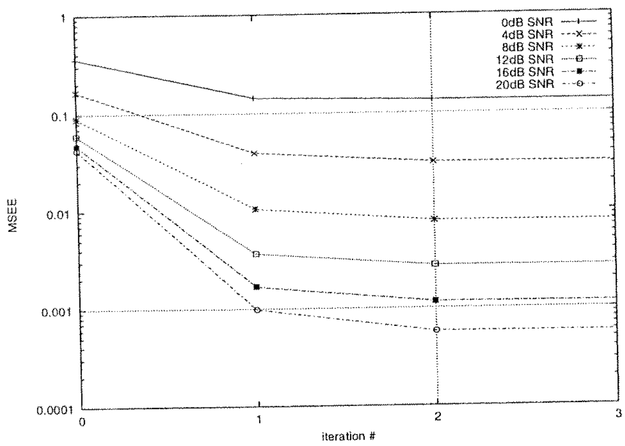

Fig. 5. Mean square estimation error for $f_D T = 0.03$

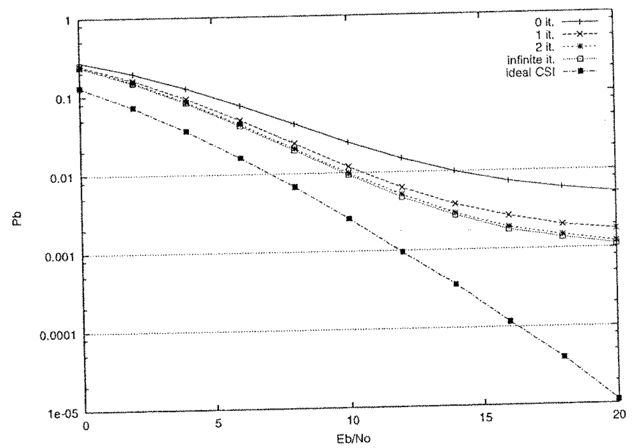

Fig. 4. Bit error performance of trellis coded 8-PSK code for $f_D T = 0.03$

transmit diversity. This algorithm performs an iterative estimation of the transmitted sequence of data symbols according to the ML criterion, using the EM algorithm employing M-PSK modulation scheme with additive Gaussian noise. The discrete multipath channel was represented in terms of the channel gains from each transmit antenna to the receive antenna. EM algorithm derived estimates jointly the complex channel parameters of each channel and the data sequence transmitted, iteratively, which converges to the true ML solution. The algorithm is applied to the trellis coded 8-PSK modulated wireless systems and efficiency of the algorithm proposed has been shown by the computer simulations. Simulation results show that the EM algorithm converges quickly for fast fading channels. The performance of the EM-based decoder approaches that of the ML receiver which has perfect knowledge of the channel. In addition, the EM-based detector is rather simple to implement since the maximization step of the algorithm can be done using the Viterbi algorithm.

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ÖZET

Gezgin telsiz kanallarda iletişim kalitesini düşürecek çeşitli etkiler bulunmaktadır. Bunlar toplamsal beyaz Gauss gürültüsü, çok-yollu iletimden kaynaklanan sönümleme etkisi ve kanallararası girişim olarak sıralanabilir. Yüksek gezginliğe sahip bir iletişim kanalı üzerinden iletimde hata başarımı kanaldaki sönümleme etkisinden önemli biçimde etkilenir. Vericiden iletilen işaretin kopyalarının alıcıya değişik yollardan farklı gecikme ve kazanç değerleri ile ulaşması durumunda ortaya çıkan sönümleme etkisi yüksek hata başarımına sahip iletişim sistemi tasarımında göz önüne alınacak en önemli bozucu etkidir.

Sönümleme etkisinden kurtulmak amacıyla çeşitleme tekniklerinden yararlanılması gerekmektedir. Çeşitleme teknikleri, iletim ortamında bağımsız kanallar yaratılarak iletilecek bilginin kopyalarının bu kanallar üzerinden alıcıya ulaştırılması ilkesine dayanmaktadır. Böylece, bağımsız kanallardan biri derin sönümlemeden etkilense bile alıcıya bir diğer kanaldan ulaşabilecek (daha az zayıflamış) bir kopya iletilen işaretin hatasız kestirilmesini sağlamaktadır. Çeşitleme zamanda, frekansta ve/veya uzayda sağlanabilmektedir. Zaman çeşitlemesinde bağımsız kanallar değişik zaman aralıklarında işaret iletimi ile sağlanırken, frekans çeşitlemesinde bu kanallar değişik frekans bandlarının birlikte kullanılması ile sağlanır. Uzay çeşitlemesinde ise verici ve/veya alıcıda birden çok kanal kullanılıp bunlar birbirlerinden yeteri kadar uzağa yerleştirilerek elde edilen kanalların istatistiksel bağımsız sönümlemelerden etkilenmesi sağlanır.

Alıcı anten çeşitlemesi literatürde oldukça yoğun şekilde incelenmiş iken verici çeşitlemesi konusunda yeteri kadar çalışma yapılmamıştır. Gezgin iletişim sistemlerinde kullanılan baz istasyon–gezgin birim kanalı için alıcıda birden çok anten kullanmak gezgin birimin boyutunu ve maliyetini oldukça arttırmaktadır. Her bir gezgin birimde birden çok anten kullanarak maliyetlerini arttırmak yerine verici çeşitlemesi kullanılarak sadece baz istasyonda birden çok anten kullanılarak aynı kazanç sağlanabilmektedir. Literatürde yakın zamanda ortaya atılan, bu düşünceye dayalı bir yapı uzay-zaman kodlaması olarak adlandırılmaktadır. Uzay-zaman kodlamasında birden çok verici ve/veya alıcı anten kullanılarak her bir verici anten için, tüm iletişim sisteminin başarımı göz önünde tutularak, ayrı ayrı kodlar tasarlanmaktadır.

Yakın zamanda ortaya atılan uzay-zaman kodlama tekniği literatürde genellikle faz kaydırmalı anahtarlamaya (PSK) ve dördül genlik modülasyonuna (QAM) uygulanmıştır. Bu çalışmada ise uzay-zaman kodlama tekniği ile çoklu kafes kodlamalı minimum kaydırmalı anahtarlama (MSK) tekniği birleştirilerek yüksek

hata başarımına sahip bir iletişim sistemi önerilmiştir.

Elde edilen bu iletişim sistemi uzay-zaman kodlama tekniğinin beraberinde getirdiği uzay ve zaman çeşitlemeleri sayesinde yüksek hata başarımına sahipken, sürekli faz modülasyonunun özel bir biçimi olan MSK modülasyonu nedeniyle yüksek band verimliliğine sahiptir.

Bu çalışmada, uzay-zaman kodlama tekniği MSK modülasyonuna uygulanmış, iki verici ve bir alıcı anten için iki, dört ve sekiz durumlu uzay-zaman kodlamalı çoklu MSK sistemler önerilmiştir. Bu sistemlerin tasarımlarında, hızlı ve durağumsu sönümlemeli kanallarda uzay-zaman kodlarının tasarım ölçütlerinin en iyileştirilmesi yoluna gidilmiş ve bu amaçla geliştirilen bir kod arama algoritmasından yararlanılmıştır. Önerilen kodların hata başarımları geliştirilen bilgisayar benzetimleri yardımıyla incelenmiş referans yapılara göre olan üstünlükleri Rayleigh sönümlemeli kanallar için ortaya konulmuştur. Önerilen yapıların hızlı sönümlemeli kanallar üzerindeki hata başarımının analitik yolla kestirilmesi amacıyla tam doğru hata hesabı kullanılmış ve yüksek işaret–gürültü oranlarında analitik kestirim ile bilgisayar benzetim sonuçlarının yakınsadığı gözlenmiştir.

SPACE–TIME CODED MULTIPLE MSK SCHEMES

SUMMARY

Mobile wireless channel has severe problems that do not allow high–speed reliable data communications. Some of these problems are the additive white Gaussian noise, multipath fading, and interchannel interference. At high mobility conditions, the performance of wireless communication system is severely degraded by the multipath fading effects. Multipath fading is caused by the summation of the replicas of the transmitted signal over different paths with different delays and gains. It is the primary destructive effect that should be considered while designing a communication system with high error performance.

Diversity techniques should be used in order to increase the error performance of communication systems in fading channels. These techniques form independent subchannels and transmit replicas of data over these subchannels. Thus, even when one of the subchannels is in deep fade, a less–attenuated copy of the data arriving to the receiver can increase the error performance. Diversity can be achieved in time, frequency and/or space. In temporal diversity technique, independent subchannels are formed by transmitting the replicas in different time intervals, while, in frequency diversity technique, independent subchannels can be formed over different frequency regions. Space diversity technique generally employs multiple transmit and/or receive antennas. These antennas are located at enough distance of each other to ensure that the formed channels are independent.

While the receiver diversity techniques are widely investigated in the literature, transmit diversity techniques received much attention recently. For a base station–mobile unit channel, employing multiple antennas at each mobile unit results in an increase of the size and cost of the mobile units. The same performance improvement can be achieved by using multiple antennas at the transmitter (base station) resulting in a lower cost of the overall communication system. This new scheme, recently proposed in the literature, is known as space–time coding. Main idea of this technique is using multiple transmit antennas and designing different codes for each transmit antenna by jointly considering the overall system error performance.

Until now, space–time codes have been applied to phase shift keying (PSK) and quadrature amplitude modulation (QAM) schemes, where, in this work, it is applied to multiple trellis coded minimum shift keying (MSK) modulation to

achieve high error performance.

The proposed scheme has high power–efficiency due to the space–time coding technique, while the bandwidth–efficiency comes from the minimum shift keying modulation.

In this work, space–time coding technique has been applied to MSK modulation and space–time coded multiple MSK schemes employing two transmit and one receive antennas with 2, 4, and 8 states are proposed. A computer–based code search algorithm has been developed to find schemes maximizing the error performance criteria of the space–time coding technique over rapid and quasi–static fading channels. Error performances of the proposed codes have been investigated via computer simulations for the Rayleigh fading channels. Bit and frame error probability curves have been obtained and comparisons with the corresponding reference schemes have been made to emphasize the performance improvement. The exact error probability calculations have been made for rapid fading channels. Computer simulations have been shown to converge with the analytical error curves at high signal–to–noise ratios.

İÇİNDEKİLER

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8. *Özet:* Projede uzay-zaman kodlama tekniği ile OFDM tekniğinin birleştirildiği ve sürekli faz modulasyonunun kullanıldığı yeni bir uzay-zaman kodlamalı OFDM tümleşik, geniş bandlı gezgin ileşitim sisteminin verici ve alıcı kısımlarının tasarlanması ve böyle bir sistem için gerekli eşzamanlama algoritmalarının geliştirilmesi öngörülmektedir. Daha sonra tasarlanan sistemin hata başarımları gerek analitik yöntemlerle ve gerekse benzetim yoluyla incelenmektedir. Başarım analizlerinde bit hata olasılıklarının üst sınırlarının belirlenmesi amaçlanmakta ve diğer sistemlerle karşılaştırılarak tasarlanan sistemin üstünlüğü gösterilmektedir.

Abstract: The main objective of this project is to design receiver and transmitter structures and develop synchronization algorithms for a new "space-time-coded OFDM integrated, wide band mobile communication system" that utilizes a combination of space-time coding and OFDM with continuous phase modulation. The performance of these developed structures will be examined by analytical means and computer simulations. In performance analysis the definition of upper bit error rate boundaries will be employed and the BER performences are compared with other systems.

Anahtar Kelimeler: Gezgin iletişim, uzay-zaman kodlaması, OFDM, eşzamanlama, kanal kestirimi.

Keywords: Mobile communications, space-time coding, OFDM, synchronization, channel estimation.

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