HARMONIC RECIPROCAL STATUS INDEX AND COINDEX OF GRAPHS

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ABSTRACT. The reciprocal status of a vertex u is defined as the sum of reciprocal of the distances between u and all other vertices of a graph G. In this paper we have defined the harmonic reciprocal status index and coindex of a graph and obtained the bounds for it. Further the harmonic reciprocal status index and coindex of some graphs are obtained.

Keywords: Distance in graph, Reciprocal status of a vertex, Harmonic reciprocal status index.

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1. INTRODUCTION

The harmonic index, based on the degrees of the vertices is well studied in the literature [2, 3, 6, 8, 9, 15, 17, 18]. In this paper we study the harmonic index, based on the reciprocal distances in graphs.

Let G be a connected, nontrivial graph on n vertices and m edges. Let V(G) be the vertex set and E(G) be the edge set of G. The edge joining the vertices u and v is denoted by uv. The degree of a vertex u is the number of edges incident to it and is denoted by d(u). If all the vertices of G have same degree equal to r, then G is called a regular graph of degree r. The distance between the vertices u and v, denoted by d(u, v), is the length of the shortest path joining u and v in G. The eccentricity of a vertex u in a graph G is defined as e(u) = max{d(u, v) | v ∈ V(G)}. The maximum distance between any pair of vertices in G is called the diameter of G and is denoted by diam(G) [1].

The status [5] of a vertex u is defined as the sum of its distances from every other vertex of G and is denoted by σ(u). That is,

\[ \sigma(u) = \sum_{v \in V(G)} d(u, v). \]
In [14], the first and second status connectivity indices of a connected graph $G$ are defined respectively as

$$S_1(G) = \sum_{uv \in E(G)} [\sigma(u) + \sigma(v)] \quad \text{and} \quad S_2(G) = \sum_{uv \in E(G)} \sigma(u)\sigma(v).$$

The reciprocal status of a vertex $u$ is defined as the sum of reciprocal of its distances from every other vertex of $G$ and is denoted by $rs(u)$. That is,

$$rs(u) = \sum_{v \in V(G), u \neq v} \frac{1}{d(u, v)}.$$

The Harary index $HI(G)$ of a connected graph $G$ is defined as the sum of reciprocal of the distances between all pairs of vertices of $G$ [7]. That is,

$$HI(G) = \sum_{\{u, v\} \subseteq V(G), u \neq v} \frac{1}{d(u, v)} = \frac{1}{2} \sum_{u \in V(G)} rs(u).$$

For more about Harary index one can refer [10, 16].

The first reciprocal status connectivity index $RS_1(G)$ and second reciprocal status connectivity index $RS_2(G)$ of a connected graph $G$ are defined respectively as [12, 13]

$$RS_1(G) = \sum_{uv \in E(G)} [rs(u) + rs(v)] \quad \text{and} \quad RS_2(G) = \sum_{uv \in E(G)} rs(u)rs(v).$$

The harmonic index of a graph $G$ is defined as [4]

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d(u) + d(v)}.$$

Recent results on the harmonic index can be found in [2, 3, 6, 8, 9, 15, 17, 18].

The Harmonic status index of a graph $G$ is defined as [11]

$$HS(G) = \sum_{uv \in E(G)} \frac{2}{\sigma(u) + \sigma(v)}.$$

Motivated by the harmonic index and harmonic status index of a graph, we introduce and study here the harmonic reciprocal status index and harmonic reciprocal status co-index of connected graphs.

The harmonic reciprocal status index of a connected graph $G$ is defined as

$$HRS(G) = \sum_{uv \in E(G)} \frac{2}{rs(u) + rs(v)}$$

and harmonic reciprocal status coindex of a connected graph $G$ is defined as

$$HRS(G) = \sum_{uv \notin E(G)} \frac{2}{rs(u) + rs(v)}.$$

For a graph given in Fig. 1, $HRS(G) = \frac{913}{420} \approx 2.1738$ and $HRS(G) = \frac{8}{13} \approx 0.6153$. 
2. Harmonic Reciprocal Status Index

First we give bounds for the harmonic reciprocal status index.

**Theorem 2.1.** Let $G$ be a connected graph with $n$ vertices and let $\text{diam}(G) = D$. Then

$$\sum_{uv \in E(G)} \frac{2}{n-1 + \frac{1}{2} [d(u) + d(v)]} \leq HRS(G) \leq \frac{2}{D(n-1) + (1 - \frac{1}{D}) [d(u) + d(v)]}.$$  

Equality on both sides holds if and only if $\text{diam}(G) \leq 2$.

**Proof.** Upper bound: For any vertex $u$ of $G$, there are $d(u)$ vertices which are at distance 1 from $u$ and the remaining $n-1-d(u)$ vertices are at distance at most $D$. Therefore for any vertex $u \in V(G)$,

$$rs(u) \geq d(u) + \frac{1}{D}(n-1-d(u)) = \frac{1}{D}(n-1 + d(u))\left(1 - \frac{1}{D}\right).$$

Therefore

$$HRS(G) = \sum_{uv \in E(G)} \frac{2}{rs(u) + rs(v)} \leq \sum_{uv \in E(G)} \frac{2}{\frac{2}{D}(n-1) + (1 - \frac{1}{D}) (d(u) + d(v))}.$$  

Lower bound: For any vertex $u$ of $G$, there are $d(u)$ vertices which are at distance 1 from $u$ and the remaining $n-1-d(u)$ vertices are at distance at least 2. Therefore for any vertex $u \in V(G)$,

$$rs(u) \leq d(u) + \frac{1}{2}(n-1-d(u)) = \frac{1}{2}[d(u) + n - 1].$$

Therefore

$$HRS(G) = \sum_{uv \in E(G)} \frac{2}{rs(u) + rs(v)} \geq \sum_{uv \in E(G)} \frac{2}{(n-1) + \frac{1}{2} [d(u) + d(v)]}.$$  

For equality: If the diameter of $G$ is 1 or 2 then the equality holds.

Conversely, let

$$HRS(G) = \sum_{uv \in E(G)} \frac{2}{(n-1) + \frac{1}{2} [d(u) + d(v)]}.$$
Suppose, \( \text{diam}(G) \geq 3 \), then there exists at least one pair of vertices, say \( u_1 \) and \( u_2 \) such that \( d(u_1, u_2) \geq 3 \).

Therefore

\[
rs(u_1) \leq d(u_1) + \frac{1}{3} + \frac{1}{2}(n - 2 - d(u_1)) = \frac{n}{2} - \frac{2}{3} + \frac{d(u_1)}{2}.
\]

Similarly \( rs(u_2) \leq \frac{n}{2} - \frac{2}{3} + \frac{d(u_2)}{2} \) and for all other vertices \( u \) of \( G \), \( rs(u) \leq \frac{n}{2} - \frac{1}{2} + \frac{d(u)}{2} \).

Partition the edge set of \( G \) into three sets \( E_1 \), \( E_2 \) and \( E_3 \), such that

\[
E_1 = \left\{ u_1v \mid rs(u_1) \leq \frac{n}{2} - \frac{2}{3} + \frac{d(u_1)}{2} \text{ and } rs(v) \leq \frac{n}{2} - \frac{1}{2} + \frac{d(v)}{2} \right\},
\]

\[
E_2 = \left\{ u_2v \mid rs(u_2) \leq \frac{n}{2} - \frac{2}{3} + \frac{d(u_2)}{2} \text{ and } rs(v) \leq \frac{n}{2} - \frac{1}{2} + \frac{d(v)}{2} \right\}
\]

and

\[
E_3 = \left\{ uv \mid rs(u) \leq \frac{n}{2} - \frac{1}{2} + \frac{d(u)}{2} \text{ and } rs(v) \leq \frac{n}{2} - \frac{1}{2} + \frac{d(v)}{2} \right\}.
\]

It is easy to check that \( |E_1| = d(u_1), \ |E_2| = d(u_2) \) and \( |E_3| = m - d(u_1) - d(u_2) \).

Therefore

\[
HRS(G) = \sum_{uv \in E(G)} \frac{2}{rs(u) + rs(v)}
\]

\[
= \sum_{u_1v \in E_1} \frac{2}{rs(u_1) + rs(v)} + \sum_{u_2v \in E_2} \frac{2}{rs(u_2) + rs(v)} + \sum_{uv \in E_3} \frac{2}{rs(u) + rs(v)}
\]

\[
\geq \sum_{u_1v \in E_1} \frac{2}{n - \frac{7}{6} + \frac{1}{2}(d(u_1) + d(v))} + \sum_{u_2v \in E_2} \frac{2}{n - \frac{7}{6} + \frac{1}{2}(d(u_2) + d(v))}
\]

\[
+ \sum_{uv \in E_3} \frac{2}{n - 1 + \frac{1}{2}[d(u) + d(v)]}
\]

which is a contradiction. Hence \( \text{diam}(G) \leq 2 \). \( \square \)

**Corollary 2.1.** Let \( G \) be a connected graph with \( n \) vertices, \( m \) edges and \( \text{diam}(G) = D \). Let \( \delta \) and \( \Delta \) be the minimum and maximum degree of the vertices of \( G \) respectively. Then

\[
\frac{2m}{n - 1 + \Delta} \leq HRS(G) \leq \frac{m}{\frac{n-1}{D} + \left(1 - \frac{1}{D}\right)\delta}.
\]

**Proof.** For any vertex \( u \) of \( G \), \( \delta \leq d(u) \leq \Delta \). Therefore substituting \( d(u) + d(v) \geq 2\delta \) in the upper bound and \( d(u) + d(v) \leq 2\Delta \) in the lower bound of Theorem 2.1, we get the results. \( \square \)

**Corollary 2.2.** Let \( G \) be a connected regular graph of degree \( r \) on \( n \) vertices and \( m \) edges and let \( \text{diam}(G) = D \). Then

\[
\frac{2m}{n - 1 + r} \leq HRS(G) \leq \frac{m}{\frac{n-1}{D} + \left(1 - \frac{1}{D}\right)r}.
\]

Equality on both side holds if and only if \( \text{diam}(G) \leq 2 \).

**Proof.** For any vertex \( u \) of \( G \), \( d(u) = r \). Therefore the results follows by the Theorem 2.1. \( \square \)
Now we compute the harmonic reciprocal status index of some specific graphs.

**Proposition 2.1.** For a complete graph $K_n$ on $n$ vertices, $HRS(K_n) = \frac{n}{2}$.

**Proof.** For any vertex $u$ of $K_n$, $rs(u) = n - 1$. Therefore by the definition of harmonic reciprocal status index, $HRS(K_n) = \frac{n}{2}$. \hfill $\square$

**Proposition 2.2.** For a complete bipartite graph $K_{p,q}$, $HRS(K_{p,q}) = \frac{4pq}{3(p+q) - 2}$.

**Proof.** The vertex set $V(K_{p,q})$ can be partitioned into two independent sets $V_1$ and $V_2$ such that for every edge $uv$ of $K_{p,q}$, the vertex $u \in V_1$ and $v \in V_2$. Therefore $d(u) = q$ and $d(v) = p$, where $|V_1| = p$ and $|V_2| = q$. The graph $K_{p,q}$ has $n = p + q$ vertices and $m = pq$ edges. Also $diam(K_{p,q}) \leq 2$. Therefore by the equality part of Theorem 2.1,

$$HRS(K_{p,q}) = \sum_{uv \in E(K_{p,q})} \frac{2}{p + q - 1 + \frac{1}{2}[p + q]} = \frac{4pq}{3(p+q) - 2}.$$ \hfill $\square$

**Proposition 2.3.** For a path $P_n$ on $n$ vertices,

$$HRS(P_n) = \left[ \frac{4}{n-1 + 2\sum_{i=1}^{n-2} \frac{1}{i}} \right] + \sum_{i=2}^{n-2} \left[ \frac{2}{\sum_{i=1}^{n-1} \frac{1}{i} + \sum_{j=1}^{i-1} \frac{1}{j}} \right] + \sum_{i=1}^{n-2} \left[ \frac{2}{\sum_{j=1}^{i-1} \frac{1}{j} + \sum_{j=1}^{i} \frac{1}{j} + \sum_{j=1}^{n-i-1} \frac{1}{j}} \right].$$

**Proof.** Let $v_1, v_2, \ldots, v_n$ be the vertices of $P_n$, where $v_i$ is adjacent to $v_{i+1}$, $i = 1, 2, \ldots, n-1$. Therefore for $i = 1, 2, \ldots, n$,

$$rs(v_1) = \sum_{i=1}^{n-1} \frac{1}{i},$$

$$rs(v_i) = \sum_{j=1}^{i-1} \frac{1}{j} + \sum_{j=1}^{n-1} \frac{1}{j}, \text{ for } 2 \leq i \leq n-1$$

and

$$rs(v_n) = \sum_{i=1}^{n-1} \frac{1}{i}.$$ 

Therefore,

$$HRS(P_n) = \sum_{uv \in E(P_n)} \frac{2}{rs(u) + rs(v)}$$

$$= \left[ \frac{2}{rs(v_1) + rs(v_2)} \right] + \sum_{i=2}^{n-2} \left[ \frac{2}{rs(v_i) + rs(v_{i+1})} \right] + \left[ \frac{2}{rs(v_{n-1}) + rs(v_n)} \right]$$

$$= \left[ \frac{2}{\sum_{i=1}^{n-1} \frac{1}{i} + \sum_{j=1}^{n-2} \frac{1}{j}} \right] + \sum_{i=2}^{n-2} \left[ \frac{2}{\sum_{j=1}^{i-1} \frac{1}{j} + \sum_{j=1}^{n-i-1} \frac{1}{j}} \right]$$

$$+ \left[ \frac{2}{\sum_{j=1}^{n-2} \frac{1}{j} + \sum_{i=1}^{n-1} \frac{1}{i}} \right]$$

$$= \left[ \frac{4}{n-1 + 2\sum_{i=1}^{n-2} \frac{1}{i}} \right] + \sum_{i=2}^{n-2} \left[ \frac{2}{\sum_{j=1}^{i} \frac{1}{j} + \sum_{j=1}^{n-i-1} \frac{1}{j}} \right].$$ \hfill $\square$
Proposition 2.4. For a cycle $C_n$ on $n \geq 3$ vertices,

$$HRS(C_n) = \begin{cases} 
\frac{2}{n} + \frac{n}{2} \sum_{i=1}^{(n-2)/2} \frac{1}{i}, & \text{if } n \text{ is even} \\
\frac{n}{2} \sum_{i=1}^{(n-1)/2} \frac{1}{i}, & \text{if } n \text{ is odd}.
\end{cases}$$

Proof. Case (i): If $n$ is even number then for any vertex $u$ of $C_n$,

$$rs(u) = 2 \left[ 1 + \frac{1}{2} + \cdots + \frac{1}{\frac{n-2}{2}} \right] + \frac{1}{\frac{n}{2}} = \frac{2}{n} + 2 \sum_{i=1}^{(n-2)/2} \frac{1}{i}.$$ 

Therefore,

$$HRS(C_n) = \sum_{uv \in E(C_n)} \frac{2}{n} \frac{rs(u) + rs(v)}{rs(u) + rs(v)} = \sum_{uv \in E(C_n)} \left[ \frac{2}{n} + 2 \sum_{i=1}^{(n-2)/2} \frac{1}{i} + \frac{2}{n} + 2 \sum_{i=1}^{(n-2)/2} \frac{1}{i} \right] = \frac{n}{2} + 2 \sum_{i=1}^{(n-2)/2} \frac{1}{i}.$$ 

Case (ii): If $n$ is odd then for any vertex $u$ of $C_n$,

$$rs(u) = 2 \left[ 1 + \frac{1}{2} + \cdots + \frac{1}{\frac{n-1}{2}} \right] = 2 \sum_{i=1}^{(n-1)/2} \frac{1}{i}.$$ 

Therefore

$$HRS(C_n) = \sum_{uv \in E(C_n)} \frac{2}{n} \frac{rs(u) + rs(v)}{rs(u) + rs(v)} = \sum_{uv \in E(C_n)} \frac{2}{n} \frac{\sum_{i=1}^{(n-1)/2} \frac{1}{i} + 2 \sum_{i=1}^{(n-1)/2} \frac{1}{i}}{\sum_{i=1}^{(n-1)/2} \frac{1}{i}} = \frac{2 \sum_{i=1}^{(n-1)/2} \frac{1}{i}}{2 \sum_{i=1}^{(n-1)/2} \frac{1}{i}}.$$ 

\[\square\]

A wheel $W_{k+1}$ is a graph obtained from the cycle $C_k$, $k \geq 3$, by adding a new vertex and making it adjacent to all the vertices of $C_k$. The degree of a central vertex of $W_{k+1}$ is $k$ and the degree of all other vertices is 3.

Proposition 2.5. For a wheel $W_{k+1}$, $k \geq 3$,

$$HRS(W_{k+1}) = \frac{2k(5k + 9)}{3k^2 + 12k + 9}.$$ 

Proof. Partition the edge set $E(W_{k+1})$ into two sets $E_1$ and $E_2$, such that $E_1 = \{uv \mid d(u) = k \text{ and } d(v) = 3\}$ and $E_2 = \{uv \mid d(u) = 3 \text{ and } d(v) = 3\}$. It is easy to check that $|E_1| = |E_2| = k$. Also $\text{diam}(W_{k+1}) = 2$. Therefore by the equality part of Theorem 2.1,
\[ HRS(W_{k+1}) = \sum_{uv \in E(W_{n+1})} \frac{2}{k + \frac{1}{2}[d(u) + d(v)]} \]

\[ = \sum_{uv \in E_1} \frac{2}{k + \frac{1}{2}[d(u) + d(v)]} + \sum_{uv \in E_2} \frac{2}{k + \frac{1}{2}[d(u) + d(v)]} \]

\[ = \sum_{uv \in E_1} \frac{2}{k + \frac{1}{2}(k + 3)} + \sum_{uv \in E_2} \frac{2}{k + \frac{1}{2}(3 + 3)} \]

\[ = \frac{2k}{k + \frac{1}{2}(k + 3)} + \frac{2k}{k + 3} \]

\[ = \frac{2k(5k + 9)}{3k^2 + 12k + 9}. \]

A windmill graph \( F_k, k \geq 2 \), is a graph that can be constructed by coalescence \( k \) copies of the cycle \( C_3 \) of length 3 with a common vertex. It has \( 2k + 1 \) vertices and \( 3k \) edges. The degree of a coalescence vertex of \( F_k \) is \( 2k \) and the degree of all other vertices is 2.

**Proposition 2.6.** For a windmill graph \( F_k, k \geq 2 \),

\[ HRS(F_k) = \frac{k(7k + 5)}{3k^2 + 4k + 1}. \]

**Proof.** Partition the edge set \( E(F_k) \) into two sets \( E_1 \) and \( E_2 \), such that \( E_1 = \{uv \mid d(u) = 2k \text{ and } d(v) = 2\} \) and \( E_2 = \{uv \mid d(u) = 2 \text{ and } d(v) = 2\} \). It is easy to check that \( |E_1| = 2k \text{ and } |E_2| = k \). Also \( \text{diam}(F_k) = 2 \). Therefore by the equality part of Theorem 2.1,

\[ HRS(F_k) = \sum_{uv \in E(F_k)} \frac{2}{2k + \frac{1}{2}[d(u) + d(v)]} \]

\[ = \sum_{uv \in E_1} \frac{2}{2k + \frac{1}{2}[d(u) + d(v)]} + \sum_{uv \in E_2} \frac{2}{2k + \frac{1}{2}[d(u) + d(v)]} \]

\[ = \sum_{uv \in E_1} \frac{2}{2k + \frac{1}{2}[2k + 2]} + \sum_{uv \in E_2} \frac{2}{2k + \frac{1}{2}[2 + 2]} \]

\[ = \frac{4k}{3k + 1} + \frac{2k}{2k + 2} \]

\[ = \frac{k(7k + 5)}{3k^2 + 4k + 1}. \]

□

3. Harmonic Reciprocal Status Coindex of Graphs

**Theorem 3.1.** Let \( G \) be a connected graph on \( n \) vertices and let \( \text{diam}(G) = D \). Then

\[ \sum_{uv \in E(G)} \frac{2}{n - 1 + \frac{1}{2}[d(u) + d(v)]} \leq HRS(G) \leq \sum_{uv \in E(G)} \frac{2}{D(n - 1 + (1 - \frac{1}{D})[d(u) + d(v)]}. \]

Equality on both sides holds if and only if \( \text{diam}(G) \leq 2 \).

**Proof.** Proof is analogous to that of Theorem 2.1. □
Corollary 3.1. Let $G$ be a connected graph with $n$ vertices, $m$ edges and $\text{diam}(G) = D$. Let $\delta$ and $\Delta$ be the minimum and maximum degree of the vertices of $G$ respectively. Then
\[
\frac{n(n-1) - 2m}{n-1 + \Delta} \leq HRS(G) \leq \frac{n(n-1) - 2m}{2 \left[ \frac{n-1}{\Delta} + (1 - \frac{1}{\Delta}) \delta \right]}.
\]

Proof. For any vertex $u \in V(G)$, $\delta \leq d(u) \leq \Delta$. Therefore $2\delta \leq d(u) + d(v) \leq 2\Delta$. The graph $G$ has $\frac{n(n-1)}{2} - m$ pair of non adjacent vertices. Substituting $d(u) + d(v) \geq 2\delta$ in the upper bound and $d(u) + d(v) \leq 2\Delta$ in the lower bound of Theorem 3.1 we get the results. \hspace{1cm} \Box

Corollary 3.2. Let $G$ be a connected $r$-regular graph on $n$ vertices and let $\text{diam}(G) = D$. Then
\[
\frac{n(n-1) - nr}{n-1 + r} \leq HRS(G) \leq \frac{n(n-1) - nr}{2 \left[ \frac{n-1}{\Delta} + (1 - \frac{1}{\Delta}) r \right]}.\]

Equality on both sides holds if and only if $\text{diam}(G) \leq 2$. Proof. Substituting $d(u) = r$ for all $u \in V(G)$ in Theorem 3.1, we get the results. \hspace{1cm} \Box

Proposition 3.1. For a complete graph $K_n$, $HRS(K_n) = 0$.

Proposition 3.2. For a complete bipartite graph $K_{p,q}$,
\[
HRS(K_{p,q}) = \frac{p(p-1)}{2q + p - 1} + \frac{q(q-1)}{2p + q - 1}.
\]

Proof. Let $V_1$ and $V_2$ be the partite sets of $V(K_{p,q})$, where $|V_1| = p$ and $|V_2| = q$ such that for every edge of $K_{p,q}$, one end in $V_1$ and other end in $V_2$. If $u \in V_1$ then $rs(u) = q + \frac{1}{2}(p-1)$ and if $u \in V_2$ then $rs(u) = p + \frac{1}{2}(q-1)$. Therefore for $u, v \in V_1$, $rs(u) + rs(v) = 2q + (p -1)$ and for $u, v \in V_2$, $rs(u) + rs(v) = 2p + (q -1)$. Therefore,
\[
HRS(K_{p,q}) = \sum_{uv \in E(K_{p,q})} \frac{2}{rs(u) + rs(v)} = \sum_{\{u,v\} \subseteq V_1} \frac{2}{rs(u) + rs(v)} + \sum_{\{u,v\} \subseteq V_2} \frac{2}{rs(u) + rs(v)} = \frac{p(p-1)}{2q + p - 1} + \frac{q(q-1)}{2p + q - 1}.
\]

\hspace{1cm} \Box

Proposition 3.3. For a cycle $C_n$ on $n \geq 3$ vertices,
\[
HRS(C_n) = \begin{cases} 
\frac{n^2-3n}{4 + 4 \sum_{i=1}^{\frac{n-1}{2}} \frac{1}{i}}, & \text{if } n \text{ is even} \\
\frac{n^3-3n}{4 \sum_{i=1}^{\frac{n-1}{2}} \frac{1}{i}}, & \text{if } n \text{ is odd}.
\end{cases}
\]

Proof. There are $\frac{n(n-1)}{2} - n$ pairs of non-adjacent vertices in $C_n$. As seen in Proposition 2.4, we have for a vertex $u$ of $C_n$,
\[
rs(u) = \begin{cases} 
\frac{2}{n} + 2 \sum_{i=1}^{\frac{n-2}{2}} \frac{1}{i}, & \text{if } n \text{ is even} \\
2 \sum_{i=1}^{\frac{n-1}{2}} \frac{1}{i}, & \text{if } n \text{ is odd}.
\end{cases}
\]
Therefore by the definition of harmonic reciprocal status coinindex, we get the results. \hspace{1cm} \Box
Proposition 3.4. For a wheel $W_{k+1}$, $k \geq 3$,
\[
\text{HRS}(W_{k+1}) = \frac{k(k - 3)}{k + 3}.
\]

Proof. The non adjacent pairs of vertices of the wheel $W_{k+1}$ has degree 3 and there are $\binom{k+1}{2} - 2k$ pairs of non adjacent vertices in $W_{k+1}$. Also $\text{diam}(W_{k+1}) = 2$. Therefore by the equality part of Theorem 3.1, we get the result.

Proposition 3.5. For a windmill graph $F_k$, $k \geq 2$,
\[
\text{HRS}(F_k) = \frac{2k(k - 1)}{k + 1}.
\]

Proof. The non adjacent pairs of vertices of the windmill graph $F_k$ has degree 2 and there are $2k(2k+1) - 3k$ such pairs in $F_k$. Also $\text{diam}(F_k) = 2$. Therefore by the equality part of Theorem 3.1, we get the result.

4. Conclusion

We have introduced harmonic reciprocal status index and coindex of connected graphs and obtained bounds for these indices. Also these indices of certain standard graphs have been obtained.

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References


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