TWMS J. App. and Eng. Math. V.11, N.3, 2021, pp. 814-825

CERTAIN TYPES OF GRAPHS IN INTERVAL-VALUED INTUITIONISTIC FUZZY SETTING

S. NAZ[1], M. A. MALIK[2], §

Abstract. Interval-valued intuitionistic fuzzy graph (IVIFG) as a generalization of intuitionistic fuzzy graph (IFG) increases its elasticity drastically. In this paper, some important types of IVIFGs such as regular, irregular, neighbourly irregular, highly irregular and strongly irregular IVIFGs are discussed. The relation among neighbourly irregular, highly irregular and strongly irregular IVIFGs is proved. The notion of interval-valued intuitionistic fuzzy clique (IVIFC) is introduced. A complete characterization of the structure of the IVIFC is presented. Finally, an application of interval-valued intuitionistic fuzzy digraph (IVIFDG) in vulnerability assessment of water supply network is provided.

Keywords: Interval-valued intuitionistic fuzzy set, interval-valued intuitionistic fuzzy graph, irregular interval-valued intuitionistic fuzzy graph, interval-valued intuitionistic fuzzy clique.

AMS Subject Classification: 05C99

1. Introduction

In 1965, Zadeh [27] originally introduced the concept of fuzzy set. Its prominent characteristic is that a membership degree in $[0, 1]$ is assigned to each element in the set. When it is difficult to give the accurate judgments to the things, fuzzy set shows great advantages in expressing uncertain or vague information and depict the indeterminacy of things. It was later understood that a single membership function could not capture the ambiguity existing in human mind and the complexity of data. To overcome this shortcoming of the fuzzy set, Atanassov [6] proposed an extension of fuzzy set by introducing non-membership function, and defined IFS. As IFS can describe the uncertainty of an object more reasonably and comprehensively than the FS, lots of research on the IFS have been done, in recent decades. However, in some cases, membership degree or non-membership degree cannot be indicated by using a value, but using an interval. That is why, intuitionistic fuzzy set (IFS) was extended to the interval-valued intuitionistic fuzzy

[1] Department of Mathematics, Division of Science and Technology, University of Education, Lahore, Pakistan.
e-mail: sumera.naz@ue.edu.pk; ORCID: https://orcid.org/0000-0001-6550-9607.
[2] Department of Mathematics, University of the Punjab, Quaid-e-Azam Campus, Lahore, Pakistan.
e-mail: aslam.math@pu.edu.pk; ORCID: https://orcid.org/0000-0003-4434-7893.
§ Manuscript received: January 19, 2017; accepted: April 24, 2017.

set (IVIFS) by Atanassov and Gargov [7] as a combining concept of IFS and IVFS. It greatly furnishes the additional capability to deal with imprecise information and model non-statistical uncertainty by expressing the variations of membership function and non-membership function and has played a vital role in the vague system and received much attention from researchers. IVIFS has been widely used in many areas, such as decision making [8], pattern recognition [28], medical diagnosis [1], and graph theory [11].

It is natural that when there is fuzziness in the description of the items (vertices) or in their relationships (edges) or in both, a fuzzy graph model is designed. Obtaining analogs of several basic graph theoretical concepts, Rosenfeld [21] considered fuzzy relations on fuzzy sets and defined the structure of fuzzy graphs. Applications of fuzzy graphs cover an extensive range such as control theory, information theory, neural networks, expert systems, medical diagnosis, cluster analysis, database theory, decision making and optimization of networks. The notion of irregular fuzzy graphs was defined by Gani and Latha [9]. The concept of strongly irregular fuzzy graphs was initiated by Nandhini and Nandhini [15]. Nair and Cheng [14] defined the concept of a fuzzy clique in fuzzy graphs. The concept of cycles and cocycles of fuzzy graphs was introduced by Mordeson and Nair [12]. Akram and Davvaz [3] put forward the concept of intuitionistic fuzzy graphs. Sahoo and Pal [23, 24] introduced many concepts of intuitionistic fuzzy graphs. The concept of interval-valued fuzzy graphs was initiated by Hongmei and Lianhua [10]. Product of IVIFGs has been proposed by Mishra and Pal [13]. The concept of strong IVIFGs was defined by Ismayil and Ali [11]. Rashmanlou and Borzooei [20] introduced the concept of interval-valued intuitionistic (S, T)-fuzzy graphs. Naz et al. [16, 17, 18, 19] introduced several new concepts related to extended structure of fuzzy graphs. Akram et al. [2, 25] put forward many new concepts, including m-polar fuzzy graphs and fuzzy soft graphs. Ashraf et al. [4, 5] put forward some novel concepts of fuzzy and generalized fuzzy graphs and provided their applications in decision making.

This paper is organized as follows: In Section 2, basic concepts related to IVIFSs and IVIFGs are reviewed. In Section 3, we define certain types of IVIFGs like, neighbourly irregular, highly irregular and strongly irregular IVIFGs. In Section 4, we propose the concept of IVIFCs consistent with interval-valued intuitionistic fuzzy cycles in IVIFGs. Section 5 is devoted to the application of IVIFDGs in vulnerability assessment of water supply network and finally we draw conclusions in section 6.

2. Preliminaries

In the following, some basic concepts are reviewed to facilitate next sections.

A graph is a pair of sets $G = (V, E)$, satisfying $E \subseteq V \times V$. The elements of V and E are the vertices and the edges of the graph G, respectively. A vertex joined by an edge to a vertex x is called a neighbor of x. The (open) neighborhood $\mathcal{N}(x)$ of a vertex x in a graph G is the set of all the neighbors of x, while closed neighborhood $\mathcal{N}[x]$ of x is given by $\mathcal{N}[x] = \mathcal{N}(x) \cup \{x\}$. In graph theory, clique is an important concept. A clique in a graph G is a complete subgraph of G. A subgraph H of a graph G is a disjoint union of cliques if $V(H)$ can be partitioned into $H_1, H_2, \ldots, H_k$ such that $xy \in E(H)$ for all $x, y \in V(H)$ if and only if $\{x, y\} \subseteq H_i$, for some $i,\ i = 1, 2, \ldots, k$ [14].

Definition 2.1. [21] *A fuzzy graph $\mathcal{G} = (\eta, \mu)$ is a pair of functions $\eta : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$ such that $\mu(xy) \leq \eta(x) \wedge \eta(y)$ for all $x, y \in V$. An edge xy of a fuzzy graph is called an effective edge* [22] *if $\mu(xy) = \eta(x) \wedge \eta(y)$. In a fuzzy graph, the path ρ is a sequence of distinct vertices $x_0, x_1, \ldots, x_n$ such that $\mu(x_{i-1}, x_i) > 0,\ i = 1, 2, \ldots, n$.*

Definition 2.2. [13] *An IVIFG on a non-empty set V is a pair $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$, where $\tilde{X}$ is an IVIFS on V and $\tilde{Y}$ is an interval-valued intuitionistic fuzzy relation on V such that*

$$\begin{aligned}\tilde{\mu}^L_{\tilde{Y}}(xy) &\leq \min\{\tilde{\mu}^L_{\tilde{X}}(x), \tilde{\mu}^L_{\tilde{X}}(y)\}, \tilde{\mu}^U_{\tilde{Y}}(xy) \leq \min\{\tilde{\mu}^U_{\tilde{X}}(x) \wedge \tilde{\mu}^U_{\tilde{X}}(y)\},\\ \tilde{\nu}^L_{\tilde{Y}}(xy) &\geq \max\{\tilde{\nu}^L_{\tilde{X}}(x), \tilde{\nu}^L_{\tilde{X}}(y)\}, \tilde{\nu}^U_{\tilde{Y}}(xy) \geq \max\{\tilde{\nu}^U_{\tilde{X}}(x), \tilde{\nu}^U_{\tilde{X}}(y)\}\end{aligned}$$

and $(\tilde{\mu}^U_{\tilde{Y}}(xy))^2 + (\tilde{\nu}^U_{\tilde{Y}}(xy))^2 \leq 1$ for all $x, y \in V$. We call $\tilde{X}$ the interval-valued intuitionistic fuzzy vertex set of $\tilde{\mathcal{G}}$ and $\tilde{Y}$ the interval-valued intuitionistic fuzzy edge set of $\tilde{\mathcal{G}}$. Here, $\tilde{Y}$ is a symmetric interval-valued intuitionistic fuzzy relation on $\tilde{X}$. If $\tilde{Y}$ is not symmetric on $\tilde{X}$, then $\tilde{\mathcal{D}} = (\tilde{X}, \overrightarrow{\tilde{Y}})$ is called an interval-valued intuitionistic fuzzy digraph.

3. Interval-valued intuitionistic fuzzy graphs (IVIFGs)

Definition 3.1. *The degree of a vertex $x \in V$ in an IVIFG $\tilde{\mathcal{G}}$ is defined as* $\deg(x) = \langle[\deg_{\tilde{\mu}^L}(x), \deg_{\tilde{\mu}^U}(x)], [\deg_{\tilde{\nu}^L}(x), \deg_{\tilde{\nu}^U}(x)]\rangle$, *where*

$$\begin{aligned}\deg_{\tilde{\mu}^L}(x) &= \Sigma_{x,y\neq x\in V}\tilde{\mu}^L_{\tilde{Y}}(xy), \deg_{\tilde{\mu}^U}(x) = \Sigma_{x,y\neq x\in V}\tilde{\mu}^U_{\tilde{Y}}(xy),\\ \deg_{\tilde{\nu}^L}(x) &= \Sigma_{x,y\neq x\in V}\tilde{\nu}^L_{\tilde{Y}}(xy) \text{ and } \deg_{\tilde{\nu}^U}(x) = \Sigma_{x,y\neq x\in V}\tilde{\nu}^U_{\tilde{Y}}(xy).\end{aligned}$$

For an IVIFG, the degree of a vertex can be generalized in different ways.

Definition 3.2. *The sum of the weights of the effective edges incident at a vertex x in an IVIFG is called the effective degree of x. That is, $\mathcal{E}\deg(x) = \langle[\mathcal{E}\deg_{\tilde{\mu}^L}(x), \mathcal{E}\deg_{\tilde{\mu}^U}(x)], [\mathcal{E}\deg_{\tilde{\nu}^L}(x), \mathcal{E}\deg_{\tilde{\nu}^U}(x)]\rangle$, where for all effective edges $xy \in E$*

$$\begin{aligned}\mathcal{E}\deg_{\tilde{\mu}^L}(x) &= \Sigma_{x,y\neq x\in V}\tilde{\mu}^L_{\tilde{Y}}(xy), \mathcal{E}\deg_{\tilde{\mu}^U}(x) = \Sigma_{x,y\neq x\in V}\tilde{\mu}^U_{\tilde{Y}}(xy),\\ \mathcal{E}\deg_{\tilde{\nu}^L}(x) &= \Sigma_{x,y\neq x\in V}\tilde{\nu}^L_{\tilde{Y}}(xy) \text{ and } \mathcal{E}\deg_{\tilde{\nu}^U}(x) = \Sigma_{x,y\neq x\in V}\tilde{\nu}^U_{\tilde{Y}}(xy).\end{aligned}$$

Definition 3.3. *The neighbourhood degree of a vertex $x \in V$ in an IVIFG $\tilde{\mathcal{G}}$ is defined as $\mathcal{N}\deg(x) = \langle[\mathcal{N}\deg_{\tilde{\mu}^L}(x), \mathcal{N}\deg_{\tilde{\mu}^U}(x)], [\mathcal{N}\deg_{\tilde{\nu}^L}(x), \mathcal{N}\deg_{\tilde{\nu}^U}(x)]\rangle$, where*

$$\begin{aligned}\mathcal{N}\deg_{\tilde{\mu}^L}(x) &= \Sigma_{y\in\mathcal{N}(x)}\tilde{\mu}^L_{\tilde{X}}(y), \mathcal{N}\deg_{\tilde{\mu}^U}(x) = \Sigma_{y\in\mathcal{N}(x)}\tilde{\mu}^U_{\tilde{X}}(y),\\ \mathcal{N}\deg_{\tilde{\nu}^L}(x) &= \Sigma_{y\in\mathcal{N}(x)}\tilde{\nu}^L_{\tilde{X}}(y) \text{ and } \mathcal{N}\deg_{\tilde{\nu}^U}(x) = \Sigma_{y\in\mathcal{N}(x)}\tilde{\nu}^U_{\tilde{X}}(y).\end{aligned}$$

Definition 3.4. *The closed neighbourhood degree of a vertex $x \in V$ in an IVIFG $\tilde{\mathcal{G}}$ is defined by $\mathcal{N}\deg[x] = \langle[\mathcal{N}\deg_{\tilde{\mu}^L}[x], \mathcal{N}\deg_{\tilde{\mu}^U}[x]], [\mathcal{N}\deg_{\tilde{\nu}^L}[x], \mathcal{N}\deg_{\tilde{\nu}^U}[x]]\rangle$, where*

$$\begin{aligned}\mathcal{N}\deg_{\tilde{\mu}^L}[x] &= \mathcal{N}\deg_{\tilde{\mu}^L}(x) + \tilde{\mu}^L_{\tilde{X}}(x), \mathcal{N}\deg_{\tilde{\mu}^U}[x] = \mathcal{N}\deg_{\tilde{\mu}^U}(x) + \tilde{\mu}^U_{\tilde{X}}(x),\\ \mathcal{N}\deg_{\tilde{\nu}^L}[x] &= \mathcal{N}\deg_{\tilde{\nu}^L}(x) + \tilde{\nu}^L_{\tilde{X}}(x) \text{ and } \mathcal{N}\deg_{\tilde{\nu}^U}[x] = \mathcal{N}\deg_{\tilde{\nu}^U}(x) + \tilde{\nu}^U_{\tilde{X}}(x).\end{aligned}$$

Definition 3.5. *The vertices of $\tilde{\mathcal{G}}$ which are incident with effective edges are said to be the effective vertices. The sum of the weights of the effective vertices adjacent to a vertex x of an IVIFG $\tilde{\mathcal{G}}$ is called the effective neighbourhood degree of x.*

The types of IVIFGs are introduced according to their (open) neighbourhood and closed neighbourhood degree.

Definition 3.6. *An IVIFG $\tilde{\mathcal{G}}$ on G, in which each vertex has the same neighbourhood degree is called an interval-valued intuitionistic fuzzy regular graph. If each vertex has degree $\langle[j,k],[s,t]\rangle$, $\tilde{\mathcal{G}}$ is called $\langle[j,k],[s,t]\rangle$-regular.*

Example 3.1. *Consider a graph $G = (V, E)$, where $V = \{v_1,\ v_2,\ v_3\}$ and $E = \{v_1v_2,\ v_2v_3,\ v_1v_3\}$. Let $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ be an IVIFG of a graph G, given in Fig. 1. Here* $\deg_{\tilde{\mathcal{G}}}(v_i) = \langle[0.4, 0.6], [0.8, 1.2]\rangle$ *for all $i = 1, 2, 3$. Hence $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ is a regular IVIFG.*

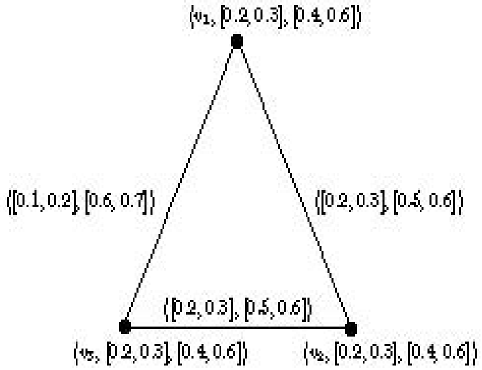

FIGURE 1. Regular IVIFG.

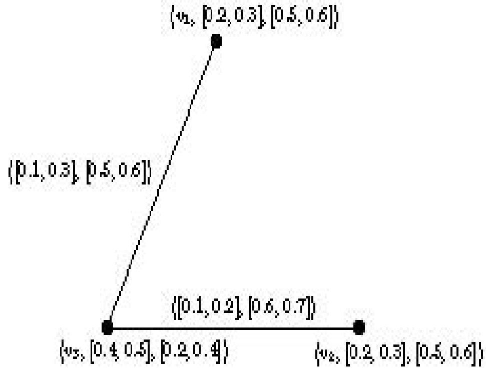

FIGURE 2. Irregular IVIFG.

Definition 3.7. *An IVIFG $\tilde{\mathcal{G}}$ is said to be irregular, if there is a vertex which is adjacent to vertices with distinct neighbourhood degrees. That is,* $\deg_{\tilde{\mathcal{G}}}(l) \neq \langle [j,k],[s,t] \rangle$ *for all* $l \in V$.

Example 3.2. *Consider an IVIFG $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ on $V = \{v_1, v_2, v_3\}$, as given in Fig. 2. Clearly,* $\deg_{\tilde{\mathcal{G}}}(v_1) = \deg_{\tilde{\mathcal{G}}}(v_2) = \langle [0.4, 0.5], [0.2, 0.4] \rangle$ *and* $\deg_{\tilde{\mathcal{G}}}(v_3) = \langle [0.4, 0.6], [1.0, 1.2] \rangle$. *Hence $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ is an irregular IVIFG.*

Definition 3.8. *Let $\tilde{\mathcal{G}}$ be a connected IVIFG on G. $\tilde{\mathcal{G}}$ is called neighbourly irregular, if no two adjacent vertices of $\tilde{\mathcal{G}}$ have same neighbourhood degree. That is,* $\deg_{\tilde{\mathcal{G}}}(l) \neq deg_{\tilde{\mathcal{G}}}(m)$ *for all $lm \in E$.*

Example 3.3. *Consider an IVIFG $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ on $V = \{v_1, v_2, v_3, v_4\}$, as given in Fig. 3, where* $\deg_{\tilde{\mathcal{G}}}(v_1) = \deg_{\tilde{\mathcal{G}}}(v_4) = \langle [0.5, 1.0], [0.6, 0.8] \rangle$ *and* $\deg_{\tilde{\mathcal{G}}}(v_2) = \deg_{\tilde{\mathcal{G}}}(v_3) = \langle [0.5, 0.9], [0.4, 0.8] \rangle$. *Hence $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ is a neighbourly irregular IVIFG.*

Definition 3.9. *A connected IVIFG $\tilde{\mathcal{G}}$ is said to be a highly irregular if every vertex of $\tilde{\mathcal{G}}$ is adjacent to vertices with distinct neighbourhood degrees. That is, $l, m \in \mathcal{N}(x)$, $l \neq m$* $\implies \deg_{\tilde{\mathcal{G}}}(l) \neq \deg_{\tilde{\mathcal{G}}}(m)$ *for all $x \in V$.*

Example 3.4. *Consider an IVIFG $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ on $V = \{v_1, v_2, v_3, v_4, v_5\}$, as shown in Fig. 4, where* $\deg_{\tilde{\mathcal{G}}}(v_1) = \langle [0.3, 0.5], [0.3, 0.6] \rangle$, $\deg_{\tilde{\mathcal{G}}}(v_2) = \langle [0.7, 1.3], [0.9, 1.4] \rangle$, $\deg_{\tilde{\mathcal{G}}}(v_3) = \langle [0.4, 0.6], [0.5, 0.9] \rangle$ *and* $\deg_{\tilde{\mathcal{G}}}(v_4) = \deg_{\tilde{\mathcal{G}}}(v_5) = \langle [0.5, 0.9], [0.6, 1.0] \rangle$. *Therefore, $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ is a highly irregular IVIFG.*

Remark 3.1. *A neighbourly irregular IVIFG may not be a highly irregular IVIFG.*

Remark 3.2. *A highly irregular IVIFG may not be a neighbourly irregular IVIFG.*

Definition 3.10. *A connected IVIFG $\tilde{\mathcal{G}}$ on G is called strongly irregular if every pair of vertices in $\tilde{\mathcal{G}}$ have distinct neighborhood degrees. That is,* $\deg_{\tilde{\mathcal{G}}}(l) \neq \deg_{\tilde{\mathcal{G}}}(m)$ *for all $l, m \in V$.*

Example 3.5. *Consider an IVIFG $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ on $V = \{v_1, v_2, v_3\}$, shown in Fig. 5. where* $\deg_{\tilde{\mathcal{G}}}(v_1) = \langle [1.1, 1.4], [0.3, 0.6] \rangle$, $\deg_{\tilde{\mathcal{G}}}(v_2) = \langle [0.8, 1.5], [0.2, 0.5] \rangle$ *and* $\deg_{\tilde{\mathcal{G}}}(v_3) = \langle [0.7, 1.3], [0.3, 0.7] \rangle$. *Therefore $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ is a strongly regular IVIFG.*

Theorem 3.1. *Every strongly irregular IVIFG is both neighbourly irregular IVIFG and highly irregular IVIFG.*

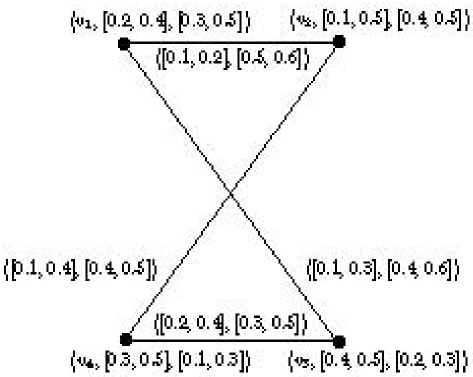

Figure 3. Neighbourly irregular IVIFG.

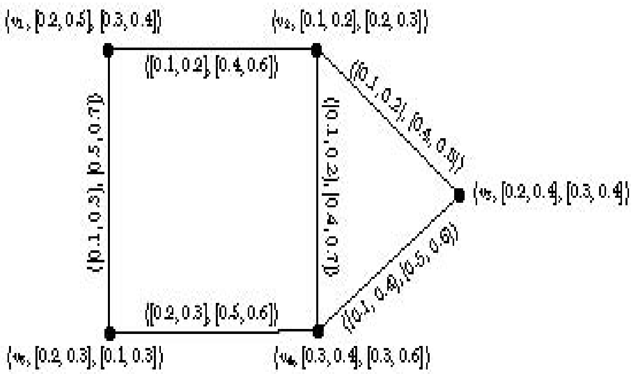

Figure 4. Highly irregular IVIFG

Proof. Suppose that $\tilde{\mathcal{G}}$ is a strongly irregular IVIFG. That is, degrees of every pair of vertices in $\tilde{\mathcal{G}}$ are distinct. Then every two adjacent vertices of $\tilde{\mathcal{G}}$ have distinct degrees and every vertex of $\tilde{\mathcal{G}}$ is adjacent to vertices with distinct degrees . Hence $\tilde{\mathcal{G}}$ is neighbourly irregular IVIFG and highly irregular IVIFG. □

The converse of above statement does not hold. That is, a highly irregular IVIFG and neighbourly irregular IVIFG may not be a strongly irregular IVIFG. The following example illustrate the assertion above.

Example 3.6. *Consider an IVIFG* $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ *on* $V = \{v_1, v_2, v_3, v_4\}$*, as in Fig. 6. Clearly* $\tilde{\mathcal{G}}$ *is neighbourly irregular IVIFG and highly irregular IVIFG, but not strongly irregular IVIFG, as* $\deg_{\tilde{\mathcal{G}}}(v_2) = \deg_{\tilde{\mathcal{G}}}(v_3)$.

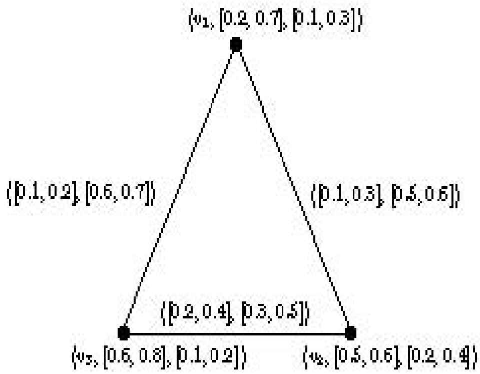

Figure 5. Strongly irregular IVIFG.

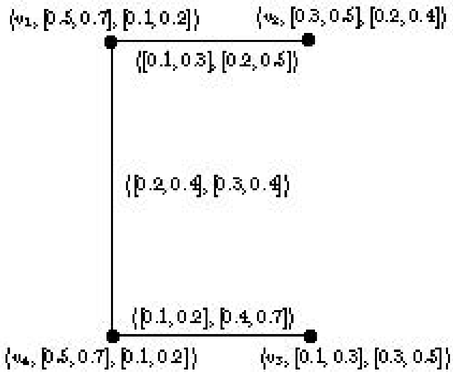

Figure 6. Neighbourly irregular and highly irregular IVIFG

Definition 3.11. *An IVIFG* $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ *is said to be complete if*

$$\begin{aligned} \tilde{\mu}^L_{\tilde{Y}}(xy) &= \min(\tilde{\mu}^L_{\tilde{X}}(x), \tilde{\mu}^L_{\tilde{X}}(y)), \tilde{\mu}^U_{\tilde{Y}}(xy) = \min(\tilde{\mu}^U_{\tilde{X}}(x), \tilde{\mu}^U_{\tilde{X}}(y)), \\ \tilde{\nu}^L_{\tilde{Y}}(xy) &= \max(\tilde{\nu}^L_{\tilde{X}}(x), \tilde{\nu}^L_{\tilde{X}}(y)), \tilde{\nu}^U_{\tilde{Y}}(xy) = \max(\tilde{\nu}^U_{\tilde{X}}(x), \tilde{\nu}^U_{\tilde{X}}(y)) \end{aligned}$$

such that $0 < \tilde{\mu}^U_{\tilde{Y}}(xy) + \tilde{\nu}^U_{\tilde{Y}}(xy) \leq 1$ *for all* $x, y \in V$.

Proposition 3.1. *A complete IVIFG may not be a strongly irregular IVIFG.*

Definition 3.12. *An IVIFG $\tilde{\mathcal{G}}$ is said to be totally irregular, if there is a vertex which is adjacent to vertices with distinct closed neighbourhood degrees. That is,* $\deg_{\tilde{\mathcal{G}}}[l] \neq \langle [j,k],[s,t]\rangle$ *for all* $l \in V$*. An IVIFG in Fig. 2. is totally irregular as,* $\deg_{\tilde{\mathcal{G}}}[v_1] = \deg_{\tilde{\mathcal{G}}}[v_2] = \langle [0.6,0.8],[0.7,1.0]\rangle$ *and* $\deg_{\tilde{\mathcal{G}}}[v_3] = \langle [0.8,1.1],[1.2,1.6]\rangle$.

Definition 3.13. *A connected IVIFG $\tilde{\mathcal{G}}$ is said to be a neighbourly totally irregular, if no two adjacent vertices of $\tilde{\mathcal{G}}$ have same closed neighbourhood degree. That is,* $\deg_{\tilde{\mathcal{G}}}[l] \neq \deg_{\tilde{\mathcal{G}}}[m]$ *for all* $lm \in E$.

Definition 3.14. *A connected IVIFG $\tilde{\mathcal{G}}$ is said to be a highly totally irregular if every vertex of $\tilde{\mathcal{G}}$ is adjacent to vertices with distinct closed neighbourhood degrees. That is,* $l, m \in \mathcal{N}(x)$, $l \neq m \implies \deg_{\tilde{\mathcal{G}}}[l] \neq \deg_{\tilde{\mathcal{G}}}[m]$ *for all* $x \in V$.

Definition 3.15. *A connected IVIFG $\tilde{\mathcal{G}}$ on G is called strongly totally irregular if every pair of vertices in $\tilde{\mathcal{G}}$ have distinct closed neighborhood degrees.*

Remark 3.3. *A neighbourly irregular IVIFG need not be a neighbourly totally irregular IVIFG. A neighbourly totally irregular IVIFG need not be a neighbourly irregular IVIFG.*

Theorem 3.2. *Let $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ be an IVIFG. If $\tilde{\mathcal{G}}$ is neighbourly irregular and $\langle [\tilde{\mu}^L_{\tilde{X}}, \tilde{\mu}^U_{\tilde{X}}], [\tilde{\nu}^L_{\tilde{X}}, \tilde{\nu}^U_{\tilde{X}}]\rangle$ is a constant function. Then $\tilde{\mathcal{G}}$ is a neighbourly totally irregular IVIFG.*

Proof. Suppose that $\tilde{\mathcal{G}}$ is a neighbourly irregular IVIFG. That is, no two adjacent vertices of $\tilde{\mathcal{G}}$ have same neighbourhood degree. Let x and y be the adjacent vertices of $\tilde{\mathcal{G}}$ with distinct neighborhood degrees $\langle [j_1,k_1],[s_1,t_1]\rangle$ and $\langle [j_2,k_2],[s_2,t_2]\rangle$, respectively. Also take $\langle [\tilde{\mu}^L_{\tilde{X}}(x_i), \tilde{\mu}^U_{\tilde{X}}(x_i)], [\tilde{\nu}^L_{\tilde{X}}(x_i), \tilde{\nu}^U_{\tilde{X}}(x_i)]\rangle = \langle [c_1,c_2],[c_3,c_4]\rangle$ for all $x_i \in V$, where $c_1, c_2, c_3, c_4 \in [0,1]$ are constants. Therefore,

$$
\begin{aligned}
\deg[x] &= \langle [\deg_{\tilde{\mu}^L}[x], \deg_{\tilde{\mu}^U}[x]], [\deg_{\tilde{\nu}^L}[x], \deg_{\tilde{\nu}^U}[x]]\rangle \\
&= \langle [\deg_{\tilde{\mu}^L}(x) + \tilde{\mu}^L_{\tilde{X}}(x), \deg_{\tilde{\mu}^U}(x) + \tilde{\mu}^U_{\tilde{X}}(x)], [\deg_{\tilde{\nu}^L}(x) + \tilde{\nu}^L_{\tilde{X}}(x), \deg_{\tilde{\nu}^U}(x) + \tilde{\nu}^U_{\tilde{X}}(x)]\rangle \\
&= \langle [j_1 + c_1, k_1 + c_2], [s_1 + c_3, t_1 + c_4]\rangle, \\
\deg[y] &= \langle [\deg_{\tilde{\mu}^L}[y], \deg_{\tilde{\mu}^U}[y]], [\deg_{\tilde{\nu}^L}[y], \deg_{\tilde{\nu}^U}[y]]\rangle \\
&= \langle [\deg_{\tilde{\mu}^L}(y) + \tilde{\mu}^L_{\tilde{X}}(y), \deg_{\tilde{\mu}^U}(y) + \tilde{\mu}^U_{\tilde{X}}(y)], [\deg_{\tilde{\nu}^L}(y) + \tilde{\nu}^L_{\tilde{X}}(y), \deg_{\tilde{\nu}^U}(y) + \tilde{\nu}^U_{\tilde{X}}(y)]\rangle \\
&= \langle [j_2 + c_1, k_2 + c_2], [s_2 + c_3, t_2 + c_4]\rangle.
\end{aligned}
$$

To prove that closed neighborhood degrees of every two adjacent vertices are distinct.
Assume that, $\deg[x] = \deg[y]$.
$\langle [j_1 + c_1, k_1 + c_2], [s_1 + c_3, t_1 + c_4]\rangle = \langle [j_2 + c_1, k_2 + c_2], [s_2 + c_3, t_2 + c_4]\rangle$
$\Rightarrow \langle [j_1,k_1],[s_1,t_1]\rangle = \langle [j_2,k_2],[s_2,t_2]\rangle$,
a contradiction. Therefore, no two adjacent vertices of $\tilde{\mathcal{G}}$ have same closed neighbourhood degree. Hence $\tilde{\mathcal{G}}$ is a neighbourly totally irregular IVIFG. □

Theorem 3.3. *Let $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ be an IVIFG. If $\tilde{\mathcal{G}}$ is a neighbourly totally irregular and $\langle [\tilde{\mu}^L_{\tilde{X}}, \tilde{\mu}^U_{\tilde{X}}], [\tilde{\nu}^L_{\tilde{X}}, \tilde{\nu}^U_{\tilde{X}}]\rangle$ is a constant function, then $\tilde{\mathcal{G}}$ is a neighbourly irregular IVIFG.*

Proof. Suppose that $\tilde{\mathcal{G}}$ is a neighbourly totally irregular IVIFG. That is, no two adjacent vertices of $\tilde{\mathcal{G}}$ have same closed neighbourhood degrees. Let x and y be the adjacent vertices of $\tilde{\mathcal{G}}$ with distinct closed neighborhood degrees $\langle [j_1+c_1, k_1+c_2], [s_1+c_3, t_1+c_4]\rangle$ and $\langle [j_2+c_1, k_2+c_2], [s_2+c_3, t_2+c_4]\rangle$, respectively. Also take $\langle [\tilde{\mu}^L_{\tilde{X}}(x_i), \tilde{\mu}^U_{\tilde{X}}(x_i)], [\tilde{\nu}^L_{\tilde{X}}(x_i), \tilde{\nu}^U_{\tilde{X}}(x_i)]\rangle = \langle [c_1,c_2],[c_3,c_4]\rangle$ for all $x_i \in V$, where $c_1, c_2, c_3, c_4 \in [0,1]$ are constants. We show that no two adjacent vertices of $\tilde{\mathcal{G}}$ have same neighbourhood degrees.

As $\deg[x] \neq \deg[y]$
$\Rightarrow \langle [j_1 + c_1, k_1 + c_2], [s_1 + c_3, t_1 + c_4] \rangle \neq \langle [j_2 + c_1, k_2 + c_2], [s_2 + c_3, t_2 + c_4] \rangle$
$\Rightarrow \langle [j_1, k_1], [s_1, t_1] \rangle \neq \langle [j_2, k_2], [s_2, t_2] \rangle$.
Therefore, no two adjacent vertices of $\tilde{\mathcal{G}}$ have same neighbourhood degrees. Hence $\tilde{\mathcal{G}}$ is a neighbourly irregular IVIFG. □

Proposition 3.2. *If an IVIFG $\tilde{\mathcal{G}}$ is both neighbourly irregular and neighbourly totally irregular, then $\langle [\tilde{\mu}^L_{\tilde{X}}, \tilde{\mu}^U_{\tilde{X}}], [\tilde{\nu}^L_{\tilde{X}}, \tilde{\nu}^U_{\tilde{X}}] \rangle$ may not be a constant function.*

Proposition 3.3. *The interval-valued intuitionistic fuzzy subgraph $\tilde{H} = (\tilde{X}', \tilde{Y}')$ of a neighbourly (totally) irregular IVIFG $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ may not be neighbourly (totally) irregular.*

4. Interval-valued intuitionistic fuzzy cliques

In this section, we propose the notion of IVIFC consistent with interval-valued intuitionistic fuzzy cycles in IVIFGs and present a complete characterization of the structure of the IVIFC. To do this, we firstly introduce the concept of interval-valued intuitionistic fuzzy cycles.

Definition 4.1. *Let $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ be an IVIFG. Then*

1. *$\tilde{\mathcal{G}}$ is a cycle if and only if $G = (V, E)$ is a cycle.*
2. *$\tilde{\mathcal{G}}$ is called an interval-valued intuitionistic fuzzy cycle if and only if G is a cycle and there does not exist unique edge lm of G such that*

$$\tilde{\mu}^L_{\tilde{Y}}(lm) = \min\{\tilde{\mu}^L_{\tilde{Y}}(xy) \mid xy \in E\}, \tilde{\mu}^U_{\tilde{Y}}(lm) = \min\{\tilde{\mu}^U_{\tilde{Y}}(xy) \mid xy \in E\},$$
$$\tilde{\nu}^L_{\tilde{Y}}(lm) = \max\{\tilde{\nu}^L_{\tilde{Y}}(xy) \mid xy \in E\}, \tilde{\nu}^U_{\tilde{Y}}(lm) = \max\{\tilde{\nu}^U_{\tilde{Y}}(xy) \mid xy \in E\}.$$

Definition 4.2. *Let $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ be an IVIFG of a graph $G = (V, E)$ and $\tilde{H} = (\tilde{X}', \tilde{Y}')$ be a subgraph induced by $S \subseteq V$. Then $\tilde{H}$ is a clique if $H^* = (S, T)$ is a clique and $\tilde{H}$ is an IVIFC if $\tilde{H}$ is a clique and every cycle in $\tilde{H}$ is an interval-valued intuitionistic fuzzy cycle.*

Example 4.1. *Consider an IVIFG $\tilde{\mathcal{G}}$ as in Fig. 1. Take $S = V$, then $\tilde{H}$ is the same as $\tilde{\mathcal{G}}$. Clearly, $\tilde{H}$ is a cycle but not an interval-valued intuitionistic fuzzy cycle. Hence $\tilde{H}$ is a clique but not an IVIFC.*

Further if $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ is an IVIFG on $V = \{v_1, v_2, v_3, v_4\}$ with $\tilde{\mu}^L_{\tilde{X}}(v) = 0.4, \tilde{\mu}^U_{\tilde{X}}(v) = 0.6, \tilde{\nu}^L_{\tilde{X}}(v) = 0.1$ and $\tilde{\nu}^U_{\tilde{X}}(v) = 0.2$ for all $v \in V$, as in Fig. 7. Take $S = V$, then $\tilde{H}$ is the same as $\tilde{\mathcal{G}}$. Clearly, every cycle in $\tilde{H}$ is an interval-valued intuitionistic fuzzy cycle. Hence $\tilde{H}$ is a clique and is also an IVIFC.

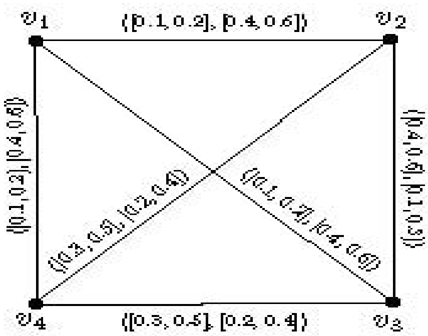

Figure 7. IVIFC

Theorem 4.1. *Let $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ be an IVIFG of a graph $G = (V, E)$ and $\tilde{H} = (\tilde{X}', \tilde{Y}')$ be a subgraph induced by $S \subseteq V$. Then $\tilde{H}$ is an IVIFC if and only if every cycle of length 3 in $\tilde{H}$ is an interval-valued intuitionistic fuzzy cycle.*

Proof. Suppose that $\tilde{H}$ is an IVIFC. Then by above definition every cycle in $\tilde{H}$ is an interval-valued intuitionistic fuzzy cycle and so every cycle of length 3 in $\tilde{H}$ is also an interval-valued intuitionistic fuzzy cycle.

Conversely, assume that every cycle of length 3 is an interval-valued intuitionistic fuzzy cycle. To prove that $\tilde{H}$ is an IVIFC, we have to show that every cycle in $\tilde{H}$ of length $n \geq 3$ is an interval-valued intuitionistic fuzzy cycle. The proof is by induction on the length of interval-valued intuitionistic fuzzy cycles in $\tilde{H}$. By assumption, every cycle of length 3 is an interval-valued intuitionistic fuzzy cycle. Induction hypothesis is that every cycle of length n is an interval-valued intuitionistic fuzzy cycle. Let $v_0, v_1, ..., v_n, v_{n+1}$ be any cycle C_{n+1} of length $n+1$ in $\tilde{H}$. Since $\tilde{H}$ is a clique, $\tilde{H}$ contains a cycle C_n of length n i.e $v_0, v_1, ..., v_n$ and is an interval-valued intuitionistic fuzzy cycle in $\tilde{H}$. Therefore $\exists$ at least two edges, say e_1 and e_2 in an interval-valued intuitionistic fuzzy cycle C_n such that

$$\begin{aligned}
\tilde{\mu}^L_{\tilde{Y}}(e_1) &= \tilde{\mu}^L_{\tilde{Y}}(e_2) = \min\{\tilde{\mu}^L_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_n\},\\
\tilde{\mu}^U_{\tilde{Y}}(e_1) &= \tilde{\mu}^U_{\tilde{Y}}(e_2) = \min\{\tilde{\mu}^U_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_n\},\\
\tilde{\nu}^L_{\tilde{Y}}(e_1) &= \tilde{\nu}^L_{\tilde{Y}}(e_2) = \max\{\tilde{\nu}^L_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_n\},\\
\tilde{\nu}^U_{\tilde{Y}}(e_1) &= \tilde{\nu}^U_{\tilde{Y}}(e_2) = \max\{\tilde{\nu}^U_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_n\}.
\end{aligned}$$

Also v_0, v_n, v_{n+1} is an interval-valued intuitionistic fuzzy cycle and hence $\exists$ at least two edges, say e_3 and e_4 in an interval-valued intuitionistic fuzzy cycle v_0, v_n, v_{n+1} such that

$$\begin{aligned}
\tilde{\mu}^L_{\tilde{Y}}(e_3) &= \tilde{\mu}^L_{\tilde{Y}}(e_4) = \min\{\tilde{\mu}^L_{\tilde{Y}}(e) \mid e \text{ is an edge in } v_0, v_n, v_{n+1}\},\\
\tilde{\mu}^U_{\tilde{Y}}(e_3) &= \tilde{\mu}^U_{\tilde{Y}}(e_4) = \min\{\tilde{\mu}^U_{\tilde{Y}}(e) \mid e \text{ is an edge in } v_0, v_n, v_{n+1}\},\\
\tilde{\nu}^L_{\tilde{Y}}(e_3) &= \tilde{\nu}^L_{\tilde{Y}}(e_4) = \max\{\tilde{\nu}^L_{\tilde{Y}}(e) \mid e \text{ is an edge in } v_0, v_n, v_{n+1}\},\\
\tilde{\nu}^U_{\tilde{Y}}(e_3) &= \tilde{\nu}^U_{\tilde{Y}}(e_4) = \max\{\tilde{\nu}^U_{\tilde{Y}}(e) \mid e \text{ is an edge in } v_0, v_n, v_{n+1}\}.
\end{aligned}$$

Then two cases arise, firstly, if one of the edges e_1 or e_2 is the same as one of the edges e_3 or e_4. In this case, take $e_1 = e_3$. Then e_2 and e_4 are the edges in C_{n+1} such that

$$\begin{aligned}
\tilde{\mu}^L_{\tilde{Y}}(e_2) &= \tilde{\mu}^L_{\tilde{Y}}(e_4) = \min\{\tilde{\mu}^L_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\},\\
\tilde{\mu}^U_{\tilde{Y}}(e_2) &= \tilde{\mu}^U_{\tilde{Y}}(e_4) = \min\{\tilde{\mu}^U_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\},\\
\tilde{\nu}^L_{\tilde{Y}}(e_2) &= \tilde{\nu}^L_{\tilde{Y}}(e_4) = \max\{\tilde{\nu}^L_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\},\\
\tilde{\nu}^U_{\tilde{Y}}(e_2) &= \tilde{\nu}^U_{\tilde{Y}}(e_4) = \max\{\tilde{\nu}^U_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\}
\end{aligned}$$

as required.

Secondly, all four edges e_1, e_2, e_3, e_4 are edges in C_{n+1} and either

$$\begin{aligned}
\tilde{\mu}^L_{\tilde{Y}}(e_1) &= \tilde{\mu}^L_{\tilde{Y}}(e_2) = \min\{\tilde{\mu}^L_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\},\\
\tilde{\mu}^U_{\tilde{Y}}(e_1) &= \tilde{\mu}^U_{\tilde{Y}}(e_2) = \min\{\tilde{\mu}^U_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\},\\
\tilde{\nu}^L_{\tilde{Y}}(e_1) &= \tilde{\nu}^L_{\tilde{Y}}(e_2) = \max\{\tilde{\nu}^L_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\},\\
\tilde{\nu}^U_{\tilde{Y}}(e_1) &= \tilde{\nu}^U_{\tilde{Y}}(e_2) = \max\{\tilde{\nu}^U_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\}
\end{aligned}$$

or

$$\begin{aligned}
\tilde{\mu}^L_{\tilde{Y}}(e_3) &= \tilde{\mu}^L_{\tilde{Y}}(e_4) = \min\{\tilde{\mu}^L_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\},\\
\tilde{\mu}^U_{\tilde{Y}}(e_3) &= \tilde{\mu}^U_{\tilde{Y}}(e_4) = \min\{\tilde{\mu}^U_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\},\\
\tilde{\nu}^L_{\tilde{Y}}(e_3) &= \tilde{\nu}^L_{\tilde{Y}}(e_4) = \max\{\tilde{\nu}^L_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\},\\
\tilde{\nu}^U_{\tilde{Y}}(e_3) &= \tilde{\nu}^U_{\tilde{Y}}(e_4) = \max\{\tilde{\nu}^U_{\tilde{Y}}(e) \mid e \text{ is an edge in } C_{n+1}\}.
\end{aligned}$$

Hence in both cases, $\tilde{H}$ is an IVIFC. □

Lemma 4.1. *Let $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ be an IVIFG of a graph $G = (V, E)$ and $\tilde{H} = (\tilde{X}', \tilde{Y}')$ be a subgraph induced by $S \subseteq V$. Then every cycle of length 3 in $\tilde{H}$ is an interval-valued intuitionistic fuzzy cycle if and only if for any three vertices u, v, w in $\tilde{H}$ such that the edges $uv, vw \in E(H_t)$ implies $uw \in E(H_t)$ for all $t \in [0, 1]$.*

Lemma 4.2. *Let $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ be an IVIFG of a graph $G = (V, E)$ and $\tilde{H} = (\tilde{X}', \tilde{Y}')$ be a subgraph induced by $S \subseteq V$. Then H_t is a disjoint union of cliques if and only if for any three vertices u, v, w in $\tilde{H}$ such that the edges $uv, vw \in E(H_t)$ implies $uw \in E(H_t)$ for all $t \in [0, 1]$.*

As a consequence of Lemmas 4.1 and 4.2, we obtain

Theorem 4.2. *Let $\tilde{\mathcal{G}} = (\tilde{X}, \tilde{Y})$ be an IVIFG of a graph $G = (V, E)$ and $\tilde{H} = (\tilde{X}', \tilde{Y}')$ be a subgraph induced by $S \subseteq V$. Then $\tilde{H}$ is an IVIFC if and only if every cut set of $\tilde{H}$ is a disjoint union of cliques.*

5. Interval-valued intuitionistic fuzzy digraph in vulnerability assessment of water supply network

In modern age, we have been able to observe increasing development of different types of networks. For example, telecommunication, energetic, gas or water networks systems whose main purpose is media transportation from the source to the target place of use.

Vulnerability assessment of water supply network can be categorized into structural components reliability, flow performance reliability, connectivity reliability, and independent reliability. These reliabilities depended on the type of pipe and fittings used, their aging, and the connection between pipe and fitting. Usually, the exact age and condition of connectivity is unknown. These factors can be represented as an IVIFS. Any water supply network can be represented as an IVIFDG $\tilde{\mathcal{D}} = (\tilde{\mathcal{F}}, \tilde{\mathcal{P}})$, where $\tilde{\mathcal{F}}$ is an IVIFS of pipe fittings (vertices) presenting their ages and connectivity conditions as interval-valued intuitionistic fuzzy membership degree $\tilde{\mu}_{\tilde{\mathcal{F}}}(x)$ and interval-valued intuitionistic fuzzy non-membership degree $\tilde{\nu}_{\tilde{\mathcal{F}}}(x)$, and $\tilde{\mathcal{P}}$ is an IVIFS of pipelines (edges) between fittings, whose interval-valued intuitionistic fuzzy membership $\tilde{\mu}_{\tilde{\mathcal{P}}}(xy)$ and non-membership degree $\tilde{\nu}_{\tilde{\mathcal{P}}}(xy)$ can be calculated as:

$$\tilde{\mu}^L_{\tilde{\mathcal{P}}}(xy) \leq \min\{\tilde{\mu}^L_{\tilde{\mathcal{F}}}(x), \tilde{\mu}^L_{\tilde{\mathcal{F}}}(y)\}, \tilde{\mu}^U_{\tilde{\mathcal{P}}}(xy) \leq \min\{\tilde{\mu}^U_{\tilde{\mathcal{F}}}(x), \tilde{\mu}^U_{\tilde{\mathcal{F}}}(y)\}$$
$$\tilde{\nu}^L_{\tilde{\mathcal{P}}}(xy) \geq \max\{\tilde{\nu}^L_{\tilde{\mathcal{F}}}(x), \tilde{\nu}^L_{\tilde{\mathcal{F}}}(y)\} \text{ and } \tilde{\nu}^U_{\tilde{\mathcal{P}}}(xy) \geq \max\{\tilde{\nu}^U_{\tilde{\mathcal{F}}}(x), \tilde{\nu}^U_{\tilde{\mathcal{F}}}(y)\} \text{ for all } xy \in E.$$

The IVIFDG $\tilde{\mathcal{D}} = (\tilde{\mathcal{F}}, \tilde{\mathcal{P}})$ of the water supply network is given in Fig. 8. The interval-valued intuitionistic fuzzy out neighbourhoods are given in Table 1. The final weights on edges can be calculated by finding the score functions of interval-valued intuitionistic fuzzy edges as $s_i = \frac{1}{2}\left((\tilde{\mu}^L_{\tilde{\mathcal{P}}})_i + (\tilde{\mu}^U_{\tilde{\mathcal{P}}})_i - (\tilde{\nu}^L_{\tilde{\mathcal{P}}})_i - (\tilde{\nu}^U_{\tilde{\mathcal{P}}})_i\right)$ [26]. The final weighted digraph given in Fig. 9 and weighted relations are given in Table 2.

Algorithm

```
void interval-valued intuitionistic fuzzy pipeline vulnerability(){
F̃ =IVIFS of pipeline fitting;
number of fittings=count(F̃);
P̃ =Empty IVIFS;
for (int f = 0; f < numberoffittings; f + +){
for (int f' = 0; f' < numberoffittings; f' + +){
if (F̃(x) is adjacent to F̃(y)){
μ̃^L_P̃(ff') ≤ min{μ̃^L_F̃(f), μ̃^L_F̃(f')};
μ̃^U_P̃(ff') ≤ min{μ̃^U_F̃(f), μ̃^U_F̃(f')};
ν̃^L_P̃(ff') ≥ max{ν̃^L_F̃(f), ν̃^L_F̃(f')};
ν̃^U_P̃(ff') ≥ max{ν̃^U_F̃(f), ν̃^L_F̃(f')};
}
}
}
P̃ =IVIFS of egdes;
R = Interval-valued intuitionistic fuzzy relation;
```

WR =Weighted relation;
no of edges= $count(\tilde{\mathcal{P}})$;
for (int $i = 0$;i <no of edges;$i++$){
$s_i = \frac{1}{2}\left((\tilde{\mu}^L_{\tilde{\mathcal{P}}})_i + (\tilde{\mu}^U_{\tilde{\mathcal{P}}})_i - (\tilde{\nu}^L_{\tilde{\mathcal{P}}})_i - (\tilde{\nu}^U_{\tilde{\mathcal{P}}})_i\right)$;
f =Adjacent from Node of $\tilde{\mathcal{P}}_i$;
f' =Adjacent to Node of $\tilde{\mathcal{P}}_i$;
$WR_{ff'} = s_i$;
}
print WR;
Calculate vulnerability using WR;
}

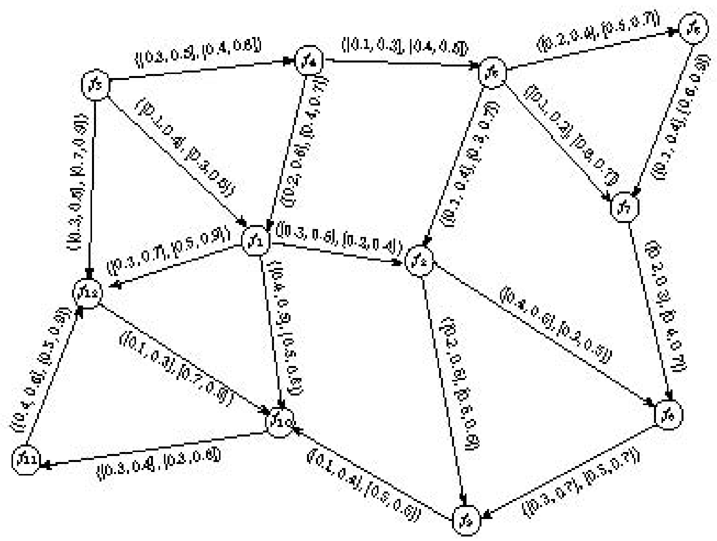

Figure 8. IVIFDG of water supply network.

Table 1. Interval-valued intuitionistic fuzzy out and in neighbourhoods of pipe fittings.

Pipe fittings	$\mathcal{N}$(Pipe fittings)
f_1	$\{f_2\langle[0.3, 0.5], [0.2, 0.4]\rangle, f_{10}\langle[0.4, 0.5], [0.5, 0.8]\rangle, f_{12}\langle[0.3, 0.7], [0.5, 0.9]\rangle\}$
f_2	$\{f_8\langle[0.4, 0.6], [0.2, 0.5]\rangle, f_9\langle[0.2, 0.5], [0.5, 0.6]\rangle\}$
f_3	$\{f_1\langle[0.1, 0.4], [0.3, 0.6]\rangle, f_4\langle[0.3, 0.5], [0.4, 0.6]\rangle, f_{12}\langle[0.3, 0.5], [0.7, 0.9]\rangle\}$
f_4	$\{f_1\langle[0.2, 0.6], [0.4, 0.7]\rangle, f_5\langle[0.1, 0.3], [0.4, 0.5]\rangle\}$
f_5	$\{f_2\langle[0.1, 0.4], [0.3, 0.7]\rangle, f_6\langle[0.2, 0.4], [0.5, 0.7]\rangle, f_7\langle[0.1, 0.2], [0.6, 0.7]\rangle\}$
f_6	$\{f_7\langle[0.1, 0.4], [0.6, 0.9]\rangle\}$
f_7	$\{f_8\langle[0.2, 0.3], [0.4, 0.7]\rangle\}$
f_8	$\{f_9\langle[0.3, 0.7], [0.5, 0.7]\rangle\}$
f_9	$\{f_{10}\langle[0.1, 0.4], [0.5, 0.6]\rangle\}$
f_{10}	$\{f_{11}\langle[0.3, 0.4], [0.3, 0.8]\rangle\}$
f_{11}	$\{f_{12}\langle[0.4, 0.6], [0.5, 0.9]\rangle\}$
f_{12}	$\{f_{10}\langle[0.1, 0.3], [0.7, 0.8]\rangle\}$

6. Conclusions

IVIFG is an extended structure of a fuzzy graph which gives more precision, flexibility, and compatibility to the system when compared with the classical, fuzzy and intuitionistic

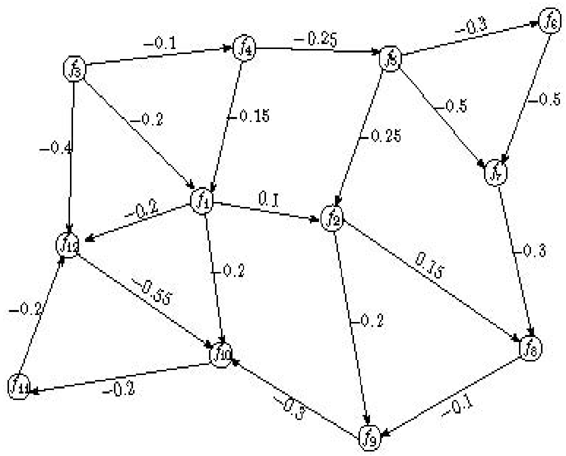

Figure 9. Weighted digraph of water supply network.

Table 2. Weighted out and in neighbourhoods of pipe fittings.

Pipe fittings	$\mathcal{N}$(Pipe fittings)
f_1	$\{f_2(0.1), f_{10}(-0.2), f_{12}(-0.2)\}$
f_2	$\{f_8(0.15), f_9(-0.2)\}$
f_3	$\{f_1(-0.2), f_4(-0.1), f_{12}(-0.4)\}$
f_4	$\{f_1(-0.15), f_5(-0.25)\}$
f_5	$\{f_2(-0.25), f_6(-0.3), f_7(-0.5)\}$
f_6	$\{f_7(-0.5)\}$
f_7	$\{f_8(-0.3)\}$
f_8	$\{f_9(-0.1)\}$
f_9	$\{f_{10}(-0.3)\}$
f_{10}	$\{f_{11}(-0.2)\}$
f_{11}	$\{f_{12}(-0.2)\}$
f_{12}	$\{f_{10}(-0.4)\}$

fuzzy models. In this paper, we have mainly provided specific types of IVIFGs. Firstly, regular, irregular, neighbourly irregular, highly irregular and strongly irregular IVIFGs have been introduced and some of their properties have been investigated. Then, the concept of IVIFC consistent with interval-valued intuitionistic fuzzy cycles in IVIFGs is proposed along with a complete characterization of its structure. Also, we have provided an application of IVIFDG in vulnerability assessment of water supply network. In further work, it is necessary and meaningful to extend the discussed concepts of IVIFGs to interval neutrosophic graphs.

References

[1] Ahn, J. Y., Han, K. S., Oh, S. Y., Lee, C. D., (2011), An application of interval-valued intuitionistic fuzzy sets for medical diagnosis of headache, International Journal of Innovative Computing, Information and Control, 7 (5), pp. 2755-2762.

[2] Akram, M. and Adeel, A., (2017), m-polar fuzzy graphs and m-polar fuzzy line graphs, Journal of Discrete Mathematical Sciences and Cryptography, 20 (8), pp. 1597-1617.

[3] Akram, M. and Davvaz, B., (2012), Strong intuitionistic fuzzy graphs, Filomat, 26 (1), pp. 177-196.

[4] Ashraf, S., Naz, S. and Kerre, E. E., (2018), Dombi fuzzy graphs, Fuzzy Information and Engineering, 10 (1), pp. 58-79.

[5] Ashraf, S., Naz, S., Rashmanlou, H. and Malik, M. A., (2017), Regularity of graphs in single-valued neutrosophic environment, Journal of Intelligent & Fuzzy Systems, 33 (1), pp. 529-542.

[6] Atanassov, K. T., (1986), Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1), pp. 87-96.

[7] Atanassov, K. and Gargov, G., (1989), Interval-valued intuitionistic fuzzy sets, Fuzzy Sets and Systems, 31 (3), pp. 343-349.
[8] Dogenci, M., (2016), A new distance measure for interval valued intuitionistic fuzzy sets and its application to group decision making problems with incomplete weights information, Applied Soft Computing, 41, pp. 120-134.
[9] Gani, A. N., Latha, S. R., (2012), On irregular fuzzy graphs, Applied Mathematical Sciences, 6(11), pp. 517-523.
[10] Hongmei, J. and Lianhua, W., (2009), Interval-valued fuzzy subsemigroups and subgroups associated by interval-valued fuzzy graphs, in: 2009 WRI Global Congress on Intelligent Systems, pp. 484-487.
[11] Ismayil, A. M., Ali, A. M., (2014), On strong interval-valued intuitionistic fuzzy graph, International Journal of Fuzzy Mathematics and Systems, 4(2), pp. 161-168.
[12] Mordeson, J. N. and Nair, P. S., (1996), Cycles and cocycles of fuzzy graphs, Information Sciences, 90(1-4), pp. 39-49.
[13] Mishra, S. N. and Pal, A., (2013), Product of interval-valued intuitionistic fuzzy graph, Annals of Pure and Applied Mathematics, 5(1), pp. 37-46.
[14] Nair, P. S., Cheng, S.C., (2001), Cliques and fuzzy cliques in fuzzy graphs, IEEE.
[15] Nandhini, S. P. and Nandhini, E., (2014), Strongly irregular fuzzy graphs, International Journal of Mathematical Archive-5 (5), pp. 110-114.
[16] Naz, S., Ashraf, S. and Karaaslan, F., (2018), Energy of a bipolar fuzzy graph and its application in decision making, Italian Journal of Pure and Applied Mathematics, 40, pp. 339-352.
[17] Naz, S., Rashmanlou, H. and Malik, M. A., (2017), Operations on single-valued neutrosophic graphs with application, Journal of Intelligent & Fuzzy Systems, 32(3), pp. 2137-2151.
[18] Naz, S., and Malik, M. A., (2018), Single valued neutrosophic line graphs, TWMS Journal of Applied and Engineering Mathematics, 8(2), pp. 483-494.
[19] Naz, S., Malik, M. A. and Rashmanlou, H., (2018), Hypergraphs and transversals of hypergraphs in interval-valued intuitionistic fuzzy setting, The Journal of Multiple-Valued Logic and Soft Computing, 30 (4-6), pp. 399-417.
[20] Rashmanlou, H., and Borzooei, R. A., (2016), New concepts of interval-valued intuitionistic (S, T)-fuzzy graphs, Journal of Intelligent and Fuzzy Systems, 30 (4), pp. 1893–1901.
[21] Rosenfeld, A., (1975), Fuzzy graphs, Fuzzy Sets and their Applications (L. A. Zadeh, K. S. Fu, M. Shimura, Eds.) Academic Press, New York, pp. 77-95.
[22] Somasundaram, A., and Somasundaram, S., (1998), Domination in fuzzy graphs-I. Pattern Recognition letters, 19, pp. 787-791.
[23] Sahoo, S. and Pal, M., (2017), Intuitionistic fuzzy tolerance graphs with application, Journal of Applied Mathematics and Computing, 55(1-2), pp. 495-511.
[24] Sahoo, S. and Pal, M., (2016), Product of intuitionistic fuzzy graphs and degree, Journal of Intelligent and Fuzzy Systems, 32 (1), pp. 1059-1067.
[25] Shahzadi, S. and Akram, M., (2016), Edge regular intuitionistic fuzzy soft graphs, Journal of Intelligent and Fuzzy Systems, 31(3), 1881-1895.
[26] Xu, Z., (2007), Methods for aggregating interval-valued intuitionistic fuzzy information and their application to decision making, Control and Decision, 22, pp. 215-219.
[27] Zadeh, L. A., (1965), Fuzzy sets, Information and Control, 8(3), pp. 338-353.
[28] Zhang, Z., Wang, M., Hu, Y., Yang, J., Ye, Y. and Li, Y., (2013), A dynamic interval-valued intuitionistic fuzzy sets applied to pattern recognition, Mathematical Problems in Engineering.

Sumera Naz for the photography and short autobiography, see TWMS J. App. Eng. Math., V.8, N.2, 2018.

M. Aslam Malik for the photography and short autobiography, see TWMS J. App. Eng. Math., V.8, N.2, 2018.