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NON-DATA AIDED EM-BASED CHANNEL ESTIMATION FOR OFDM SYSTEMS WITH TIME-VARYING FADING CHANNELS(*)

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Abstract- In this paper, a computationally efficient algorithm is presented for maximum *a posteriori* (MAP) channel estimation for OFDM systems employing M-PSK modulation scheme with additive Gaussian noise, based on the Expectation Maximization (EM) method. A non-data-aided scheme is considered for the estimation of a multipath time-varying channel by averaging over the M-PSK signal constellation. For this, an EM algorithm is derived which estimates the complex channel parameters of each subcarriers iteratively in frequency domain and which converges to the true MAP estimation of the unknown channel. The algorithm requires a convenient representation of the discrete multipath fading channel based on the Karhunen-Loeve orthogonal expansion. The algorithm is applied to the QPSK modulated OFDM systems and efficiency of the method proposed is shown by the computer simulations.

1. INTRODUCTION

OFDM signaling is proven to be an efficient way to overcome the effects of fading channel and multi-path by dividing the frequency selective channel into a number of sub-channels corresponding to the OFDM sub-carrier frequencies. OFDM has already been accepted for the new wireless local area network (WLAN) standards (IEEE 802.11), the ETSI High Performance Local Area Network type 2 (HIPERLAN/2) and Japan's Mobil Multimedia Access Communications (MMAC) systems [1].

In OFDM, channel state information between transmit and receive antenna pairs is required for coherent decoding. Therefore, several channel parameter estimation techniques were proposed in literature. In [2-3] a channel estimator for OFDM systems has been proposed based on the singular-value decomposition or frequency-domain filtering. Time domain filtering has been proposed in [4].

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To further improve the channel estimator performance, a MMSE channel estimator, which makes full use of the time-frequency correlation of the time-varying dispersive channel were proposed in [5]. This technique has been extended later in [6] to develop a channel estimation in OFDM systems with transmitter diversity using space time coding. In this paper we apply the method of Siala [7] to the estimation of time-varying fading channels for OFDM systems. This algorithm performs an iterative channel estimation according to the maximum *a posteriori* (MAP) criterion, using the Expectation-Maximization (EM) algorithm. It uses profitably not only pilot symbols but also information-carrying symbols on the optimization of the channel estimation. It requires a conventional representation of the multipath Doppler channel, based on a discrete Karhunen-Loeve(KL) orthogonal expansion of the discrete multipath Doppler channel seen by the OFDM receiver. The channel estimator makes full use of the correlation of the channel frequency response at different times and frequencies. In particular, for mobile wireless channels, the correlation of the channel frequency response at different times and frequencies can be separated into the multiplication of the time-and frequency-domain correlation functions and this would decrease the computational complexity of the channel estimation substantially [5]. Computer simulations demonstrate that the computational complexity of our channel estimation algorithm is significantly improved.

2. OFDM SYSTEMS WITH TIME-VARYING CHANNEL ESTIMATOR

The received signal after demodulation (performing a DFT), can be expressed as

$$R(n,k) = H(n,k)A(n,k) + W(n,k), \quad k = 0,1,....K-1; \quad n = 0,1,....N-1 \qquad (1)$$

where $A(n,k)$ is the signal modulating the $k$the subcarrier during time $nT_s \leq t \leq (n+1)T_s$, T_s being the OFDM symbol duration. They are assumed to have unit variance and be independent for different k's and n's. Since the phase of each subchannel can be obtained by the channel estimator, coherent phase-shift keying (PSK) modulation is used here to enhance the system performance. $W(n,k)$ is represents the additive complex Gaussian noise with variance σ^2, entering the system. $H(n,k)$ is the frequency response of the fading channel at the kth subcarrier at time n. They are correlated samples, both in time and frequency, of a complex Gaussian process.

In the absence of absence of channel state information, decoder must estimate the channel states and there has been extensive affords in the direction of channel parameter estimation. However, most of the works done tries to achieve this goal with employing the training symbols. For OFDM systems channel estimation is challenging if we assume that this should be implemented in a non-data aided fashion [8, 9]. In this paper a novel time-varying channel estimation algorithm is presented by representing the discrete multipath channel based on the Karhunen-Loeve orthogonal representation and make use of the Expectation Maximization technique.

3. REPRESENTATION OF DISCRETE MOBILE RADIO

For an OFDM system with block length T_s and subchannel spacing Δf, the discrete correlation function for different blocks and subcarriers of the frequency response of the time-varying multi-path radio channel for different discrete times and frequencies defined by $r(n,k;n',k')=E[H(n,k)H^*(n',k')]$ can be written as

$$r(n,k;n',k') = \sigma_H^2 r_1(n,n') r_2(k,k'), \quad n,n' = 0,1,...,N-1; \quad k,k' = 0,1,....K-1 \tag{2}$$

where $\sigma_H^2(t)$ is the total average power of the channel impulse response defined as $\sigma_H^2 = \sum_\ell \sigma_\ell^2$. $\sigma_l^2(t)$ is the average power of the lth path and

$$r_f(k,k') = \left(1/\sigma_H^2 \sum_\ell \sigma_\ell^2 e^{-j2\pi(k-k')\Delta f \tau_\ell} \right)$$

From Jakes' model [11]

$$r_l(n,n') = J_0(2\pi(n-n')f_d T_s) \tag{3}$$

where J_0 is the zeroth-order Bessel function of the first kind and f_d is the Doppler frequency which is related to the vehicle speed v and the carrier frequency f_c by $f_d = v f_c / c$, where c is the speed of light.

Discrete frequency response of the wireless channel, $H(n,k)$ can be expressed as

$$H(n,k) = \sum_{i=0}^{N-1} \sum_{j=0}^{K-1} G(i,j)\psi_{i,j}(n,k), \quad n = 0,1,...,N-1; \quad k = 0,1,...,K-1 \tag{4}$$

where the random variables $\{ G(i,j) \}$ are independent complex zero-mean Gaussian coefficients. The variance of these coefficients, arranged in decreasing order, are equal to the eigenvalues $\{ \lambda_{i,j} \}$ of the Karhunen Loeve (KL) transformation with the orthogonalized eigenfunctions $\psi_{i,j}(n,k)$'s of the discrete autocorrelation function $r(n,k;n',k')$ defined by

$$\sum_{n'=0}^{N-1} \sum_{k'=0}^{K-1} r(n,k;n',k')\psi_{i,j}(n',k') = \lambda_{i,j}\psi_{i,j}(n,k), \quad n = 0,1,...,N-1; \quad k = 0,1,...,K-1. \tag{5}$$

Note that when the autocorrelation function is separable as in (2), then it can be shown that $\{\psi_{i,j}(n,k)\}$'s become also separable. That is,

$$\psi_{i,j}(n,k)=\phi_1(n,i)\phi_2(k,j) \tag{6}$$

where $\phi_1(n,i)$ and $\phi_2(k,j)$ are the components of the normalized eigenvectors of the autocorrelations $r_1(n,n')$ and $r_2(k,k')$, respectively. The corresponding eigenvalues are β_i, and γ_j, $i=0,1,\ldots,N-1$; $j=0,1,\ldots,K-1$. From (1) and (5), they satisfy the following relationship.

$$\lambda_{i,j}=\beta_i\gamma_j/\sigma_H^2 . \tag{7}$$

The advantage in having the autocorrelation function by a separable function is that instead of solving NK x NK matrix eigenvalue problem of (5), only two N x N and K x K matrix eigenvalue problems need to solved. Since the required computations to solve these problems are $O(NK^3)$ and $O(N^3)+O(K^3)$, respectively, the reduction in dimensionality achieved by the separable model is quite significant.

4. EM-BASED MAP CHANNEL ESTIMATION

The MAP criterion is used in the fading channel as seen at the FFT output of the OFDM receiver since the joint probability density function of the random variables are known by the receiver and can be expressed as

$$p(\mathbf{G})\approx\prod_i\prod_j\exp\left(-\frac{|G(i,j)|^2}{\lambda_{i,j}}\right) \tag{8}$$

where $\boldsymbol{G}=\{G(i,j)\}$. Given the transmitted signal $\boldsymbol{A}=\{A(n,k)\}$, and the discrete channel representation $\boldsymbol{G}$, and taking into account the independence of the noise components, we can express the conditional probability density function of the received signal $\boldsymbol{R}=\{R(n,k)\}$ as

$$p(\mathbf{R}|\mathbf{A},\mathbf{G})\approx\prod_n\prod_k\exp\left\{-\frac{1}{\sigma^2}\left|R(n,k)-A(n,k)\sum_i\sum_j G(i,j)\psi_{i,j}(n,k)\right|^2\right\} \tag{9}$$

The MAP estimate $\{\mathbf{G}\}$ is given by

$$\hat{\mathbf{G}}=\arg\max_{\mathbf{G}} p(\mathbf{G}|\mathbf{R}). \tag{10}$$

Directly solving this equation is mathematically intractable. However, the solution can be obtained easily by means of the iterative EM algorithm. This algorithm inductively reestimate $\boldsymbol{G}$ so that a monotonic increase in the *a posteriori* conditional pdf in (9) is guaranteed. The monotonic increase is realized via the maximization of the auxiliary function

$$Q\left(\mathbf{G}\middle|\mathbf{G}^{(m)}\right)=\sum_A p(\mathbf{R},\mathbf{A},\mathbf{G})\log p\left(\mathbf{R},\mathbf{A},\mathbf{G}^{(m)}\right) \tag{11}$$

where sum is taken over all possible transmitted data coded signals and $\boldsymbol{G}^{(m)}$ is the estimation of $\boldsymbol{G}$ at the mth iteration. Given the received signal $\boldsymbol{R}$, the EM algorithm starts with an initial value $G^{(0)}$ of the unknown channel parameters G. The $(m+1)$th estimate of G is obtained by the maximization step described by

$$\mathbf{G}^{(m+1)} = \arg\max_{\mathbf{G}} Q\left(\mathbf{G}|\mathbf{G}^{(m)}\right).$$

After long algebraic manipulations the expression of the (p,q)th component $G^{(m)}(p, q)$, $(p=0,1,...,N-1;\ q=0,1,...,K-1)$ of the re-estimate $G^{(m+1)}$ can be obtained as follows:

$$G^{(m+1)}(p,q) = \frac{1}{\left(1+\sigma^2/\lambda_{p,q}\right)} \sum_n \sum_k \Gamma^{(m)}(n,k) R(n,k) \psi^*_{p,q}(n,k) \tag{12}$$

where,

$$\Gamma^{(m)}(n,k) = \sum_{a \in S_{n,k}} a^* P\left(A(n,k)=a|\mathbf{R},\mathbf{G}^{(m)}\right) \tag{13}$$

and $S_{n,k}$ denotes alphabet set taken by the (n,k)th OFDM symbol. In order to be able to choose good initial values for the unknown channel parameters and to ensure a fast start up in the equalization/detection operation following the channel estimation process, the leading L data symbols $D(n,k)$, $k=0,1,...,L-1$ in each OFDM frame are generally used as pilot symbols known by the receiver. When K is large, however, this does not create a significant degradation in spectrum efficiency since L takes small values with respect to the total number of subcarriers carrying the data. Therefore for PSK modulated alphabet set, the initial value of the channel parameters can be selected according to the following data-aided estimates.

$$G^{(0)}(p,q) = \frac{1}{\left(1+\sigma^2/\lambda_{p,q}\right)} \sum_n \sum_{k=0}^{L-1} D^*(n,k) R(n,k) \psi^*_{p,q}(n,k) \tag{14}$$

4.1 Computation of $\Gamma^{(m)}(n,k)$ for QPSK Signaling

If a=(±1 ± j) represents independent identically distributed data sequence modulating the QPSK carrier, $\Gamma^{(m)}(n,k)$ in (13) can be expressed as follows.

$$\Gamma^{(m)}(n,k) = \frac{\sum_{a \in S_{n,k}} a^* P\left(R(n,k)|A(n,k)=a,\mathbf{G}^{(m)}\right) P\left(A(n,k)=a\right)}{\sum_{a \in S_{n,k}} P\left(R(n,k)|A(n,k)=a,\mathbf{G}^{(m)}\right) P\left(A(n,k)=a\right)} \tag{15}$$

From (11) it follows that

$$\Gamma^{(m)}(n,k) = \frac{\sum_{a \in S_{n,k}} a^* exp\left(\frac{2}{\sigma^2} Re\left[a^* Z^m(n,k)\right]\right)}{\sum_{a \in S_{n,k}} exp\left(\frac{2}{\sigma^2} Re\left[a^* Z^m(n,k)\right]\right)} \tag{16}$$

where

$$Z^{m}(n,k)=R(n,k)\sum_{l}\sum_{l}G^{(m)*}(i,j)\psi^{*}_{i,l}(n,k).$$

Then, taking summations in the numerator and the denumerator of (16) over the values of QPSK symbols a, we have the final result as follows.

$$\Gamma^{(m)}(n,k)=\tanh\left[\frac{2}{\sigma^{2}}\operatorname{Re}\left(Z^{m}(n,k)\right)\right]-j\tanh\left[\frac{2}{\sigma^{2}}\operatorname{Im}\left(Z^{m}(n,k)\right)\right] \tag{17}$$

Note that the Modified-Cramer-Rao-Bound (MCRM) can be derived for the estimated random parameters *{G(i,j)}* as follows. Performing the derivatives in (8) and (9) with respect to *{G(i,j)}*, taking expectations over ***R***, ***A*** and ***G*** and then taking into fact that the eigenfunctions $\psi_{i,l}(n,k)$ are orthogonal, it follows that

$$\text{MCRB(G(p,q))}=2\,(1/\sigma^{2}+1/\lambda_{p,q})^{-1} \tag{18}$$

where σ^2 is the noise variance and $\lambda_{p,q}$ are the eigenvalues of the discrete autocorrelation function $r(n,k;n',k')$

5. SIMULATION RESULTS

The performance of the proposed EM based ML channel estimation technique was evaluated as a function of signal-to-noise ratio (SNR) based on the Monte Carlo simulations. We considered the fading multipath channel with an exponentially decaying power delay profile $\Theta(\tau_l)=C\exp(-\tau_l/\tau_{max})$ per path delays τ_l that are uniformly and independently distributed over the length of the cyclic prefix. C is a normalizing constant. Note that the normalized discrete channel-correlations for different subcarriers of this channel model was presented in [3] as follows:

$$r_2(k,k')=\frac{1-\exp\left[\frac{1}{\tau_{rms}}+\frac{2\pi j(k-k')}{N}\right]}{\tau_{rms}\left(1-\exp(-L/\tau_{rms})\right)\left(\frac{1}{\tau_{rms}}+\frac{j2\pi(k-k')}{N}\right)}.$$

The discrete channel correlations for different block is given by (3) The scenario for our simulation study consists of a wireless QPSK OFDM system operating with a *500* kHz bandwidth and is divided into 16 tones with a total symbol period (T_s) of *136* µs, of which *27* µs constitute the cyclic prefix ($L=4$). The uncoded data rate of the system is *0.24* Mbit/s. We assume that the *rms* width is $\tau_{rms}=1$ sample (6.8 µs) for the power-delay profile and the dopler frequency is $f_d=100$ *Hz*.

The proposed algorithm was tested for 100 Monte Carlo trials per SNR point accross a range of SNRs (5-15 dB). The average SNR was defined as $E[|H(n,k)|^2]E[|A(n,k)|^2]/\sigma^2$. Since $E[|A(n,k)|^2]=1$ for QPS signaling and $E[|H(n,k)|^2]=1$ for the normalized frequency response of the fading channel, the normalized SNR simply becomes $1/\sigma^2$, where σ^2 is the variance of the complex white Gaussian noise entering the system. The initial values of $G^{(0)}(n,k)$'s were according to (14). Root Mean-square-error (RMSE) is defined as the difference between the matrices

$\boldsymbol{G}=[G(n,k)]$ and $\hat{\boldsymbol{G}}=[\ \hat{G}(n,k)\ ]$, representing the true and the estimated values of channel parameters, respectively . Namely,

$$RMSE=\left\|\mathrm{G}-\hat{\mathrm{G}}\right\|=\left(\sum_{n=0}^{N-1}\sum_{k=0}^{K-1}\left(G(n,k)-G(\hat{n},k)\right)^{2}\right)^{\frac{1}{2}}.$$

In each trial, the RMS of the estimation error for the channel parameters were recorded. In Fig. 1, we have plotted the experimental estimation RMS error as well as the corresponding modified CRBs. Fig. 2 shows the estimation RMS error experienced by the proposed technique at each iteration (SNR=10dB and SNR=20 dB respectively). Based on the experimental results, we made the following observations:

-Since MCRB given by (18) provides an approximate bound, it is not tight however it is much easier to compute.

-For low SNR, the proposed approach requires more iterations to converge. It is concluded from Fig.2 that the MSE performance of the EM-based algorithm converges within 3-10 iterations, depending on the SNR.

6. CONCLUSIONS

In this paper, we proposed an optimum channel estimation algorithm for OFDM systems. This algorithm performs an iterative estimation of the channel according to the MAP criterion, using the EM algorithm employing M-PSK modulation scheme with additive Gaussian noise. The discrete multipath channel was represented in terms of a Karhunen-Loeve expansion which makes full use of time and frequency-domain correlations of the frequency response of the time-varying dispersive fading channel. A non-data aided estimation scheme was considered for time-varying channel estimation by taking averaging over the M-PSK signal constellation. For this, an EM algorithm is derived which estimates the complex channel parameters of each sub carriers iteratively in frequency domain and which converges to the true MAP estimation of the unknown channel. The algorithm is applied to the QPSK modulated OFDM systems and efficiency of the algorithm proposed is shown by the computer simulations.

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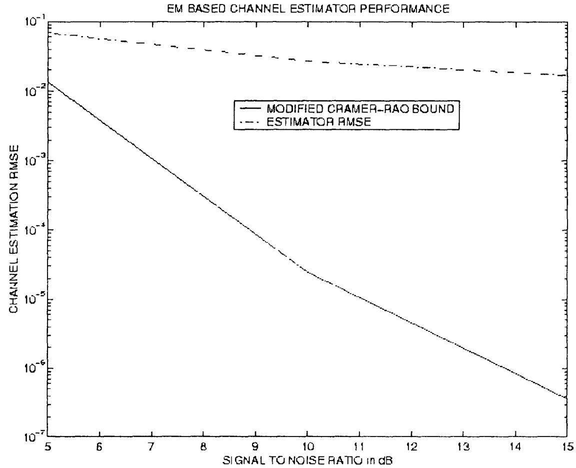

Figure 1. Performance of the proposed method

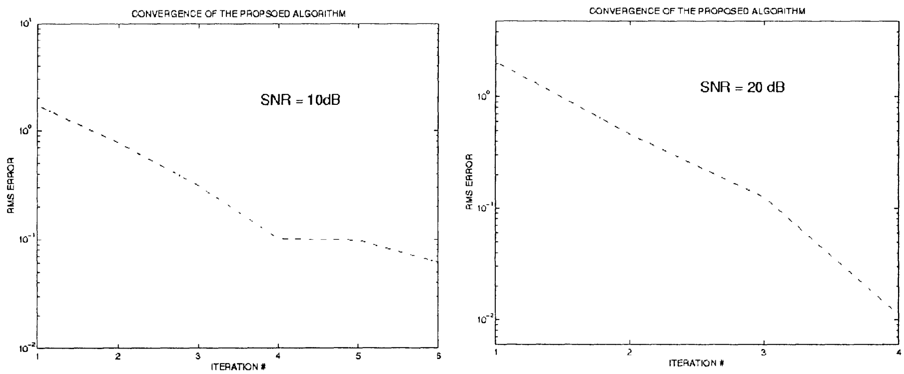

Figure 2. Convergence of the proposed method (SNR = 10 & 20 dB)

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