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ON SOFT $\widetilde{\gamma}$ GENERALIZED CLOSED SETS

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Abstract. The purpose of this paper is to define some new classes of soft generalized closed sets, namely soft β-$\widetilde{\gamma}$g.closed, soft b-$\widetilde{\gamma}$g.closed, soft p-$\widetilde{\gamma}$g.closed, soft s-$\widetilde{\gamma}$g.closed, soft α-$\widetilde{\gamma}$g.closed and soft $\widetilde{\gamma}$g.closed by using soft $\widetilde{\gamma}$ open set. The relationships between these soft sets and the related concepts are investigated. Basic properties of soft β-$\widetilde{\gamma}$g.closed, soft b-$\widetilde{\gamma}$g.closed, soft p-$\widetilde{\gamma}$g.closed, soft s-$\widetilde{\gamma}$g.closed, soft α-$\widetilde{\gamma}$g.closed and soft $\widetilde{\gamma}$g.closed sets are analyzed. Finally, the new soft separation axiom, namely soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$ space is introduced and its basic properties are investigated.

Keywords: Soft topology, soft open set, soft closed set, soft $\widetilde{\gamma}$ open set, soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$.

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1. Introduction

Soft set is a parameterized general mathematical tool which deals with a collection of approximate descriptions of objects. Each approximate description has two parts, a predicate and an approximate value set. The concept of soft sets was first introduced by Molodtsov [18] in 1999 as a general mathematical tool for dealing with uncertain objects. He also successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. Shabir and Naz [20] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Chen [7] investigated soft semiopen sets in soft topological spaces and studied some properties of it. Arockiarani and Arokialancy [4] defined soft preopen and soft β-open sets and continued to study weak forms of soft open sets in soft topological space. Akdag and Ozkan [5] defined soft α-open sets and also they defined soft b-open sets [6]. Kalavathia [13] introduced and discussed an operation of a family of all soft open sets in soft topological space. Many researchers defined and studied the weak and strong forms of soft closed sets that are important for the study of characterizations of soft topological spaces. Ibrahim [11] introduced and studied the concept of γgb-closed sets in topological spaces. Soft $T_{\frac{1}{2}}$ space was introduced by Kannan [14].

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2. Preliminaries

Definition 2.1. [18] *Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denotes the power set of X and (A, E) is a non-empty subset of E. A pair (F, A) is called a soft set over X and is defined by the set $(F, A) = \{(e, F(e)) : e \in A, F(e) \in P(X)\}$, where F is a mapping given by $F : A \to P(X)$ such that $F(e) = \phi$ if $e \notin A$. In other words, a soft set over X is a parametrized family of subsets over the universe X. For $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A). The value of $F(e)$ may be arbitrary, some of them may be empty, some may have non-empty intersection.*

Definition 2.2. [16] *For two soft sets (F, A) and (G, B) over a common universe X, (F, A) is a soft subset of (G, B), denoted by $(F, A) \widetilde{\subseteq} (G, B)$, if*

(1) *$A \subseteq B$ and*
(2) *$F(e) \subseteq G(e)$, for all $e \in A$.*

(F, A) is said to be a soft superset of (G, B) if (G, B) is a soft subset of (F, A) and denoted it by $(F, A) \widetilde{\supseteq} (G, B)$.

Definition 2.3. [16] *Two soft sets (F, A) and (G, B) over the common universe X are said to be soft equal, if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).*

Definition 2.4. [16] *The relative soft complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to P(X)$ is a mapping given by $F^c(e) = X \setminus F(e)$ for all $e \in A$.*

Definition 2.5. [16] *A soft set (F, A) over X is said to be a null soft set and denoted by $\widetilde{\Phi}$, if $F(e) = \phi$ for all $e \in A$.*

Definition 2.6. [16] *A soft set (F, A) over X is said to be an absolute soft set and denoted by $\widetilde{X}$, if $F(e) = X$ for all $e \in A$. Clearly $\widetilde{X}^c = \widetilde{\Phi}$ and $\widetilde{\Phi}^c = \widetilde{X}$.*

Definition 2.7. [16] *The soft union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,*

$$(H, C) = \begin{cases} F(e) & \text{if } e \in A \setminus B \\ G(e) & \text{if } e \in B \setminus A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.8. [16] *The soft intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for each $e \in C$, $H(e) = F(e) \cap G(e)$.*

Definition 2.9. [22] *A soft subset (F, A) over X is called a soft point in $\widetilde{X}$ denoted by e_F, if for the element $e \in A$, $F(e) \neq \phi$ and $F(e') = \phi$ for every $e' \in A \setminus \{e\}$. The soft point e_F is said to be in the soft set (G, A) denoted by $e_F \widetilde{\in} (G, A)$, if the element $e \in A$ and $F(e) \subseteq G(e)$.*

Throughout this paper, consider X is an universe set, E is a fixed set of parameters. We consider only soft sets (F, E) over the universe X in which all the parameter set E are same. The collection of all soft sets over X is denoted by $S(X)$.

Definition 2.10. [20] *Let $\widetilde{\tau}$ be the collection of soft sets over X. Then $\widetilde{\tau}$ is said to be a soft topology on $\widetilde{X}$ if*

(1) $\widetilde{\Phi}$ *and* $\widetilde{X}$ *belongs to* $\widetilde{\tau}$.
(2) *The soft union of any number of soft sets in* $\widetilde{\tau}$ *belongs to* $\widetilde{\tau}$.
(3) *The soft intersection of any two soft sets in* $\widetilde{\tau}$ *belongs to* $\widetilde{\tau}$.

The triplet $(\widetilde{X}, \widetilde{\tau}, E)$ *is called a soft topological space over* X. *The members of* $\widetilde{\tau}$ *are said to be soft open sets over* X. *A soft set* (F, E) *over* X *is said to be soft closed in* $\widetilde{X}$, *if its relative soft complement* (F^c, E) *belong to* $\widetilde{\tau}$.

Definition 2.11. *Let* $(\widetilde{X}, \widetilde{\tau}, E)$ *be a soft topological space over* X *and* $(F, E) \widetilde{\in} S(X)$.

(1) *The soft interior of* (F, E) *denoted by* $int(F, E)$ *is defined as the soft union of all soft open subsets of* (F, E) [22].
(2) *The soft closure of* (F, E) *denoted by* $cl(F, E)$ *is defined as the soft intersection of all soft closed supersets of* (F, E) [20].

Definition 2.12. *A soft set* (F, E) *in a soft topological space* $(\widetilde{X}, \widetilde{\tau}, E)$ *is called:*

(1) *soft* α*-open, if* $(F, E) \widetilde{\subseteq} int(cl(int(F, E)))$ *and soft* α*-closed if* $cl(int(cl(F, E))) \widetilde{\subseteq} (F, E)$ [5].
(2) *soft preopen, if* $(F, E) \widetilde{\subseteq} int(cl(F, E))$ *and soft preclosed if* $cl(int(F, E)) \widetilde{\subseteq} (F, E)$ [4].
(3) *soft semiopen, if* $(F, E) \widetilde{\subseteq} cl(int(F, E))$ *and soft semiclosed if* $int(cl(F, E)) \widetilde{\subseteq} (F, E)$ [7].
(4) *soft b-open, if* $(F, E) \widetilde{\subseteq} cl(int(F, E)) \widetilde{\cup} int(cl(F, E))$ *and soft b-closed if* $cl(int(F, E)) \widetilde{\cap} int(cl(F, E)) \widetilde{\subseteq} (F, E)$ [6].
(5) *soft* β*-open, if* $(F, E) \widetilde{\subseteq} cl(int(cl(F, E)))$ *and soft* β*-closed if* $int(cl(int(F, E))) \widetilde{\subseteq} (F, E)$ [4].
(6) *soft regular open, if* $(F, E) = int(cl(F, E))$ [4].

Definition 2.13. *Let* $(\widetilde{X}, \widetilde{\tau}, E)$ *be a soft topological space over* X *and* $(F, E) \widetilde{\in} S(X)$.

(1) *The soft* α*-closure of* (F, E) *denoted by* $\alpha cl(F, E)$ *is defined as the soft intersection of all soft* α*-closed supersets of* (F, E) [5].
(2) *The soft pre closure of* (F, E) *denoted by* $pcl(F, E)$ *is defined as the soft intersection of all soft preclosed supersets of* (F, E) [4].
(3) *The soft semi closure of* (F, E) *denoted by* $scl(F, E)$ *is defined as the soft intersection of all soft semiclosed supersets of* (F, E) [7].
(4) *The soft b-closure of* (F, E) *denoted by* $bcl(F, E)$ *is defined as the soft intersection of all soft b-closed supersets of* (F, E) [4].
(5) *The soft* β*-closure of* (F, E) *denoted by* $\beta cl(F, E)$ *is defined as the soft intersection of all soft* β*-closed supersets of* (F, E) [4].

Definition 2.14. [13] *Let* $(\widetilde{X}, \widetilde{\tau}, E)$ *be a soft topological space over* X. *An operation* $\widetilde{\gamma}$ *on the soft topology* $\widetilde{\tau}$ *is a mapping from a soft topology* $\widetilde{\tau}$ *into the soft power set* $S(X)$ *of* X *such that* $(V, E) \widetilde{\subseteq} (V, E)^{\widetilde{\gamma}}$ *for each* $(V, E) \widetilde{\in} \widetilde{\tau}$, *where* $(V, E)^{\widetilde{\gamma}}$ *denote the value of* $\widetilde{\gamma}$ *at* (V, E). *It is denoted by* $\widetilde{\gamma}: \widetilde{\tau} \to S(X)$.

Definition 2.15. [13] *A soft subset* (F, E) *of a soft topological space* $(\widetilde{X}, \widetilde{\tau}, E)$ *is said to be soft* $\widetilde{\gamma}$ *open, if for each* $e_F \widetilde{\in} (F, E)$, *there exists a soft open set* (U, E) *such that* $e_F \widetilde{\in} (U, E)$ *and* $(U, E)^{\widetilde{\gamma}} \widetilde{\subseteq} (F, E)$. *A soft subset* (F, E) *of* $(\widetilde{X}, \widetilde{\tau}, E)$ *is said to be soft* $\widetilde{\gamma}$

closed if and only if $(F,E)^c$ *is soft* $\widetilde{\gamma}$ *open in* $(\widetilde{X},\widetilde{\tau},E)$. *The family of all soft* $\widetilde{\gamma}$ *open sets denotes by* $\widetilde{\tau}_{\widetilde{\gamma}}$.

Remark 2.1. [13] *Let* $(\widetilde{X},\widetilde{\tau},E)$ *be a soft topological space. Then,* $\widetilde{\tau}_{\widetilde{\gamma}}\widetilde{\subseteq}\widetilde{\tau}$.

Definition 2.16. [13] *A soft topological space* $(\widetilde{X},\widetilde{\tau},E)$ *is said to be soft* $\widetilde{\gamma}$ *regular if for each* $e_F\,\widetilde{\in}\widetilde{X}$ *and for each soft open set* (V,E) *such that* $e_F\,\widetilde{\in}\,(V,E)$, *there exists a soft open set* (U,E) *such that* $e_F\,\widetilde{\in}\,(U,E)$ *and* $(U,E)^{\widetilde{\gamma}}\,\widetilde{\subseteq}\,(V,E)$.

Theorem 2.1. [13] *A soft topological space* $(\widetilde{X},\widetilde{\tau},E)$ *is soft* $\widetilde{\gamma}$ *regular if and only if* $\widetilde{\tau}=\widetilde{\tau}_{\widetilde{\gamma}}$.

Definition 2.17. [13] *The soft* $\widetilde{\tau}_{\widetilde{\gamma}}$ *closure of* (F,E) *denoted by* $\widetilde{\tau}_{\widetilde{\gamma}}$-*cl*$(F,E)$ *is defined as the soft intersection of all soft* $\widetilde{\gamma}$ *closed supersets of* (F,E).

Definition 2.18. *A soft set* (F,E) *in a soft topological space* $(\widetilde{X},\widetilde{\tau},E)$ *is called:*

(1) *soft g-closed if cl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft open in* $\widetilde{X}$ [14].

(2) *soft gαb-closed if bcl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft* α-*open in* $\widetilde{X}$ [1].

(3) *soft sgb-closed if bcl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft semiopen in* $\widetilde{X}$ [1].

(4) *soft gb-closed if bcl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft open in* $\widetilde{X}$ [19].

(5) *soft gβb-closed if bcl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft* β-*open in* $\widetilde{X}$ [19].

(6) *soft gβ-closed if* β*cl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft open in* $\widetilde{X}$ [2].

(7) *soft pre generalized closed (in short soft pg-closed*) if pcl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft preopen in* $\widetilde{X}$ [21].

(8) *soft pg-closed if pcl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft open in* $\widetilde{X}$ [3].

(9) *soft gp-closed if cl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft preopen in* $\widetilde{X}$ [3].

(10) *soft sg-closed if scl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft semiopen in* $\widetilde{X}$ [2].

(11) *soft gs-closed if scl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft open in* $\widetilde{X}$ [2].

(12) *soft s*g-closed if cl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$, *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft semiopen in* $\widetilde{X}$ [15].

(13) *soft αg-closed if* α*cl*$(F,E)\,\widetilde{\subseteq}\,(U,E)$ *whenever* $(F,E)\,\widetilde{\subseteq}\,(U,E)$ *and* (U,E) *is soft open in* $\widetilde{X}$ [2].

(14) *soft gα-closed if* $cl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft α-open in* $\widetilde{X}$ [1].

(15) *soft gsβ-closed if* $\beta cl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$, *whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft semiopen in* $\widetilde{X}$ [2].

Definition 2.19. [14] *A soft set* (F,E) *is called soft g-open if the relative complement* $(F,E)^c$ *is soft g-closed.*

Definition 2.20. *A soft set* (F,E) *in a soft topological space* $(\widetilde{X}, \widetilde{\tau}, E)$ *is called:*

(1) *soft g*-closed if* $cl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$, *whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft g-open in* $\widetilde{X}$ [9].

(2) *soft wg-closed if* $cl(int(F,E)) \mathrel{\widetilde{\subseteq}} (U,E)$, *whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft open in* $\widetilde{X}$ [12].

(3) *soft rg-closed if* $cl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$, *whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft regular open in* $\widetilde{X}$ [2].

(4) *soft rwg-closed if* $cl(int(F,E)) \mathrel{\widetilde{\subseteq}} (U,E)$, *whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft regular open in* $\widetilde{X}$ [12].

(5) *soft gpr-closed if* $pcl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$, *whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft regular open in* $\widetilde{X}$ [10].

(6) *soft-gsr-closed if* $scl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$, *whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft regular open in* $\widetilde{X}$ [17].

(7) *soft rgα-closed if* $\alpha cl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$, *whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft regular open in* $\widetilde{X}$ [1].

(8) *soft rgb-closed if* $bcl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$, *whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft regular open in* $\widetilde{X}$ [1].

Theorem 2.2. [14] *If a soft set* (F,E) *is soft g-closed, then* $cl((A,E)) \mathrel{\widetilde{\setminus}} (A,E)$ *contains only null soft closed.*

3. Soft $\widetilde{\gamma}$g.closed sets

Definition 3.1. *A soft set* (F,E) *in a soft topological space* $(\widetilde{X}, \widetilde{\tau}, E)$ *is called soft β-*$\widetilde{\gamma}$*g.closed (resp. soft b-*$\widetilde{\gamma}$*g.closed, soft p-*$\widetilde{\gamma}$*g.closed, soft s-*$\widetilde{\gamma}$*g.closed, soft α-*$\widetilde{\gamma}$*g.closed and soft* $\widetilde{\gamma}$*g.closed) if* $\beta cl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *(resp.* $bcl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$, $pcl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$, $scl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$, $\alpha cl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* $cl(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$*), whenever* $(F,E) \mathrel{\widetilde{\subseteq}} (U,E)$ *and* (U,E) *is soft* $\widetilde{\gamma}$ *open in* $\widetilde{X}$.

Theorem 3.1. (1) *Every soft b-*$\widetilde{\gamma}$*g.closed set is soft β-*$\widetilde{\gamma}$*g.closed.*

(2) *Every soft p-*$\widetilde{\gamma}$*g.closed set is soft b-*$\widetilde{\gamma}$*g.closed.*

(3) *Every soft s-*$\widetilde{\gamma}$*g.closed set is soft b-*$\widetilde{\gamma}$*g.closed.*

(4) *Every soft α-*$\widetilde{\gamma}$*g.closed set is soft p-*$\widetilde{\gamma}$*g.closed.*

(5) *Every soft α-*$\widetilde{\gamma}$*g.closed set is soft s-*$\widetilde{\gamma}$*g.closed.*

(6) *Every soft* $\widetilde{\gamma}$*g.closed set is soft α-*$\widetilde{\gamma}$*g.closed.*

Proof. Let (F,E) be any soft set in a soft topological space $(\widetilde{X},\widetilde{\tau},E)$ and $(F,E)\widetilde{\subseteq}(U,E)$, where (U,E) is soft $\widetilde{\gamma}$ open.

(1) If (F,E) is soft b-$\widetilde{\gamma}$g.closed, then $bcl(F,E)\widetilde{\subseteq}(U,E)$ and since $\beta cl(F,E)\widetilde{\subseteq} bcl(F,E)$. Thus, (F,E) is a soft β-$\widetilde{\gamma}$g.closed set in $\widetilde{X}$.
(2) If (F,E) is soft p-$\widetilde{\gamma}$g.closed, then $pcl(F,E)\widetilde{\subseteq}(U,E)$ and since $bcl(F,E)\widetilde{\subseteq} pcl(F,E)$. Thus, (F,E) is a soft b-$\widetilde{\gamma}$g.closed set in $\widetilde{X}$.
(3) If (F,E) is soft s-$\widetilde{\gamma}$g.closed, then $scl(F,E)\widetilde{\subseteq}(U,E)$ and since $bcl(F,E)\widetilde{\subseteq} scl(F,E)$. Thus, (F,E) is a soft b-$\widetilde{\gamma}$g.closed set in $\widetilde{X}$.
(4) If (F,E) is soft α-$\widetilde{\gamma}$g.closed, then $\alpha cl(F,E)\widetilde{\subseteq}(U,E)$ and since $pcl(F,E)\widetilde{\subseteq}\alpha cl(F,E)$. Thus, (F,E) is a soft p-$\widetilde{\gamma}$g.closed set in $\widetilde{X}$.
(5) If (F,E) is soft α-$\widetilde{\gamma}$g.closed, then $\alpha cl(F,E)\widetilde{\subseteq}(U,E)$ and since $scl(F,E)\widetilde{\subseteq}\alpha cl(F,E)$. Thus, (F,E) is a soft s-$\widetilde{\gamma}$g.closed set in $\widetilde{X}$.
(6) If (F,E) is soft $\widetilde{\gamma}$g.closed, then $cl(F,E)\widetilde{\subseteq}(U,E)$ and since $\alpha cl(F,E)\widetilde{\subseteq} cl(F,E)$. Thus, (F,E) is a soft α-$\widetilde{\gamma}$g.closed set in $\widetilde{X}$.

□

Remark 3.1. *In general, soft p-$\widetilde{\gamma}$g.closed and soft s-$\widetilde{\gamma}$g.closed are independent. It is shown by the following examples.*

Example 3.1. *Let $X=\{p_1,p_2,p_3,p_4,p_5\}$, $E=\{e\}$ and $\widetilde{\tau}=\{\widetilde{X},\widetilde{\Phi},(F_1,E),(F_2,E),(F_3,E)\}$ be a soft topological space over X, where $(F_1,E)=\{(e,\{p_1,p_2\})\},(F_2,E)=\{(e,\{p_3,p_4\})\}$ and $(F_3,E)=\{(e,\{p_1,p_2,p_3,p_4\})\}$ are soft open sets over X. Define an operation $\widetilde{\gamma}$ on $\widetilde{\tau}$ by*

$$(A,E)^{\widetilde{\gamma}}=\begin{cases}(A,E) & \text{if } (A,E)=(F_3,E)\\ \widetilde{X} & \text{otherwise.}\end{cases}$$

Then, $\widetilde{\tau}_{\widetilde{\gamma}}=\{\widetilde{X},\widetilde{\Phi},(F_3,E)\}$. If $(G,E)=\{(e,\{p_1,p_4\})\}$, then (G,E) is both soft p-$\widetilde{\gamma}$g.closed and soft b-$\widetilde{\gamma}$g.closed but (G,E) is neither soft α-$\widetilde{\gamma}$g.closed nor soft s-$\widetilde{\gamma}$g.closed.

Example 3.2. *Let $X=\{p_1,p_2,p_3,p_4\}$, $E=\{e\}$ and $\widetilde{\tau}=\{\widetilde{X},\widetilde{\Phi},(F_1,E),(F_2,E),(F_3,E)\}$ be a soft topological space over X, where $(F_1,E)=\{(e,\{p_1\})\},(F_2,E)=\{(e,\{p_2,p_4\})\}$ and $(F_3,E)=\{(e,\{p_1,p_2,p_4\})\}$ are soft open sets over X. Define an operation $\widetilde{\gamma}$ on $\widetilde{\tau}$ by $(A,E)^{\widetilde{\gamma}}=(A,E)$ for all $(A,E)\widetilde{\in}\widetilde{\tau}$. Then, $\widetilde{\tau}_{\widetilde{\gamma}}=\widetilde{\tau}$.*

(1) *If $(G,E)=\{(e,\{p_1\})\}$, then (G,E) is both soft s-$\widetilde{\gamma}$g.closed and soft b-$\widetilde{\gamma}$g.closed but (G,E) is neither soft α-$\widetilde{\gamma}$g.closed nor soft p-$\widetilde{\gamma}$g.closed.*
(2) *If $(G,E)=\{(e,\{p_1,p_2\})\}$, then (G,E) is soft β-$\widetilde{\gamma}$g.closed but (G,E) is not soft b-$\widetilde{\gamma}$g.closed.*

Theorem 3.2. (1) *Every soft $\widetilde{\gamma}$ closed set is soft $\widetilde{\gamma}$g.closed.*
(2) *Every soft closed set is soft $\widetilde{\gamma}$g.closed.*
(3) *Every soft α-closed set is soft α-$\widetilde{\gamma}$g.closed.*
(4) *Every soft preclosed set is soft p-$\widetilde{\gamma}$g.closed.*
(5) *Every soft semiclosed set is soft s-$\widetilde{\gamma}$g.closed.*

(6) *Every soft b-closed set is soft b-$\widetilde{\gamma}$g.closed.*
(7) *Every soft β-closed set is soft β-$\widetilde{\gamma}$g.closed.*
(8) *Every soft g-closed set is soft $\widetilde{\gamma}$g.closed.*
(9) *Every soft g*-closed set is soft $\widetilde{\gamma}$g.closed.*
(10) *Every soft gp-closed set is soft $\widetilde{\gamma}$g.closed.*
(11) *Every soft s*g-closed set is soft $\widetilde{\gamma}$g.closed.*
(12) *Every soft gα-closed set is soft $\widetilde{\gamma}$g.closed.*

Proof. (1) Let (F,E) be a soft $\widetilde{\gamma}$ closed set in $\widetilde{X}$ such that $(F,E) \widetilde{\subseteq} (U,E)$, where (U,E) is soft $\widetilde{\gamma}$ open. Since (F,E) is soft $\widetilde{\gamma}$ closed, then $\widetilde{\tau}_{\widetilde{\gamma}}$-$cl(F,E) = (F,E)$ and $cl(F,E) \widetilde{\subseteq} \widetilde{\tau}_{\widetilde{\gamma}}$-$cl(F,E) = (F,E)$. Therefore, $cl(F,E) \widetilde{\subseteq} (U,E)$. Hence, (F,E) is a soft $\widetilde{\gamma}$g.closed set in $\widetilde{X}$.

The proofs of the other parts are similar. □

Remark 3.2. *The converse of the Theorem 3.2 need not be true in general as it is shown below.*

Example 3.3. *Let $X = \{p_1, p_2, p_3\}$, $E = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\widetilde{X}, \widetilde{\Phi}, (F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E), (F_6,E), (F_7,E)\}$ be a soft topological space over X, where $\widetilde{X} = \{(e_1,\{p_1,p_2,p_3\}), (e_2,\{p_1,p_2,p_3\})\}$, $\widetilde{\Phi} = \{(e_1,\phi), (e_2,\phi)\}$, $(F_1,E) = \{(e_1,\{p_1\}), (e_2,\{p_2\})\}$, $(F_2,E) = \{(e_1,X), (e_2,\{p_2\})\}$, $(F_3,E) = \{(e_1,\{p_2,p_3\}), (e_2,\{p_1,p_3\})\}$, $(F_4,E) = \{(e_1,X), (e_2,\{p_1,p_3\})\}$, $(F_5,E) = \{(e_1,\{p_1\}), (e_2,\phi)\}$, $(F_6,E) = \{(e_1,\{p_2,p_3\}), (e_2,\phi)\}$ and $(F_7,E) = \{(e_1,X), (e_2,\phi)\}$ are soft open sets over X. Define an operation $\widetilde{\gamma}$ on $\widetilde{\tau}$ by*

$$(A,E)^{\widetilde{\gamma}} = \begin{cases} (A,E) & \text{if } (A,E) = (F_1,E) \text{ or } (F_3,E) \\ \widetilde{X} & \text{otherwise.} \end{cases}$$

Then, $\widetilde{\tau}_{\widetilde{\gamma}} = \{\widetilde{X}, \widetilde{\Phi}, (F_1,E), (F_3,E)\}$. If $(G,E) = \{(e_1,\{p_2\}), (e_2,\{p_1,p_3\})\}$, then the only soft $\widetilde{\gamma}$ open supersets of (G,E) are (F_3,E) and $\widetilde{X}$, then (G,E) is a soft $\widetilde{\gamma}$g.closed set but (G,E) is neither soft $\widetilde{\gamma}$ closed nor soft closed. Also, it is both soft α-$\widetilde{\gamma}$g.closed and soft s-$\widetilde{\gamma}$g.closed but (G,E) is neither soft α-closed nor soft semiclosed.

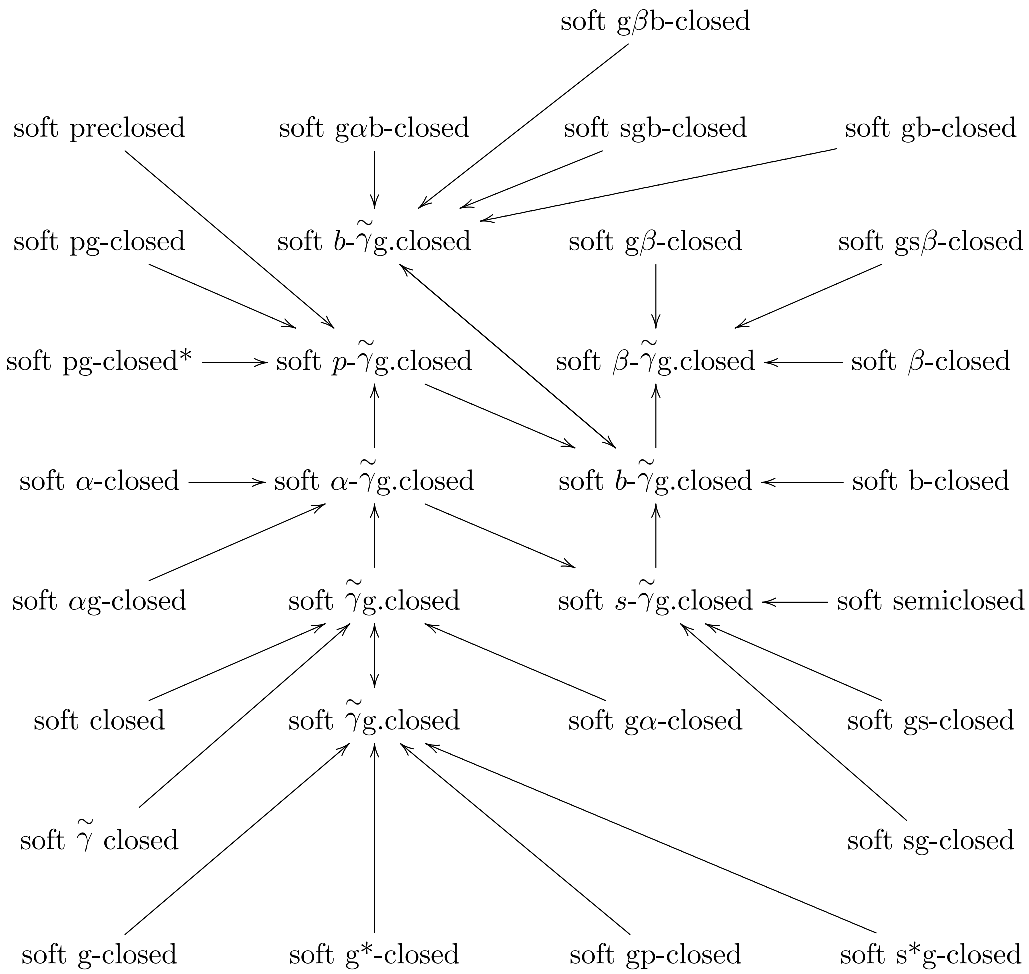
Example 3.4. *Let $X = \{p_1, p_2\}$, $E = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\widetilde{X}, \widetilde{\Phi}, (F_1,E), (F_2,E), (F_3,E)\}$ be a soft topological space over X, where $(F_1,E) = \{(e_1,\{p_2\}), (e_2,\{p_1\})\}$, $(F_2,E) = \{(e_1,\{p_2\}), (e_2,\{p_1,p_2\})\}$ and $(F_3,E) = \{(e_1,\{p_1,p_2\}), (e_2,\{p_1\})\}$ are soft open sets over X. Define an operation $\widetilde{\gamma}$ on $\widetilde{\tau}$ by $(A,E)^{\widetilde{\gamma}} = \widetilde{X}$ for all $(A,E) \widetilde{\in} \widetilde{\tau}$. Then, $\widetilde{\tau}_{\widetilde{\gamma}} = \{\widetilde{X}, \widetilde{\Phi}\}$. Let $(G,E) = \{(e_1,X), (e_2,\{p_1\})\}$, since the only soft $\widetilde{\gamma}$ open superset of (G,E) is $\widetilde{X}$, then (G,E) is soft p-$\widetilde{\gamma}$g.closed, soft b-$\widetilde{\gamma}$g.closed and soft β-$\widetilde{\gamma}$g.closed but (G,E) is not soft preclosed, soft b-closed and soft β-closed.*

Example 3.5. *From Example 3.3, we define an operation $\widetilde{\gamma}$ on $\widetilde{\tau}$ by $(A,E)^{\widetilde{\gamma}} = \widetilde{X}$ for all $(A,E) \widetilde{\in} \widetilde{\tau}$. Then, $\widetilde{\tau}_{\widetilde{\gamma}} = \{\widetilde{X}, \widetilde{\Phi}\}$. If $(G,E) = \{(e_1,X), (e_2,\{p_2\})\}$, then (G,E) is soft $\widetilde{\gamma}$g.closed but (G,E) is not soft g-closed, soft g*-closed, soft gp-closed, soft s*g-closed and soft gα-closed.*

Theorem 3.3. (1) *Every soft αg-closed set is soft soft α-$\widetilde{\gamma}g$.closed.*
(2) *Every soft pg-closed* set is soft p-$\widetilde{\gamma}g$.closed.*
(3) *Every soft pg-closed set is soft p-$\widetilde{\gamma}g$.closed.*
(4) *Every soft sg-closed set is soft s-$\widetilde{\gamma}g$.closed.*
(5) *Every soft gs-closed set is soft s-$\widetilde{\gamma}g$.closed.*
(6) *Every soft $g\alpha b$-closed set is soft b-$\widetilde{\gamma}g$.closed.*
(7) *Every soft sgb-closed set is soft b-$\widetilde{\gamma}g$.closed.*
(8) *Every soft gb-closed set is soft b-$\widetilde{\gamma}g$.closed.*
(9) *Every soft $g\beta b$-closed set is soft b-$\widetilde{\gamma}g$.closed.*
(10) *Every soft $g\beta$-closed set is soft β-$\widetilde{\gamma}g$.closed.*
(11) *Every soft $gs\beta$-closed set is soft β-$\widetilde{\gamma}g$.closed.*

Proof. The proofs are obvious. ☐

Remark 3.3. *From Theorems 3.1,3.2 and 3.3, the following diagram of implications is obtain:*



Theorem 3.4. *Let $(\widetilde{X}, \widetilde{\tau}, E)$ be soft $\widetilde{\gamma}$ regular. Then:*
(1) *Every soft $\widetilde{\gamma}g$.closed set is soft rg-closed.*

(2) *Every soft $\widetilde{\gamma}$g.closed set is soft rwg-closed.*
(3) *Every soft $\widetilde{\gamma}$g.closed set is soft wg-closed.*
(4) *Every soft $\widetilde{\gamma}$g.closed set is soft gpr-closed.*
(5) *Every soft $\widetilde{\gamma}$g.closed set is soft rgα-closed.*
(6) *Every soft $\widetilde{\gamma}$g.closed set is soft rgb-closed.*
(7) *Every soft $\widetilde{\gamma}$g.closed set is soft-gsr-closed.*

Proof. Since every soft regular open set is soft open, then the proofs are obvious by Theorem 2.1. □

Remark 3.4. *Let $(\widetilde{X}, \widetilde{\tau}, E)$ be soft $\widetilde{\gamma}$ regular. Then:*

(1) *Every soft p-$\widetilde{\gamma}$g.closed set in $\widetilde{X}$ is soft gpr-closed.*
(2) *Every soft α-$\widetilde{\gamma}$g.closed set in $\widetilde{X}$ is soft rgα-closed.*
(3) *Every soft b-$\widetilde{\gamma}$g.closed set in $\widetilde{X}$ is soft rgb-closed.*
(4) *Every soft s-$\widetilde{\gamma}$g.closed set in $\widetilde{X}$ is soft-gsr-closed.*

Theorem 3.5. *If (F, E) is soft open and soft wg-closed, then (F, E) is soft $\widetilde{\gamma}$g.closed.*

Proof. Let (F, E) be soft open and soft wg-closed in $\widetilde{X}$ such that $(F, E) \widetilde{\subseteq} (U, E)$, where (U, E) is soft $\widetilde{\gamma}$ open. Since every soft $\widetilde{\gamma}$ open set is soft open and (F, E) is wg-closed, then $cl(F, E) = cl(int(F, E)) \widetilde{\subseteq} (U, E)$. Thus, (F, E) is soft $\widetilde{\gamma}$g.closed. □

Corollary 3.1. *Let $(\widetilde{X}, \widetilde{\tau}, E)$ be soft $\widetilde{\gamma}$ regular and (F, E) be a soft set in $\widetilde{X}$. Then,*

(1) *(F, E) is soft g-closed if and only if (F, E) is soft $\widetilde{\gamma}$g.closed.*
(2) *(F, E) is soft αg-closed if and only if (F, E) is soft α-$\widetilde{\gamma}$g.closed.*
(3) *(F, E) is soft pg-closed if and only if (F, E) is soft p-$\widetilde{\gamma}$g.closed.*
(4) *(F, E) is soft gs-closed if and only if (F, E) is soft s-$\widetilde{\gamma}$g.closed.*
(5) *(F, E) is soft gb-closed if and only if (F, E) is soft b-$\widetilde{\gamma}$g.closed.*
(6) *(F, E) is soft gβ-closed if and only if (F, E) is soft β-$\widetilde{\gamma}$g.closed.*

Proof. The proofs are obvious by Theorem 2.1. □

Theorem 3.6. *Let (F, E) be a soft $\widetilde{\gamma}$ open set in a soft topological space $(\widetilde{X}, \widetilde{\tau}, E)$. Then,*

(1) *(F, E) is soft closed if (F, E) is soft $\widetilde{\gamma}$g.closed.*
(2) *(F, E) is soft α-closed if (F, E) is soft α-$\widetilde{\gamma}$g.closed.*
(3) *(F, E) is soft preclosed if (F, E) is soft p-$\widetilde{\gamma}$g.closed.*
(4) *(F, E) is soft semiclosed if (F, E) is soft s-$\widetilde{\gamma}$g.closed.*
(5) *(F, E) is soft b-closed if (F, E) is soft b-$\widetilde{\gamma}$g.closed.*
(6) *(F, E) is soft β-closed if (F, E) is soft β-$\widetilde{\gamma}$g.closed.*

Proof. (1) Let (F, E) be soft $\widetilde{\gamma}$ open and soft $\widetilde{\gamma}$g.closed. As $(F, E) \widetilde{\subseteq} (F, E)$, we have $cl(F, E) \widetilde{\subseteq} (F, E)$ and since $(F, E) \widetilde{\subseteq} cl(F, E)$, therefore $cl(F, E) = (F, E)$. Thus, (F, E) is soft closed.
The proofs of (2), (3), (4), (5) and (6) are similar to (1). □

Theorem 3.7. *The intersection of a soft $\widetilde{\gamma}$g.closed set and a soft $\widetilde{\gamma}$ closed set is always soft $\widetilde{\gamma}$g.closed.*

Proof. Let (A,E) be soft $\widetilde{\gamma}$g.closed and (F,E) be soft $\widetilde{\gamma}$ closed. Assume that (U,E) is a soft $\widetilde{\gamma}$ open set such that $(A,E)\ \widetilde{\cap}\ (F,E)\ \widetilde{\subseteq}\ (U,E)$, set $(G,E)=(F,E)^c$. Then, $(A,E)\ \widetilde{\subseteq}\ (U,E)\ \widetilde{\cup}\ (G,E)$, and since (G,E) is soft $\widetilde{\gamma}$ open, then $(U,E)\ \widetilde{\cup}\ (G,E)$ is soft $\widetilde{\gamma}$ open. Since (A,E) is soft $\widetilde{\gamma}$g.closed, then $cl(A,E)\ \widetilde{\subseteq}\ (U,E)\ \widetilde{\cup}\ (G,E)$. Now, $cl((A,E)\ \widetilde{\cap}\ (F,E))\ \widetilde{\subseteq}\ cl(A,E)\ \widetilde{\cap}\ cl(F,E) = cl(A,E)\ \widetilde{\cap}\ (F,E)\ \widetilde{\subseteq}\ ((U,E)\ \widetilde{\cup}\ (G,E))\ \widetilde{\cap}\ (F,E) = ((U,E)\ \widetilde{\cap}\ (F,E))\ \widetilde{\cup}\ ((G,E)\ \widetilde{\cap}\ (F,E)) = ((U,E)\ \widetilde{\cap}\ (F,E))\ \widetilde{\cup}\widetilde{\Phi}\widetilde{\subseteq}\ (U,E)$. Thus, $(A,E)\ \widetilde{\cap}\ (F,E)$ is soft $\widetilde{\gamma}$g.closed. □

Theorem 3.8. *Let (F,E) be any soft $\widetilde{\gamma}$ closed set and (A,E) be any soft set in a soft topological space $(\widetilde{X},\widetilde{\tau},E)$. Then,*

(1) *$(A,E)\ \widetilde{\cap}\ (F,E)$ is soft α-$\widetilde{\gamma}$g.closed if (A,E) is α-$\widetilde{\gamma}$g.closed.*
(2) *$(A,E)\ \widetilde{\cap}\ (F,E)$ is soft p-$\widetilde{\gamma}$g.closed if (A,E) is p-$\widetilde{\gamma}$g.closed.*
(3) *$(A,E)\ \widetilde{\cap}\ (F,E)$ is soft s-$\widetilde{\gamma}$g.closed if (A,E) is s-$\widetilde{\gamma}$g.closed.*
(4) *$(A,E)\ \widetilde{\cap}\ (F,E)$ is soft b-$\widetilde{\gamma}$g.closed if (A,E) is b-$\widetilde{\gamma}$g.closed.*
(5) *$(A,E)\ \widetilde{\cap}\ (F,E)$ is soft β-$\widetilde{\gamma}$g.closed if (A,E) is β-$\widetilde{\gamma}$g.closed.*

Proof. The proof is similar to the proof of Theorem 3.7. □

Theorem 3.9. *Let $(\widetilde{X},\widetilde{\tau},E)$ be any soft topological space. Then,*

(1) *the finite union of soft $\widetilde{\gamma}$g.closed sets in $\widetilde{X}$ is always a soft $\widetilde{\gamma}$g.closed set.*
(2) *the finite union of soft α-$\widetilde{\gamma}$g.closed sets in $\widetilde{X}$ is always a soft α-$\widetilde{\gamma}$g.closed set.*

Proof. (1) Let (A,E) and (B,E) be two soft $\widetilde{\gamma}$g.closed sets and $(A,E)\ \widetilde{\cup}\ (B,E)\ \widetilde{\subseteq}\ (U,E)$, where (U,E) is soft $\widetilde{\gamma}$ open. Since (A,E) and (B,E) are soft $\widetilde{\gamma}$g.closed, then $cl(A,E)\ \widetilde{\subseteq}\ (U,E)$ and $cl(B,E)\ \widetilde{\subseteq}\ (U,E)$ implies $cl(A,E)\ \widetilde{\cup}\ cl(B,E)\ \widetilde{\subseteq}\ (U,E)$. Since $cl(A,E)\ \widetilde{\cup}\ cl(B,E) = cl((A,E)\ \widetilde{\cup}\ (B,E))$, therefore $cl((A,E)\ \widetilde{\cup}\ (B,E))\ \widetilde{\subseteq}\ (U,E)$. Thus, $(A,E)\ \widetilde{\cup}\ (B,E)$ is soft $\widetilde{\gamma}$g.closed.
(2) The proof is similar to (1).

□

Remark 3.5. *The union of two soft p-$\widetilde{\gamma}$g.closed sets need not be soft p-$\widetilde{\gamma}$g.closed in general. It is shown by the following example.*

Example 3.6. *From Example 3.2, if $(A,E)=\{(e,\{p_2\})\}$ and $(B,E)=\{(e,\{p_4\})\}$, then (A,E) and (B,E) are soft p-$\widetilde{\gamma}$g.closed but $(A,E)\ \widetilde{\cup}\ (B,E)=\{(e,\{p_2,p_4\})\}$ is not soft p-$\widetilde{\gamma}$g.closed.*

Remark 3.6. *The union of two soft s-$\widetilde{\gamma}$g.closed sets need not be soft s-$\widetilde{\gamma}$g.closed in general. It is shown by the following example.*

Example 3.7. *From Example 3.2, if $(A,E)=\{(e,\{p_1\})\}$ and $(B,E)=\{(e,\{p_2,p_4\})\}$, then (A,E) and (B,E) are soft s-$\widetilde{\gamma}$g.closed but $(A,E)\ \widetilde{\cup}\ (B,E)=\{(e,\{p_1,p_2,p_4\})\}$ is not soft s-$\widetilde{\gamma}$g.closed.*

Remark 3.7. *The union of two soft b-$\widetilde{\gamma}$g.closed sets need not be soft b-$\widetilde{\gamma}$g.closed in general. It is shown by the following example.*

Example 3.8. *From Example 3.2, if $(A, E) = \{(e, \{p_1\})\}$ and $(B, E) = \{(e, \{p_2\})\}$, then (A, E) and (B, E) are soft b-$\widetilde{\gamma}$g.closed but $(A, E) \widetilde{\cup} (B, E) = \{(e, \{p_1, p_2\})\}$ is not soft b-$\widetilde{\gamma}$g.closed.*

Remark 3.8. *The union of two soft β-$\widetilde{\gamma}$g.closed sets need not be soft β-$\widetilde{\gamma}$g.closed in general. It is shown by the following example.*

Example 3.9. *From Example 3.2, if $(A, E) = \{(e, \{p_4\})\}$ and $(B, E) = \{(e, \{p_1, p_2\})\}$, then (A, E) and (B, E) are soft β-$\widetilde{\gamma}$g.closed but $(A, E) \widetilde{\cup} (B, E) = \{(e, \{p_1, p_2, p_4\})\}$ is not soft β-$\widetilde{\gamma}$g.closed.*

Remark 3.9.
1. *The intersection of two soft $\widetilde{\gamma}$g.closed sets need not be soft $\widetilde{\gamma}$g.closed in general.*
2. *The intersection of two soft α-$\widetilde{\gamma}$g.closed sets need not be soft α-$\widetilde{\gamma}$g.closed in general.*
3. *The intersection of two soft p-$\widetilde{\gamma}$g.closed sets need not be soft p-$\widetilde{\gamma}$g.closed in general.*

It is shown by the following example.

Example 3.10. *From Example 3.2, we define an operation $\widetilde{\gamma}$ on $\widetilde{\tau}$ by*

$$(A, E)^{\widetilde{\gamma}} = \begin{cases} (A, E) & \text{if } (A, E) = (F_1, E) \\ \widetilde{X} & \text{otherwise.} \end{cases}$$

Then, $\widetilde{\tau}_{\widetilde{\gamma}} = \{\widetilde{X}, \widetilde{\Phi}, (F_1, E)\}$. Let $(A, E) = \{(e, \{p_1, p_3\})\}$ and $(B, E) = \{(e, \{p_1, p_2\})\}$, then (A, E) and (B, E) are soft $\widetilde{\gamma}$g.closed (resp. soft α-$\widetilde{\gamma}$g.closed and soft p-$\widetilde{\gamma}$g.closed) but $(A, E) \widetilde{\cap} (B, E) = (F_1, E)$ is not soft $\widetilde{\gamma}$g.closed (resp. soft α-$\widetilde{\gamma}$g.closed and soft p-$\widetilde{\gamma}$g.closed).

Remark 3.10.
1. *The intersection of two soft s-$\widetilde{\gamma}$g.closed sets need not be soft s-$\widetilde{\gamma}$g.closed in general.*
2. *The intersection of two soft b-$\widetilde{\gamma}$g.closed sets need not be soft b-$\widetilde{\gamma}$g.closed in general.*
3. *The intersection of two soft β-$\widetilde{\gamma}$g.closed sets need not be soft β-$\widetilde{\gamma}$g.closed in general.*

It is shown by the following example.

Example 3.11. *Let $X = \{p_1, p_2, p_3\}$, $E = \{e\}$ and $\widetilde{\tau} = \{\widetilde{X}, \widetilde{\Phi}, (F, E)\}$ be a soft topological space over X, where $(F, E) = \{(e, \{p_1\})\}$. Define an operation $\widetilde{\gamma}$ on $\widetilde{\tau}$ by $(A, E)^{\widetilde{\gamma}} = (A, E)$ for all $(A, E) \widetilde{\in} \widetilde{\tau}$. Then, $\widetilde{\tau}_{\widetilde{\gamma}} = \widetilde{\tau}$. Let $(A, E) = \{(e, \{p_1, p_2\})\}$ and $(B, E) = \{(e, \{p_1, p_3\})\}$, then (A, E) and (B, E) are soft s-$\widetilde{\gamma}$g.closed (resp. soft b-$\widetilde{\gamma}$g.closed and soft β-$\widetilde{\gamma}$g.closed) but $(A, E) \widetilde{\cap} (B, E) = (F, E)$ is not soft s-$\widetilde{\gamma}$g.closed (resp. soft b-$\widetilde{\gamma}$g.closed and soft β-$\widetilde{\gamma}$g.closed).*

Definition 3.2. [8] *A soft set (F, E) over X is said to be a soft point, if there is exactly one $e \in E$ such that $F(e) = \{x\}$ for some $x \in X$ and $F(e^{'}) = \phi$ for all $e^{'} \in E \setminus \{e\}$. It will be denoted by F_e^x. The soft point F_e^x is said to be in the soft set (G, E) denoted by $F_e^x \widetilde{\in} (G, E)$, if the element $e \in E$ and $F(e) \subseteq G(e)$.*

Theorem 3.10. *Let $(\widetilde{X}, \widetilde{\tau}, E)$ be a soft topological space over X. Then, for each $x \in X$,*

(1) *either F_e^x is soft $\widetilde{\gamma}$ closed or F_e^{xc} is soft $\widetilde{\gamma}g$.closed in $\widetilde{X}$.*
(2) *either F_e^x is soft $\widetilde{\gamma}$ closed or F_e^{xc} is soft α-$\widetilde{\gamma}g$.closed in $\widetilde{X}$.*
(3) *either F_e^x is soft $\widetilde{\gamma}$ closed or F_e^{xc} is soft p-$\widetilde{\gamma}g$.closed in $\widetilde{X}$.*
(4) *either F_e^x is soft $\widetilde{\gamma}$ closed or F_e^{xc} is soft s-$\widetilde{\gamma}g$.closed in $\widetilde{X}$.*
(5) *either F_e^x is soft $\widetilde{\gamma}$ closed or F_e^{xc} is soft b-$\widetilde{\gamma}g$.closed in $\widetilde{X}$.*
(6) *either F_e^x is soft $\widetilde{\gamma}$ closed or F_e^{xc} is soft β-$\widetilde{\gamma}g$.closed in $\widetilde{X}$.*

Proof. (1) Suppose that F_e^x is not soft $\widetilde{\gamma}$ closed, then F_e^{xc} is not soft $\widetilde{\gamma}$ open. Let (U, E) be any soft $\widetilde{\gamma}$ open set such that $F_e^{xc} \widetilde{\subseteq} (U, E)$ implies $(U, E) = \widetilde{X}$. Therefore, $clF_e^{xc} \widetilde{\subseteq} (U, E)$. Thus, F_e^{xc} is soft $\widetilde{\gamma}$g.closed.
The proofs of (2), (3), (4), (5) and (6) are similar to (1). $\square$

4. Soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$ space

Definition 4.1. *A soft topological space $(\widetilde{X}, \widetilde{\tau}, E)$ is called soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$, if every soft $\widetilde{\gamma}g$.closed set is soft $\widetilde{\gamma}$ closed.*

Example 4.1. *Let $X = \{p_1, p_2, p_3\}$, $E = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\widetilde{X}, \widetilde{\Phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E)\}$ be a soft topological space over X, where $\widetilde{X} = \{(e_1, X), (e_2, X)\}, \widetilde{\Phi} = \{(e_1, \phi), (e_2, \phi)\}$, $(F_1, E) = \{(e_1, \{p_1\}), (e_2, \{p_1\})\}, (F_2, E) = \{(e_1, \{p_2\}), (e_2, \{p_2\})\}$, $(F_3, E) = \{(e_1, \{p_3\}), (e_2, \{p_3\})\}, (F_4, E) = \{(e_1, \{p_1, p_2\}), (e_2, \{p_1, p_2\})\}$, $(F_5, E) = \{(e_1, \{p_1, p_3\}), (e_2, \{p_1, p_3\})\}$ and $(F_6, E) = \{(e_1, \{p_2, p_3\}), (e_2, \{p_2, p_3\})\}$ are soft open sets over X. Define an operation $\widetilde{\gamma}$ on $\widetilde{\tau}$ by*

$$(A, E)^{\widetilde{\gamma}} = \begin{cases} (A, E) & \text{if } (A, E) = (F_2, E) \text{ or } (F_3, E) \\ \widetilde{X} & \text{otherwise.} \end{cases}$$

Then, $\widetilde{\tau}_{\widetilde{\gamma}} = \{\widetilde{X}, \widetilde{\Phi}, (F_2, E), (F_3, E), (F_6, E)\}$ and $(\widetilde{X}, \widetilde{\tau}, E)$ is not soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$, because $\{(e_1, \{p_2\}), (e_2, \{p_3\})\}$ is a soft $\widetilde{\gamma}g$.closed set but it is not soft $\widetilde{\gamma}$ closed.

Example 4.2. *Let $X = \{p_1, p_2\}$, $E = \{e_1, e_2\}$ and $\widetilde{\tau} = \{\widetilde{X}, \widetilde{\Phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (F_5, E), (F_6, E), (F_7, E), (F_8, E), (F_9, E), (F_{10}, E), (F_{11}, E), (F_{12}, E)\}$ be a soft topological space over X, where $(F_1, E) = \{(e_1, \phi), (e_2, \{p_1\})\}, (F_2, E) = \{(e_1, \phi), (e_2, \{p_2\})\}$, $(F_3, E) = \{(e_1, \{p_1\}), (e_2, \phi)\}, (F_4, E) = \{(e_1, \{p_2\}), (e_2, \phi)\}$, $(F_5, E) = \{(e_1, \{p_1\}), (e_2, \{p_2\})\}, (F_6, E) = \{(e_1, \{p_2\}), (e_2, \{p_1\})\}$, $(F_7, E) = \{(e_1, \{p_1, p_2\}), (e_2, \{p_1\})\}, (F_8, E) = \{(e_1, \{p_1, p_2\}), (e_2, \{p_2\})\}$, $(F_9, E) = \{(e_1, \{p_1\}), (e_2, \{p_1, p_2\})\}, (F_{10}, E) = \{(e_1, \{p_2\}), (e_2, \{p_1, p_2\})\}$, $(F_{11}, E) = \{(e_1, \phi), (e_2, \{p_1, p_2\})\}$ and $(F_{12}, E) = \{(e_1, \{p_1, p_2\}), (e_2, \phi)\}$ are soft open sets over X. Define an operation $\widetilde{\gamma}$ on $\widetilde{\tau}$ by $(A, E)^{\widetilde{\gamma}} = (A, E)$ for all $(A, E) \widetilde{\in} \widetilde{\tau}$. Then, $(\widetilde{X}, \widetilde{\tau}, E)$ is soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$.*

Theorem 4.1. *Let $(\widetilde{X}, \widetilde{\tau}, E)$ be a soft topological space. If $(\widetilde{X}, \widetilde{\tau}, E)$ is soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$, then F_e^x is soft $\widetilde{\gamma}$ closed or soft $\widetilde{\gamma}$ open for each $x \in X$.*

Proof. Suppose that F_e^x is not soft $\widetilde{\gamma}$ closed. By Theorem 3.10 (1), F_e^{xc} is soft $\widetilde{\gamma}$g.closed. Since $(\widetilde{X}, \widetilde{\tau}, E)$ is soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$, then F_e^{xc} is soft $\widetilde{\gamma}$ closed and hence F_e^x is soft $\widetilde{\gamma}$ open. □

Theorem 4.2. *Let $(\widetilde{X}, \widetilde{\tau}, E)$ be a soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$ topological space. If $\widetilde{\cap}\ \{(L, E) \in \widetilde{\tau}_{\widetilde{\gamma}}\colon F_e^x \widetilde{\subseteq} (L, E)\} \neq \widetilde{X}$ for $x \in X$, then there exist two soft $\widetilde{\gamma}$ open sets (L_1, E) and (L_2, E) such that $(L_1, E) \neq \widetilde{X}$ and $F_e^x = (L_1, E) \widetilde{\setminus} (L_2, E)$.*

Proof. Let $\widetilde{\cap}\ \{(L, E) \in \widetilde{\tau}_{\widetilde{\gamma}}\colon F_e^x \widetilde{\subseteq} (L, E)\} \neq \widetilde{X}$ for $x \in X$, then there exists soft $\widetilde{\gamma}$ open (L, E) such that $F_e^x \widetilde{\subseteq} (L, E)$ and $(L, E) \neq \widetilde{X}$. By Theorem 4.1, we have F_e^x is soft $\widetilde{\gamma}$ open or soft $\widetilde{\gamma}$ closed. If F_e^x is soft $\widetilde{\gamma}$ open, then $F_e^x = F_e^x \widetilde{\setminus} \widetilde{X}$. But if F_e^x is soft $\widetilde{\gamma}$ closed, then F_e^{xc} is soft $\widetilde{\gamma}$ open. Put $(L_1, E) = (L, E)$ and $(L_2, E) = \widetilde{X} \widetilde{\cap}\ F_e^{xc}$. Then, $F_e^x = (L_1, E) \widetilde{\setminus} (L_2, E)$ and $(L_1, E) \neq \widetilde{X}$. □

Definition 4.2. [23] *A soft mapping $f_{pu} : (\widetilde{X}, \widetilde{\tau}, E) \to (\widetilde{Y}, \widetilde{\tau_1}, E_1)$ is called soft closed, if $f_{pu}((A, E))$ is soft closed over Y for each soft closed set (A, E) over X.*

Theorem 4.3. [22, 23] *Let $f_{pu} : (\widetilde{X}, \widetilde{\tau}, E) \to (\widetilde{Y}, \widetilde{\tau_1}, E_1)$ be a soft mapping. Then, the following conditions are equivalent:*

(1) *f_{pu} is soft continuous.*
(2) *$f_{pu}^{-1}(B, E_1)$ is a soft open set over X, for each soft open set (B, E_1) over Y.*
(3) *$f_{pu}^{-1}(K, E_1)$ is a soft closed set over X, for each soft closed set (K, E_1) over Y.*
(4) *$f_{pu}(cl(A, E)) \widetilde{\subseteq} cl(f_{pu}((A, E)))$, for each soft set (A, E) over X.*

Theorem 4.4. *Let $f_{pu} : (\widetilde{X}, \widetilde{\tau}, E) \to (\widetilde{Y}, \widetilde{\tau_1}, E_1)$ be soft continuous and soft closed. Then:*

(1) *$f_{pu}((A, E))$ is soft $\widetilde{\delta}$g.closed in $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$, for every soft g.closed (A, E) in $(\widetilde{X}, \widetilde{\tau}, E)$. Where $\widetilde{\delta}$ is an operation on the soft topology $\widetilde{\tau_1}$.*
(2) *$f_{pu}^{-1}((B, E_1))$ is soft $\widetilde{\gamma}$g.closed in $(\widetilde{X}, \widetilde{\tau}, E)$, for every soft g.closed (B, E_1) in $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$.*

Proof. (1) Let (K, E_1) be any soft $\widetilde{\delta}$ open set in $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$ such that f_{pu}((A,E)) $\widetilde{\subseteq}$ (K, E_1). Since f_{pu} is soft continuous and every soft $\widetilde{\delta}$ open set is soft open, then $f_{pu}^{-1}((K, E_1))$ is soft open in $(\widetilde{X}, \widetilde{\tau}, E)$. Since (A, E) is soft g.closed and $(A, E) \widetilde{\subseteq} f_{pu}^{-1}((K, E_1))$, then $cl(A, E) \widetilde{\subseteq} f_{pu}^{-1}((K, E_1))$, and hence $f_{pu}(cl(A, E)) \widetilde{\subseteq} (K, E_1)$. Since f_{pu} is soft closed, so $f_{pu}(cl(A, E))$ is soft closed in $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$ and hence $cl(f_{pu}(A, E)) \widetilde{\subseteq} cl(f_{pu}(cl(A, E))) = f_{pu}(cl(A, E)) \widetilde{\subseteq} (K, E_1)$. This implies $f_{pu}((A, E))$ is soft $\widetilde{\delta}$g.closed.

(2) Let (L, E) be a soft $\widetilde{\gamma}$ open set in $(\widetilde{X}, \widetilde{\tau}, E)$ such that $f_{pu}^{-1}((B, E_1)) \widetilde{\subseteq} (L, E)$. Let $(F, E) = cl(f_{pu}^{-1}(B, E_1)) \widetilde{\cap} (L, E)^c$, then (F, E) is soft closed. Since f_{pu} is soft closed, this implies $f_{pu}((F, E))$ is soft closed in $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$. By Theorem 4.3, we have $f_{pu}((F, E)) \subseteq f_{pu}(cl(f_{pu}^{-1}(B, E_1))) \widetilde{\cap} f_{pu}((L, E)^c) \widetilde{\subseteq} cl(f_{pu}(f_{pu}^{-1}(B, E_1))) \widetilde{\cap} f_{pu}((f_{pu}^{-1}(B, E_1))^c) \widetilde{\subseteq} cl(B, E_1) \widetilde{\cap} (B, E_1)^c$ and by Theorem 2.2, $f_{pu}((F, E)) = \widetilde{\Phi}$

and hence $(F, E) \widetilde{=} \widetilde{\Phi}$. Thus, $cl(f_{pu}^{-1}(B, E_1)) \widetilde{\subseteq} (L, E)$ and therefore $f_{pu}^{-1}((B, E_1))$ is soft $\widetilde{\gamma}$g.closed.

□

Definition 4.3. [14] *A soft topological space $(\widetilde{X}, \widetilde{\tau}, E)$ is called soft $T_{\frac{1}{2}}$ if every soft g-closed set is soft closed.*

Theorem 4.5. *Let $f_{pu} : (\widetilde{X}, \widetilde{\tau}, E) \to (\widetilde{Y}, \widetilde{\tau_1}, E_1)$ be soft continuous and soft closed. Then:*

(1) *If f_{pu} is injective and $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$ is soft $\widetilde{\delta}$-$T_{\frac{1}{2}}$, then $(\widetilde{X}, \widetilde{\tau}, E)$ is soft $T_{\frac{1}{2}}$.*

(2) *If f_{pu} is surjective and $(\widetilde{X}, \widetilde{\tau}, E)$ is soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$, then $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$ is soft $T_{\frac{1}{2}}$.*

Proof. (1) Let (A, E) be soft g.closed in $(\widetilde{X}, \widetilde{\tau}, E)$. By Theorem 4.4 (1), $f_{pu}(A, E)$ is soft $\widetilde{\delta}$g.closed in $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$. Since $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$ is soft $\widetilde{\delta}$-$T_{\frac{1}{2}}$, this implies that $f_{pu}(A, E)$ is soft $\widetilde{\delta}$ closed and hence it is soft closed. By Theorem 4.3, we have $(A, E) = f_{pu}^{-1}(f_{pu}(A, E))$ is soft closed in $(\widetilde{X}, \widetilde{\tau}, E)$. Thus, $(\widetilde{X}, \widetilde{\tau}, E)$ is soft $T_{\frac{1}{2}}$.

(2) Let (B, E_1) be soft g.closed in $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$. By Theorem 4.4 (2), $f_{pu}^{-1}(B, E_1)$ is soft $\widetilde{\gamma}$g.closed in $(\widetilde{X}, \widetilde{\tau}, E)$. Since $(\widetilde{X}, \widetilde{\tau}, E)$ is soft $\widetilde{\gamma}$-$T_{\frac{1}{2}}$, then $f_{pu}^{-1}(B, E_1)$ is soft $\widetilde{\gamma}$ closed and hence it is soft closed. Also, since f_{pu} is both surjective and soft closed, so $f_{pu}(f_{pu}^{-1}(B, E_1)) = (B, E_1)$ is soft closed in $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$. Thus, $(\widetilde{Y}, \widetilde{\tau_1}, E_1)$ is soft $T_{\frac{1}{2}}$.

□

References

[1] Al-Salem, S. M., (2014), Soft regular generalized b-closed sets in soft topological spaces, Journal of Linear and Topological Algebra, 3 (4), pp. 195-204.

[2] Arockiarani, I., (2013), Generalized soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces, International Journal of Mathematical Archive, 4 (2), pp. 17-23.

[3] Arockiarani, I. and Albinaa, T. A., (2014), Soft generalized preclosed sets and space, Proceedings of ICMSCA, pp. 183-187.

[4] Arockiarani, I. and Arokialancy, A., (2013), Generalized soft gβ-closed sets and soft gsβ-closed sets in soft topological spaces, Int. J. Math. Arch., 4 (2), pp. 1-7.

[5] Akdag, M. and Ozkan, A., (2014), Soft α-open sets and soft α-continuous functions, Abstract and Applied Analysis, pp. 1-7.

[6] Akdag, M. and Ozkan, A., (2014), Soft b-open sets and soft b-continuous functions, Math Sci, 8 (2), pp. 1-9.

[7] Chen, B., (2013), Soft semi-open sets and related properties in soft topological spaces, Appl. Math. Inf. Sci., 7 (1), pp. 287-294.

[8] Das, S. and Samanta, S. K., (2013), Soft metric, Annals of Fuzzy Mathematics and Informatics, 6 (1), pp. 77-94.

[9] Devika, A. and Elvina, L., (2015), Soft g*-closed sets in soft topological spaces, International Journal of Mathematics Trends and Technology, 18 (1), pp. 32-39.

[10] Guzel, Z. E., Yksel, S. and Tozlu, N., (2014), On soft generalized preregular closed and open sets in soft topological spaces, Applied Mathematical Sciences, 8 (158), pp. 7875-7884.

[11] Ibrahim, H. Z., (2013), γgb-closed sets in topological spaces, Journal of Advanced Studies in Topology, 4 (1), pp. 73-79.

[12] Janaki, C. and Savithiri, D., (2014), Soft rg-closed sets in soft topological spaces, International Journal of Engineering Sciences & Research Technology, 3 (4), pp. 2084-2091.

[13] Kalavathia, A., (2017), Studies on generalizations of soft closed sets and their operation approaches in soft topological spaces, Doctor of philosophy, University of Chennai.
[14] Kannan, K., (2012), Soft generalized closed sets in soft topological spaces, Journal of Theoretical and Applied Information Technology, 37 (1), pp. 17-21.
[15] Kannan, K. and Rajalakshmi, D., (2015), Soft semi star generalized closed sets, Malaysian Journal of Mathematical Sciences, 9 (1), pp. 77-89.
[16] Maji, P. K., Biswas, R. and Roy, R., (2003), Soft set theory, Comput. Math. Appl., 45, pp. 555-562.
[17] Mohana, K., Anitha, S. and Radhika, V., (2017), On soft GSR-closed sets in soft topological spaces, International Journal of Engineering Science and Computing, 7 (1), pp. 4116-4120.
[18] Molodtsov, D., (1999), Soft set theory first results, Comput. Math. Appl., 37, pp. 19-31.
[19] Palanisamy, M., Sentamilselvi, M. and Sitheeswari, A, (2019), On soft generalized βb-closed sets in soft topological spaces, Young Scientist-Tomorrow's Science Begins Today, 3 (1), pp. 15-25.
[20] Shabir, M. and Naz, M., (2011), On soft topological spaces, Computers and Mathematics with Applications, 6, pp. 1786-1799.
[21] Subhashinin, J. and Sekar, C., (2014), Soft pre generalized - closed sets in a soft topological space, International Journal of Engineering Trends and Technology, 12 (7), pp. 356-364.
[22] Zorlutuna, I., Akdag, Min, M. W. K. and Atmaca, S., (2012), Remarks on soft topological spaces, Annals of Fuzzy Mathematics and Informatics, 3 (2), pp. 171-185.
[23] Zorlutuna, I. and Cakir H., (2015), On continuity of soft mappings, Appl. Math. Inf. Sci., 9 (1), pp. 403-409.

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