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# EDGE PRODUCT CORDIAL LABELING OF SWITCHING OPERATION ON SOME GRAPHS

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ABSTRACT. Here we discuss and prove that the graphs attained by switching of any vertex with degree two which is adjacent to a vertex with degree two in triangular snake  $T_m$ , switching of any vertex with degree one in path  $P_m$  for  $m \ge 3$  and m odd, Switching of vertex with degree two in  $P_m$  except vertices  $u_2$  or  $u_{m-1}$  with m > 4 and switching of any vertex in cycle  $C_m$  are an edge product cordial graphs.

Keywords: Graph labeling, Product cordial labeling, Switching Operation, Edge Product Cordial Labeling.

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#### 1. INTRODUCTION

The graph labeling is an important area in graph theory which have many applications in the communication networks, coding theory, X-Ray Crystallography, chemistry, social sciences etc. there are several types of graph labelings available. For a study of various type of graph labeling we refer to Gallian [2].

Consider G as finite, simple and undirected graph with U(G) as vertex set and and F(G)as edge set, having no any vertex of degree zero. Let |F(G)| and |U(G)| be the number of edges and vertices of G respectively. We follow Gross and Yellen [1] for all other terminology. Cahit [3] in 1987, first established cordial labeling. Then after Sundaram et al.[4] presented Product cordial labeling. Barasara and Vaidya [5] have presented edge product cordial labeling in 2012. In 2013, Vaidya and Barasara [6] have discussed edge product cordial labeling in the context of some graph operation. In 2015, Thamizharasi and Rajeswari [10] have shown the existence of edge product cordial labeling for regular diagraph. In 2016, Prajapati and Patel [7] have discussed some results on edge product cordial labeling. in 2019, Prajapati and Patel [9] have discussed edge product cordial labeling of  $W_n^{(t)}$ ,  $PS_n$  and  $DPS_n$ .

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**Definition 1.1.** Let G be a graph with U(G) as the vertex set and F(G) as the edge set. Let  $h: F(G) \to \{0,1\}$  be a function such that  $|f_h(1) - f_h(0)| \le 1$ , where  $f_h(k) = |\{f \in F(G)|h(f) = k\}|$  for  $k \in \{0,1\}$ . Define the induced vertex labeling as

$$h^*(u) = \prod_{j=1}^m h(f_j) \text{ for } \{u \in U(G), f_j \in F(G), f_j \text{ incident to } u\}.$$

If  $|u_h(1) - u_h(0)| \leq 1$ , where  $u_h(k) = |\{u \in U(G)|h^*(u) = k\}|$ , for  $k \in \{0,1\}$  then h is called an edge product cordial labeling.

A graph G which has an edge product cordial labeling is said to be an edge product cordial graph.

**Definition 1.2.** [8] Graph derived by fetching a vertex u of G, eliminating all edges joining u to their adjacent vertices and by adding new edges joining u to their non-adjacent vertices in G is called vertex switching  $G_u$  of G.

**Definition 1.3.** Graph derived from path  $P_m$  by substituting every edge of  $P_m$  by  $C_3$  is called Triangular Snake  $T_m$ .

## 2. MAIN RESULTS

**Theorem 2.1.** The Graph derived from switching of any vertex in  $C_m (m \ge 4)$  is an edge product cordial graph.

*Proof.* Consider  $u_k$  for  $1 \le k \le m$  are successive vertices of  $C_m$ . consider  $G_{u_1}$  is the graph derived from switching of a  $u_1$  in  $C_m$ . So in  $G_{u_1}$  every vertex  $u_i$  other than  $u_2$  and  $u_n$  is adjacent to  $u_1$ . Thus  $|U(G_{u_1})| = m$  and  $|F(G_{u_1})| = 2m - 5$ . Define  $h : F(G_{u_1}) \to \{0, 1\}$  as:

$$h(f) = \begin{cases} 1 & \text{if } f = u_k u_{k+1} \text{ for } 2 \le k \le \left\lfloor \frac{m}{2} \right\rfloor; \\ 0 & \text{if } f = u_k u_{k+1} \text{ for } \left\lfloor \frac{m}{2} \right\rfloor + 1 \le k \le m - 2; \\ 1 & \text{if } f = u_1 u_k \text{ for } 3 \le k \le \left\lceil \frac{m}{2} \right\rceil; \\ 0 & \text{if } f = u_1 u_k \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1; \\ 1 & \text{if } f = u_m u_{m-1}. \end{cases}$$

Thus  $h^*: U(G_{u_1}) \to \{0, 1\}$  is,

$$h^{*}(u_{1}) = \prod_{k=3}^{m-1} h(u_{1}u_{k}) = 0,$$
  

$$h^{*}(u_{2}) = h(u_{2}u_{3}) = 1,$$
  

$$h^{*}(u_{m}) = h(u_{m}u_{m-1}) = 1,$$
  

$$h^{*}(u_{k}) = h(u_{k}u_{k-1})h(u_{k}u_{k+1})h(u_{k}u_{1}) = 1 \quad \text{for } 3 \le k \le \left\lfloor \frac{m}{2} \right\rfloor,$$
  

$$h^{*}(u_{k}) = h(u_{k}u_{k-1})h(u_{k}u_{k+1})h(u_{k}u_{1}) = 0 \quad \text{for } \left\lfloor \frac{m}{2} \right\rfloor + 1 \le k \le m - 1.$$

Hence,  $u_h(1) = \left| \left\{ u_2, u_3, \dots, u_{\lfloor \frac{m}{2} \rfloor}, u_m \right\} \right|$  and  $u_h(0) = \left| \left\{ u_1, u_{\lfloor \frac{m}{2} \rfloor + 1}, u_{\lfloor \frac{m}{2} \rfloor + 2}, \dots, u_{m-1} \right\} \right|$ . So  $u_h(0) = \left\lceil \frac{m}{2} \right\rceil$ ,  $u_h(1) = \left\lfloor \frac{m}{2} \right\rfloor$  and  $f_h(0) = \left\lfloor \frac{2m - 5}{2} \right\rfloor$ ,  $f_h(1) = \left\lceil \frac{2m - 5}{2} \right\rceil$ . Thus  $|u_h(0) - u_h(1)| = 1 \le 1$  and  $|f_h(0) - f_h(1)| = 1 \le 1$ . Hence  $G_{u_1}$  is an edge product cordial graph.  $\Box$  **Example 2.1.** Edge product cordial labeling of  $G_{u_1}$  derived from  $C_9$  reveal in the following figure 1.



FIGURE 1. Graph  $G_{u_1}$  derived from  $C_9$ 

**Theorem 2.2.** The graph derived from switching of a vertex with degree one in  $P_m$  is an edge product cordial graph if and only if  $m \ge 3$  and m odd.

*Proof.* Consider  $u_k$  for  $1 \le k \le m$  are successive vertices of path  $P_m$ . Let  $G_{u_1}$  be the graph derived by switching of a vertex with degree one say  $u_1$  of  $P_m$ . So in  $G_{u_1}$ , every vertex  $u_k$  for  $k = 3, 4, \ldots, m$  is adjacent to  $u_1$ . Thus  $|U(G_{u_1})| = m$  and  $|F(G_{u_1})| = 2m - 4$ . Here we cosider two cases:

**Case 1** If m > 4 then define  $h : F(G_{u_1}) \to \{0, 1\}$  as:

$$h(f) = \begin{cases} 1 & \text{if } f = u_k u_{k+1} \text{ for } 2 \le k \le \left| \frac{m}{2} \right|; \\ 0 & \text{if } f = u_k u_{k+1} \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1; \\ 1 & \text{if } f = u_1 u_k \text{ for } 3 \le k \le \left\lceil \frac{m}{2} \right\rceil; \\ 0 & \text{if } f = u_1 u_k \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m. \end{cases}$$

Thus  $h^*$  is given by,

$$\begin{aligned} h^*(u_1) &= \prod_{k=3}^m h(u_1 u_k) = 0, \\ h^*(u_2) &= h(u_2 u_3) = 1, \\ h^*(u_n) &= h(u_1 u_m) h(u_m u_{m-1}) = 0, \\ h^*(u_k) &= h(u_k u_{k-1}) h(u_k u_{k+1}) h(u_k u_1) = 1 \quad \text{for } 3 \le k \le \left\lceil \frac{m}{2} \right\rceil, \\ h^*(u_k) &= h(u_k u_{k-1}) h(u_k u_{k+1}) h(u_k u_1) = 0 \quad \text{for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1. \end{aligned}$$

Hence, 
$$u_h(1) = \left| \left\{ u_2, u_3, \dots, u_{\left\lceil \frac{m}{2} \right\rceil} \right\} \right|$$
 and  $u_h(0) = \left| \left\{ u_1, u_{\left\lceil \frac{m}{2} \right\rceil + 1}, u_{\left\lceil \frac{m}{2} \right\rceil + 2}, \dots, u_{m-1}, u_m \right\} \right|$ .  
So  $u_h(0) = \left\lceil \frac{m}{2} \right\rceil$ ,  $u_h(1) = \left\lfloor \frac{m}{2} \right\rfloor$  and  $f_h(0) = f_h(1) = \left\lfloor \frac{2m - 4}{2} \right\rfloor$ .  
**Case 2**: For  $m = 3$ , labeling is shown in the figure 2.

Hence  $|u_h(0) - u_h(1)| = 1 \leq 1$  and  $|f_h(0) - f_h(1)| = 0 \leq 1$ . So  $G_{u_1}$  is an edge product cordial graph.



FIGURE 2.  $G_{u_1}$  derived from  $P_3$ 

**Example 2.2.** Edge product cordial labeling of  $G_{u_1}$  derived from  $P_7$  reveal in the following figure 3.



FIGURE 3.  $G_{u_1}$  derived from  $P_7$ 

**Theorem 2.3.** The graph derived from switching of a vertex with degree two in  $P_m$  except  $u_2$  or  $u_{m-1}$  with m > 4 is an edge product cordial graph.

Proof. Consider  $u_k$  for  $1 \leq k \leq m$  are successive vertices of path  $P_m$ . Let  $G_{u_i}$ ,  $3 \leq i \leq m-2$  be the graph derived by switching of a vertex  $u_i$ . So in  $G_{u_i}$ , every vertex  $u_k$  for  $1 \leq k \leq m$  and  $k \neq i-1, i+1, i$  is adjacent to  $u_i$ . Thus  $|U(G_{u_i})| = m$  and  $|F(G_{u_i})| = 2m-6$ . To prove this theorem we will cosider the case for switching of vertex  $u_i$ ,  $3 \leq i \leq \lfloor \frac{m}{2} \rfloor$ . For rest of the vertices  $u_i$ ,  $\lfloor \frac{m}{2} \rfloor + 1 \leq i \leq m-3$  proof is similar. Define  $h: F(G_{u_i}) \to \{0,1\}$  as:

$$h(f) = \begin{cases} 1 & \text{if } f = u_i u_k \text{ for } 3 \le i \le \left\lceil \frac{m}{2} \right\rceil, k < i - 1 \text{ and } i + 1 < k \le \left\lceil \frac{m}{2} \right\rceil + 1; \\ 0 & \text{if } f = u_i u_k \text{ for } 3 \le i \le \left\lceil \frac{m}{2} \right\rceil, \left\lceil \frac{m}{2} \right\rceil + 1 < k \le m; \\ 1 & \text{if } f = u_k u_{k+1} \text{ for } 1 \le k < i - 1, i + 1 \le k \le \left\lfloor \frac{m}{2} \right\rfloor + 1; \\ 0 & \text{if } f = u_k u_{k+1} \text{ for } \left\lfloor \frac{m}{2} \right\rfloor + 1 < k \le m. \end{cases}$$

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Thus  $h^*$  is given by,

$$\begin{split} h^*(u_i) &= \prod_{\substack{k \neq i-1, i, i+1}}^m h(u_i u_k) = 0, \\ h^*(u_1) &= h(u_1 u_i) h(u_1 u_2) = 1, \\ h^*(u_{i-1}) &= h(u_{i-1} u_{i-2}) = 1, \\ h^*(u_{i+1}) &= \begin{cases} h(u_{i+1} u_{i+2}) = 0 \text{ for } m \text{ odd and } i = \left\lceil \frac{m}{2} \right\rceil, \\ h(u_{i+1} u_{i+2}) = 1; \text{ otherwise,} \end{cases} \\ h^*(u_m) &= h(u_i u_m) h(u_m u_{m-1}) = 0, \\ h^*(u_k) &= h(u_k u_{k-1}) h(u_k u_{k+1}) h(u_k u_i) = 1 \text{ for } 2 \le k \le i-2, \\ h^*(u_k) &= h(u_k u_{k-1}) h(u_k u_{k+1}) h(u_k u_i) = 1 \text{ for } i+2 \le k \left\lfloor \frac{m}{2} \right\rfloor + 1, \\ h^*(u_k) &= h(u_k u_{k-1}) h(u_k u_{k+1}) h(u_k u_i) = 0 \text{ for } \left\lfloor \frac{m}{2} \right\rfloor + 2 \le k \le m-1. \end{split}$$

Hence,

$$u_{h}(1) = \begin{cases} \left| \{u_{1}, u_{2}, \dots, u_{i-1}\} \right|; i = \left\lceil \frac{m}{2} \right\rceil \text{ and } m \text{ odd}; \\ \left\{u_{1}, u_{2}, \dots, u_{i-1}, u_{i+1}, u_{i+2}, \dots, u_{\lfloor \frac{m}{2} \rfloor + 1} \right\} \right|; \text{ otherwise.} \\ u_{h}(0) = \begin{cases} \left| \{u_{i}, u_{i+1}, u_{i+2}, \dots, u_{m} \} \right|; i = \left\lceil \frac{m}{2} \right\rceil \text{ and } m \text{ odd}; \\ \left\{u_{i}, u_{\lfloor \frac{m}{2} \rfloor + 2}, u_{\lceil \frac{m}{2} \rceil + 3}, \dots, u_{m} \right\} \right|; \text{ otherwise.} \end{cases}$$
$$u_{h}(1) = \left| \left\{u_{1}, u_{2}, \dots, u_{i-1}, u_{i+1}, u_{i+2}, \dots, u_{\lfloor \frac{m}{2} \rfloor + 1}; i \neq \left\lceil \frac{m}{2} \right\rceil \text{ and } m \text{ odd} \right\} \right| \text{ and} \\ u_{h}(0) = \left| \left\{u_{i}, u_{\lfloor \frac{m}{2} \rfloor + 2}, u_{\lceil \frac{m}{2} \rceil + 3}, \dots, u_{m} \right\} \right|. \text{ So } u_{h}(0) = \left\lceil \frac{m}{2} \right\rceil, u_{h}(1) = \left\lfloor \frac{m}{2} \right\rfloor \text{ and } f_{h}(0) = \\ f_{h}(1) = \left\lfloor \frac{2m - 6}{2} \right\rfloor. \text{ Hence } |u_{h}(0) - u_{h}(1)| = 1 \leq 1 \text{ and } |f_{h}(0) - f_{h}(1)| = 0 \leq 1. \text{ So } G_{u_{1}} \text{ is} \\ \text{an edge product cordial graph.} \qquad \Box$$

**Example 2.3.** Edge product cordial labeling of  $G_{u_5}$  derived from  $P_{11}$  reveal in the following figure 4.



FIGURE 4.  $G_{u_5}$  derived from  $P_{11}$ 

**Theorem 2.4.** The graph derived from switching of a vertex with degree two which is adjacent to a vertex with degree two in triangular snake  $T_m$  is an edge product cordial graph.

Proof. Consider  $u_k$  for  $1 \le k \le m$  are the successive vertices of  $P_m$ . Let  $w_1, w_2, \ldots, w_{m-1}$  be the vertices of triangle other than the vertices of  $P_m$  in  $T_m$ . Consider  $G_w$  graph derived by switching of any vertex with degree two which is adjacent to a vertex with degree two in  $T_m$ . Thus  $|U(G_w)| = 2m - 1$  and  $|F(G_w)| = 5m - 9$ . Then there are four cases arise: **Case 1** If  $w = w_1$ , then in  $G_{w_1}$ , every vertex  $w_k$  for  $k = 2, 3, \ldots, m - 1$  and  $u_k$  for  $k = 3, 4, \ldots, m$  are adjacent to  $w_1$ . Define mapping  $r : F(G_{w_1}) \to \{0, 1\}$  by,

$$r(f) = \begin{cases} 1 & \text{if } f = u_k u_{k+1} \text{ for } 1 \le k \le \left\lceil \frac{m}{2} \right\rceil; \\ 0 & \text{if } f = u_k u_{k+1} \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1; \\ 1 & \text{if } f = w_1 u_k \text{ for } 3 \le k \le \left\lceil \frac{m}{2} \right\rceil; \\ 0 & \text{if } f = w_1 u_k \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m; \\ 1 & \text{if } f \in \{w_1 w_k, w_k u_{k+1}\} \text{ for } 2 \le k \le \left\lfloor \frac{m}{2} \right\rfloor; \\ 0 & \text{if } f \in \{w_1 w_k, w_k u_{k+1}\} \text{ for } \left\lfloor \frac{m}{2} \right\rfloor + 1 \le k \le m - 1; \\ 1 & \text{if } f = w_k u_k \text{ for } 2 \le k \le \left\lceil \frac{m}{2} \right\rceil; \\ 0 & \text{if } f = w_k u_k \text{ for } 2 \le k \le \left\lceil \frac{m}{2} \right\rceil; \\ 0 & \text{if } f = w_k u_k \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1. \end{cases}$$

Thus  $r^*: U(G_{w_1}) \to \{0, 1\}$  is obtained as follows,

$$\begin{aligned} r^*(w_1) &= \prod_{k=3}^m r(w_1 u_k) \prod_{k=2}^{m-1} r(w_1 w_k) = 0, \\ r^*(w_k) &= r(w_k u_k) r(w_k u_{k+1}) r(w_1 w_k) = 1 \quad \text{for } 2 \le k \le \left\lfloor \frac{m}{2} \right\rfloor, \\ r^*(w_k) &= r(w_k u_k) r(w_k u_{k+1}) r(w_1 w_k) = 0 \quad \text{for } \left\lfloor \frac{m}{2} \right\rfloor + 1 \le k \le m - 1, \\ r^*(u_1) &= r(u_1 u_2) = 1, \\ r^*(u_2) &= r(u_1 u_2) r(u_2 u_3) r(w_2 u_2) = 1, \\ r^*(u_k) &= r(u_k u_{k-1}) r(u_k u_{k+1}) r(w_{k-1} u_k) r(w_k u_k) r(w_1 u_k) = 1 \quad \text{for } 3 \le k \le \left\lceil \frac{m}{2} \right\rceil, \\ r^*(u_k) &= r(u_k u_{k-1}) r(u_k u_{k+1}) r(u_k w_{k-1}) r(w_k u_k) r(w_1 u_k) = 0 \quad \text{for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1, \\ r^*(u_m) &= r(u_{m-1} u_m) r(w_{m-1} u_m) r(w_1 u_m) = 0. \end{aligned}$$

Hence, 
$$u_r(1) = \left| \left\{ u_1, u_2, \dots, u_{\lceil \frac{m}{2} \rceil}, w_2, w_3, \dots, w_{\lfloor \frac{m}{2} \rfloor} \right\} \right|$$
 and  
 $u_r(0) = \left| \left\{ u_{\lceil \frac{m}{2} \rceil + 1}, u_{\lceil \frac{m}{2} \rceil + 2}, \dots, u_m, w_1, w_{\lfloor \frac{m}{2} \rfloor} + 1, w_{\lfloor \frac{m}{2} \rfloor} + 2, \dots, w_{m-1} \right\} \right|$ .  
So  $u_r(0) = u_r(1) + 1 = m$  and  $f_r(1) = \left\lfloor \frac{5m - 9}{2} \right\rfloor, f_r(0) = \left\lceil \frac{5m - 9}{2} \right\rceil$ .

If  $w = w_m$ , then proof is similar.

**Case 2** If  $w = u_1$ , then in  $G_{u_1}$ , every vertex  $u_k$  for  $3 \le k \le m$  and  $w_k$  for  $2 \le k \le m - 1$ 

are adjacent to  $u_1$ . Define mapping  $r: F(G_{u_1}) \to \{0, 1\}$  by,

$$r(f) = \begin{cases} 1 & \text{if } f \in \{u_k u_{k+1}, w_k u_k\} \text{ for } 2 \le k \le \left\lceil \frac{m}{2} \right\rceil; \\ 0 & \text{if } f \in \{u_k u_{k+1}, w_k u_k\} \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1; \\ 1 & \text{if } f = u_1 w_k \text{ for } 2 \le k \le \left\lfloor \frac{m}{2} \right\rceil; \\ 0 & \text{if } f = u_1 u_k \text{ for } 3 \le k \le \left\lceil \frac{m}{2} \right\rceil; \\ 0 & \text{if } f = u_1 u_k \text{ for } 1 \le k \le m; \\ 1 & \text{if } f = w_k u_{k+1} \text{ for } 1 \le k \le m; \\ 1 & \text{if } f = w_k u_{k+1} \text{ for } 1 \le k \le m - 1. \end{cases}$$
  
Thus  $r^* : U(G_{u_1}) \to \{0, 1\}$  is obtained as follows,  
 $r^*(w_1) = r(w_1 u_2) = 1,$   
 $r^*(w_k) = r(w_k u_k) r(w_k u_{k+1}) r(u_1 w_k) = 1 \text{ for } 2 \le k \le \left\lfloor \frac{m}{2} \right\rfloor,$   
 $r^*(w_k) = r(w_k u_k) r(w_k u_{k+1}) r(u_1 w_k) = 0 \text{ for } \left\lfloor \frac{m}{2} \right\rfloor + 1 \le k \le m - 1,$   
 $r^*(u_2) = r(w_1 u_2) r(w_2 u_2) r(u_2 u_3) = 1,$   
 $r^*(u_1) = \prod_{k=2}^{m-1} r(u_1 w_k) \prod_{k=3}^m r(u_1 u_k) = 0,$   
 $r^*(u_k) = r(u_k w_{k-1}) r(u_k w_k) r(u_{k-1} u_k) r(u_k u_{k+1}) r(u_1 u_k) = 1 \text{ for } 3 \le k \le \left\lceil \frac{m}{2} \right\rceil,$   
 $r^*(u_k) = r(u_k w_{k-1}) r(u_k w_k) r(u_{k-1} u_k) r(u_k u_{k+1}) r(u_1 u_k) = 0 \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1,$   
 $r^*(u_k) = r(u_k w_{k-1}) r(u_k w_k) r(u_{k-1} u_k) r(u_k u_{k+1}) r(u_1 u_k) = 1 \text{ for } 3 \le k \le \left\lceil \frac{m}{2} \right\rceil,$   
 $r^*(u_k) = r(u_k w_{k-1}) r(u_k w_k) r(u_{k-1} u_k) r(u_k u_{k+1}) r(u_1 u_k) = 1 \text{ for } 3 \le k \le \left\lceil \frac{m}{2} \right\rceil,$   
 $r^*(u_k) = r(u_k w_{k-1}) r(u_k w_k) r(u_{k-1} u_k) r(u_k u_{k+1}) r(u_1 u_k) = 0 \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1,$   
 $r^*(u_k) = r(w_k m_{k-1}) r(u_k m_k) r(u_{k-1} u_k) r(u_k u_{k+1}) r(u_1 u_k) = 0 \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1,$   
 $r^*(u_k) = r(u_k w_{k-1}) r(u_k m_k) r(u_{k-1} u_k) r(u_k u_{k+1}) r(u_1 u_k) = 0 \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1,$   
 $r^*(u_k) = r(u_k m_{k-1}) r(u_k m_k) r(u_{k-1} u_k) r(u_k u_{k+1}) r(u_1 u_k) = 0 \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1,$   
 $r^*(u_k) = r(u_k m_{k-1}) r(u_k m_k) r(u_k u_{k+1}) r(u_1 u_k) = 0 \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1,$   
 $r^*(u_k) = r(u_k m_{k-1}) r(u_k m_k) r(u_k u_{k+1}) r(u_k u_{k+1}) r(u_1 u_k) = 0 \text{ for } \left\lceil \frac{m}{2} \right\rceil + 1 \le k \le m - 1,$   
 $r^*(u_k) = r(u_k m_{k-1}) r(u_k m_k) r$ 

Thus from the aboves cases  $|u_h(0) - u_h(1)| = 1 \le 1$  and  $|f_h(0) - f_h(1)| = 1 \le 1$ . Hence  $G_w$  is an edge product cordial graph.

**Example 2.4.** Edge product cordial labeling of  $G_{w_1}$  derived by  $T_6$  and  $G_{u_1}$  obtained by  $T_7$  shown in the following figure 5 and figure 6 respectively.

## 3. Conclusions

We examine four results on graph derived by a vertex switching with degree one in  $P_m$  if and only if  $m \ge 3$  and m is odd, Switching of vertex with degree two in  $P_m$  except  $u_2$  or  $u_{m-1}$  with m > 4, switching of any vertex in  $C_m$  and a vertx switching with degree two which is adjacent to a vertex with degree two in  $T_m$  are edge product cordial graph.

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FIGURE 5.  $G_{w_1}$  obtained from  $T_6$ 



FIGURE 6.  $G_{u_1}$  obtained from  $T_7$ 

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