# EDGE PRODUCT CORDIAL LABELING OF SWITCHING OPERATION ON SOME GRAPHS 

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#### Abstract

Here we discuss and prove that the graphs attained by switching of any vertex with degree two which is adjacent to a vertex with degree two in triangular snake $T_{m}$, switching of any vertex with degree one in path $P_{m}$ for $m \geq 3$ and $m$ odd, Switching of vertex with degree two in $P_{m}$ except vertices $u_{2}$ or $u_{m-1}$ with $m>4$ and switching of any vertex in cycle $C_{m}$ are an edge product cordial graphs.


Keywords: Graph labeling, Product cordial labeling, Switching Operation, Edge Product Cordial Labeling.

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## 1. Introduction

The graph labeling is an important area in graph theory which have many applications in the communication networks, coding theory, X-Ray Crystallography, chemistry, social sciences etc. there are several types of graph labelings available. For a study of various type of graph labeling we refer to Gallian [2].
Consider $G$ as finite, simple and undirected graph with $U(G)$ as vertex set and and $F(G)$ as edge set, having no any vertex of degree zero. Let $|F(G)|$ and $|U(G)|$ be the number of edges and vertices of $G$ respectively. We follow Gross and Yellen [1] for all other terminology. Cahit [3] in 1987, first established cordial labeling. Then after Sundaram et al. [4] presented Product cordial labeling. Barasara and Vaidya [5] have presented edge product cordial labeling in 2012. In 2013, Vaidya and Barasara [6] have discussed edge product cordial labeling in the context of some graph operation. In 2015, Thamizharasi and Rajeswari [10] have shown the existence of edge product cordial labeling for regular diagraph. In 2016, Prajapati and Patel [7] have discussed some results on edge product cordial labeling. in 2019, Prajapati and Patel [9] have discussed edge product cordial labeling of $W_{n}^{(t)}, P S_{n}$ and $D P S_{n}$.

[^0]Definition 1.1. Let $G$ be a graph with $U(G)$ as the vertex set and $F(G)$ as the edge set. Let $h: F(G) \rightarrow\{0,1\}$ be a function such that $\left|f_{h}(1)-f_{h}(0)\right| \leq 1$, where $f_{h}(k)=\mid\{f \in$ $F(G) \mid h(f)=k\} \mid$ for $k \in\{0,1\}$. Define the induced vertex labeling as

$$
h^{*}(u)=\prod_{j=1}^{m} h\left(f_{j}\right) \text { for }\left\{u \in U(G), f_{j} \in F(G), f_{j} \text { incident to } u\right\} .
$$

If $\left|u_{h}(1)-u_{h}(0)\right| \leq 1$, where $u_{h}(k)=\left|\left\{u \in U(G) \mid h^{*}(u)=k\right\}\right|$, for $k \in\{0,1\}$ then $h$ is called an edge product cordial labeling.
A graph $G$ which has an edge product cordial labeling is said to be an edge product cordial graph.

Definition 1.2. [8] Graph derived by fetching a vertex u of G, eliminating all edges joining $u$ to their adjacent vertices and by adding new edges joining u to their non-adjacent vertices in $G$ is called vertex switching $G_{u}$ of $G$.

Definition 1.3. Graph derived from path $P_{m}$ by substituting every edge of $P_{m}$ by $C_{3}$ is called Triangular Snake $T_{m}$.

## 2. Main Results

Theorem 2.1. The Graph derived from switching of any vertex in $C_{m}(m \geq 4)$ is an edge product cordial graph.

Proof. Consider $u_{k}$ for $1 \leq k \leq m$ are successive vertices of $C_{m}$. consider $G_{u_{1}}$ is the graph derived from switching of a $u_{1}$ in $C_{m}$. So in $G_{u_{1}}$ every vertex $u_{i}$ other than $u_{2}$ and $u_{n}$ is adjacent to $u_{1}$. Thus $\left|U\left(G_{u_{1}}\right)\right|=m$ and $\left|F\left(G_{u_{1}}\right)\right|=2 m-5$. Define $h: F\left(G_{u_{1}}\right) \rightarrow\{0,1\}$ as:

$$
h(f)=\left\{\begin{array}{lll}
1 & \text { if } f=u_{k} u_{k+1} \text { for } 2 \leq k \leq\left\lfloor\frac{m}{2}\right\rfloor \\
0 & \text { if } \quad f=u_{k} u_{k+1} \text { for }\left\lfloor\frac{m}{2}\right\rfloor+1 \leq k \leq m-2 \\
1 & \text { if } f=u_{1} u_{k} \text { for } 3 \leq k \leq\left\lceil\frac{m}{2}\right\rceil ; \\
0 & \text { if } f=u_{1} u_{k} \text { for } \quad\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m-1 ; \\
1 & \text { if } f=u_{m} u_{m-1} .
\end{array}\right.
$$

Thus $h^{*}: U\left(G_{u_{1}}\right) \rightarrow\{0,1\}$ is,

$$
\begin{aligned}
h^{*}\left(u_{1}\right) & =\prod_{k=3}^{m-1} h\left(u_{1} u_{k}\right)=0, \\
h^{*}\left(u_{2}\right) & =h\left(u_{2} u_{3}\right)=1, \\
h^{*}\left(u_{m}\right) & =h\left(u_{m} u_{m-1}\right)=1, \\
h^{*}\left(u_{k}\right) & =h\left(u_{k} u_{k-1}\right) h\left(u_{k} u_{k+1}\right) h\left(u_{k} u_{1}\right)=1 \\
h^{*}\left(u_{k}\right) & \text { for } 3 \leq k \leq\left\lfloor\frac{m}{2}\right\rfloor, \\
\left.u_{k} u_{k-1}\right) h\left(u_{k} u_{k+1}\right) h\left(u_{k} u_{1}\right)=0 & \text { for }\left\lfloor\frac{m}{2}\right\rfloor+1 \leq k \leq m-1 .
\end{aligned}
$$

Hence, $u_{h}(1)=\left|\left\{u_{2}, u_{3}, \ldots, u_{\left\lfloor\frac{m}{2}\right\rfloor}, u_{m}\right\}\right|$ and $u_{h}(0)=\left|\left\{u_{1}, u_{\left\lfloor\frac{m}{2}\right\rfloor+1}, u_{\left\lfloor\frac{m}{2}\right\rfloor+2}, \ldots, u_{m-1}\right\}\right|$. So $u_{h}(0)=\left\lceil\frac{m}{2}\right\rceil, u_{h}(1)=\left\lfloor\frac{m}{2}\right\rfloor$ and $f_{h}(0)=\left\lfloor\frac{2 m-5}{2}\right\rfloor, f_{h}(1)=\left\lceil\frac{2 m-5}{2}\right\rceil$. Thus $\left|u_{h}(0)-u_{h}(1)\right|=1 \leq 1$ and $\left|f_{h}(0)-f_{h}(1)\right|=1 \leq 1$. Hence $G_{u_{1}}$ is an edge product cordial graph.

Example 2.1. Edge product cordial labeling of $G_{u_{1}}$ derived from $C_{9}$ reveal in the following figure 1.


Figure 1. Graph $G_{u_{1}}$ derived from $C_{9}$

Theorem 2.2. The graph derived from switching of a vertex with degree one in $P_{m}$ is an edge product cordial graph if and only if $m \geq 3$ and $m$ odd.

Proof. Consider $u_{k}$ for $1 \leq k \leq m$ are successive vertices of path $P_{m}$. Let $G_{u_{1}}$ be the graph derived by switching of a vertex with degree one say $u_{1}$ of $P_{m}$. So in $G_{u_{1}}$, every vertex $u_{k}$ for $k=3,4, \ldots, m$ is adjacent to $u_{1}$. Thus $\left|U\left(G_{u_{1}}\right)\right|=m$ and $\left|F\left(G_{u_{1}}\right)\right|=2 m-4$. Here we cosider two cases:
Case 1 If $m>4$ then define $h: F\left(G_{u_{1}}\right) \rightarrow\{0,1\}$ as:

$$
h(f)=\left\{\begin{array}{lll}
1 & \text { if } & f=u_{k} u_{k+1} \text { for } 2 \leq k \leq\left\lceil\frac{m}{2}\right\rceil \\
0 & \text { if } & f=u_{k} u_{k+1} \text { for }\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m-1 \\
1 & \text { if } f=u_{1} u_{k} \text { for } 3 \leq k \leq\left\lceil\frac{m}{2}\right\rceil \\
0 & \text { if } \quad f=u_{1} u_{k} \text { for }\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m
\end{array}\right.
$$

Thus $h^{*}$ is given by,

$$
\begin{aligned}
& h^{*}\left(u_{1}\right)=\prod_{k=3}^{m} h\left(u_{1} u_{k}\right)=0 \\
& h^{*}\left(u_{2}\right)=h\left(u_{2} u_{3}\right)=1 \\
& h^{*}\left(u_{n}\right)=h\left(u_{1} u_{m}\right) h\left(u_{m} u_{m-1}\right)=0 \\
& h^{*}\left(u_{k}\right)=h\left(u_{k} u_{k-1}\right) h\left(u_{k} u_{k+1}\right) h\left(u_{k} u_{1}\right)=1 \quad \text { for } 3 \leq k \leq\left\lceil\frac{m}{2}\right\rceil \\
& h^{*}\left(u_{k}\right)=h\left(u_{k} u_{k-1}\right) h\left(u_{k} u_{k+1}\right) h\left(u_{k} u_{1}\right)=0 \quad \text { for } \quad\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m-1
\end{aligned}
$$

Hence, $u_{h}(1)=\left|\left\{u_{2}, u_{3}, \ldots, u_{\left\lceil\frac{m}{2}\right\rceil}\right\}\right|$ and $u_{h}(0)=\left|\left\{u_{1}, u_{\left\lceil\frac{m}{2}\right\rceil+1}, u_{\left\lceil\frac{m}{2}\right\rceil+2}, \ldots, u_{m-1}, u_{m}\right\}\right|$.
So $u_{h}(0)=\left\lceil\frac{m}{2}\right\rceil, u_{h}(1)=\left\lfloor\frac{m}{2}\right\rfloor$ and $f_{h}(0)=f_{h}(1)=\left\lfloor\frac{2 m-4}{2}\right\rfloor$.
Case 2: For $m=3$, labeling is shown in the figure 2 .
Hence $\left|u_{h}(0)-u_{h}(1)\right|=1 \leq 1$ and $\left|f_{h}(0)-f_{h}(1)\right|=0 \leq 1$. So $G_{u_{1}}$ is an edge product cordial graph.


Figure 2. $G_{u_{1}}$ derived from $P_{3}$

Example 2.2. Edge product cordial labeling of $G_{u_{1}}$ derived from $P_{7}$ reveal in the following figure 3.


Figure 3. $G_{u_{1}}$ derived from $P_{7}$
Theorem 2.3. The graph derived from switching of a vertex with degree two in $P_{m}$ except $u_{2}$ or $u_{m-1}$ with $m>4$ is an edge product cordial graph.

Proof. Consider $u_{k}$ for $1 \leq k \leq m$ are successive vertices of path $P_{m}$. Let $G_{u_{i}}, 3 \leq$ $i \leq m-2$ be the graph derived by switching of a vertex $u_{i}$. So in $G_{u_{i}}$, every vertex $u_{k}$ for $1 \leq k \leq m$ and $k \neq i-1, i+1, i$ is adjacent to $u_{i}$. Thus $\left|U\left(G_{u_{i}}\right)\right|=m$ and $\left|F\left(G_{u_{i}}\right)\right|=2 m-6$. To prove this theoram we will cosider the case for switching of vertex $u_{i}, 3 \leq i \leq\left\lceil\frac{m}{2}\right\rceil$. For rest of the vertices $u_{i},\left\lceil\frac{m}{2}\right\rceil+1 \leq i \leq m-3$ proof is similar. Define $h: F\left(G_{u_{i}}\right) \rightarrow\{0,1\}$ as:

$$
h(f)=\left\{\begin{array}{lll}
1 & \text { if } & f=u_{i} u_{k} \text { for } 3 \leq i \leq\left\lceil\frac{m}{2}\right\rceil, k<i-1 \quad \text { and } i+1<k \leq\left\lceil\frac{m}{2}\right\rceil+1 \\
0 & \text { if } & f=u_{i} u_{k} \text { for } 3 \leq i \leq\left\lceil\frac{m}{2}\right\rceil,\left\lceil\frac{m}{2}\right\rceil+1<k \leq m \\
1 & \text { if } f=u_{k} u_{k+1} \text { for } 1 \leq k<i-1, i+1 \leq k \leq\left\lfloor\frac{m}{2}\right\rfloor+1 \\
0 & \text { if } f=u_{k} u_{k+1} \text { for }\left\lfloor\frac{m}{2}\right\rfloor+1<k \leq m
\end{array}\right.
$$

Thus $h^{*}$ is given by,

$$
\begin{aligned}
h^{*}\left(u_{i}\right) & =\prod_{\substack{k=1 \\
k \neq i-1, i, i+1}}^{m} h\left(u_{i} u_{k}\right)=0, \\
h^{*}\left(u_{1}\right) & =h\left(u_{1} u_{i}\right) h\left(u_{1} u_{2}\right)=1, \\
h^{*}\left(u_{i-1}\right) & =h\left(u_{i-1} u_{i-2}\right)=1, \\
h^{*}\left(u_{i+1}\right. & =\left\{\begin{array}{l}
h\left(u_{i+1} u_{i+2}\right)=0 \text { for } m \text { odd and } i=\left\lceil\frac{m}{2}\right\rceil \\
h\left(u_{i+1} u_{i+2}\right)=1 ; \text { otherwise, } \\
h^{*}\left(u_{m}\right)
\end{array}\right)=h\left(u_{i} u_{m}\right) h\left(u_{m} u_{m-1}\right)=0, \\
h^{*}\left(u_{k}\right) & =h\left(u_{k} u_{k-1}\right) h\left(u_{k} u_{k+1}\right) h\left(u_{k} u_{i}\right)=1 \quad \text { for } 2 \leq k \leq i-2, \\
h^{*}\left(u_{k}\right) & =h\left(u_{k} u_{k-1}\right) h\left(u_{k} u_{k+1}\right) h\left(u_{k} u_{i}\right)=1 \quad \text { for } i+2 \leq k\left\lfloor\frac{m}{2}\right\rfloor+1, \\
h^{*}\left(u_{k}\right) & =h\left(u_{k} u_{k-1}\right) h\left(u_{k} u_{k+1}\right) h\left(u_{k} u_{i}\right)=0 \quad \text { for }\left\lfloor\frac{m}{2}\right\rfloor+2 \leq k \leq m-1 .
\end{aligned}
$$

Hence,

$$
\begin{gathered}
u_{h}(1)=\left\{\begin{array}{l}
\left\lvert\, \begin{array}{l}
\left\{u_{1}, u_{2}, \ldots, u_{i-1}\right\} \left\lvert\, ; i=\left\lceil\frac{m}{2}\right\rceil\right. \text { and } m \text { odd } \\
\left.\left\{u_{1}, u_{2}, \ldots, u_{i-1}, u_{i+1}, u_{i+2}, \ldots, u_{\left\lfloor\frac{m}{2}\right\rfloor+1}\right\} \right\rvert\, ; \text { otherwise. }
\end{array}\right. \\
u_{h}(0)=\left\{\begin{array}{l}
\left|\left\{u_{i}, u_{i+1}, u_{i+2}, \ldots, u_{m}\right\}\right| ; i=\left\lceil\frac{m}{2}\right\rceil \text { and } m \text { odd } \\
\left.\left\{u_{i}, u_{\left\lfloor\frac{m}{2}\right\rfloor+2}, u_{\left\lceil\frac{m}{2}\right\rceil+3}, \ldots, u_{m}\right\} \right\rvert\, ; \text { otherwise. }
\end{array}\right.
\end{array} .\right.
\end{gathered}
$$

$\left.u_{h}(1)=\left\lvert\,\left\{u_{1}, u_{2}, \ldots, u_{i-1}, u_{i+1}, u_{i+2}, \ldots, u_{\left\lfloor\frac{m}{2}\right\rfloor+1} ; i \neq\left\lceil\frac{m}{2}\right\rceil\right.$ and $m$ odd $\}\right. \right\rvert\,$ and
$u_{h}(0)=\left|\left\{u_{i}, u_{\left\lfloor\frac{m}{2}\right\rfloor+2}, u_{\left\lceil\frac{m}{2}\right\rceil+3}, \ldots, u_{m}\right\}\right|$. So $u_{h}(0)=\left\lceil\frac{m}{2}\right\rceil, u_{h}(1)=\left\lfloor\frac{m}{2}\right\rfloor$ and $f_{h}(0)=$
$f_{h}(1)=\left\lfloor\frac{2 m-6}{2}\right\rfloor$. Hence $\left|u_{h}(0)-u_{h}(1)\right|=1 \leq 1$ and $\left|f_{h}(0)-f_{h}(1)\right|=0 \leq 1$. So $G_{u_{1}}$ is an edge product cordial graph.

Example 2.3. Edge product cordial labeling of $G_{u_{5}}$ derived from $P_{11}$ reveal in the following figure 4.


Figure 4. $G_{u_{5}}$ derived from $P_{11}$

Theorem 2.4. The graph derived from switching of a vertex with degree two which is adjacent to a vertex with degree two in triangular snake $T_{m}$ is an edge product cordial graph.

Proof. Consider $u_{k}$ for $1 \leq k \leq m$ are the successive vertices of $P_{m}$. Let $w_{1}, w_{2}, \ldots, w_{m-1}$ be the vertices of triangle other than the vertices of $P_{m}$ in $T_{m}$. Consider $G_{w}$ graph derived by switching of any vertex with degree two which is adjacent to a vertex with degree two in $T_{m}$. Thus $\left|U\left(G_{w}\right)\right|=2 m-1$ and $\left|F\left(G_{w}\right)\right|=5 m-9$. Then there are four cases arise:
Case 1 If $w=w_{1}$, then in $G_{w_{1}}$, every vertex $w_{k}$ for $k=2,3, \ldots, m-1$ and $u_{k}$ for $k=3,4, \ldots, m$ are adjacent to $w_{1}$. Define mapping $r: F\left(G_{w_{1}}\right) \rightarrow\{0,1\}$ by,

$$
r(f)=\left\{\begin{array}{lll}
1 & \text { if } & f=u_{k} u_{k+1} \text { for } 1 \leq k \leq\left\lceil\frac{m}{2}\right\rceil ; \\
0 & \text { if } f=u_{k} u_{k+1} \text { for }\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m-1 ; \\
1 & \text { if } f=w_{1} u_{k} \text { for } 3 \leq k \leq\left\lceil\frac{m}{2}\right\rceil ; \\
0 & \text { if } f=w_{1} u_{k} \text { for }\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m ; \\
1 & \text { if } f \in\left\{w_{1} w_{k}, w_{k} u_{k+1}\right\} \text { for } 2 \leq k \leq\left\lfloor\frac{m}{2}\right\rfloor ; \\
0 & \text { if } f \in\left\{w_{1} w_{k}, w_{k} u_{k+1}\right\} \text { for }\left\lfloor\frac{m}{2}\right\rfloor+1 \leq k \leq m-1 ; \\
1 & \text { if } f=w_{k} u_{k} \text { for } 2 \leq k \leq\left\lceil\frac{m}{2}\right\rceil ; \\
0 & \text { if } f=w_{k} u_{k} \text { for }\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m-1 .
\end{array}\right.
$$

Thus $r^{*}: U\left(G_{w_{1}}\right) \rightarrow\{0,1\}$ is obtained as follows,

$$
\begin{aligned}
& r^{*}\left(w_{1}\right)=\prod_{k=3}^{m} r\left(w_{1} u_{k}\right) \prod_{k=2}^{m-1} r\left(w_{1} w_{k}\right)=0 \\
& r^{*}\left(w_{k}\right)=r\left(w_{k} u_{k}\right) r\left(w_{k} u_{k+1}\right) r\left(w_{1} w_{k}\right)=1 \quad \text { for } 2 \leq k \leq\left\lfloor\frac{m}{2}\right\rfloor \\
& r^{*}\left(w_{k}\right)=r\left(w_{k} u_{k}\right) r\left(w_{k} u_{k+1}\right) r\left(w_{1} w_{k}\right)=0 \text { for }\left\lfloor\frac{m}{2}\right\rfloor+1 \leq k \leq m-1 \\
& r^{*}\left(u_{1}\right)=r\left(u_{1} u_{2}\right)=1 \\
& r^{*}\left(u_{2}\right)=r\left(u_{1} u_{2}\right) r\left(u_{2} u_{3}\right) r\left(w_{2} u_{2}\right)=1, \\
& r^{*}\left(u_{k}\right)=r\left(u_{k} u_{k-1}\right) r\left(u_{k} u_{k+1}\right) r\left(w_{k-1} u_{k}\right) r\left(w_{k} u_{k}\right) r\left(w_{1} u_{k}\right)=1 \quad \text { for } 3 \leq k \leq\left\lceil\frac{m}{2}\right\rceil, \\
& r^{*}\left(u_{k}\right)=r\left(u_{k} u_{k-1}\right) r\left(u_{k} u_{k+1}\right) r\left(u_{k} w_{k-1}\right) r\left(w_{k} u_{k}\right) r\left(w_{1} u_{k}\right)=0 \text { for }\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m-1, \\
& r^{*}\left(u_{m}\right)=r\left(u_{m-1} u_{m}\right) r\left(w_{m-1} u_{m}\right) r\left(w_{1} u_{m}\right)=0 .
\end{aligned}
$$

Hence, $u_{r}(1)=\left|\left\{u_{1}, u_{2}, \ldots, u_{\left\lceil\frac{m}{2}\right\rceil}, w_{2}, w_{3}, \ldots, w_{\left\lfloor\frac{m}{2}\right\rfloor}\right\}\right|$ and
$u_{r}(0)=\left|\left\{u_{\left\lceil\frac{m}{2}\right\rceil+1}, u_{\left\lceil\frac{m}{2}\right\rceil+2}, \ldots, u_{m}, w_{1}, w_{\left\lfloor\frac{m}{2}\right\rfloor}+1, w_{\left\lfloor\frac{m}{2}\right\rfloor}+2, \ldots, w_{m-1}\right\}\right|$.
So $u_{r}(0)=u_{r}(1)+1=m$ and $f_{r}(1)=\left\lfloor\frac{5 m-9}{2}\right\rfloor, f_{r}(0)=\left\lceil\frac{5 m-9}{2}\right\rceil$.
If $w=w_{m}$, then proof is similar.
Case 2 If $w=u_{1}$, then in $G_{u_{1}}$, every vertex $u_{k}$ for $3 \leq k \leq m$ and $w_{k}$ for $2 \leq k \leq m-1$
are adjacent to $u_{1}$. Define mapping $r: F\left(G_{u_{1}}\right) \rightarrow\{0,1\}$ by,

$$
r(f)=\left\{\begin{array}{lll}
1 & \text { if } & f \in\left\{u_{k} u_{k+1}, w_{k} u_{k}\right\} \quad \text { for } \quad 2 \leq k \leq\left\lceil\frac{m}{2}\right\rceil ; \\
0 & \text { if } & f \in\left\{u_{k} u_{k+1}, w_{k} u_{k}\right\} \quad \text { for }\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m-1 ; \\
1 & \text { if } f=u_{1} w_{k} \text { for } 2 \leq k \leq\left\lfloor\frac{m}{2}\right\rfloor ; \\
0 & \text { if } f=u_{1} w_{k} \text { for }\left\lfloor\frac{m}{2}\right\rfloor+1 \leq k \leq m-1 ; \\
1 & \text { if } f=u_{1} u_{k} \text { for } 3 \leq k \leq\left\lceil\frac{m}{2}\right\rceil ; \\
0 & \text { if } f=u_{1} u_{k} \text { for }\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m ; \\
1 & \text { if } f=w_{k} u_{k+1} \text { for } 1 \leq k \leq\left\lfloor\frac{m}{2}\right\rfloor ; \\
0 & \text { if } f=w_{k} u_{k+1} \text { for }\left\lfloor\frac{m}{2}\right\rfloor+1 \leq k \leq m-1 .
\end{array}\right.
$$

Thus $r^{*}: U\left(G_{u_{1}}\right) \rightarrow\{0,1\}$ is obtained as follows, $r^{*}\left(w_{1}\right)=r\left(w_{1} u_{2}\right)=1$,
$r^{*}\left(w_{k}\right)=r\left(w_{k} u_{k}\right) r\left(w_{k} u_{k+1}\right) r\left(u_{1} w_{k}\right)=1$ for $2 \leq k \leq\left\lfloor\frac{m}{2}\right\rfloor$,
$r^{*}\left(w_{k}\right)=r\left(w_{i} u_{k}\right) r\left(w_{k} u_{k+1}\right) r\left(u_{1} w_{k}\right)=0 \quad$ for $\quad\left\lfloor\frac{m}{2}\right\rfloor+1 \leq k \leq m-1$,
$r^{*}\left(u_{2}\right)=r\left(w_{1} u_{2}\right) r\left(w_{2} u_{2}\right) r\left(u_{2} u_{3}\right)=1$,
$r^{*}\left(u_{1}\right)=\prod_{k=2}^{m-1} r\left(u_{1} w_{k}\right) \prod_{k=3}^{m} r\left(u_{1} u_{k}\right)=0$,
$r^{*}\left(u_{k}\right)=r\left(u_{k} w_{k-1}\right) r\left(u_{k} w_{k}\right) r\left(u_{k-1} u_{k}\right) r\left(u_{k} u_{k+1}\right) r\left(u_{1} u_{k}\right)=1 \quad$ for $3 \leq k \leq\left\lceil\frac{m}{2}\right\rceil$,
$r^{*}\left(u_{k}\right)=r\left(u_{k} w_{k-1}\right) r\left(u_{k} w_{k}\right) r\left(u_{k-1} u_{k}\right) r\left(u_{k} u_{k+1}\right) r\left(u_{1} u_{k}\right)=0 \quad$ for $\quad\left\lceil\frac{m}{2}\right\rceil+1 \leq k \leq m-1$,
$r^{*}\left(u_{m}\right)=r\left(w_{m-1} u_{m}\right) r\left(u_{m-1} u_{m}\right) r\left(u_{1} u_{m}\right)=0$.
Hence, $u_{r}(1)=\left|\left\{u_{1}, u_{2}, \ldots, u_{\left\lceil\frac{m}{2}\right\rceil}, w_{2}, w_{3}, \ldots, w_{\left\lfloor\frac{m}{2}\right\rfloor}\right\}\right|$ and
$u_{r}(0)=\left|\left\{u_{\left\lceil\frac{m}{2}\right\rceil+1}, u_{\left\lceil\frac{m}{2}\right\rceil+2}, \ldots, u_{m}, w_{1}, w_{\left\lfloor\frac{m}{2}\right\rfloor+1}, w_{\left\lfloor\frac{m}{2}\right\rfloor+2}, \ldots, w_{m-1}\right\}\right|$.
So $u_{r}(0)=u_{r}(1)+1=m$ and $f_{r}(1)=\left\lfloor\frac{5 m-9}{2}\right\rfloor, f_{r}(0)=\left\lceil\frac{5 m-9}{2}\right\rceil$.
If $w=u_{m}$, then proof is similar.
Thus from the aboves cases $\left|u_{h}(0)-u_{h}(1)\right|=1 \leq 1$ and $\left|f_{h}(0)-f_{h}(1)\right|=1 \leq 1$. Hence $G_{w}$ is an edge product cordial graph.

Example 2.4. Edge product cordial labeling of $G_{w_{1}}$ derived by $T_{6}$ and $G_{u_{1}}$ obtained by $T_{7}$ shown in the following figure 5 and figure 6 respectively.

## 3. Conclusions

We examine four results on graph derived by a vertex switching with degree one in $P_{m}$ if and only if $m \geq 3$ and $m$ is odd, Switching of vertex with degree two in $P_{m}$ except $u_{2}$ or $u_{m-1}$ with $m>4$, switching of any vertex in $C_{m}$ and a vertx switching with degree two which is adjacent to a vertex with degree two in $T_{m}$ are edge product cordial graph.

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Figure 5. $G_{w_{1}}$ obtained from $T_{6}$


Figure 6. $G_{u_{1}}$ obtained from $T_{7}$

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