TWMS J. App. Eng. Math. V.5, N.1, 2015, pp. 1-9.

ON GENERALIZED $\phi-$RECURRENT KENMOTSU MANIFOLDS

S. K. CHAUBEY[1], C. S. PRASAD[2], §

ABSTRACT. The aim of the present paper is to study the properties of generalized $\phi-$recurrent and concircular $\phi-$recurrent Kenmotsu manifolds.

Keywords: Generalized $\phi-$recurrent, generalized concircular $\phi-$recurrent, $\eta-$Einstein and Kenmotsu manifolds.

AMS Subject Classification: $53C25$, $53C35$, $53D10$.

1. INTRODUCTION

Let M_n be an $n-$dimensional differentiable manifold of differentiability class C^{r+1} with a $(1,1)$ tensor field ϕ, the associated vector field ξ, a contact form η and the associated Riemannian metric g. In 1958, Boothby and Wong [12] studied odd dimensional manifolds with contact and almost contact structures from topological point of view. Sasaki and Hatakeyama [16] re-investigated them using tensor calculus in 1961. In 1972, K. Kenmotsu studied a class of contact Riemannian manifolds and call them Kenmotsu manifold [4]. He proved that if a Kenmotsu manifold satisfies the condition $R(X,Y).R = 0$, then the manifold is of negative curvature -1, where R is the Riemannian curvature tensor of type $(1,3)$ and $R(X,Y)$ denotes the derivation of the tensor algebra at each point of the tangent space. It is well known that odd dimensional spheres admit Sasakian structure whereas odd dimensional hyperbolic spaces can not admit Sasakian structures, but have so called Kenmotsu structure. Kenmotsu manifolds are locally isometric to warped product spaces with one dimensional base and Kaehler fiber. In 1963, Kobayashi and Nomizu [12] shown that any two simply connected complete Riemannian manifolds of constant curvature k are isometric to each other. A space form (i.e., a complete simply connected Riemannian manifold of constant curvature) is said to be elliptic, hyperbolic or Euclidean according as the sectional curvature tensor is positive, negative or zero [6]. The properties of Kenmotsu manifolds have studied by several authors such as De and Pathak [3], Sinha and Srivastava [13], Jun, De and Pathak [14], De, Yildiz and Yaliniz [15], Chaubey and Ojha [17], De [18], Cihan [19] and many others.

The notion of locally $\phi-$symmetric Sasakian manifold was introduced by T. Takahashi [2] in 1977 and obtained some interesting properties. Some authors like De and Pathak [3], Venkatesha and Bagewadi [11] have extended this notion to $3-$dimensional Kenmotsu, Trans-Sasakian manifolds respectively.

[1] Department of Mathematics, Galgotias University, Greater Noida U.P.201308, India.
e-mail: sk22_math@yahoo.co.in;
[2] Department of Mathematics, D.B.S.(P.G.) College, Kanpur 208006, India.
e-mail: csprasd@gmail.com;
§ Manuscript received: September 03, 2013.

2. Preliminaries

If on an $n-$dimensional differentiable manifold M_n, $(n = 2m + 1)$, of differentiability class C^{r+1}, there exists a vector valued real linear function ϕ, a $1-$form η, the associated vector field ξ and the Riemannian metric g satisfying

$$(a) \quad \phi^2 X = -X + \eta(X)\xi, \quad (b) \quad \eta(\phi X) = 0, \tag{1}$$

and

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2}$$

for arbitrary vector fields X and Y, then (M_n, g) is said to be an almost contact metric manifold and the structure $\{\phi, \eta, \xi, g\}$ is called an almost contact metric structure to M_n [1].

In view of (1) (a), (1) (b) and (2), we find

$$\eta(\xi) = 1, \quad g(X, \xi) = \eta(X), \quad \phi(\xi) = 0 \tag{3}$$

and

$$g(X, \phi Y) + g(\phi X, Y) = 0. \tag{4}$$

An almost contact metric manifold $(M_n, \phi, \xi, \eta, g)$ is called a Kenmotsu manifold [4] if the relation

$$(D_X \phi)(Y) = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{5}$$

satisfies for arbitrary vector fields X and Y. Also, the following relations hold in a Kenmotsu manifold [3], [4]:

$$D_X \xi = X - \eta(X)\xi, \tag{6}$$

$$(D_X \eta)(Y) = g(\phi X, \phi Y), \tag{7}$$

$$\eta(R(X, Y)Z) = \eta(Y)g(X, Z) - \eta(X)g(Y, Z), \tag{8}$$

$$R(X, Y)\xi = \eta(X)Y - \eta(Y)X, \tag{9}$$

$$R(\xi, X)Y = \eta(Y)X - g(X, Y)\xi, \tag{10}$$

$$S(\phi X, \phi Y) = S(X, Y) + 2m\eta(X)\eta(Y), \tag{11}$$

$$S(X, \xi) = -2m\eta(X), \tag{12}$$

$$(D_Z R)(X, Y)\xi = g(Z, X)Y - g(Z, Y)X - R(X, Y)Z, \tag{13}$$

for arbitrary vector fields X, Y, Z.

A Riemannian manifold M_n is said to be $\eta-$Einstein if its Ricci tensor S takes the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y), \tag{14}$$

for arbitrary vector fields X and Y, where a and b are smooth functions on (M_n, g) [1], [4]. If $b = 0$, then $\eta-$Einstein manifold becomes Einstein manifold.

3. Generalized $\phi-$recurrent Kenmotsu manifolds

Analogous of consideration of generalized recurrent manifolds [5], we give the following definition.

Definition 3.1 A Kenmotsu manifold is said to be generalized $\phi-$recurrent if its curvature tensor R satisfies the relation

$$\phi^2((D_W R)(X,Y)Z) = A(W)R(X,Y)Z + B(W)\{g(Y,Z)X - g(X,Z)Y\}, \qquad (15)$$

where A and B are $1-$forms, B is non-zero and these are defined by

$$A(W) = g(W,\rho_1), \quad B(W) = g(W,\rho_2), \qquad (16)$$

and ρ_1, ρ_2 are unit vector fields associated with $1-$forms A, B respectively. If the vector fields X, Y, Z, W are orthogonal to ξ, then the manifold is called local generalized $\phi-$recurrent manifold. In [21], [22], authors studied the properties of generalized recurrent manifolds whereas the properties of generalized $\phi-$recurrent manifolds have studied in [8], [9], [10] and [20]. If the $1-$form B in (15) becomes zero, then the manifold reduces to a $\phi-$recurrent Kenmotsu manifold. The properties of $\phi-$recurrent Kenmotsu manifolds have studied by De, Yildiz and Yaliniz [15] and many others. In consequence of (1), equation (15) becomes

$$\begin{aligned} -(D_W R)(X,Y)Z \quad &+ \quad \eta((D_W R)(X,Y)Z)\xi \\ &= A(W)R(X,Y)Z + B(W)\{g(Y,Z)X - g(X,Z)Y\} \end{aligned} \qquad (17)$$

from which it follows by taking inner product with U that

$$\begin{aligned} -g((D_W R)(X,Y)Z,U) \quad &+ \quad \eta((D_W R)(X,Y)Z)\eta(U) = A(W)g(R(X,Y)Z,U) \\ &+B(W)\{g(Y,Z)g(X,U) - g(X,Z)g(Y,U)\}. \end{aligned} \qquad (18)$$

Let $\{e_i\}$, $i = 1,2,...,n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X = U = e_i$ in (18) and taking summation over i, $1 \le i \le n$, we get

$$-(D_W S)(Y,Z)+\sum_{i=1}^{n}\eta((D_W R)(e_i,Y)Z)\eta(e_i) = A(W)S(Y,Z)+(n-1)B(W)g(Y,Z). \qquad (19)$$

Again replacing Z by ξ in (19) and using (3) and (12), we get

$$-(D_W S)(Y,\xi) + \sum_{i=1}^{n}\eta((D_W R)(e_i,Y)\xi)\eta(e_i) = 2m\{-A(W) + B(W)\}\eta(Y). \qquad (20)$$

The second term of left hand side in (20) takes the form

$$\eta((D_W R)(e_i,Y)\xi)\eta(e_i) = g((D_W R)(e_i,Y)\xi,\xi)g(e_i,\xi),$$

which is denoted by E. In this case, we have proved that E vanishes. Namely, we have

$$\begin{aligned} g((D_W R)(e_i,Y)\xi,\xi) \quad &= \quad g(D_W R(e_i,Y)\xi,\xi) - g(R(D_W e_i,Y)\xi,\xi) \\ &-g(R(e_i,D_W Y)\xi,\xi) - g(R(e_i,Y)D_W\xi,\xi) \end{aligned} \qquad (21)$$

at $p \in M_n$. In local coordinates $D_X e_i = X^j\Gamma^h_{ji}e_h$, where Γ^h_{ji} are Christofell symbols. Since $\{e_i\}$ is an orthonormal basis, the metric tensor $g_{ij} = \delta_{ij}$, where δ_{ij} is Kronecker delta and hence the Christofell symbols are zero. Therefore, $D_X e_i = 0$. Also we have

$$g(R(e_i,D_W Y)\xi,\xi) = 0, \qquad (22)$$

because R is skew-symmetric. Using (22) and $D_X e_i = 0$ in (21), we find

$$g((D_W R)(e_i,Y)\xi,\xi) = g(D_W R(e_i,Y)\xi,\xi) - g(R(e_i,Y)D_W\xi,\xi).$$

In consequence of $g(R(e_i, Y)\xi, \xi) = -g(R(\xi, \xi)Y, e_i) = 0$, above equation gives

$$g(D_W R(e_i, Y)\xi, \xi) + g(R(e_i, Y)\xi, D_W\xi) = 0, \tag{23}$$

which implies

$$g((D_W R)(e_i, Y)\xi, \xi) = -g(R(e_i, Y)D_W\xi, \xi) - g(R(e_i, Y)\xi, D_W\xi). \tag{24}$$

By the skew-symmetric properties of R, (24) becomes

$$g((D_W R)(e_i, Y)\xi, \xi) = 0. \tag{25}$$

In view of (25), (20) becomes

$$(D_W S)(Y, \xi) = 2m(A(W) - B(W))\eta(Y). \tag{26}$$

It is well known that

$$(D_W S)(Y, \xi) = D_W S(Y, \xi) - S(D_W Y, \xi) - S(Y, D_W\xi).$$

The last equation with (3), (6), (7) and (12) gives

$$(D_W S)(Y, \xi) = -2mg(Y, W) - S(Y, W). \tag{27}$$

From equations (26) and (27), we find

$$S(Y, W) = -2mg(Y, W) + 2m(B(W) - A(W))\eta(Y). \tag{28}$$

Replacing Y by ϕY in (28) and then using (1) (b), we get

$$S(\phi Y, W) = -2mg(\phi Y, W). \tag{29}$$

Again replacing W by ϕW in (29) and using (2) and (11), we find

$$S(Y, W) = -2mg(Y, W), \quad for\ all\ Y, W \in \chi(M_n). \tag{30}$$

Hence, we can state the following corollary:

Corollary 3.1. *A generalized $\phi-$recurrent Kenmotsu manifold is an Einstein manifold.*

Two vector fields P and Q are said to be co-directional if $P = fQ$, where f is a non-zero scalar, i.e., $g(P, X) = fg(Q, X)$ for all $X \in \chi(M_n)$. Now from (17), we have

$$\begin{aligned} -(D_W R)(X, Y)Z \quad &= \quad -\eta((D_W R)(X, Y)Z)\xi + A(W)R(X, Y)Z \\ &\quad +B(W)\{g(Y, Z)X - g(X, Z)Y\}. \end{aligned} \tag{31}$$

Taking inner product of (31) with ξ and then using (3) and (8), we get

$$A(W) - B(W) = 0. \tag{32}$$

Thus, we can state the following corollary:

Corollary 3.2. *In a generalized $\phi-$recurrent Kenmotsu manifold (M_n, g), the associated vector fields ρ_1 and ρ_2 of the $1-$forms A and B respectively are co-directional.*

By virtue of (3), (17) and Bianchi's identity, we get

$$\begin{aligned} A(W)\eta(R(X, Y)Z) \quad + \quad & A(X)\eta(R(Y, W)Z) + A(Y)\eta(R(W, X)Z) \\ & +B(W)\{\eta(X)g(Y, Z) - \eta(Y)g(X, Z)\} \\ & +B(X)\{\eta(Y)g(W, Z) - \eta(W)g(Y, Z)\} \\ & +B(Y)\{\eta(W)g(X, Z) - \eta(X)g(W, Z)\} = 0. \end{aligned} \tag{33}$$

In consequence of (8), (33) gives

$$\{-A(W)+B(W)\}[\eta(X)g(Y,Z)-\eta(Y)g(X,Z)] \\ +\{-A(X)+B(X)\}[\eta(Y)g(W,Z)-\eta(W)g(Y,Z)] \\ +\{-A(Y)+B(Y)\}[\eta(W)g(X,Z)-\eta(X)g(W,Z)]=0. \tag{34}$$

Putting $Y=Z=e_i$ in (34) and taking summation over i, $1\leq i\leq 2m+1$, we get

$$\{-A(W)+B(W)\}\eta(X)=\{-A(X)+B(X)\}\eta(W), \tag{35}$$

for all vector fields X, $W\in\chi(M_n)$. Again replacing X by ξ in (35) and then using (3), we get

$$(-A(W)+B(W))=(-\eta(\rho_1)+\eta(\rho_2))\eta(W), \tag{36}$$

for any vector field W, where $A(\xi)=g(\rho_1,\xi)=\eta(\rho_1)$ and $B(\xi)=g(\rho_2,\xi)=\eta(\rho_2)$. Thus, in view of equations (35) and (36), we state the following theorem:

Theorem 3.1. *In a generalized $\phi-$recurrent Kenmotsu manifold (M_n,g) $(n>2)$, the characteristic vector field ξ and the vector field $\rho_1+\rho_2$ associated to the $1-$form $A+B$ are co-directional.*

From (17), we have

$$(D_WR)(X,Y)Z = \eta((D_WR)(X,Y)Z)\xi-A(W)R(X,Y)Z \\ -B(W)\{g(Y,Z)X-g(X,Z)Y\}. \tag{37}$$

Taking cyclic sum of (37) in W, X, Y and then using Bianchi's second identity and (32), we have

$$A(X)R(Y,W)Z+A(Y)R(W,X)Z+A(W)R(X,Y)Z \\ +B(W)[g(Y,Z)X-g(X,Z)Y]+B(X)[g(W,Z)Y \\ -g(Y,Z)W]+B(Y)[g(X,Z)W-g(W,Z)X]=0. \tag{38}$$

Contracting above equation with respect to X and then replacing Y and Z by e_i, $1\leq i\leq n$, we find

$$r=-2m(2m-1)\frac{B(W)}{A(W)}, \tag{39}$$

where $r\overset{\text{def}}{=}\sum_{i=1}^{n}S(e_i,e_i)$ is the scalar curvature of the manifold. Hence we can state the following theorem:

Theorem 3.2. *In a generalized $\phi-$recurrent Kenmotsu manifold (M_{2m+1},g), the scalar curvature satisfies the relation (39).*

Taking inner product of (38) with U, we get

$$A(X)'R(Y,W,Z,U)+A(Y)'R(W,X,Z,U)+A(W)'R(X,Y,Z,U) \\ +B(W)[g(Y,Z)g(X,U)-g(X,Z)g(Y,U)]+B(X)[g(W,Z)g(Y,U) \\ -g(Y,Z)g(W,U)]+B(Y)[g(X,Z)g(W,U)-g(W,Z)g(X,U)]=0. \tag{40}$$

Now putting $Y=Z=e_i$ and $X=U=e_i$ respectively in the above equation and then taking summation over i, $1\leq i\leq n$, we get

$$S(W,\rho_1)=\frac{r+(n-1)(n-2)}{2}A(W).$$

Hence, we can state the following theorem:

Theorem 3.3. *In a generalized $\phi-$recurrent Kenmotsu manifold (M_n, g), $\frac{r+(n-1)(n-2)}{2}$ is the eigen value of the Ricci tensor corresponding to the eigen vector ρ_1, where ρ_1 is defined as in (16).*

From (13), we have

$$(D_W R)(X,Y)\xi = g(X,W)Y - g(Y,W)X - R(X,Y)W. \tag{41}$$

Taking inner product of (41) with ξ and using (3) and (8), we get

$$\eta((D_W R)(X,Y)\xi) = 0. \tag{42}$$

In consequence of (9) and (42), (17) becomes

$$-(D_W R)(X,Y)\xi = [A(W) - B(W)]R(X,Y)\xi. \tag{43}$$

Hence from (41) and (43), we find

$$R(X,Y)W = g(X,W)Y - g(Y,W)X - [A(W) - B(W)]R(X,Y)\xi. \tag{44}$$

Thus if X and Y are orthogonal to ξ, then (44) gives

$$R(X,Y)W = g(X,W)Y - g(Y,W)X. \tag{45}$$

for all X, Y, $W \in \chi(M_n)$. A space form (i.e., a complete simply connected Riemannian manifold of constant curvature) is said to be elliptic, hyperbolic or Euclidean according as the sectional curvature tensor is positive, negative or zero [6]. Hence we can state the following theorem:

Theorem 3.4. *A locally generalized $\phi-$recurrent Kenmotsu manifold (M_n, g) is locally isometric to a hyperbolic space $H^n(-1)$.*

4. Three dimensional locally generalized $\phi-$recurrent Kenmotsu manifolds

The curvature tensor in a three dimensional Kenmotsu manifold assume the form [3]

$$R(X,Y)Z = \frac{(r+4)}{2}\{g(Y,Z)X - g(X,Z)Y\}$$
$$-\frac{(r+6)}{2}\{g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y\}, \tag{46}$$

where r is the scalar curvature of the manifold (M_n, g). The covariant differentiation of (46) along W consider the form

$$(D_W R)(X,Y)Z = \frac{dr(W)}{2}\{g(Y,Z)X - g(X,Z)Y\}$$
$$-\frac{dr(W)}{2}\{g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y\}$$
$$-\frac{(r+6)}{2}[g(Y,Z)(D_W\eta)(X)\xi + g(Y,Z)\eta(X)D_W\xi - g(X,Z)(D_W\eta)(Y)\xi$$
$$-g(X,Z)\eta(Y)D_W\xi + (D_W\eta)(Y)\eta(Z)X + (D_W\eta)(Z)\eta(Y)X$$
$$-(D_W\eta)(X)\eta(Z)Y - (D_W\eta)(Z)\eta(X)Y]. \tag{47}$$

Operating ϕ^2 on either sides of (47) and using (1), (3), (6) and (7), we get

$$\phi^2((D_WR)(X,Y)Z) = -\frac{dr(W)}{2}[g(Y,Z)X - g(X,Z)Y - g(Y,Z)\eta(X)\xi$$
$$+g(X,Z)\eta(Y)\xi - \eta(Y)\eta(Z)X + \eta(X)\eta(Z)Y] + \frac{(r+6)}{2}[(D_W\eta)(Y)\eta(Z)X$$
$$+\eta(Y)(D_W\eta)(Z)X - (D_W\eta)(X)\eta(Z)Y + g(Y,Z)\eta(X)W$$
$$-(D_W\eta)(Z)\eta(X)Y - (D_W\eta)(Y)\eta(Z)\eta(X)\xi - (D_W\eta)(X)\eta(Z)\eta(Y)\xi$$
$$-g(Y,Z)\eta(X)\eta(W)\xi - g(X,Z)\eta(Y)W + g(X,Z)\eta(Y)\eta(W)\xi]. \tag{48}$$

If X, Y, Z, W are orthogonal to ξ, then in view of (15), (48) assumes the form

$$A(W)R(X,Y)Z = -\left\{B(W) + \frac{dr(W)}{2}\right\}[g(Y,Z)X - g(X,Z)Y]. \tag{49}$$

Replacing W by $\{e_i\}$ in (49), where $\{e_i\}$, $(i = 1,2,3)$, is an orthonormal basis of the tangent space at any point of the manifold and taking summation over i, $1 \le i \le 3$, we obtain

$$R(X,Y)Z = \lambda\{g(Y,Z)X - g(X,Z)Y\},$$

where $\lambda = -\left\{\frac{2B(e_i)+dr(e_i)}{2A(e_i)}\right\}$ is a scalar, since A is non-zero 1−form. Then by Schur's theorem λ will be a constant on the manifold. Therefore (M_3, g) is of constant curvature λ. Hence we state the following theorem:

Theorem 4.1. *A three dimensional locally generalized ϕ−recurrent Kenmotsu manifold (M_3, g) is of constant curvature.*

From theorem (4.1), we can easily prove that a locally generalized ϕ−recurrent Kenmotsu manifold is conformally flat and hence $divC = 0$, where $'div'$ denote the divergence and C is the conformal curvature tensor defined as

$$V(X,Y)Z = R(X,Y)Z - \frac{1}{n-2}[S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY]$$
$$+\frac{r}{(n-1)(n-2)}[g(Y,Z)X - g(X,Z)Y].$$

Also it is known that [7] if a conformally flat Riemannian manifold (M_n, g) $(n > 3)$ admits a proper concircular vector field, then the manifold is subprojective manifold in the sense of Kagan. Hence we state the following theorem:

Theorem 4.2. *A locally generalized ϕ−recurrent Kenmotsu manifold (M_n, g) $(n > 3)$ is a subprojective manifold in the sense of Kagan.*

In 1944, K. Yano [7] proved that in order that a Riemannian manifold admits a concircular vector field, it is necessary and sufficient that there exists a coordinate system with respect to which the fundamental quadratic differential form assumes the form

$$ds^2 = (dx^1)^2 + e^p g^*_{ij} dx^i dx^j,$$

where $g^*_{ij} = g_{ij}[x^k]$ are functions of x^k only $(i,j,k = 2,3,..........n)$ and $p = p(x^1)$ is a non constant function of x^1 only. Hence, we can state the following theorem:

Theorem 4.3. *A locally generalized ϕ−recurrent Kenmotsu is warped product $I \times_{e^p} M^*$, where (M^*, g^*) is an $(n-1)$ dimensional Kenmotsu manifold.*

References

[1] Sasaki, S., (1965, 1967, 1968), Almost contact manifolds, I, II, III, A Lecture note, Tohoku University.
[2] Takahashi, T., (1977), Sasakian $\phi-$symmetric spaces, Tohoku Math. J., 29, pp. 91-113.
[3] De, U. C. and Pathak, G., (2004), On $3-$dimensional Kenmotsu manifolds, Indian J. pure Appl. Math., 35, pp. 159-165.
[4] Kenmotsu, K., (1972), A class of almost contact Riemannian manifolds, Tohoku Math. J., 24, pp. 93-103.
[5] De, U. C. and Guha, N., (1991) , On generalized recurrent manifolds, Proceedings of the Mathematical Society, 7, pp. 7-11.
[6] Chen, B. Y., (1973), Geometry of submanifolds, M. Dekker Inc., New York.
[7] Yano, K., (1944), On the torseforming direction in Riemannian spaces, Proc. Imp. Acad. Tokyo, 20, pp. 340-345.
[8] Chaubey, S. K., (2013), On generalized $\phi-$recurrent trans-Sasakian manifolds, (to appear).
[9] Patil, D. A., Prakasha, D. G. and Bagewadi, C. S., (2009), On generalized $\phi-$recurrent Sasakian manifolds, Bull. of Math. Anal. and Appl., 1 (3), pp. 42-48.
[10] Jaiswal, J. P. and Ojha, R. H., (2009) , On generalized $\phi-$recurrent LP-Sasakian manifolds, Kyungpook Math. J., 49, pp. 779-788.
[11] Venkatesha and Bagewadi, C. S., (2005), On $3-$dimensional trans-Sasakian manifolds, AMSE, 42(5), pp. 63-73.
[12] Kobayashi, K. and Nomizu, K., (1963), Foundations of Differential Geometry, I, II, Wiley-Interscience, New York.
[13] Sinha, B. B. and Srivastava, A. K., (1991), Curvatures on Kenmotsu manifold, Indian J. Pure Appl. Math., 22, (1), pp. 23-28.
[14] Jun, J. B., De, U. C. and Pathak, G., (2005), On Kenmotsu manifolds, J. Korean Math. Soc., 42, pp. 435-445.
[15] De, U. C., Yildiz, A. and Yaliniz, Funda, (2008), On $\phi-$recurrent Kenmotsu manifolds, Turk J. Math., 32, pp. 1-12.
[16] Sasaki, S. and Hatakeyama, Y., (1961), On differentiable manifolds with certain structures which are closely related to almost contact structure, Tohoku Math. J., 13, pp. 281-294.
[17] Chaubey, S. K. and Ojha, R. H., (2010), On m-projective curvature tensor of a Kenmotsu manifold, Diff. Geom. Dyn. Sys., 12, pp. 52-60.
[18] De, U. C., (2008), On $\phi-$symmetric Kenmotsu manifolds, International Electronic J. of Geom., 1 (1), pp. 33-38.
[19] Cihan, Ö . and De, U. C., (2006), On the quasi-conformal curvature tensor of a Kenmotsu manifolds, Mathematica Pannonica, 17/2, pp. 221-228.
[20] Basari, A. and Murathan, C., (2008), On generalized $\phi-$recurrent Kenmotsu manifolds, Fen Derg. 3(1), pp. 91-97.
[21] De, U. C. and Guha, N., (1991), On generalized recurrent manifolds, J. Nat. Acad. Math. India, 9, pp. 85-92.
[22] Cihan, Ö ., (2007), On generalized recurrent Kenmotsu manifolds, Word Applied Sci. J., 2(1), pp. 29-33.

Sudhakar Kumar Chaubey is an Assistant Professor in the Department of Mathematics at Galgotias University, India. He received his Ph. D. (2008) degree in Differential geometry of differentiable manifolds and its applications from Banaras Hindu University, India. His research interests include differential geometry of differentiable manifolds, semi-Riemannian manifolds and linear connections.

Chandra Shekhar Prasad is Associate Professor in the Department of Mathematics D.B.S.P.G. College Kanpur (U.P.) India. He graduated from Lucknow University Lucknow India in 1980 and received his Ph.D. degree from the same University in 1987. He worked on three research projects and guided five students for the award of Ph.D. and M.Phil. degree in mathematics. His research work is in the field of Differential Geometry of Differentiable Manifolds and their Applications.