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SOMEWHAT NEUTROSOPHIC δ-IRRESOLUTE CONTINUOUS MAPPINGS IN NEUTROSOPHIC TOPOLOGICAL SPACES

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Abstract. One of the strongest form in a neutrosophic open sets is a neutrosophic δ-open sets. Based on this open sets, the continuous mapping, open mapping and closed mapping functions are already defined in neutrosophic topological spaces. Also, the extension of these mappings called somewhat neutrosophic δ-continuous (open) mapping functions are defined in previous paper. The concept of somewhat neutrosophic δ-irresolute continuous (open) mapping which is stronger than a somewhat neutrosophic δ-continuous (open) mapping in a netrosophic topological spaces have been introduced and studied in this paper. Alongside, some interesting properties of those mappings are given in neutrosophic topological spaces.

Keywords: Neutrosophic δ-open set, Somewhat neutrosophic δ-irresolute continuous mapping, Somewhat neutrosophic irresolute δ-open mapping.

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1. Introduction

In mathematics, concept of fuzzy set between the real standard intervals was first introduced by Zadeh [28] in discipline of logic and set theory. The general topology has been framework with fuzzy set was undertaken by Chang [4] as fuzzy topological space. In 1983, Atanassov [3] initiated intuitionistic fuzzy set which is a combination of membership and non-membership values. Coker [5] created intuitionistic fuzzy set in a topology entitled as intuitionistic fuzzy topological space. In classical topology, the class of somewhat continuous functions was introduced and studied by Karl. R. Gentry and Hughes B. Hoyle [12]. Later, the concept of somewhat in classical topology has been extended to fuzzy topological spaces. Besides giving characterizations of these functions, several interesting properties of these functions are studied. M. N. Mukherjee and S. P. Shina in [15] was introduced and studied the concept of fuzzy irresolute continuous mappings on a fuzzy topological space. The concepts of somewhat fuzzy irresolute continuous mappings [10],

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somewhat fuzzy γ-irresolute continuous mappings [11] and somewhat fuzzy α-irresolute continuous mappings [12] on a fuzzy topological space are successively introduced and studied by Y. B. and others.

The new concepts of neutrosophy and neutrosophic set was introduced Smarandache [21, 22] at the beginning of 20^{th} century which has truth value, neutral value and false value. It has laid the foundation for a whole family of new mathematical theories in both crisp set and fuzzy set. Salama and Alblowi [17, 18] in 2012, originated neutrosophic set in a neutrosophic topological space. This gave the way for investigation in terms of neutrosophic topology and its application in decision making problems [13, 20, 23]. In [2], the properties of neutrosophic open sets, neutrosophic closed sets, neutrosophic interior operator and neutrosophic closure operator such as semi open, pre open, α open and their mappings gave the way for applying neutrosophic topology. M.L. Thivagar et al. [14] introduced the notion of N_n-open (closed) sets and N-neutrosophic topological spaces and R.K Al-Hamido et al. [1] introduced Neutrosophic crisp topology via N-Topology. Neutrosophic closed sets as well as Neutrosophic continuous mappings were developed in [19]. R. Dhavaseelan et al. [6, 7, 8, 9] introduced generalized neutrosophic closed sets, neutrosophic generalized contra continuous, neutrosophic generalized α-contra continuous and neutrosophic Almost α-contra-continuous functions and studied their properties. Recently, Rajesh et al. [16] introduced Neutrosophic pre irresolute functions via α and β.

S. Saha [24] defined δ-open sets in fuzzy topological spaces. Recently, Vadivel et al. introduced the neutrosophic regular open sets, neutrosophic δ open sets and neutrosophic δ continuous functions in [25]. In his paper [26], neutrosophic δ-open mappings and neutrosophic δ-closed mappings are introduced. Also, the concept of somewhat neutrosophic δ-continuous functions and somewhat neutrosophic δ-open sets are introduced and studied in [27].

The main work in this paper is introduced the concepts of somewhat neutrosophic δ-irresolute continuous mappings in a neutosophic topological spaces in third section. Also, in section four, we discussed about somewhat neutrosophic irresolute δ-open mappings on a neutrosophic topological spaces and further some properties of the mappings are also discussed.

2. Preliminaries

Definition 2.1. [17] Let Y be a non-empty set. A neutrosophic set (briefly, $N_s s$) L is an object having the form $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$ where $\mu_L \to [0,1]$ denote the degree of membership function, $\sigma_L \to [0,1]$ denote the degree of indeterminacy function and $\nu_L \to [0,1]$ denote the degree of non-membership function respectively of each element $y \in Y$ to the set L and $0 \leq \mu_L(y) + \sigma_L(y) + \nu_L(y) \leq 3$ for each $y \in Y$.

Remark 2.1. [17] A $N_s s$ $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$ can be identified to an ordered triple $\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle$ in $[0,1]$ on Y.

Definition 2.2. [17] Let Y be a non-empty set and the $N_s s$'s L and M in the form $L = \{\langle y, \mu_L(y), \sigma_L(y), \nu_L(y)\rangle : y \in Y\}$, $M = \{\langle y, \mu_M(y), \sigma_M(y), \nu_M(y)\rangle : y \in Y\}$, then

(i) $0_N = \langle y, 0, 0, 1\rangle$ and $1_N = \langle y, 1, 1, 0\rangle$,
(ii) $L \subseteq M$ iff $\mu_L(y) \leq \mu_M(y)$, $\sigma_L(y) \leq \sigma_M(y)$ & $\nu_L(y) \geq \nu_M(y) : y \in Y$,
(iii) $L = M$ iff $L \subseteq M$ and $M \subseteq L$,
(iv) $1_N - L = \{\langle y, \nu_L(y), 1 - \sigma_L(y), \mu_L(y)\rangle : y \in Y\} = L^c$,
(v) $L \cup M = \{\langle y, \max(\mu_L(y), \mu_M(y)), \max(\sigma_L(y), \sigma_M(y)), \min(\nu_L(y), \nu_M(y))\rangle : y \in Y\}$,
(vi) $L \cap M = \{\langle y, \min(\mu_L(y), \mu_M(y)), \min(\sigma_L(y), \sigma_M(y)), \max(\nu_L(y), \nu_M(y))\rangle : y \in Y\}$.

Definition 2.3. [17] A neutrosophic topology (briefly, N_st) on a non-empty set Y is a family Ψ_N of neutrosophic subsets of Y satisfying

(i) 0_N, $1_N \in \Psi_N$.
(ii) $L_1 \cap L_2 \in \Psi_N$ for any $L_1, L_2 \in \Psi_N$.
(iii) $\bigcup L_x \in \Psi_N, \forall \ L_x : x \in X \subseteq \Psi_N$.

Then (Y, Ψ_N) is called a neutrosophic topological space (briefly, N_sts) in Y. The Ψ_N elements are called neutrosophic open sets (briefly, N_sos) in Y. A N_ss C is called a neutrosophic closed sets (briefly, N_scs) iff its complement C^c is N_sos.

Definition 2.4. [17] Let (Y, Ψ_N) be N_sts on Y and L be an N_ss on Y, then the neutrosophic interior of L (briefly, $N_sint(L)$) and the neutrosophic closure of L (briefly, $N_scl(L)$) are defined as

$$N_sint(L) = \bigcup\{I : I \subseteq L \text{ and } I \text{ is a } N_sos \text{ in } Y\}$$

$$N_scl(L) = \bigcap\{J : L \subseteq J \text{ and } J \text{ is a } N_scs \text{ in } Y\}.$$

Definition 2.5. [2] Let (Y, Ψ_N) be N_sts on Y and L be an N_ss on Y. Then L is said to be a neutrosophic regular open set (briefly, N_sros) if $L = N_sint(N_scl(L))$.

The complement of a N_sros is called a neutrosophic regular closed set (briefly, N_srcs) in Y.

Definition 2.6. [25] A set K is said to be a neutrosophic

(i) δ interior of G (briefly, $N_s\delta int(K)$) is defined by $N_s\delta int(K) = \bigcup\{B : B \subseteq K \text{ and } B \text{ is a } N_sros \text{ in } Y\}$.
(ii) δ closure of K (briefly, $N_s\delta cl(K)$) is defined by $N_s\delta cl(K) = \bigcap\{J : K \subseteq J \text{ and } J \text{ is a } N_srcs \text{ in } Y\}$.

Definition 2.7. [25] A set L is said to be a neutrosophic δ-open set (briefly, $N_s\delta os$) if $L = N_s\delta int(L)$. The complement of an $N_s\delta os$ is called a neutrosophic δ closed set (briefly, $N_s\delta cs$) in Y.

Definition 2.8. [19] Let (Y, τ_N) and (Z, σ_N) be any two Nts's. A map $h : (Y, \tau_N) \to (Z, \sigma_N)$ is said to be a neutrosophic (resp. δ) continuous (briefly, N_sCts (resp. $N_s\delta Cts$)) iff inverse image of every N_sos in (Z, σ_N) is a N_sos (resp. $N_s\delta os$) in (Y, τ_N).

Definition 2.9. [26] Let (Y, τ_N) and (Z, σ_N) be any two Nts's. A map $h : (X, \tau_N) \to (Y, \sigma_N)$ is said to be neutrosophic (resp. δ) open map (briefly, N_sO (resp. $N_s\delta O$)) if the image of every N_sos in (Y, τ_N) is an N_sos (resp. $N_s\delta os$) in (Z, σ_N).

Definition 2.10. [27] A function $h : (X, \tau_N) \to (Y, \sigma_N)$ from a N_sts (X, τ_N) into another N_sts (Y, σ_N) is called somewhat neutrosophic δ-continuous (briefly, $swN_s\delta Cts$) if $\xi \in \sigma_N$ and $h^{-1}(\xi) \neq 0$ implies that there exist a $N_s\delta os$ ς in (X, τ_N) such that $\varsigma \neq 0$ and $\varsigma \leq h^{-1}(\xi)$. That is, $N_s\delta int[h^{-1}(\xi)] \neq 0$.

Definition 2.11. [27] A function $h : (X, \tau_N) \to (Y, \sigma_N)$ from a N_sts (X, τ_N) into another N_sts (Y, σ_N) is called somewhat neutrosophic δ-open (briefly, $swN_s\delta O$) function if $\xi \in \tau_N$ and $\xi \neq 0$ implies that there exists a $N_s\delta os$ ς in (Y, σ_N) such that $\varsigma \neq 0$ and $\varsigma \leq h(\xi)$. That is, $N_s\delta int[h(\xi)] \neq 0$.

Definition 2.12. [27] A N_ss ξ in a N_sts (X, τ_N) is called neutrosophic δ-dense (briefly, $N_s\delta D$) set if there exists no $N_s\delta cs$ ς in (X, τ_N) such that $\xi < \varsigma < 1$. That is, $N_s\delta cl(\xi) = 1$.

3. Somewhat neutrosophic δ-irresolute continuous mappings

Definition 3.1. A mapping $h : (Y, \tau_N) \to (Z, \sigma_N)$ is called neutrosophic δ-irresolute continuous (briefly, $N_s\delta irrCts$) if $h^{-1}(\varsigma)$ is a $N_s\delta os$ on (Y, τ_N) for any $N_s\delta os$ ς on (Z, σ_N).

Definition 3.2. A mapping $h : (Y, \tau_N) \to (Z, \sigma_N)$ is called somewhat neutrosophic δ-irresolute continuous (briefly, $swN_s\delta irrCts$) if there exists a $N_s\delta os$ $\xi \neq 0_N$ on (Y, τ_N) such that $\xi \leq h^{-1}(\varsigma) \neq 0_N$ for any $N_s\delta os$ $\varsigma \neq 0_Y$ on (Z, σ_N).

Remark 3.1. The implications contained in the following diagram are true and the reverse implications need not be true.

$$\begin{array}{c} \boxed{N_s\delta irrCts} \\ \downarrow \\ \boxed{swN_s\delta irrCts} \\ \downarrow \\ \boxed{N_s\delta Cts} \\ \downarrow \\ \boxed{swN_s\delta Cts} \end{array}$$

Figure 1. $swN_s\delta Cts$ function in Nts.

Example 3.1. Let $X = \{l, m, n\}$ & $Y = \{x, y, z\}$ and define $N_s s$'s X_1, X_2 and X_3 in X and Y_1, Y_2 and Y_3 in Y by

$$\begin{aligned}
X_1 &= \langle X, (\frac{\mu_l}{0.3}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.3}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.7}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.7})\rangle, \\
X_2 &= \langle X, (\frac{\mu_l}{0.7}, \frac{\mu_m}{0.7}, \frac{\mu_n}{0.7}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.3}, \frac{\nu_m}{0.3}, \frac{\nu_n}{0.3})\rangle, \\
X_3 &= \langle X, (\frac{\mu_l}{0.5}, \frac{\mu_m}{0.5}, \frac{\mu_n}{0.5}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.5}, \frac{\nu_m}{0.5}, \frac{\nu_n}{0.5})\rangle, \\
Y_1 &= \langle Y, (\frac{\mu_x}{0.2}, \frac{\mu_y}{0.3}, \frac{\mu_z}{0.2}), (\frac{\sigma_x}{0.5}, \frac{\sigma_y}{0.5}, \frac{\sigma_z}{0.5}), (\frac{\nu_x}{0.8}, \frac{\nu_y}{0.7}, \frac{\nu_z}{0.8})\rangle, \\
Y_2 &= \langle Y, (\frac{\mu_x}{0.8}, \frac{\mu_y}{0.7}, \frac{\mu_z}{0.8}), (\frac{\sigma_x}{0.5}, \frac{\sigma_y}{0.5}, \frac{\sigma_z}{0.5}), (\frac{\nu_x}{0.2}, \frac{\nu_y}{0.3}, \frac{\nu_z}{0.2})\rangle, \\
Y_3 &= \langle Y, (\frac{\mu_x}{0.5}, \frac{\mu_y}{0.5}, \frac{\mu_z}{0.5}), (\frac{\sigma_x}{0.5}, \frac{\sigma_y}{0.5}, \frac{\sigma_z}{0.5}), (\frac{\nu_x}{0.5}, \frac{\nu_y}{0.5}, \frac{\nu_z}{0.5})\rangle.
\end{aligned}$$

Then we have $\tau_N = \{0_N, X_1, X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, Y_2, Y_3, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be defined by

$$h(l) = y, h(m) = y, h(n) = y.$$

Then we have $h^{-1}(Y_1) = X_1$, $X_1 \leq h^{-1}(Y_2) = X_2$ and $X_1 \leq h^{-1}(Y_3) = X_3$. Since X_1 is a $N_s\delta os$ on (X, τ_N), h is $swN_s\delta irrCts$. But $h^{-1}(Y_3) = X_3$ is not $N_s\delta os$ on (X, τ_N). Thus h is not a $N_s\delta irrCts$ mapping.

Example 3.2. Let $X = \{l, m, n\}$ & $Y = \{x, y, z\}$ and define $N_s s$'s X_1 in X and Y_1 in Y by

$$X_1 = \langle X, (\frac{\mu_l}{0.2}, \frac{\mu_m}{0.2}, \frac{\mu_n}{0.2}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.8}, \frac{\nu_m}{0.8}, \frac{\nu_n}{0.8})\rangle,$$
$$Y_1 = \langle Y, (\frac{\mu_x}{0.4}, \frac{\mu_y}{0.0}, \frac{\mu_z}{0.4}), (\frac{\sigma_x}{0.5}, \frac{\sigma_y}{0.5}, \frac{\sigma_z}{0.5}), (\frac{\nu_x}{0.6}, \frac{\nu_y}{1.0}, \frac{\nu_z}{0.6})\rangle.$$

Then we have $\tau_N = \{0_N, X_1, X_1^c, 1_N\}$ and $\sigma_N = \{0_N, Y_1, Y_1^c, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be defined by

$$h(l) = y, h(m) = y, h(n) = y.$$

Then we have $h^{-1}(Y_1) = 0_N$ and $h^{-1}({Y_1}^c) = 1_N$, h is $N_s\delta Cts$. But for a $N_s\delta os$ $Y_1 = 0_N$ on Y, $h^{-1}(Y_1) = 0_N$. Thus h is not a $swN_s\delta irrCts$ mapping.

Example 3.3. In Example 3.1, for a $N_s os$ on Y, $h^{-1}(Y_1) = X_1, X_1 \leq h^{-1}(Y_2) = X_2$ and $X_1 \leq h^{-1}(Y_3) = X_3$. Since X_1 is a $N_s\delta os$ on X, h is $swN_s\delta Cts$. But $h^{-1}(Y_3) = X_3$ is not a $N_s\delta os$ on X. Thus h is not a $N_s\delta Cts$ mapping.

Theorem 3.1. Let $h : (Y, \tau_N) \to (Z, \sigma_N)$ be a mapping. Then the statements

(i) h is $swN_s\delta irrCts$.
(ii) If ς is a $N_s\delta cs$ of $Z \ni h^{-1}(\varsigma) \neq 1_Y$ then there exists a $N_s\delta cs$ $\xi \neq 1_Y$ of $Y \ni h^{-1}(\varsigma) \leq \xi$.
(iii) If ξ is a $N_s\delta D$ set on Y, then $h(\xi)$ is a $N_s\delta D$ set on Z

are equivalent.

Proof. (i) $\Rightarrow$ (ii): Let ς be a $N_s\delta cs$ on $Z \ni h^{-1}(\varsigma) \neq 1_Y$. Then ς^c is a $N_s\delta os$ on Z and $h^{-1}(\varsigma^c) = h^{-1}(\varsigma)^c \neq 0_Y$. Since h is $swN_s\delta irrCts$, $\exists$ $N_s\delta os$ $\lambda \neq 0_Y$ on $Y \ni \lambda \leq h^{-1}(\varsigma)^c$. Let $\xi = \lambda^c$. Then $\xi \neq 1_Y$ is $N_s\delta C \ni$

$$h^{-1}(\varsigma) = 1 - h^{-1}(\varsigma)^c \leq 1 - \xi^c = \xi.$$

(ii) $\Rightarrow$ (iii): Let ξ be a $N_s\delta D$ set on Y and suppose $h(\xi)$ is not $N_s\delta D$ set on Z. Then $\exists$ $N_s\delta cs$ ς on $Z \ni h(\xi) < \varsigma < 1$. Since $\varsigma < 1$ and $h^{-1}(\varsigma) \neq 1_Y$, there exists a $N_s\delta cs$ $\eta \neq 1_X$ $\ni$

$$\xi \leq h^{-1}(h(\xi)) < h^{-1}(\varsigma) \leq \eta.$$

$\not\Rightarrow$ ξ is a $N_s\delta D$ set on Y. Thus $h(\xi)$ is a $N_s\delta D$ set on Z.

(iii) $\Rightarrow$ (i): Let $\varsigma \neq 0_Z$ be a $N_s\delta os$ on Z and $h^{-1}(\varsigma) \neq 0_Y$. Suppose there exists no $N_s\delta o$ $\xi \neq 0_Y$ on $Y \ni \xi \leq h^{-1}(\varsigma)$. Then $(h^{-1}(\varsigma)^c)$ is a $N_s s$ on $Y \ni$ there is no $N_s\delta cs$ η on Y with $(h^{-1}(\varsigma)^c) < \eta < 1$. In fact, if $\exists$ $N_s\delta os$ $\delta^c \ni \delta^c \leq h^{-1}(\varsigma)$, then it is a contradiction. Thus $(h^{-1}(\varsigma))^c$ is a $N_s\delta D$ set on Y. So $h((h^{-1}(\varsigma))^c)$ is a $N_s\delta D$ set on Z. But $h((h^{-1}(\varsigma))^c) = h((h^{-1}(\varsigma))^c) \neq \varsigma^c < 1$. This contradicts to the fact that $h((h^{-1}(\varsigma))^c)$ is $N_s\delta D$ on Z. Hence $\exists$ $N_s\delta os$ $\xi \neq 0_Y$ on $Y \ni \xi \leq h^{-1}(\varsigma)$. Consequently, h is $swN_s\delta irrCts$. $\square$

Theorem 3.2. Let Y_1 be product related to Y_2 and let Z_1 be product related to Z_2. Then the product $h_1 \times h_2 : Y_1 \times Y_2 \to Z_1 \times Z_2$ of $swN_s\delta irrCts$ mappings $h_1 : Y_1 \to Z_1$ and $h_2 : Y_2 \to Z_2$ is also $swN_s\delta irrCts$.

Proof. Let $\lambda = \bigvee_{i,j} (\xi_i \times \varsigma_j)$ be a $N_s\delta os$ on $Z_1 \times Z_2$ where $\xi_i \neq 0_{Z_1}$ and $\varsigma_j \neq 0_{Z_2}$ are $N_s\delta os$'s on Z_1 and Z_2 respectively. Then

$$(h_1 \times h_2)^{-1}(\lambda) = \bigvee_{i,j} (h_1^{-1}(\xi_i) \times h_2^{-1}(\varsigma_j)).$$

since h_1 is $swN_s\delta irrCts$, $\exists$ $N_s\delta os$ $\delta_i \neq 0_{Y_1} \ni \delta_i \leq h_1^{-1}(\xi_i) \neq 0_{Y_1}$. And, since h_2 is $swN_s\delta irrCts$, $\exists$ $N_s\delta os$ $\eta_j \neq 0_{Y_2}$ such that $\eta_j \leq h_2^{-1}(\varsigma_j) \neq 0_{Y_2}$. Now $\delta_i \times \eta_j \leq h_1^{-1}(\xi_i) \times$

$h_2^{-1}(\varsigma_j) = (h_1 \times h_2)^{-1}(\xi_i \times \varsigma_j)$ and $\delta_i \times \eta_j \neq 0_{Y_1} \times 0_{Y_2}$ is a $N_s\delta os$ on $Y_1 \times Y_2$. Thus $\bigvee_{i,j}(\delta_i \times \eta_j) \neq 0_{Y_1} \times 0_{Y_2}$ is a $N_s\delta os$ on $Y_1 \times Y_2 \ni$

$$\begin{aligned}\bigvee_{i,j} (\delta_i \times \eta_j) &\leq \bigvee_{i,j} \left(h_1^{-1}(\xi_i) \times h_2^{-1}(\xi_i)\right) \\ &= (h_1 \times h_2)^{-1} \left(\bigvee_{i,j} (\xi_i \times \varsigma_j)\right) \\ &= (h_1 \times h_2)^{-1}(\lambda) \neq 0_{Y_1 \times Y_2}\end{aligned}$$

So, $h_1 \times h_2$ is $swN_s\delta irrCts$. □

Theorem 3.3. Let $h : (Y, \tau_N) \to (Z, \sigma_N)$ be a mapping. If the graph $g_\delta : Y \to Y \times Z$ of h is a $swN_s\delta irrCts$ mapping, then h is also $swN_s\delta irrCts$.

Proof. Let ς be a $N_s\delta os$ on Z. Then $h^{-1}(\varsigma) = 1 \wedge h^{-1}(\varsigma) = g_\delta^{-1}(1 \times \varsigma)$. Since g_δ is $swN_s\delta irrCts$ and $(1 \times \varsigma)$ is a $N_s\delta os$ on $Y \times Z$, $\exists$ $N_s\delta os$ $\xi \neq 0_Y$ on $Y \ni$

$$\xi \leq g_\delta^{-1}(1 \times \varsigma) = h^{-1}(\varsigma) \neq 0_X$$

Thus, h is $swN_s\delta irrCts$. □

4. Somewhat neutrosophic irresolute δ-open mappings

Definition 4.1. A mapping $h : (Y, \tau_N) \to (Z, \sigma_N)$ is called neutrosophic irresolute δ-open (briefly, $N_s irr\delta O$) if $h(\xi)$ is a $N_s\delta os$ on Z for any $N_s\delta os$ ξ on Y.

Definition 4.2. A fuzzy mapping $h : (Y, \tau_N) \to (Z, \sigma_N)$ is called somewhat neutrosophic irresolute δ-open (briefly, $swN_s irr\delta O$) if there exists a $N_s\delta os$ $\varsigma \neq 0_Z$ on Z such that $\varsigma \leq h(\xi) \neq 0_Z$ for any $N_s\delta os$ $\xi \neq 0_Y$ on Y.

Theorem 4.1. Let $h : (Y, \tau_N) \to (Z, \sigma_N)$ be a bijection. Then the statements

- (i) h is $swN_s irr\delta O$.
- (ii) If ξ is a $N_s\delta cs$ on Y such that $h(\xi) \neq 1_Z$, then there exists a $N_s\delta cs$ $\varsigma \neq 1_Z$ on Z such that $h(\xi) < \varsigma$

are equivalent.

Proof. (i) $\Rightarrow$ (ii): Let ξ be a $N_s\delta cs$ on Y such that $h(\xi) \neq 1_Z$. Since h is bijective and ξ^c is a $N_s\delta os$ on Y, $h(\xi^c) = (h(\xi))^c \neq 0_Z$. And, since h is $swN_s irr\delta O$, $\exists$ $N_s\delta os$ $\delta \neq 0_Z$ on Z $\ni \delta < h(\xi^c) = (h(\xi))^c$. Consequently, $h(\xi) < \delta^c = \varsigma \neq 1_Z$ and ς is a $N_s\delta cs$ on Z.

(ii) $\Rightarrow$ (i): Let ξ be a $N_s\delta os$ on Y such that $h(\xi) \neq 0_Z$. Then ξ^c is a $N_s\delta cs$ on Y and $h(\xi^c) \neq 1_Z$. Thus there exists a $N_s\delta cs$ $\varsigma \neq 1_Z$ on Z such that $h(\varsigma^c) < \varsigma$. Since h is bijective, $h(\xi^c) = (h(\xi))^c < \varsigma$. So $\varsigma^c < h(\xi)$ and $\varsigma^c \neq 0_Y$ is a $N_s\delta os$ on Z. Hence, h is $swN_s irr\delta O$. □

Remark 4.1. The implications contained in the following diagram are true and the reverse implications need not be true.

$$\begin{tikzcd}
\boxed{N_s irr\delta O \text{ mapping}} \arrow[d] \\
\boxed{swN_s irr\delta O \text{ mapping}} \arrow[d] \\
\boxed{N_s\delta O \text{ mapping}} \arrow[d] \\
\boxed{swN_s\delta O \text{ mapping}}
\end{tikzcd}$$

FIGURE 2. $swN_s\delta O$ mapping in N_sts.

Example 4.1. Let $X = \{l, m, n\}$ & $Y = \{x, y, z\}$ and define N_s's X_1 and X_2 in X and Y_1 and Y_2 in Y by

$$\begin{aligned}
X_1 &= \langle X, (\frac{\mu_l}{0.3}, \frac{\mu_m}{0.3}, \frac{\mu_n}{0.3}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.7}, \frac{\nu_m}{0.7}, \frac{\nu_n}{0.7})\rangle,\\
X_2 &= \langle X, (\frac{\mu_l}{0.7}, \frac{\mu_m}{0.7}, \frac{\mu_n}{0.7}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.3}, \frac{\nu_m}{0.3}, \frac{\nu_n}{0.3})\rangle,\\
Y_1 &= \langle Y, (\frac{\mu_x}{0.0}, \frac{\mu_y}{0.3}, \frac{\mu_z}{0.0}), (\frac{\sigma_x}{0.5}, \frac{\sigma_y}{0.5}, \frac{\sigma_z}{0.5}), (\frac{\nu_x}{1.0}, \frac{\nu_y}{0.7}, \frac{\nu_z}{1.0})\rangle,\\
Y_2 &= \langle Y, (\frac{\mu_x}{0.0}, \frac{\mu_y}{0.7}, \frac{\mu_z}{0.0}), (\frac{\sigma_x}{0.5}, \frac{\sigma_y}{0.5}, \frac{\sigma_z}{0.5}), (\frac{\nu_x}{1.0}, \frac{\nu_y}{0.3}, \frac{\nu_z}{1.0})\rangle.
\end{aligned}$$

Then we have $\tau_N = \{0_N, X_1, X_2, 1_N\}$ and $\sigma_N = \{0_N, Y_1, Y_2, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be defined by

$$h(l) = y, h(m) = y, h(n) = y.$$

Then we have $Y_1 \leq h(X_1) = Y_1$ and $Y_2 \leq h(X_2) = Y_2$. Since Y_1 is a $N_s\delta os$ on (Y, σ_N), h is $swN_s irr\delta O$. But for a $N_s\delta os$ X_2 on X, $h(X_2) = Y_2$ is not $N_s\delta os$ on (Y, σ_N). Thus h is not a $N_s irr\delta O$ mapping.

Example 4.2. Let $X = \{l, m, n\}$ & $Y = \{x, y, z\}$ and define N_s's X_1 and X_2 in X and Y_1 in Y by

$$\begin{aligned}
X_1 &= \langle X, (\frac{\mu_l}{0.3}, \frac{\mu_m}{0.0}, \frac{\mu_n}{0.3}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.7}, \frac{\nu_m}{1.0}, \frac{\nu_n}{0.7})\rangle,\\
X_2 &= \langle X, (\frac{\mu_l}{0.5}, \frac{\mu_m}{0.5}, \frac{\mu_n}{0.4}), (\frac{\sigma_l}{0.5}, \frac{\sigma_m}{0.5}, \frac{\sigma_n}{0.5}), (\frac{\nu_l}{0.5}, \frac{\nu_m}{0.5}, \frac{\nu_n}{0.6})\rangle,\\
Y_1 &= \langle Y, (\frac{\mu_x}{0.5}, \frac{\mu_y}{0.5}, \frac{\mu_z}{0.5}), (\frac{\sigma_x}{0.5}, \frac{\sigma_y}{0.5}, \frac{\sigma_z}{0.5}), (\frac{\nu_x}{0.5}, \frac{\nu_y}{0.5}, \frac{\nu_z}{0.5})\rangle.
\end{aligned}$$

Then we have $\tau_N = \{0_N, X_1, X_1^c, 1_N\}$ and $\sigma_N = \{0_N, Y_1, 1_N\}$. Let $h : (X, \tau_N) \to (Y, \sigma_N)$ be defined by

$$h(l) = y, h(m) = y, h(n) = y.$$

Since $h(X_1) = 0_N$, $h(X_1{}^c) = 1_N$ and $h(X_2) = Y_1$ are $N_s\delta os$'s on (Y, σ_N), h is $N_s\delta o$. But for a $N_s\delta os$ X_1, $h(X_1) = 0_N$. Thus h is not a $swN_s irr\delta O$ mapping.

Example 4.3. In Example 4.1, we have $Y_1 \leq h(X_1) = Y_1$ and $Y_2 \leq h(X_2) = Y_2$. Since Y_1 is a $N_s\delta os$ on X, h is $swN_s\delta O$ mapping. But $h(X_2) = Y_2$ is not $N_s\delta os$ on (Y, σ_N). Thus h is not a $N_s\delta O$ mapping.

Theorem 4.2. Let $h : (Y, \tau_N) \to (Z, \sigma_N)$ be a surjection. Then the statements

(i) h is $swN_s irr\delta O$.
(ii) If ς is a $N_s\delta D$ set on Z, then $h^{-1}(\varsigma)$ is a $N_s\delta D$ set on Y
are equivalent.

Proof. (i) $\Rightarrow$ (ii): Let ς be a $N_s\delta D$ set on Z. Suppose $h^{-1}(\varsigma)$ is not $N_s\delta D$ on Y. Then $\exists$ $N_s\delta cs$ ξ on Y such that $h^{-1}(\varsigma) < \xi < 1$. Since h is $swN_s irr\delta O$ and ξ^c is a $N_s\delta os$ on Y, $\exists$ $N_s\delta os$ $\delta \neq 0_Z$ on $Z \ni \delta \leq h\left(N_S int\xi^c\right) \leq h\left(\xi^c\right)$. Since h is surjective, $\delta \leq h\left(\xi^c\right) < h\left(h^{-1}\left(\varsigma^c\right)\right) = \varsigma^c$. Thus $\exists$ $N_s\delta cs$ δ^c on Z such that $\varsigma < \delta^c < 1$. This is a contradiction. So $h^{-1}(\varsigma)$ is $N_s\delta D$ on Y.

(ii) $\Rightarrow$ (i): Let ξ be a $N_s\delta os$ on Y and $h(\xi) \neq 0_Z$. Suppose $\exists$ no $N_s\delta O$ $\varsigma \neq 0_Z$ on $Z \ni \varsigma \leq h(\xi)$. Then $(h(\xi))^c$ is a $N_s s$ on $Z \ni \exists$ no $N_s\delta cs$ δ on Z with $(h(\xi))^c < \delta < 1$. This means that $(h(\xi))^c$ is $N_s\delta D$ on Z. Thus $h^{-1}\left((h(\xi))^c\right)$ is $N_s\delta D$ on Y. But $h^{-1}\left((h(\xi))^c\right) = \left(h^{-1}(h(\xi))\right)^c \leq \xi^c < 1$. This contradicts to the fact that $h^{-1}(h(\xi))^c$ is $N_s\delta D$ on Y. So, hence $\exists$ $N_s\delta os$ $\varsigma \neq 0_Z$ on $Z \ni \varsigma \leq h(\xi)$. Hence, h is $swN_s irr\delta O$. □

5. Conclusions

We have studied the concepts of somewhat neutrosophic δ-irresolute continuous mappings in $N_s ts$ is introduced and studied in this paper. Also, we discuss about a somewhat neutrosophic irresolute δ-open mappings in a $N_s ts$. In future, neutrosophic δ-irresolute mapping in a $N_s ts$ can be extended. Also, this can be carried out to be neutrosophic $\delta\mathcal{S}$ mapping, neutrosophic $\delta\mathcal{P}$ mapping, neutrosophic $\delta\beta$ mapping such as continuous and irresolute in a neutrosophic topological spaces.

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References

[1] Al-Hamido, R.K., Gharibah, T., Jafari, S. and Smarandache, F., (2018), On neutrosophic crisp topology via N-topology, Neutrosophic Sets and Systems, 23, pp. 96-109.
[2] Arokiarani, I., Dhavaseelan, R., Jafari, S. and Parimala, M., (2017), On some new notions and functions in neutrosophic topological spaces, Neutrosophic Sets and Systems, 16, pp. 16-19 .
[3] Atanassov, K., (1986), Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20, pp. 87-96.
[4] Chang, C.L., (1968), Fuzzy topological spaces, J. Math. Anal. Appl., 24, pp. 182-190.
[5] Coker, D., (1997), An introduction to intuitionistic fuzzy topological spaces, Fuzzy sets and systems, 88, pp. 81-89.
[6] Dhavaseelan, R. and Jafari, S., (2017), Generalized Neutrosophic closed sets, New Trends in Neutrosophic Theory and Applications, 2, pp. 261-273.
[7] Dhavaseelan, R., Jafari, S., Ozel, C. and Al-Shumrani, M.A., (2017), Generalized Neutrosophic Contra-Continuity, New Trends in Neutrosophic Theory and Applications, 2, pp. 1-17.
[8] Dhavaseelan, R., Jafari, S. and Hanif PAGE, Md., (2018), Neutrosophic Generalized α-contra continuity, Creat. Math. Inform., 27 (2), pp. 133-139.
[9] Dhavaseelan, R. and Hanif PAGE, Md., (2019), Neutrosophic Almost α-contra continuous function, Neutrosophic Sets and Systems, 29, pp. 71-77.
[10] Im, Y.B., Lee, J.S. and Cho, Y.D., (2012), Somewhat fuzzy irresolute continuous mappings, Int. J. of Fuzzy Logic and Intelligent Systems, 12 (4), pp. 319-323.
[11] Im, Y.B., Lee, J.S. and Cho, Y.D., (2012), Somewhat fuzzy γ -irresolute continuous mappings, J. Appl. Math. & Informatics, 32, pp. 203-209.
[12] Im, Y.B., Lee, J.S. and Cho, Y.D., (2012), Somewhat fuzzy α -irresolute continuous mappings, Far East J. Math. Sci., 70, pp. 145-155.
[13] Jayaparthasarathy, G., Little Flower, V.F. and Arockia Dasan, M., (2019), Neutrosophic supra topological applications in data mining process, Neutrosophic Sets and Systems, 27, pp. 80-97.
[14] Lellis Thivagar, M., Jafari, S., Antonysamy, V. and Sutha Devi, V., (2018), The ingenuity of neutrosophic topology via N-topology, Neutrosophic Sets and Systems, 19, pp. 91-100.

[15] Mukherjee, M.N. and Sinha, S.P., (1989), Irresolute and almost open functions between fuzzy topological spaces, Fuzzy Sets and Systems, 29, pp. 381-388.
[16] Rajesh Kannan, T. and Chandrasekar, S., (2021), Neutrosophic Pre-α, Semi-α & Pre-β Irresolute Functions, Neutrosophic Sets and Systems, 39, pp. 70-85.
[17] Salama, A.A. and Alblowi, S.A., (2012), Neutrosophic Set and Neutrosophic Topological Spaces, IOSR Journal of Mathematics, 3 (4), pp. 31-35.
[18] Salama, A.A. and Smarandache, F., (2015), Neutrosophic crisp set theory, USA: Educational Publisher.
[19] Salama, A.A., Smarandache. F. and Kromov, V., (2014), Neutrosophic closed set and netrosophic continuous functions, Neutrosophic Sets and Systems, 4, pp. 04-08.
[20] Saranya, S. and Vigneshwaran, M., (2019), Design and Development of .Net Framework to Deal with Neutrosophic $*g\alpha$ Sets, International Journal of Engineering and Advanced Technology, 8 (3S), pp. 852-857.
[21] Smarandache, F., (1999), A Unifying field in logics: neutrosophic logic, neutrosophy, neutrosophic set, neutrosophic probability, Rehoboth: American Research Press.
[22] Smarandache, F., (2002), Neutrosophy and neutrosophic logic, 1st Int. Conf. on Neutrosophy, Neutrosophic Logic, Set, Probability and Statistics, University of New Mexico, Gallup, NM, 87301 USA.
[23] Smarandache, F., (2020), The Score, Accuracy, and Certainty Functions determine a Total Order on the Set of Neutrosophic Triplets (T, I, F), Neutrosophic Sets and Systems, 38, pp. 1-14.
[24] Saha, S., (1987), Fuzzy δ-continuous mappings, Journal of Mathematical Analysis and Applications, 126, 130-142.
[25] Vadivel, A., Seenivasan, M. and Sundar, C.J., (2021), An Introduction to δ-open sets in a Neutrosophic Topological Spaces, Journal of Physics: Conference Series, 1724, 012011.
[26] Vadivel, A. and Sundar, C.J., (2021), Neutrosophic δ-open maps and neutrosophic δ-closed maps, International Journal of Neutrosophic Science, 13 (2), pp. 66-74.
[27] Vadivel, A., Sundar, C.J., Seenivasan, M. and Thangaraja, P., Somewhat Neutrosopic δ-Continuous Functions in Neutrosophic Topological Spaces, Manuscript.
[28] Zadeh, L.A., (1965), Fuzzy sets, Information and control, 8, pp. 338-353.

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