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HIGHLY TOTAL PRIME LABELING FOR SOME DUPLICATE GRAPH

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Abstract. Let $G = (V, E)$ be a graph with p vertices and q edges. A bijection $f : V \cup E \to \{1, 2, \cdots, p+q\}$ is said to be a highly total prime labeling if (i) for each edge $e = uv$, the labels assigned to u and v are relatively prime (ii) any pair of adjacent edges receives relatively prime labeling. A graph which admits highly total prime labeling is called highly total prime graph. In this paper we investigate the existence of highly total prime labeling of some duplicate graphs related to path P_n, cycle C_n and star S_n.

Keywords: Prime labeling, vertex labeling, total prime labeling, highly total prime labeling, highly total prime graph.

AMS Subject Classification: 05C78.

1. Introduction

Graph theory has obvious utility in its applications in science. However, when viewed apart from these applications, it yields beautiful mathematical gems, and can be used as a lens to study other area of mathematics. We use graph theory and specifically graph labeling as a lens to study prime numbers. Here, we consider only simple, finite, undirected and non-trivial graph $G = (V(G), E(G))$ with vertex set $V(G)$ and edge set $E(G)$. The set of vertices adjacent to a vertex u of G is denoted by $N(u)$. For notations and terminology we refer to Bondy and Murthy[1]. For latest dynamical survey on graph labeling we refer to Gallian [3].

Two integers a and b are said to be relatively prime if there greatest common divisor is 1. The notion of the prime labeling was introduced by Tout, Dabboucy and Howalla in 1982 [11]. Deretsky, Lee and Mitchem considered the labeling of edges rather than vertices and proposed vertex prime labeling in 1991 [2]. In vertex prime labeling, the edges are labeled with distinct positive integer less than or equal to the number of edges in the graph such that for each vertex degree of atleast 2 the greatest common divisor of the labels of its incident edges is 1.

In 2012, M. Ravi (a) Ramasubramanian and R. Kala combined the ideas of prime labeling and vertex labeling into total prime labeling [6]. In total prime labeling, the vertices and edges are labeled with distinct positive integers less than or equal to the sum of the

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number of vertices and edges in the graph such that two conditions are there, i.e., The labels of any two adjacent vertices are relatively prime and the greatest common divisor of the labels of all edges incident the same vertex is 1. In [5] S. Meena and A. Ezhil have investigate the total prime labeling of some graph.

R.B. Gnanajothi and S. Suganya expanded on this work and introduced highly total prime labeling in 2016 [4]. Highly total prime lableling adds the restriction to total prime labeling that for each vertex of degree atleast 2, any two edges that are incident to the same vertex have labels that are relatively prime. Note that highly total prime graphs are the same total prime graphs except that instead of requiring simply that the labels of the edges incident to the same vertex have a greatest common divisor of 1, highly total prime graphs require the stronger conditions that the edges be pairwise relatively prime. Thus all highly total prime graphs are total prime, but the converse is not true. Robert scholle [7] investigated the consecutive prime and highly total prime labeling in graphs.

The concept of duplicate graph was introduced by E. Sampathkumar and he prove many results on it [8]. K. Thirusangu, P.P. Ulaganathan and B. Selvam have proved that the duplicate graph of path graph P_n is cordial [9]. K. Thirusangu, P.P. Ulaganathan and P. Vijayakumar proved that the duplicate graph of ladder graph is cordial, total cordial and prime cordial [10]. Motivated by this study, in this paper, we prove that the duplicate graph of path P_n, the duplicate graph of cycle C_n, the duplicate graph of disjoint union of path and cycle $P_k \cup C_n$ are highly total prime labeling. We also show that the splitting graph of a star and the duplicate graph of splitting graph of a star are do not have a highly total prime labeling.

2. Preliminaries

Definition 2.1. *A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (or edges), then the labeling called a vertex labeling (or an edge labeling).*

Definition 2.2. *Let $G = (V, E)$ be a graph with p vertices and q edges. A bijection $f : V \cup E \to \{1, 2, \cdots, p + q\}$ is said to be a highly total prime labeling if*
(i) for each degree $e = uv$, the labels assigned to u and v are relatively prime.
(ii) any pair of adjacent edges receives relatively prime labeling.
A graph which admits highly total prime labeling is called Highly Total Prime Graph.

Definition 2.3. *A set of points in G is independent if no two of them are adjacent. The largest number of points in such a set is called the point independence number of G and is denoted by $\beta_0(G)$ or β_0.*

Definition 2.4. *An independent set of lines of G has no two of its lines adjacent and the maximum cardinality of scuh a set is the line independence number $\beta_1(G)$ or β_1.*

Definition 2.5. *Let $G = (V, E)$ be a simple graph. A duplicate graph of G is $DG = (V_1, E_1)$ where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and $f : V \to V'$ is bijective (for $v \in V$, we write $f(v) = v'$) and the edge set E_1 of DG is defined as: The edge ab is in E if and only if both ab' and $a'b$ are edges in E_1.*

Definition 2.6. *The union of two graphs G_1 and G_2 is a graph $G_1 \cup G_2$ with $V(G_1 \cup G_2) = V(G_1) \cup V(G_2)$ and $E(G_1 \cup G_2) = E(G_1) \cup E(G_2)$.*

Definition 2.7. *For a graph G, the splitting graph is obtained by adding to each vertex v, a new vertex v' so that v' is adjacent to every vertex that is adjacent to v in G.*

Remark 2.1. *If a connected graph G is highly total prime, then $\beta_0 + \beta_1 \geq \left[\dfrac{p+q}{2}\right]$ where β_0 is the point independence number of G, β_1 is the line independence number of G, p is the number of vertices of G and q is the number of edges of G.*

3. Theorems

Theorem 3.1. *The duplicate graph of path P_n is highly total prime where $n \geq 2$.*

Proof. Let P_n denote the path of length n.
Let $V(P_n) = \{v_i/1 \leq i \leq n\}$; $E(P_n) = \{v_i v_{i+1}/1 \leq i \leq n-1\}$.
Let G be the duplicate graph of path P_n. Now $v_1, v_2, \cdots, v_n, v_1^{'}, v_2^{'}, \cdots, v_n^{'}$ be the new set of vertices and $e_1, e_2, \cdots, e_{n-1}, e_1^{'}, e_2^{'}, \cdots, e_{n-1}^{'}$ be the new set of edges of the duplicate graph of path P_n.
Let $V(G) = \{v_i, v_i^{'}/1 \leq i \leq n\}; E(G) = \{v_i, v_{i+1}^{'}/1 \leq i \leq n-1\} \cup \{v_i^{'} v_{i+1}/1 \leq i \leq n-1\}$.
Then $p = 2n$ and $q = 2(n-1)$.
To define labeling $f : V(G) \cup E(G) \to \{1, 2, \cdots, p+q\}$ as

$$f(v_i) = i + 2(n-1) \quad \text{if } i = \begin{cases} 1, 3, 5, \cdots, (n-1) \\ \text{for n is even} \\ \text{(or)} 1, 3, 5, \cdots, n \end{cases}$$

$$f(v_i) = i + 3n - 2 \quad \text{if } i = \begin{cases} 2, 4, 6, \cdots, n \\ \text{for n is even} \\ \text{(or)} 2, 4, 6, \cdots, n-1 \end{cases}$$

$$f(v_i^{'}) = i + 3n - 2 \quad \text{if } i = \begin{cases} 1, 3, 5, \cdots, (n-1) \\ \text{for n is even} \\ \text{(or)} 1, 3, 5, \cdots, n \end{cases}$$

$$f(v_i^{'}) = i + 2(n-1) \quad \text{if } i = \begin{cases} 2, 4, 6, \cdots, n \\ \text{for n is even} \\ \text{(or)} 2, 4, 6, \cdots, n-1 \end{cases}$$

$$f(v_i v_{i+1}^{'}) = i \quad \text{if } i = \begin{cases} 1, 3, 5, \cdots, (n-1) \\ \text{for n is even} \\ \text{(or)} 1, 3, 5, \cdots, n-2 \end{cases}$$

$$f(v_i v_{i+1}^{'}) = n + i - 1 \quad \text{if } i = \begin{cases} 2, 4, 6, \cdots, n-2 \\ \text{for n is even} \\ \text{(or)} 2, 4, 6, \cdots, n-1 \end{cases}$$

$$f(v_i^{'} v_{i+1}) = n + i - 1 \quad \text{if } i = \begin{cases} 1, 3, 5, \cdots, (n-1) \\ \text{for n is even} \\ \text{(or)} 1, 3, 5, \cdots, n-2 \end{cases}$$

$$f(v_i^{'} v_{i+1}) = i \quad \text{if } i = \begin{cases} 2, 4, 6, \cdots, n-2 \\ \text{for n is even} \\ \text{(or)} 2, 4, 6, \cdots, n-1 \end{cases}$$

According to this pattern, the

$$gcd\{f(v_i), f(v'_{i+1})\} = gcd\{2(n-1)+i, 2(n-1)+i+1\} = 1 \quad \text{if } i = \begin{cases} 1, 3, 5, \cdots, (n-1) \\ \text{for n is even} \\ \text{(or)}1, 3, 5, \cdots, n-2 \end{cases}$$

$$gcd\{f(v_i), f(v'_{i+1})\} = gcd\{3n-2+i, 3n-1+i\} = 1 \quad \text{if } i = \begin{cases} 2, 4, 6, \cdots, n-2 \\ \text{for n is even} \\ \text{(or)}2, 4, 6, \cdots, n-1 \end{cases}$$

$$gcd\{f(v'_i), f(v_{i+1})\} = gcd\{3n-2+i, 3n-1+i\} = 1 \quad \text{if } i = \begin{cases} 1, 3, 5, \cdots, (n-1) \\ \text{for n is even} \\ \text{(or)}1, 3, 5, \cdots, n-2 \end{cases}$$

$$gcd\{f(v'_i), f(v_{i+1})\} = gcd\{2n-2+i, 2n-1+i\} = 1 \quad \text{if } i = \begin{cases} 2, 4, 6, \cdots, n-2 \\ \text{for n is even} \\ \text{(or)}2, 4, 6, \cdots, n-1 \end{cases}$$

Since all the above vertex labels are consecutive integers.

$$f^*(v_i) = gcd\{f(v_iv'_{i-1}), f(v_iv'_{i+1})\} = gcd(i-1, i) = 1 \quad \text{if } i = \begin{cases} 3, 5, 7, \cdots, (n-1) \\ \text{for n is even} \\ \text{(or)}3, 5, 7, \cdots, n-2 \end{cases}$$

$$\begin{aligned} f^*(v_i) &= gcd\{f(v_iv'_{i-1}), f(v_iv'_{i+1})\} \\ &= gcd(n+i-2, n+i-1) = 1 \quad \text{if } i = \begin{cases} 2, 4, 6, \cdots, n-2 \\ \text{for n is even} \\ \text{(or)}2, 4, 6, \cdots, n-1 \end{cases} \end{aligned}$$

$$\begin{aligned} f^*(v'_i) &= gcd\{f(v'_iv_{i+1}), f(v'_iv'_{i-1})\} \\ &= gcd(n+i-1, n+i-2) = 1 \quad \text{if } i = \begin{cases} 3, 5, 7, \cdots, (n-1) \\ \text{for n is even} \\ \text{(or)}3, 5, 7, \cdots, n-2 \end{cases} \end{aligned}$$

$$\begin{aligned} f^*(v'_i) &= gcd\{f(v'_iv_{i+1}), f(v'_iv_{i-1})\} \\ &= gcd(i, i-1) = 1 \quad \text{if } i = \begin{cases} 2, 4, 6, \cdots, n-2 \\ \text{for n is even} \\ \text{(or)}2, 4, 6, \cdots, n-1 \end{cases} \end{aligned}$$

Since all the above edge labels are consecutive integers.
Therefore for each edge $e = uv$, where u and v are relatively prime. And adjacent edges receive pairwise relatively prime labels.
Hence G is a highly total prime labeling. □

Theorem 3.2. *The duplicate graph of cycle $C_n, n \geq 3$ is a highly total prime graph.*

Proof. Let C_n denote the cycle graph of length n.
Let $V(C_n) = \{v_i/1 \leq i \leq n\}; E(C_n) = \{v_iv_{i+1}/1 \leq i \leq n-1\} \cup \{v_nv_1\}$.
Let G be the duplicate graph of cycle C_n. Let $v_1, v_2, \cdots, v_n, v'_1, v'_2, \cdots, v'_n$ and $e_1, e_2, \cdots, e_n, e'_1, e'_2, \cdots, e'_n$ respectively be the new vertices and edges of the duplicate graph of cycle C_n.
Then $V(G) = \{v_i, v'_i/1 \leq i \leq n\}, E(G) = \{v_iv'_{i+1}/1 \leq i \leq n-1\} \cup \{v_1v'_n\} \cup \{v'_iv_{i+1}/1 \leq i \leq n-1\} \cup \{v'_1v_n\}$.
Now $p = 2n, q = 2n$.

To define $f: V(G) \cup E(G) \to \{1, 2, c \ldots, p+q\}$ as
Case (i): For 'n' is odd:
$f(v_i) = 3n + i$ for $i = 2, 4, 6, \cdots, n-1$
$f(v_i) = 2n + i$ for $i = 3, 5, 7, \cdots, n$
$f(v_1) = 1$
$f(v_i^{'}) = 3n + i$ for $i = 1, 3, 5, \cdots, n$
$f(v_i^{'}) = 2n + i$ for $i = 2, 4, 6, \cdots, n-1$
Now in edge labeling,
$f(v_1 v_n^{'}) = 2n + 1$; $f(v_1^{'} v_n) = n + 1$
$f(v_i v_{i+1}^{'}) = i + 1$ for $i = 1, 3, 5, \cdots, n-2$
$f(v_i v_{i+1}^{'}) = n + i + 1$ for $i = 2, 4, 6, \cdots, n-1$
$f(v_i^{'} v_{i+1}) = n + i + 1$ for $i = 1, 3, 5, \cdots, n-2$
$f(v_i^{'} v_{i+1}) = i + 1$ for $i = 2, 4, 6, \cdots, n-1$
According to this pattern,
$gcd(f(v_1), f(v_n^{'})) = gcd(1, f(v_n^{'}) = 1$
$gcd(f(v_1), f(v_2^{'})) = gcd(1, f(v_2^{'}) = 1$
$gcd(f(v_1^{'}), f(v_n)) = gcd(3n+1, 3n) = 1$
$gcd(f(v_i), f(v_{i-1}^{'})) = gcd(3n+i, 3n+i-1) = 1$ for $i = 2, 4, 6, \cdots, n-1$
$gcd(f(v_i), f(v_{i+1}^{'})) = gcd(3n+i, 3n+i+1) = 1$ for $i = 2, 4, 6, \cdots, n-1$
$gcd(f(v_i), f(v_{i-1}^{'})) = gcd(2n+i, 2n+i-1) = 1$ for $i = 3, 5, 7, \cdots, n$
and $gcd(f(v_i), f(v_{i+1}^{'})) = gcd(2n+i, 2n+i+1) = 1$ for $i = 3, 5, 7, \cdots, n-2$
Since the above vertex labels are positive consecutive integers.

$$
\begin{aligned}
\text{Now,}\quad f^*(v_1) &= gcd\{f(v_1 v_2^{'}), f(v_1, v_n^{'})\} = gcd(2, 2n+1) = 1 \\
f^*(v_1^{'}) &= gcd\{f(v_1^{'} v_2), f(v_1^{'} v_n)\} = gcd(n+2, n+1) = 1 \\
f^*(v_i) &= gcd\{f(v_i v_{i-1}^{'}), f(v_i v_{i+1}^{'})\} \text{ for } i = 2, 4, 6, \cdots, n-1 \\
&= gcd(n+i, n+i+1) = 1 \text{ for } i = 2, 4, 6, \cdots, n-1 \\
f^*(v_i) &= gcd\{f(v_i v_{i-1}^{'}), f(v_i v_{i+1}^{'})\} \text{ for } i = 3, 5, 7, \cdots, n-2 \\
&= gcd(i, i+1) = 1 \text{ for } i = 3, 5, 7, \cdots, n-2 \\
f^*(v_i^{'}) &= gcd\{f(v_i^{'} v_{i-1}), f(v_i^{'} v_{i+1})\} \text{ for } i = 2, 4, 6, \cdots, n-1 \\
&= gcd(i, i+1) = 1 \text{ for } i = 2, 4, 6, \cdots, n-1 \\
f^*(v_i^{'}) &= gcd\{f(v_i^{'} v_{i-1}), f(v_i^{'} v_{i+1})\} \text{ for } i = 3, 5, 7, \cdots, n-2 \\
&= gcd(n+i, n+i+1) = 1 \; for \, i = 3, 5, 7, \cdots, n-2 \\
f^*(v_n) &= gcd\{f(v_n v_1^{'}), f(v_n v_{n-1}^{'})\} = gcd(n+1, n) = 1 \\
f^*(v_n^{'}) &= gcd\{f(v_n^{'} v_1), f(v_n^{'} v_{n-1})\} = gcd(2n+1, 2n) = 1
\end{aligned}
$$

Case (ii): For n is even.
If $n \not\equiv 2(\mod 3)$ then
$f(v_i) = 3n + i$ for $i = 3, 5, 7, \cdots, n-1$
$f(v_i^{'}) = 3n + i$ for $i = 2, 4, 6, \cdots, n$
If $n \equiv 2(\mod 3)$ then
$f(v_i) = n + i + 2$ for $i = 3, 5, 7, \cdots, n-1$
$f(v_i^{'}) = n + i + 2$ for $i = 2, 4, 6, \cdots, n$
For all n:
$f(v_1) = 1$; $f(v_1^{'}) = 2$
$f(v_i) = 2n + i + 1$ for $i = 2, 4, 6, \cdots, n$
$f(v_i^{'}) = 2n + i + 1$ for $i = 3, 5, 7, \cdots, n-1$
If $n \not\equiv 2(\mod 3)$ then
$f(v_1 v_n^{'}) = 2n + 2$

$f(v_i v'_{i+1}) = n + i + 2 \quad$ for $i = 3, 5, 7, \cdots, n-1$
$f(v'_i v_{i+1}) = n + i + 2 \quad$ for $i = 2, 4, 6, \cdots, n-2$
If $n \equiv 2(\mod 3)$ then
$f(v_1 v'_n) = 4n$
$f(v_i v'_{i+1}) = 3n + i \quad$ for $i = 3, 5, 7, \cdots, n-1$
$f(v'_i v_{i+1}) = 3n + i \quad$ for $i = 2, 4, 6, \cdots, n-2$
For all n
$f(v_1 v'_2) = 3; f(v'_1 v_n) = n + 3$
$f(v_i v'_{i+1}) = i + 3 \quad$ for $i = 2, 4, 6, \cdots, n-2$
$f(v'_i v_{i+1}) = i + 3 \quad$ for $i = 1, 3, 5, \cdots, n-1$
According to this pattern,
For $n \not\equiv 2(\mod 3)$ then
$gcd(f(v_i), f(v'_{i-1})) = gcd(3n + i, 3n + i - 1) = 1 \quad$ for $i = 3, 5, 7, \cdots, n-1$
$gcd(f(v_i), f(v'_{i+1})) = gcd(3n + i, 3n + i + 1) = 1 \quad$ for $i = 3, 5, 7, \cdots, n-1$
For $n \equiv 2(\mod 3)$
$gcd(f(v_i), f(v'_{i-1})) = gcd(n + i + 2, n + i + 1) = 1 \quad$ for $i = 3, 5, 7, \cdots, n-1$
$gcd(f(v_i), f(v'_{i+1})) = gcd(n + i + 2, n + i + 3) = 1 \quad$ for $i = 3, 5, 7, \cdots, n-1$
For all n,
$gcd(f(v_i), f(v'_{i-1})) = gcd(2n + i + 1, 2n + i) = 1 \quad$ for $i = 4, 6, 8, \cdots, n-2$
$gcd(f(v_i), f(v'_{i+1})) = gcd(2n + i + 1, 2n + i + 2) = 1 \quad$ for $i = 4, 6, 8, \cdots, n-2$
Since all the above vertex labels are consecutive positive integers.
$gcd(f(v_1), f(v'_2)) = gcd(1, f(v'_2) = 1$
$gcd(f(v_1), f(v'_n)) = gcd(1, f(v'_n) = 1$
$gcd(f(v'_1), f(v_2)) = gcd(2, 2n + 3) = 1$
$gcd(f(v'_1), f(v_n)) = gcd(2, 3n + 1) = 1$ here $3n + 1$ is odd.
$gcd(f(v_n), f(v'_{n-1})) = gcd(3n + 1, 3n) = 1$
$gcd(f(v_2), f(v'_3)) = gcd(2n + 3, 2n + 4) = 1$
For $n \not\equiv 2(\mod 3)$

$$
\begin{aligned}
f^*(v_1) &= gcd\{f(v_1 v'_2), f(v_1 v'_n)\} = gcd\{3, 2n + 2\} = 1 \\
f^*(v'_i) &= gcd\{f(v'_i v_{i-1}), f(v'_i v_{i+1})\} \text{ for } i = 4, 6, 8, \cdots, n-2 \\
&= gcd(n + i + 1, n + i + 2) = 1 \\
f^*(v'_2) &= gcd\{f(v'_2 v_1), f(v'_2 v_3)\} = gcd\{3, n + 4\} = 1. \text{ Since here } n + 4 \text{ is not multiple of 3.} \\
f^*(v_i) &= gcd\{f(v_i v'_{i-1}), f(v_i v'_{i+1})\} \text{ for } i = 3, 5, 7, \cdots, n-1 \\
&= gcd(n + i + 1, n + i + 2) = 1 \text{ for } i = 3, 5, 7, \cdots, n-1 \\
f^*(v'_n) &= gcd\{f(v'_n v_1), f(v'_n v_{n-1})\} = gcd\{2n + 2, 2n + 1\} = 1
\end{aligned}
$$

For $n \equiv 2(\mod 3)$:

$$
\begin{aligned}
f^*(v'_n) &= gcd\{f(v'_n v_1), f(v'_n v_{n-1})\} = gcd\{4n, 4n - 1\} = 1 \\
f^*(v_i) &= gcd\{f(v_i v'_{i-1}), f(v_i v'_{i+1})\} \quad \text{for } i = 3, 5, 7, \cdots, n-1 \\
&= gcd\{3n + i - 1, 3n + i\} = 1 \\
f^*(v'_i) &= gcd\{f(v'_i v_{i-1}), f(v'_i v_{i+1})\} \quad \text{for } i = 4, 6, 8, \cdots, n-2 \\
&= gcd\{3n + i - 1, 3n + i\} = 1
\end{aligned}
$$

Since the above labels are consecutive positive integers.

$$
\begin{aligned}
f^*(v_1) &= gcd\{f(v_1 v'_2), f(v_1 v'_n)\} \\
&= gcd\{3, 4n\} = 1. \text{ Since } 4n \text{ is not multiple of 3.} \\
f^*(v'_2) &= gcd\{f(v'_2 v_1), f(v'_2 v_3)\} = gcd\{3, 3n + 2\} = 1
\end{aligned}
$$

For all n:
$f^*(v_1^{'}) \ = gcd\{f(v_1^{'}v_2), f(v_1^{'}v_n)\} = gcd\{4, n+3\} = 1$. Since n is even.
$f^*(v_i) \ = gcd\{f(v_iv_{i-1}^{'}), f(v_iv_{i+1}^{'})\}$ for $i = 2, 4, 6, \cdots, n-2$
$= gcd\{i+2, i+3\} = 1$
$f^*(v_i^{'}) \ = gcd\{f(v_i^{'}v_{i-1}), f(v_i^{'}v_{i+1})\}$ for $i = 3, 5, 7, \cdots, n-1$
$= gcd\{i+2, i+3\} = 1$
Since the above edge labels are consecutive positive integers.
$f^*(v_n) = gcd\{f(v_nv_1^{'}), f(v_nv_{n-1}^{'})\} = gcd\{n+3, n+2\} = 1$
Therefore for each edge $e = uv$ where u and v are relatively prime and adjacent edges receive pairwise relatively prime labels.
Hence G is a highly total prime labeling. □

Theorem 3.3. *The duplicate graph of disjoint union of path and cycle $P_k \cup C_n$ are highly total prime where $k \geq 2, n \geq 3$.*

Proof. Consider $P_k \cup C_n$ be the disjoint union of path and cycle of length k and n respectively.
Let $V(P_k \cup C_n) = \{v_1, v_2, \cdots, v_k, u_1, u_2, \cdots, u_n\}$ and
$E(P_k \cup C_n) = \{v_iv_{i+1}/1 \leq i \leq k-1\} \cup \{u_iu_{i+1}/1 \leq i \leq n-1\} \cup \{u_1u_n\}$.
Let G be the duplicate graph of disjoint union of path and cycle $P_k \cup C_n$ where $k \geq 2, n \geq 3$.
Now $v_1, v_2, \cdots, v_k, u_1, u_2, \cdots, u_n, v_1^{'}, v_2^{'}, \cdots, v_k^{'}, u_1^{'}, u_2^{'}, \cdots, u_n^{'}$ and $e_1, e_2, \cdots, e_{k-1}$, $E_1, E_2, \cdots, E_n, e_1^{'}, e_2^{'}, \cdots, e_{k-1}^{'}, E_1^{'}, E_2^{'}, \cdots, E_n^{'}$ respectively be the new set of vertices and edges of the duplicate graph of disjoint union of $P_k \cup C_n$.
Now $V(G) = \{v_i, v_i^{'}, u_j, u_j^{'}/1 \leq i \leq k, 1 \leq j \leq n\}$ and
$E(G) = \{v_iv_{i+1}^{'}, v_i^{'}v_{i+1}/1 \leq i \leq k-1\} \cup \{u_ju_{j+1}^{'}, u_j^{'}u_{j+1}/1 \leq j \leq n-1\} \cup \{u_1u_n^{'}\} \cup \{u_1^{'}u_n\}$
Then $p = 2k + 2n \Rightarrow 2(k+n)$ and $q = 2(k+n-1)$.
To define labeling $f : V(G) \cup E(G) \to \{1, 2, \cdots, p+q\}$ as

$$f(v_i) = 4n + 2(k-1) + i \qquad \text{if } i = \begin{cases} 1, 3, 5, \cdots, k-1 \\ \text{for k is even} \\ \text{(or)}1, 3, 5, \cdots, k \end{cases}$$

$$f(v_i) = 4n + 3(k-1) + i + 1 \quad \text{if } i = \begin{cases} 2, 4, 6, \cdots, k \\ \text{for k is even} \\ \text{(or)}2, 4, 6, \cdots, k-1 \end{cases}$$

$$f(v_i^{'}) = 4n + 3(k-1) + i + 1 \quad \text{if } i = \begin{cases} 1, 3, 5, \cdots, k-1 \\ \text{for k is even} \\ \text{(or)}1, 3, 5, \cdots, k \end{cases}$$

$$f(v_i^{'}) = 4n + 2(k-1) + i \qquad \text{if } i = \begin{cases} 2, 4, 6, \cdots, k \\ \text{for k is even} \\ \text{(or)}2, 4, 6, \cdots, k-1 \end{cases}$$

$$f(v_iv_{i+1}^{'}) = 4n + i \qquad \text{if } i = \begin{cases} 1, 3, 5, \cdots, k-1 \\ \text{for k is even} \\ \text{(or)}1, 3, 5, \cdots, k-2 \end{cases}$$

$$f(v_iv_{i+1}^{'}) = 4n + k + i - 1 \qquad \text{if } i = \begin{cases} 2, 4, 6, \cdots, k-2 \\ \text{for k is even} \\ \text{(or)}2, 4, 6, \cdots, k-1 \end{cases}$$

$$f(v_i^{'}v_{i+1}) = 4n+k+i-1 \quad \text{if } i = \begin{cases} 1,3,5,\cdots,k-1 \\ \text{for k is even} \\ \text{(or)}1,3,5,\cdots,k-2 \end{cases}$$

$$f(v_i^{'}v_{i+1}) = 4n+i \qquad\qquad \text{if } i = \begin{cases} 2,4,6,\cdots,k-2 \\ \text{for k is even} \\ \text{(or)}2,4,6,\cdots,k-1 \end{cases}$$

According to this pattern,

$$gcd(f(v_i), f(v_{i+1}^{'})) = gcd(4n+2(k-1)+i, 4n+2(k-1)+i+1) = 1 \quad \text{if } i = \begin{cases} 1,3,5,\cdots,k-1 \\ \text{for k is even} \\ \text{(or)}1,3,5,\cdots,k-2 \end{cases}$$

$$gcd(f(v_i), f(v_{i+1}^{'})) = gcd(4n+3k-2+i, 4n+3k-1+i) = 1 \quad \text{if } i = \begin{cases} 2,4,6,\cdots,k-2 \\ \text{for k is even} \\ \text{(or)}2,4,6,\cdots,k-1 \end{cases}$$

$$gcd(f(v_i^{'}), f(v_{i+1})) = gcd(4n+3k-2+i, 4n+3k-1+i) = 1 \quad \text{if } i = \begin{cases} 1,3,5,\cdots,k-1 \\ \text{for k is even} \\ \text{(or)}1,3,5,\cdots,k-2 \end{cases}$$

$$gcd(f(v_i^{'}), f(v_{i+1})) = gcd(4n+2(k-1)+i, 4n+2k-1+i) = 1 \quad \text{if } i = \begin{cases} 2,4,6,\cdots,k-2 \\ \text{for k is even} \\ \text{(or)}2,4,6,\cdots,k-1 \end{cases}$$

$$\begin{aligned} f^*(v_i^{'}) &= gcd\{f(v_i^{'}f(v_{i+1}), f(v_i^{'}v_{i-1})\} \\ &= gcd\{4n+i, 4n+i-1\} = 1 \quad \text{if } i = \begin{cases} 2,4,6,\cdots,k-2 \\ \text{for k is even} \\ \text{(or)}2,4,6,\cdots,k-1 \end{cases} \\ f^*(v_i) &= gcd\{f(v_iv_{i+1}^{'}), f(v_iv_{i-1}^{'})\} \\ &= gcd\{4n+i-1+k, 4n+i+k-2\} = 1 \quad \text{if } i = \begin{cases} 2,4,6,\cdots,k-2 \\ \text{for k is even} \\ \text{(or)}2,4,6,\cdots,k-1 \end{cases} \\ f^*(v_i^{'}) &= gcd\{f(v_i^{'}v_{i+1}), f(v_i^{'}v_{i-1})\} \\ &= gcd\{4n+k+i-1, 4n+k+i-2\} = 1 \quad \text{if } i = \begin{cases} 3,5,7,\cdots,k-1 \\ \text{for k is even} \\ \text{(or)}1,3,5,\cdots,k-2 \end{cases} \\ f^*(v_i) &= gcd\{f(v_iv_{i+1}^{'}), f(v_iv_{i-1}^{'})\} \\ &= gcd\{4n+i, 4n+i-1\} = 1 \quad \text{if } i = \begin{cases} 3,5,7,\cdots,k-1 \\ \text{for k is even} \\ \text{(or)}3,5,7,\cdots,k-2 \end{cases} \end{aligned}$$

Since the above all edge labels are consecutive integers.

Case (i): For n is odd.

$f(u_j) = 3n+j \quad$ for $j = 2,4,6,\cdots,n-1$

$f(u_j) = 2n+j \quad$ for $j = 3,5,7,\cdots,n$

$f(u_1) = 1$

$f(u_j^{'}) = 3n+j \quad$ for $j = 1,3,5,\cdots,n$

$f(u_j^{'}) = 2n+j \quad$ for $j = 2,4,6,\cdots,n-1$

$f(u_1u_n^{'}) = 2n+1$
$f(u_1^{'}u_n) = n+1$
$f(u_ju_{j+1}^{'}) = j+1 \quad$ for $j = 1,3,5,\cdots,n-2$
$f(u_ju_{j+1}^{'}) = n+j+1 \quad$ for $j = 2,4,6,\cdots,n-1$
$f(u_j^{'}u_{j+1}) = n+j+1 \quad$ for $j = 1,3,5,\cdots,n-2$
$f(u_j^{'}u_{j+1}) = j+1 \quad$ for $j = 2,4,6,\cdots,n-1$
$gcd(f(u_1), f(u_n^{'})) = gcd(1, f(u_n^{'}) = 1$
$gcd(f(u_1), f(u_2^{'})) = gcd(1, f(u_2^{'}) = 1$
$gcd(f(u_1^{'}), f(u_n)) = gcd(3n+1, 3n) = 1$
$gcd(f(u_i), f(u_{i-1}^{'})) = gcd(3n+i, 3n+i-1) = 1$ for $i = 2,4,6,\cdots,n-1$
$gcd(f(u_i), f(u_{i+1}^{'})) = gcd(3n+i, 3n+i+1) = 1$ for $i = 2,4,6,\cdots,n-1$
$gcd(f(u_i), f(u_{i-1}^{'})) = gcd(2n+i, 2n+i-1) = 1$ for $i = 3,5,7,\cdots,n$
and $gcd(f(u_i), f(u_{i+1}^{'})) = gcd(2n+i, 2n+i+1) = 1$ for $i = 3,5,7,\cdots,n-2$
Since the above vertex labels are positive consecutive integers.
Now $f^*(u_1) = gcd\{f(u_1u_2^{'}), f(u_1u_n^{'})\} = gcd\{2, 2n+1\} = 1$

$$
\begin{aligned}
f^*(u_1^{'}) &= gcd\{f(u_1^{'}u_2), f(u_1^{'}u_n),\} = gcd\{n+2, n+1\} = 1\\
f^*(u_i) &= gcd\{f(u_iu_{i-1}^{'}), f(u_iu_{i+1}^{'})\} \quad \text{for } i = 2,4,6,\cdots,n-1\\
&= gcd\{n+i, n+i+1\} = 1 \quad \text{for } i = 2,4,6,\cdots,n-1\\
f^*(u_i) &= gcd\{f(u_iu_{i-1}^{'}), f(u_iu_{i+1}^{'})\} \quad \text{for } i = 3,5,7,\cdots,n-2\\
&= gcd\{i, i+1\} = 1 \quad \text{for } i = 3,5,7,\cdots,n-2\\
f^*(u_i^{'}) &= gcd\{f(u_i^{'}u_{i-1}), f(u_i^{'}u_{i+1})\} \quad \text{for } i = 2,4,6,\cdots,n-1\\
&= gcd\{i, i+1\} = 1 \quad \text{for } i = 2,4,6,\cdots,n-1\\
f^*(u_i^{'}) &= gcd\{f(u_i^{'}u_{i-1}), f(u_i^{'}u_{i+1})\} \quad \text{for } i = 3,5,7,\cdots,n-2\\
&= gcd\{n+i, n+i+1\} = 1 \quad \text{for } i = 3,5,7,\cdots,n-2\\
f^*(u_n) &= gcd\{f(u_nu_{n-1}^{'}), f(u_nu_1^{'})\} = gcd\{n, n+1\} = 1\\
f^*(u_n^{'}) &= gcd\{f(u_n^{'}u_{n-1}), f(u_n^{'}u_1)\} = gcd\{2n, 2n+1\} = 1
\end{aligned}
$$

Since the above edge labels are consecutive positive integers.
Case (ii): For n is even.
If $n \not\equiv 2(\mod 3)$ then
$f(u_j) = 3n+j \quad$ for $j = 3,5,7,\cdots,n-1$
$f(u_j^{'}) = 3n+j \quad$ for $j = 2,4,6,\cdots,n$
If $n \equiv 2(\mod 3)$ then
$f(u_j) = n+j+2 \quad$ for $j = 3,5,7,\cdots,n-1$
$f(u_j^{'}) = n+j+2 \quad$ for $j = 2,4,6,\cdots,n$
For all n:
$f(u_1) = 1; f(u_1^{'}) = 2$
$f(u_j) = 2n+j+1 \quad$ for $j = 2,4,6,\cdots,n$
$f(u_j^{'}) = 2n+j+1 \quad$ for $j = 3,5,7,\cdots,n-1$
For $n \not\equiv 2(\mod 3)$ then
$f(u_1u_n^{'}) = 2n+2$
$f(u_ju_{j+1}^{'}) = n+j+2 \quad$ for $j = 3,5,7,\cdots,n-1$
$f(u_j^{'}u_{j+1}) = n+j+2 \quad$ for $j = 2,4,6,\cdots,n-2$
For $n \equiv 2(\mod 3)$ then
$f(u_1u_n^{'}) = 4n$
$f(u_ju_{j+1}^{'}) = 3n+j \quad$ for $j = 1,3,5,\cdots,n-1$

$f(u_j^{'}u_{j+1}) = 3n + j \quad$ for $j = 2, 4, 6, \cdots, n-2$

For all n:

$f(u_1u_2^{'}) = 3; f(u_1^{'}u_n) = n + 3$

$f(u_ju_{j+1}^{'}) = j + 3 \quad$ for $j = 2, 4, 6, \cdots, n-2$

$f(u_j^{'}u_{j+1}) = j + 3 \quad$ for $j = 1, 3, 5, \cdots, n-1$

According to this pattern,

For $n \equiv 2(mod3)$

$gcd(f(u_i), f(u_{i-1}^{'})) = gcd(n + i + 2, n + i + 1) = 1 \quad$ for $i = 3, 5, 7, \cdots, n-1$

$gcd(f(u_i), f(u_{i+1}^{'})) = gcd(n + i + 2, n + i + 3) = 1 \quad$ for $i = 3, 5, 7, \cdots, n-1$

For $n \not\equiv 2(\mod 3)$

$gcd(f(u_i), f(u_{i-1}^{'})) = gcd(3n + i, 3n + i - 1) = 1$ for $i = 3, 5, 7, \cdots, n-1$

$gcd(f(u_i), f(u_{i+1}^{'})) = gcd(3n + i, 3n + i - 1) = 1$ for $i = 3, 5, 7, \cdots, n-1$

For all n

$gcd(f(u_i), f(u_{i-1}^{'})) = gcd(2n + i + 1, 2n + i) = 1$ for $i = 4, 6, 8, \cdots, n$

$gcd(f(u_i), f(u_{i+1}^{'})) = gcd(2n + i + 1, 2n + i + 2) = 1$ for $i = 2, 4, 6, \cdots, n-2$

Since the above vertex labels are consecutive integers.

$gcd(f(u_1), f(u_2^{'})) = gcd(1, f(u_2^{'}) = 1$

$gcd(f(u_1), f(u_n^{'})) = gcd(1, f(u_n^{'}) = 1$

$gcd(f(u_1^{'}), f(u_2)) = gcd(2, 2n + 3) = 1$

$gcd(f(u_1^{'}), f(u_n)) = gcd(2, 3n + 1) = 1$. Since here $3n + 1$ is odd.

$gcd(f(u_n), f(u_{n-1}^{'})) = gcd(3n + 1, 3n) = 1$

$gcd(f(u_2), f(u_3^{'})) = gcd(2n + 3, 2n + 4) = 1$

For $n \equiv 2(\mod 3)$

$$\begin{aligned}
f^*(u_n^{'}) &= gcd\{f(u_n^{'}u_1), f(u_n^{'}u_{n-1})\} = gcd\{4n, 4n-1\} = 1 \\
f^*(u_i) &= gcd\{f(u_iu_{i-1}^{'}), f(u_iu_{i+1}^{'})\} \quad \text{for } i = 3, 5, 7, \cdots, n-1 \\
&= gcd\{3n + i - 1, 3n + i\} = 1 \quad \text{for } i = 3, 5, 7, \cdots, n-1 \\
f^*(u_i^{'}) &= gcd\{f(u_i^{'}u_{i-1}), f(u_i^{'}u_{i+1})\} \quad \text{for } i = 4, 6, 8, \cdots, n-2 \\
&= gcd\{3n + i - 1, 3n + i\} = 1 \quad \text{for } i = 4, 6, 8, \cdots, n-2 \\
f^*(u_1) &= gcd\{f(u_1u_2^{'}), f(u_1u_n^{'})\} = gcd\{3, 4n\} = 1. \text{ Since } 4n \text{ is not a multiple of 3.} \\
f^*(u_2^{'}) &= gcd\{f(u_2^{'}u_1), f(u_2^{'}u_3)\} = gcd\{3, 3n+2\} = 1. \text{ Since here } 3n+2 \text{ is not a multiple of 3.}
\end{aligned}$$

For $n \not\equiv 2(\mod 3)$

$$\begin{aligned}
f^*(u_1) &= gcd\{f(u_1u_2^{'}), f(u_1u_n^{'})\} = gcd\{3, 2n+2\} = 1. \text{ Since here } 2n+2 \text{ is not a multiple of 3.} \\
f^*(u_i^{'}) &= gcd\{f(u_i^{'}u_{i-1}), f(u_i^{'}u_{i+1})\} \quad \text{for } i = 4, 6, 8, \cdots, n-2 \\
&= gcd\{n + i + 1, n + i + 2\} = 1 \quad \text{for } i = 4, 6, 8, \cdots, n-2 \\
f^*(u_2^{'}) &= gcd\{f(u_2^{'}u_1), f(u_2^{'}u_3)\} = gcd\{3, n+4\} = 1. \text{ Since here } n+4 \text{ is not a multiple of 3.} \\
f^*(u_i) &= gcd\{f(u_iu_{i-1}^{'}), f(u_iu_{i+1}^{'})\} \quad \text{for } i = 3, 5, 7, \cdots, n-1 \\
&= gcd\{n + i + 1, n + i + 2\} = 1 \quad \text{for } i = 3, 5, 7, \cdots, n-1 \\
f^*(u_n^{'}) &= gcd\{f(u_n^{'}u_1), f(u_n^{'}u_{n-1})\} = gcd\{2n+2, 2n+1\} = 1
\end{aligned}$$

For all n.

$$\begin{aligned}
f^*(u_1^{'}) &= gcd\{f(u_1^{'}u_2), f(u_1^{'}u_n)\} = \{4, n+3\} = 1. \text{Since } n \text{ is even.} \\
f^*(u_i) &= gcd\{f(u_iu_{i-1}^{'}), f(u_iu_{i+1}^{'})\} \quad \text{for } i = 2, 4, 6, \cdots, n-2 \\
&= gcd\{i + 2, i + 3\} = 1 \quad \text{for } i = 2, 4, 6, \cdots, n-2 \\
f^*(u_i^{'}) &= gcd\{f(u_i^{'}u_{i-1}), f(u_i^{'}u_{i+1})\} \quad \text{for } i = 3, 5, 7, \cdots, n-1 \\
&= gcd\{i + 2, i + 3\} = 1 \quad \text{for } i = 3, 5, 7, \cdots, n-1 \\
f^*(u_n) &= gcd\{f(u_nu_1^{'}), f(u_nu_{n-1}^{'})\} = gcd\{n+3, n+2\} = 1
\end{aligned}$$

Therefore for each edge $e = uv$ where u and v are relatively prime and adjacent edges receive pairwise relatively prime labels.
Hence G is a highly total prime labeling. $\square$

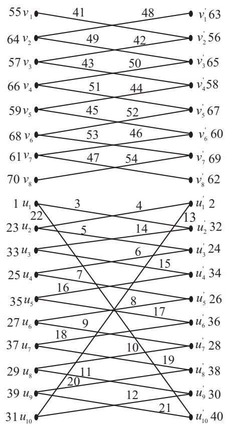

FIGURE 1. The duplicate graph of disjoint union of $P_8 \cup C_{10}$.

Theorem 3.4. *Splitting graph of a star S_n are not highly total prime for $n \geq 4$.*

Proof. Let S_n denote the splitting graph of a star of size n.
$V(S_n) = \{u_i, u_i^{'}/0 \leq i \leq n\}$ and the edge set
$E(S_n) = \{u_0 u_i/1 \leq i \leq n\} \cup \{u_0 u_i^{'}/1 \leq i \leq n\} \cup \{u_0^{'} u_i^{'}/1 \leq i \leq n\}$
$|V(S_n)| = 2(n+1)$ and $|E(S_n)| = 3n$.
Then $p = 2(n+1)$, $q = 3n$.
$\beta_0(S_n) = 2n$ and $\beta_1(S_n) = 2$
$\beta_0 + \beta_1 = 2(n+1) < \left[(2n+1) + \dfrac{n}{2}\right] = \dfrac{p+q}{2}$ for $n \geq 4$.
By using remark (2.1), we get S_n is not highly total prime. $\square$

Note 3.1. *Splitting graph of star S_3 is highly total prime.*

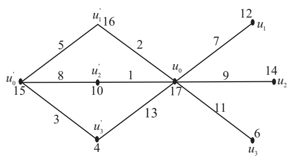

FIGURE 2. Splitting graph of a star graph S_3

Theorem 3.5. *The duplicate graph of splitting graph of a star S_n where $n \geq 3$ is not highly total prime.*

Proof. S_n be the Splitting graph of star with vertex set,
$V(S_n) = \{u_i, u_i^{'}/0 \leq i \leq n\}$ and the edge set
$E(S_n) = \{u_0 u_i/1 \leq i \leq n\} \cup \{u_0 u_i^{'}/1 \leq i \leq n\} \cup \{u_0^{'} u_i^{'}/1 \leq i \leq n\}$
$|V(S_n)| = 2(n+1)$ and $|E(S_n)| = 3n$.
Let G be the duplicate graph of splitting graph of a star S_n.

Let $u_0, u_1, u_2, \cdots, u_n, u_0^{'}, u_1^{'}, u_2^{'}, \cdots, u_n^{'}, v_0^{'}, v_1^{'}, v_2^{'}, \cdots, v_n^{'}, v_0^{''}, v_1^{''}, v_2^{''}, \cdots, v_n^{''}$ be the new vertices and new edges are $e_1, e_2, \cdots, e_{3n}$, $e_1^{'}, e_2^{'}, \cdots, e_{3n}^{'}$ respectively.
Then $V(G) = \{u_i, u_i^{'}, v_i^{'}, v_i^{''}/0 \leq i \leq n\}$ and the edge set
$E(G) = \{u_0v_i^{'}, u_0v_i^{''}/1 \leq i \leq n\} \cup \{u_0^{'}v_i^{''}/1 \leq i \leq n\} \cup \{v_0^{'}u_i, v_0^{'}u_i^{'}/1 \leq i \leq n\}$
$\cup\{v_0^{''}u_i^{'}/1 \leq i \leq n\}$
Here $p = 4(n+1)$ and $q = 6n$
$\beta_0(G) = 4n$ and $\beta_1(G) = 4$

$$\beta_0 + \beta_1 = 4(n+1) \tag{1}$$

$$\begin{aligned} p+q &= 4(n+1)+6n \\ &= 2[(2n+2)+3n] \\ &= 2[2n+2+3n] \\ &= 2[5n+2] \\ \frac{p+q}{2} &= 5n+2 \end{aligned} \tag{2}$$

From (1) and (2), we get
$\beta_0 + \beta_1 = 4(n+1) < \dfrac{p+q}{2} = 5n+2$ for $n \geq 3$.
By using remark (2.1), we get G is not highly total prime, where $n \geq 3$. □

4. Conclusion

Here we investigate five corresponding results on highly total prime labeling analogues. This work can be carried out for other families also.

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References

[1] Bondy, J. A. and Murthy, U. S. R., (1976), Graph Theory and Applications (North-Holland), New York.
[2] Deretsky, T., Lee, S. M. and Mitchem, J., (1991), On vertex prime labelings of graphs, in Graph Theory, Combinatorics and Applications, J. Alavi, G. Chartrand, O. Oellerman, and A. Schwenk, eds., Proceedings 6th International Conference Theory and Applications of Graphs (Wiley, New York), 1, pp. 359–369.
[3] Gallian, J. A., (2014), A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 17, DS6.
[4] Gnanajothi, R. B. and Suganya, S., (2016), Highly Total Prime Labeling, International Journal of Informative and Futuristic Research, 3, (9), pp. 3364-3374.
[5] Meena, S. and Ezhil, A., (2019), Total Prime labeling for Some Graphs, International Journal of Research in Advent Technology, 7, (1), pp. 115-123.
[6] Ravi, M., Ramasubramanian and Kala, R., (2012), Total Prime Graph, International Journal of Computational Engineering Research (ijceronline.com), 2, (5), pp. 1588–1593.
[7] Robert Scholle, (2019), Consecutive Prime and Highly Total Prime Labeling in Graphs, Rose-Hulman Undergraduate Mathematics Journal, 20, (2).
[8] Sampathkumar, E., (1973), On Duplicate Graphs, Journal of the Indian Math. Soc., 37, pp. 285-293.
[9] Thirusangu, K., Ulaganathan, P. P. and Selvam, B., (2010), Cordial labeling in duplicate graphs, Int. J. Computer Math. Sci. Appl., 4, (1-2), pp. 179-186.
[10] Thirusangu, K., Ulaganathan, P. P. and Vijaya Kumar, P., (2014), Some Cordial Labeling of Duplicate Graph of Ladder Graph, Annals of Pure and Applied Mathematics, 8, (2), pp. 43-50.

[11] Tout, A., Dabboucy, A. N. and Howalla, K., (1982), Prime labeling of graphs, Nat. Acad. Sci letters, 11, pp. 365-368.

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