TWMS J. App. and Eng. Math. V.13, N.1, 2023, pp. 309-326

DEGREE BASED TOPOLOGICAL INDICES OF LINE GRAPH OF A CAYLEY TREE Γ_n^k

U. POOJARI[1*], B. S. DURGI[1], R. M. BANAKAR[2], §

Abstract. Several topological indices and their chemical applicability have been studied in chemical graph theory. Some of the degree based topological indices, namely Zagreb index, Modified Zagreb index, Randić index, Atom-bond connectivity index, the fourth version of atom-bond connectivity index, Geometric arithmetic index, the fifth version of geometric arithmetic index, Sum connectivity index have been obtained for Cayley tree Γ_n^k for $k = 2$. In this paper, we have computed these topological indices for the line graph of a Cayley tree $L(\Gamma_n^k)$.

Keywords: Degree based topological index, Zagreb index, atom-bond connectivity index, Randić index, Cayley tree.

AMS Subject Classification: 05C12, 05C35, 05C90.

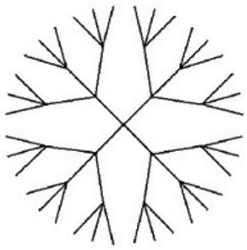
1. Introduction

Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. Let $d_u(G)$ denotes the degree of a vertex u in G, $d_G(u,v)$ denotes the shortest distance between the vetices u and v and $S_u(G)$ denotes the degree sum of the neighbourhood of the vertex u in G. $|V(G)|$ and $|E(G)|$ respectively denote the number of vertices and the number of edges in the graph G. The line graph of a graph G is denoted by $L(G)$, and is the graph whose vertices are the edges of G and two vertices in $L(G)$ are adjacent if and only if they have a common vertex in G[11].

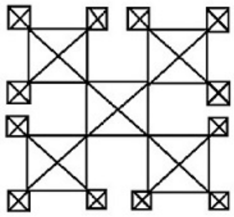
Topological index is the numerical value, which is used to characterize the physical and chemical nature of chemical molecules. Wiener[25] introduced the concept of topological indices in 1947, while he was working on the boiling point of paraffin molecules. He defined Wiener index as $W(G) = \frac{1}{2}\sum_{u,v\in V} d_G(u,v)$ [25].

[1] Department of Mathematics, KLE Dr. M.S.Sheshgiri College of Engineering and Technology, Belagavi-590008, India.
e-mail: ucphal@gmail.com; ORCID: https://orcid.org/0000-0002-4990-8242.
e-mail: bsdurgi@gmail.com; ORCID: https://orcid.org/0000-0002-7258-0089.
[*] Corresponding author.
[2] Department of Mathematics, Siddaganga Institute of Technology, Tumakuru-572103, India.
e-mail: raghumb@sit.ac.in; ORCID: https://orcid.org/0000-0001-6804-2977.
§ Manuscript received: November 18, 2020; accepted: August 09, 2021.

The Cayley tree Γ_n^k, with n levels from the root, is an infinite and symmetric regular tree, that is, a graph without cycles, from each vertex of which exactly $k+1$ edges are issued [17].The number of vertices in Γ_n^k when $k \geq 2$ is $|V|= 1+\frac{(k+1)(k^n-1)}{k-1}$ and the number of edges is $|E|= \frac{(k+1)(k^n-1)}{k-1}$. The Cayley graph with $n = 3$ and $k = 3$ is shown in Figure 1(a).



(a) Cayley tree Γ_3^3



(b) Line graph of Cayley tree Γ_3^3

FIGURE 1. Cayley tree and line graph of Cayley tree Γ_3^3

2. Preliminary

2.1. **Definitions.** Following are some of the degree based topological indices:

Definition 2.1. *The atom-bond connectivity index* (ABC) *was introduced by Estrada et. al.*[10]. *The* ABC *index is given by*

$$ABC(G) = \sum_{uv \in E} \sqrt{\frac{d_u + d_v - 2}{d_u \cdot d_v}}. \quad (1)$$

Definition 2.2. *The geometric arithmetic index* $GA(G)$ *was introduced by D. Vukicevic and B. Furtula* [9]. *The* $GA(G)$ *index is given by*

$$GA(G) = \sum_{uv \in E} \frac{2\sqrt{d_u \cdot d_v}}{(d_u + d_v)}. \quad (2)$$

Definition 2.3. *The Randić index, which is regarded as the oldest degree based topological index* [21], *is given by*

$$R_{-\frac{1}{2}}(G) = \sum_{uv \in E} \frac{1}{\sqrt{d_u \cdot d_v}}. \quad (3)$$

Definition 2.4. *The general sum connectivity index was introduced by Zhou and Trinajstic* [7] *and is given by*

$$\chi_\alpha(G) = \sum_{uv \in E} (d_u + d_v)^\alpha. \quad (4)$$

Definition 2.5. *The Zagreb indices were first defined by Gutman and Trinajstic* [15, 16] *in studying pi-electron energy of chemical compounds. For a graph G, the first, second, third and the modified Zagreb indices are given by,*

$$First\ Zagreb\ index: \qquad M_1(G) = \sum_{uv \in E} (d_u + d_v), \quad (5)$$

$$\textit{Second Zagreb index}: \qquad M_2(G) = \sum_{uv\in E} (d_u \cdot d_v)\,, \tag{6}$$

$$\textit{Third Zagreb index}: \qquad M_3(G) = \sum_{uv\in E} |d_u - d_v|\,, \tag{7}$$

$$\textit{Modified Zagreb index}: \qquad M_2^*(G) = \sum_{uv\in E} \frac{1}{d_u \cdot d_v}. \tag{8}$$

Definition 2.6. *The fourth version of atom-bond connectivity index $ABC_4(G)$ was introduced by M. Ghorbani et. al.* [13]. *The ABC_4 index is given by*

$$ABC_4(G) = \sum_{uv\in E} \sqrt{\frac{S_u + S_v - 2}{S_u \cdot S_v}}. \tag{9}$$

Definition 2.7. *The fifth version of geometric arithmetic index $GA_5(G)$ was introduced by Graovac et.al.*[14]. *The fifth version of geometric arithmetic index $GA_5(G)$ is given by*

$$GA_5(G) = \sum_{uv\in E} \frac{2\sqrt{S_u \cdot S_v}}{(S_u + S_v)}. \tag{10}$$

3. Main Results

In this paper, we compute the first, second, third and the modified Zagreb indices, Randić index, atom bond Connectivity index, the fourth version of atom-bond connectivity index, geometric arithmetic index, the fifth version of geometric arithmetic index, the sum connectivity index of $L(\Gamma_n^k)$. The line graph of a Cayley tree with $n = 3$ and $k = 3$, $L(\Gamma_n^k)$ is shown in Figure 1(b). The number of vertices and the number of edges of $L(\Gamma_n^k)$, when $k \geq 2$, are given by $|V| = \frac{(k+1)(k^n-1)}{k-1}$ and $|E| = \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-1} - 2]$ respectively.

Edge partition of $L(\Gamma_n^k)$ on the basis of degree of end vertices of each edge, when $n \geq 2$ and $k \geq 2$:

$(d_u, d_v)\, where\, uv \in E$	Number of edges
(k, k)	$\frac{k^{n+1}-k^{n-1}}{2}$
$(k, 2k)$	$(k+1)k^{n-1}$
$(2k, 2k)$	$\frac{k(k+1)}{2(k-1)}\ [(k+1)k^{n-2} - 2]$

Table 1. Edge partition of $L(\Gamma_n^k)$, when $n \geq 2$ and $k \geq 2$.

Edge partition of $L(\Gamma_n^k)$ on the basis of degree of end vertices of each edge, when $n = 1$ and $k \geq 2$:

$(d_u, d_v)\, where\, uv \in E$	Number of edges
(k, k)	$\frac{k(k+1)}{2}$

TABLE 2. Edge partition of $L(\Gamma_n^k)$, when $n = 1$ and $k \geq 2$.

Theorem 3.1. *The ABC index, GA index, Randić index, sum connectivity index and Zagreb indices for the line graph of Cayley tree Γ_n^k, when $n \geq 2$ and $k \geq 2$ are given by,*
(i) $ABC(L(\Gamma_n^k)) = \frac{(k^n - k^{n-2})\sqrt{k-1}}{\sqrt{2}} + (k+1)k^{n-2}\sqrt{\frac{3k-2}{2}} + \frac{(k+1)}{(k-1)}\frac{\sqrt{2k-1}}{2\sqrt{2}}[(k+1)k^{n-2} - 2]$.
(ii) $GA(L(\Gamma_n^k)) = \frac{k^{n+1} - k^{n-1}}{2} + (k+1)k^{n-1}\frac{2\sqrt{2}}{3} + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2} - 2]$.
(iii) $R_{-\frac{1}{2}}(L(\Gamma_n^k)) = \frac{k^n - k^{n-2}}{2} + \frac{(k+1)}{\sqrt{2}}k^{n-2} + \frac{(k+1)}{4(k-1)}[(k+1)k^{n-2} - 2]$.
(iv) $\chi_\alpha(L(\Gamma_n^k)) = 2^{\alpha-1}[k^{n+1+\alpha} - k^{n-1+\alpha}] + 3^\alpha(k+1)k^{n-1+\alpha} + \frac{4^\alpha}{2}k^{1+\alpha}(k+1)[(k+1)k^{n-2} - 2]$.
(v) $M_1(L(\Gamma_n^k)) = k^{n+2} + 2k^n + 3k^{n+1} + \frac{2k^2(k+1)}{(k-1)}[(k+1)k^{n-2} - 2]$.
(vi) $M_2(L(\Gamma_n^k)) = \frac{k^{n+3}}{2} + 2k^{n+2} + \frac{3}{2}k^{n+1} + \frac{2k^3(k+1)}{(k-1)}[(k+1)k^{n-2} - 2]$.
(vii) $M_3(L(\Gamma_n^k)) = (k+1)k^n$.
(viii) $M_2^*(L(\Gamma_n^k)) = \frac{k^{n-1} + k^{n-2}}{2} + \frac{k+1}{8k(k-1)}[(k+1)k^{n-2} - 2]$.

Proof. Let $uv \in E$ is an edge in $L(\Gamma_n^k)$, with u and v as end vertices. From Table(1), it is clear that, in $L(\Gamma_n^k)$, on the basis of degree of end vertices of each edge, when $n \geq 2$ and $k \geq 2$, there exist three types of edges. In first type, there are $\frac{k^{n+1} - k^{n-1}}{2}$ edges with each of the edges $uv \in E$ is such that $d_u = k$ and $d_v = k$. In second type, there are $(k+1)k^{n-1}$ edges with each of the edges $uv \in E$ is such that $d_u = k$ and $d_v = 2k$. And in the third type, there are $\frac{k(k+1)}{2(k-1)}$ $[(k+1)k^{n-2} - 2]$ edges with each of the edges $uv \in E$ is such that $d_u = 2k$ and $d_v = 2k$. We use this information in formulae (1) - (8) to obtain the results of Theorem 3.1.

(i) From Eq.(1), we have $ABC(G) = \sum_{uv \in E} \sqrt{\frac{d_u + d_v - 2}{d_u \cdot d_v}}$,

using Table (1), we get

$$ABC(L(\Gamma_n^k) = \left(\frac{k^{n+1} - k^{n-1}}{2}\right)\sqrt{\frac{k+k-2}{k \cdot k}} + (k+1)k^{n-1}\sqrt{\frac{k+2k-2}{k \cdot 2k}} + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2} - 2]\sqrt{\frac{2k+2k-2}{2k \cdot 2k}}.$$

$$\therefore ABC(L(\Gamma_n^k)) = \frac{(k^n - k^{n-2})\sqrt{k-1}}{\sqrt{2}} + (k+1)k^{n-2}\sqrt{\frac{3k-2}{2}} + \frac{(k+1)}{(k-1)}\frac{\sqrt{2k-1}}{2\sqrt{2}}[(k+1)k^{n-2} - 2].$$

(ii) From Eq.(2), we have $GA(G) = \sum_{uv \in E} \frac{2\sqrt{d_u \cdot d_v}}{(d_u + d_v)}$,

using Table (1), we get

$$GA(L(\Gamma_n^k)) = \left(\frac{k^{n+1} - k^{n-1}}{2}\right) \frac{2\sqrt{k \cdot k}}{(k+k)} + (k+1)k^{n-1} \frac{2\sqrt{k \cdot 2k}}{(k+2k)} + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2} - 2] \frac{2\sqrt{2k \cdot 2k}}{(2k+2k)}.$$

$$\therefore GA(L(\Gamma_n^k)) = \frac{k^{n+1} - k^{n-1}}{2} + (k+1)k^{n-1} \frac{2\sqrt{2}}{3} + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2} - 2].$$

$$\therefore GA(L(\Gamma_n^k)) = \frac{k^{n+1} - k^{n-1}}{2} + (k+1)k^{n-1} \frac{2\sqrt{2}}{3} + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2} - 2].$$

(iii) From Eq.(3), we have $R_{-\frac{1}{2}}(G) = \sum_{uv \in E} \frac{1}{\sqrt{d_u \cdot d_v}}$,

using Table (1), we get

$$R_{-\frac{1}{2}}(L(\Gamma_n^k)) = \left(\frac{k^{n+1} - k^{n-1}}{2}\right) \frac{1}{\sqrt{k \cdot k}} + (k+1)k^{n-1} \frac{1}{\sqrt{k \cdot 2k}} + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2} - 2] \frac{1}{\sqrt{2k \cdot 2k}}.$$

$$\therefore R_{-\frac{1}{2}}(L(\Gamma_n^k)) = \frac{k^n - k^{n-2}}{2} + \frac{(k+1)}{\sqrt{2}} k^{n-2} + \frac{(k+1)}{4(k-1)}[(k+1)k^{n-2} - 2].$$

(iv) From Eq.(4), we have $\chi_\alpha(G) = \sum_{uv \in E} (d_u + d_v)^\alpha$,

using Table (1), we get

$$\chi_\alpha(L(\Gamma_n^k)) = \left(\frac{k^{n+1} - k^{n-1}}{2}\right) (k+k)^\alpha + (k+1)k^{n-1}(k+2k)^\alpha + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2} - 2](2k+2k)^\alpha.$$

$$\therefore \chi_\alpha(L(\Gamma_n^k)) = 2^{\alpha-1}[k^{n+1+\alpha} - k^{n-1+\alpha}] + 3^\alpha(k+1)k^{n-1+\alpha} + \frac{4^\alpha}{2} k^{1+\alpha}(k+1)[(k+1)k^{n-2} - 2].$$

(v) From Eq.(5), we have $M_1(G) = \sum_{uv \in E} (d_u + d_v)$,

using Table (1), we get

$$M_1(L(\Gamma_n^k)) = \left(\frac{k^{n+1} - k^{n-1}}{2}\right) (2k) + (k+1)k^{n-1}(3k) + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2} - 2](4k).$$

$$\therefore M_1(L(\Gamma_n^k)) = k^{n+2} + 2k^n + 3k^{n+1} + \frac{2k^2(k+1)}{(k-1)}[(k+1)k^{n-2} - 2].$$

(vi) From Eq.(6), we have $M_2(G) = \sum_{uv\in E}(d_u \cdot d_v)$,
using Table (1), we get

$$M_2(L(\Gamma_n^k)) = \left(\frac{k^{n+1}-k^{n-1}}{2}\right)(k\cdot k) + (k+1)k^{n-1}(k\cdot 2k) + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2}-2](2k\cdot 2k).$$

$$\therefore M_2(L(\Gamma_n^k)) = \frac{k^{n+3}}{2} + 2k^{n+2} + \frac{3}{2}k^{n+1} + \frac{2k^3(k+1)}{(k-1)}[(k+1)k^{n-2}-2].$$

(vii) From Eq.(7), we have $M_3(G) = \sum_{uv\in E}|d_u - d_v|$,
using Table (1), we get

$$M_3(L(\Gamma_n^k)) = \left(\frac{k^{n+1}-k^{n-1}}{2}\right)|k-k| + (k+1)k^{n-1}|k-2k| + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2}-2]|2k-2k|.$$

$$\therefore M_3(L(\Gamma_n^k)) = (k+1)k^n.$$

(viii) From Eq.(8), we have $M_2^*(G) = \sum_{uv\in E}\frac{1}{d_u\cdot d_v}$,
using Table (1), we get

$$M_2^*(L(\Gamma_n^k)) = \left(\frac{k^{n+1}-k^{n-1}}{2}\right)\frac{1}{k^2} + (k+1)k^{n-1}\frac{1}{2k^2} + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-2}-2]\frac{1}{4k^2}.$$

$$\therefore M_2^*(L(\Gamma_n^k)) = \frac{k^{n-1}+k^{n-2}}{2} + \frac{k+1}{8k(k-1)}[(k+1)k^{n-2}-2].$$ ■

Theorem 3.2. *The ABC index, GA index, Randić index, sum connectivity index and Zagreb indices for the line graph of cayley tree Γ_n^k, when $n=1$ and $k\geq 2$ are:*

(i) $ABC(L(\Gamma_n^k)) = \frac{(k+1)\sqrt{k-1}}{\sqrt{2}}$.
(ii) $GA(L(\Gamma_n^k)) = \frac{k(k+1)}{2}$.
(iii) $R_{-\frac{1}{2}}(L(\Gamma_n^k)) = \frac{k+1}{2}$.
(iv) $\chi_\alpha(L(\Gamma_n^k)) = (k+1)k^{1+\alpha}2^{\alpha-1}$.
(v) $M_1(L(\Gamma_n^k)) = k^2(k+1)$.
(vi) $M_2(L(\Gamma_n^k)) = \frac{k^3(k+1)}{2}$.
(vii) $M_3(L(\Gamma_n^k)) = 0$.
(viii) $M_2^*(L(\Gamma_n^k)) = \frac{k+1}{2k}$.

Proof.

(i) From Eq.(1), we have $ABC(G) = \sum_{uv\in E} \sqrt{\frac{d_u + d_v - 2}{d_u \cdot d_v}}$,

using Table (2), we get

$$ABC(L(\Gamma_n^k)) = \left(\frac{k(k+1)}{2}\right)\sqrt{\frac{k+k-2}{k\cdot k}}.$$

$$\therefore ABC(L(\Gamma_n^k)) = \frac{(k+1)\sqrt{k-1}}{\sqrt{2}}.$$

(ii) From Eq.(2), we have $GA(G) = \sum_{uv\in E} \frac{2\sqrt{d_u \cdot d_v}}{(d_u + d_v)}$,

using Table (2), we get

$$GA(L(\Gamma_n^k)) = \left(\frac{k(k+1)}{2}\right)\frac{2\sqrt{k\cdot k}}{k+k}.$$

$$\therefore GA(L(\Gamma_n^k)) = \frac{k(k+1)}{2}.$$

(iii) From Eq.(3), we have $R_{-\frac{1}{2}}(G) = \sum_{uv\in E} \frac{1}{\sqrt{d_u \cdot d_v}}$,

using Table (2), we get

$$R_{-\frac{1}{2}}(L(\Gamma_n^k)) = \left(\frac{k(k+1)}{2}\right)\frac{1}{\sqrt{k\cdot k}}.$$

$$\therefore R_{-\frac{1}{2}}(L(\Gamma_n^k)) = \frac{k+1}{2}.$$

(iv) From Eq.(4), we have $\chi_\alpha(G) = \sum_{uv\in E} (d_u + d_v)^\alpha$,

using Table (2), we get

$$\chi_\alpha(L(\Gamma_n^k)) = \left(\frac{k(k+1)}{2}\right)[k+k]^\alpha.$$

$$\therefore \chi_\alpha(L(\Gamma_n^k)) = (k+1)k^{1+\alpha}2^{\alpha-1}.$$

(v) From Eq.(5), we have $M_1(G) = \sum_{uv\in E} (d_u + d_v)$,

using Table (2), we get

$$M_1(L(\Gamma_n^k)) = \left(\frac{k(k+1)}{2}\right)(k+k).$$

$$\therefore M_1(L(\Gamma_n^k)) = k^2(k+1).$$

(vi) From Eq.(6), we have $M_2(G) = \sum_{uv\in E} (d_u \cdot d_v)$,

using Table (2), we get

$$M_2(L(\Gamma_n^k)) = \left(\frac{k(k+1)}{2}\right)(k\cdot k).$$

$$\therefore M_2(L(\Gamma_n^k)) = \frac{k^3(k+1)}{2}.$$

(vii) From Eq.(7), we have $M_3(G) = \sum_{uv\in E} |d_u - d_v|$,

using Table (2), we get

$$M_3(L(\Gamma_n^k)) = \left(\frac{k(k+1)}{2}\right)|k-k|.$$

$$\therefore M_3(L(\Gamma_n^k)) = 0.$$

(viii) From Eq.(8), we have $M_2^*(G) = \sum_{uv\in E} \frac{1}{d_u \cdot d_v}$,

using Table (2), we get

$$M_2^*(L(\Gamma_n^k)) = \left(\frac{k(k+1)}{2}\right)\frac{1}{k\cdot k}.$$

$$\therefore M_2^*(L(\Gamma_n^k)) = \frac{k+1}{2k}.$$ ■

Edge partition of $L(\Gamma_n^k)$ on the basis of degree sum of neighbours of end vertices of each edge, when $n = 1$ and $k \geq 2$:

$(S_u, S_v)\, where\, uv \in E$	Number of edges
(k^2, k^2)	$\frac{k(k+1)}{2}$

Table 3. Edge partition of L(Γ_n^k), when $n = 1$ and $k \geq 2$.

Edge partition of $L(\Gamma_n^k)$ on the basis of degree sum of neighbours of end vertices of each edge, when $n = 2$ and $k \geq 2$:

$(S_u, S_v)\, where\, uv \in E$	Number of edges
$(k(k+1), k(k+1))$	$\frac{k(k^2-1)}{2}$
$(k(k+1), 3k^2)$	$k(k+1)$
$(3k^2, 3k^2)$	$\frac{k(k+1)}{2}$

Table 4. Edge partition of L(Γ_n^k), when $n = 2$ and $k \geq 2$.

Edge partition of $L(\Gamma_n^k)$ on the basis of degree sum of neighbours of end vertices of each edge, when $n \geq 3$ and $k \geq 2$:

$(S_u, S_v)\, where\, uv \in E$	Number of edges
$(k(k+1), k(k+1))$	$\frac{k^{n-1}(k^2-1)}{2}$
$(k(k+1), 3k^2)$	$k^{n-1}(k+1)$
$(3k^2, 3k^2)$	$\frac{k^{n-2}(k^2-1)}{2}$
$(3k^2, 4k^2)$	$k^{n-2}(k+1)$
$(4k^2, 4k^2)$	$\frac{k(k+1)}{2(k-1)}[(k+1)k^{n-3} - 2]$

Table 5. Edge partition of L(Γ_n^k), when $n \geq 3$ and $k \geq 2$.

Theorem 3.3. *The ABC_4 index and GA_5 index for the line graph of cayley tree Γ_n^k, when $n = 1$ and $k \geq 2$ are:*

(i) $ABC_4(L(\Gamma_n^k)) = \frac{(k+1)\sqrt{k^2-1}}{k\sqrt{2}}$.

(ii) $GA_5(L(\Gamma_n^k)) = \frac{k(k+1)}{2}$.

Proof.

(a) From Eq.(9), we have $ABC_4(G) = \sum_{uv \in E} \sqrt{\frac{S_u + S_v - 2}{S_u \cdot S_v}}$,

using Table(3), we get

$$ABC_4(L(\Gamma_n^k)) = \left(\frac{k(k+1)}{2}\right)\sqrt{\frac{k^2+k^2-2}{k^2 \cdot k^2}}.$$

$$\therefore ABC_4(L(\Gamma_n^k)) = \frac{(k+1)\sqrt{k^2-1}}{k\sqrt{2}}.$$

(b) From Eq.(10), we have $GA_5(G) = \sum_{uv \in E} \frac{2\sqrt{S_u \cdot S_v}}{(S_u + S_v)}$,

using Table(3), we get

$$GA_5(L(\Gamma_n^k)) = \left(\frac{k(k+1)}{2}\right)\frac{2\sqrt{k^2 \cdot k^2}}{k^2+k^2}.$$

$$\therefore GA_5(L(\Gamma_n^k)) = \frac{k(k+1)}{2}. \qquad \blacksquare$$

Theorem 3.4. *The ABC_4 index and GA_5 index for the line graph of cayley tree Γ_n^k, when $n = 2$ and $k \geq 2$ are:*

(i) $ABC_4(L(\Gamma_n^k)) = (k-1)\sqrt{\frac{k^2+k-1}{2}} + \sqrt{\frac{4k^3+5k^2-k-2}{3k}} + \frac{(k+1)}{3k}\sqrt{\frac{3k^2-1}{2}}$.

(ii) $GA_5(L(\Gamma_n^k)) = \frac{k(k+1)}{2}\left(k + \frac{4\sqrt{3k(k+1)}}{4k+1}\right)$.

Proof.

(i) From Eq.(9), we have $ABC_4(G) = \sum_{uv \in E} \sqrt{\frac{S_u + S_v - 2}{S_u \cdot S_v}}$,

using Table(4), we get

$$ABC_4(L(\Gamma_n^k)) = \left(\frac{k(k^2-1)}{2}\right)\sqrt{\frac{k^2+k+k^2+k-2}{k(k+1)k(k+1)}} + k(k+1)\sqrt{\frac{k(k+1)+3k^2-2}{k(k+1)3k^2}} + \frac{k(k+1)}{2}\sqrt{\frac{6k^2-2}{9k^4}}.$$

$$\therefore ABC_4(L(\Gamma_n^k)) = (k-1)\sqrt{\frac{k^2+k-1}{2}} + \sqrt{\frac{4k^3+5k^2-k-2}{3k}} + \frac{(k+1)}{3k}\sqrt{\frac{3k^2-1}{2}}.$$

(ii) From Eq.(10), we have $GA_5(G) = \sum_{uv\in E} \frac{2\sqrt{S_u \cdot S_v}}{(S_u + S_v)}$,
using Table(4), we get

$$GA_5(L(\Gamma_n^k)) = \frac{k(k^2-1)}{2}\left(\frac{2\sqrt{k^2(k+1)^2}}{k(k+1)+k(k+1)}\right) + k(k+1)\left(\frac{2\sqrt{k(k+1)3k^2}}{k(k+1)+3k^2}\right) + \frac{k(k+1)}{2}\left(\frac{2\sqrt{3k^2 \cdot 3k^2}}{6k^2}\right).$$

$$\therefore GA_5(L(\Gamma_n^k)) = \frac{k(k+1)}{2}\left(k + \frac{4\sqrt{3k(k+1)}}{4k+1}\right).$$

■

Theorem 3.5. *The ABC_4 index and GA_5 index for the line graph of cayley tree Γ_n^k, when $n \geq 3$ and $k \geq 2$ are:*

$$\textit{(i)}\ ABC_4(L(\Gamma_n^k)) = (k-1)k^{n-2}\sqrt{\frac{k^2+k-1}{2}} + k^{n-\frac{5}{2}}\sqrt{\frac{4k^3+5k^2-k-2}{3}} + (k+1)k^{n-4}\left[\frac{(k-1)}{3}\sqrt{\frac{3k^2-1}{2}} + \sqrt{\frac{7k^2-2}{12}}\right] + \frac{(k+1)}{4k(k-1)}[(k+1)k^{n-3}-2]\sqrt{\frac{4k^2-1}{2}}.$$

$$\textit{(ii)}\ GA_5(L(\Gamma_n^k)) = (k+1)k^{n-1}\left(\frac{k-1}{2} + \frac{2\sqrt{3k(k+1)}}{4k+1}\right) + (k+1)k^{n-2}\left(\frac{k}{2} + \frac{1}{k-1} + \frac{4\sqrt{3}}{7}\right) - \frac{k(k+1)}{k-1}.$$

Proof.

(i) From Eq.(9), we have $ABC_4(G) = \sum_{uv\in E} \sqrt{\frac{S_u + S_v - 2}{S_u \cdot S_v}}$,
using Table(5), we get

$$ABC_4(L(\Gamma_n^k)) = \frac{(k^2-1)k^{n-1}}{2}\sqrt{\frac{2k(k+1)-2}{k^2(k+1)^2}} + (k+1)k^{n-1}\sqrt{\frac{k(k+1)+3k^2-2}{k(k+1)3k^2}} + \frac{(k^2-1)k^{n-2}}{2}\sqrt{\frac{6k^2-2}{9k^4}} + (k+1)k^{n-2}\sqrt{\frac{7k^2-2}{12k^4}} + \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-3}-2]\sqrt{\frac{8k^2-2}{16k^4}}.$$

$$\therefore ABC_4(L(\Gamma_n^k)) = (k-1)k^{n-2}\sqrt{\frac{k^2+k-1}{2}} + k^{n-\frac{5}{2}}\sqrt{\frac{4k^3+5k^2-k-2}{3}}$$
$$+ (k+1)k^{n-4}\left[\frac{(k-1)}{3}\sqrt{\frac{3k^2-1}{2}} + \sqrt{\frac{7k^2-2}{12}}\right]$$
$$+ \frac{(k+1)}{4k(k-1)}[(k+1)k^{n-3}-2]\sqrt{\frac{4k^2-1}{2}}.$$

(ii) From Eq.(10), we have $GA_5(G) = \sum_{uv\in E} \frac{2\sqrt{S_u \cdot S_v}}{(S_u+S_v)}$,

using Table(5), we get

$$GA_5(L(\Gamma_n^k)) = \frac{k^{n-1}(k^2-1)}{2}\left(\frac{2\sqrt{k^2(k+1)^2}}{2k(k+1)}\right) + (k+1)k^{n-1}\left(\frac{2\sqrt{k(k+1)3k^2}}{k(k+1)+3k^2}\right)$$
$$+ \frac{k^{n-2}(k^2-1)}{2}\left(\frac{2\sqrt{3k^2\cdot 3k^2}}{6k^2}\right) + (k+1)k^{n-2}\left(\frac{2\sqrt{3k^2 4k^2}}{7k^2}\right)$$
$$+ \frac{k(k+1)}{2(k-1)}[(k+1)k^{n-3}-2]\frac{2\sqrt{4k^2\cdot 4k^2}}{8k^2}.$$

$$\therefore GA_5(L(\Gamma_n^k)) = (k+1)k^{n-1}\left(\frac{k-1}{2} + \frac{2\sqrt{3k(k+1)}}{4k+1}\right) + (k+1)k^{n-2}\left(\frac{k}{2} + \frac{1}{k-1} + \frac{4\sqrt{3}}{7}\right)$$
$$- \frac{k(k+1)}{k-1}.$$

Edge partition of $L(\Gamma_n^k)$ on the basis of degree of end vertices of each edge, when $k=1$: The number of vertices and edges of $L(\Gamma_n^k)$ when $k=1$ are $|V|= 2n$ and $|E|= 2n-1$ respectively.

(d_u, d_v) *where* $uv \in E$	Number of edges
$(1,1)$	1

TABLE 6. Edge partition of $L(\Gamma_n^k)$, when $n=1$ and $k=1$.

(d_u, d_v) *where* $uv \in E$	Number of edges
$(1,2)$	2
$(2,2)$	$2n-3$

TABLE 7. Edge partition of $L(\Gamma_n^k)$, when $n \geq 2$ and $k=1$.

Theorem 3.6. *The ABC index, GA index, Randić index, sum connectivity index and Zagreb indices for the line graph of cayley tree Γ_n^k, when $n=1$ and $k=1$ are:*

(i) $ABC(L(\Gamma_n^k)) = 0$.

(ii) $GA(L(\Gamma_n^k)) = 1$.
(iii) $R_{-\frac{1}{2}}(L(\Gamma_n^k)) = 1$.
(iv) $\chi_\alpha(L(\Gamma_n^k)) = 2^\alpha$.
(v) $M_1(L(\Gamma_n^k)) = 2$.
(vi) $M_2(L(\Gamma_n^k)) = 1$.
(vii) $M_3(L(\Gamma_n^k)) = 0$.
(viii) $M_2^*(L(\Gamma_n^k)) = 1$.

Proof.

(i) From Eq.(1), we have $ABC(G) = \sum_{uv\in E} \sqrt{\frac{d_u + d_v - 2}{d_u \cdot d_v}}$, using Table(6), we get

$$ABC(L(\Gamma_n^k)) = \sqrt{\frac{1+1-2}{1}}.$$
$$\therefore ABC(L(\Gamma_n^k)) = 0.$$

(ii) From Eq.(2), we have $GA(G) = \sum_{uv\in E} \frac{2\sqrt{d_u \cdot d_v}}{(d_u + d_v)}$, using Table(6), we get

$$GA(L(\Gamma_n^k)) = \frac{2\sqrt{1}}{(1+1)}.$$
$$\therefore GA(L(\Gamma_n^k)) = 1.$$

(iii) From Eq.(3), we have $R_{-\frac{1}{2}}(G) = \sum_{uv\in E} \frac{1}{\sqrt{d_u \cdot d_v}}$, using Table(6), we get

$$R_{-\frac{1}{2}}(L(\Gamma_n^k)) = \frac{1}{\sqrt{1}}.$$
$$\therefore R_{-\frac{1}{2}}(L(\Gamma_n^k)) = 1.$$

(iv) From Eq.(4), we have $\chi_\alpha(G) = \sum_{uv\in E} (d_u + d_v)^\alpha$, using Table(6), we get

$$\chi_\alpha(L(\Gamma_n^k)) = (1+1)^\alpha.$$
$$\therefore \chi_\alpha(L(\Gamma_n^k)) = 2^\alpha.$$

(v) From Eq.(5), we have $M_1(G) = \sum_{uv\in E} (d_u + d_v)$, using Table(6), we get

$$M_1(L(\Gamma_n^k)) = (1+1).$$
$$\therefore M_1(L(\Gamma_n^k)) = 2.$$

(vi) From Eq.(6), we have $M_2(G) = \sum_{uv \in E} (d_u \cdot d_v)$,

using Table(6), we get

$$M_2(L(\Gamma_n^k)) = 1.$$
$$\therefore M_2(L(\Gamma_n^k)) = 1.$$

(vii) From Eq.(7), we have $M_3(G) = \sum_{uv \in E} |d_u - d_v|$,

using Table(6), we get

$$M_3(L(\Gamma_n^k)) = |1 - 1|.$$
$$\therefore M_3(L(\Gamma_n^k)) = 0.$$

(viii) From Eq.(8), we have $M_2^*(G) = \sum_{uv \in E} \frac{1}{d_u \cdot d_v}$,

using Table(6), we get

$$M_2^*(L(\Gamma_n^k)) = \frac{1}{1}.$$
$$\therefore M_2^*(L(\Gamma_n^k)) = 1.$$

■

Theorem 3.7. *The ABC index, GA index, Randić index, sum connectivity index and Zagreb indices for the line graph of cayley tree Γ_n^k, when $n \geq 2$ and $k = 1$ are:*

(i) $ABC(L(\Gamma_n^k)) = \frac{2n-1}{\sqrt{2}}$.

(ii) $GA(L(\Gamma_n^k)) = \frac{4\sqrt{2}-9+6n}{3}$.

(iii) $R_{-\frac{1}{2}}(L(\Gamma_n^k)) = \sqrt{2} + \frac{2n-3}{2}$.

(iv) $\chi_\alpha(L(\Gamma_n^k)) = 2(3^\alpha) + (2n-3)(4^\alpha)$.

(v) $M_1(L(\Gamma_n^k)) = 8n - 6$.

(vi) $M_2(L(\Gamma_n^k)) = 8(n-1)$.

(vii) $M_3(L(\Gamma_n^k)) = 2$

(viii) $M_2^*(L(\Gamma_n^k)) = \frac{2n+1}{4}$.

Proof.

(i) From Eq.(1), we have $ABC(G) = \sum_{uv \in E} \sqrt{\frac{d_u + d_v - 2}{d_u \cdot d_v}}$,

using Table(7), we get

$$ABC(L(\Gamma_n^k)) = 2\sqrt{\frac{1+2-2}{2}} + (2n-3)\sqrt{\frac{2+2-2}{4}}.$$
$$\therefore ABC(L(\Gamma_n^k)) = \frac{2n-1}{\sqrt{2}}.$$

(ii) From Eq.(2), we have $GA(G) = \sum_{uv \in E} \frac{2\sqrt{d_u \cdot d_v}}{(d_u + d_v)}$,

using Table(7), we get

$$GA(L(\Gamma_n^k)) = \frac{4\sqrt{2}}{(1+2)} + (2n-3)\frac{2\sqrt{4}}{(2+2)}.$$
$$\therefore GA(L(\Gamma_n^k)) = \frac{4\sqrt{2} - 9 + 6n}{3}.$$

(iii) From Eq.(3), we have $R_{-\frac{1}{2}}(G) = \sum_{uv \in E} \frac{1}{\sqrt{d_u \cdot d_v}}$,

using Table(7), we get

$$R_{-\frac{1}{2}}(L(\Gamma_n^k)) = 2\frac{1}{\sqrt{2}} + (2n-3)\frac{1}{\sqrt{4}}.$$

$$\therefore R_{-\frac{1}{2}}(L(\Gamma_n^k)) = \sqrt{2} + \frac{2n-3}{2}.$$

(iv) From Eq.(4), we have $\chi_\alpha(G) = \sum_{uv \in E} (d_u + d_v)^\alpha$,

using Table(7), we get

$$\chi_\alpha(L(\Gamma_n^k)) = 2((1+2)^\alpha) + (2n-3)((2+2)^\alpha.$$

$$\therefore \chi_\alpha(L(\Gamma_n^k)) = 2(3^\alpha) + (2n-3)(4^\alpha).$$

(v) From Eq.(5), we have $M_1(G) = \sum_{uv \in E} (d_u + d_v)$,

using Table(7), we get

$$M_1(L(\Gamma_n^k)) = 2(1+2) + (2n-3)(2+2).$$

$$\therefore M_1(L(\Gamma_n^k)) = 8n - 6.$$

(vi) From Eq.(6), we have $M_2(G) = \sum_{uv \in E} (d_u \cdot d_v)$,

using Table(7), we get

$$M_2(L(\Gamma_n^k)) = 2(2) + (2n-3)(4).$$

$$\therefore M_2(L(\Gamma_n^k)) = 8(n-1).$$

(vii) From Eq.(7), we have $M_3(G) = \sum_{uv \in E} |d_u - d_v|$,

using Table(7), we get

$$M_3(L(\Gamma_n^k)) = 2|2-1| + (2n-3)|2-2|.$$

$$\therefore M_3(L(\Gamma_n^k)) = 2.$$

(viii) From Eq.(8), we have $M_2^*(G) = \sum_{uv \in E} \frac{1}{d_u \cdot d_v}$,

using Table(7), we get

$$M_2^*(L(\Gamma_n^k)) = 2\frac{1}{2} + (2n-3)\frac{1}{4}.$$

$$\therefore M_2^*(L(\Gamma_n^k)) = \frac{2n+1}{4}.$$

■

Edge partition of $L(\Gamma_n^k)$ on the basis of degree sum of neighbours of end vertices of each edge, when $k = 1$:

(S_u, S_v) *where* $uv \in E$	Number of edges
$(1, 1)$	1

TABLE 8. Edge partition of L(Γ_n^k), when $n = 1$ and $k = 1$.

(S_u, S_v) *where* $uv \in E$	Number of edges
$(2, 3)$	2
$(3, 3)$	1

TABLE 9. Edge partition of L(Γ_n^k), when $n = 2$ and $k = 1$.

(S_u, S_v) *where* $uv \in E$	Number of edges
$(2, 3)$	2
$(3, 4)$	2
$(4, 4)$	$2n - 5$

TABLE 10. Edge partition of L(Γ_n^k), when $n \geq 3\, and\, k = 1$.

Theorem 3.8. *The ABC_4 index and GA_5 index for the line graph of cayley tree Γ_n^k, when $n = 1$ and $k = 1$ are:*

(i) $ABC_4(L(\Gamma_n^k)) = 0$.
(ii) $GA_5(L(\Gamma_n^k)) = 1$.

Proof.

(i) From Eq.(9), we have $ABC_4(G) = \sum_{uv \in E} \sqrt{\frac{S_u + S_v - 2}{S_u \cdot S_v}}$,

using Table(8), we get

$$ABC_4(L(\Gamma_n^k)) = \sqrt{\frac{1 + 1 - 2}{1}}.$$
$$\therefore ABC_4(L(\Gamma_n^k)) = 0.$$

(ii) From Eq.(10), we have $GA_5(G) = \sum_{uv \in E} \frac{2\sqrt{S_u \cdot S_v}}{(S_u + S_v)}$,

using Table(8), we get

$$GA_5(L(\Gamma_n^k)) = \frac{2\sqrt{1}}{2}.$$
$$\therefore GA_5(L(\Gamma_n^k)) = 1.$$

■

Theorem 3.9. *The ABC_4 index and GA_5 index for the line graph of cayley tree Γ_n^k, when $n = 2$ and $k = 1$ are:*

(i) $ABC_4(L(\Gamma_n^k)) = \sqrt{2} + \frac{2}{3}$.
(ii) $GA_5(L(\Gamma_n^k)) = \frac{1}{2} + \frac{4\sqrt{6}}{5}$.

Proof.

(i) From Eq.(9), we have $ABC_4(G) = \sum_{uv\in E} \sqrt{\frac{S_u + S_v - 2}{S_u \cdot S_v}}$,

using Table(9), we get

$$ABC_4(L(\Gamma_n^k)) = 2\sqrt{\frac{2+3-2}{6}} + \sqrt{\frac{3+3-2}{9}}.$$

$$\therefore ABC_4(L(\Gamma_n^k)) = \sqrt{2} + \frac{2}{3}.$$

(ii) From Eq.(10), we have $GA_5(G) = \sum_{uv\in E} \frac{2\sqrt{S_u \cdot S_v}}{(S_u + S_v)}$,

using Table(9), we get

$$GA_5(L(\Gamma_n^k)) = 2(\frac{2\sqrt{6}}{5}) + 2(\frac{\sqrt{9}}{6}).$$

$$\therefore GA_5(L(\Gamma_n^k)) = \frac{1}{2} + \frac{4\sqrt{6}}{5}.$$ ■

Theorem 3.10. *The ABC_4 index and GA_5 index for the line graph of cayley tree Γ_n^k, when $n \geq 3$ and $k = 1$ are:*

(i) $ABC_4(L(\Gamma_n^k)) = \sqrt{2} + \sqrt{\frac{5}{3}} + (2n-5)\frac{\sqrt{6}}{4}$.

(ii) $GA_5(L(\Gamma_n^k)) = \frac{4}{5}\sqrt{6} + \frac{4}{7}\sqrt{3} + n - \frac{5}{2}$.

Proof:

(i) From Eq.(9), we have $ABC_4(G) = \sum_{uv\in E} \sqrt{\frac{S_u + S_v - 2}{S_u \cdot S_v}}$,

using Table(10), we get

$$ABC_4(L(\Gamma_n^k)) = 2\sqrt{\frac{2+3-2}{6}} + 2\sqrt{\frac{3+4-2}{12}} + (2n-5)\sqrt{\frac{4+4-2}{16}}.$$

$$\therefore ABC_4(L(\Gamma_n^k)) = \sqrt{2} + \sqrt{\frac{5}{3}} + (2n-5)\frac{\sqrt{6}}{4}.$$

(ii) From Eq.(10), we have $GA_5(G) = \sum_{uv\in E} \frac{2\sqrt{S_u \cdot S_v}}{(S_u + S_v)}$,

using Table(10), we get

$$GA_5(L(\Gamma_n^k)) = 2(\frac{2\sqrt{6}}{5}) + 2(\frac{2\sqrt{12}}{7}) + 2(2n-5)\frac{4}{8}.$$

$$\therefore GA_5(L(\Gamma_n^k)) = \frac{4}{5}\sqrt{6} + \frac{4}{7}\sqrt{3} + n - \frac{5}{2}.$$ ■

4. Conclusions

In this paper, generalized formulae for Zagreb index, Modified Zagreb index, Randic index, Atom-bond connectivity index, the fourth version of atom-bond connectivity index, geometric arithmetic index, the fifth version of geometric arithmetic index and the Sum connectivity index for line graph of Cayley tree Γ_n^k are computed.

5. Acknowledgements

All the authors are grateful to the anonymous referees for their helpful comments and suggestions.

References

[1] Ashra, A. R., Doslić,T., and Hamzeh, A., (2010), The Zagreb coindices of graph operations, Discrete Appl. Math., 158, pp. 1571-1578.

[2] Furtula, B., Graovac, A., Vukicevic, D., (2009), Atom-bond connectivity index of trees, Discrete Applied Mathematics., vol. 157, no. 13, pp. 2828-2835.

[3] Durgi, B. S., Hampiholi, P. R., (2013), On the spectra and energy of some cluster graphs, Int. J. Math. Sci. Engg. Appl 7(2), 219-228.

[4] Durgi, B. S., Ramane, S., Hampiholi, R., Mekkalike, S. M., (2016), On the Wiener Index of Some Edge Deleted Graphs, Iranian Journal of Mathematical Sciences and Informatics, 11 (2), pp. 139-148.

[5] Durgi, B. S., Mekkalike, S. M., Ramane, H. S., Hande, S. P., (2016), On the Zagreb indices of semi-total point graphs of some graphs, Annals of Pure and Applied Mathematics, 12 (1), pp. 49-57.

[6] Durgi, B. S., Mekkalike, S. M., Hosamani, S. M., (2020), On the modified Weiner index and its chemical applicability, Advances in Mathematics: Scientific Journal, 9, pp. 7689–7698.

[7] Zhou, B. and Trinajstic, N., (2010), On general sum-connectivity index, Journal of Mathematical Chemistry, 47(1), pp. 210-218.

[8] Das K. C., (2010), On comparing Zagreb indices of graphs. MATCH Commun Math Comput Chem,63: 433-440 12.

[9] Vukicević, D., Furtula, B., (2009), Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges, Journal of Mathematical Chemistry, 46(4), pp. 1369-1376.

[10] Estrada, E., Torres, L., Rodriguez, L.and Gutman, I., (1998), An atom bond connectivity index: modelling the enthalpy of formation of alkanes, Indian Journal of Chemistry-Section A: Inorganic, Physical, Theoretical and Analytical Chemistry, 37(10), pp. 849-855.

[11] Harary, F., (1969), Graph Theory, Addison-Wesley Publishing Company, Boston.

[12] Gao, Y., Liang, L., Gao, W., (2015), Fourth ABC Index and Fifth GA Index of Certain Special Molecular Graphs. Int. J. Eng. Appl. Sci. 2, 6-12.

[13] Ghorbani,M. and HosseinzadehM.A., (2010), Computing ABC_4 index of nanostar dendrimers, Optoelectron. Adv. Mater.-Rapid Commun., 4(9), pp. 1419-1422.

[14] Graovac,A., Ghorbani,M. and Hosseinzadeh,M.A., (2011), Computing fifth geometric-arithmetic index for nanostar dendrimers., J. Math. Nanosci, 1, pp. 33-42.

[15] Gutman I, Ruscić B, Trinajstić N, Wilcox C F., (1975), Graph theory and molecular orbitals. XII. Acyclic polyenes, J Chem Phys, 62, pp. 3399-3405.

[16] Gutman I, Trinajstić N., (1972), Graph theory and molecular orbitals. Total pi-electron energy of alternant hydrocarbons, Journal of Chem Phys Lett, 17(4), pp. 535-538.

[17] M. Salaheldeen Abdelgader, C. Wang, and S. Abdalrhman Mohammed, (2018), Computation of topological indices of some special graphs, Mathematics, vol. 6, no. 3, p. 33.

[18] Mukhamedov, F.; Rozikov, U., (2005), On inhomogeneous p-adic Potts model on a Cayley tree. Infin. Dimens, Anal. Quantum Probab. Relat. Top., 8, pp. 277-290.

[19] Nilanjan De, (2019), General Zagreb indices of some cactus chains, Open Journal of Discrete Applied Mathematics, Vol. 2, Issue 1, pp. 24-31.

[20] Prashant V. Patil, Girish G. Yattinahalli, (2020), Second Zagreb indices of transformation graohs and total transformation graphs, Open Journal of Discrete Applied Mathematics, Vol. 3, Issue 1, pp. 1-7.

[21] Randić,M., (1974), On characterization of molecular branching, J. Am. Chem. Soc., 97, pp. 6609-6615.

[22] Trinajstić, N., (1992), Chemical Graph Theory, 2nd ed.; CRC Press: Boca Raton, FL, USA.

[23] Vukicevic,D. and Furtula,B., (2009), Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges, J. Math. Chem., 46, pp. 1369-1376.

[24] Wan Nor Nabila Nadia Wan Zuki, Zhibin Du, Muhammad Kamran Jamil, Roslan Hasni, (2020), Extremal Trees with Respect to the Difference between Atom-Bond Connectivity Index and Randic Index, Symmetry,12,1591; doi:10.3390/sym12101591.

[25] Wiener, H., (1947), Structural determination of paraffin boiling points, J. Am. Chem. Soc., 69, pp. 17-20.

Umesh Poojari has obtained his UG and PG from Karnataka University Dharwad. He was awarded gold medal in M.Sc. He is pursuing Ph.D. under the guidance of Dr. B. S. Durgi under Visvesvaraya Technological University Belagavi, Karnataka, India. His area of research is Graph theory. Currently, he is working as an Assistant Professor in the department of Mathematics at Vishwanathrao Deshpande Institute of Technology, Haliayal, Karnataka, India. He has the Teaching experience of 10 years.

B. S. Durgi has obtained his UG and PG from Karnatak University Dharwad, India and PhD form Visweswaraya Technological University, Karnataka, India. Currently, he is Associate Professor at KLE Dr. M. S. Sheshgiri College of Engineering and Technology. He has the Teaching experience of 20 years. His area of research includes Graph theory, spectra of graphs, Energy of graphs, Topological indices.

Raghu M Banakar received his M.Sc. from Karnataka University Dharwad in 2012. He is at present working as an Assistant Professor in Department of Mathematics, Siddaganga Institute of Technology, Tumakuru. His area of research is Graph theory..