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Modulation of electron-acoustic waves in a plasma with kappa distribution

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In the present work, employing a one dimensional model of an unmagnetized collisionless plasma consisting of a cold electron fluid, hot electrons obeying κ velocity distribution, and stationary ions, we study the amplitude modulation of an electron-acoustic waves by use of the conventional reductive perturbation method. Employing the field equations of such a plasma, we obtained the nonlinear Schrödinger equation as the evolution equation. Seeking a harmonic wave solution with progressive wave amplitude to the evolution equation, as opposed to the plasma with vortex distribution, the amplitude wave assumes a shock wave type of solution. Finally, the modulational stability of the wave is studied and it is observed that the wave is modulationally stable for all admissible wave numbers. © *2016 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4943279]

I. INTRODUCTION

The concept of electro-acoustic mode had been conceived by Fried and Gould[1] during the numerical solutions of the linear electrostatic dispersion equation in an unmagnetized, homogeneous plasma. Besides the well-known Langmuir and ion-acoustic waves, they noticed the existence of a heavily damped acoustic-like solution of dispersion equation. It was later shown that in the presence of two distinct groups (cold and hot) of electrons and immobile ions, one indeed obtains a weakly damped electron-acoustic mode (Watanabe and Taniuti[2]), the properties of which significantly differ from those of the Langmuir waves.

To study the properties of electron-acoustic solitary wave structure, Dubouloz *et al.*[3] considered a one-dimensional, unmagnetized collisionless plasma consisting of cold electrons, Maxwellian hot electrons, and stationary ions. However, in practice, the hot electrons may not follow a Maxwellian distribution due to the formation of phase space holes caused by the trapping of hot electrons in a wave potential. Accordingly, in most space plasma the hot electrons follow the kappa type of distribution (Hellberg and Mace,[4] El-Shewy,[5] Abbasi *et al.*[6]). Therefore, in the present work, we shall consider a plasma model consisting of a cold electron fluid, hot electrons obeying a non-isothermal kappa-like distribution, and stationary ions.

The propagation of small-but-finite amplitude waves in a one-dimensional ion-acoustic model had been studied by several researchers (see, for instance, Washimi and Taniuti[7] and one dimensional electron-acoustic model by Schamel,[8,9] Mamun and Shukla[10]) by use of the classical reductive perturbation method (Taniuti[11]) and Demiray[12] by use of the modified PLK (Poincaré, Lighthill-Kuo) method, wherein the contribution of higher order terms is also investigated.

Due to its central importance to the theory of quantum mechanics, the nonlinear equations of Schrödinger type are of great interest. They arise in many nonlinear problems such as water waves,[13–17] waves in plasmas,[18–23] nonlinear waves in fluid-filled elastic or viscoelastic tubes,[24–26] and other nonlinear waves of similar nature.

In the present work, by utilizing a one dimensional model of a plasma composed of a cold electron fluid, hot electrons obeying the kappa type of distribution, and stationary ions, we study the amplitude modulation of electron-acoustic waves (EAW) through the use the reductive perturbation method. Employing the nonlinear field equations of such a plasma and the classical reductive perturbation method, we obtained the nonlinear Schrödinger equation as the evolution equation. Seeking a harmonic wave solution with progressing amplitude wave to the evolution equation, as opposed to the result of plasma with vortex distribution, it is found that the amplitude wave assumes a shock wave type of solution. Finally, the modulational stability of the wave is studied and it is observed that the wave is modulationally stable for all admissible wave numbers.

II. GOVERNING EQUATIONS

We consider a homogeneous system of an unmagnetized collisionless plasma consisting of a cold electron fluid and hot electrons obeying κ velocity distribution, and stationary ions. The dynamics of such a system in one dimension is governed by the following normalized equations:[6]

$$\frac{\partial n_c}{\partial t}+\frac{\partial}{\partial x}(n_c u)=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}-\alpha\frac{\partial \phi}{\partial x}=0, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2}-\frac{n_c}{\alpha}-n_h+\left(1+\frac{1}{\alpha}\right)=0, \tag{3}$$

where u is the cold electron fluid velocity normalized by $C_e = (k_B T_h/\alpha M)^{1/2}$, n_c (n_h is the cold (superthermal hot) electron density normalized by the equilibrium value, n_{c0} (n_{h0}), ϕ is the electric potential normalized by $k_B T_h/e$, $\alpha = n_{h0}/n_{c0}$, M is the mass of electron, e is the electron charge, x is the space coordinate normalized to the hot electron Debye length $\lambda_{Dh} = (k_B T_h/4\pi n_{h0} e^2)^{1/2}$, t is the time variable normalized

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by the inverse of cold electron plasma frequency $\omega_{pc}^{-1} = (M/4\pi n_{c0}e^2)^{1/2}$, and k_B is the Boltzmann constant.

To model the hot electron distribution, we shall employ the three-dimensional isotropic kappa distribution presented by Hellberg and Mace,[4] which is given by

$$n_h = \left[1 - \frac{\phi}{\kappa - 3/2}\right]^{-\kappa+1/2}, \tag{4}$$

where, for a realistic thermal speed, we require that $\kappa > 3/2$.

Introducing the fluctuation of the cold electron number density from its equilibrium value by n, i.e., $n_c = 1 + n$, the field equations take the following form:

$$\frac{\partial n}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial}{\partial x}(nu) = 0, \tag{5}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - \alpha\frac{\partial \phi}{\partial x} = 0, \tag{6}$$

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{n}{\alpha} + 1 - \left[1 - \frac{\phi}{\kappa - 3/2}\right]^{-\kappa+1/2} = 0. \tag{7}$$

III. MODULATION OF NONLINEAR WAVES

In this section, we shall study the amplitude modulation of nonlinear waves propagating in such a medium. For this purpose, we introduce the following slow variables:

$$\xi = \epsilon(x - \lambda t), \quad \tau = \epsilon^2 t, \tag{8}$$

where ϵ is a small parameter characterizing the band width of the superposed waves and λ is an unknown constant to be determined from the solution. The field variables are assumed to be functions of the fast variables (x, t) as well as the slow variables (ξ, τ). Then the following differential relations hold true:

$$\frac{\partial}{\partial x} \to \frac{\partial}{\partial x} + \epsilon\frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial t} \to \frac{\partial}{\partial t} - \epsilon\lambda\frac{\partial}{\partial \xi} + \epsilon^2\frac{\partial}{\partial \tau}. \tag{9}$$

Equation (9) is to be regarded as a multiple scale space/time expansion where on the right side of the arrow, x/t is to be regarded as x_0/t_0, ξ is to be regarded as $x_1 - \lambda t_1$, and τ is to be regarded as t_2. The differential relation (9) can be obtained through the use of chain rule for differentiation.

Introducing (9) into the field equations (5)–(7), the following equations are obtained:

$$\frac{\partial n}{\partial t} - \epsilon\lambda\frac{\partial n}{\partial \xi} + \epsilon^2\frac{\partial n}{\partial \tau} + \frac{\partial u}{\partial x} + \epsilon\frac{\partial u}{\partial \xi} + \frac{\partial}{\partial x}(nu) + \epsilon\frac{\partial}{\partial \xi}(nu) = 0, \tag{10}$$

$$\frac{\partial u}{\partial t} - \epsilon\lambda\frac{\partial u}{\partial \xi} + \epsilon^2\frac{\partial u}{\partial \tau} + u\frac{\partial u}{\partial x} + \epsilon u\frac{\partial u}{\partial \xi} - \alpha\frac{\partial \phi}{\partial x} - \epsilon\alpha\frac{\partial \phi}{\partial \xi} = 0, \tag{11}$$

$$\frac{\partial^2 \phi}{\partial x^2} + 2\epsilon\frac{\partial^2 \phi}{\partial x \partial \xi} + \epsilon^2\frac{\partial^2 \phi}{\partial \xi^2} - \frac{n}{\alpha} + 1 - \left[1 - \frac{\phi}{\kappa - 3/2}\right]^{-\kappa+1/2} = 0. \tag{12}$$

For our future purposes, it is convenient to assume that the field quantities can be expanded into a power series in ϵ of the following form:

$$\begin{aligned} n &= \epsilon n^{(1)} + \epsilon^2 n^{(2)} + \epsilon^3 n^{(3)} + \cdots, \\ u &= \epsilon u^{(1)} + \epsilon^2 u^{(2)} + \epsilon^3 u^{(3)} + \cdots, \\ \phi &= \epsilon \phi^{(1)} + \epsilon^2 \pi + \epsilon^3 \phi^{(3)} + \cdots, \end{aligned} \tag{13}$$

where $n^{(1)}, \ldots, \phi^{(3)}$ are functions of the fast (x, t) as well as the slow (ξ, τ) variables.

Introducing the expansion (13) into the field equations (10)–(12) and setting the coefficients of like powers of ϵ equal to zero, the following sets of differential equations are obtained:

$O(\epsilon)$ equations:

$$\begin{gathered} \frac{\partial n^{(1)}}{\partial t} + \frac{\partial u^{(1)}}{\partial x} = 0, \quad \frac{\partial u^{(1)}}{\partial t} - \alpha\frac{\partial \phi^{(1)}}{\partial x} = 0, \\ \frac{\partial^2 \phi^{(1)}}{\partial x^2} - \frac{n^{(1)}}{\alpha} - \left(\frac{2\kappa - 1}{2\kappa - 3}\right)\phi^{(1)} = 0. \end{gathered} \tag{14}$$

$O(\epsilon^2)$ equations:

$$\begin{aligned} &\frac{\partial n^{(2)}}{\partial t} + \frac{\partial u^{(2)}}{\partial x} - \lambda\frac{\partial n^{(1)}}{\partial \xi} + \frac{\partial u^{(1)}}{\partial \xi} + \frac{\partial}{\partial x}\left(u^{(1)}n^{(1)}\right) = 0, \\ &\frac{\partial u^{(2)}}{\partial t} - \alpha\frac{\partial \phi^{(2)}}{\partial x} - \lambda\frac{\partial u^{(1)}}{\partial \xi} - \alpha\frac{\partial \phi^{(1)}}{\partial \xi} + \frac{\partial}{\partial x}\left(\frac{u^{(1)^2}}{2}\right) = 0, \\ &\frac{\partial^2 \phi^{(2)}}{\partial x^2} + 2\frac{\partial^2 \phi^{(1)}}{\partial x \partial \xi} - \frac{n^{(2)}}{\alpha} - \frac{(4\kappa^2 - 1)}{2(2\kappa - 3)^2}\left(\phi^{(1)}\right)^2 \\ &\quad - \left(\frac{2\kappa - 1}{2\kappa - 3}\right)\phi^{(2)} = 0. \end{aligned} \tag{15}$$

$O(\epsilon^3)$ equations:

$$\begin{aligned} &\frac{\partial n^{(3)}}{\partial t} + \frac{\partial u^{(3)}}{\partial x} - \lambda\frac{\partial n^{(2)}}{\partial \xi} + \frac{\partial u^{(2)}}{\partial \xi} + \frac{\partial n^{(1)}}{\partial \tau} + \frac{\partial}{\partial x}\left(u^{(1)}n^{(2)} + u^{(2)}n^{(1)}\right) + \frac{\partial}{\partial \xi}\left(u^{(1)}n^{(1)}\right) = 0, \\ &\frac{\partial u^{(3)}}{\partial t} - \alpha\frac{\partial \phi^{(3)}}{\partial x} - \lambda\frac{\partial u^{(2)}}{\partial \xi} - \alpha\frac{\partial \pi}{\partial \xi} + \frac{\partial u^{(1)}}{\partial \tau} + \frac{\partial}{\partial x}\left(u^{(1)}u^{(2)}\right) + \frac{\partial}{\partial \xi}\left[\frac{u^{(1)2}}{2}\right] = 0, \\ &\frac{\partial^2 \phi^{(3)}}{\partial x^2} + 2\frac{\partial^2 \phi^{(2)}}{\partial x \partial \xi} + \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} - \frac{n^{(3)}}{\alpha} - \left(\frac{2\kappa - 1}{2\kappa - 3}\right)\phi^{(3)} - \frac{(4\kappa^2 - 1)}{(2\kappa - 3)^2}\phi^{(1)}\phi^{(2)} - \frac{(4\kappa^2 - 1)(2\kappa + 3)}{6(2\kappa - 3)^3}\left(\phi^{(1)}\right)^3 = 0. \end{aligned} \tag{16}$$

A. Solution of the field equations

For the set of Equation (14), we shall seek a solution of the following form:

$$[n^{(1)}, u^{(1)}, \phi^{(1)}] = [N^{(1)}, U^{(1)}, \varphi_1] \exp(i\theta) + c.c., \quad \theta = \omega t - kx, \tag{17}$$

where ω is the angular frequency, k is the wave number, $N^{(1)}, U^{(1)}, \varphi_1$ are some complex functions of the slow variables (ξ, τ), and $c.c.$ stands for the complex conjugate of the corresponding quantities. Introducing (17) into the field equations (14), we obtain

$$N^{(1)} = -\alpha \frac{k^2}{\omega^2}\varphi_1, \quad U^{(1)} = -\alpha \frac{k}{\omega}\varphi_1, \tag{18}$$

provided that the following dispersion relation is satisfied:

$$\omega = k\left[k^2 + \frac{2\kappa - 1}{2\kappa - 3}\right]^{-1/2}, \tag{19}$$

where $\varphi_1(\xi, \tau)$ is an unknown complex function whose governing equation will be obtained later.

Introducing (17) and (18) into Equation (15), we obtain

$$\begin{aligned}
&\frac{\partial n^{(2)}}{\partial t} + \frac{\partial u^{(2)}}{\partial x} + \alpha \frac{k}{\omega}\left(\lambda \frac{k}{\omega} - 1\right)\frac{\partial \varphi_1}{\partial \xi} e^{i\theta} \\
&\quad - 2i\alpha^2 \frac{k^4}{\omega^3}\varphi_1^2 e^{2i\theta} + c.c. = 0, \\
&\frac{\partial u^{(2)}}{\partial t} - \alpha \frac{\partial \phi^{(2)}}{\partial x} + \alpha\left(\lambda \frac{k}{\omega} - 1\right)\frac{\partial \varphi_1}{\partial \xi} e^{i\theta} \\
&\quad - i\alpha^2 \frac{k^3}{\omega^2}\varphi_1^2 e^{2i\theta} + c.c. = 0, \\
&\frac{\partial^2 \phi^{(2)}}{\partial x^2} - \frac{2\kappa - 1}{2\kappa - 3}\phi^{(2)} - \frac{n^{(2)}}{\alpha} - \frac{(4\kappa^2 - 1)}{(2\kappa - 3)^2}|\varphi_1|^2 \\
&\quad -2ik\frac{\partial \varphi_1}{\partial \xi} e^{i\theta} - \frac{(4\kappa^2 - 1)}{2(2\kappa - 3)^2}\varphi_1^2 e^{2i\theta} + c.c. = 0.
\end{aligned} \tag{20}$$

The form of Equation (20) suggests us to seek a solution to the variables $n^{(2)}, u^{(2)}$, and $\phi^{(2)}$ of the following form:

$$[n^{(2)}, u^{(2)}, \phi^{(2)}] = (N_0, U_0, \Phi_0) + \left(\sum_{m=1}^{2}[N^{(2m)}, U^{(2m)}, \Phi^{(2m)}]e^{im\theta} + c.c.\right), \tag{21}$$

where N_0, U_0, Φ_0 are real functions of (ξ, τ), $N^{(2m)}, U^{(2m)}, \Phi^{(2m)} (m = 1, 2)$ are some complex functions of the slow variables (ξ, τ). Introducing (21) into Equation (20), the following equations are obtained:

$$N_0 = -\alpha\left(\frac{2\kappa - 1}{2\kappa - 3}\right)\Phi_0 - \alpha\left(\frac{4\kappa^2 - 1}{(2\kappa - 3)^2}\right)|\varphi_1|^2, \tag{22}$$

where $|\varphi_1|^2 = \varphi_1 \varphi_1^*$, φ_1^* being the complex conjugate of φ_1:

$$\begin{aligned}
&\omega N^{(21)} - kU^{(21)} - i\alpha \frac{k}{\omega}\left(\lambda \frac{k}{\omega} - 1\right)\frac{\partial \varphi_1}{\partial \xi} = 0, \\
&\omega U^{(21)} + \alpha k \Phi^{(21)} - i\alpha\left(\lambda \frac{k}{\omega} - 1\right)\frac{\partial \varphi_1}{\partial \xi} = 0, \\
&\frac{k^2}{\omega^2}\Phi^{(21)} + \frac{N^{(21)}}{\alpha} + 2ik\frac{\partial \varphi_1}{\partial \xi} = 0,
\end{aligned} \tag{23}$$

$$\begin{aligned}
&\omega N^{(22)} - kU^{(22)} - \alpha^2 \frac{k^4}{\omega^3}\varphi_1^2 = 0, \\
&\omega U^{(22)} + \alpha k \Phi^{(22)} - \alpha^2 \frac{k^3}{2\omega^2}\varphi_1^2 = 0, \\
&\left(3k^2 + \frac{k^2}{\omega^2}\right)\Phi^{(22)} + \frac{N^{(22)}}{\alpha} + \frac{(4\kappa^2 - 1)}{2(2\kappa - 3)^2}\varphi_1^2 = 0.
\end{aligned} \tag{24}$$

Using the dispersion relation (19) from the solution of Equation (23), we obtain

$$\begin{aligned}
&N^{(21)} = -\alpha \frac{k^2}{\omega^2}\Phi^{(21)} - 2i\alpha k \frac{\partial \varphi_1}{\partial \xi}, \\
&U^{(21)} = -\alpha \frac{k}{\omega}\Phi^{(21)} + i\frac{\alpha}{\omega}\left(\lambda \frac{k}{\omega} - 1\right)\frac{\partial \varphi_1}{\partial \xi}, \\
&\left(k + \lambda \frac{k^2}{\omega^3} - \frac{k}{\omega^2}\right)\frac{\partial \varphi_1}{\partial \xi} = 0.
\end{aligned} \tag{25}$$

In order to have a ξ-dependent solution for φ_1, the coefficient of $\partial \varphi_1/\partial \xi$ in Equation (25) must vanish, which yields

$$k + \lambda \frac{k^2}{\omega^3} - \frac{k}{\omega^2} = 0, \quad \text{or} \quad \lambda = \frac{\omega}{k} - \frac{\omega^3}{k}. \tag{26}$$

Here, as expected, $\lambda = d\omega/dk$ is the group velocity of the wave packet.

From the solution of Equation (24), we obtain

$$\Phi^{(22)} = -A\varphi_1^2, \quad U^{(22)} = B\varphi_1^2, \quad N^{(22)} = C\varphi_1^2, \tag{27}$$

where the coefficients A, B, C are defined by

$$\begin{aligned}
&A = \frac{4\kappa^2 - 1}{6k^2(2\kappa - 3)^2} + \alpha \frac{k^2}{2\omega^4}, \\
&B = \alpha \frac{k}{\omega}\left[\frac{4\kappa^2 - 1}{6k^2(2\kappa - 3)^2} + \frac{\alpha k^2}{2\omega^4}\right] + \frac{\alpha^2 k^3}{2\omega^3}, \\
&C = \alpha^2\left(3k^2 + \frac{k^2}{\omega^2}\right)\frac{k^2}{2\omega^4} + \frac{\alpha(4\kappa^2 - 1)}{6\omega^2(2\kappa - 3)^2}.
\end{aligned} \tag{28}$$

For our future use, we need the equations proportional to $\exp(im\theta)$, $m = 0, \ldots, 3$ of the $O(\epsilon^3)$ equations. For that purpose, we shall seek a solution of the following form:

$$[u^{(3)}, n^{(3)}, \phi^{(3)}] = [U_1, N_1, \Phi_1] + \left(\sum_{m=1}^{3}[U^{(3m)}, N^{(3m)}, \Phi^{(3m)}]e^{im\theta} + c.c.\right), \tag{29}$$

where U_1, N_1, Φ_1 are some real functions, and $U^{(3m)}, N^{(3m)}, \Phi^{(3m)} (m = 1, 2, 3)$ are some complex functions of the slow variables (ξ, τ). Introducing (18), (25), and (29) into Equation (16), we obtain the following equations:

$$-\lambda\frac{\partial N_0}{\partial\xi}+\frac{\partial U_0}{\partial\xi}+2\alpha^2\frac{k^3}{\omega^3}\frac{\partial}{\partial\xi}|\varphi_1|^2=0,$$
$$-\lambda\frac{\partial U_0}{\partial\xi}-\alpha\frac{\partial\Phi_0}{\partial\xi}+\alpha^2\frac{k^2}{\omega^2}\frac{\partial}{\partial\xi}|\varphi_1|^2=0,$$
$$\frac{N_1}{\alpha}+\frac{(2\kappa-1)}{(2\kappa-3)}\Phi_1+\frac{(4\kappa^2-1)}{(2\kappa-3)^2}\left(\Phi^{(21)}\varphi_1^*+\Phi^{*(21)}\varphi_1\right)=0, \tag{30}$$

$$\omega N^{(31)}-kU^{(31)}+i\frac{\partial}{\partial\xi}\left(\lambda N^{(21)}-U^{(21)}\right)+i\alpha\frac{k^2}{\omega^2}\frac{\partial\varphi_1}{\partial\tau}$$
$$+\alpha\frac{k^2}{\omega}\left(N_0\varphi_1+N^{(22)}\varphi_1^*\right)+\alpha\frac{k^3}{\omega^2}\left(U_0\varphi_1+U^{(22)}\varphi_1^*\right)=0,$$
$$\omega U^{(31)}+\alpha k\Phi^{(31)}+i\frac{\partial}{\partial\xi}\left(\lambda U^{(21)}+\alpha\Phi^{(21)}\right)+i\alpha\frac{k}{\omega}\frac{\partial\varphi_1}{\partial\tau}+\alpha\frac{k^2}{\omega}\left(\varphi_1U_0+\varphi_1^*U^{(22)}\right)=0,$$
$$-\frac{k^2}{\omega^2}\Phi^{(31)}-\frac{N^{(31)}}{\alpha}-2ik\frac{\partial\Phi^{(21)}}{\partial\xi}+\frac{\partial^2\varphi_1}{\partial\xi^2}-\frac{(4\kappa^2-1)}{(2\kappa-3)^2}\left(\varphi_1\Phi_0+\varphi_1^*\Phi^{(22)}\right)$$
$$-\frac{(4\kappa^2-1)(2\kappa+3)}{2(2\kappa-3)^3}|\varphi_1|^2\varphi_1=0. \tag{31}$$

From the solution of the first two equations of (30) and (22), we obtain

$$\Phi_0=D|\varphi_1|^2,\quad N_0=-E|\varphi_1|^2,\quad U_0=-F|\varphi_1|^2, \tag{32}$$

where the coefficients D, E, F are defined by

$$D=\left(\frac{1}{\lambda^2}-\frac{2\kappa-1}{2\kappa-3}\right)^{-1}\left[\alpha\left(\frac{2k^3}{\lambda\omega^3}+\frac{k^2}{\lambda^2\omega^2}\right)+\frac{4\kappa^2-1}{(2\kappa-3)^2}\right],$$
$$E=\frac{\alpha}{\lambda^2}D-\alpha^2\left(\frac{2k^3}{\lambda\omega^3}+\frac{k^2}{\lambda^2\omega^2}\right),\quad F=\frac{\alpha}{\lambda}D-\frac{\alpha^2k^2}{\lambda\omega^2}. \tag{33}$$

Eliminating $N^{(31)}, U^{(31)}$, and $\Phi^{(31)}$ between Equation (31) and utilizing the dispersion relation, we obtain the following evolution equation:

$$i\frac{\partial\varphi_1}{\partial\tau}+\mu_1\frac{\partial^2\varphi_1}{\partial\xi^2}+\mu_2|\varphi_1|^2\varphi_1=0, \tag{34}$$

where the coefficients μ_1 and μ_2 are defined by

$$\mu_1=\frac{3}{2}\frac{\omega^3}{k^2}(1-\omega^2)=-\frac{1}{2}\frac{d\lambda}{dk},\quad \mu_2=(B-F)k+\frac{\omega}{2}(C-E)+\frac{\omega^3}{2k^2}\frac{(4\kappa^2-1)}{(2\kappa-3)^2}(A-D)-\frac{\omega^3}{4k^2}\frac{(4\kappa^2-1)(2\kappa+3)}{(2\kappa-3)^3}. \tag{35}$$

B. Progressive wave solution

In this sub-section, we shall give a progressive wave solution to the evolution equation (34). As is well-known, the form of the progressive wave solution of the nonlinear Schröndinger (NLS) equation depends on the sign of the product of the coefficients μ_1 and μ_2. For the purpose of numerical illustration, we shall select the parameters α and κ as $\alpha=1$ and $\kappa=2$. In this case from Equations (19), (26), (27), and (33), we have

$$\omega=k/(k^2+3)^{1/2},\quad \lambda=3/(k^2+3)^{3/2},\quad A=(k^4+6k^2+14)/2k^2,$$
$$B=(k^2+3)^{1/2}(2k^4+9k^2+14)/(2k^2),\quad C=(k^2+3)(4k^4+15k^2+14)/(2k^2),$$
$$D=[135+(k^2+9)(k^2+3)^3)]/(k^6+9k^4+27k^2),$$
$$E=(k^2+3)^3(3k^2+42)/(k^6+9k^4+27k^2),$$
$$F=(k^2+3)^{3/2}[(k^2+3)(2k^4+12k^2+27)+45]/(k^6+9k^4+27k^2), \tag{36}$$

and the coefficients μ_1 and μ_2 take the following forms:

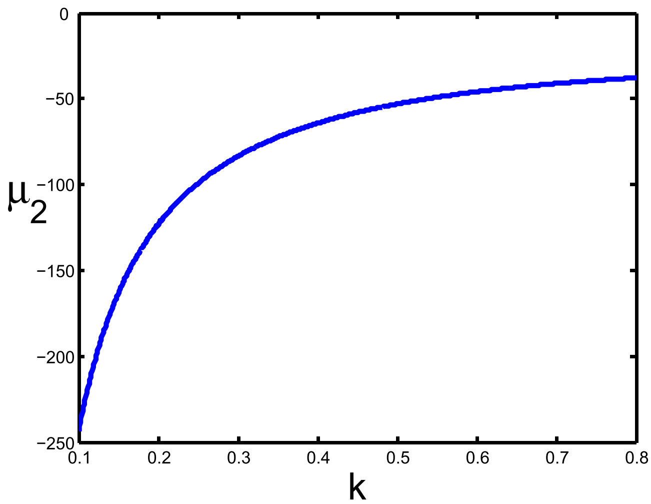

FIG. 1. The variation of μ_2 with wave number k.

$$\mu_1 = \frac{9}{2}k(k^2+3)^{5/2},$$
$$\mu_2 = k\left\{B - F + \left[\frac{(C-E)}{2} + \frac{15}{2}(A-D) - \frac{105}{4}\right](k^2+3)^{-3/2}\right\}. \tag{37}$$

As is seen from the expression of μ_1 and Figure 1, $\mu_1 > 0$ and $\mu_2 < 0$ for all admissible values of k. As is well-known, the solution of the nonlinear Schrödinger equation depends on the sign of the product $\mu_1\mu_2$, which is negative. Thus, we shall seek a progressive wave solution to the evolution equation of the form

$$\varphi_1 = f(\zeta)\exp[i(\Omega\tau - K\xi)], \quad \zeta = c(\xi + 2\mu_1 K\tau), \tag{38}$$

where $f(\zeta)$ is a real function of its argument and c, Ω, and K are some constants. Introducing (38) into (34), one has

$$\mu_1 c^2 f'' - (\Omega + \mu_1 K^2)f + \mu_2 f^3 = 0. \tag{39}$$

Here, a prime denotes the differentiation of the corresponding quantity with respect to ζ. Since the coefficients μ_1 and μ_2 satisfy the inequality $\mu_1\mu_2 < 0$, the solution for $f(\zeta)$ may be given by

$$f(\zeta) = a\tanh\zeta, \tag{40}$$

where a is the constant amplitude of the solitary wave and other quantities are defined by

$$c = \left(-\frac{\mu_2}{2\mu_1}\right)^{1/2} a, \quad \Omega = \mu_2 a^2 - \mu_1 K^2. \tag{41}$$

This result shows that the NLS equation derived from the field equations of an unmagnetized plasma consisting of a cold electron fluid, hot electrons obeying κ distribution, and stationary ions assumes an envelope shock type of solution as opposed to the plasma with vortex electron distribution, which assumes an envelope solitary wave. One should also note that the frequency of the harmonic wave is proportional to the square of the amplitude of the shock wave.

C. Stability of electron-acoustic waves

In this sub-section, we shall study the stability of EAW modulated on the wave amplitude (packet). For that purpose, we consider the dynamical solution of Equation (34) and separate the amplitude φ_1 into two parts

$$\varphi_1 = [\varphi_1^0 + \delta\varphi_1(\xi,\tau)]\exp(-i\Delta\tau), \tag{42}$$

where φ_1^0 is the real constant amplitude of the bump carrier wave, $\delta\varphi_1(\ll\varphi_1^0)$ is the small amplitude modulation, and Δ is a nonlinear frequency shift. After substituting (42) into (34) and collecting the terms of the same order, we obtain

$$\Delta = -\mu_2|\varphi_1^0|^2, \tag{43}$$

and

$$i\frac{\partial\delta\varphi_1}{\partial\tau} + \mu_1\frac{\partial^2\delta\varphi_1}{\partial\xi^2} + \mu_2|\varphi_1^0|^2 1(\delta\varphi_1 + \delta\varphi_1^*) = 0, \tag{44}$$

where $\delta\varphi_1^*$ is the complex conjugate of $\delta\varphi_1$. Letting $\delta\varphi_1 = U + iV$ in Equation (44), the real and imaginary parts U and V satisfy the following coupled linear differential equations:

$$\frac{\partial V}{\partial\tau} = \mu_1\frac{\partial^2 U}{\partial\xi^2} + 2\mu_2|\varphi_1^0|^2 U, \tag{45}$$

and

$$\frac{\partial U}{\partial\tau} = -\mu_1\frac{\partial^2 V}{\partial\xi^2}. \tag{46}$$

Assuming that the amplitude perturbation $\delta\varphi_1$ varies as $\sim\exp[i(K\xi - \Omega\tau)]$, we obtain from Equations (45) and (46) the following nonlinear dispersion relation for the amplitude modulation of EAW modes

$$\Omega^2 = \mu_1 K^2(\mu_1 K^2 + 2\Delta). \tag{47}$$

As pointed out before $\mu_1 > 0$ and $\mu_2 < 0$ accordingly $\Delta > 0$. From Equation (47), it is seen that $\Omega^2 > 0$, and hence, the wave packet is modulationally stable.

IV. CONCLUSIONS

In the present work, by utilizing a one dimensional model of a plasma composed of a cold electron fluid, hot electrons obeying the kappa type of distribution, and stationary ions, we study the amplitude modulation of electron-acoustic waves through the use the reductive perturbation method. Employing the nonlinear field equations of such a plasma and the classical reductive perturbation method, we obtained the nonlinear Schrödinger equation as the evolution equation. Seeking a harmonic wave solution with progressing amplitude wave to the evolution equation, as opposed to the result of plasma with vortex electron distribution, it is found that the amplitude wave assumes a shock wave type of solution. Finally, the modulational stability of the wave is studied and it is observed that the wave is modulationally stable for all admissible wave numbers.

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