

COMPUTING VE TOPOLOGICAL INDICES OF TICKYSIM SPINNAKER MODEL

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ABSTRACT. Topological parameters are employed to calculate the biological activity, thermodynamic properties, chemical reactivity and physical features of different structures, since various studies show a strong bond between the molecular structure and its properties [2, 4, 8, 14, 23, 25]. Therefore, helping the researchers to make up for the shortage of laboratory experiments and can provide theoretical ground for the manufacturing of diverse products. The aim of this paper is to calculate ve-version of sum-connectivity index, harmonic index, atom bond connectivity index and geometric arithmetic index of Tickysim SpiNNaker Model Sheet, by considering edge partitioning method. These can be used for better understanding of architectural techniques and innovative designs

Keywords: ve-Degree, ve-Topological Indices, Tickysim SpiNNaker Model.

AMS Subject Classification: 05-XX, 94XX, 94C15, 05CXX, 05C10.

1. INTRODUCTION

Topological indices are numerical parameters of molecular graphs associated with quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR) [3], [4], [5] and [11]. The history of topological indices are traced back from 1947 by Wiener, while he was working on the boiling point of paraffin [28]. Various topological indices, topological polynomials and certain bounds are calculated by the authors for different chemical structures and networks in [1], [2], [3], [4], [5], [6], [8], [11], [13], [14], [15], [20], [21], [22], [23], [24], [25] and [31]. In this paper, we compute a variety of topological indices for the molecular structure of Tickysim SpiNNaker Model sheet. Moreover, analytically closed formulas for the indices are given which will be helpful in

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§ Manuscript received: November 14, 2019; accepted: June 03, 2020.

TWMS Journal of Applied and Engineering Mathematics, Vol.11, No.4 © Işık University, Department of Mathematics, 2021; all rights reserved.

studying the underlying topologies.

The general randic index was defined as [26],

$$R_\alpha(G) = \sum_{uv \in E(G)} (d_u d_v)^\alpha. \tag{1}$$

The sum connectivity index was defined in 2009 [32],

$$\chi(G) = \sum_{uv \in E(G)} (d_u + d_v)^{-\frac{1}{2}}. \tag{2}$$

The general sum connectivity index was defined by Zhou [33],

$$\chi_\alpha(G) = \sum_{uv \in E(G)} (d_u + d_v)^\alpha. \tag{3}$$

Moreover, the harmonic index is defined as follows [29],

$$H(G) = \sum_{uv \in E(G)} \frac{2}{d_u + d_v}. \tag{4}$$

In 2015 the general version of harmonic index was defined [30],

$$H_k(G) = \sum_{uv \in E(G)} \left(\frac{2}{d_u + d_v} \right)^k. \tag{5}$$

Ranjini in 2013 stated the redefined first, second and third Zareb indices [27],

$$ReZG_1(G) = \sum_{uv \in E(G)} \left(\frac{d_u + d_v}{d_u d_v} \right), \tag{6}$$

$$ReZG_2(G) = \sum_{uv \in E(G)} \left(\frac{d_u d_v}{d_u + d_v} \right), \tag{7}$$

$$ReZG_3(G) = \sum_{uv \in E(G)} (d_u d_v)(d_u + d_v). \tag{8}$$

Whereas in 2010 the atomic bond connectivity index was defined [9],

$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}. \tag{9}$$

Furtula stated geometric arithmetic index as [12],

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}. \tag{10}$$

And the augmented Zagreb index is:

$$A(G) = \sum_{uv \in E(G)} \left[\frac{d_u d_v}{d_u + d_v - 2} \right]^3.$$

The 4th version of atomic bond connectivity index is defined as [16],

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{s_u + s_v - 2}{s_u s_v}}. \quad (11)$$

The 5th version of geometric arithmetic index is defined as [18],

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{s_u s_v}}{s_u + s_v}. \quad (12)$$

The sanskruti index is defined as [19],

$$S(G) = \sum_{uv \in E(G)} \left(\frac{s_u s_v}{s_u + s_v - 2} \right)^3. \quad (13)$$

The 5th version of atomic bond connectivity index is defined as [10],

$$ABC_5(G) = \sum_{uv \in E(G)} \sqrt{\frac{\epsilon_u + \epsilon_v - 2}{\epsilon_u \epsilon_v}}. \quad (14)$$

The 4th version of geometric arithmetic index is defined as [17],

$$GA_4(G) = \sum_{uv \in E(G)} \frac{2\sqrt{\epsilon_u \epsilon_v}}{\epsilon_u + \epsilon_v}. \quad (15)$$

In 2017, e_v -degree for the edges and v_e -degree for the vertices of a graph are defined [7].

Based on the novel concept some v_e -degree indices are defined as follows [8],

The general sum connectivity index was defined by,

$$\chi^{ve}(G) = \sum_{uv \in E(G)} (d_{ve}(u) + d_{ve}(v))^{-\frac{1}{2}}. \quad (16)$$

Moreover, the harmonic index is defined as follows,

$$H^{ve}(G) = \sum_{uv \in E(G)} \frac{2}{d_{ve}(u) + d_{ve}(v)}. \quad (17)$$

The v_e -degree version of atomic bond connectivity index is defined as,

$$ABC^{ve}(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_{ve}(u) + d_{ve}(v) - 2}{d_{ve}(u)d_{ve}(v)}}. \quad (18)$$

The 4th version of geometric arithmetic index is defined as,

$$GA^{ve}(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_{ve}(u)d_{ve}(v)}}{d_{ve}(u) + d_{ve}(v)}. \quad (19)$$

Topological descriptors have gained more importance over the past two decades, as these numerical parameters are helpful for modelling and prediction of certain physicochemical properties of different molecules and networks, usually by means of correlation coefficient. There are many molecular topological studies present in literature which showed the importance of topological indices in QSAR/QSPR studies, [1, 4, 8, 9, 10, 14, 20]. Recently in 2017, Chellali et al studied two new graph degree based invariants, the authors found that these indices are closely related to the degree based topological index; first Zagreb index. In 2018, Ediz defined some v_e -degree based indices and studied the correlation between

TABLE 1. Ve-degree Based Edges

$(d_{ve}u, d_{ve}v)$	$ (d_{ve}u, d_{ve}v) $
(7, 13)	4
(12, 16)	4
(12, 23)	2
(13, 13)	2
(13, 17)	4
(13, 22)	4
(16, 17)	4
(16, 23)	4
(16, 26)	4
(17, 17)	$2m+2n-20$
(17, 22)	4
(17, 26)	$4m+4n-36$
(22, 26)	4
(23, 26)	4
(23, 30)	2
(26, 26)	$2m+2n-18$
(26, 30)	$4m+4n-36$
(30, 30)	$3mn-14m-14n+65$

indices. These interesting results motivated us to calculate ve-degree based indices for Tickysim SpiNNaker Model sheet.

2. MAIN RESULTS AND DISCUSSION

In this section we calculate the ve-topological indices for the molecular structure of Tickysim SpiNNaker Model sheet. Further, we provide the closed form formulas for the defined topological descriptors. Whereas, at the end conclusion has been drawn and some future work is defined.

Tickysim is basically a timing based simulator of the inter-chip interconnection network of the SpiNNaker architecture. The Tickysim simulator is written in C and implements a synchronous model of SpiNNaker asynchronous interconnection network. A clock tick in the simulator is defined to correspond with a clock tick in a SpiNNaker router. The model consists of network of nodes, which represent individual SpiNNaker chips, connected via models of the slow chip to chip links. Consider $TSM(V, E)$ be the molecular graph of Tickysim SpiNNaker Model sheet in which $V(TSM)$ represent a non-empty set of vertices and $E(TSM)$ represent a set of edges. The ve-degree of any vertex u is defined as, deg_{ve} =sum of neighborhood degree of u- number of triangles in which u contribute, [7]. The graph of TSM is simple and connected with order, $|V| = mn$ and size, $|E| = 3mn - 2m - 2n + 1$. It has the above defined edge partitions depending upon ve-vertex degree.

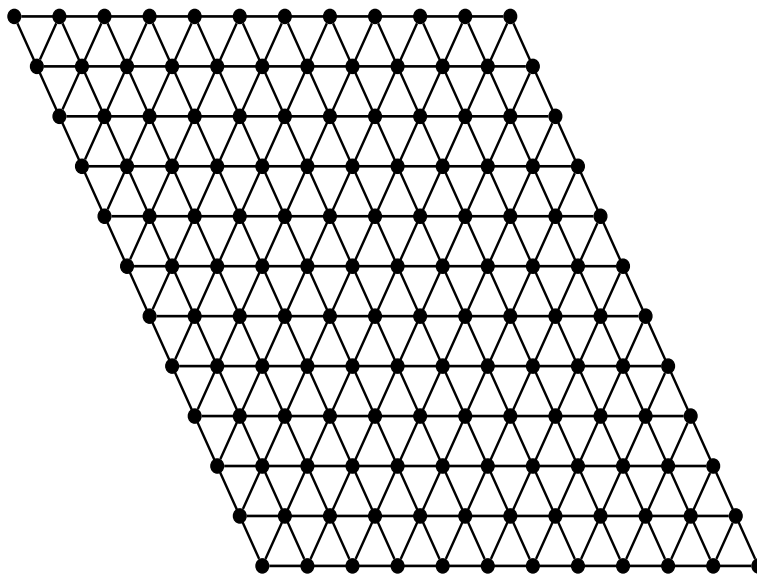


FIGURE 1. Tickkysim SpiNNaker Model Sheet

Theorem 2.1. *The sum connectivity index of Tickkysim SpiNNaker Model sheet is,*

$$\begin{aligned} \chi^{ve}(TSM) = & \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{5}} + \frac{2}{\sqrt{7}} + \frac{2}{\sqrt{26}} + \frac{2}{\sqrt{53}} + \frac{4}{\sqrt{30}} + \frac{4}{\sqrt{33}} + \frac{4}{\sqrt{42}} + \frac{6}{\sqrt{35}} \\ & + \frac{8}{\sqrt{39}} + \frac{4}{7} + \frac{m+n-9}{\sqrt{13}} + \frac{2m+2n-18}{\sqrt{14}} + \frac{2m+2n-20}{\sqrt{34}} \\ & + \frac{4m+4n-36}{\sqrt{43}} + \frac{3mn-14m-14n+65}{\sqrt{60}}. \end{aligned}$$

Proof. By using the information in Table (1) and inserting the values in Formula (16), we get

$$\begin{aligned}
 \chi^{ve}(TSM) &= \sum_{uv \in E(TSM)} \frac{1}{(d_{ve}(u) + d_{ve}(v))^{\frac{1}{2}}}, \\
 &= 4 \frac{1}{(7 + 13)^{\frac{1}{2}}} + 4 \frac{1}{(12 + 16)^{\frac{1}{2}}} + 2 \frac{1}{(12 + 23)^{\frac{1}{2}}} + 2 \frac{1}{(13 + 13)^{\frac{1}{2}}} \\
 &\quad + 4 \frac{1}{(13 + 17)^{\frac{1}{2}}} + 4 \frac{1}{(13 + 22)^{\frac{1}{2}}} + 4 \frac{1}{(16 + 17)^{\frac{1}{2}}} + 4 \frac{1}{(16 + 23)^{\frac{1}{2}}} \\
 &\quad + 4 \frac{1}{(16 + 26)^{\frac{1}{2}}} + (2m + 2n - 20) \frac{1}{(17 + 17)^{\frac{1}{2}}} + 4 \frac{1}{(17 + 22)^{\frac{1}{2}}} \\
 &\quad + (4m + 4n - 36) \frac{1}{(17 + 26)^{\frac{1}{2}}} + 4 \frac{1}{(22 + 26)^{\frac{1}{2}}} + 4 \frac{1}{(23 + 26)^{\frac{1}{2}}} \\
 &\quad + 2 \frac{1}{(23 + 30)^{\frac{1}{2}}} + (2m + 2n - 18) \frac{1}{(26 + 26)^{\frac{1}{2}}} \\
 &\quad + (4m + 4n - 36) \frac{1}{(26 + 30)^{\frac{1}{2}}} \\
 &\quad + (3mn - 14m - 14n + 65) \frac{1}{(30 + 30)^{\frac{1}{2}}}, \\
 &= \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{5}} + \frac{2}{\sqrt{7}} + \frac{2}{\sqrt{26}} + \frac{2}{\sqrt{53}} + \frac{4}{\sqrt{30}} + \frac{4}{\sqrt{33}} + \frac{4}{\sqrt{42}} + \frac{6}{\sqrt{35}} \\
 &\quad + \frac{8}{\sqrt{39}} + \frac{4}{7} + \frac{m+n-9}{\sqrt{13}} + \frac{2m+2n-18}{\sqrt{14}} + \frac{2m+2n-20}{\sqrt{34}} \\
 &\quad + \frac{4m+4n-36}{\sqrt{43}} + \frac{3mn-14m-14n+65}{\sqrt{60}}.
 \end{aligned}$$

□

Theorem 2.2. *The harmonic index of Tickkysim SpiNNaker Model sheet is,*

$$\begin{aligned}
 H^{ve}(TSM) &= \frac{1}{6} + \frac{2}{5} + \frac{2}{7} + \frac{2}{13} + \frac{4}{35} + \frac{4}{53} + \frac{8}{29} + \frac{8}{33} + \frac{8}{35} + \frac{8}{49} + \frac{16}{39} \\
 &\quad + \frac{m+n-9}{7} + \frac{m+n-9}{13} + \frac{2m+2n-20}{17} + \frac{8m+8n-72}{43} \\
 &\quad + \frac{3mn-14m-14n+65}{30}.
 \end{aligned}$$

Proof. By using the information in Table (1) and inserting the values in Formula (17), we get

$$\begin{aligned}
 H^{ve}(TSM) &= \sum_{uv \in E(TSM)} \frac{2}{d_{ve}(u) + d_{ve}(v)}, \\
 &= 4 \frac{2}{7+13} + 4 \frac{2}{12+16} + 2 \frac{2}{12+23} + 2 \frac{2}{13+13} + 4 \frac{2}{13+17} \\
 &\quad + 4 \frac{2}{13+22} + 4 \frac{2}{16+17} + 4 \frac{2}{16+23} + 4 \frac{2}{16+26} \\
 &\quad + (2m+2n-20) \frac{2}{17+17} + 4 \frac{2}{17+22} + (4m+4n-36) \frac{2}{17+26} \\
 &\quad + 4 \frac{2}{22+26} + 4 \frac{2}{23+26} + 2 \frac{2}{23+30} + (2m+2n-18) \frac{2}{26+26} \\
 &\quad + (4m+4n-36) \frac{2}{26+30} + (3mn-14m-14n+65) \frac{2}{30+30}, \\
 &= \frac{1}{6} + \frac{2}{5} + \frac{2}{7} + \frac{2}{13} + \frac{4}{35} + \frac{4}{53} + \frac{8}{29} + \frac{8}{33} + \frac{8}{35} + \frac{8}{49} + \frac{16}{39} \\
 &\quad + \frac{m+n-9}{7} + \frac{m+n-9}{13} + \frac{2m+2n-20}{17} + \frac{8m+8n-72}{43} \\
 &\quad + \frac{3mn-14m-14n+65}{30}.
 \end{aligned}$$

□

Theorem 2.3. *The atom bond connectivity index of Tickkysim SpiNNaker Model sheet is,*

$$\begin{aligned}
 ABC^{ve}(TSM) &= 4 \left[\sqrt{\frac{5}{52}} + \sqrt{\frac{13}{96}} + \sqrt{\frac{18}{91}} + \sqrt{\frac{23}{286}} + \sqrt{\frac{28}{221}} + \sqrt{\frac{31}{272}} + \sqrt{\frac{33}{286}} \right. \\
 &\quad \left. + \sqrt{\frac{37}{368}} + \sqrt{\frac{37}{374}} + \sqrt{\frac{47}{598}} \right] + 2 \left[\sqrt{\frac{11}{92}} + \sqrt{\frac{17}{230}} + \sqrt{\frac{24}{169}} \right] \\
 &\quad + (2m+2n-18) \sqrt{\frac{25}{338}} + (2m+2n-20) \sqrt{\frac{32}{289}} \\
 &\quad + (4m+4n-36) \left[\sqrt{\frac{9}{130}} + \sqrt{\frac{41}{442}} \right] \\
 &\quad + (3mn-14m-14n+65) \sqrt{\frac{29}{450}}.
 \end{aligned}$$

Proof. By using the information in Table (1) and inserting the values in Formula (18), we get

$$\begin{aligned}
 ABC^{ve}(TSM) &= \sum_{uv \in E(TSM)} \sqrt{\frac{d_{ve}(u) + d_{ve}(v) - 2}{d_{ve}(u)d_{ve}(v)}}, \\
 &= 4\sqrt{\frac{7 + 13 - 2}{7 \times 13}} + 4\sqrt{\frac{12 + 16 - 2}{12 \times 16}} + 2\sqrt{\frac{12 + 23 - 2}{12 \times 23}} \\
 &\quad + 2\sqrt{\frac{13 + 13 - 2}{13 \times 13}} + 4\sqrt{\frac{13 + 17 - 2}{13 \times 17}} + 4\sqrt{\frac{13 + 22 - 2}{13 \times 22}} \\
 &\quad + 4\sqrt{\frac{16 + 17 - 2}{16 \times 17}} + 4\sqrt{\frac{16 + 23 - 2}{16 \times 23}} + 4\sqrt{\frac{16 + 26 - 2}{16 \times 26}} \\
 &\quad + (2m + 2n - 20)\sqrt{\frac{17 + 17 - 2}{17 \times 17}} + 4\sqrt{\frac{17 + 22 - 2}{17 \times 22}} \\
 &\quad + (4m + 4n - 36)\sqrt{\frac{17 + 26 - 2}{17 \times 26}} + 4\sqrt{\frac{22 + 26 - 2}{22 \times 26}} \\
 &\quad + 4\sqrt{\frac{23 + 26 - 2}{23 \times 26}} + 2\sqrt{\frac{23 + 30 - 2}{23 \times 30}} \\
 &\quad + (2m + 2n - 18)\sqrt{\frac{26 + 26 - 2}{26 \times 26}} + (4m + 4n - 36)\sqrt{\frac{26 + 30 - 2}{26 \times 30}} \\
 &\quad + (3mn - 14m - 14n + 65)\sqrt{\frac{30 + 30 - 2}{30 \times 30}}, \\
 &= 4\left[\sqrt{\frac{5}{52}} + \sqrt{\frac{13}{96}} + \sqrt{\frac{18}{91}} + \sqrt{\frac{23}{286}} + \sqrt{\frac{28}{221}} + \sqrt{\frac{31}{272}} + \sqrt{\frac{33}{286}}\right. \\
 &\quad \left. + \sqrt{\frac{37}{368}} + \sqrt{\frac{37}{374}} + \sqrt{\frac{47}{598}}\right] + 2\left[\sqrt{\frac{11}{92}} + \sqrt{\frac{17}{230}} + \sqrt{\frac{24}{169}}\right] \\
 &\quad + (2m + 2n - 18)\sqrt{\frac{25}{338}} + (2m + 2n - 20)\sqrt{\frac{32}{289}} \\
 &\quad + (4m + 4n - 36)\left[\sqrt{\frac{9}{130}} + \sqrt{\frac{41}{442}}\right] \\
 &\quad + (3mn - 14m - 14n + 65)\sqrt{\frac{29}{450}}.
 \end{aligned}$$

□

Theorem 2.4. *The geometric arithmetic index of Tickkysim SpiNNaker Model sheet is,*

$$\begin{aligned}
 GA^{ve}(TSM) &= \frac{\sqrt{572}}{6} + \frac{2\sqrt{91}}{5} + \frac{2\sqrt{192}}{7} + \frac{4\sqrt{221}}{15} + \frac{4\sqrt{416}}{21} + \frac{4\sqrt{276}}{35} + \frac{4\sqrt{690}}{53} \\
 &\quad + 8\frac{2\sqrt{272}}{33} + \frac{8\sqrt{286}}{35} + \frac{8\sqrt{368}}{39} + \frac{8\sqrt{374}}{39} + \frac{8\sqrt{598}}{49} \\
 &\quad + (m + n - 9)\frac{\sqrt{780}}{7} + (8m + 8n - 72)\frac{\sqrt{442}}{43} \\
 &\quad + 3mn - 10m - 10n + 29.
 \end{aligned}$$

Proof. By using the information in Table (1) and inserting the values in Formula (19), we get

$$\begin{aligned}
GA^{ve}(TSM) &= \sum_{uv \in E(TSM)} \frac{2\sqrt{d_{ve}(u)d_{ve}(v)}}{d_{ve}(u) + d_{ve}(v)} \\
&= 4\frac{2\sqrt{7 \times 13}}{7 + 13} + 4\frac{2\sqrt{12 \times 16}}{12 + 16} + 2\frac{2\sqrt{12 \times 23}}{12 + 23} + 2\frac{2\sqrt{13 \times 13}}{13 + 13} \\
&\quad + 4\frac{2\sqrt{13 \times 17}}{13 + 17} + 4\frac{2\sqrt{13 \times 22}}{13 + 22} + 4\frac{2\sqrt{16 \times 17}}{16 + 17} + 4\frac{2\sqrt{16 \times 23}}{16 + 23} \\
&\quad + 4\frac{2\sqrt{16 \times 26}}{16 + 26} + (2m + 2n - 20)\frac{2\sqrt{17 \times 17}}{17 + 17} + 4\frac{2\sqrt{17 \times 22}}{17 + 22} \\
&\quad + (4m + 4n - 36)\frac{2\sqrt{17 \times 26}}{17 + 26} + 4\frac{2\sqrt{22 \times 26}}{22 + 26} + 4\frac{2\sqrt{23 \times 26}}{23 + 26} \\
&\quad + 2\frac{2\sqrt{23 \times 30}}{23 + 30} + (2m + 2n - 18)\frac{2\sqrt{26 \times 26}}{26 + 26} \\
&\quad + (4m + 4n - 36)\frac{2\sqrt{26 \times 30}}{26 + 30} + (3mn - 14m - 14n + 65)\frac{2\sqrt{30 \times 30}}{30 + 30}, \\
&= \frac{\sqrt{572}}{6} + \frac{2\sqrt{91}}{5} + \frac{2\sqrt{192}}{7} + \frac{4\sqrt{221}}{15} + \frac{4\sqrt{416}}{21} + \frac{4\sqrt{276}}{35} + \frac{4\sqrt{690}}{53} \\
&\quad + 8\frac{2\sqrt{272}}{33} + \frac{8\sqrt{286}}{35} + \frac{8\sqrt{368}}{39} + \frac{8\sqrt{374}}{39} + \frac{8\sqrt{598}}{49} \\
&\quad + (m + n - 9)\frac{\sqrt{780}}{7} + (8m + 8n - 72)\frac{\sqrt{442}}{43} \\
&\quad + 3mn - 10m - 10n + 29.
\end{aligned}$$

□

3. CONCLUSION

The objective of this paper is to define the closed formulas for ve-degree based topological indices. We determined ve version of sum-connectivity index, harmonic index, atom bond connectivity index and geometric arithmetic index of Tickysim SpiNNaker Model Sheet. These indices are helpful to better study the underlying topologies. In future, some additional structures can be studied for improved architectural designs.

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