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Enhancing the W State Fusion Process With a Toffoli Gate and a CNOT Gate via One-Way Quantum Computation and Linear Optics

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Creation of large-scale W state quantum networks is a key step for realization of various quantum information tasks. Regarding the photonics technology, a simple optical setup was proposed for the fusion of two W states. Recently it was shown that via a single Fredkin gate, this basic so-called "fusion setup" can be enhanced. However the main problem was that the probability of success of realization of Fredkin gate with linear optics is too low. In this work, we show that the same enhancement can be made possible via one Toffoli and one CNOT gate, instead of a Fredkin gate. Not only the probability of success of the combination of these two gates is much higher, than that of a single Fredkin gate via linear optics, but also there is another method for implementing our setup with current photonics technology, almost with a unity success probability: A hybrid circuit consisting of a Toffoli gate which can be implemented via one-way quantum computation on a weighted graph state of 8 qubits with a unity success probability and a linear optical CNOT gate which has a success probability close to unity. Therefore the preparation of polarization based encoded multi particle entangled W states of arbitrary sizes becomes considerably more efficient.

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1. Introduction

For many quantum information tasks, creation of multipartite entangled quantum states is a vital step. Since classes of multipartite entangled states are inequivalent, meaning they cannot be converted to each other by local operations and classical communications, for specific tasks it is required to have a specific class of multipartite entangled states. Due to the sophisticated structures of multipartite entangled states, there creation becomes even more challenging. Regarding the photonic systems, theoretical proposals and experimental realizations [1] of GHZ [2] and Cluster states [3] have been achieved. On the other hand, it is more difficult to find efficient methods to create an arbitrary size W state, because its structure and entanglement dynamics are more sophisticated [4–6]. An efficient way to fuse two W states [7] via the "fusion" setup has been found, which is composed of one polarizing beam splitter (PBS), one half-wave plate (HWP) and two photon detectors. This setup is realizable with currently available linear optics. The basic fusion gate accepts one photon as an input qubit from each of two parties, holding W states of size n and m, where n and m are larger than 2, as given in the Eq. 1 and Eq. 2,

$$|W_n\rangle_A = \frac{1}{\sqrt{n}}[|(n-1)_H\rangle_a|1_V\rangle_1 + \sqrt{n-1}|W_{n-1}\rangle_a|1_H\rangle_1], \tag{1}$$

$$|W_m\rangle_A = \frac{1}{\sqrt{m}}[|(m-1)_H\rangle_a|1_V\rangle_1 + \sqrt{m-1}|W_{m-1}\rangle_a|1_H\rangle_1]. \tag{2}$$

There are three possible outcomes, recycle, success and failure with the probabilities $(n-1)(m-1)/nm$, $(n+m-2)/nm$ and $1/nm$, respectively. In the failure case, both of the W states are destroyed. The successful fusion means that the two states are fused to form a W state of size ($n+m-2$), as given in Eq. 3, since one photon is destroyed at each detector.

$$|W_{n+m-2}\rangle = \frac{1}{\sqrt{n+m-2}}[|(n+m-3)_H\rangle|1_V\rangle + \sqrt{n+m-3}|W_{n+m-3}\rangle|1_H\rangle]. \tag{3}$$

Recycle means that the states are not fused, but not destroyed either, each state loses one photon, resulting in W states of sizes $n-1$ and $m-1$, such that one can choose to use the resultant smaller W states for a new fusion process. By integration of a Fredkin gate into this basic fusion gate, the failure case was turned into a success case [8]. Hence, in the setup the total success probability has increased by $1/nm$, and also the size of the resultant W state has increased by adding an ancillary photon to this state. Moreover, another advantage of this enhanced setup is that one can also fuse W states of sizes $n=2$ and/or $m=2$, corresponding to EPR pairs. Fusing several W states was also made possible via the basic fusion gate with Fredkin and Toffoli gates [9, 10].

In this work we revisit the quantum circuit of [8]. In this circuit, since the basic fusion gate can be implemented effectively by using optical tools, we now look at the physical realization of the Fredkin gate which can actually be implemented with current photonics technol-

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ogy, but not with a high probability of success. It can be implemented with a probability of about 0.6% [11]. This low probability reduces the applicability of the circuit, by increasing the cost, since it is very likely that we will not be able to implement the Fredkin gate. Therefore, we ask whether it is possible to use a different kind of setup to make it experimentally more realizable. Along this direction, we interchange the Fredkin gate with a Toffoli and a CNOT gate as can be seen in Fig. 1. The Toffoli gate acts on qubits only when the photons in mode 1 and 2 have vertical (V) polarizations, changing the horizontal (H) polarized photon into the vertical one. The CNOT gate's action depends purely on whether the Toffoli gate operates, since the photon in mode 3 must be V polarized. We deduce from this, that two-gate system operates only when the photons in mode 1 and 2 are V polarized, resulting in the same outcomes as in the case of Bugu's et al. strategy [8]. We achieve the same success probability that is $(n+m-1)/nm$.

2. Implementation of the circuit

We propose two ways to implement this setup: (i) a linear optical circuit on which linear optical Toffoli and CNOT gates are constructed, and (ii) a hybrid circuit consisting of a Toffoli gate implemented via one-way quantum computation on weighted graph states [12] and a linear optical CNOT gate with a probability of almost unity [13]. In the first method, we implement both Toffoli and CNOT gates by using linear optical tools. Toffoli gate, that is a three-qubit gate, is universal for quantum computation and it can be made by using linear optics with the probability of 1/32 [14]. Also, the linear optical CNOT gate is proven to operate with almost unit probability. Therefore the probability of success of our circuit (one Toffoli and one CNOT) is much higher than that of Fredkin gate.

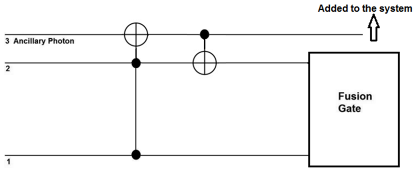

Fig. 1. The proposed setup for fusion of two W states.

In the second method, we propose to construct a hybrid circuit as mentioned before, since a Toffoli gate can be implemented via one-way quantum computation, whereas our CNOT gate is implemented in the same linear optical way, as it was in the first method. Measurement-based one-way quantum computation on cluster states is the revolutionary theoretical framework of universal computation [3]. To take this method a step further, cluster states, which are a specific type of graph states, were used to implement the Toffoli gate. By introducing unequal weighted graph states, Toffoli gate can be implemented by using 6, 7 and 8 qubits, with the probabilities 1/4, 1/2 and 1, respectively [12]. Together with the linear optical CNOT gate, the second setup is much more applicable than a linear optical Fredkin gate.

3. Conclusions

An increasing effort is devoted to various aspects of W states [15–19]. Efficient methods are required to create W state in each technology, such as photonics and cavity QED. We have proposed a new setup for creating large scale polarization-based encoded W states, which uses one Toffoli gate and one CNOT gate, instead of a Fredkin gate. This setup has a higher probability of success with linear optics and almost unity success with a hybrid setup, including a weighted graph state and a linear optical CNOT gate. Note that the graph states have been widely demonstrated in photonics in various setups [20–21]. This work can be extended to other technologies, where there is a similar effort for preparing W states [22–24].

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