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A NOVEL SIMPLE 5D HYPERCHAOTIC SYSTEM DERIVED FROM THE 3D SPROTT C SYSTEM

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Abstract. A novel simple 5D hyperchaotic system with two non-hyperbolic equilibria points is presented. The proposed system is designed by coupling between a 3D Sprott C system and a 2D linear system via a coupling strategy. Compared to the traditional systems, a new system is considered simply because it consists of five first-order ordinary differential equations with nine terms: seven linear terms and two quadratic nonlinearities. Due to the nature of equilibria points, this system belongs to the group of self-excited attractors. The attractors have been described as hyperchaotic. Finally, the projective synchronization problem of the new system has been realized through both Lyapunov stability theory and numerical simulation. The numerical simulation confirmed the validity of our analytical results. This work throws light on the great significance of the new system via designing controllers with the minimum terms possible are helpful in some practical applications.

Keywords: Sprott C system, equilibrium points, Lyapunov stability theory, Self-excited attractors, Multistability.

AMS Subject Classification: 34Cxx, 34C28

1. Introduction

In (1994), Sprott introduce a 3D system with a hidden attractor which is termed as Sprott A system [1], but the phenomenon of hidden attractor was not known until 2010 when Kuznetsov et al., introduced the first definition of hidden attractors. Thereafter, Leonov et al. worked on developing Cua's circuit[2],[3],[4]. The hidden attractors are divided according to the equilibrium points; (i) without equilibrium points [5],[6],[7], [8],[9], (ii) stable equilibrium points [10],[11], (iii) curve of equilibrium [12], [13], [14], [15], [16] and circles of equilibrium points[17]. An equilibrium point plays an important part in classifying the dynamical systems into two parts: hidden attractors and self-excited attractors [14].

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Historically, American meteorologist Lorenz finds the first chaotic attractor in the world [18]. Then 3D Rossler, 4D-Rossler, Chen and Liu, 4D Sprott S systems in 1976, 1979, 1999, 2004 and 2021, respectively are reported[19]. In 2018, Wang et al. presented a novel system by introducing a flux-controlled memristor-based new 5D hyperchaotic system, this system consists of seventeen terms, eight of them are non-linear [20], with two Lyapunov exponents (LEs) as $(+, +, -, -, -)$ i.e., vanished of zero Lyapunov exponents which are necessary for calculating these exponents [21].

Recently, Nguyen et al. created a new 5D hyperchaotic system with eleven terms, two nonlinearity, and has two positive Lyapunov exponents($+veLEs$), thus it fulfills the feature $(n-3)$[22]. But, in practical applications such as encryption [23], [24] and fractional ordinary differential equations [25], it is preferable to deal with systems that have $(n-2)+ve$ LEs [26]. Table (1) summarized many previous works for a 5D hyperchaotic system. Compared to these available systems in Table (1), it is clear that the new system has the fewest numbers of terms (nine terms only) and we believed it is one of the simple systems according to the third criterion of the scientist Sprott [27],[28],[29],[30].

In 1990, chaos control and chaos synchronization for the aforementioned simple systems have been intensively studied due to their usefulness in great potential in many aspects. Especially, chaos synchronization has received a lot of attention, and numerous phenomena synchronization discover such as complete synchronization (CS)[30], [31], anti- synchronization (AS)[32], [33], hybrid synchronization(HS)[34],[35], and projective synchronization (PS)[36]. Among these phenomena, projective synchronization has considerable generalization for complete synchronization, and anti- synchronization, whereas hybrid synchronization is considered a special form of hybrid projective synchronization(HPS). This paper deals with PS and a suitable control is designed to achieve synchronization between identical proposed systems.

Table 1. Results for distinct 5-D hyperchaotic systems.

No.	Nature of equilibria	Attractors behavior	Total of terms	No. $+veLE$s	References
1	Line of equilibrium	Hidden	11	$n-3$	2021 [22]
2	One stable saddle point	Hidden	13	$n-3$	2018 [11]
3	Curve of equilibrium points	Hidden	17	$n-3$	2019 [13]
4	Unstable saddle-node point	Self-excited	12	$n-2$	2009 [28]
5	Unstable/Line of equilibrium	Self-excited	17	$n-3$	2018 [20]
6	Unstable equilibria point	Self-excited	12	$n-2$	2013 [8]
7	No equilibria point	Hidden	15	$n-2$	2015 [6]
8	No equilibria point	Hidden	10	$n-3$	2016 [29]
9	Stable equilibria point	Hidden	12	$n-2$	2019 [3]
10	No equilibria point	Hidden	13	$n-3$	2019 [5]
11	Unstable equilibria point	Self-excited	9	$n-2$	This work

The main contribution for this paper is summarized as follows:

• A new 5D hyperchaotic system with three positive Lyapunov exponents is constructed from the well-known 3D Sprott C system.

• The proposed system is considered simple due to consists of (9) terms: two non-linear and one parameter.

• dynamical analysis for the 5D-hyperchaotic system is investigated including Jacobian matrix, equilibria points, and its categorized, Lyapunov exponent and Kaplan-Yorke dimension as shown in Table (1).
• projective synchronization of the new system was realized theoretical and numerical.
The structure of this paper is organized into six sections as follows. In Section 2, a new 4-D hyperchaotic system is introduced from the Sprott C system, whereas Section 3 deals with analyzing the dynamical properties of this system. Multistability and projective synchronization are given in Sections 4 and 5, respectively. Finally, the conclusion of this paper gives in Section 6.

2. Description of the new system

The researcher Sprott (1994) display nineteen systems which consider the simplest systems, one of them is a 3D Sprott C system which defined by a system of first order ordinary differential equations(ODE's) [1]:

$$\begin{cases} \dot{x} = yz \\ \dot{y} = x - y \\ \dot{z} = 1 - x^2 \end{cases} \tag{1}$$

Obviously, this system consists of five terms with one positive Lyapunov exponent $LE_1 = 0.163$ (largest exponent) such as ($LE_1 = 0.163$, $LE_2 = 0$, $LE_3 = -1.163$).
Relying on coupling strategy [37], a 2D system with four terms system is added to the system(1), which construct a novel simple 5D Sprott C hyperchaotic system as:

$$\begin{cases} \dot{x} = yz \\ \dot{y} = x - y \\ \dot{z} = 1 - x^2 \\ \dot{w} = -3s - x \\ \dot{s} = w - ds \end{cases} \tag{2}$$

where d is a positive parameter ($d \neq 0$) and it is called a coupling parameter. Clearly, the novel system consists of nine terms only i.e, a simple system. Thus, it meets one of Sprott conditions [27]. The proposed system has two non-hyperbolic equilibria points, therefore classified as self-excited attractors as shown in Fig. 1 and parameter d plays an important role in classifying the novel system into a dissipative or conservative system.

3. System analysis

Some of the basic properties of the new system (2) have been analyzed numerically and theoretically, i.e., equilibria points, dissipative, Lyapunov exponents, Kaplan-Yorke dimension, and sensitivity to the initial condition.

3.1. Equilibria Points.

Equilibria Points. To get the equilibria points [38] for system (2), let $\dot{x}, \dot{y}, \dot{z}, \dot{w}, \dot{s} = 0$ i.e.,

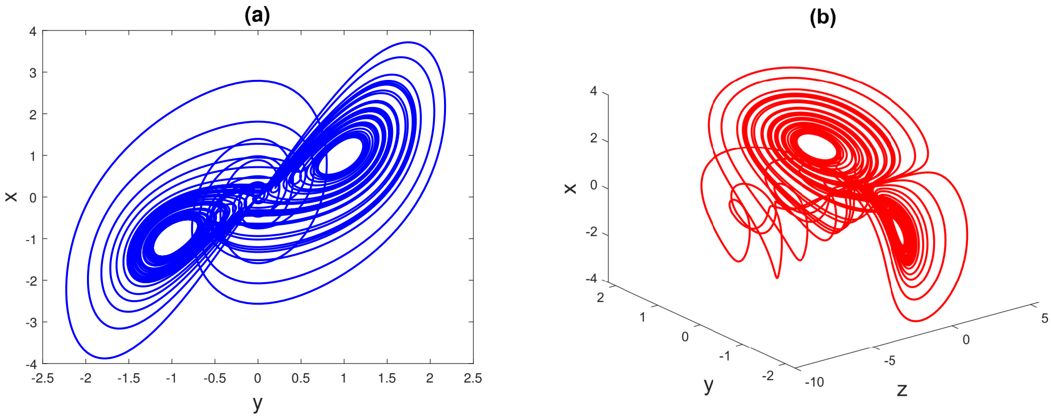

FIGURE 1. The phase portrait for the new system with $d = 0.998$ and IC $(0.2, 0.2, 0.1, 0.2, 0.2)$.

$$\begin{aligned} yz &= 0 \\ x - y &= 0 \\ 1 - x^2 &= 0 \\ -3s - x &= 0 \\ w - ds &= 0 \end{aligned} \tag{3}$$

Solving the above system, two equilibria points are obtain as E_1 (1, 1, 0, $d/3$,-1/3), E_2 (-1,-1, 0, $-d/3$, 1/3). The Jacobian matrix of a new system at E_1 and the characteristic equation is given in Eq.(4) and Eq.(5), respectively.

$$J = \begin{bmatrix} 0 & z & y & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ -2x & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & d \end{bmatrix} \overset{J(E_1)}{=\longrightarrow} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ -2 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & d \end{bmatrix} \tag{4}$$

$$\lambda^5 + \underbrace{(1-d)}_{A}\lambda^4 + \underbrace{(5-d)}_{B}\lambda^3 + \underbrace{(5-2d)}_{C}\lambda^2 + \underbrace{(6-2d)}_{D}\lambda + \underbrace{6}_{E} = 0 \tag{5}$$

There are two methods to determine the stability, theoretical (Routh-Hurwitz criteria), and numerical mathematical program based on MATLAB software. According to Routh-Hurwitz criteria and Eq.(5), Hurwitz matrix is defined:

$$H_5 = \begin{bmatrix} 1-d & 1 & 0 & 0 & 0 \\ 5-2d & 5-d & 1-d & 1 & 0 \\ 6 & 6-2d & 5-2d & 5-d & 1-d \\ 0 & 0 & 6 & 6-2d & 5-2d \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \tag{6}$$

Eq.(5) has eigenvalues with negative real part if the determinants of all Hurwitz matrices are positive i.e.,

$$\Delta_1 = \left|1 - d\right| > 0,$$

$$\Delta_2 = \begin{vmatrix} 1-d & 1 \\ 5-2d & 5-d \end{vmatrix} > 0,$$

$$\Delta_3 = \begin{vmatrix} 1-d & 1 & 0 \\ 5-2d & 5-d & 1-d \\ 6 & 6-2d & 5-2d \end{vmatrix} > 0,$$

$$\Delta_4 = \begin{vmatrix} 1-d & 1 & 0 & 0 \\ 5-2d & 5-d & 1-d & 1 \\ 6 & 6-2d & 5-2d & 5-d \\ 0 & 0 & 6 & 6-2d \end{vmatrix} > 0, \Delta_5 = 6\Delta_4 > 0$$

In other word, the Routh-Hurwitz criteria for characteristic equation(5) of degree five are summarized as

- $A > 0$, $B > 0$, $C > 0$, $D > 0$, $E > 0$
- $ABC > C^2 + A^2 D$
- $(AD - E)(ABC - C^2 - A^2 D) > E(AB - C)^2 + AE^2$

The condition $A > 0$, leads to $d < 1$. But, the second condition $ABC > C^2 + A^2 D$ is not satisfy.

$\implies (1-d)(5-d)(5-2d) > (5-2d)^2 + (1-d)^2(6-2d)$

$\implies (25 - 40d + 17d^2 - 2d^3 > 31 - 34d + 14d^2 - 2d^3$

$\implies d^2 - 2d - 2 > 0$

either $d > 1 + \sqrt{3} \notin (0, 1)$ or $d > 1 - \sqrt{3} \notin (0, 1)$. Thus, the system (2) is unstable.

3.2. **Dissipative and Conservative.** System (2) can be classified as dissipative or conservative based on divergence as

$$\nabla V = \left(\frac{\partial \dot{x}}{\partial x}\right) + \left(\frac{\partial \dot{y}}{\partial y}\right) + \left(\frac{\partial \dot{z}}{\partial z}\right) + \left(\frac{\partial \dot{w}}{\partial w}\right) + \left(\frac{\partial \dot{s}}{\partial s}\right) = -1 + d \tag{7}$$

- if $d = 1$, then the new system is conservative,
- if $d < 1$, or $d \in (0, 1)$, then the new system is dissipative.

Whether the system (2) is conservative or dissipative, for any value of d within interval $(0, 1)$, then it is unstable always, therefore its classified as a system with self-excited attractors.Table 2 show the characteristic polynomials and corresponding eigenvalues under vary parameter d. In this work, all results established under the control parameter $d \in (0, 1)$.

TABLE 2. Vary parameter d with corresponding characteristic polynomial and eigenvalues for system.

d	Equilibria	Eigenvalues	State
0.2	$(\pm1, \pm1, 0, \pm0.0666, \mp0.3333)$	$(-1, 0.1 \pm 1.7292i, \pm1.4142i)$	unstable
0.4	$(\pm1, \pm1, 0, \pm0.1333, \mp0.3333)$	$(-1, 0.2 \pm 1.7205i, \pm1.4142i)$	unstable
0.9	$(\pm1, \pm1, 0, \pm0.3, \mp0.3333)$	$(-1, 0.45 \pm 1.6726i, \pm1.4142i)$	unstable
1	$(\pm1, \pm1, 0, \pm0.3333, \mp0.3333)$	$(-1, 0.5 \pm 1.6583i, \pm1.4142i)$	unstable

It clear that from the Table 2, the fourth and fifth eigenvalues with zero real parts, thus the new system known as a system with non hyperbolic equilibria (not under the class of Silnikov sense chaos).

3.3. **Lyapunov Exponents and Kaplan-Yorke dimension.** Lyapunov exponents (LE_s) are one of the tools which categorized the behaviors of the dynamical system whether the chaotic or hyperchaotic. Moreover, the maximum Lyapunov exponents (MLE) lead to the additional complex behavior of the system. According to Wolf's algorithm and ode45 [39], and based on mathematical program (MATLAB R2020a), the proposed system has three positive Lyapunov exponents under the coupling parameters $d = 0.998$ and IC $(0.2, 0.2, 0.1, 0.2, 0.2)$ at time $t = 300$ as depicted in Eq.(8) and Fig. 2.

$$\begin{cases} LE_1 = 0.4976 \\ LE_2 = 0.4942 \\ LE_3 = 0.1737 \\ LE_4 = -0.0002 \\ LE_5 = -1.1673 \end{cases} \tag{8}$$

It's clear that $\sum_{i=1}^{5} LE_i = -0.002$ is equal to the divergence at $d = 0.998$. Compare with the original system, the maximum Lyapunov exponents for the new system is LE_1 (new system)= 0.4976 which is the largest form of the original system LE_1 (original system)= 0.163, this indicates that the proposed system is more efficient than the original system according to Ref. [5]. In addition, it satisfied the feature $(n-2)$ positive LEs. The second important aspect of this study is Lyapunov dimension (Kaplan-Yorke dimension) which can becalculated as in Eq. (9). If varying the parameter d within the interval $(0, 1)$ and IC $x(0) = (0.2, 0.2, 0.1, 0.2, 0.2)$, the a new system has different behaviors: chaotic, chaotic with 2-tour and hyperchaotic as given in Table 3.

$$D_{LE} = J + \frac{1}{|LE_{J+1}|} \sum_{i=1}^{4} LE_i = 4 + \frac{0.4976 + 0.4942 + 0.1737 - 0.0002}{|LE_5|} = 4.9983, J = 1, 2, 3, 4 \tag{9}$$

TABLE 3. LE_S of new system with varying the d and $x(0) = (0.2, 0.2, 0.1, 0.2, 0.2)^T$.

d	LE_1	LE_2	LE_3	LE_4	LE_5	$\sum_{i=1}^{5} LE_i$	∇V	Sign of LE_s	Behavior
0.001	0.1836	-0.0003	-0.0010	-0.0098	-1.1714	-0.9989	-0.999	(+,0,-,-,-)	Chaotic
0.002	0.1921	0.0005	-0.0049	-0.0017	-1.1840	-0.998	-0.998	(+,0,-,-,-)	Chaotic
0.003	0.1591	0.0006	0.0001	-0.0111	-1.1457	-0.997	-0.997	(+,0,0,-,-)	Chaotic 2-tour
0.0037	0.1880	0.0001	-0.0002	-0.0029	-1.1813	-0.9963	-0.9963	(+,0,-,-,-)	chaotic 2-tour
0.009	0.2046	0.0031	0.0000	-0.0052	-1.1935	-0.991	-0.991	(+,+,0,-,-)	Hyperchaotic
0.011	0.1944	0.0024	0.0004	-0.0127	-1.1736	-0.9891	-0.989	(+,+,0,-,-)	Hyperchaotic
0.029	0.1955	0.0134	0.0050	0.0001	-1.1849	-0.9709	-0.971	(+,+,+,0,-)	Hyperchaotic
0.112	0.1752	0.0551	0.0490	-0.0002	-1.1672	-0.8881	-0.888	(+,+,+,0,-)	Hyperchaotic

3.4. **Sensitive to initial condition.** One of the conditions that characterizes a dynamical system is that it is sensitive to initial conditions. For the new system and under the coupling parameter $d = 0.001$ with three different initial conditions $(0.1, 0.1, 0.1, 0.1, 0.1)$, $(0.4, 0.4, 0.4, 0.4, 0.4)$ and $(0.1, 0.1, 0.5, 0.1, 0.1)$, a three different dynamical behavior: chaotic 2-tour, chaotic and hyperchaotic are obtained, respectively. Dynamical behavior and Lyapunov exponents are showed in Table 4.

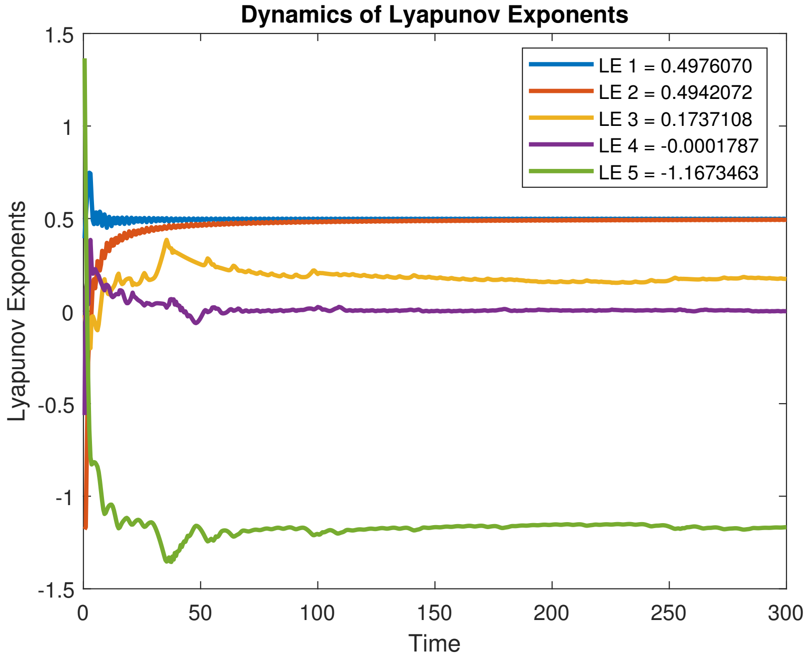

FIGURE 2. The LE_s for new system where $t = 300$ and $x_0 = (0.2, 0.2, 0.1, 0.2, 0.2)$.

TABLE 4. Lyapunov exponents of system (2) the $d = 0.001$ with varying initial conditions.

Initial conditions	LE_i	Sign of LE_s	Behavior
$(0.1, 0.1, 0.1, 0.1, 0.1)$	(0.1576, 0.0000, 0.0001, -0.0069, -1.1498)	(+,0,-,-,-)	Chaotic 2-tour
$(0.4, 0.4, 0.4, 0.4, 0.4)$	(0.1597, 0.0001, -0.0021, -0.0041, -1.1526)	(+,0,-,-,-)	Chaotic
$(0.1, 0.1, 0.5, 0.1, 0.1)$	(0.1808, 0.0037, -0.0001, -0.0160, -1.1675)	(+,+,0,-,-)	Hyperchaotic

4. MULTISTABILITY

Multistability or coexisting attractors means that a system has two or more solutions together under the like group of parameters with different initial conditions [12]. Notice that the coexisting attractors for proposed system under parameters $d = 0.998$ with IC $(1, 0.1, 0.9, 0.6, 0.2)$ (red), and $(-0.1, -0.1, 0.9, -0.6, -0.2)$ (blue) as shown in Fig. 3(a) whereas Fig. 3(b) depict the coexisting attractors with ICs $(0.1, 0.1, 0.9, 0.6, 0.2)$ red, $(-0.1, 0.1, 0.9, 0.6, -0.2)$ blue. If parameters $d = 0.1$, ICs $(0.3, 0.1, -0.8, 0.1, 0.1)$ red and $(0.5, 0.5, 0.5, 0.5, 0.5)$blue, the coexisting attractors as shown in Fig. 4(a) whilst Fig. 4(b) describe the attractors at IC $(0.1, 0.3, -0.6, 0.4, 0.5)$ red and $(0.1, -0.3, 0.6, 0.4, 0.5)$ blue.

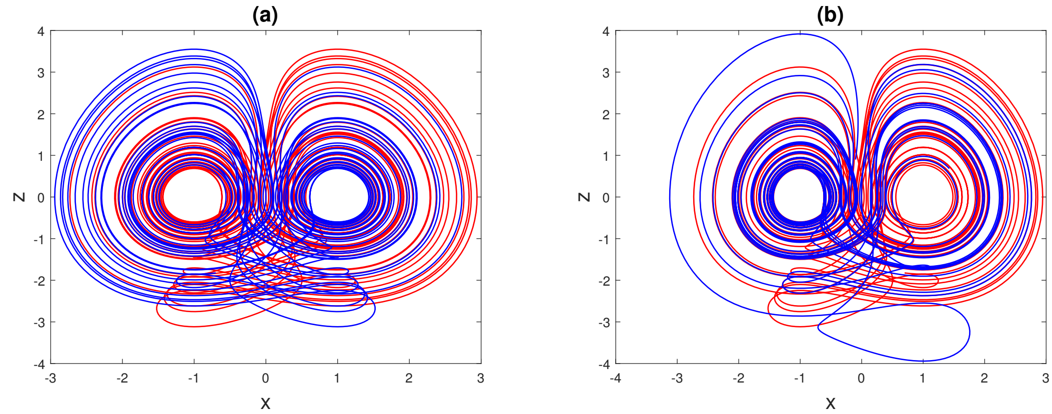

FIGURE 3. Coexisting attractors of new system with parameter $d = 0.998$ (a): at (0.1,0.1,0.9,0.6,0.2) red and (-0.1,-0.1,0.9,-0.6,-0.2) blue, (b): at (0.1,0.1,0.9,0.6,0.2) red and (-0.1,0.1,0.9,0.6,-0.2) blue.

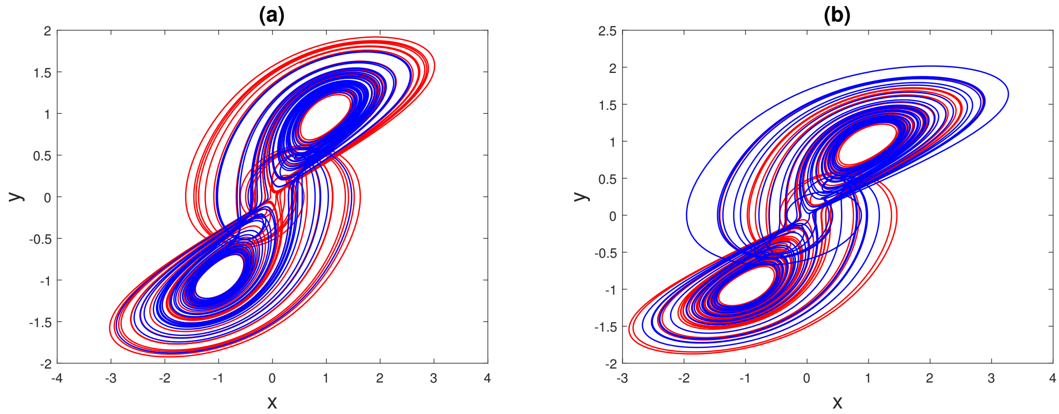

FIGURE 4. Coexisting attractors of new system with parameter $d = 0.1$, (a) at (0.3,0.1,-0.8,0.1,0.1) red and (0.5,0.5,0.5,0.5,0.5) blue, (b): at (0.1,0.3,-0.6,0.4,0.5) red and (0.1,-0.3,0.6,0.4,0.5) blue.

5. Projective Synchronization

For a novel hyperchaotic system (2), the master and slave systems can be represented in (10)and (11) as:

$$\begin{cases} \dot{x_1} = y_1 z_1 \\ \dot{y_1} = x_1 - y_1 \\ \dot{z_1} = 1 - x_1^2 \\ \dot{w_1} = -3s_1 - x_1 \\ \dot{s_1} = w_1 + ds_1 \end{cases} \tag{10}$$

$$\begin{cases} \dot{x_2} = y_2 z_2 + u_1 \\ \dot{y_2} = x_2 - y_2 + u_2 \\ \dot{z_2} = 1 - x_2^2 + u_3 \\ \dot{w_2} = -3s_2 - x_2 + u_4 \\ \dot{s_2} = w_2 + ds_2 + u_5 \end{cases} \tag{11}$$

Define the projective synchronization error [40] between the systems (10) and(11) as:

$$\begin{cases} e_1(t) = x_2(t) - 2x_1(t) \\ e_2(t) = y_2(t) - 2y_1(t) \\ e_3(t) = z_2(t) - 2z_1(t) \\ e_4(t) = w_2(t) - 2w_1(t) \\ e_5(t) = s_2(t) - 2s_1(t) \end{cases} \tag{12}$$

where $\lim_{t\to\infty} e_i(t) = 0, i = 1, 2, ...5$

Adding system (10) with system (11), the error dynamics system is obtain as:

$$\begin{cases} \dot{e_1} = e_2 e_3 + 2y_1 e_3 + 2z_1(y_2 - y_1) + u_1 \\ \dot{e_2} = e_1 - e_2 + u_2 \\ \dot{e_3} = -1 - x_2^2 + 2x_1^2 + u_3 \\ \dot{e_4} = -3e_5 - e_1 + u_4 \\ \dot{e_5} = e_4 + de_5 + u_5 \end{cases} \tag{13}$$

Theorem 5.1. *If the designed controller is in Eq. (14), then the error dynamics system (13) tends to zero.*

$$\begin{cases} u_1 = -e_1 - 2z_1(y_2 - y_1) - e_2 + e_4 \\ u_2 = -e_1 e_3 \\ u_3 = 1 - 2y_1 e_3 + x_2^2 - 2x_1^2 - e_3 \\ u_4 = 2e_5 - e_4 \\ u_5 = -2de_5 \end{cases} \tag{14}$$

Proof. Substitute control(14) in Eq. (13) as the follows:

$$\begin{cases} \dot{e_1} = -e_1 + e_2 e_3 + 2y_1 e_3 - e_2 + e_4 \\ \dot{e_2} = e_1 - e_2 - e_1 e_3 \\ \dot{e_3} = -2y_1 e_1 - e_3 \\ \dot{e_4} = -e_1 - e_4 - e_5 \\ \dot{e_5} = e_4 - de_5 \end{cases} \tag{15}$$

By choosing Lyapunov function as $V(e_i) = e_i^T P e_i$

$$V(e_i) = \begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 \end{bmatrix} \times \begin{bmatrix} 0.5 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} \tag{16}$$

$$\implies V(\dot{e_i}) = e_1\dot{e_1} + 4e_2\dot{e_2} + e_3\dot{e_3} + e_4\dot{e_4} + e_5\dot{e_5}$$

$$\implies V(\dot{e_i}) = e_1(-e_1 + e_2e_3 + 2y_1e_3 - e_2 + e_4) + 4e_2(e_1 - e_2 - e_1e_3)$$

$$+e_3(-2y_1e_1 - e_3) + e_4(-e_1 - e_4 - e_5) + e_5(e_4 - de_5)$$

$$\implies V(\dot{e_i}) = -7e_1^2 - e_2^2 + e_3^2 - e_4^2 - de_5^2$$

$$V(\dot{e_i}) = -\begin{bmatrix} e_1 & e_2 & e_3 & e_4 & e_5 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.4 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} = -e_i^T Q e_i \tag{17}$$

where $d = 0.4$, $Q = diag(1, 4, 1, 1, 0.4)$, which leads to $Q > 0$. Consequently, $V(\dot{e_i})$ on R^5. The proposed controller achieves projective synchronization theoretically. Numerically, the attractors of the error dynamics system (13) tend to zero as shown in Fig. 5 with the initial conditions as $(0.2, 0.2, 0.1, 0.3, 0.3)$ and $(0.3, 0.8, 0.7, 0.9, 0.5)$ whereas Fig.6 depict the attractor of systems (10) and (11) with projective synchronization.

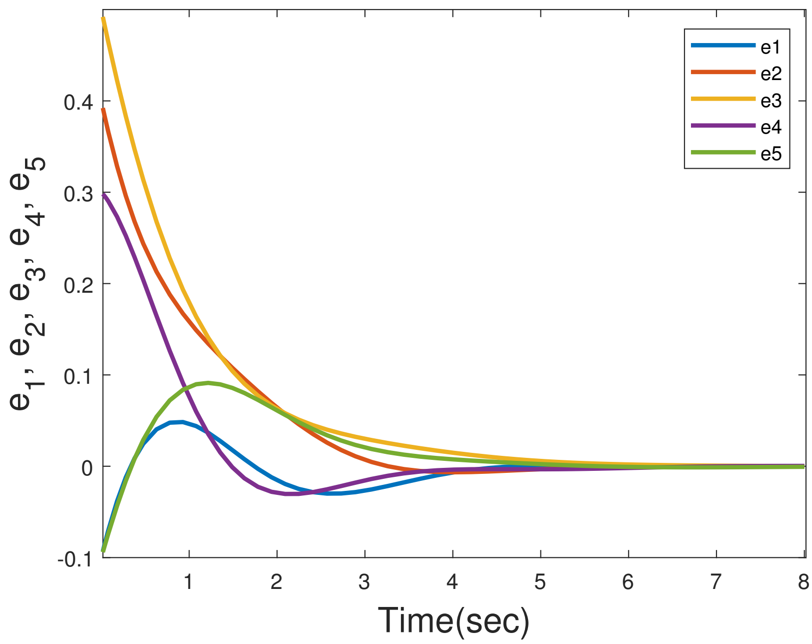

FIGURE 5. The attractors of the error dynamics system (13) convergent to zero under the new control (14).

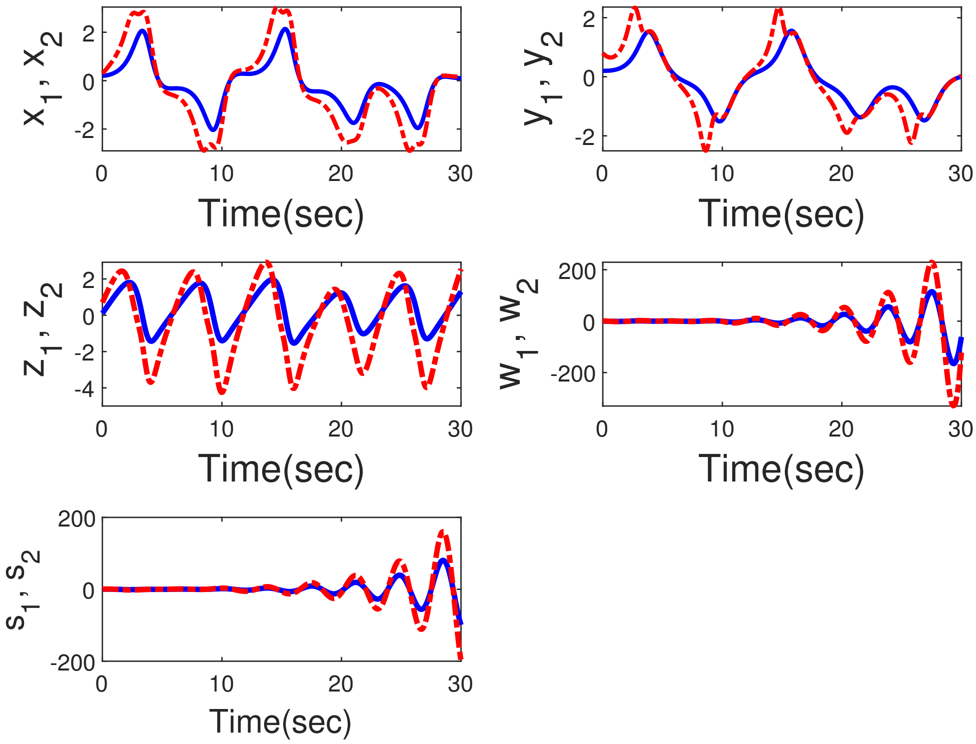

FIGURE 6. Projective synchronization between attractor of systems (10) and (11).

6. Conclusion

Based on the famous Sprott C system and the coupling strategy, a new simple 5D hyperchaotic system with nine terms has been introduced. A novel system has two non-hyperbolic equilibria points and belongs to a class of self-excited attractors, in addition, it has $(n-2)$ positive Lyapunov exponents, with the maximal Lyapunov exponent is largest than the maximal Lyapunov exponent of the original system. We believe that this work should be useful and can be set to contribute to the science on the methods of projective synchronization which may have applications in diverse fields of engineering and secure communication. In the forthcoming study, such a new system with two positive Lyapunov exponents for the suitable physical will form the basis of more methodology studies of self-excited chaos.

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