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COMPUTATION OF CONNECTIVITY INDICES OF KULLI PATH WINDMILL GRAPH

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Abstract. The Kulli path windmill graph $P_{n+1}^{(m)}$ is the graph obtained by taking $m \geq 2$ copies of the graph $K_1 + P_n$ for $n \geq 4$ with a vertex K_1 in common. In this paper, we determine Zagreb, hyper-Zagreb, sum connectivity, general sum connectivity, Randic connectivity, General Randic connectivity, atom-bond connectivity, geometric-arithmetic, harmonic and symmetric division deg indices of Kulli path windmill graph.

Keywords: Topological indices; Degree based connectivity indices; Windmill graph and Kulli path windmill graph.

AMS Subject Classification: 05C05, 05C07, 05C35.

1. Introduction

Throughout this paper, we consider simple graphs which are finite, undirected without loops and multiple edges. Let $G = (V, E)$ be a connected graph with vertex set $V = V(G)$ and edge set $E = E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. For other undefined notations and terminologies from graph theory, the reader are referred to [7].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Chemical graph theory is a branch of Mathematical chemistry which has an important effect on the development of the chemical sciences. A single number that can be used to characterize some property of the graph of a molecular is called a topological index for that graph. There are numerous topological descriptors that have found some applications in theoretical chemistry, especially in QSPR/QSAR research.

In [6], the first and second Zagreb indices were introduced to take account of the contributions of pairs of adjacent vertices. The first and second Zagreb indices of a graph G are defined as $M_1(G) = \sum_{v\in V(G)} d_G(v)^2$ or $M_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]$ and $M_2(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]$.

In [11], Shirdel et al., introduced the first hyper Zagreb index $HM_1(G)$ of a graph G. This index is defined as $HM_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]^2$. In [4], the second hyper Zagreb index $HM_2(G)$ of a graph G is defined as $HM_2(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]^2$.

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The Randic index or product connectivity index of a graph G is defined as $\chi(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}}$. This topological index was proposed by Randic in [10].

The sum connectivity index of a graph G is defined as $X(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)+d_G(v)}}$. This topological index was proposed by Zhou and Trinajstic in [14].

The general Randic connectivity index or second K_a index of a graph G is defined as $\chi^a(G) = \sum_{uv\in E(G)}[d_G(u)d_G(v)]^a$. The general sum connectivity index or first K_a index of a graph G is defined as $X^a(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]^a$. The above two topological indices were proposed in [1], [6] and [8].

In [2], Estrada et al. introduced the atom-bond connectivity index, which is defined as $ABC(G) = \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}}$.

The geometric-arithmetic index of a graph G is defined as $GA(G) = \sum_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)}$. This index was proposed by Vukicevic and Furtula in [12].

The harmonic index of a graph G is defined on the arithmetic mean as $H(G) = \sum_{uv\in E(G)} \frac{2}{d_G(u)+d_G(v)}$. This index was first appeared in [3].

In [13], Vukicevic and Gasperov posed the symmetric division deg index of a graph G, which is defined as

$$SDD(G) = \sum_{uv\in E(G)} \frac{max\,(d_G(u), d_G(v))}{min(d_G(u), d_G(v))} + \frac{min\,(d_G(u), d_G(v))}{max(d_G(u), d_G(v))} = \sum_{uv\in E(G)} \frac{d_G(u)^2 + d_G(v)^2}{d_G(u)d_G(v)}.$$

The Kulli path windmill graph $P_{n+1}^{(m)}$ is the graph obtained by taking $m \geq 2$ copies of the graph $K_1 + P_n$ for $n \geq 4$ with a vertex K_1 in common. This graph is shown in Figure-1. The Kulli path windmill graph $P_{2+1}^{(m)}$ is a friendship graph and it is denoted by $F_3^{(m)}$. The Kulli path windmill graph $P_{3+1}^{(m)}$ is the first Kulli path windmill graph. For more details on french windmill graph $F_n^{(m)}$ and Kulli cycle windmill graph $C_{n+1}^{(m)}$, refer to [5] and [9], respectively. In this paper, we consider only the Kulli path windmill graphs $P_{n+1}^{(m)}$ for $m \geq 2$ and $n \geq 4$.

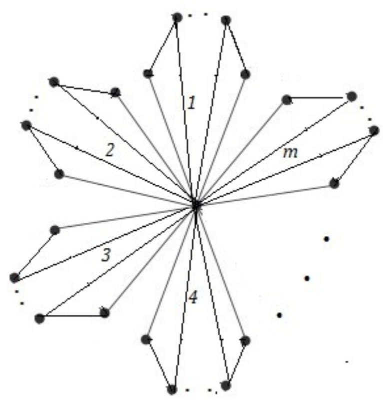

FIGURE 1. Kulli path windmill graph $P_{n+1}^{(m)}$.

2. Results

Theorem 2.1. *The sum connectivity index of Kulli path windmill graph is*

$$\begin{aligned} X(P_{n+1}^{(m)}) &= \left[\frac{2}{\sqrt{5}} - \frac{3}{\sqrt{6}} + \frac{2}{\sqrt{mn+2}} - \frac{2}{\sqrt{mn+3}}\right] m \\ &+ \left[\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{mn+3}}\right] mn. \end{aligned}$$

Proof. Let $G = P_{n+1}^{(m)}$, where $P_{n+1}^{(m)}$ is a Kulli path windmill graph. By algebraic method, we have $|V(G)| = mn + 1$ and $|E(G)| = 2mn - m$. We have three partitions of the vertex set $V(G)$ as follows:
$V_2 = \{v \in V(G) : d_G(v) = 2\}; |V_2| = 2m$,
$V_3 = \{v \in V(G) : d_G(v) = 3\}; |V_3| = mn - 2m$, and
$V_{mn} = \{v \in V(G) : d_G(v) = mn\}, |V_{mn}| = 1$.
Also we have four partitions of the edge set $E(G)$ as follows:
$E_5 = \{uv \in E(G) : d_G(u) = 2, d_G(v) = 3\}; |E_5| = 2m$,
$E_6 = \{uv \in E(G) : d_G(u) = 3, d_G(v) = 3\}; |E_6| = mn - 3m$,
$E_{mn+2} = \{uv \in E(G) : d_G(u) = mn, d_G(v) = 2\}; |E_{mn+2}| = 2m$, and
$E_{mn+3} = \{uv \in E(G) : d_G(u) = mn, d_G(v) = 3\}; |E_{mn+3}| = mn - 2m$. Now

$$\begin{aligned} X(G) &= \sum_{uv \in E(G)} \frac{1}{\sqrt{d_G(u) + d_G(v)}} \\ &= \sum_{uv \in E_5} \frac{1}{\sqrt{2+3}} + \sum_{uv \in E_6} \frac{1}{\sqrt{3+3}} + \sum_{uv \in E_{mn+2}} \frac{1}{\sqrt{mn+2}} \\ &+ \sum_{uv \in E_{mn+3}} \frac{1}{\sqrt{mn+3}} \\ &= \frac{1}{\sqrt{5}} \times 2m + \frac{1}{\sqrt{6}} \times (mn - 3m) + \frac{1}{\sqrt{mn+2}} \times 2m \\ &+ \frac{1}{\sqrt{mn+3}} \times (mn - 2m) \\ &= \left[\frac{2}{\sqrt{5}} - \frac{3}{\sqrt{6}} + \frac{2}{\sqrt{mn+2}} - \frac{2}{\sqrt{mn+3}}\right] m \\ &+ \left[\frac{1}{\sqrt{6}} + \frac{1}{\sqrt{mn+3}}\right] mn. \end{aligned}$$

□

Theorem 2.2. *The general sum connectivity index of Kulli path windmill graph is*

$$\begin{aligned} X^a(P_{n+1}^{(m)}) &= [2(5^a) - 3(6^a) + 2(mn+2)^a - 2(mn+3)^a]\, m \\ &+ [6^a + (mn+3)^a]\, mn. \end{aligned}$$

Proof. Let $G = P_{n+1}^{(m)}$ be a Kulli path windmill graph. Now

$$\begin{aligned} X^a(G) &= \sum_{uv\in E(G)} [d_G(u)+d_G(v)]^a \\ &= \sum_{uv\in E_5} [2+3]^a + \sum_{uv\in E_6} [3+3]^a \\ &+ \sum_{uv\in E_{mn+2}} [mn+2]^a + \sum_{uv\in E_{mn+3}} [mn+3]^a \\ &= 5^a \times 2m + 6^a \times (mn-3m) + (mn+2)^a \times 2m \\ &+ (mn+3)^a \times (mn-2m) \\ &= [2(5^a) - 3(6^a) + 2(mn+2)^a - 2(mn+3)^a]\, m \\ &+ [6^a + (mn+3)^a]\, mn. \end{aligned}$$

□

From the above Theorem, the following results are immediate

Corollary 2.1. *The first Zagreb index of $P_{n+1}^{(m)}$ is*

$$M_1(P_{n+1}^m) = (mn)^2 + 9mn - 10m.$$

Corollary 2.2. *The first hyper Zagreb index of $P_{n+1}^{(m)}$ is*

$$HM_1(P_{n+1}^m) = (mn)^3 + 6(mn)^2 + 41mn - 68.$$

Theorem 2.3. *The Randic index of Kulli path windmill graph is*

$$\chi(P_{n+1}^{(m)}) = \left[\frac{\sqrt{2}}{\sqrt{3}} - 1 + \frac{\sqrt{2}}{\sqrt{mn}} - \frac{2}{\sqrt{3mn}}\right] m + \left[\frac{1}{3} + \frac{1}{\sqrt{3mn}}\right] mn.$$

Proof. Let $G = P_{n+1}^{(m)}$, where $P_{n+1}^{(m)}$ is a Kulli path windmill graph. Now

$$\begin{aligned} \chi(G) &= \sum_{uv\in E(G)} \frac{1}{\sqrt{d_G(u)d_G(v)}} \\ &= \sum_{uv\in E_5} \frac{1}{\sqrt{2\times 3}} + \sum_{uv\in E_6} \frac{1}{\sqrt{3\times 3}} + \sum_{uv\in E_{mn+2}} \frac{1}{\sqrt{mn\times 2}} \\ &+ \sum_{uv\in E_{mn+3}} \frac{1}{\sqrt{mn\times 3}} \\ &= \frac{1}{\sqrt{6}} \times 2m + \frac{1}{\sqrt{9}} \times (mn-3m) + \frac{1}{\sqrt{2mn}} \times 2m \\ &+ \frac{1}{\sqrt{3mn}} \times (mn-2m) \\ &= \left[\frac{\sqrt{2}}{\sqrt{3}} - 1 + \frac{\sqrt{2}}{\sqrt{mn}} - \frac{2}{\sqrt{3mn}}\right] m + \left[\frac{1}{3} + \frac{1}{\sqrt{3mn}}\right] mn. \end{aligned}$$

□

Theorem 2.4. *The general Randic index of Kulli path windmill graph is*

$$\chi^a(P_{n+1}^{(m)}) = [2\times 6^a - 3^{2a+1} + 2^{a+1}(mn)^a - 2(3mn)^a]m + [9^a + (3mn)^a]mn.$$

Proof. Let $G = P_{n+1}^{(m)}$ be a Kulli path windmill graph. Now

$$\begin{aligned}
\chi^a(G) &= \sum_{uv\in E(G)} [d_G(u)d_G(v)]^a \\
&= \sum_{uv\in E_5} [2\times 3]^a + \sum_{uv\in E_6} [3\times 3]^a + \sum_{uv\in E_{mn+2}} [mn\times 2]^a \\
&+ \sum_{uv\in E_{mn+3}} [mn\times 3]^a \\
&= 6^a\times(2m) + 9^a\times(mn-3m) + (2mn)^a\times(2m) \\
&+ (3mn)^a(mn-2m) \\
&= [2\times 6^a - 3^{2a+1} + 2^{a+1}(mn)^a - 2(3mn)^a]m \\
&+ [9^a + (3mn)^a]mn.
\end{aligned}$$

□

From Theorem 2.4, we have the following results.

Corollary 2.3. *The second Zagreb index of* $P_{n+1}^{(m)}$ *is*

$$M_2(P_{n+1}^m) = 3(mn)^2 + 9mn - 2m^2n - 15m.$$

Corollary 2.4. *The second hyper Zagreb index of* $P_{n+1}^{(m)}$ *is*

$$HM_2(P_{n+1}^m) = 9(mn)^3 - 10m^3n + 81mn - 171m.$$

Theorem 2.5. *The atom-bond connectivity index of Kulli path windmill graph is*

$$ABC(P_{n+1}^{(m)}) = (\sqrt{2}-2)m + \frac{2}{3}mn + \sqrt{\frac{2m}{n}} + \sqrt{\frac{mn(mn+1)}{3}} - 2\sqrt{\frac{m^2n+m}{3n}}.$$

Proof. Let $G = P_{n+1}^{(m)}$, where $P_{n+1}^{(m)}$ is a Kulli path windmill graph. Now

$$\begin{aligned}
ABC(G) &= \sum_{uv\in E(G)} \sqrt{\frac{d_G(u)+d_G(v)-2}{d_G(u)d_G(v)}} \\
&= \sum_{uv\in E_5} \sqrt{\frac{2+3-2}{2\times 3}} + \sum_{uv\in E_6} \sqrt{\frac{3+3-2}{3\times 3}} \\
&+ \sum_{uv\in E_{mn+2}} \sqrt{\frac{mn+2-2}{mn\times 2}} + \sum_{uv\in E_{mn+3}} \sqrt{\frac{mn+3-2}{mn\times 3}} \\
&= \frac{1}{\sqrt{2}}2m + \frac{2}{3}(mn-3m) + \frac{1}{\sqrt{mn}}2m + \left(\frac{mn+1}{3mn}\right)(mn-2m) \\
&= (\sqrt{2}-2)m + \frac{2}{3}mn + \sqrt{\frac{2m}{n}} + \sqrt{\frac{mn(mn+1)}{3}} - 2\sqrt{\frac{m^2n+m}{3n}}.
\end{aligned}$$

□

Theorem 2.6. *The geometric-arithmetic index of Kulli path windmill graph is*

$$\begin{aligned} GA(P_{n+1}^{(m)}) &= \left(\frac{4\sqrt{6}}{5} - 3\right) m + mn + \frac{4\sqrt{2}m\sqrt{mn}}{mn+2} \\ &+ \left(\frac{2\sqrt{3}\sqrt{mn}}{mn+3}\right)(mn - 2m). \end{aligned}$$

Proof. Let $G = P_{n+1}^{(m)}$, where $P_{n+1}^{(m)}$ is a Kulli path windmill graph. Now

$$\begin{aligned} GA(G) &= \sum_{uv\in E(G)} \frac{2\sqrt{d_G(u)d_G(v)}}{d_G(u)+d_G(v)} = \sum_{uv\in E_5} \frac{2\sqrt{2\times 3}}{2+3} + \sum_{uv\in E_6} \frac{2\sqrt{3\times 3}}{3+3} \\ &+ \sum_{uv\in E_{mn+2}} \frac{2\sqrt{mn\times 2}}{mn+2} + \sum_{uv\in E_{mn+3}} \frac{2\sqrt{mn\times 3}}{mn+3} \\ &= \frac{2\sqrt{6}}{5}\times 2m + (1)\times(mn-3m) + \left(\frac{2\sqrt{2}\sqrt{mn}}{mn+2}\right)2m \\ &+ \left(\frac{2\sqrt{3}\sqrt{mn}}{mn+3}\right)(mn-2m) \\ &= \left(\frac{4\sqrt{6}}{5} - 3\right) m + mn + \frac{4\sqrt{2}m\sqrt{mn}}{mn+2} \\ &+ \left(\frac{2\sqrt{3}\sqrt{mn}}{mn+3}\right)(mn-2m). \end{aligned}$$

□

Theorem 2.7. *The harmonic index of Kulli path windmill graph is*

$$H(P_{n+1}^{(m)}) = \left(\frac{1}{3} + \frac{2}{mn+3}\right) mn + \left(\frac{1}{mn+2} - \frac{1}{mn+3} - \frac{1}{20}\right) 4m.$$

Proof. Let $G = P_{n+1}^{(m)}$, where $P_{n+1}^{(m)}$ is a Kulli path windmill graph. Now

$$\begin{aligned} H(G) &= \sum_{uv\in E(G)} \frac{2}{d_G(u)+d_G(v)} \\ &= \sum_{uv\in E_5} \frac{2}{2+3} + \sum_{uv\in E_6} \frac{2}{3+3} + \sum_{uv\in E_{mn+2}} \frac{2}{mn+2} \\ &+ \sum_{uv\in E_{mn+3}} \frac{2}{mn+3} \\ &= \frac{2}{5}\times 2m + \frac{1}{3}\times(mn-3m) + \left(\frac{2}{mn+2}\right)\times 2m \\ &+ \left(\frac{2}{mn+3}\right)\times(mn-2m) \\ &= \left(\frac{1}{3} + \frac{2}{mn+3}\right) mn + \left(\frac{1}{mn+2} - \frac{1}{mn+3} - \frac{1}{20}\right) 4m. \end{aligned}$$

□

Corollary 2.5. *Let $P_{n+1}^{(m)}$ be a Kulli path windmill graph with $n \geq 2$. Then*

(i) $H(P_{n+1}^{(m)}) = 2X^{(-1)}(P_{n+1}^{(m)})$,

(ii) $H(P_{n+1}^{(m)}) < \chi(P_{n+1}^{(m)})$.

Theorem 2.8. *The symmetric division deg index of Kulli path windmill graph is*

$$SDD(P_{n+1}^{(m)}) = \Big(\frac{mn}{3} + \frac{m}{3} + 2\Big) mn - \frac{5}{3}m - \frac{2}{n} + 3.$$

Proof. Let $G = P_{n+1}^{(m)}$ be a Kulli path windmill graph. Now

$$\begin{aligned} SDD(P_{n+1}^{(m)}) &= \sum_{uv \in E(G)} \frac{d_G(u)^2 + d_G(v)^2}{d_G(u)d_G(v)} \\ &= \sum_{uv \in E_5} \frac{2^2 + 3^2}{2 \times 3} + \sum_{uv \in E_6} \frac{3^2 + 3^2}{3 \times 3} + \sum_{uv \in E_{mn+2}} \frac{(mn)^2 + 2^2}{mn \times 2} \\ &+ \sum_{uv \in E_{mn+3}} \frac{(mn)^2 + 3^2}{mn \times 3} \\ &= \frac{13}{6} \times 2m + 2 \times (mn - 3m) + \left(\frac{(mn)^2 + 4}{2mn}\right) \times 2m \\ &+ \left(\frac{(mn)^2 + 9}{3mn}\right) \times (mn - 2m) \\ &= \Big(\frac{mn}{3} + \frac{m}{3} + 2\Big) mn - \frac{5}{3}m - \frac{2}{n} + 3. \end{aligned}$$

□

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